CAREERS 360
PREPARATION Series

JEE Advanced 2024

Question Paper with Solutions
By FIITJEE

Note: ***For the benefit of the students, specially the aspiring ones, the question of JEE(advanced), 2024 are also given in this booklet. Keeping the interest of students studying in class XI, the questions based on topics from class XI have been marked with '*', which can be attempted as a test. For this test the time allocated in Mathematics, Physics and Chemistry are 30 minutes, 20 minutes and 25 minutes respectively.***

FIITJEE

SOLUTIONS TO JEE (ADVANCED) – 2024
(PAPER-1)

Mathematics

SECTION 2 (Maximum Marks: 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	: +3	If **ONLY** the correct option is chosen;
Zero Marks	: 0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	: –1	In all other cases.

Q.1 Let f(x) be a continuously differentiable function on the interval $(0, \infty)$ such that f(1) = 2 and $\lim\limits_{t\to x}\frac{t^{10}f(x)-x^{10}f(t)}{t^9-x^9}=1$ for each x > 0. Then, for all x > 0, f(x) is equal to

(A) $\frac{31}{11x}-\frac{9}{11}x^{10}$ (B) $\frac{9}{11x}+\frac{31}{11}x^{10}$

(C) $\frac{-9}{11x}+\frac{31}{11}x^{10}$ (D) $\frac{13}{11x}+\frac{9}{11}x^{10}$

Q.2 A student appears for a quiz consisting of only true-false type questions and answers all the questions. The student knows the answers of some questions and guesses the answers for the remaining questions. Whenever the student knows the answer of a question, he gives the correct answer. Assume that the probability of the student giving the correct answer for a question, given that he has guess it, is $\frac{1}{2}$. Also assume that the probability of the answer for a question being guessed, given that the student's answer is correct, is $\frac{1}{6}$. Then the probability that the student knows the answer of a randomly chosen question is

(A) $\frac{1}{12}$ (B) $\frac{1}{7}$

(C) $\frac{5}{7}$ (D) $\frac{5}{12}$

*Q.3 Let $\frac{\pi}{2} < x < \pi$ be such that $\cot x = \frac{-5}{\sqrt{11}}$. Then

$\left(\sin\frac{11x}{2}\right)(\sin 6x - \cos 6x) + \left(\cos\frac{11x}{2}\right)(\sin 6x + \cos 6x)$ is equal to

(A) $\frac{\sqrt{11}-1}{2\sqrt{3}}$ (B) $\frac{\sqrt{11}+1}{2\sqrt{3}}$

(C) $\frac{\sqrt{11}+1}{3\sqrt{2}}$ (D) $\frac{\sqrt{11}-1}{3\sqrt{2}}$

*Q.4 Consider the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$. Let S(p, q) be a point in the first quadrant such that $\frac{p^2}{9} + \frac{q^2}{4} > 1$. Two tangents are drawn from S to the ellipse, of which one meets the ellipse at one end point of the minor axis and the other meets the ellipse at a point T in the fourth quadrant. Let R be the vertex of the ellipse with positive x-coordinate and O be the centre of the ellipse. If the area of the triangle ΔORT is $\frac{3}{2}$, then which of the following options is correct ?

(A) $q = 2, p = 3\sqrt{3}$ (B) $q = 2, p = 4\sqrt{3}$

(C) $q = 1, p = 5\sqrt{3}$ (D) $q = 1, p = 6\sqrt{3}$

SECTION 1 (Maximum Marks: 12)

- This section contains **THREE (03)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated <u>according to the following marking scheme:</u>

Full Marks	: + 4	**ONLY** if (all) the correct option(s) is(are) chosen;
Partial Marks	: + 3	If all the four options are correct but **ONLY** three options are chosen;
Partial Marks	: + 2	If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
Partial Marks	: + 1	If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
Zero Marks	: 0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	: – 2	In all other cases.

- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then
  choosing ONLY (A), (B) and (D) will get +4 marks;
  choosing ONLY (A) and (B) will get +2 marks;
  choosing ONLY (A) and (D) will get +2 marks;
  choosing ONLY (B) and (D) will get +2 marks;
  choosing ONLY (A) will get +1 mark;
  choosing ONLY (B) will get +1 mark;
  choosing ONLY (D) will get +1 mark;
  choosing no option (i.e. the question is unanswered) will get 0 marks; and
  choosing any other combination of options will get –2 marks.

*Q.5 Let $S = \{a + b\sqrt{2} : a, b \in Z\}$, $T_1 = \{(-1+\sqrt{2})^n : n \in Z\}$, and $T_2 = \{(1+\sqrt{2})^n : n \in N\}$. Then which of the following statements is(are) TRUE ?

(A) $Z \cup T_1 \cup T_2 \subset S$

(B) $T_1 \cap \left(0, \frac{1}{2024}\right) = \phi$, where ϕ denotes the empty set

(C) $T_2 \cap (2024, \infty) \neq \phi$

(D) For any given a, b $\in$ Z, $\cos(\pi(a + b\sqrt{2})) + i\sin(\pi(a + b\sqrt{2})) \in Z$ if and only if b = 0, where $i = \sqrt{-1}$

Q.6 Let R^2 denote $R \times R$.

Let $S = \{(a,b,c) : a,b,c \in R \text{ and } ax^2 + 2bxy + cy^2 > 0 \text{ for all } (x,y) \in R^2 - \{(0,0)\}\}$

Then which of the following statements is (are) TRUE?

(A) $\left(2,\frac{7}{2},6\right) \in S$

(B) If $\left(3,b,\frac{1}{12},6\right) \in S$, then $|2b| < 1$

(C) For any given (a, b, c) $\in$ S, then the system of linear equations

$ax + by = 1$

$bx + cy = -1$

has a unique solution.

(D) For any given (a, b, c) $\in$ S, then the system of linear equations

$(a + 1)x + by = 0$

$bx + (c + 1)y = 0$

has a unique solution.

Q.7 Let R^3 denote the three dimensional space. Take two points P = (1, 2, 3) and Q = (4, 2, 7). Let dist(X, Y) denote the distance between two points X and Y in R^3. Let

$S = \{X \in R^3 : (\text{dist}(X, P))^2 - (\text{dist}(X, Q))^2 = 50\}$ and

$T = \{Y \in R^3 : (\text{dist}(Y, Q))^2 - (\text{dist}(Y, P))^2 = 50\}$.

Then which of the following statements is(are) TRUE ?

(A) There is a triangle whose area is 1 and all of whose vertices are from S.

(B) There are two distinct points L and M in T such that each point on the line segments LM is also in T.

(C) There are infinitely many rectangles of perimeter 48, two of whose vertices are from S and the other two vertices are from T.

(D) There is a square of perimeter 48, two of whose vertices are from S and the other two vertices are from T.

SECTION 3 (Maximum Marks: 24)

- This section contains **SIX (06)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER**.
- For each question, enter the correct integer corresponding to the answer using the mouse and the onscreen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 If **ONLY** the correct integer is entered;

Zero Marks : 0 In all other cases.

*Q.8 Let $a = 3\sqrt{2}$ and $b = \frac{1}{5^{1/6}\sqrt{6}}$. If x, y $\in$ R are such that

$3x + 2y = \log_a(18)^{5/4}$ and $2x - y = \log_b\left(\sqrt{1080}\right)$,

then 4x + 5y is equal to _____ .

*Q.9 Let $f(x) = x^4 + ax^3 + bx^2 + c$ be a polynomial with real coefficients such that f(1) = –9. Suppose that $i\sqrt{3}$ is a root of the equation $4x^3 + 3ax^2 + 2bx = 0$, where $i = \sqrt{-1}$. If α_1, α_2, α_3, and α_4 are all the roots of the equation $f(x) = 0$, then $|\alpha_1|^2 + |\alpha_2|^2 + |\alpha_3|^2 + |\alpha_4|^2$ is equal to _____ .

Q.10 Let $S = \left\{ A\begin{pmatrix} 0 & 1 & c \\ 1 & a & d \\ 1 & b & e \end{pmatrix} : a,b,c,d,e \in \{0,1\} \text{ and } |A| \in \{-1,1\} \right\}$, where |A| denotes the determinant of A.

Then the number of elements in S is _____ .

*Q.11 A group of 9 students s_1, s_2, …… s_9 is to be divided to form three teams X, Y and Z of sizes 2, 3 and 4 respectively. Suppose that s_1 cannot be selected for the team X, and s_2 cannot be selected for team Y. Then the number of ways to form such teams, is _____ .

Q.12 Let $\overrightarrow{OP} = \frac{\alpha-1}{\alpha}\hat{i}+\hat{j}+\hat{k}, \overrightarrow{OQ} = \hat{i}+\frac{\beta-1}{\beta}\hat{j}+\hat{k}$ and $\overrightarrow{OR} = \hat{i}+\hat{j}+\frac{1}{2}\hat{k}$ be three vectors, where $\alpha, \beta \in$ R–{0} and O denotes the origin. If $(\overrightarrow{OP}\times\overrightarrow{OQ})\cdot\overrightarrow{OR} = 0$ and the point $(\alpha, \beta, 2)$ lies on the plane $3x + 3y - z + l = 0$, then the value of l is _____ .

Q.13 Let X be a random variable, and let P(X = x) denote the probability that X takes the values x. Suppose that the points (x, P(X = x)), x = 0, 1, 2, 3, 4, lie on a fixed straight line in the xy-plane, and P(X = x) = 0 for all $x \in R - \{0, 1, 2, 3, 4\}$. If the mean of X is $\frac{5}{2}$, and the variance of X is α, then the value of 24α is _____ .

SECTION 4 (Maximum Marks: 12)

- This section contains **FOUR (04)** Matching List Sets.
- Each set has **ONE** Multiple Choice Question.
- Each set has **TWO** lists: **List-I** and **List-II**.
- **List-I** has **Four** entries (P), (Q), (R) and (S) and **List-II** has **Five** entries (1), (2), (3), (4) and (5).
- **FOUR** options are given in each Multiple Choice Question based on **List-I** and **List-II** and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated <u>according to the following marking scheme</u>:

Full Marks	: +3	**ONLY** if the option corresponding to the correct combination is chosen;
Zero Marks	: 0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	: –1	In all other cases.

Q.14 Let α and β be the distinct roots of the equation $x^2 + x - 1 = 0$. Consider the set $T = \{1, \alpha, \beta\}$. For a 3×3 matrix $M = (a_{ij})_{3\times3}$, define $R_i = a_{i1} + a_{i2} + a_{i3}$ and $C_j = a_{1j} + a_{2j} + a_{3j}$ for i = 1, 2, 3 and j = 1, 2, 3.

Match each entry in **List-I** to the correct entries in **List-II.**

	List – I		**List – II**
(P)	The number of matrices $M = (a_{ij})_{3\times3}$ with all entries in T such that $R_i = C_j = 0$ for all i, j, is	(1)	1
(Q)	The number of symmetric matrices $M = (a_{ij})_{3\times3}$ with all entries in T such that $C_j = 0$ for all j, is	(2)	12
(R)	Let $M = (a_{ij})_{3\times3}$ be a skew symmetric matrix such that $a_{ij} \in T$ for $i > j$. Then the number of elements in the set $\left\{\begin{pmatrix}x\\y\\z\end{pmatrix} : x,y,z \in R, M\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}a_{12}\\0\\-a_{23}\end{pmatrix}\right\}$ is	(3)	infinite
(S)	Let $M = (a_{ij})_{3\times3}$ be a matrix with all entries in T such that $R_i = 0$ for all i. Then the absolute value of determinant of M is	(4)	6
		(5)	0

The correct option is:

(A) (P) → (4) (Q) → (2) (R) → (5) (S) → (1)
(B) (P) → (2) (Q) → (4) (R) → (1) (S) → (5)
(C) (P) → (2) (Q) → (4) (R) → (3) (S) → (5)
(D) (P) → (1) (Q) → (5) (R) → (3) (S) → (4)

*Q.15 Let the straight line y = 2x touch a circle with centre $(0, \alpha)$, $\alpha > 0$, and radius r at a point A_1. Let B_1 be the point on the circle such the line segment A_1B_1 is a diameter of the circle. Let $\alpha + r = 5 + \sqrt{5}$.

Match each entry in **List-I** to the correct entries in **List-II.**

List – I		**List – II**	
(P)	α equals	(1)	$(-2, 4)$
(Q)	r equals	(2)	$\sqrt{5}$
(R)	A_1 equals	(3)	$(-2, 6)$
(S)	B_1 equals	(4)	5
		(5)	(2, 4)

The correct option is:
(A) (P) → (4) (Q) → (2) (R) → (1) (S) → (3)
(B) (P) → (2) (Q) → (4) (R) → (1) (S) → (3)
(C) (P) → (4) (Q) → (2) (R) → (5) (S) → (3)
(D) (P) → (2) (Q) → (4) (R) → (3) (S) → (5)

Q.16 Let $\gamma \in R$ be such that the lines $L_1 : \frac{x+11}{1} = \frac{y+21}{2} = \frac{z+29}{3}$ and $L_2 : \frac{x+16}{3} = \frac{y+11}{2} = \frac{z+4}{\gamma}$ intersect. Let R_1 be the point of intersection of L_1 and L_2. Let O = (0, 0, 0), and $\hat{n}$ denote a unit normal vector to the plane containing both the lines L_1 and L_2.

Match each entry in **List-I** to the correct entries in **List-II.**

List – I		**List – II**	
(P)	γ equals	(1)	$-\hat{i} - \hat{j} + \hat{k}$
(Q)	A possible choice for $\hat{n}$ is	(2)	$\sqrt{\frac{3}{2}}$
(R)	$\overrightarrow{OR_1}$ equals	(3)	1
(S)	A possible value of $\overrightarrow{OR_1} . \hat{n}$ is	(4)	$\frac{1}{\sqrt{6}}\hat{i} - \frac{2}{\sqrt{6}}\hat{j} + \frac{1}{\sqrt{6}}\hat{k}$
		(5)	$\sqrt{\frac{2}{3}}$

The correct option is:
(A) (P) → (3) (Q) → (4) (R) → (1) (S) → (2)
(B) (P) → (5) (Q) → (4) (R) → (1) (S) → (2)
(C) (P) → (3) (Q) → (4) (R) → (1) (S) → (5)
(D) (P) → (3) (Q) → (1) (R) → (4) (S) → (5)

Q.17 Let $f : R \to R$ and $g : R \to R$ be functions defined by

$$f(x) = \begin{cases} x|x|\sin\left(\frac{1}{x}\right), & x \neq 0, \\ 0, & x = 0, \end{cases} \quad \text{and} \quad g(x) = \begin{cases} 1-2x, & 0 \leq x \leq \frac{1}{2}, \\ 0, & \text{otherwise.} \end{cases}$$

Let $a, b, c, d \in R$. Define the function $h : R \to R$ by

$$h(x) = af(x) + b\left(g(x) + g\left(\frac{1}{2} - x\right)\right) + c\left(x - g(x)\right) + d\,g(x), x \in R$$

Match each entry in **List-I** to the correct entries in **List-II.**

List – I		**List – II**	
(P)	If a = 0, b = 1, c = 0 and d = 0, then	(1)	h is one-one.
(Q)	If a = 1, b = 0, c = 0 and d = 0, then	(2)	h is onto.
(R)	If a = 0, b = 0, c = 1 and d = 0, then	(3)	h is differentiable on R.
(S)	If a = 0, b = 0, c = 0 and d = 1, then	(4)	the range of h is [0, 1].
		(5)	the range of h is {0, 1}.

The correct option is:

(A) (P) $\to$ (4) (Q) $\to$ (3) (R) $\to$ (1) (S) $\to$ (2)
(B) (P) $\to$ (5) (Q) $\to$ (2) (R) $\to$ (4) (S) $\to$ (3)
(C) (P) $\to$ (5) (Q) $\to$ (3) (R) $\to$ (2) (S) $\to$ (4)
(D) (P) $\to$ (4) (Q) $\to$ (2) (R) $\to$ (1) (S) $\to$ (3)

Physics

SECTION 1 (Maximum Marks: 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 If **ONLY** the correct option is chosen;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : −1 In all other cases.

Q.1 A dimensionless quantity is constructed in terms of electronic charge e, permittivity of free space ε_0, Planck's constant h, and speed of light c. If the dimensionless quantity is written as $e^{\alpha}\varepsilon_0^{\beta}h^{\gamma}c^{\delta}$ and n is a non-zero integer, then (α, β, γ, δ) is given by

(A) (2n, – n, – n, – n) (B) (n, – n, – 2n, – n)
(C) (n, – n, – n, – 2n) (D) (2n, – n, – 2n, – 2n)

Q.2 An infinitely long wire, located on the z-axis, carries a current I along the +z-direction and produces the magnetic field $\vec{B}$. The magnitude of the line integral $\int \vec{B}.\vec{dl}$ along a straight line from the point $\left(-\sqrt{3}a,a,0\right)$ to (a, a, 0) is given by

[μ_0 is the magnetic permeability of free space.]

(A) $7\mu_0 I/24$ (B) $7\mu_0 I/12$
(C) $\mu_0 I/8$ (D) $\mu_0 I/6$

Q.3 Two beads, each with charge q and mass m, are on a horizontal, frictionless, non-conducting, circular hoop of radius R. One of the beads is glued to the hoop at some point, while the other one performs small oscillations about its equilibrium position along the hoop. The square of the angular frequency of the small oscillations is given by

[ε_0 is the permittivity of free space.]

(A) $q^2/\left(4\pi\varepsilon_0 R^3 m\right)$ (B) $q^2/\left(32\pi\varepsilon_0 R^3 m\right)$
(C) $q^2/\left(8\pi\varepsilon_0 R^3 m\right)$ (D) $q^2/\left(16\pi\varepsilon_0 R^3 m\right)$

*Q.4 A block of mass 5 kg moves along the x-direction subject to the force F = (–20x + 10) N, with the value of x in metre. At time t = 0 s, it is at rest at position x = 1 m. The position and momentum of the block at t = (π/4) s are

(A) – 0.5 m, 5 kg m/s (B) 0.5 m, 0 kg m/s
(C) 0.5 m, –5 kg m/s (D) – 1 m, 5 kg m/s

SECTION 2 (Maximum Marks: 12)

- This section contains **THREE (03)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 **ONLY** if (all) the correct option(s) is(are) chosen;
Partial Marks : +3 If all the four options are correct but **ONLY** three options are chosen;
Partial Marks : +2 If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
Partial Marks : +1 If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : −2 In all other cases.

- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then
choosing ONLY (A), (B) and (D) will get +4 marks;
choosing ONLY (A) and (B) will get +2 marks;
choosing ONLY (A) and (D) will get +2 marks;
choosing ONLY (B) and (D) will get +2 marks;
choosing ONLY (A) will get +1 mark;
choosing ONLY (B) will get +1 mark;
choosing ONLY (D) will get +1 mark;
choosing no option (i.e. the question is unanswered) will get 0 marks; and
choosing any other combination of options will get −2 marks.

