CAREERS360

PREPARATION Series

# CAT 2025

# Permutation & Combination: Videos and Questions

# contents

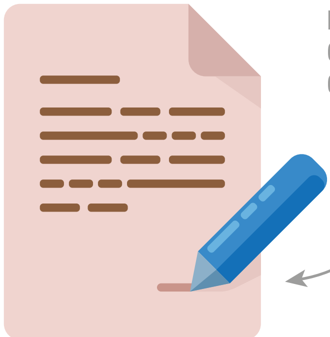
# INTRODUCTION

The chapter 'Permutations and Combinations' is concerned with determining the number of different ways of arranging and selecting objects out of a given number of objects, without actually listing them. These are generally referred to as "PnC". This chapter is all about logic and "counting". PnC tests your ability to observe the pattern, your mathematical reasoning, and creativity. From the exam point of view, PnC is one of the important chapters. Concepts of Permutations and combinations are mostly used while solving the problems from probability. The problems of probability are no longer as simple as they used to be in elementary standards. However, if you have a command on Permutations and Combinations, the chapter probability will be a piece of cake for you.

**In this chapter, we are going to study the concepts and applications of Permutations and combinations.**
**Following are the sub-topics that we are going to study:**

1. Fundamental Principle of Counting: Addition Rule (OR Rule) and Multiplication Rule (AND Rule)
2. Permutations
3. Combinations
4. Grouping
5. Distribution
6. Derangements

For More See Video

Practice Questions Click Here

# FUNDAMENTAL PRINCIPLE OF COUNTING

Fundamental principle of counting is a rule used to find the total number of outcomes possible in a given situation. Fundamental principle of counting can be classified into two types

1. **Multiplication Rule (AND rule)**
2. **Addition Rule (OR rule)**

## Multiplication Rule

If a certain work W can be completed by doing 2 tasks, first doing task A AND then doing task B. A can be done in m ways and following that B can be done in n ways, then the number of ways of doing the work W is **(m x n) ways**.

**For example,** let's say a person wants to travel from Noida to Gurgaon, and he has to travel via New Delhi. It is given that the person can travel from Noida to New Delhi in 3 different ways and from New Delhi to Gurgaon in 5 different ways.

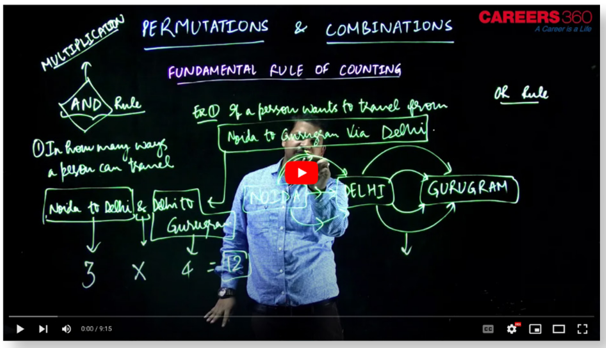
So, in this case, to complete his work (reach Gurgaon) he has to do two tasks **one after the other**, first traveling from Noida to New Delhi (task A) **and then** from New Delhi to Gurgaon (task B), as he has 3 different ways of reaching New Delhi (doing task A), and he has 5 different ways to reach Gurgaon from New Delhi (doing task B), so in that way, he has a total of 3×5 = 15 different ways to reach Gurgaon from Noida.

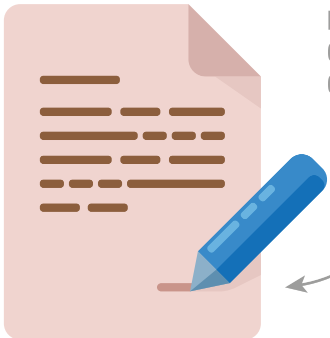
### Addition Rule

If work W can be completed by doing task A **OR** task B, and A can be done in m ways and B can be done in n ways (and both cannot occur simultaneously: in this case we call tasks A and B as **mutually exclusive**), then work W can be done in **(m + n) ways**.

**For example**, let's say that a person can travel from New Delhi to Noida in 3 different types of buses, and 2 different types of trains, so, he can complete the work of going from New Delhi to Noida in 3 + 2 = 5 ways (As work can be completed by going by bus (A) **OR** by going by train (B))

For More See Video



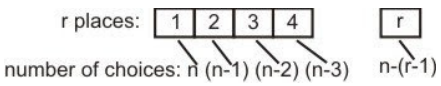



Practice Questions Click Here

## PERMUTATION AS AN ARRANGEMENT

Permutation basically means the arrangement of things. And when we talk about arrangement then the order becomes important if the things to be arranged are different from each other (when things to be arranged are the same then order doesn't have any role to play). So in permutations order of objects becomes important.

Arranging n objects in r places (Same as arranging n objects taken r at a time) is equivalent to filling r places from n things.




So the number of ways of arranging n objects taken r at a time = n(n - 1) (n - 2) ... (n - r + 1)

$$\frac{n(n-1)(n-2)\ldots(n-r+1)(n-r)!}{(n-r)!} = \frac{n!}{(n-r)!} = {}^{n}P_{r}$$

Where $r \leq n$ and $r \in W$

So, the number of ways arranging n different objects taken all at a time = ${}^{n}P_{n} = n!$

**Example:** In how many ways can 5 people be seated at 3 places?

**Solution:** Basically this question is about arranging 5 people at 3 different places

Let's think that we are given 3 places, so for the first place we have 5 people to choose from, hence this can be done in 5 ways as all 5 are available.

Now for 2nd place we have 4 people to choose from, hence this can be done in 4 ways.

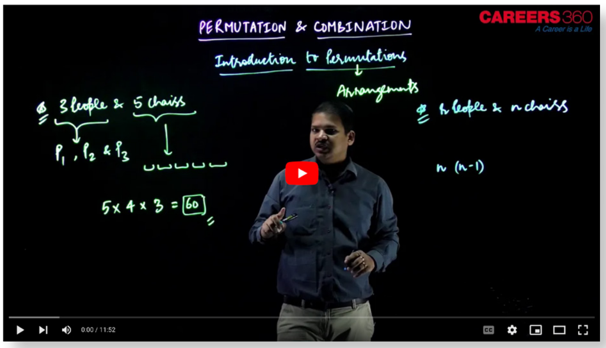
Similarly for 3rd place we have 3 choices.
Since we have to choose for all 3 places, so multiplication rule is applicable, and the total number of ways $5\times4\times3=120$,
This can also be done directly from the notation or formula

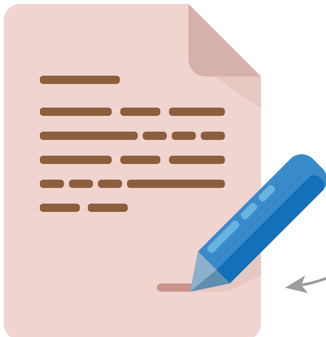
$$^{n}P_{r} \text{ where } n = 5, r = 3, \text{ so } ^{5}P_{3} = \frac{5!}{2!} = 5 \times 4 \times 3 = 120$$

**Example:** Find the number of ways the letters of the word "BIRTHDAY" can be arranged taken all at a time.
**Solution:** From the above concept directly using the formula $^{n}P_{n}$ we have

$$^{8}P_{8} = 8! = 40,320$$

For More
See Video



Practice
Questions
Click Here

## APPLICATION OF PERMUTATION - I

**Arrangement of Digits (Part 1)**
**To make 5 digit number from 1,2,3,4,5,6,7**

**1. If the repetition of digits is not allowed**
In this case,
First place of digit (from left) can be filled in 7 ways
Second place can be filled in 6 ways (as one number is already used at first place)
Third place can be filled in 5 ways
Fourth in 4 ways
Fifth in 3 ways
Hence the total number of ways is 7 x 6 x 5 x 4 x 3 = 2520.

**2. If the repetition of digits is allowed**
In this case,
First place can be filled in 7 ways
Second place can be filled in 7 ways (as repetition is allowed)
Similarly all other places can be filled in 7 ways
Hence the total number of ways is 7 x 7 x 7 x 7 x 7 = $7^5$.

**3. If the number is EVEN**

In this case,

One's place can be filled by three numbers (2,4,6) as even number can only end with (0,2,4,6,8). Hence we have three different possibilities.