Q.5 A particle of mass m is moving in a circular orbit under the influence of the central force $F(r) = -kr$, corresponding to the potential energy $V(r) = kr^2/2$, where k is a positive force constant and r is the radial distance from the origin. According to the Bohr's quantization rule, the angular momentum of the particle is given by $L = n\hbar$, where $\hbar = h/(2\pi)$, h is the Planck's constant, and n a positive integer. If v and E are the speed and total energy of the particle, respectively, then which of the following expression(s) is(are) correct?

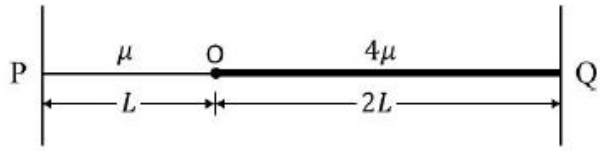
(A) $r^2 = n\hbar\sqrt{\dfrac{1}{mk}}$

(B) $v^2 = n\hbar\sqrt{\dfrac{k}{m^3}}$

(C) $\dfrac{L}{mr^2} = \sqrt{\dfrac{k}{m}}$

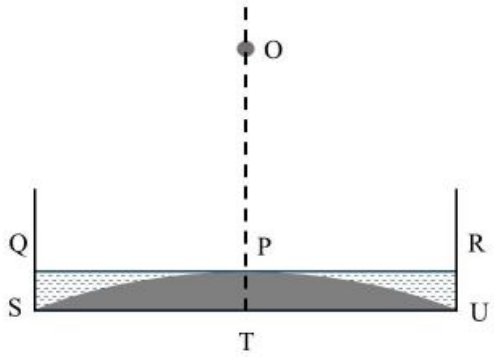
(D) $E = \dfrac{n\hbar}{2}\sqrt{\dfrac{k}{m}}$

*Q.6 Two uniform strings of mass per unit length μ and 4μ, and length L and 2L, respectively, are joined at point O, and tied at two fixed ends P and Q, as shown in the figure. The strings are under a uniform tension T. If we define the frequency $\nu_0 = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$, which of the following statement(s) is(are) correct?



(A) With a node at O, the minimum frequency of vibration of the composite string is ν_0.
(B) With an antinode at O, the minimum frequency of vibration of the composite string is $2\nu_0$.
(C) When the composite string vibrates at the minimum frequency with a node at O, it has 6 nodes, including the end nodes.
(D) No vibrational mode with an antinode at O is possible for the composite string.

Q.7 A glass beaker has a solid, plano-convex base of refractive index 1.60, as shown in the figure. The radius of curvature of the convex surface (SPU) is 9 cm, while the planar surface (STU) acts as a mirror. This beaker is filled with a liquid of refractive index n up to the level QPR. If the image of a point object O at a height of h (OT in the figure) is formed onto itself, then, which of the following option(s) is(are) correct?



(A) For n = 1.42, h = 50 cm.
(B) For n = 1.35, h = 36 cm.
(C) For n = 1.45, h = 65 cm.
(D) For n = 1.48, h = 85 cm.

SECTION 3 (Maximum Marks: 24)

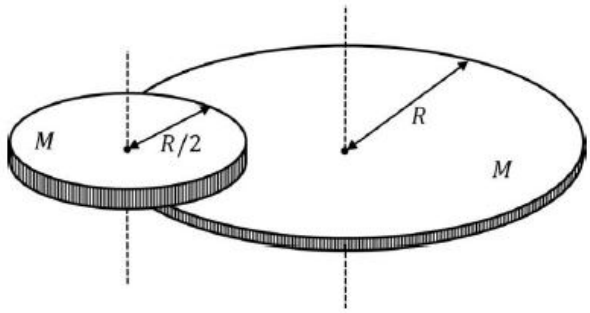
- This section contains **SIX (06)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER**.
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 If **ONLY** the correct integer is entered;
Zero Marks : 0 In all other cases.

*Q.8 The specific heat capacity of a substance is temperature dependent and is given by the formula C = kT, where k is a constant of suitable dimensions in SI units, and T is the absolute temperature. If the heat required to raise the temperature of 1 kg of the substance from –73°C to 27°C is nk, the value of n is _____.

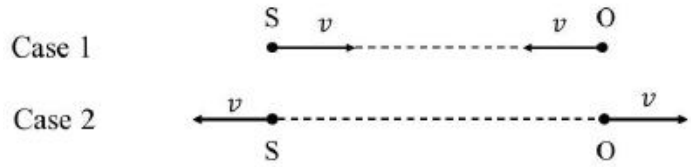
[Given: 0 K = –273°C]

*Q.9 A disc of mass M and radius R is free to rotate about its vertical axis as shown in the figure. A battery operated motor of negligible mass is fixed to this disc at a point on its circumference. Another disc of the same mass M and radius R/2 is fixed to the motor's thin shaft. Initially, both the discs are at rest. The motor is switched on so that the smaller disc rotates at a uniform angular speed ω. If the angular speed at which the large disc rotates is ω/n, then the value of n is_____.



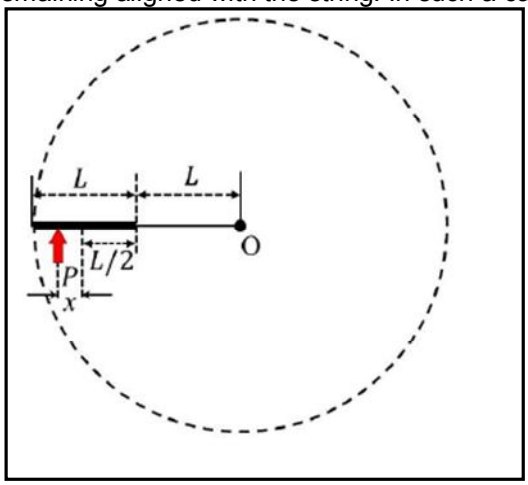
Q.10 A point source S emits unpolarized light uniformly in all directions. At two points A and B, the ratio $r = I_A/I_B$ of the intensities of light is 2. If a set of two polaroids having 45° angle between their pass-axes is placed just before point B, then the new value of r will be _____.

*Q.11 A source (S) of sound has frequency 240 Hz. When the observer (O) and the source move towards each other at a speed v with respect to the ground (as shown in Case 1 in the figure), the observer measures the frequency of the sound to be 288 Hz. However, when the observer and the source move away from each other at the same speed v with respect to the ground (as shown in Case 2 in the figure), the observer measures the frequency of sound to be n Hz. The value of n is _____.



*Q.12 Two large, identical water tanks, 1 and 2, kept on the top of a building of height H, are filled with water up to height h in each tank. Both the tanks contain an identical hole of small radius on their sides, close to their bottom. A pipe of the same internal radius as that of the hole is connected to tank 2, and the pipe ends at the ground level. When the water flows from the tanks 1 and 2 through the holes, the times taken to empty the tanks are t_1 and t_2, respectively. If $H = \left(\frac{16}{9}\right)h$, then the ratio t_1/t_2 is_____.

*Q.13 A thin uniform rod of length L and certain mass is kept on a frictionless horizontal table with a massless string of length L fixed to one end (top view is shown in the figure). The other end of the string is pivoted to a point O. If a horizontal impulse P is imparted to the rod at a distance x = L/n from the mid-point of the rod (see figure), then the rod and string revolve together around the point O, with the rod remaining aligned with the string. In such a case, the value of n is _____.

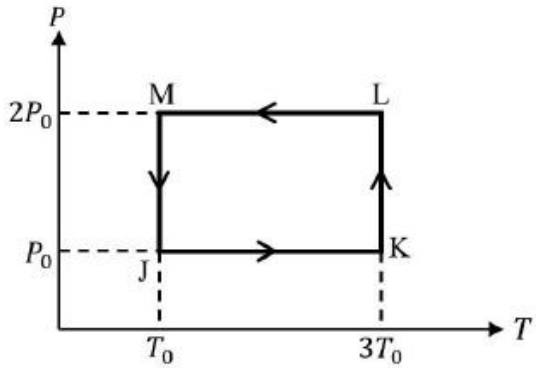


SECTION 4 (Maximum Marks: 12)

- This section contains **FOUR (04)** Matching List Sets.
- Each set has **ONE** Multiple Choice Question.
- Each set has **TWO** lists: **List-I** and **List-II**.
- **List-I** has **Four** entries (P), (Q), (R) and (S) and **List-II** has **Five** entries (1), (2), (3), (4) and (5).
- **FOUR** options are given in each Multiple Choice Question based on **List-I** and **List-II** and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 **ONLY** if the option corresponding to the correct combination is chosen;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : −1 In all other cases.

*Q.14 One mole of a monatomic ideal gas undergoes the cyclic process J → K → L → M → J, as shown in the P-T diagram.

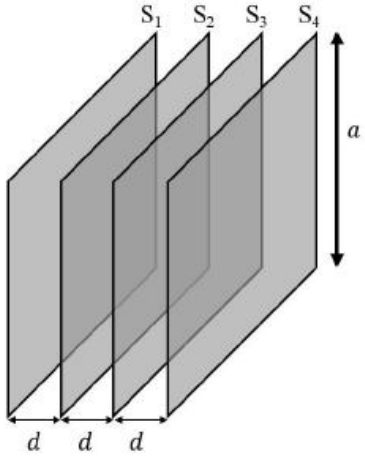


Match the quantities mentioned in List-I with their values in List-II and choose the correct option. [R is the gas constant.]

	List-I		List-II
(P)	Work done in the complete cyclic process	(1)	$RT_0 - 4RT_0 \ln 2$
(Q)	Change in the internal energy of the gas in the process JK	(2)	0
(R)	Heat given to the gas in the process KL	(3)	$3RT_0$
(S)	Change in the internal energy of the gas in the process MJ	(4)	$-2RT_0 \ln 2$
		(5)	$-3RT_0 \ln 2$

(A) P $\rightarrow$ 1; Q $\rightarrow$ 3; R $\rightarrow$ 5; S $\rightarrow$ 4
(B) P $\rightarrow$ 4; Q $\rightarrow$ 3; R $\rightarrow$ 5; S $\rightarrow$ 2
(C) P $\rightarrow$ 4; Q $\rightarrow$ 1; R $\rightarrow$ 2; S $\rightarrow$ 2
(D) P $\rightarrow$ 2; Q $\rightarrow$ 5; R $\rightarrow$ 3; S $\rightarrow$ 4

Q.15 Four identical thin, square metal sheets, S_1, S_2, S_3 and S_4, each of side a are kept parallel to each other with equal distance d (<< a) between them, as shown in the figure. Let $C_0 = \varepsilon_0 a^2/d$, where ε_0 is the permittivity of free space.



Match the quantities mentioned in List-I with their values in List-II and choose the correct option.

	List-I		List-II
(P)	The capacitance between S_1 and S_4, with S_2 and S_3 not connected, is	(1)	$3C_0$
(Q)	The capacitance between S_1 and S_4, with S_2 shorted to S_3, is	(2)	$C_0/2$
(R)	The capacitance between S_1 and S_3, with S_2 shorted to S_4, is	(3)	$C_0/3$
(S)	The capacitance between S_1 and S_2, with S_3 shorted to S_1, and S_2 shorted to S_4, is	(4)	$2C_0/3$
		(5)	$2C_0$

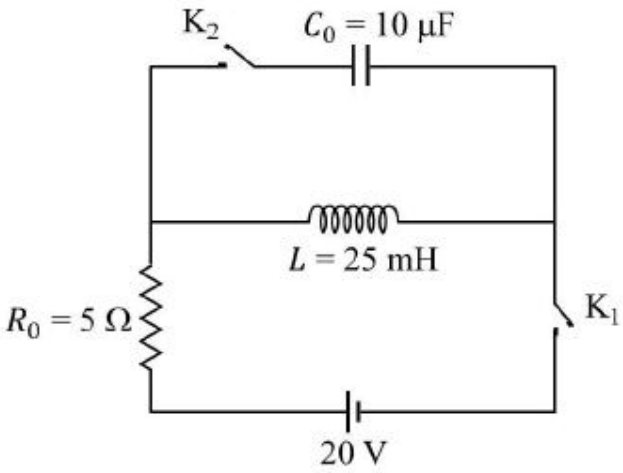
(A) P $\rightarrow$ 3; Q $\rightarrow$ 2; R $\rightarrow$ 4; S $\rightarrow$ 5
(B) P $\rightarrow$ 2; Q $\rightarrow$ 3; R $\rightarrow$ 2; S $\rightarrow$ 1
(C) P $\rightarrow$ 3; Q $\rightarrow$ 2; R $\rightarrow$ 4; S $\rightarrow$ 1
(D) P $\rightarrow$ 3; Q $\rightarrow$ 2; R $\rightarrow$ 2; S $\rightarrow$ 5

Q.16 A light ray is incident on the surface of a sphere of refractive index n at an angle of incidence θ_0. The ray partially refracts into the sphere with angle of refraction ϕ_0 and then partly reflects from the back surface. The reflected ray then emerges out of the sphere after a partial refraction. The total angle of deviation of the emergent ray with respect to the incident ray is α. Match the quantities mentioned in List-I with their values in List-II and choose the correct option.

	List-I		List-II
(P)	If n = 2 and α = 180°, then all the possible values of θ_0 will be	(1)	30° and 0^0
(Q)	If $n = \sqrt{3}$ and α = 180°, then all the possible values of θ_0 will be	(2)	60° and 0^0
(R)	If $n = \sqrt{3}$ and α = 180°, then all the possible values of ϕ_0 will be	(3)	45° and 0^0
(S)	If $n = \sqrt{2}$ and θ_0 = 45°, then all the possible values of α will be	(4)	150°
		(5)	0°

(A) P → 5; Q → 2; R → 1; S → 4
(B) P → 5; Q → 1; R → 2; S → 4
(C) P → 3; Q → 2; R → 1; S → 4
(D) P → 3; Q → 1; R → 2; S → 5

Q.17 The circuit shown in the figure contains an inductor L, a capacitor C_0, a resistor R_0 and an ideal battery. The circuit also contains two keys K_1 and K_2. Initially, both the keys are open and there is no charge on the capacitor. At an instant, key K_1 is closed and immediately after this the current in R_0 is found to be I_1. After a long time, the current attains a steady state value I_2. Thereafter, K_2 is closed and simultaneously K_1 is opened and the voltage across C_0 oscillates with amplitude V_0 and angular frequency ω_0



Match the quantities mentioned in List-I with their values in List-II and choose the correct option.

	List-I		List-II
(P)	The value of I_1 in Ampere is	(1)	0
(Q)	The value of I_2in Ampere is	(2)	2
(R)	The value of ω_0 in kilo-radians/s	(3)	4
(S)	The value of V_0 in Volt is	(4)	20
		(5)	200

(A) P → 1; Q → 3; R → 2; S → 5
(B) P → 1; Q → 2; R → 3; S → 5
(C) P → 1; Q → 3; R → 2 S → 4
(D) P → 2 Q → 5 R → 3 S → 4

Chemistry

SECTION 1 (Maximum Marks: 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 If **ONLY** the correct option is chosen;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : –1 In all other cases.

***Q.1** A closed vessel contains 10 g of an ideal gas **X** at 300 K, which exerts 2 atm pressure. At the same temperature, 80 g of another ideal gas **Y** is added to it and the pressure becomes 6 atm. The ratio of root mean square velocities of **X** and **Y** at 300 K is

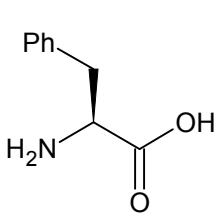
(A) $2\sqrt{2} : \sqrt{3}$ (B) $2\sqrt{2} : 1$
(C) 1 : 2 (D) 2 : 1

Q.2 At room temperature, disproportionation of an aqueous solution of *in situ* generated nitrous acid (HNO_2) gives the species

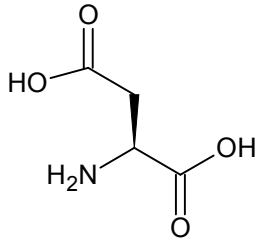
(A) H_3O^+, NO_3^- and NO (B) H_3O^+, NO_3^- and NO_2
(C) H_3O^+, NO^- and NO_2 (D) H_3O^+, NO_3^- and N_2O

Q.3 Aspartame, an artificial sweetener, is a dipeptide aspartyl phenylalanine methyl ester. The structure of aspartame is

Structure of phenylalanine and aspartic acid are given below.

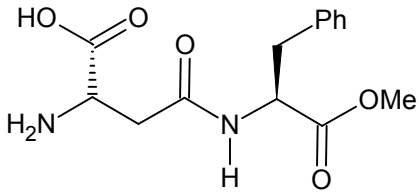


Phenylalanine

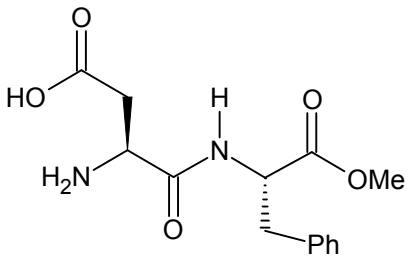


Aspartic acid

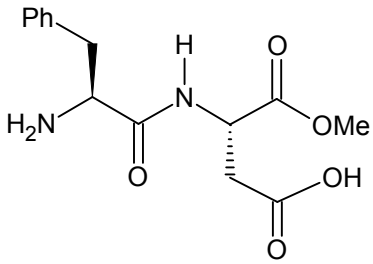
(A)



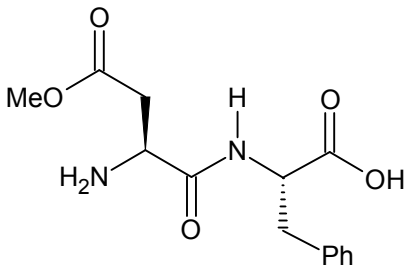
(B)



(C)



(D)



Q.4 Among the following options, select the option in which each complex in **Set-I** shows geometrical isomerism and the two complexes in **Set-II** are ionization isomers of each other.
[en = $H_2NCH_2CH_2NH_2$]

(A) **Set – I:** $[Ni(CO)_4]$ and $[PdCl_2(PPh_3)_2]$
Set – II: $[Co(NH_3)_5Cl]SO_4$ and $[Co(NH_3)_5(SO_4)]Cl$

(B) **Set – I:** $[Co(en)(NH_3)_2Cl_2]$ and $[PdCl_2(PPh_3)_2]$
Set – II: $[Co(NH_3)_6][Cr(CN)_6]$ and $[Cr(NH_3)_6[Co(CN)_6]$

(C) **Set – I:** $[Co(NH_3)_3(NO_2)_3]$ and $[Co(en)_2Cl_2]$
Set – II: $[Co(NH_3)_5Cl]SO_4$ and $[Co(NH_3)_5(SO_4)]Cl$

(D) **Set – I:** $[Cr(NH_3)_5Cl]Cl_2$ and $[Co(en)(NH_3)_2Cl_2]$
Set – II: $[Cr(H_2O)_6]Cl_3$ and $[Cr(H_2O)_5Cl]Cl_2.H_2O$

SECTION 2 (Maximum Marks: 12)

- This section contains **THREE (03)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +4 **ONLY** if (all) the correct option(s) is(are) chosen;
  Partial Marks : +3 If all the four options are correct but **ONLY** three options are chosen;
  Partial Marks : +2 If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
  Partial Marks : +1 If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
  Zero Marks : 0 If unanswered;
  Negative Marks : –2 In all other cases.
- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then
  choosing ONLY (A), (B) and (D) will get +4 marks;
  choosing ONLY (A) and (B) will get +2 marks;
  choosing ONLY (A) and (D) will get +2marks;
  choosing ONLY (B) and (D) will get +2 marks;
  choosing ONLY (A) will get +1 mark;
  choosing ONLY (B) will get +1 mark;
  choosing ONLY (D) will get +1 mark;
  choosing no option(s) (i.e. the question is unanswered) will get 0 marks and
  choosing any other option(s) will get –2 marks.