If one's place is occupied by 2 then remaining 6 digits can be arranged in remaining 4 places in $^6P_4$ ways OR,

If one's place is occupied by 4 hence remaining can be arranged in $^6P_4$ ways OR,

If one's place is occupied by 6 hence remaining can be arranged in $^6P_4$ ways

Hence the total possibilities are $^6P_4 +^6 P_4 +^6 P_4 = 1080$

**Let us take another example**

**Using 0,1,2,3,4,5 how many 4-digit numbers can be formed if the repetition of digits is not allowed.**

Here 0 cannot be at a leading place as in such a case the number will become a 3-digit number.

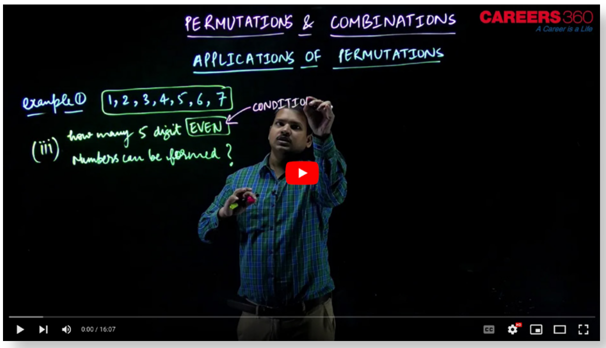
Hence the first place can be filled in 5 ways and the second place left with 5 possibilities (now we have one lesser digit available as it is used up at leading place, but we have one more digit '0' that can be filled at second place) that's why it can also be filled with 5 ways and the third-place left with 4 possibilities and the fourth place left with 3 possibilities.

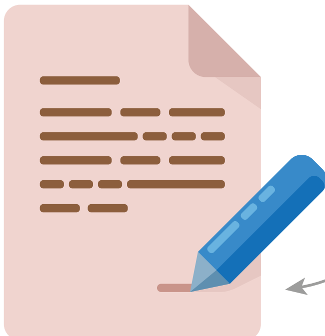
So the total number of 4 digit numbers = 5×5×4×3=300

**Solving the same question if repetition of digits was allowed**

In that case, the first place (from left) has 5 choices (excluding 0 otherwise it will not be 4 digit number). 2nd, 3rd, and 4th place will have 6 choices (all choices available, repetition allowed), so the total number, in this case, = 5×6×6×6

For More See Video



Practice Questions Click Here

# APPLICATION OF PERMUTATION - II

**Arrangements of Digits (Part 2)**

**Find the number of Distinct 4-digits numbers that can be formed using 1,2,3,4,5,6,7 for each of the following condition:**

**1. If the number is greater than 3000.**

The thousand's digit can be filled in 5 ways (any one of 3, 4, 5, 6 or 7). Now one is left with 6 digits and has to arrange 3 of them. This can be done in $^6P_3 = 6 \times 5 \times 4$ ways. Thus a total of $5 \times (6 \times 5 \times 4) = 600$ such numbers can be formed.

**2. If the number is EVEN.**

The unit's digit can be filled in three ways (2, 4 or 6). Now we are left with 6 digits and have to arrange 3 of them. This can be done in

$^6P_3 = 6 \times 5 \times 4$ ways. Thus a total of $(6 \times 5 \times 4) \times 3 = 360$ such numbers can be formed.

**3. If the number is EVEN and greater than 3000.**

The unit digit could be 2 OR 4 OR 6

Considering the case that the units digit is 2:

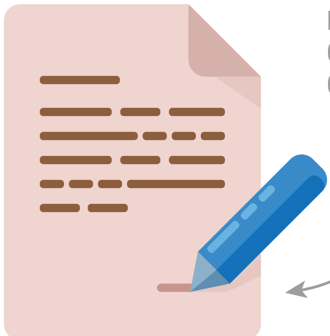
The units digit can be filled in only 1 way, with a 2. Having filled this, all of 3, 4, 5, 6 and 7 are available (so that the number is greater than 3000) for the thousands place and thus it can be filled in 5 ways (3,4,5,6,7). Next, we are left with 5 digits and two of which have to be arranged i.e. can be done in $5 \times 4$ ways. Thus total possible numbers with units digit being 2 will be $5 \times (5 \times 4) \times 1 = 100$

Considering the case that the units digit is 4: The units digit can be filled in 1 way. Having filled this, the digits available for the thousands place are 3, 5, 6, 7 i.e. there are further 4 possibilities. Next, we are left with 5 digits and two of which have to be arranged i.e. can be done in $5 \times 4$ ways. Thus total possible numbers with unit's digit being 4 or 6 will be $4 \times (5 \times 4) \times 1 = 80$.

Considering the case that the units digit is 6: The units digit can be filled in 1 way. Having filled this, the digits available for the thousands place are 3, 4, 5, 7 i.e. there are further 4 possibilities. Next, we are left with 5 digits and two of which have to be arranged i.e. can be done in $5 \times 4$ ways. Thus total possible numbers with unit's digit being 4 or 6 will be $4 \times (5 \times 4) \times 1 = 80$.

Thus the total number of even numbers greater than 3000 that can be formed will be $100 + 80 + 80 = 260$

For More
See Video

Practice
Questions
Click Here

# APPLICATION OF PERMUTATION - III

ARRANGEMENT OF PEOPLE IN A ROW

**Arranging 4 boys and 4 girls such that 4 girls have to be together:**

Considering all the four girls as one unit, we have to arrange 5 units in 5 places and this can be done in 5! ways. Now one of the units is of 4 girls, who can be arranged amongst themselves in 4! ways. Thus the total number of ways of arranging is $5! \times 4!$.

**Arranging 4 boys and 4 girls such that no two girls should stand together:**

The four boys can first be arranged in 4! ways. Now there will be 5 places (Three between 4 boys and one at each end) for the 4 girls and they could be arranged in $5 \times 4 \times 3 \times 2$ ways. Thus the total number of arrangements possible is $4! \times 5 \times 4 \times 3 \times 2$.

**Arranging 4 boys and 4 girls such that girls and boys have to be alternate:**

When girls and boys have to be alternate, it would just be either G B G B G B G B or B G B G B G B G. In each of these ways, there are 4 places for the boys and 4 places for the girls and thus they can be arranged in $4! \times 4!$ in each of these. Thus, the total number of arrangements possible is $2 \times 4! \times 4!$.

For More
See Video


Practice
Questions
Click Here

# APPLICATION OF PERMUTATION - IV

**QUESTION-BASED ON FACTORS**

Factors of a number N refer to all the numbers which divide N completely. These are also called **divisors of a number.**

**Basic formula related to factors of a number:**

These are certain basic formulas pertaining to factors of a number N, such that,

$$N = p^a q^b r^c$$

Where, p, q, and r are prime factors of the number n. a, b and c are non-negative powers/ exponents

- Number of factors of N = (a+1)(b+1)(c+1)
- Sum of factors: ( pa+1-1)( qb+1-1)(rc+1-1)/ (p-1) (q-1) (r-1)

**Let us take an example:**

Find the Number and Sum of the factor of 18.

Factors of 18 are 1,2,3,6,9,18

Number of factors = 6

And their Sum is 39 (=1+2+3+6+9+18)

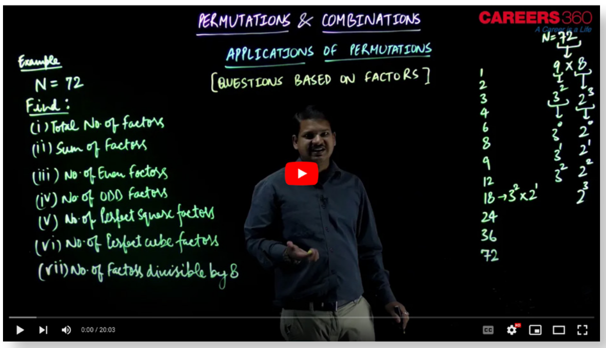
**Using the Formula**

$$18 = 2^1 \times 3^2$$

Number of factors of N = (1+1)(2+1)=6

Sum of factors: $( 2^2-1)( 3^3-1)/ (2-1)(3-1)=39$

For More
See Video




Practice
Questions
Click Here

# APPLICATION OF PERMUTATION - V

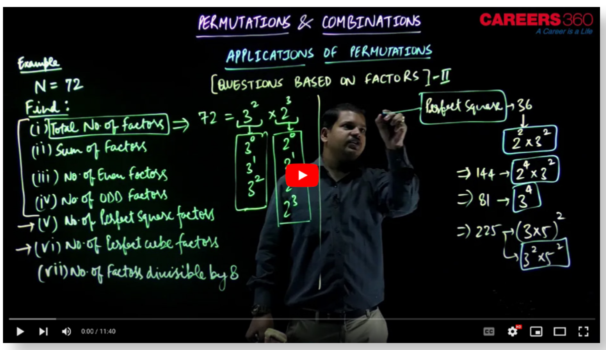
**Number of odd and even factors:**
Let us understand this concept with an example:

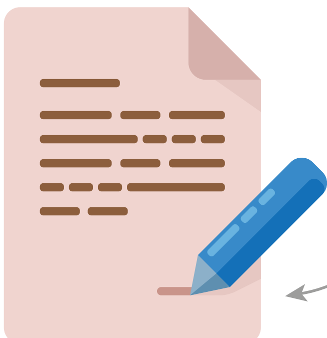
**Find the number of Even and Odd factors of 72**
Prime factorization of $72 = 2^3 \times 3^2$
We know the fact that all the factors of 72 will be in the form of $2^a \times 3^b$.
**For ODD factors,** the exponent of 2 i.e., a has to be 0 always. Or, the number of ways using 2 for making the combination is 1, i.e., $2^0$.
Also, the number of values that the exponent of 3 i.e., b can take is 3 (0, 1 or 2)
Hence the number of odd factors of 72 = 1x3 = 3.
Extending the logic **for EVEN factors**, we can say that for a factor to be Even, it must contain 2 at least once.
So, a can take values 3 values (1, 2, or 3, note: 0 is not possible for the number to be even)
b can take 3 values (0, 1 or 2)
So, the number of values a, and b can take are 3 and 3 respectively.
Therefore, the total number of even factors of 72 = 3x3 = 9.

**Number of factors which are perfect squares:**
**Find the number of factors that are perfect squares of 72.**
Prime factorization of $72 = 2^3 \times 3^2$
If we prime factorize any number which is a perfect square, we would observe that in all cases the exponent of all the prime factors of the number to be even only.
**For example,** 36 is a perfect square $36 = 2^2 \times 3^2$, here we can see that the exponent of both 2 and 3 are even.
Again, any factor 72 will be in the form of $2^a \times 3^b$. For the factors to be perfect squares, all the values a, and b has to be even only.
Or, the possible values which a can take = 0, 2 i.e. 2 values only. Similarly, b can take 0, 2 i.e. 2 values.
Therefore, the different combinations we can have = 2x2 = 4.
Hence, 72 has 4 factors which are perfect squares.

**Number of factors which are perfect cube:**
**Find the number of factors that are perfect cubes of 72.**
Prime factorization of $72 = 2^3 \times 3^2$
If we prime factorize any number which is a perfect cube, we would observe that in all cases the exponent of all the prime factors of the number to be a multiple of 3.
For example, 27 is a perfect cube $27 = 3^3$, here we can see that the exponent of 3 is a multiple of three.
Again, any factor 72 will be in the form of $2^a \times 3^b$. For the factors to be perfect cubes, all the values a, and b has to be divisible by 3.
Or, the possible values which a can take = 0, 3 i.e. 2 values only. Similarly, b can take 0 i.e. 1 value.
Therefore, the different combinations we can have = 2x1 = 2.
Hence, 72 has 2 factors which are perfect cubes.

For More
See Video



Practice
Questions
Click Here

# APPLICATION OF PERMUTATION - VI

**DIVISIBILITY OF A FACTOR BY A NUMBER**

Let us understand with an example:

**Find the number of factors of 72 which are divisible by 8**

Prime factorization of $72=2^3\times3^2$

Prime factorization of $8=2^3$

For the factors to be divisible by 8 the factor should be multiple of 8 or $2^3$.

Or we can say that the values a should be equal to 3 or greater than 3, and b can take any value.

Hence, the possible values which a can take = 3, i.e. 1 value only. Similarly, b can take 0, 1 and 2 i.e. 3 values.

Therefore, the different combinations we can have = 1x3 = 3.

Hence, 72 has 3 factors which are divisible by 8 i.e. 8, 24, 72.

**Product of factors:**

Let a number be N

It's Prime factorization be $N=2^a x 3^b x 5^c$

$$\text{Product of factors} = \mathrm{N}^{\frac{\text{no. of factors}}{2}}$$

**For example: Find the Product of the factors of 72**

$$\text{Product of factors of } 72 = 72^{\frac{(3+1)(2+1)}{2}}$$

**Exponent of Prime P in n!**

Where [x] stands for greatest integer value of $x\epsilon R$

If m is the index of highest power of a prime p that divides n! then

$$m=\left[\frac{n}{p}\right]+\left[\frac{n}{p^2}\right]+\left[\frac{n}{p^3}\right]+\ldots\ldots\ldots\ldots$$

**Example:** Find the number of trailing zeros in 20!

**Solution:**

10 can be written as 2 x 5

If you want to figure out the exact number of zeroes, you would have to check how many times the number N is divisible by 10.

When dividing N by 10, factors of 10 will be the smaller of the power of 2 or 5.

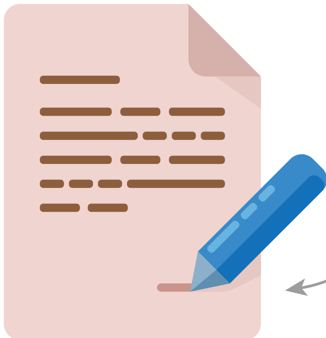
**Number of trailing zeros is going to be the power of 2 or 5, whichever is lesser.**

$$m=\left[\frac{20}{2}\right]+\left[\frac{20}{2^2}\right]+\left[\frac{20}{2^3}\right]+\left[\frac{20}{2^4}\right]+\left[\frac{20}{2^4}\right]\ldots\ldots\ldots\ldots=18$$

$$m=\left[\frac{20}{5}\right]+\left[\frac{20}{5^2}\right]+\left[\frac{20}{5^3}\right]+\ldots\ldots\ldots\ldots=4$$

Hence number of trailling zeros are 4

For More
See Video

Practice
Questions
Click Here

# PERMUTATION OF OBJECTS WHEN FEW ARE IDENTICAL

If there n objects of which p objects are of one type, q objects of another type,
r objects of yet another type and all others are distinct, the total number of ways of arranging all the objects is $\frac{n!}{p!q!r!}$

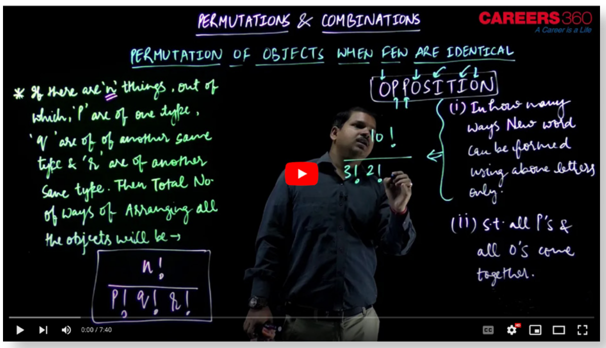
**Proof:**
Suppose the total number of permutations are x, now if we replace all p identical objects by p different objects then we have $x \times p!$ arrangements. The number of arrangements, if we do the same thing with q and r, will be $x \times p! \times q! \times r!$
Now, we have replaced all identical objects and we are left with n different object which can be arranged in n! Ways.
Hence, $x \times p! \times q! \times r! = n!$

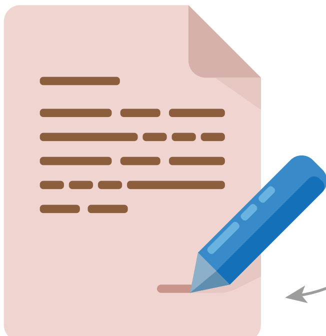
So, $\frac{n!}{p!q!r!}$

**Example:** In how many ways can the letters of the word "MISSISSIPPI" be arranged?
**Solution:** repeated letters I = 4 times, S = 4 times and P=2 times