***Q5.** Among the following, the correct statement(s) for electrons in an atom is(are)

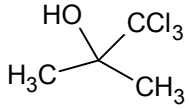
(A) Uncertainty principle rules out the existence of definite paths for electrons.

(B) The energy of an electrons in 2s orbital of an atom is lower than the energy of an electron that is infinitely far away from the nucleus.

(C) According to Bohr's model, the most negative energy value for an electron is given by n = 1, which corresponds to the most stable orbit.

(D) According to Bohr's model, the magnitude of velocity of electrons increases with increase in values of n.

Q.6 Reaction of *iso*-propylbenzene with O_2 followed by the treatment with H_3O^+ forms phenol and a by product **P**. Reaction of **P** with 3 equivalents of Cl_2 gives compound **Q**. Treatement of **Q** with $Ca(OH)_2$ produces compound **R** and calcium salt **S**.
The correct statement(s) regarding **P, Q, R** and **S** is(are)
(A) Reaction of **P** with **R** in the presence of KOH followed by acidification gives



(B) Reaction of **R** with O_2 in the presence of light gives phosgene gas
(C) **Q** reacts with aqueous NaOH to produce Cl_3CCH_2OH and $Cl_3CCOONa$
(D) **S** on heating gives **P**

***Q7.** The option(s) in which at least three molecules follow Octet Rule is(are)
(A) CO_2, C_2H_4, NO and HCl (B) NO_2, O_3, HCl and H_2SO_4
(C) BCl_3, NO, NO_2 and H_2SO_4 (D) CO_2, BCl_3, O_3 and C_2H_4

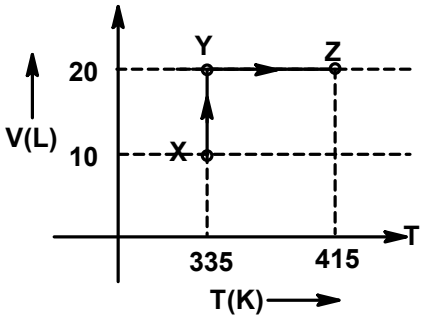
SECTION 3 (Maximum Marks: 24)

- This section contains **SIX (06)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER**.
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated <u>according to the following marking scheme:</u>

  Full Marks : +4 If **ONLY** the correct integer is entered;

  Zero Marks : 0 In all other cases.

***Q8.** Consider the following volume – temperature (V – T) diagram for the expansion of 5 moles of an ideal monoatomic gas.



Consider only P – V work is involved, the total change in enthalpy (in Joule) for the transformation of state in the sequence **X** $\rightarrow$ **Y** $\rightarrow$ **Z** is _____.
[Use the given data: Molar heat capacity of the gas for the given temperature range, $C_{v,m}$ = 12 J K^{-1} mol^{-1} and gas constant, R = 8.3 J K^{-1} mol^{-1}]

***Q.9** Consider the following reaction,

$$2H_2(g) + 2NO(g) \longrightarrow N_2(g) + 2H_2O(g)$$

Which follows the mechanism given below?

$$2NO(g) \underset{k_{-1}}{\overset{k_1}{\rightleftharpoons}} N_2O_2(g) \quad \text{(fast equilibrium)}$$

$$N_2O_2(g) + H_2(g) \xrightarrow{k_2} N_2O(g) + H_2O(g) \quad \text{(slow reaction)}$$

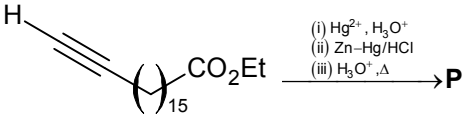
$$N_2O(g) + H_2(g) \xrightarrow{k_3} N_2(g) + H_2O(g) \quad \text{(fast reaction)}$$

The order of the reaction is _____.

Q.10 Complete reaction of acetaldehyde with excess formaldehyde, upon heating with conc. NaOH solution, gives **P** and **Q**. Compound **P** does not give Tollen's test, whereas **Q** on acidification gives positive Tollen's test. Treatment of **P** with excess cyclohexanone in the presence of catalytic amount of p-tolunesulfonic acid (PTSA) gives product **R**.
Sum of the number of methylene groups ($-CH_2-$) and oxygen atoms in **R** is _____.

Q.11 Among $V(CO)_6$, $Cr(CO)_5$, $Mn(CO)_5$, $Fe(CO)_5$, $[Co(CO)_3]^{3-}$, $[Cr(CO)_4]^{4-}$ and $Ir(CO)_3$, the total number of species isoelectronic with $Ni(CO)_4$ is _____.
[Given, atomic number: V = 23, Cr = 24, Mn = 25, Fe = 26, Co = 27, Ni = 28, Cu = 29, Ir = 77]

Q.12 In the following reaction sequence, major product **P** is formed.



Glycerol reacts completely with excess **P** in the presence of an acid catalyst to form **Q**. Reaction of **Q** with excess NaOH followed by the treatment with $CaCl_2$ yields Ca – soap **R**, quantitatively. Starting with one mole of **Q**, the amount of **R** produced in gram is _____.
[Given, atomic weight: H = 1, C = 12, N = 14, O = 16, Na = 23, Cl = 35, Ca = 40]

Q. 13 Among the following complexes, the total number of diamagnetic species is _____.
$[Mn(NH_3)_6]^{3+}$, $[MnCl_6]^{3-}$, $[FeF_6]^{3-}$, $[CoF_6]^{3-}$, $[Fe(NH_3)_6]^{3+}$ and $[Co(en)_3]^{3+}$
[Given, atomic number: Mn = 25, Fe = 26, Co = 27; en = $H_2NCH_2CH_2NH_2$]

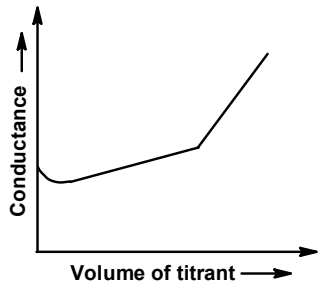
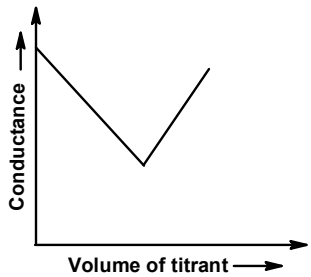
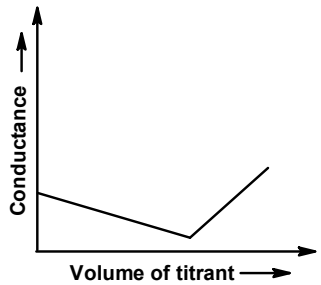
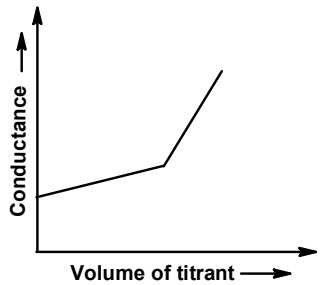
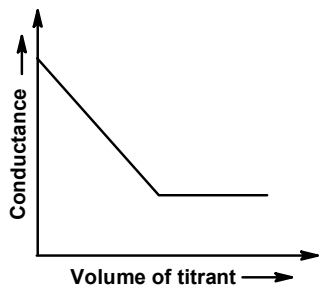
SECTION 4 (Maximum Marks: 12)

- This section contains **FOUR (04)** Matching List Sets.
- Each set has **ONE** Multiple Choice Question.
- Each set has **TWO** lists: **List-I** and **List-II**.
- **List-I** has **Four** entries (P), (Q), (R) and (S) and **List-II** has **Five** entries (1), (2), (3), (4) and (5).
- **FOUR** options are given in each Multiple Choice Question based on **List-I** and **List-II** and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +3 **ONLY** if the option corresponding to the correct combination is chosen;
  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
  Negative Marks : –1 In all other cases.

Q.14 In a conductometric titration, small volume of titrant of higher concentration is added stepwise to a larger volume of titrate of much lower concentration, and the conductance is measured after each addition.
The limiting ionic conductivity (Λ_0) values (in mS m^2 mol^{-1}) for different ions in aqueous solutions are given below:

Ions	Ag^+	K^+	Na^+	H^+	NO_3^-	Cl^-	SO_4^{2-}	OH^-	CH_3COO^-
(Λ_0)	6.2	7.4	5.0	35.0	7.2	7.6	16.0	19.9	4.1

For different combinations of titrates and titrants given in **List-I**, the graphs of 'conductance' versus 'volume of titrant' are given in **List-II**.
Match each entry **List-I** with the appropriate entry in **List-II** and choose the correct option.

List-I		List-II	
(P)	Titrate: KCl Titrant: $AgNO_3$	(1)	
(Q)	Titrate: $AgNO_3$ Titrant: KCl	(2)	
(R)	Titrate: NaOH Titrant: HCl	(3)	
(S)	Titrate: NaOH Titrant: CH_3COOH	(4)	
		(5)	

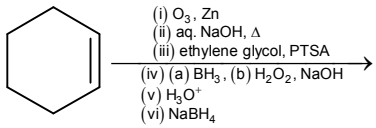
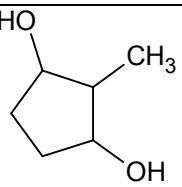
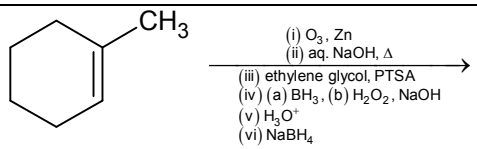
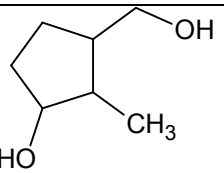
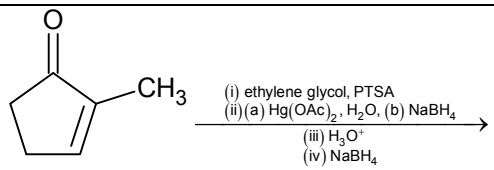
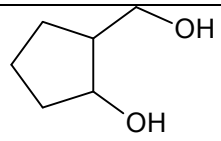
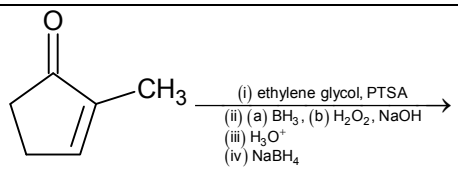
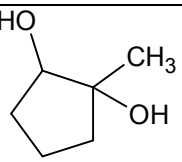
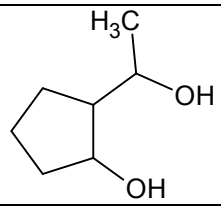
(A) (P) → (4), (Q) → (3), (R) → (2), (S) → (5)
(B) (P) → (2), (Q) → (4), (R) → (3), (S) → (1)
(C) (P) → (3), (Q) → (4), (R) → (2), (S) → (5)
(D) (P) → (4), (Q) → (3), (R) → (2), (S) → (1)

***Q.15** Based on **VSEPR** model, match the xenon compounds given in **List-I** with the corresponding geometries and the number of lone pairs on xenon given in **List-II** and choose the correct option.

List-I		List-II	
(P)	XeF_2	(1)	Trigonal bipyramidal and two lone pair of electrons
(Q)	XeF_4	(2)	Tetrahedral and one lone pair of electrons
(R)	XeO_3	(3)	Octahedral and two lone pair of electrons
(S)	XeO_3F_2	(4)	Trigonal bipyramidal and no lone pair of electrons
		(5)	Trigonal bipyramidal and three lone pair of electrons

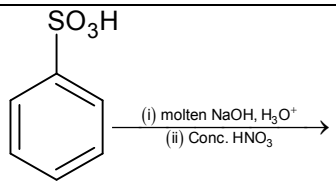
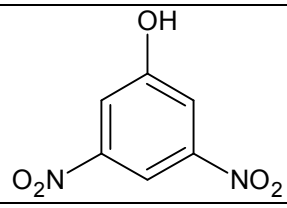
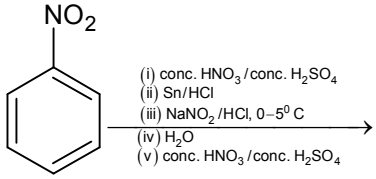
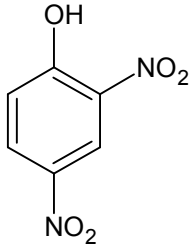
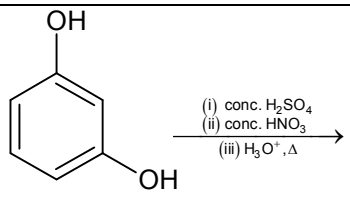
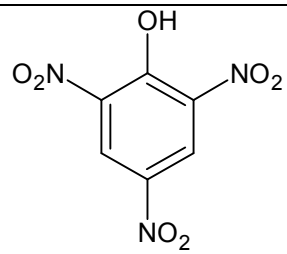
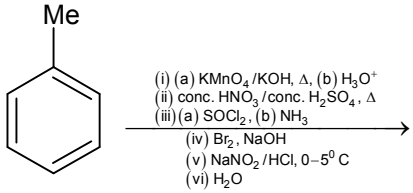
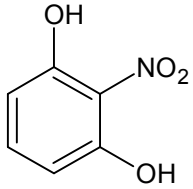
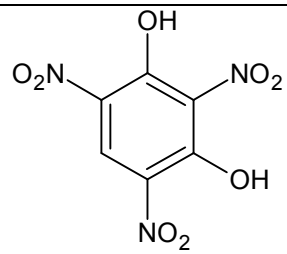
(A) (P) → (5), (Q) → (2), (R) → (3), (S) → (1)
(B) (P) → (5), (Q) → (3), (R) → (2), (S) → (4)
(C) (P) → (4), (Q) → (3), (R) → (2), (S) → (1)
(D) (P) → (4), (Q) → (2), (R) → (5), (S) → (3)

Q.16 **List-I** contains various reaction sequence and **List-II** contains the possible products. Match each entry in **List-I** with the appropriate entry in **List-II** and choose the correct option.

List-I		List-II	
(P)		(1)	
(Q)		(2)	
(R)		(3)	
(S)		(4)	
		(5)	

(A) (P) → (3), (Q) → (5), (R) → (4), (S) → (1)
(B) (P) → (3), (Q) → (2), (R) → (4), (S) → (1)
(C) (P) → (3), (Q) → (5), (R) → (1), (S) → (4)
(D) (P) → (5), (Q) → (2), (R) → (4), (S) → (1)

Q.17 **List-I** contains various reaction sequence and **List-II** contains different phenolic compounds. Match each entry in **List-I** with the appropriate entry in **List-II** and choose the correct option.

List-I		List-II	
(P)		(1)	
(Q)		(2)	
(R)		(3)	
(S)		(4)	
		(5)	

(A) (P) → (2), (Q) → (3), (R) → (4), (S) → (5)
(B) (P) → (2), (Q) → (3), (R) → (5), (S) → (1)
(C) (P) → (3), (Q) → (5), (R) → (4), (S) → (1)
(D) (P) → (3), (Q) → (2), (R) → (5), (S) → (4)

FIITJEE JEE (ADVANCED) – 2024
(PAPER-1)

ANSWER KEY

Mathematics

1.	**B**	2.	**C**	3.	**B**	4.	**A**
5.	**A, C, D**	6.	**B, C, D**	7.	**A, B, C, D**	8.	**8**
9.	**20**	10.	**16**	11.	**665**	12.	**5**
13.	**42**	14.	**C**	15.	**C**	16.	**C**
17.	**C**						

Physics

1.	**A**	2.	**A**	3.	**B**	4.	**C**
5.	**A, B, C**	6.	**A, C, D**	7.	**A, B**	8.	**25000**
9.	**12**	10.	**8**	11.	**200**	12.	**3**
13.	**18**	14.	**B**	15.	**C**	16.	**A**
17.	**A**						

Chemistry

1.	**D**	2.	**A**	3.	**B**	4.	**C**
5.	**A, B, C**	6.	**A, B, D**	7.	**A, D**	8.	**8120**
9.	**3**	10.	**18**	11.	**1**	12.	**909**
13.	**1**	14.	**C**	15.	**B**	16.	**A**
17.	**C**						

HINTS AND SOLUTIONS
Mathematics

1. $\lim\limits_{t\to x}\dfrac{f(x)t^{10}-x^{10}f(t)}{t^9-x^9}=1$ (Using L.H. Rule)

$\lim\limits_{t\to x}\dfrac{f(x)10t^9-x^{10}\cdot f'(t)}{9t^8}=1$

$\Rightarrow \dfrac{f(x)\times 10x^9-x^{10}\cdot f'(x)}{9x^8}=1$

$\Rightarrow x^2 f'(x) - 10x\, f(x) = -9 \Rightarrow f'(x)-\dfrac{10}{x}f(x)=-\dfrac{9}{x^2}$

Integrating factor $e^{-\int\frac{10}{x}dx}=\dfrac{1}{x^{10}}$

$\Rightarrow \dfrac{f(x)}{x^{10}}=\int-\dfrac{9}{x^2}\times\dfrac{1}{x^{10}}dx \Rightarrow \dfrac{f(x)}{x^{10}}=\dfrac{9}{11x^{11}}+c$

$\because$ f(1) = 2 $\Rightarrow 2=\dfrac{9}{11}+c \Rightarrow c=\dfrac{13}{11}$

$\Rightarrow f(x)=\dfrac{9}{11x}+\dfrac{13}{11}x^{10}$

2. Consider A : student's answer is correct
E_1 : He knows the answer.
E_2 : He guesses the answer.

$P(A/E_2)=\dfrac{1}{2}$, $P(A/E_1)=1$

Let $P(E_1)=1-x \Rightarrow P(E_2)=x$

Given, $P(E_2/A)=\dfrac{1}{6}=\dfrac{P(E_2)P(A/E_2)}{P(E_1)P(A/E_1)+P(E_2)P(A/E_2)}$

$\Rightarrow \dfrac{1}{6}=\dfrac{x\cdot\frac{1}{2}}{(1-x)\times 1+(x)\frac{1}{2}}=\dfrac{x}{2-x}$

$\Rightarrow x=\dfrac{2}{7}$

$\Rightarrow$ P(req.) = $\dfrac{5}{7}$

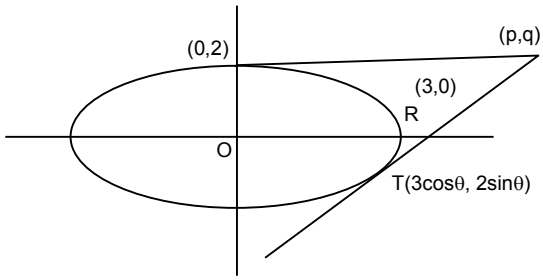
3. $\cot x=\dfrac{-5}{\sqrt{11}}, x\in\left(\dfrac{\pi}{2},\pi\right)\Rightarrow\cos x=\dfrac{-5}{6}$

$\Rightarrow \sin\dfrac{x}{2}=\dfrac{\sqrt{11}}{2\sqrt{3}}$ and $\cos\dfrac{x}{2}=\dfrac{1}{2\sqrt{3}}$

Now, $\left(\sin\dfrac{11x}{2}\right)(\sin 6x-\cos 6x)+\left(\cos\dfrac{11x}{2}\right)(\sin 6x+\cos 6x)$

$=\cos\dfrac{x}{2}+\sin\dfrac{x}{2}=\dfrac{\sqrt{11}+1}{2\sqrt{3}}$

4. Let T be $(3\cos\theta, 2\sin\theta)$



$\Rightarrow$ Ar(ΔORT) = $\frac{1}{2}\times 3\times|2\sin\theta| = \frac{3}{2}$

$\Rightarrow$ $|\sin\theta| = \frac{1}{2}$

$\Rightarrow$ $T\left(\frac{3\sqrt{3}}{2}, -1\right)$

Also $q = 2 \Rightarrow S(p, 2)$

T : $\frac{\sqrt{3}x}{6} - \frac{y}{4} = 1$

S(p, 2) lies on it $\Rightarrow$ $\frac{\sqrt{3}p}{6} - \frac{2}{4} = 1$

$p = \frac{3}{2}\times\frac{6}{\sqrt{3}}$

$p = 3\sqrt{3}$

$\Rightarrow$ $p = 3\sqrt{3}$, $q = 2$

5. (A) $T_1 \cup T_2 = \alpha + \beta\sqrt{2}$ where $\alpha, \beta \in N$

So, $Z \cup T_1 \cup T_2 \subset S$

(B) For some very large N

T_1 lies in $\left(0, \frac{1}{2024}\right)$

(C) For some large N $T_2 > 2024$

(D) $\cos\left(\pi\left(a+b\sqrt{2}\right)\right) + i\sin\left(\pi\left(a+b\sqrt{2}\right)\right) \in Z$ only if b = 0.

6. $ax^2 + 2bxy + cy^2 > 0$

a, c > 0 , $b^2 < ac$

(A) $\left(2, \frac{7}{2}, 6\right) \in$ S is incorrect

(B) $\left(3, b, \frac{1}{12}, 6\right) \in S$ then |2b| < 1

$b^2 < \frac{1}{4} \Rightarrow |2b| < 1$

(C) ax + by = 1

bx + cy = -1

$\Delta = ac - b^2 \neq 0$

unique solution.

(D) (a + 1)x + by = 0

bx + (c + 1)y = 0

$\Delta = \begin{vmatrix} a+1 & b \\ b & (c+1) \end{vmatrix}$

$= (a + 1)(c + 1) - b^2$

$\Rightarrow at - b^2 + a + c + 1 > 0$

hence unique solution.

7. Here, locus of $S \equiv 6x + 8z = 105$ i.e. a plane and locus of $T \equiv 6x + 8z = 5$ i.e. a plane.

Also, distance between these two planes = 10 units.

8. $a = 3\sqrt{2}, b = \frac{1}{5^{1/6}\sqrt{6}}$

$3x + 2y = \log_{\sqrt{18}} 18^{5/4} = \frac{5}{2}$

$2x - y = -\log_{(5^{1/6}\sqrt{6})} \sqrt{1080} = -3$

$\Rightarrow$ $6x + 4y = 5 \rightarrow (1)$

$2x - y = -3 \rightarrow (2)$

Now, subtract (2) from (1)

$4x + 5y = 8$

9. $4x^2 + 3ax + 2b = 4(x^2 + 3)$

$\Rightarrow$ $a = 0$; $b = 6$

$f(x) = x^4 + 6x^2 + c$

$f(1) = 7 + c = -9 \Rightarrow c = -16$

$\Rightarrow f(x) = x^4 + 6x^2 - 16 = (x^2 + 8)(x^2 - 2)$

$\Rightarrow$ $x = \pm 2\sqrt{2}i, \pm\sqrt{2}$

$\Rightarrow$ $|\alpha_1|^2 + |\alpha_2|^2 + |\alpha_3|^2 + |\alpha_4|^2 = 20.$

10. $|A| = (e - d) + c(b - a)$

Now for $|A| = 1$ or -1

Case : 1

$e - d = 1$ or -1 $c(b - a) = 0$ ($c = 0$, $b - a = 1$ or $-1 \rightarrow (2)$

($c = 1$, $b - a = 0 \rightarrow (2)$

($c = 0$, $b - a = 0 \rightarrow (2)$

$2 \times (2 + 2 + 2) = 12$

Case : 2

$e - d = 0$ $c(b - a) = 1$ or -1

$\Rightarrow$ $e = d$ $c = 1$, $b - a = 1$ or -1

$2 \times 2 = 4$

Total possibility = 12 + 4 = 16

11. **Case-1:** S_2 is in team X and S_1 is in team Y, ${}^7C_1 \times {}^6C_2 \times {}^4C_4 = 105$

Case-2: S_2 is in team X but S_1 is not in team Y, ${}^7C_2 \times {}^5C_2 \times {}^3C_3 = 140$

Case-3: S_2 is not in team X but S_1 is in team Y, ${}^7C_2 \times {}^5C_2 \times {}^3C_3 = 210$

Case-4: S_2 is not in team X and S_1 is not in team Y, ${}^7C_2 \times {}^5C_3 \times {}^2C_2 = 210$

$\therefore$ Total ways to form such teams = 105 + 140 + 210 + 210 = 665

12. $(\overrightarrow{OP} \times \overrightarrow{OQ}) \cdot \overrightarrow{OR} = 0$

$$\Rightarrow \begin{vmatrix} \frac{\alpha - 1}{\alpha} & 1 & 1 \\ 1 & \frac{\beta - 1}{\beta} & 1 \\ 1 & 1 & \frac{1}{2} \end{vmatrix} = 0 \Rightarrow \alpha + \beta = -1$$

Also, $3\alpha + 3\beta - 2 + l = 0$

$\Rightarrow l = 5$

13. Let $P(x_i) = mx_i + c,\ i = 0, 1, 2, 3, 4$

$$\sum_{i=0}^{4} P(x_i) = 1 \Rightarrow 2m + c = \frac{1}{5}$$

$$\sum_{i=0}^{4} x_i P(x_i) = \frac{5}{2} \Rightarrow 3m + c = \frac{1}{4} \Rightarrow m = \frac{1}{20}, c = \frac{1}{10}$$

$$\Rightarrow \alpha = \sum_{i=0}^{4} x_i^2 P(x_i) - \left(\sum_{i=0}^{4} x_i P(x_i)\right)^2 = 8 - \frac{25}{4} = \frac{7}{4} \Rightarrow 24\alpha = 42$$

14. $\alpha + \beta = -1$

(P) Only possible when each row and each column of M is made of 1, α, β. Number of ways for first row = $3 \times 2 \times 1 = 6$.
Number of ways for second row = $2 \times 1 \times 1 = 2$
Number of ways for third row = $1 \times 1 \times 1 = 1$
Number of such matrices = $6 \times 2 \times 1 = 12$

(Q) Each column should be made of 1, α, β.
Since matrix is symmetric, after making column 1, other entries would be fixed by default.
Number of ways = 6

(R) Let $a_{21} = a$, $a_{31} = b$, $a_{32} = c$, where $\{a, b, c\} \in T$

$$M = \begin{bmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{bmatrix}$$

Then $-ay - bz = -a \Rightarrow ay + bz = a$
$ax - cz = 0 \Rightarrow ax = cz$ and $bx + cy = c$
If $a = b = c$, then $y + z = 1$ $x = z$, $x + y = 1$
This has infinite solutions.

(S) Each row is made of $\{1, \alpha, \beta\}$
Also, $1 + \alpha + \beta = 0$
$C_1 \to C_1 + C_2 + C_3$ makes every element of column 1 as '0'. Hence determinant is 0.

15. $$OA_1 = \frac{|2 \times 0 - \alpha|}{\sqrt{5}} = r$$

$\Rightarrow \alpha = r\sqrt{5}$ given $\alpha + r = 5 + \sqrt{5}$

$\Rightarrow r(1 + \sqrt{5}) = \sqrt{5}(1 + \sqrt{5})$

$\Rightarrow r = \sqrt{5}$, hence $\alpha = 5$

Now, $A_1B_1 = 2r = 2\sqrt{5}$
$A_1(2, 4)$ and $B_1(-2, 6)$ from elimination.

16. $$\frac{x+11}{1} = \frac{y+21}{2} = \frac{z+29}{3} = \alpha$$

$$\frac{x+16}{3} = \frac{y+11}{2} = \frac{z+4}{4} = \beta$$

$\Rightarrow \alpha - 11 = 3\beta - 16 \Rightarrow \alpha - 3\beta = -5$
$2\alpha - 21 = 2\beta - 11 \Rightarrow \alpha - \beta = 5$
$\alpha = 10, \beta = 5$

So, $\gamma = 1$

$$\hat{n} = \begin{vmatrix} \hat{i} & \hat{j} & k \\ 1 & 2 & 3 \\ 3 & 2 & 1 \end{vmatrix} = -4\hat{i} + 8\hat{j} - 4\hat{k}$$

$\hat{n} = \left(\frac{1}{\sqrt{6}}i - \frac{2}{\sqrt{6}}\hat{j} + \frac{1}{\sqrt{6}}\hat{k}\right)$

$R_1(-1, -1, 1)$

$\overrightarrow{OR} = \left(-\hat{i} - \hat{j} + \hat{k}\right)$

$\overrightarrow{OR} \cdot \hat{n} = \sqrt{\frac{2}{3}}$

17. We have

$$\begin{cases} g(x) + g\left(\frac{1}{2} - x\right) = 0 ; & x \in R - \left[0, \frac{1}{2}\right] \\ g(x) + g\left(\frac{1}{2} - x\right) = 1; & x \in \left[0, \frac{1}{2}\right] \end{cases}$$

$$\Rightarrow \quad h(x) = \begin{cases} af(x) + (d-c)g(x) + cx & ; \quad x < 0 \\ af(x) + b + (d-c)g(x) + cx & ; \quad x \in \left[0, \frac{1}{2}\right] \\ af(x) + (d-c)g(x) + cx & ; \quad x > \frac{1}{2} \end{cases}$$

for a = c = d = 0 and b = 1 ; $h(x) = \begin{cases} 0 & ; \quad x < 0 \\ 1 & ; \quad x \in \left[0, \frac{1}{2}\right] \\ 0 & ; \quad x > \frac{1}{2} \end{cases}$

$\Rightarrow$ h(x) has range {0, 1}

for a = 1, b = c = d = 0 $\Rightarrow$ h(x) = f(x) $\forall x \in R$

$\Rightarrow$ h(x) is differentiable.

for c = 1, a = b = d = 0 $\Rightarrow$ h = x – g(x) ; $\forall x \in R$

$$\Rightarrow h(x) = \begin{cases} x & ; \quad x < 0 \\ 3x - 1 & ; \quad x \in \left[0, \frac{1}{2}\right] \\ x & ; \quad x > \frac{1}{2} \end{cases}$$

$\Rightarrow$ h(x) is onto

for d = 1 ; a = b = c = 0 $\Rightarrow$ h(x) = g(x) $\forall x \in R$

$\Rightarrow$ h(x) has range $\in$ [0, 1]

Physics

1. $e^{\alpha}\varepsilon_0{}^{\beta}h^{\gamma}c^{\delta} = k$

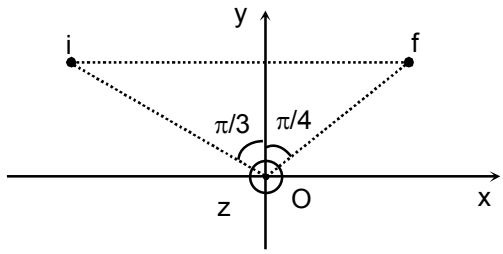
$e^{\alpha}h^{\gamma}c^{\delta} = \varepsilon_0^n$

$\left[A^{\alpha}T^{\alpha}\right]\left[ML^2T^{-1}\right]^{\gamma}\left[LT^{-1}\right]^{\delta} = \left[A^{2n}T^{4n}M^{-n}L^{-3n}\right]$

So, $\alpha = 2n$, $\gamma = -n$

2. $\oint \vec{B}\cdot\vec{d\ell} = \int_0^i \vec{B}\cdot\vec{d\ell} + \int_i^f \vec{B}\cdot\vec{d\ell} + \int_f^0 \vec{B}\cdot\vec{d\ell} = \mu_0 \frac{I}{2\pi}\frac{7\pi}{12}$

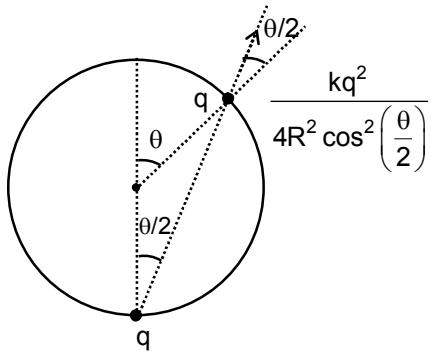
So, $\int_i^f \vec{B}\cdot\vec{d\ell} = \frac{7\mu_0 I}{24}$



3. So, a_t = tangential acceleration of bead = $\frac{kq^2\sin\left(\frac{\theta}{2}\right)}{4mR^2\cos^2\left(\frac{\theta}{2}\right)}$

So, $a_t \approx \frac{kq^2}{4mR^2}\left(\frac{\theta}{2}\right) = \frac{q^2x}{32\pi\varepsilon_0 R^3 m}$

So, $\omega^2 = \frac{q^2}{32\pi\varepsilon_0 mR^3}$



4. $a = -\frac{20x-10}{5} = -4\left(x-\frac{1}{2}\right)$

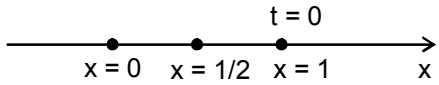
So, $x = \frac{1}{2}+\frac{1}{2}\cos(2t)$ m

At t = $\frac{\pi}{4}$ sec

x = 0.5 m

$v = -\frac{1}{2}\times 2 = -1$ m/s

So, momentum P = –5 kg-m/s



5. $\frac{mv^2}{r} = kr$

$v = \sqrt{\frac{k}{m}}r$ and L = $n\hbar = mvr$

So, $r^2 = \frac{n^2\hbar^2}{m^2v^2} = n\hbar\frac{n\hbar}{m^2v^2} = n\hbar\frac{mvr}{m^2v^2}$

So, $r^2 = n\hbar\frac{r}{mv} = n\hbar\frac{1}{mv}v\sqrt{\frac{m}{k}}$

So, $r^2 = n\hbar\sqrt{\frac{1}{mk}}$ Option (A)

$v^2 = \frac{k}{m}r^2 = \frac{k}{m}n\hbar\frac{1}{\sqrt{mk}} = n\hbar\sqrt{\frac{k}{m^3}}$ Option(B)

$\frac{L}{mr^2} = \frac{v}{r} = \sqrt{\frac{k}{m}}$ Option(C)

$E = U + K = \frac{kr^2}{2} + \frac{kr^2}{2} = kr^2$

$E = kn\hbar\sqrt{\frac{1}{mk}} = n\hbar\sqrt{\frac{k}{m}}$

6. $L = m\frac{\lambda_1}{2}$ and $2L = n\frac{\lambda_2}{2}$

So, $\frac{1}{2} = \frac{m}{n}\frac{v_1}{v_2} = 2\frac{m}{n}$

So, $\frac{m}{n} = \frac{1}{4} \Rightarrow \lambda_1 = 2L$

$\nu_0 = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$

So, there are total 6 nodes

So, $L = (2m+1)\frac{\lambda_1}{4}$

$2L = (2n+1)\frac{\lambda_2}{4}$

So, $\frac{2m+1}{2n+1} = \frac{1}{4}$

So, it is not possible

7. $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$\frac{1}{f} = (1.6-1)\left(\frac{1}{9} - \frac{1}{\infty}\right)$

$f_1 = \frac{9}{0.6} = 15\,cm$

$\frac{1}{f_2} = (n-1)\left(\frac{1}{\infty} - \frac{1}{9}\right)$

$f_2 = \frac{-9}{(n-1)}$

$P = 2P_1 + 2P_2$

$\frac{1}{f_{eq}} = \frac{2}{15} - \frac{2(n-1)}{9}$

$\frac{1}{f_{eq}} = \frac{18 - 2(n-1)\times 15}{15\times 9}$

$f_{eq} = \frac{15\times 9}{18 - 2(n-1)\times 15}$

Image V = R = $2f_e = \dfrac{2\times15\times9}{18-2(n-1)\times15}$

8. dQ = mSdT

$\int dQ = \int 1.KTdT$

$\Delta Q = K\int_{200}^{300} TdT$

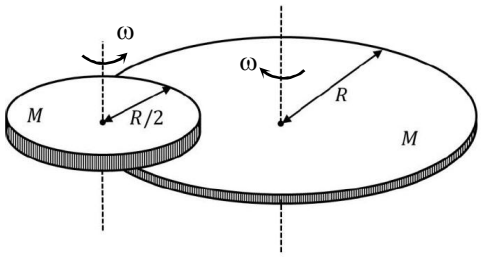
$\Delta Q = \dfrac{K}{2}\left[T^2\right]_{200}^{300}$