So using the above formula we have $x = \frac{11!}{4! \times 4! \times 2!}$

For More
See Video



Practice
Questions
Click Here

# CIRCULAR PERMUTATIONS

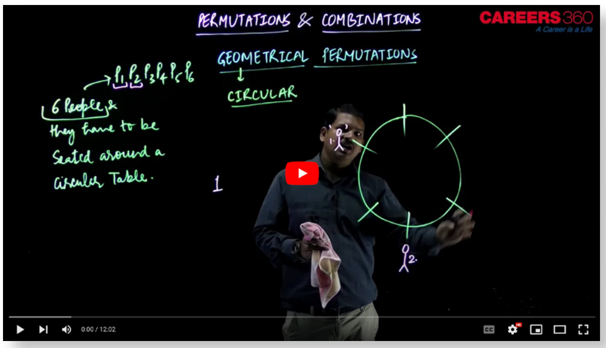
Let's say there is a round table with 6 chairs all identical and 6 persons have to sit. For the first person there is only one choice to make as all chairs are identical, so wherever he may sit doesn't matter. Now when $1^{st}$ person has sit, $2^{nd}$ person with respect to $1^{st}$ have five choices to sit, directly opposite or left or $2^{nd}$ from left or right or $2^{nd}$ from right to $1^{st}$ person, in the same ways $3^{rd}$ person will have 4 choices, $4^{th}$ person will have 3 choice, $5^{th}$ person will have 2 choices, and last person 1 choice,
so in that way total 5 x 4 x 3 x 2 x 1 = ( 6 - 1 )! permutations are possible.
We can generalize this result as an object that can be arranged along with a circular table in (n-1)! Ways

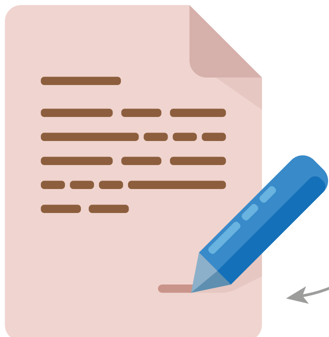
**Example:** In how many ways 7 people can be arranged along a circular table having 7 identical chairs.
**Solution:** using the above concept, it can be done in (7-1)! = 6!.

**Necklace, Garland Type Questions**
If clockwise and anticlockwise permutations are same in a circular permutation, (as in case of garlands or necklace formation), the number of permutations becomes (½)(n -1)!
Since as in the case of necklace and garland if we flip the bead or garland then anticlockwise arrangements become clockwise but they are identical because the objects are identical, hence two arrangements are reduced to one causing the total number of permutations to be halved.

**Example:** Find the ways in which 10 different beads can be arranged to form a necklace?
**Solution:** Using circular permutations the total number of permutations = (10-1)!
Now since clockwise and anticlockwise arrangements give the same permutation so the total number of permutations becomes (½)(9!)

For More
See Video



Practice
Questions
Click Here

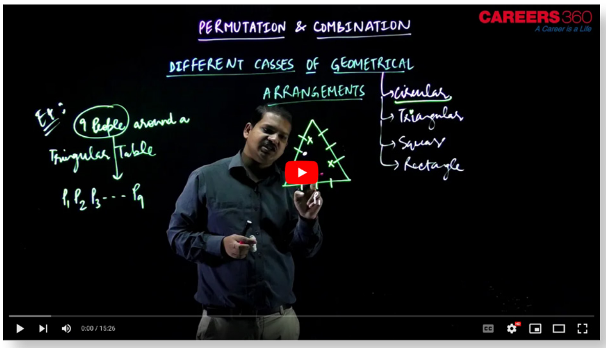
# DIFFERENT CASES OF GEOMETRICAL ARRANGEMENTS

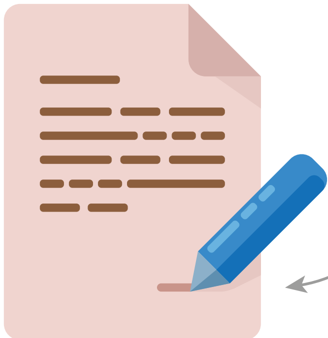
If clockwise and anticlockwise permutations are same in a circular permutation, (as in case of garlands or necklace formation), the number of permutations becomes (½)(n-1)!.
Since as in the case of necklace and garland if we flip the bead or garland then anticlockwise arrangements become clockwise but they are identical because the objects are identical, hence two arrangements are reduced to one causing the total number of permutations to be halved.

**Example:** Find the ways in which 10 different beads can be arranged to form a necklace?
**Solution:** Using circular permutations the total number of permutations = (10-1)!

Now since clockwise and anticlockwise arrangements give the same permutation so the total number of permutations becomes (½)(9!)

For More See Video



Practice Questions Click Here

## RANK OF A WORD IN A DICTIONARY

A common type of problem asked in many examinations is to find the 'rank' of a given word in a dictionary. What this means is that you are supposed to find the position of that word when all permutations of the word are written in alphabetical order.

**Rank of a word - without repetition of letters**

**Example:** Find the rank of a word MATHS in a dictionary made using its letters

**Step 1:** Write down the letters in alphabetical order.

The order will be A, H, M, S, T.

**Step 2:** Find the number of words that start with a superior letter.

Any word starting from A will be above MATHS. So, if we fix A at the first position, we have 4! = 24 words. (number of ways arranging H, M, S, T).

Similarly, there will be 24 words that will start with H.

Number of words start with MAH is 2! = 2

Number of words start with MAS is 2! = 2

Number of words start with MATHS is 1! = 1

Therefore, the overall rank of the word MATHS is 24 + 24 + 2 + 2 + 1 = 53

**Rank of a word - with repetition of letters**

**Example:** Find the rank of a word INDIA in a dictionary made using its letters

Write down the letters in alphabetical order, the order will be A, D, I, I, N.

Number of words start with A is 4!/2! = 12 (We are dividing by 2! because I is repeating itself)

Number of words start with D is 4!/2! = 12

Number of words start with IA is 3! = 6 (number of ways arranging I, D, N)

Number of words start with ID is 3! = 6

Number of words start with II is 3! = 6

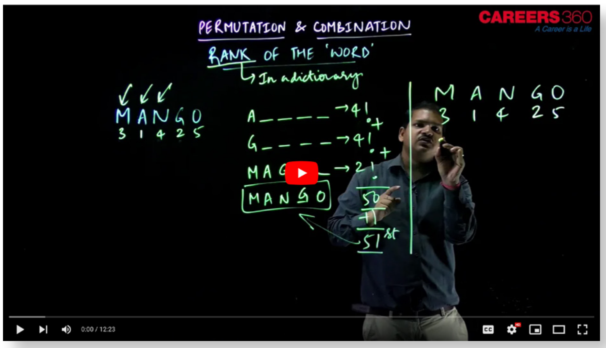
Number of words start with INA is 2! = 2

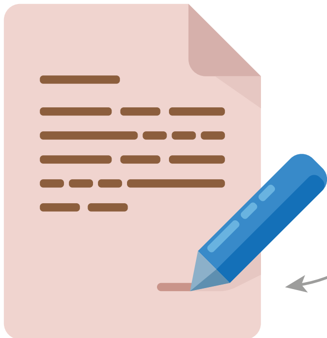
Number of words start with INDA is 1! = 1

Number of words start with INDIA is 1! = 1

Therefore, the overall rank of the word INDIA is 12 + 12 + 6 + 6 + 6 + 2 + 1 + 1= 46

For More
See Video



Practice
Questions
Click Here

# INTRODUCTION OF COMBINATIONS

So far our task was always to "arrange" objects i.e. to place them in a specific order among themselves.

Sometimes we would be interested in only "selecting" few objects out of the given objects. In this case we just need to "select" and we do not need to "arrange" them in an order.

E.g., we need to select 4 students out of the 15 students who will represent the college at a quiz or we need to form an academic committee of 3 professors from 10 professors. In this case, who is selected "first", who is selected "second" and so on does not matter. The words "first" and "second" implicitly implies an "ordering". What matters in the case of selection is only the composition of the final "group".

The notation of selecting r objects from n given object is ${}^{n}C_{r}$. Let's derive the value of ${}^{n}C_{r}$, and it's relation with permutation notation.

Let's say we want to arrange 2 objects out of 5 objects : A,B,C,D,E then using the concept of permutation we can do this in ${}^{5}P_{2}$ ways.

We can calculate the same thing by another method: by selecting 2 things out of 5, which can be done as ${}^{5}C_{2}$ and then arrange the 2 selected thing which can be done in 2! ways. So we have

$${}^{5}C_{2} \times 2! = {}^{5}P_{2}$$

$${}^{5}C_{2} = \frac{{}^{5}P_{2}}{2!}$$

$${}^{5}C_{2} = \frac{5!}{(5-2)!2!} = \frac{5!}{3!2!}$$

We can generalize this concept for r object to be selected from given n objects as

$${}^{n}C_{r} \times r! = {}^{n}P_{r}$$

$${}^{n}C_{r} = \frac{{}^{n}P_{r}}{r!}$$

$${}^{n}C_{r} = \frac{n!}{(n-r)!r!}$$

Where $0 \le r \le n$, and r is a whole number.