$\Delta Q = \dfrac{K}{2}\times(9-4)\times10^4$

$\Delta Q = 25000\ K$

n = 25000

9. Angular momentum about fixed axis remains constant.



$\dfrac{1}{2}m\dfrac{R^2}{4}\omega - mR^2\omega' - \dfrac{1}{2}mR^2\omega' = 0$

$\dfrac{mR^2\omega}{8} = mR^2\omega' + \dfrac{1}{2}mR^2\omega'$

$\dfrac{\omega}{8} = \dfrac{3}{2}\omega'$

$\dfrac{\omega}{\omega'} = 12$

10. $I \propto 1/r^2$

$I_A/I_{B=} = \dfrac{I_A}{I_B} = \left(\dfrac{r_B}{n}\right)^2 = 2$

$I'_B = \dfrac{I_B}{2}\cos45^0$

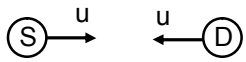
$I'_B = I_B/4$

$I_{B'} = \dfrac{I_B}{4}$

$\dfrac{I_A}{I'_B} = \dfrac{I_A}{\frac{I_B}{4}} = 8$

11. $f_0 = 240$ Hz

$f_1 = \left(\dfrac{v+u}{v-u}\right)f_0$



$288 = \left(\dfrac{v+u}{v-u}\right)240 \Rightarrow \dfrac{v+u}{v-u} = \dfrac{6}{5}$

v = 11 u ……. (i)

$f_2 = \left(\dfrac{v-u}{v+u}\right)f_0$

$$f_2 = \frac{5}{6} \times 240 = 200\,\text{Hz}.$$

12. For tank-1

$$-A\frac{dy}{dt} = a\sqrt{2gy} \qquad \ldots(i)$$

For tank-2

$$-A\frac{dy}{dt} = a\sqrt{2g(y+H)} \qquad \ldots(ii)$$

From -1

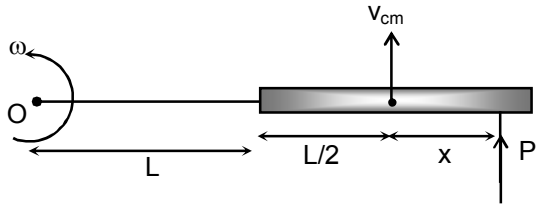
$$-\int_h^0 \frac{dy}{\sqrt{y}} = a\sqrt{2g}\int_0^{t_1} dt \Rightarrow t_1 = \frac{2A}{a}\frac{\sqrt{h}}{\sqrt{2g}}$$

From-2

$$-\int_h^0 \frac{dy}{\sqrt{y+H}} = a\sqrt{2g}\int_0^{t_2} dt \Rightarrow t_2 = \frac{2A\sqrt{h+H}}{a\sqrt{2g}} - \frac{2A\sqrt{H}}{a\sqrt{2g}}$$

$$\frac{t_1}{t_2} = 3$$

13. Conservation of angular momentum about O.



$$P\left(L + \frac{L}{2} + x\right) = \left[\frac{mL^2}{12} + m\left(\frac{3L}{2}\right)^2\right]\omega$$

$$P\left(\frac{3L}{2} + x\right) = \frac{7}{3}mL^2\omega \qquad \ldots(i)$$

$$v_{cm} = \frac{3}{2}\omega L \qquad \ldots(ii)$$

Conservation of linear momentum

$P = mv_{cm}$ …(iii)

$$\frac{P}{m} = \frac{3}{2}\left[\frac{3}{7}\left\{P\left(\frac{3L}{2} + x\right)\right\}\frac{1}{mL}\right]$$

$$14L = \frac{9}{2}(3L + 2x)$$

$\Rightarrow L = 18\,x$

$x = L/18$

14. (P) JK $\Rightarrow \Delta W_{JK} = P\Delta V = nR\Delta T = 1.R(3T_0 - T_0) = 2RT_0$

LM $\Rightarrow \Delta W_{LM} = P\Delta V = nR\Delta T = 1.R(T_0 - 3T_0) = -2RT_0$

$$KL \Rightarrow \Delta W_{KL} = nRT\ell n\left(\frac{P_i}{P_f}\right) = R3T_0\ell n\left(\frac{P_0}{2P_0}\right) = -3RT_0\ell n2$$

$$MJ \Rightarrow \Delta W_{MJ} = nRT\ell n\left(\frac{P_i}{P_f}\right) = RT_0\ell n\left(\frac{2P_0}{P_0}\right) = RT_0\ell n2$$

Total work done

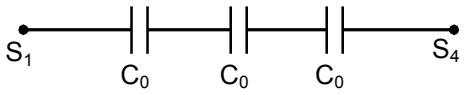
$\Delta W = -2RT_0\ell n2$

(Q) $$\Delta U_{JK} = nC_V\Delta T = 1\cdot\frac{3}{2}R(3T_0 - T_0) = 3RT_0$$

(R) $\Delta Q_{KL} = \Delta W_{KL} + \Delta U_{KL} = \Delta W_{KL} + 0 = -3RT_0 \ell n2$

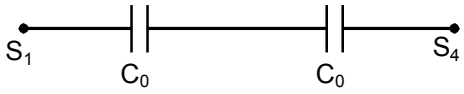
(S) $\Delta U_{MJ} = nC_V \Delta T = 0$

15. (P)



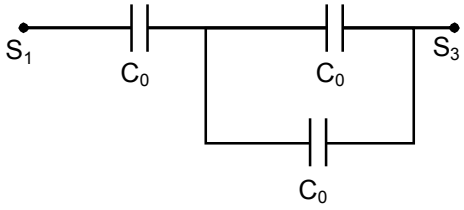
$C_{S_1S_4} = \frac{C_0}{3}$

(Q)



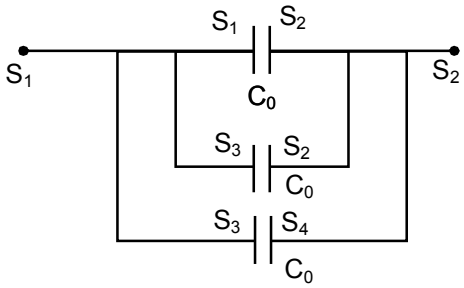
$C_{S_1S_4} = \frac{C_0}{2}$

(R)



$C_{S_1S_3} = \frac{2}{3}C_0$

(S)



$C_{S_1S_2} = 3C_0$

16. (P) $\sin\theta_0 = 2\sin\phi_0$ …(i)

$\theta_0 - \phi_0 + \theta_0 - \phi_0 + 180 - 2\phi_0 = \alpha$ …(ii)

When, $\alpha = 180$

$2\theta_0 - 4\phi_0 = 0$

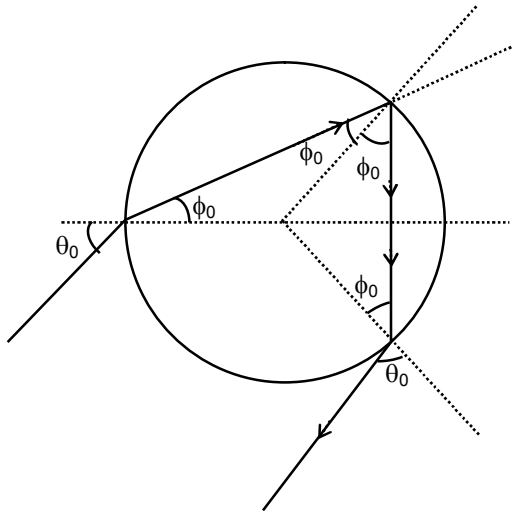
$\theta_0 = 2\phi_0$

$\sin 2\phi_0 = 2\sin\phi_0$

$2\sin\phi_0 \cos\phi_0 = 2\sin\phi_0$

$\cos\phi_0 = 1$

$\phi_0 = 0$, hence $\theta_0 = 0$



(Q) $\sin\theta_0 = \sqrt{3}\sin\phi_0$

$\cos\phi_0 = \frac{\sqrt{3}}{2}$

$\phi_0 = 30°$

$\theta_0 = 60°$

Hence, possible value of θ_0 is zero and 60°

(R) $\phi_0 = 0$ and 30°

(S) $\sin\theta_0 = \sqrt{2}\sin\phi_0$ (given $\theta_0 = 45°$)

$\sin\phi_0 = \frac{1}{2}$, $\phi_0 = 30°$

Hence, $\alpha = 180 + 2\theta_0 - 4\phi_0 = 180° + 90° - 120° = 150°$

17. (P) At t = 0, $I_1 = 0$

(Q) At $t \to \infty$, $I_2 = \frac{20}{5} = 4$ amp

(R) $\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{25\times10^{-3}\times10\times10^{-6}}}$

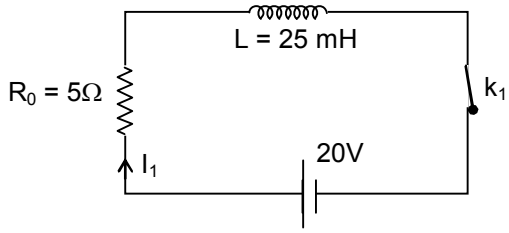
$= \frac{1}{5\times10^{-4}} = 2\times10^{3} = 2$ kilo-rad/sec

(S) $\frac{1}{2}LI_2^2 = \frac{1}{2}CV_0^2$

$V_0 = \left(\sqrt{\frac{L}{C}}\right)I_2$

$= \left(\sqrt{\frac{25\times10^{-3}}{10\times10^{-6}}}\right)4 = 5\times10\times4$

V_0 = 200 volts



Chemistry

1. $2 \times V = \frac{10}{M_x} \times R \times T \quad ...(1)$

$4 \times V = \frac{80}{M_y} \times R \times T \quad ...(2)$

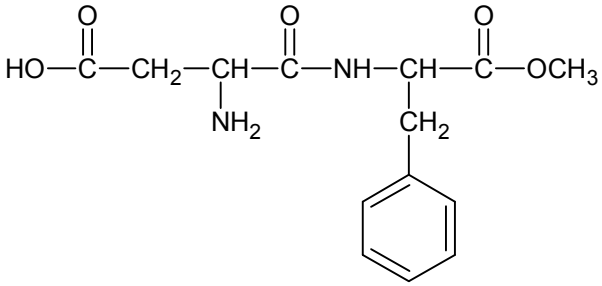
$2 = \frac{80 \times M_x}{M_y \times 10}$

$\frac{M_x}{M_y} = \frac{1}{4}$

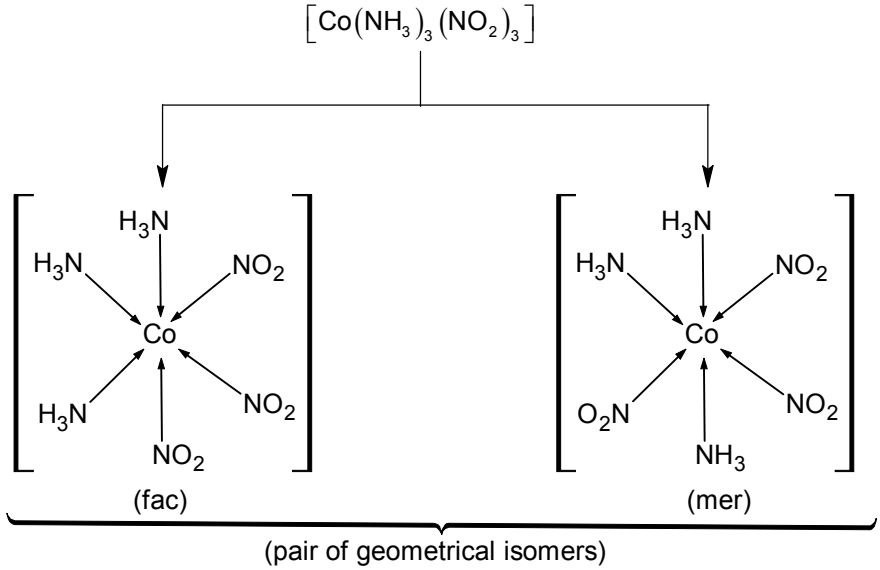
$\frac{(V_{rms})_x}{(V_{rms})_y} = \sqrt{\frac{4}{1}} = \frac{2}{1}$

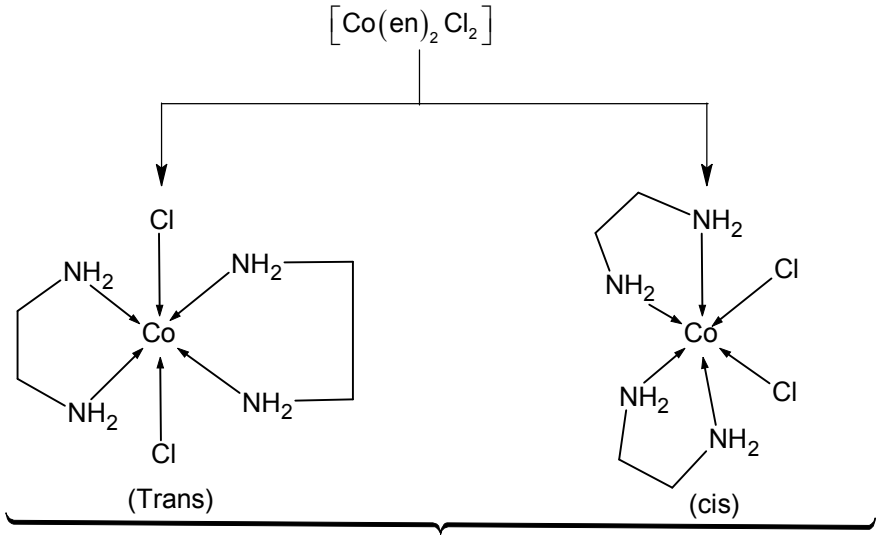
2. $3HNO_2 \longrightarrow 2NO + HNO_3 + H_2O$

3. The structure of ASPARTAME (an artificial structure) is:



4. **Set – I**





$[Co(NH_3)_5Cl]SO_4$ and $[Co(NH_3)_5(SO_4)]Cl$ are ionization isomers.

5. (A) uncertainty principle rules – out the existence of definite paths for an electron due to wave nature of electron.

(B) Energy $\propto -\frac{Z^2}{n^2}$

So, $E_{2s} < E_{\infty}$

(C) $E = -\frac{13.6 \times Z^2}{n^2}$

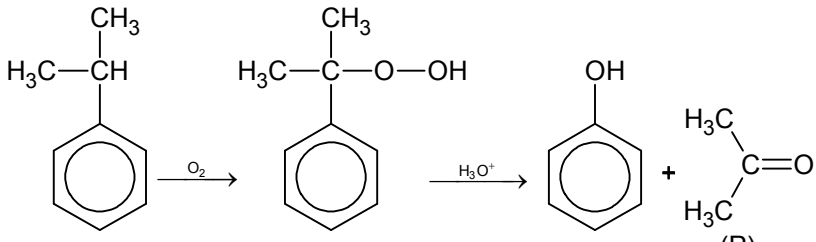
If n = 1

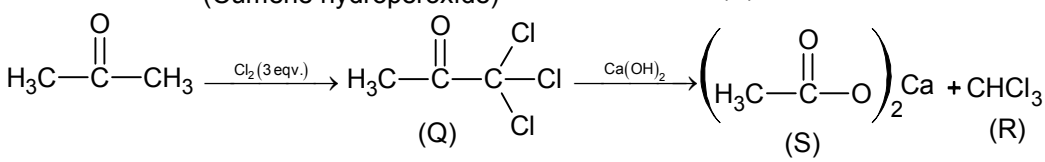
$E = -(13.6 \times Z^2)$ eV is most negative value and therefore most stable.

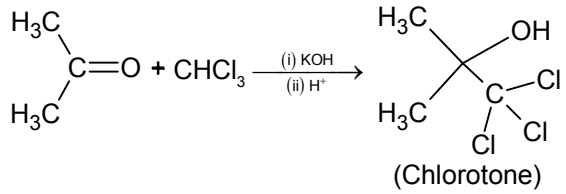
(D) $V = 2.186 \times 10^6 \times \frac{Z}{n}$

So, (D) is wrong.

6.

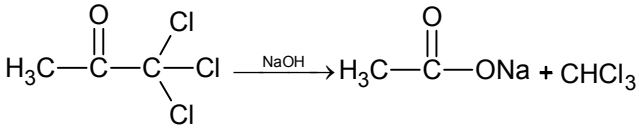




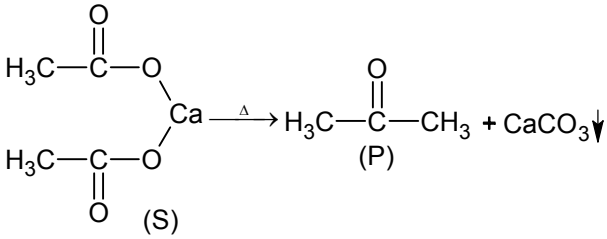
(A) 

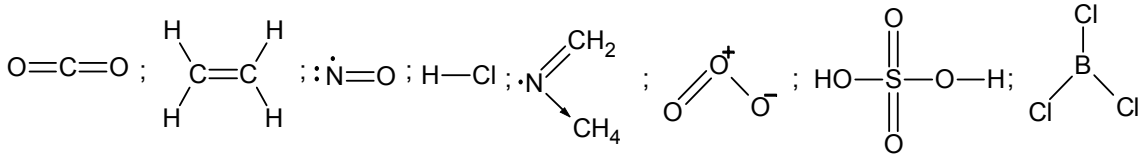
(Chlorotone)

(B) $CHCl_3 + \frac{1}{2}O_2(air) \longrightarrow \underset{(phosgene)}{COCl_2} + HCl$

(C) 

So, (C) is wrong.

(D) 

7. 

Only CO_2, C_2H_4, H – Cl and O_3 follows octet rule.

8. Since X → Y is an isothermal process (dT = 0).

So, $\Delta H_{x-y} = nC_{pm}dT = 6$

Y → Z is isochoric.

$\Delta U_{y\to z} = nC_{vm}dT = 5\times 12\times(415-335)$

$\Delta U_{y\to z}$ = 4800 J

$\Delta H_{y\to z} = \Delta U_{y\to z} + \Delta(PV)$

$= \Delta U_{y\to z} + nR\Delta T = 4800 + 5 \times 8.3 \times 80 = 8120$ J

9. Net reaction:

$2H_2(g) + 2NO(g) \longrightarrow N_2(g) + 2H_2O(g)$

(i) $2NO(g) \underset{K_{-1}}{\overset{K_1}{\rightleftharpoons}} N_2O_2(g)$ (fast)

$$\frac{k_1}{k_{-1}} = \frac{[N_2O_2]}{[NO]^2} \qquad ...(1)$$

(ii) $N_2O_2(g) + H_2(g) \xrightarrow{k_2} N_2O(g) + H_2O(g)$ (slow reaction)

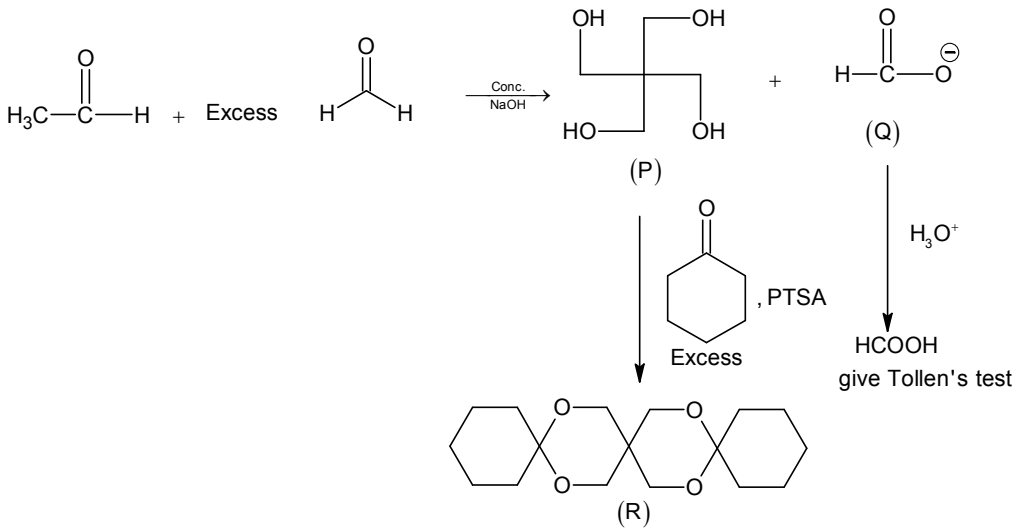
$r = k_2[N_2O_2][H_2]$

$= k_2 \times \frac{k_1}{k_{-1}}(NO)^2 \times (H_2)$

$r = \frac{k_2k_1}{k_{-1}}(NO)^2(H_2)$

So, order of reaction = 3.

10.



The number of CH_2 group in **R** = 14
The number of O atom in **R** = 4
Total = 18

11. $Ni(CO)_4 = 84$

$V(CO)_6 = 107$

$Cr(CO)_5 = 94$

$Mn(CO)_5 = 95$

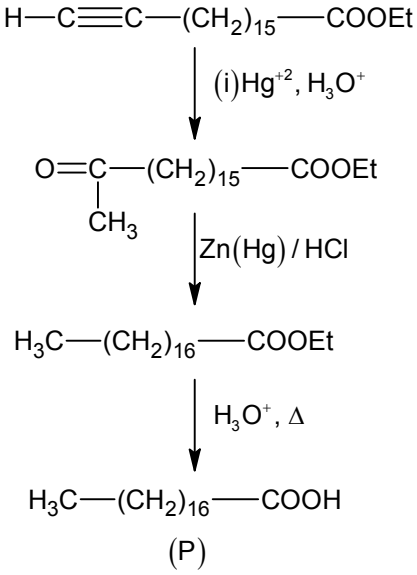
$Fe(CO)_5 = 96$

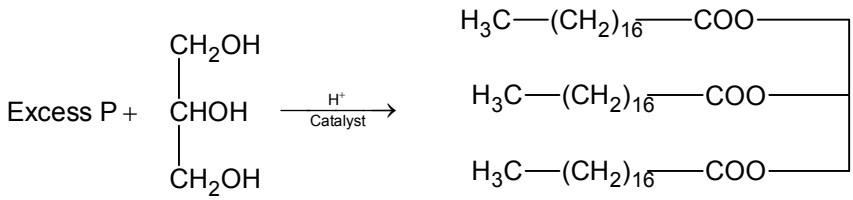
$\left[Co(CO)_3\right]^{-3} = 72$

$\left[Cr(CO)_4\right]^{-4} = 84$ $\left[\text{Isoelectronic with } Ni(CO)_4\right]$

$\left[Ir(CO)_3\right] = 119$

12.





(Q)

$$Q \xrightarrow[CaCl_2]{\text{excess NaOH}} \left[CH_3-(CH_2)_{16}-COO\right]_2 Ca$$

$$1\text{mole } Q \longrightarrow 3\text{mole } CH_3-(CH_2)_{16}-COO^-$$

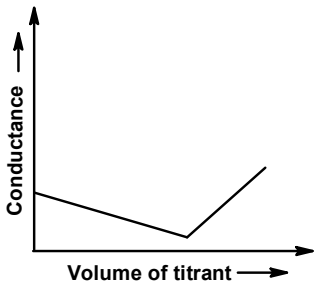
$$2CH_3-(CH_2)_{16}-COO^- + Ca^{+2} \longrightarrow \left(CH_3-(CH_2)_{16}COO\right)_2 Ca$$

(R)

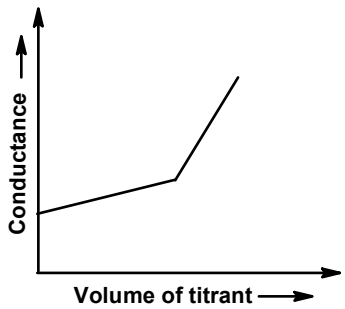
Mole of R = 1.5
Wt. of R = 1.5 × 606
Wt. of R = 909

13.
1. $\left[Mn(NH_3)_6\right]^{+3}$, n = 2, Paramagnetic
2. $\left[MnCl_6\right]^{-3}$, n = 4, Paramagnetic
3. $\left[FeF_6\right]^{-3}$, n = 5, Paramagnetic
4. $\left[CoF_6\right]^{-3}$, n = 4, Paramagnetic
5. $\left[Fe(NH_3)_6\right]^{+3}$, n = 1, Paramagnetic
6. $\left[Co(en)_3\right]^{+3}$, n = 0, Diamagnetic

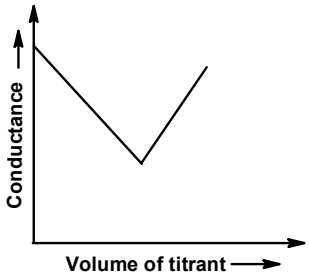
14. (P) → (3)



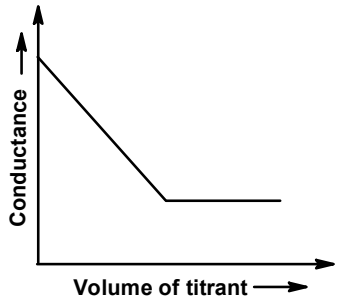
(Q) → (4)



(R) → (2)

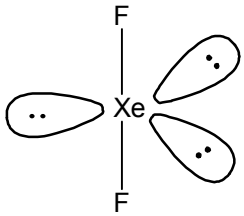


(S) → (5)

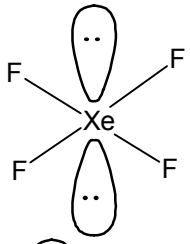


15.

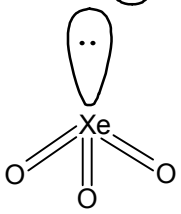
XeF_2



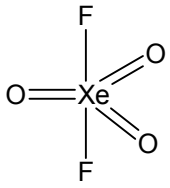
XeF_4



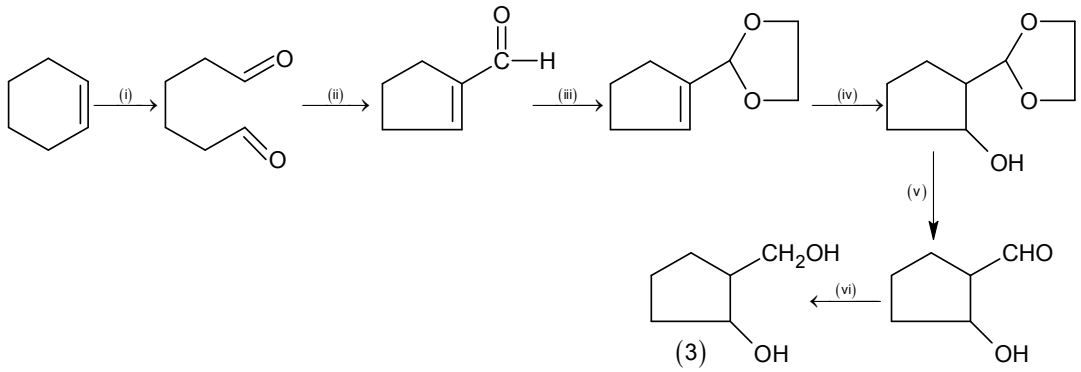
XeO_3



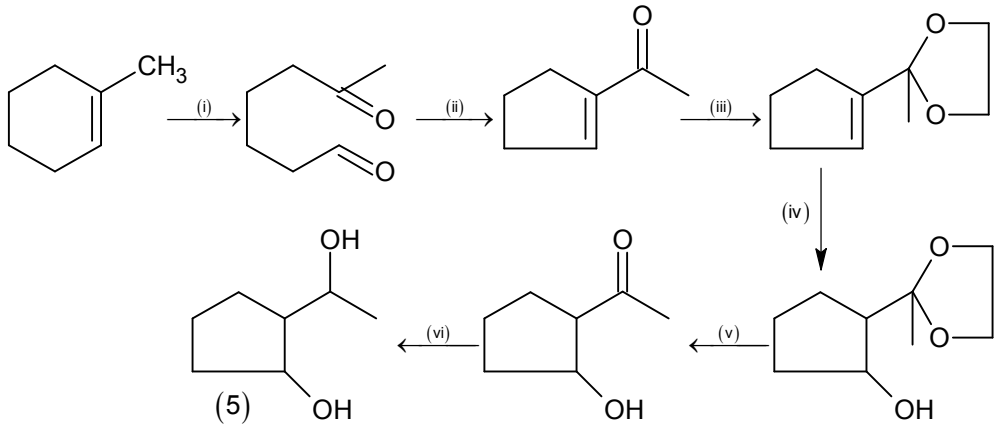
XeO_3F_2



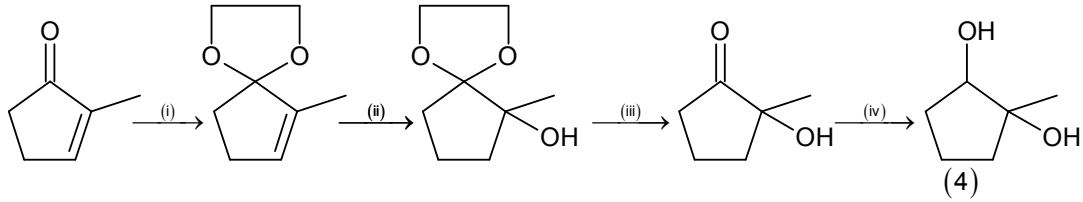
16. (P)


(i)
(ii)
(iii)
(iv)
(v)
(vi)
CH2OH
CHO
OH
(3)

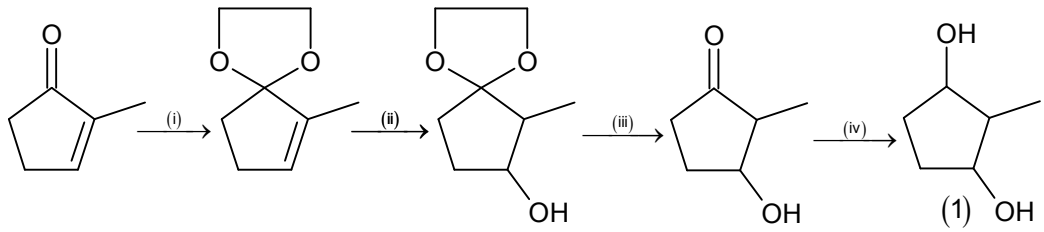
(Q)


CH3
(i)
(ii)
(iii)
(iv)
(v)
(vi)
OH
(5)

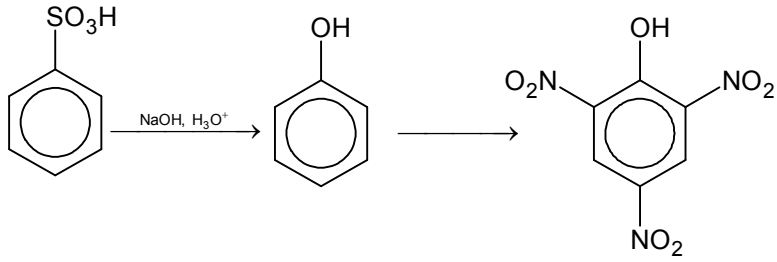
(R)


(i)
(ii)
(iii)
(iv)
OH
(4)

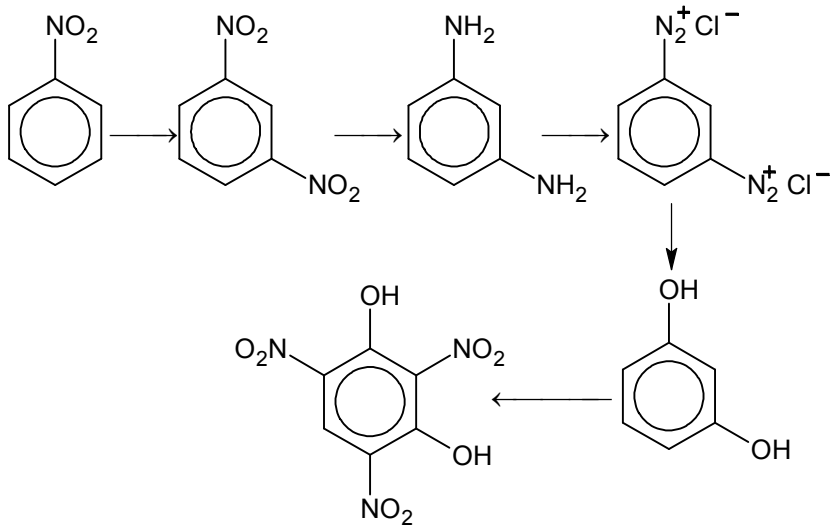
(S)


(i)
(ii)
(iii)
(iv)
OH
(1)

17. (P)

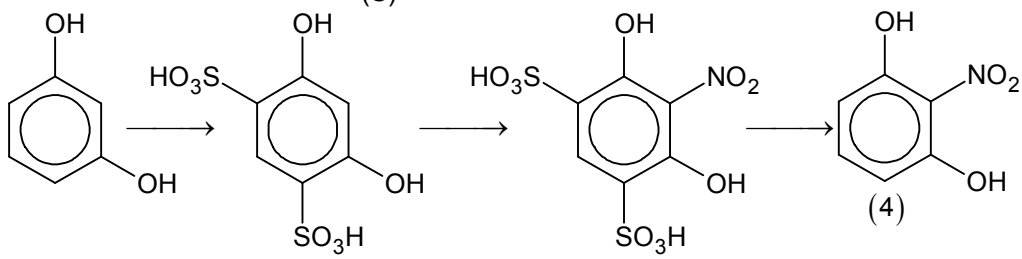

SO3H
NaOH, H3O+
OH
O2N
NO2
NO2

(Q)


NO2
NO2
NO2
NH2
NH2
N2+ Cl−
N2+ Cl−
OH
OH
O2N
NO2
OH
OH
NO2

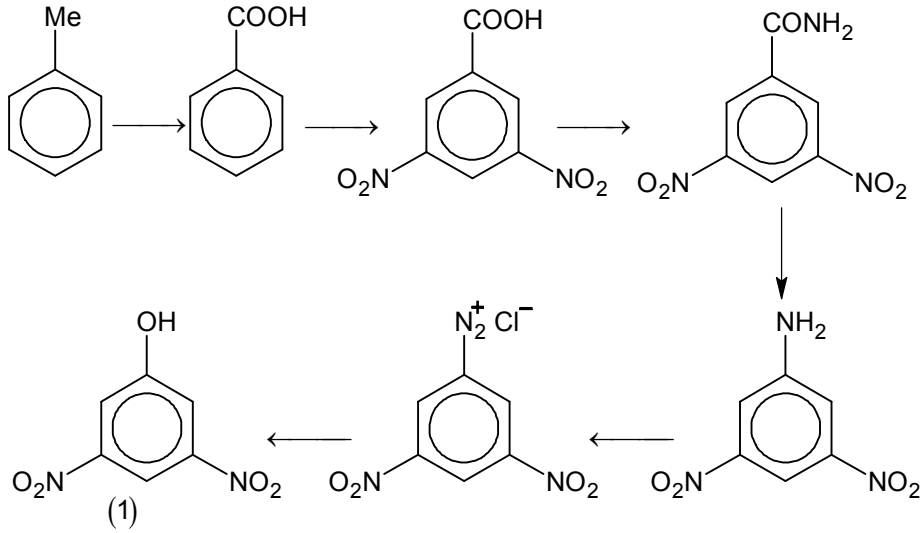
(S)

(R)


OH
OH
HO3S
OH
OH
OH
HO3S
NO2
OH
SO3H
SO3H
OH
NO2
OH

(4)

(S)


Me
COOH
COOH
O2N
NO2
CONH2
O2N
NO2
NH2
O2N
NO2
N2+ Cl−
O2N
NO2
OH
O2N
NO2

(1)

Note: ***For the benefit of the students, specially the aspiring ones, the question of JEE(advanced), 2024 are also given in this booklet. Keeping the interest of students studying in class XI, the questions based on topics from class XI have been marked with '*', which can be attempted as a test. For this test the time allocated in Mathematics, Physics and Chemistry are 30 minutes, 20 minutes and 25 minutes respectively.***

FIITJEE

SOLUTIONS TO JEE (ADVANCED) – 2024
(PAPER-2)

Mathematics

SECTION 1 (Maximum Marks: 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +3 If **ONLY** the correct option is chosen;
  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
  Negative Marks : −1 In all other cases.

*Q.1 Considering only the principal values of the inverse trigonometric functions, the value of

$\tan\left(\sin^{-1}\left(\frac{3}{5}\right)-2\cos^{-1}\left(\frac{2}{\sqrt{5}}\right)\right)$ is

(A) $\frac{7}{24}$ (B) $\frac{-7}{24}$

(C) $\frac{-5}{24}$ (D) $\frac{5}{24}$

Q.2 Let $S=\left\{(x,y)\in\mathbb{R}\times\mathbb{R}: x\geq 0, y\geq 0, y^2\leq 4x, y^2\leq 12-2x \text{ and } 3y+\sqrt{8}x\leq 5\sqrt{8}\right\}$. If the area of the region S is $\alpha\sqrt{2}$, then α is equal to

(A) $\frac{17}{2}$ (B) $\frac{17}{3}$

(C) $\frac{17}{4}$ (D) $\frac{17}{5}$

Q.3 Let $k\in\mathbb{R}$. If $\lim\limits_{x\to 0^+}(\sin(\sin kx)+\cos x+x)^{\frac{2}{x}}=e^6$, then the value of k is

(A) 1 (B) 2

(C) 3 (D) 4

Q.4 Let $f:\mathbb{R}\to\mathbb{R}$ be a function defined by

$$f(x)=\begin{cases}x^2\sin\left(\frac{\pi}{x^2}\right) & ; \text{ if } x\neq 0\\ 0 & ; \text{ if } x=0\end{cases}$$

Then which of the following statements is TRUE?