Now we have the value of ${}^{n}C_{r}$

**Example:** In ICC World Cup 2019 total 10 teams participated and each team has to play one game in the league stage with all other teams before qualifying for the semifinals, so how many total games will be played in the league stage.

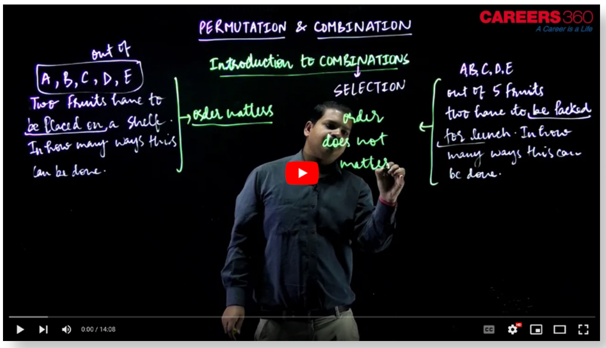
**Solution:** For playing a game we need to select two teams. So this is a simple problem of selecting two teams, so this can be done in

$$^{10}C_2 = \frac{10!}{(10-2)!2!}$$

$$\frac{10 \times 9 \times 8!}{8! \times 2!} = \frac{10 \times 9}{2} = 45$$

Hence in total 45 games will be played in the league stage.

For More See Video




Practice Questions Click Here

## APPLICATIONS OF SELECTIONS - I

Let us take an example of selecting things from two or more different groups:

**Out of 5 men and 6 women in how many ways can a committee of 5 members be selected such that at least 2 members are women?**

**Solution:** Following cases are possible for at least 2 women,

2 women + 3 men = $^6C_2 \times {}^5C_3$

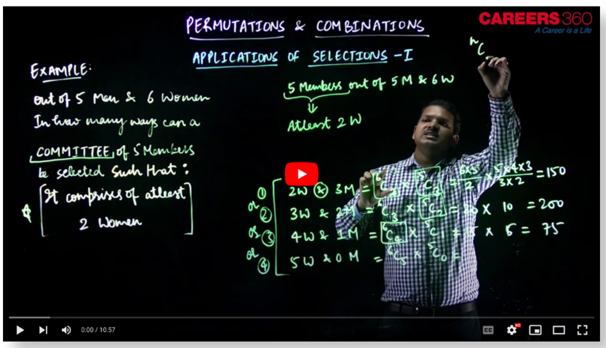
3 women + 2 men = $^6C_3 \times {}^5C_2$

4 women + 1 men = $^6C_4 \times {}^5C_1$

5 women = $^6C_5$

So, the total number of ways $= {}^6C_2 \times {}^5C_3 + {}^6C_3 \times {}^5C_2 + {}^6C_4 \times {}^5C_1 + {}^6C_5 = 431$

For more see Video




Practice Questions Click Here

# APPLICATIONS OF SELECTIONS - II

**Restricted Combination**

The number of selection of r objects from n different objects:

1. When k particular things are always included = ${}^{n-k}C_{r-k}$

This can be comprehended as taking out those k things which have to be included which can be done in 1 way and then finding the ways in which r-k objects can be selected from remaining n - k things, and putting those k things (which are already taken out) in r-k selected objects.

2. k particular things are never included = ${}^{n-k}C_{r}$

This can be comprehended as taking out k things which are not to be selected which can be done in 1 way and then finding the ways of selecting r things from n-k things.

3. The number of ways selecting r things out of n different things such that p particular objects are always included and q particular objects are always excluded = ${}^{n-p-q}C_{r-p}$

This can be comprehended as taking out the q objects which should not be selected and putting it out and then taking out p objects which have to be selected and then finding ways of selecting r-p objects out of n- p-q objects and putting back p objects in r-p selected objects.

**Example:** In how many ways a cricket team can be selected out of 16 players such that 5 certain players must be included in the team.
**Solution:** Since 5 certain player has to be included so be need to select 11-5 = 6 player from 16 - 5 = 11 player.

So we can select the team in ${}^{16-5}C_{11-5} = {}^{11}C_6 = \dfrac{11!}{5!6!}$

For More
See Video


Practice
Questions
Click Here

# APPLICATIONS OF SELECTIONS - III (Geometrical Applications)

**If there are n points in the plane and out of which no three are collinear then,**

1. Total No. of lines that can be formed using these n points = ${}^{n}C_2$
2. Total No. of triangles that can be formed using these n points = ${}^{n}C_3$

3. Total no. of Diagonals that can be formed in n sided polygon = $^nC_2$ - n

**If there are n points in the plane and out of which m points are collinear, then,**

1. Total No. of different lines that can be formed by joining these n points is $^nC_2 - ^mC_2 + 1$
2. Total No. of different triangles that can be formed by joining these n points is $^nC_3 - ^mC_3$
3. Total No. of different quadrilaterals formed by joining these n points is $^nC_4 - \left(^mC_3 \cdot ^{n-m}C_1 + ^mC_4\right)$

**Number of Parallelograms**

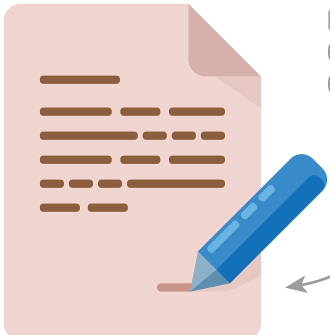
If m parallel lines in a plane are intersected by the family of other n parallel lines, then the total number of parallelograms formed is

$$^mC_2 \cdot ^nC_2 = \frac{mn(m-1)(n-1)}{4}$$

**Number of rectangles and squares**

1. Number of rectangles of any size in a square of size n x n is $\sum_{r=1}^{n} r^3$ and number of squares of any size is $\sum_{r=1}^{n} r^2$.
2. In a rectangle of size n x p (n < p) number of rectangles of any size is $\frac{np}{4}(n+1)(p+1)$.

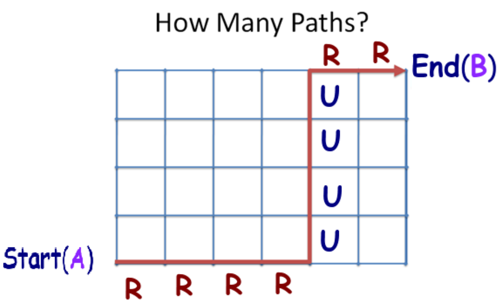
For More See Video

Practice Questions Click Here

## APPLICATIONS OF SELECTIONS - IV

**To determine the number of ways to reach in the shortest way from point A to B**



When considering the possible paths or shortest path one can observe that the total number of steps in the forward direction is 6-R(Right) and in the upward direction is 4-U(Upward)

Now, If we arrange these 6 Rs and 4 Us in any way, it comes out to be a shortest path.

Or one can say that first find all the possible steps and arrange them to get the total number of possible ways.

Using "u" and "r" we can write out a path:

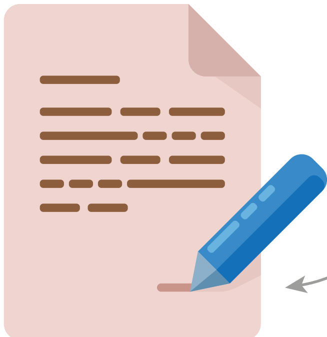
r r r r r r u u u u

r r r u u u u r r r

and others......

Hence, the total number of ways is $\frac{10!}{4!6!}$ or, ${}^{10}C_4$

For More
See Video

Practice
Questions
Click Here

# SELECTION OF ANY NUMBER OF DISTINCT OBJECTS

In certain situations, one has the liberty of selecting any number of objects from n (say) given objects. In this case, one can select 0 objects or 1 object or 2 objects or 3 objects or so on.... or all n objects.

Further, if the n objects are all different objects then not just how many objects are to be selected but a further question of which objects are selected also assumes importance. Thus there are two cases viz. the n objects being distinct or being identical.

**Selections of any number of objects out of n DISTINCT objects:**

Total no. of selections [Including Empty Selection]

$$^{n}C_0 + ^{n}C_1 + ^{n}C_2 \ldots + \ldots ^{n}C_n = 2^n$$

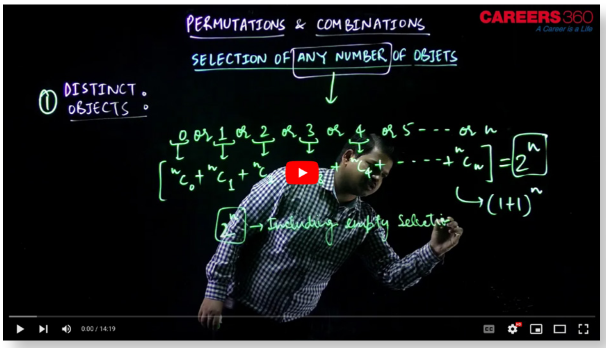
Total no. of Non Empty selection = $2^n - 1$

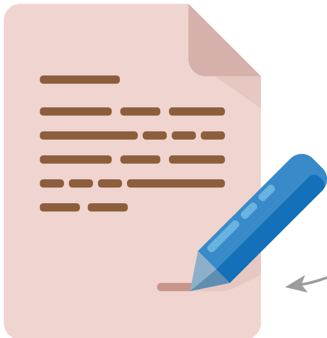
$$^{n}C_1 + ^{n}C_2 + ^{n}C_2 \ldots + \ldots ^{n}C_n = 2^n - 1$$

**Example:** A buffet dinner consists of 5 different dishes. In how many different ways can one help oneself if he has to take at least one dish?

**Solution:** The person can help himself to 1 or 2 or 3 or 4 or 5 dishes. Further, when he takes 1 or 2 or 3 or 4 or 5, he can also choose which of the dish he takes. Thus he can help himself in ${}^5C_1 + {}^5C_2 + \ldots + {}^5C_5$ i.e. 32 – 1 = 31 ways.

For More See Video



Practice Questions Click Here

## SELECTION OF ANY NUMBER OF IDENTICAL OBJECTS

**Selections of Any number of objects out of n IDENTICAL objects:**

Total no. of selections [including Empty Selection] = n+1

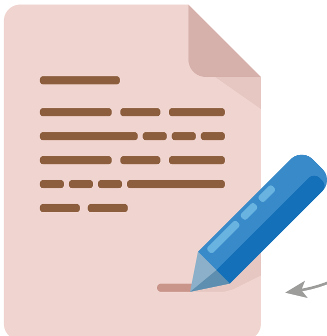
Total no. of Non Empty selections = n ways

These both cases can be justified as selecting 1 or 2 or 3...or...n objects can be done in 1 way each (as each object is identical), so total n ways and if we don't select any then it adds one more way of selecting 0 objects, hence n+1 ways

**Question:** In how many different ways can a person make a purchase from a fruit seller who has 5 mangoes, 8 apples and 10 oranges left with him and if the person has to purchase at least 1 mango, at least 1 apple and at least 1 orange?

**Solution:** Since at least 1 of each type has to be purchased, the number of ways with each of the different fruits can be purchased is 5 ways, 8 ways and 10 ways respectively. Thus, the total number of ways in which the purchase can be made is $5 \times 8 \times 10 = 400$ ways.

For More See Video

Practice Questions Click Here

## DIVISION OF OBJECTS INTO GROUPING (When Sizes of Groups are not Equal)

**Consider that 12 people have to be divided among the three groups of unequal sizes such as one group has 3 members, one group has 4 members and one group has 5 members?**

We could have formed a group of 3 members in ${}^{12}C_3$ ways. Having formed a group of three, we would be left with $12 - 3 = 9$ people. A group of 4 members can be formed from these 9 members in ${}^{9}C_{4}$ ways. For each group of 3 members formed earlier, there would be ${}^{12}C_{3}$ X ${}^{9}C_{4}$ ways of forming a group of four. Thus, the total possible number of ways of forming a group of 3 and a group of 4 would be \\\ mathrm{^{12}C_3\times^9C_4}. Now there would be 5 people left who are the third group i.e. the third group can be formed in only 1 way. To maintain consistency, we will say that the third group can be formed in ${}^{5}C_{5}$ ways (which is 1 anyways). Thus, the total number of ways of forming the groups is ${}^{12}C_3 \times {}^{9}C_4 \times {}^{5}C_5$.

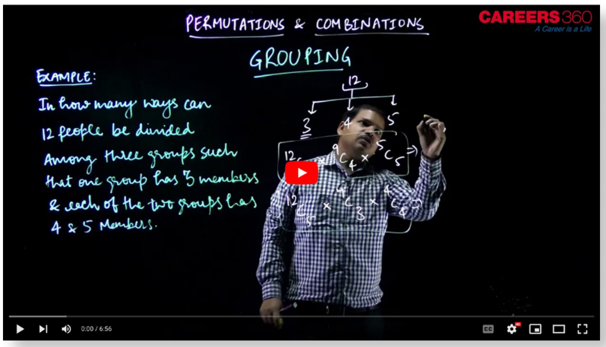
On expansion, this equals $\frac{(12)!}{3!4!5!}$.

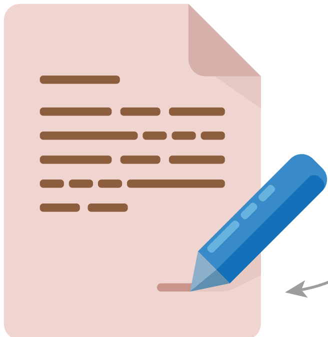
This concept can be generalized for (m+n+r) **distinct** objects which have to be grouped into three unequal groups containing m,n and r objects. So this grouping can be done in

$\frac{(m+n+r)!}{m!n!r!}$ number of ways

This same concepts will apply for (m+n) **distinct** object which has to be grouped in two unequal containing m, and n items.

For More See Video



Practice Questions Click Here

## DIVISION OF OBJECTS INTO GROUPING (When Sizes of Some Groups are Equal)

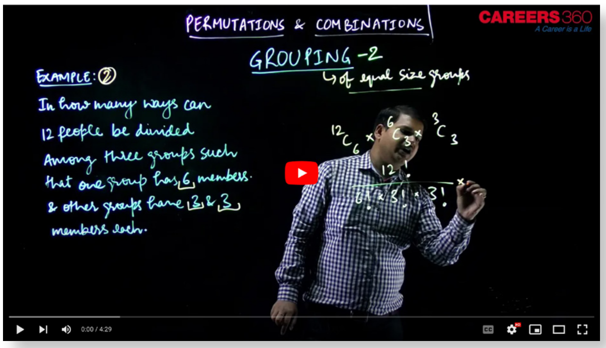
Number of ways of dividing mn object into m groups such that all groups contain n objects

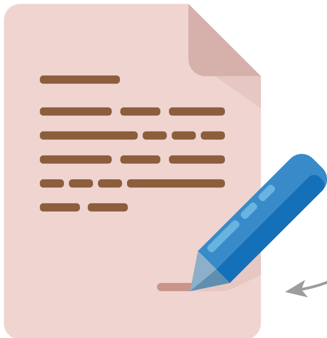
equals $\frac{(mn)!}{(n!)^m} \times \frac{1}{m!}$

**Example:** How many ways 12 people can be divided into 3 groups, such that all three of them contain 4 people each.
**Solution:** The number of ways of forming the three groups is ${}^{12}C_4 . {}^{8}C_4 . {}^{4}C_4$. Since we are multiplying these three factors, we are inadvertently also arranging the groups in a particular order. (Remember that if one position can be filled in 5 ways, another can be filled in 4 ways, third can be filled in 3 ways, when we apply the rule of AND i.e. $5 \times 4 \times 3$, we are basically finding the number of "arrangements" of the three positions)

But the question requires us to just form groups and we do not have to "arrange" the groups. Since we have arranged 3 objects which did not have to be arranged, we have counted each unique way of forming the groups 3! times i.e. 6 times. Thus, the correct answer would be found by dividing the earlier found answer by 3! This will give you the above formula itself.

For More See Video



Practice Questions Click Here

# FINDING NUMBER OF SOLUTIONS OF EQUATIONS

To find the solutions of a + b + c = 6 such that a,b,c are non-negative integers

Since zero is included, we have a ≥ 0, b ≥ 0, c ≥ 0

Number of solutions of a + b + c = 6 is equivalent to the number of ways of distributing 6 identical things into 3 different people, which can be achieved by arranging 6 objects and 3 - 1 = 2 partitions in a row. The number of objects before the first partition is given to first person, the number of objects in between the two partitions is given to second person and number of objects after second partition is given to third person.