(A) f(x) = 0 has infinitely many solutions in the interval $\left[\frac{1}{10^{10}},\infty\right)$

(B) f(x) = 0 has no solutions in the interval $\left[\frac{1}{\pi},\infty\right)$

(C) The set of solutions of f(x) = 0 in the interval $\left(0,\frac{1}{10^{10}}\right)$ is finite

(D) f(x) = 0 has more than 25 solutions in the interval $\left(\frac{1}{\pi^2},\frac{1}{\pi}\right)$

SECTION 2 (Maximum Marks: 12)

- This section contains **THREE (03)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +4 **ONLY** if (all) the correct option(s) is(are) chosen;
  Partial Marks : +3 If all the four options are correct but **ONLY** three options are chosen;
  Partial Marks : +2 If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
  Partial Marks : +1 If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
  Zero Marks : 0 If unanswered;
  Negative Marks : −2 In all other cases.
- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then
  choosing ONLY (A), (B) and (D) will get +4 marks;
  choosing ONLY (A) and (B) will get +2 marks;
  choosing ONLY (A) and (D) will get +2marks;
  choosing ONLY (B) and (D) will get +2 marks;
  choosing ONLY (A) will get +1 mark;
  choosing ONLY (B) will get +1 mark;
  choosing ONLY (D) will get +1 mark;
  choosing no option(s) (i.e. the question is unanswered) will get 0 marks and
  choosing any other option(s) will get −2 marks.

Q.5 Let S be the set of all $(\alpha, \beta) \in \mathbb{R} \times \mathbb{R}$ such that

$$\lim_{x\to\infty} \frac{\sin\left(x^2\right)\left(\log_e x\right)^{\alpha} \sin\left(\frac{1}{x^2}\right)}{x^{\alpha\beta}\left(\log_e (1+x)\right)^{\beta}} = 0 .$$

Then which of the following is(are) correct ?

(A) $(-1, 3) \in S$ (B) $(-1, 1) \in S$

(C) $(1, -1) \in S$ (D) $(1, -2) \in S$

Q.6 A straight line drawn from the point P(1, 3, 2), parallel to the line $\frac{x-2}{1} = \frac{y-4}{2} = \frac{z-6}{1}$ intersects the plane $L_1 : x - y + 3z = 6$ at the point Q. Another straight line which passes through Q and is perpendicular to the plane L_1 intersects the plane $L_2 : 2x - y + z = -4$ at the point R. then which of the following statements is(are) TRUE?

(A) The length of the line segment PQ is $\sqrt{6}$

(B) The coordinates of R are (1, 6, 3)

(C) The centroid of the triangle PQR is $\left(\frac{4}{3}, \frac{14}{3}, \frac{5}{3}\right)$

(D) The perimeter of the triangle PQR is $\sqrt{2} + \sqrt{6} + \sqrt{11}$

*Q.7 Let A_1, B_1, C_1 be three points in the xy-plane. Suppose that the lines A_1C_1 and B_1C_1 are tangents to the curve $y^2 = 8x$ at A_1 and B_1, respectively. If O = (0, 0) and C_1 = (–4, 0), then which of the following statements is(are) TRUE?

(A) The length of the line segment OA_1 is $4\sqrt{3}$
(B) The length of the line segment A_1B_1 is 16
(C) The orthocenter of the triangle $A_1B_1C_1$ is (0, 0)
(D) The orthocenter of the triangle $A_1B_1C_1$ is (1, 0)

SECTION 3 (Maximum Marks: 24)

- This section contains **SIX (06)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER**.
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +4 If **ONLY** the correct integer is entered;
  Zero Marks : 0 In all other cases

Q.8 Let $f : \mathbb{R} \to \mathbb{R}$ be a function such that f(x + y) = f(x) + f(y) for all x, $y \in \mathbb{R}$, and $g : \mathbb{R} \to (0, \infty)$ be a function such that g(x + y) = g(x)g(y) for all x, $y \in \mathbb{R}$. If $f\left(\frac{-3}{5}\right) = 12$ and $g\left(\frac{-1}{3}\right) = 2$, then the value of $\left(f\left(\frac{1}{4}\right) + g(-2) - 8\right)g(0)$ is _____

Q.9 A bag contains N balls out of which 3 balls are white, 6 balls are green, and the remaining balls are blue. Assume that the balls are identical otherwise. Three balls are drawn randomly one after the other without replacement. For i = 1, 2, 3, let W_i, G_i, and B_i denote the events that the ball drawn in the i^{th} draw is a white ball, green ball, and blue ball, respectively. If the probability $P(W_1 \cap G_2 \cap B_3) = \frac{2}{5N}$ and the conditional probability $P(B_3 \mid W_1 \cap G_2) = \frac{2}{9}$, then N equals _____

Q.10 Let the function $f : \mathbb{R} \to \mathbb{R}$ be defined by

$$f(x) = \frac{\sin x}{e^{\pi x}} \frac{(x^{2023} + 2024x + 2025)}{(x^2 - x + 3)} + \frac{2}{e^{\pi x}} \frac{(x^{2023} + 2024x + 2025)}{(x^2 - x + 3)}.$$

Then the number of solutions of f(x) = 0 in $\mathbb{R}$ is _____

Q.11 Let $\vec{p} = 2\hat{i} + \hat{j} + 3\hat{k}$ and $\vec{q} = \hat{i} - \hat{j} + \hat{k}$. If for some real numbers α, β, and γ, we have

$$15\hat{i} + 10\hat{j} + 6\hat{k} = \alpha(2\vec{p} + \vec{q}) + \beta(\vec{p} - 2\vec{q}) + \gamma(\vec{p} \times \vec{q}),$$

then the value of γ is _____

*Q.12 A normal with slope $\frac{1}{\sqrt{6}}$ is drawn from the point $(0, -\alpha)$ to the parabola $x^2 = -4ay$, where a > 0. Let L be the line passing through $(0, -\alpha)$ and parallel to the directrix of the parabola. Suppose that L intersects the parabola at two points A and B. Let r denote the length of the latus rectum and s denote the square of the length of the line segment AB. If r : s = 1 : 16, then the value of 24α is _____

Q.13 Let the function $f : [1, \infty) \to \mathbb{R}$ be defined by

$$f(t) = \begin{cases} (-1)^{n+1} 2 & ; \quad \text{if } t = 2n-1, n \in N \\ \frac{(2n+1-t)}{2} f(2n-1) + \frac{(t-(2n-1))}{2} f(2n+1) & ; \quad \text{if } 2n-1 < t < 2n+1, n \in N \end{cases}.$$

Define $g(x) = \int_1^x f(t)dt$, $x \in (1, \infty)$. Let α denote the number of solutions of the equation $g(x) = 0$ in the interval $(1, 8]$ and $\beta = \lim_{x \to 1^+} \frac{g(x)}{x-1}$. Then the value of $\alpha + \beta$ is equal to _____

SECTION 4 (Maximum Marks: 12)

- This section contains **TWO (02)** paragraphs.
- Based on each paragraph, there are **TWO (02)** questions.
- The answer to each question is a **NUMERICAL VALUE.**
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +3 If ONLY the correct numerical value is entered in the designated place;
  Zero Marks : 0 In all other cases.

PARAGRAPH "I"

Let S = {1, 2, 3, 4, 5, 6} and X be the set of all relations R from S to S that satisfy both the following properties:

i. R has exactly 6 elements.
ii. For each $(a, b) \in R$, we have $|a - b| \geq 2$.

Let Y ={$R \in X$: The range of R has exactly one element} and
Z ={$R \in X$: R is a function from S to S}.
Let n(A) denote the number of elements in a set A.

(There are two questions based on PARAGRAPH "I", the question given below is one of them)

Q.14 If $n(X) = {}^mC_6$, then the value of m is _____

PARAGRAPH "I"

Let S = {1, 2, 3, 4, 5, 6} and X be the set of all relations R from S to S that satisfy both the following properties:

i. R has exactly 6 elements.
ii. For each $(a, b) \in R$, we have $|a - b| \geq 2$.

Let Y ={$R \in X$: The range of R has exactly one element} and
Z ={$R \in X$: R is a function from S to S}.
Let n(A) denote the number of elements in a set A.

(There are two questions based on PARAGRAPH "I", the question given below is one of them)

Q.15 If the value of n(Y) + n(Z) is k^2, then |k| is _____

PARAGRAPH "II"

Let $f:\left[0,\frac{\pi}{2}\right]\to[0,1]$ be the function defined by $f(x) = \sin^2 x$ and let $g:\left[0,\frac{\pi}{2}\right]\to[0,\infty)$ be the function defined by $g=\sqrt{\frac{\pi x}{2}-x^2}$.

(There are two questions based on PARAGRAPH "II", the question given below is one of them)

Q.16 The value of $2\int_0^{\frac{\pi}{2}} f(x)g(x)dx-\int_0^{\frac{\pi}{2}} g(x)dx$ is _____

PARAGRAPH "II"

Let $f:\left[0,\frac{\pi}{2}\right]\to[0,1]$ be the function defined by $f(x) = \sin^2 x$ and let $g:\left[0,\frac{\pi}{2}\right]\to[0,\infty)$ be the function defined by $g(x)=\sqrt{\frac{\pi x}{2}-x^2}$.

(There are two questions based on PARAGRAPH "II", the question given below is one of them)

Q.17 The value of $\frac{16}{\pi^3}\int_0^{\frac{\pi}{2}} f(x)g(x)dx$ is _____

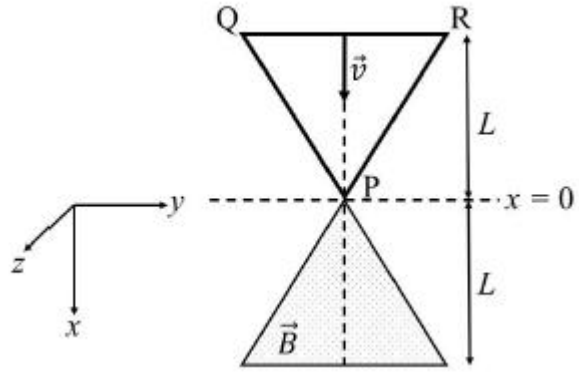
Physics

SECTION 1 (Maximum Marks: 12)

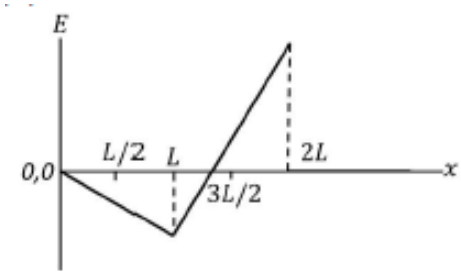
- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 If **ONLY** the correct option is chosen;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : −1 In all other cases.

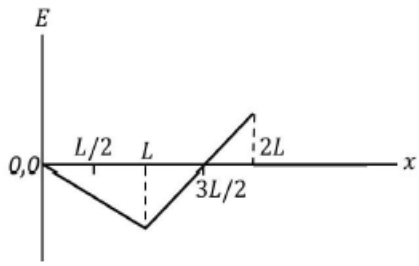
Q.1 A region in the form of an equilateral triangle (in x−y plane) of height L has a uniform magnetic field $\vec{B}$ pointing in the +z -direction. A conducting loop PQR , in the form of an equilateral triangle of the same height L , is placed in the x−y plane with its vertex P at $x = 0$ in the orientation shown in the figure. At t = 0 , the loop starts entering the region of the magnetic field with a uniform velocity $\vec{v}$ along the +x-direction. The plane of the loop and its orientation remain unchanged throughout its motion.



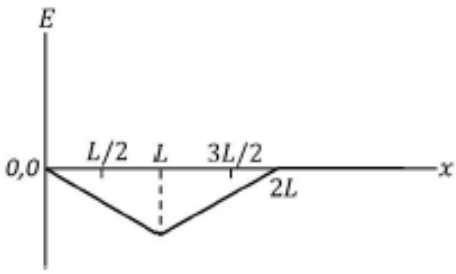
(A)



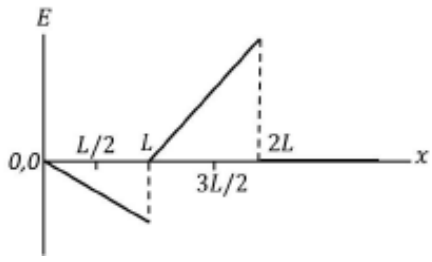
(B)



(C)



(D)



*Q.2 A particle of mass m is under the influence of the gravitational field of a body of mass $M(\gg m)$. The particle is moving in a circular orbit of radius r_0 with time period T_0 around the mass M. Then, the particle is subjected to an additional central force, corresponding to the potential energy $V_c(r) = m\alpha/r^3$, where α is a positive constant of suitable dimensions and r is the distance from the center of the orbit. If the particle moves in the same circular orbit of radius r_0 in the combined gravitational potential due to M and $V_c(r)$, but with a new time period T_1, then $(T_1^2 - T_0^2 / T_1^2$ is given by
[G is the gravitational constant.]

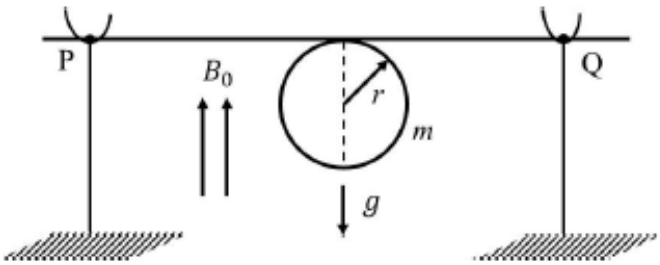
(A) $\frac{3\alpha}{GMr_0^2}$ (B) $\frac{\alpha}{2GMr_0^2}$

(C) $\frac{\alpha}{GMr_0^2}$ (D) $\frac{2\alpha}{GMr_0^2}$

Q.3 A metal target with atomic number $Z=46$ is bombarded with a high energy electron beam. The emission of X-rays from the target is analyzed. The ratio r of the wavelengths of the K_α-line and the cut-off is found to be $r = 2$. If the same electron beam bombards another metal target with $Z = 41$, the value of r will be

(A) 2.53 (B) 1.27
(C) 2.24 (D) 1.58

Q.4 A thin stiff insulated metal wire is bent into a circular loop with its two ends extending tangentially from the same point of the loop. The wire loop has mass m and radius r and it is in a uniform vertical magnetic field B_0, as shown in the figure. Initially, it hangs vertically downwards, because of acceleration due to gravity g, on two conducting supports at P and Q. When a current I is passed through the loop, the loop turns about the line PQ by an angle θ given by



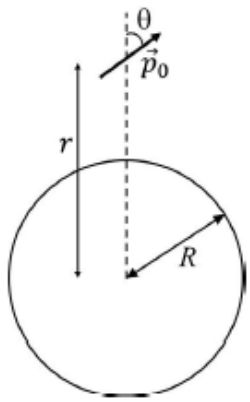
(A) $\tan\theta = \pi r I B_0/(mg)$ (B) $\tan\theta = 2\pi r I B_0/(mg)$
(C) $\tan\theta = \pi r I B_0/(2mg)$ (D) $\tan\theta = mg/(\pi r I B_0)$

SECTION 2 (Maximum Marks: 12)

- This section contains **THREE (03)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +4 **ONLY** if (all) the correct option(s) is(are) chosen;
  Partial Marks : +3 If all the four options are correct but **ONLY** three options are chosen;
  Partial Marks : +2 If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
  Partial Marks : +1 If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
  Zero Marks : 0 If unanswered;
  Negative Marks : −2 In all other cases.
- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then
  choosing ONLY (A), (B) and (D) will get +4 marks;
  choosing ONLY (A) and (B) will get +2 marks;
  choosing ONLY (A) and (D) will get +2marks;
  choosing ONLY (B) and (D) will get +2 marks;
  choosing ONLY (A) will get +1 mark;
  choosing ONLY (B) will get +1 mark;
  choosing ONLY (D) will get +1 mark;
  choosing no option(s) (i.e. the question is unanswered) will get 0 marks and
  choosing any other option(s) will get −2 marks.

Q.5 A small electric dipole $\vec{p}_0$, having a moment of inertia I about its center, is kept at a distance r from the center of a spherical shell of radius R. The surface charge density σ is uniformly distributed on the spherical shell. The dipole is initially oriented at a small angle θ as shown in the figure. While staying at a distance r, the dipole is free to rotate about its center.



If released from rest, then which of the following statement(s) is(are) correct?
[ε_0 is the permittivity of free space.]

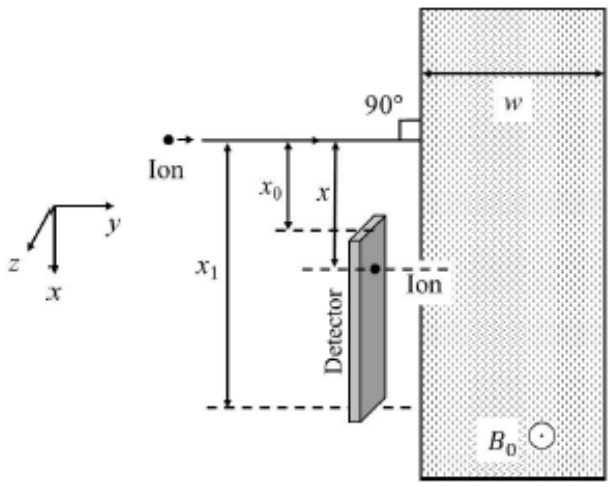
(A) The dipole will undergo small oscillations at any finite value of r.
(B) The dipole will undergo small oscillations at any finite value of $r > R$.
(C) The dipole will undergo small oscillations with an angular frequency of $\sqrt{\frac{2\sigma p_0}{\varepsilon_0 I}}$ at $r = 2R$.
(D) The dipole will undergo small oscillations with an angular frequency $\sqrt{\frac{\sigma p_0}{100\varepsilon_0 I}}$ at $r = 10R$.

*Q.6 A table tennis ball has radius $(3/2) \times 10^{-2}$m and mass $(22/7) \times 10^{-3}$kg . It is slowly pushed down into a swimming pool to a depth of $d = 0.7$m below the water surface and then released from rest. It emerges from the water surface at speed v , without getting wet, and rises up to a height H . Which of the following option(s) is(are) correct?

[Given: $\pi = 22/7$, $g = 10 ms^{-2}$, density of water $= 1 \times 10^3 kgm^{-3}$, viscosity of water $= 1 \times 10^{-3}$Pa-s.]

(A) The work done in pushing the ball to the depth d is 0.077J .
(B) If we neglect the viscous force in water, then the speed $v = 7$m/s .
(C) If we neglect the viscous force in water, then the height $H = 1.4$m .
(D) The ratio of the magnitudes of the net force excluding the viscous force to the maximum viscous force in water is 500/9.