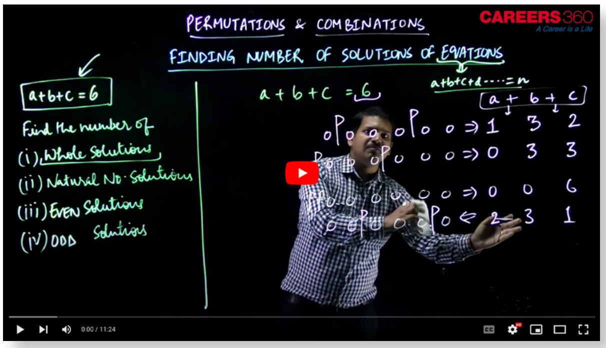
Number of ways of doing this arrangement is $\frac{8!}{6!.2!}$, which can also be written as ${}^{6+3-1}C_{3-1}$

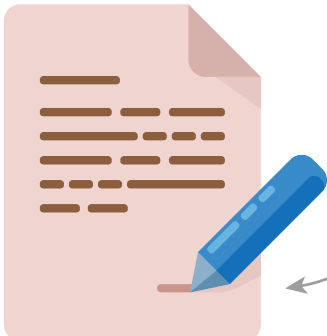
Hence the total number of solutions $= {}^{6+3-1}C_{3-1} = 28$

**Generalized formula**

For whole number solutions of $a_1 + a_2 + a_3 + \ldots.. + a_r = n$, we have the formula ${}^{n+r-1}C_{r-1}$.

For More See Video



Practice Questions Click Here

# FINDING NUMBER OF SOLUTIONS OF EQUATIONS (Special Case)

**Generalized formula**

The natural number solutions of $a_1 + a_2 + \ldots.. + a_r = n$ are ${}^{n-1}C_{r-1}$

**Q: Find the Natural number of solutions of a + b + c = 6 such that a,b,c are natural numbers**

**Solution:** Since zero is included, we have a ≥ 1, b ≥ 1, c ≥ 1

Let us define new variables

a' + 1 = a

b' + 1 = b

c' + 1 = c

Now if a', b', c' are whole numbers then a, b, c will be natural numbers.

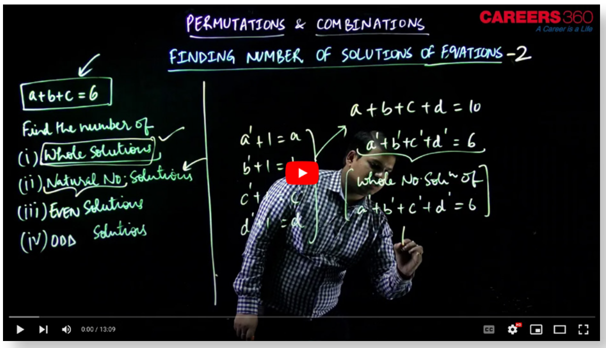
So the equation becomes a' + 1 + b' + 1 + c' + 1 = 6

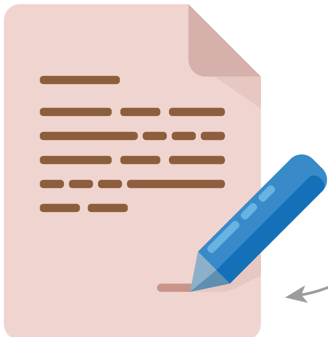
a' + b' + c' = 3

So whole number solutions of this equation equal natural number solutions of a + b + c = 6

Hence the total number of solutions $= {}^{3+3-1}C_{3-1} = 10$

For More See Video



Practice Questions Click Here

# DISTRIBUTION OF DISTINCT OBJECTS INTO DISTINCT PLACES

**No restriction**

To distribute n different things in r different boxes, such that there is no restriction on the number of objects a box can have (some boxes can remain empty as well)

We have r options for each object, so number of ways is r.r.r......n times = $r^n$

**Non-empty restriction**

To distribute n different things in r different boxes, such that none of the box is empty

Such problems can be done in following steps

1. Decide the number of objects in each group and make cases accordingly
2. In each case, first divide them into groups using grouping formula, then distibute these r groups in r people

Suppose we need to distribute 5 different hats in 3 different box such that no box is empty

**Step 1:**

To decide the group sizes, we should first distribute 1 hat to each 3 boxes so that none of them remain empty. Now we will be left with 2 hats, those 2 hats can be distributed to

1 group forming group combination as 1 1 3

OR

1-1 hat to 2 different groups giving the combination 1 2 2.
So in that way we will get two cases

**Case I**

| B1 | B2 | B3 |
|---|---|---|
| 1 | 1 | 3 |

**Case II**

| B1 | B2 | B3 |
|---|---|---|
| 1 | 2 | 2 |

**Step 2.**

**Case I:**

Make group of sizes (1 1 3) combination in $\frac{5!}{1!1!3!} \cdot \frac{1}{2!}$ ways.

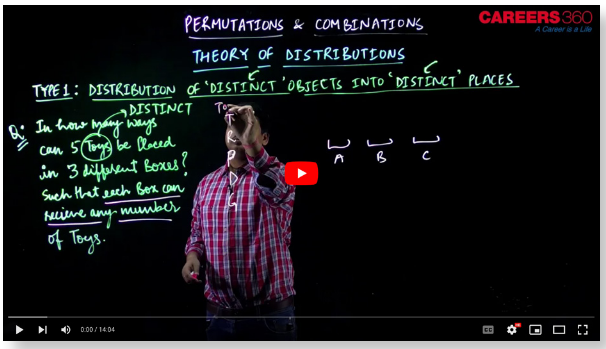
Now distribute these 3 groups in 3 people in 3! ways

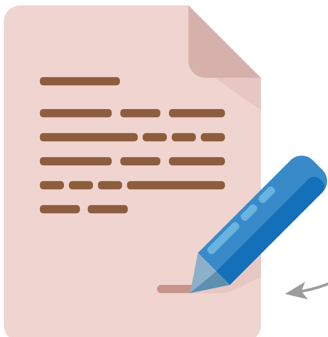
So total number of ways for case 1 = $\frac{5!}{1!1!3!} \cdot \frac{1}{2!} \cdot 3! = 60$

**Case II:**

Following the same logic in this case
Total number of possible ways of distribution $\frac{5!}{1!2!2!} \cdot \frac{1}{2!} \cdot 3! = 90$
Hence total ways = 60+90 = 150

For More
See Video



Practice
Questions
Click Here

# DISTRIBUTION OF IDENTICAL OBJECTS INTO DISTINCT PLACES

This type of concept can be comprehended as arranging all n identical objects to be distributed and (r-1) marks of partition. These (r-1) partitions will divide the objects in r groups. These r groups can be given to these r distinct people in order.

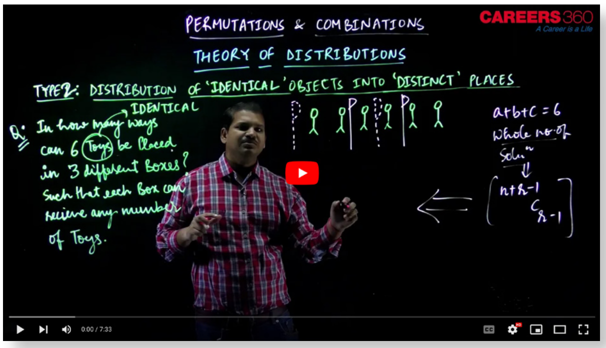
As each person can get zero or more objects in such arrangement, so number of ways is $\frac{(n+r-1)!}{(r-1)!n!} = {}^{n+r-1}C_{r-1}$

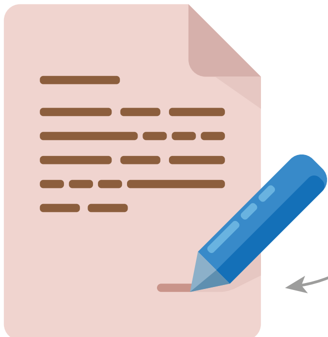
If each one has to get at least one object, then first distribute r objects to these r people (each one gets one, and any r objects can be given as these are identical), then we can distribute remaining (n - r) objects in r people in ${}^{n-r+r-1}C_{r-1} = {}^{n-1}C_{r-1}$ ways

**Alternative:**
This thing can also be comprehended as following
Let first group gets a1 objects
Second group gets a2 objects
.....
$r^{th}$ group gets ar objects
So, $a_1 + a_2 + a_3 + ... + a_r = n$ .......(i)
Now if empty groups are allowed we need to find the whole number solution of the equation (i)
So, whole number solutions of $a_1 + a_2 + a_3 + ... + a_r = n$ equals number of ways to distribute n distinct objects in r people, and these both equal ${}^{n+r-1}C_{r-1}$
If empty groups are not allowed, then each of these values has to be natural number. Number of ways of distribution will thus equal natural number solutions of equation (i), which is ${}^{n-1}C_{r-1}$

**Example:** In how many ways can 10 Identical chocolate be distributed among 3 children, such that each student can get any number and at least one?
**Solution:** using the above concept, we use direct formula for this, so we have ${}^{10-1}C_{3-1} = {}^{9}C_{2}$

For More
See Video



Practice
Questions
Click Here

# DISTRIBUTION OF DISTINCT OBJECTS INTO IDENTICAL PLACES

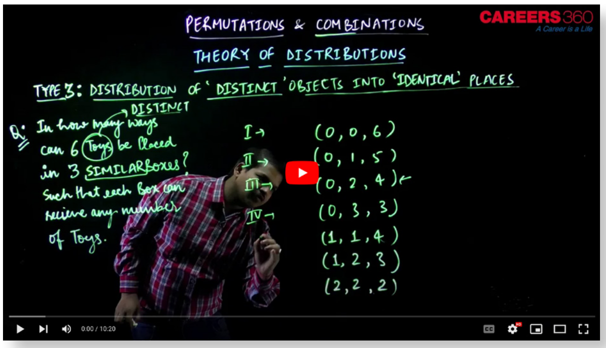
Suppose we want to distribute 5 distinct hats in 3 Identical boxes such that each box receives at least 1 hat. This can be done in 2 steps
Step 1: Decide the number of hats that each will get. Make cases depending on this
Step 2: Make groups in each case using the formula for grouping
Now,
**Step 1:** we can distribute 3 hats one-one each to all 3 groups, after that we can place remaining 2 hats in one group or 1-1 to 2 groups.
So 2 cases: $1^{st}$ = 1 1 3, $2^{nd}$ = 1 2 2
**Step 2:** Now these cases are similar to the division of groups where group sizes are given

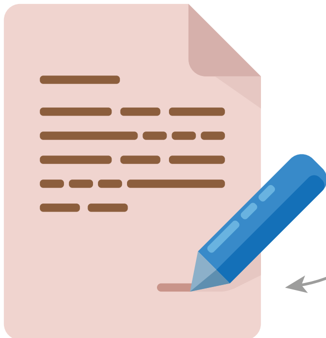
So, according to $1^{st}$ **case**, we can be group hats in $\frac{5!}{(1!)^2 3!} \cdot \frac{1}{2!}$ ways

Similarly for $2^{nd}$ **case**, $\frac{5!}{(2!)^2 1!} \cdot \frac{1}{2!}$ ways

So the total number of ways of distribution $= \dfrac{5!}{(2!)^2 1!} \cdot \dfrac{1}{2!} + \dfrac{5!}{(1!)^2 3!} \cdot \dfrac{1}{2!}$ ways

For More
See Video



Practice
Questions
Click Here

## DISTRIBUTION OF IDENTICAL OBJECTS INTO IDENTICAL PLACES

In this type, it does not matter which object goes in which group as all objects are identical, the only thing that matters is how many objects go into groups, and that means ordering or group does not matter

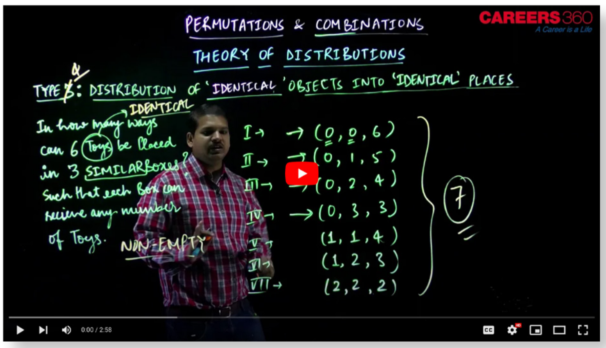
**Example:** In how many ways can 12 identical hats be put in 3 identical boxes such that each box has at least 2 hats?
**Solution:** First and foremost 2 hats should be put in each box (which hats does not matter as all are identical). Now we are left with 6 hats to be put in 3 identical boxes. As learned earlier, in the case of identical boxes, we have only concerned with dividing the 6 hats in 3 portions.

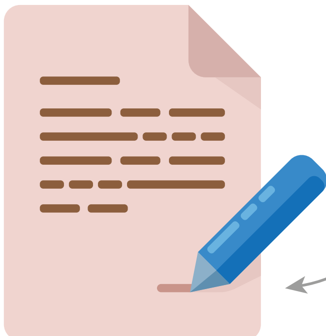
The hurdle is that the sizes could be anything. Thus again we individually consider all cases, with groups being every possible size. The possibilities are {0, 0, 6}; {0, 1, 5}; {0, 2, 4}; {0, 3, 3}; {1, 1, 4}; {1, 2, 3}; {2, 2, 2} i.e. 7 possibilities.

7 is our answer because each possible way of grouping can be done in only 1 way. Why? Because all hats are identical, so "which hat is in which group" does not matter.

Thus, the answer is 7 ways.

For More
See Video



Practice
Questions
Click Here

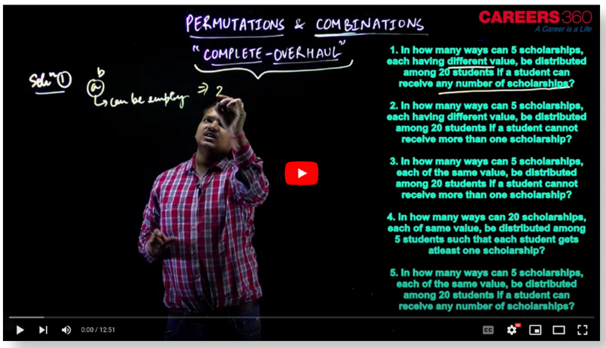
## PERMUTATION Vs COMBINATION

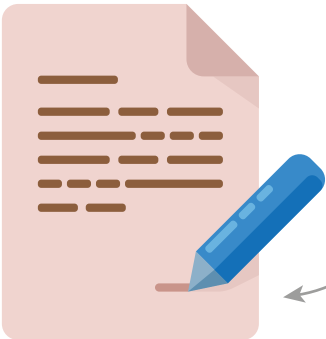
Always remember, in an arrangement, the order is always important. Whereas, in Combination, the order is not important.

Consider the following examples-

1. Selecting a team of 11 from 16 players - Selection
   Drawing a batting line-up of 11 from 16 players - Arrangement
1. Selecting 3 students out of 10 students who will receive scholarships of the same value - Selection
   Selecting 3 students out of 10 students who will receive scholarships of Rs. 500, Rs. 1000 and Rs. 2000 - Arrangement.

For More
See Video



Practice
Questions
Click Here

## DERANGEMENT

If there are n things, and n places, one correct place corresponding to each object. Then an arrangement in which none of the objects is at its right place, is called a derangement.

The number of ways of doing this is denoted by D(n) (the number of ways of deranging 'n' objects). And the formula for this is

$$D(n) = n!\left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \ldots + (-1)^n \frac{1}{n!}\right)$$

**GOLDEN TIP:**

Substituting the value of 'n' as 1, 2, 3, 4, 5, 6 we will get,

D(1)= 0

D(2)= 1

D(3)= 2

D(4)= 9

D(5)= 44

D(6)= 265

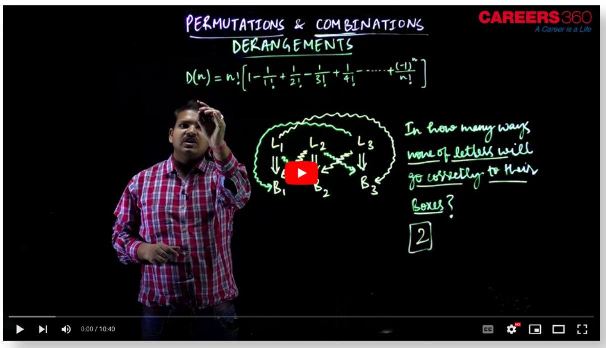
A quicker way to find out the total number of possible derangements is just to memorize the above values by heart and use them instantly in the questions.

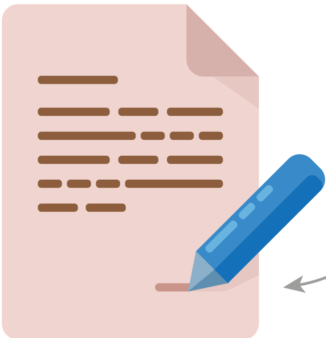
**Example:** In how many ways can you form a dancing couple from 3 boys and 3 girls so that no boy dances with his respective girlfriend?

**Solution:** This is clearly a case of derangement of 3 boys and 3 girls.

The value can be interpreted as D(3) =2 ways

For More
See Video



Practice
Questions
Click Here