Q.7 A positive, singly ionized atom of mass number A_M is accelerated from rest by the voltage 192V . Thereafter, it enters a rectangular region of width w with magnetic field $\vec{B}_0 = 0.1\hat{k}$ Tesla, as shown in the figure. The ion finally hits a detector at the distance x below its starting trajectory.
[Given: Mass of neutron/proton $= (5/3) \times 10^{-27}$kg , charge of the electron $= 1.6 \times 10^{-19}$C .]



Which of the following option(s) is(are) correct?
(A) The value of x for H^+ ion is 4 cm .
(B) The value of x for an ion with $A_M = 144$ is 48cm .
(C) For detecting ions with $1 \le A_M \le 196$, the minimum height $(x_1 - x_0)$ of the detector is 55cm .
(D) The minimum width w of the region of the magnetic field for detecting ions with $A_M = 196$ is 56cm .

SECTION 3 (Maximum Marks: 24)

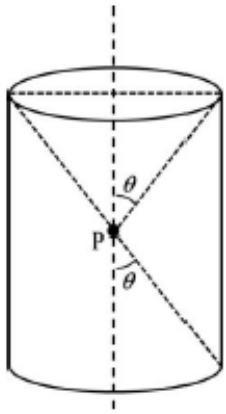
- This section contains **SIX (06)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER**.
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 If **ONLY** the correct integer is entered;
Zero Marks : 0 In all other cases.

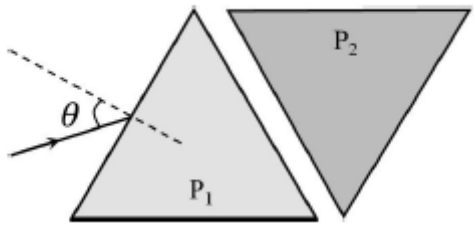
Q.8 The dimensions of a cone are measured using a scale with a least count of 2mm . The diameter of the base and the height are both measured to be 20.0cm . The maximum percentage error in the determination of the volume is -

*Q.9 A ball is thrown from the location $(x_0, y_0) = (0, 0)$ of a horizontal playground with an initial speed v_0 at an angle θ_0 from the +x-direction. The ball is to be hit by a stone, which is thrown at the same time from the location $(x_1, y_1) = (L, 0)$. The stone is thrown at an angle $(180 - \theta_1)$ from the +x -direction with a suitable initial speed. For a fixed v_0 , when $(\theta_0, \theta_1) = (45^0, 45^0)$, the stone hits the ball after time T_1 , and when $(\theta_0, \theta_1) = (60^0, 30^0)$, it hits the ball after time T_2 . In such a case, $(T_1/T_2)^2$ is _____

Q.10 A charge is kept at the central point P of a cylindrical region. The two edges subtend a half-angle θ at P , as shown in the figure. When $\theta = 30^0$, then the electric flux through the curved surface of the cylinder is Φ . If $\theta = 60^0$, then the electric flux through the curved surface becomes $\frac{\phi}{\sqrt{n}}$, where the value of n is _____

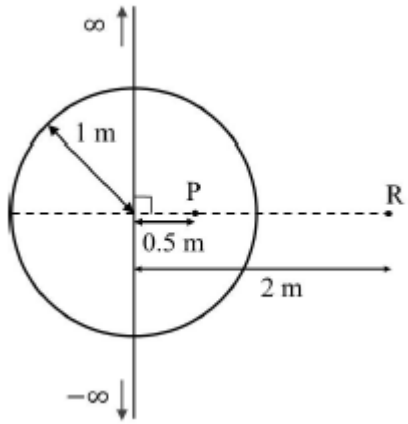


Q.11 Two equilateral-triangular prisms P_1 and P_2 are kept with their sides parallel to each other, in vacuum, as shown in the figure. A light ray enters prism P_1 at an angle of incidence θ such that the outgoing ray undergoes minimum deviation in prism P_2. If the respective refractive indices of P_1 and P_2 are $\sqrt{\frac{3}{2}}$ and $\sqrt{3}$, then $\theta = \sin^{-1}\left[\sqrt{\frac{3}{2}}\sin\left(\frac{\pi}{\beta}\right)\right]$, where the value of β is



Q.12 An infinitely long thin wire, having a uniform charge density per unit length of 5nC/m, is passing through a spherical shell of radius 1m, as shown in the figure. A 10nC charge is distributed uniformly over the spherical shell. If the configuration of the charges remains static, the magnitude of the potential difference between points P and R, in Volt, is

[Given: In SI units $\frac{1}{4\pi\varepsilon_0} = 9\times10^9$, $\ln 2 = 0.7$. Ignore the area pierced by the wire.]



*Q.13 A spherical soap bubble inside an air chamber at pressure $P_0 = 10^5$ Pa has a certain radius so that the excess pressure inside the bubble is $\Delta P = 144$Pa. Now, the chamber pressure is reduced to $8P_0/27$ so that the bubble radius and its excess pressure change. In this process, all the temperatures remain unchanged. Assume air to be an ideal gas and the excess pressure ΔP in both the cases to be much smaller than the chamber pressure. The new excess pressure ΔP in Pa is

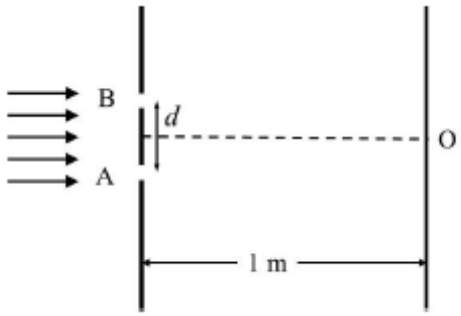
SECTION 4 (Maximum Marks: 12)

- This section contains **TWO (02)** paragraphs.
- Based on each paragraph, there are **TWO (02)** questions.
- The answer to each question is a **NUMERICAL VALUE.**
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 If ONLY the correct numerical value is entered in the designated place;
Zero Marks : 0 In all other cases.

PARAGRAPH I

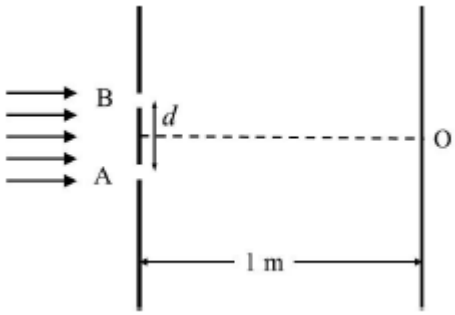
In a Young's double slit experiment, each of the two slits A and B, as shown in the figure, are oscillating about their fixed center and with a mean separation of 0.8mm. The distance between the slits at time t is given by $d = (0.8 + 0.04 \sin \omega t)\text{mm}$, where $\omega = 0.08\text{rads}^{-1}$. The distance of the screen from the slits is 1m and the wavelength of the light used to illuminate the slits is $6000\ \overset{0}{\text{A}}$. The interference pattern on the screen changes with time, while the central bright fringe (zeroth fringe) remains fixed at point O.



Q.14 The 8^{th} bright fringe above the point O oscillates with time between two extreme positions. The separation between these two extreme positions, in micrometer (μm), is

PARAGRAPH I

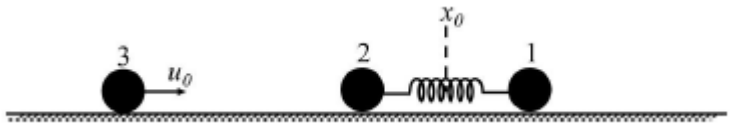
In a Young's double slit experiment, each of the two slits A and B, as shown in the figure, are oscillating about their fixed center and with a mean separation of 0.8mm. The distance between the slits at time t is given by $d = (0.8 + 0.04 \sin \omega t)\text{mm}$, where $\omega = 0.08\text{rads}^{-1}$. The distance of the screen from the slits is 1m and the wavelength of the light used to illuminate the slits is $6000\ \overset{0}{\text{A}}$. The interference pattern on the screen changes with time, while the central bright fringe (zeroth fringe) remains fixed at point O.



Q.15 The maximum speed in μm/s at which the 8^{th} bright fringe will move is

PARAGRAPH II

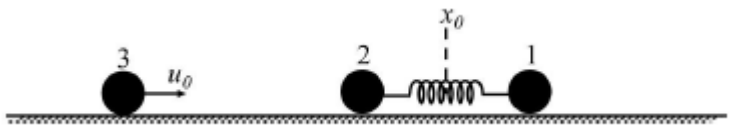
Two particles, 1 and 2, each of mass m, are connected by a massless spring, and are on a horizontal frictionless plane, as shown in the figure. Initially, the two particles, with their center of mass at x_0, are oscillating with amplitude a and angular frequency ω. Thus, their positions at time t are given by $x_1(t) = (x_0 + d) + a \sin \omega t$ and $x_2(t) = (x_0 - d) - a \sin \omega t$, respectively, where $d > 2a$. Particle 3 of mass m moves towards this system with speed $u_0 = a\omega/2$, and undergoes instantaneous elastic collision with particle 2, at time t_0. Finally, particles 1 and 2 acquire a center of mass speed v_{cm} and oscillate with amplitude b and the same angular frequency ω.



*Q.16 If the collision occurs at time $t_0 = 0$, the value of $v_{cm}/(a\omega)$ will be

PARAGRAPH II

Two particles, 1 and 2, each of mass m, are connected by a massless spring, and are on a horizontal frictionless plane, as shown in the figure. Initially, the two particles, with their center of mass at x_0, are oscillating with amplitude a and angular frequency ω. Thus, their positions at time t are given by $x_1(t) = (x_0 + d) + a \sin \omega t$ and $x_2(t) = (x_0 - d) - a \sin \omega t$, respectively, where $d > 2a$. Particle 3 of mass m moves towards this system with speed $u_0 = a\omega/2$, and undergoes instantaneous elastic collision with particle 2, at time t_0. Finally, particles 1 and 2 acquire a center of mass speed v_{cm} and oscillate with amplitude b and the same angular frequency ω.



*Q.17 If the collision occurs at time $t_0 = \pi/(2\omega)$, then the value of $4b^2/a^2$ will be

Chemistry

SECTION 1 (Maximum Marks: 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the
- correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +3 If **ONLY** the correct option is chosen;
  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
  Negative Marks : −1 In all other cases.

*Q.1 According to Bohr's model, the highest kinetic energy is associated with the electron in the

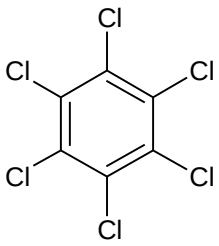
(A) first orbit of H atom
(B) first orbit of He^+
(C) second orbit of He^+
(D) second orbit of Li^{2+}

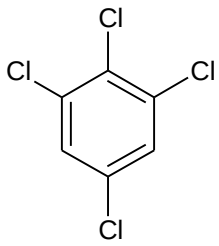
Q.2 In a metal deficient oxide sample, $M_xY_2O_4$ (**M** and **Y** are metals), **M** is present in both +2 and +3 oxidation states and **Y** is in +3 oxidation state. If the fraction of M^{2+} ions present in **M** is $\frac{1}{3}$, the value of **X** is_____.

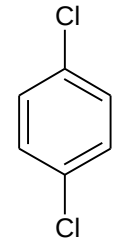
(A) 0.25
(B) 0.33
(C) 0.67
(D) 0.75

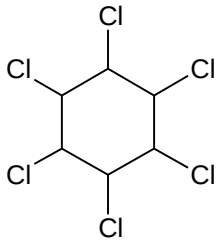
Q. 3 In the following reaction sequence, the major product **Q** is

$$L-Glucose \xrightarrow[\text{ii) } Cr_2O_3,\ 775\ K,\ 10-20\ atm]{\text{i) HI, }\Delta} P \xrightarrow[UV]{Cl_2\,(excess)} Q$$

(A) 

(B) 

(C) 

(D) 

Q.4 The species formed on fluorination of phosphorus pentachloride in a polar organic solvent are

(A) $[PF_4]^+[PF_6]^-$ and $[PCl_4]^+[PF_6]^-$
(B) $[PCl_4]^+[PCl_4F_2]^-$ and $[PCl_4]^+[PF_6]^-$
(C) PF_3 and PCl_3
(D) PF_5 and PCl_3

SECTION 2 (Maximum Marks: 12)

- This section contains **THREE (03)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +4 **ONLY** if (all) the correct option(s) is(are) chosen;
  Partial Marks : +3 If all the four options are correct but **ONLY** three options are chosen;
  Partial Marks : +2 If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
  Partial Marks : +1 If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
  Zero Marks : 0 If unanswered;
- *Negative Marks :* −2 In all other cases. For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then
  choosing ONLY (A), (B) and (D) will get +4 marks;
  choosing ONLY (A) and (B) will get +2 marks;
  choosing ONLY (A) and (D) will get +2marks;
  choosing ONLY (B) and (D) will get +2 marks;
  choosing ONLY (A) will get +1 mark;
  choosing ONLY (B) will get +1 mark;
  choosing ONLY (D) will get +1 mark;
  choosing no option(s) (i.e. the question is unanswered) will get 0 marks and
  choosing any other option(s) will get −2 marks.

Q. 5 An aqueous solution of hydrazine (N_2H_4) is electrochemically oxidized by O_2, thereby releasing chemical energy in the form of electrical energy. One of the products generated from the electrochemical reaction is $N_2(g)$.

Choose the correct statement(s) about the above process

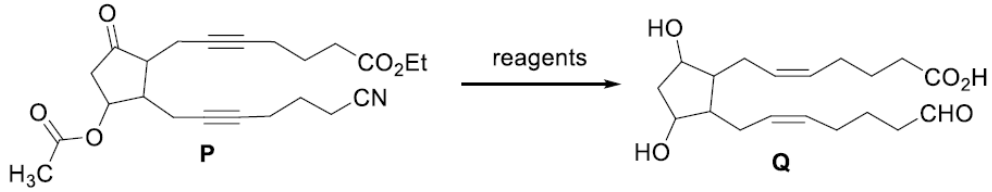
(A) OH^- ions react with N_2H_4 at the anode to form $N_2(g)$ and water, releasing 4 electrons to the anode.

(B) At the cathode, N_2H_4 breaks to $N_2(g)$ and nascent hydrogen released at the electrode reacts with oxygen to form water.

(C) At the cathode, molecular oxygen gets converted to OH^-.

(D) Oxides of nitrogen are major by-products of the electrochemical process.

Q. 6 The option(s) with correct sequence of reagents for the conversion of **P** to **Q** is(are)



(A) i) Lindlar's catalyst, H_2; ii) $SnCl_2$/HCl; iii) $NaBH_4$; iv) H_3O^+

(B) i) Lindlar's catalyst, H_2; ii) H_3O^+; iii) $SnCl_2$/HCl; iv) $NaBH_4$

(C) i) $NaBH_4$; ii) $SnCl_2$/HCl; iii) H_3O^+; iv) Lindlar's catalyst, H_2;

(D) i) Lindlar's catalyst, H_2; ii) $NaBH_4$; iii) $SnCl_2$/HCl; iv) H_3O^+

*Q. 7 The compound(s) having peroxide linkage is(are)

(A) $H_2S_2O_7$ (B) $H_2S_2O_8$

(C) $H_2S_2O_5$ (D) H_2SO_5

SECTION 3 (Maximum Marks: 24)

- This section contains **SIX (06)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER**.
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 If **ONLY** the correct integer is entered;

Zero Marks : 0 In all other cases.

Q. 8 To form a complete monolayer of acetic acid on 1g of charcoal, 100 mL of 0.5 M acetic acid was used. Some of the acetic acid remained unadsorbed. To neutralize the unadsorbed acetic acid, 40 mL of 1 M NaOH solution was required. If each molecule of acetic acid occupies $\mathbf{P}\times10^{-23}$ m^2 surface area on charcoal, the value of **P** is _____.

[Use given data : Surface area of charcoal $=1.5\times10^{2}\ m^2\ g^{-1}$; Avogadro's number $\left(N_A\right)=6.0\times10^{23}\ mol^{-1}$]

Q. 9 Vessel-1 contains $\mathbf{w_2}$ g of a non-volatile solute **X** dissolved in $\mathbf{w_1}$ g of water. Vessel-2 contains $\mathbf{w_2}$ g of another non-volatile solute **Y** dissolved in $\mathbf{w_1}$ g of water. Both the vessels are at the same temperature and pressure. The molar mass of **X** is 80% of that of **Y**. The van't Hoff factor for **X** is 1.2 times of that of **Y** for their respective concentrations.

The elevation of boiling point for solution in Vessel-1 is_____% of the solution in Vessel-2.

Q. 10 For a double strand DNA, one strand is given below:

The amount of energy required to split the double strand DNA into two single strands is _____ kcal mol^{-1}.

[Given: Average energy per H-bond for A-T base pair = 1.0 kcal mol^{-1}, G-C base pair = 1.5 kcal mol^{-1}, and A-U base pair = 1.25 kcal mol^{-1}. Ignore electrostatic repulsion between the phosphate groups.]

*Q. 11 A sample initially contains only U-238 isotope of uranium. With time, some of the U-238 radioactively decays into Pb-206 while the rest of it remains undisintegrated.

When the age of the sample is $\mathbf{P}\times10^{8}$ years, the ratio of mass of Pb-206 to that of U-238 in the sample is found to be 7. The value of **P** is_____.

[Given: Half-life of U-238 is 4.5×10^{9} years; $\log_e 2=0.693$]

Q. 12 Among $\left[Co(CN)_4\right]^{4-}$, $\left[Co(CO)_3(NO)\right]$, XeF_4, $\left[PCl_4\right]^{+}$, $\left[PdCl_4\right]^{2-}$, $\left[ICl_4\right]^{-}$, $\left[Cu(CN)_4\right]^{3-}$ and P_4 the total number of species with tetrahedral geometry is_____.

Q. 13 An organic compound **P** having molecular formula $C_6H_6O_3$ gives ferric chloride test and does not have intramolecular hydrogen bond. The compound **P** reacts with 3 equivalents of NH_2OH to produce oxime **Q**. Treatment of **P** with excess methyl iodide in the presence of KOH produces compound **R** as the major product. Reaction of **R** with excess *iso*-butylmagnesium bromide followed by treatment with H_3O^{+} gives compound **S** as the major product.

The total number of methyl $\left(-CH_3\right)$ group(s) in compound **S** is____.

SECTION 4 (Maximum Marks: 12)

- This section contains **TWO (02)** paragraphs.
- Based on each paragraph, there are **TWO (02)** questions.
- The answer to each question is a **NUMERICAL VALUE.**
- For each question, enter the correct numerical value of the answer using the mouse and the onscreen
  virtual numeric keypad in the place designated to enter the answer.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +3 If ONLY the correct numerical value is entered in the designated place;
  Zero Marks : 0 In all other cases.

"PARAGRAPH I"

An organic compound **P** with molecular formula $C_9H_{18}O_2$ decolorizes bromine water and also shows positive iodoform test. **P** on ozonolysis followed by treatment with H_2O_2 gives **Q** and **R**. While compound **Q** shows positive iodoform test, compound **R** does not give positive iodoform test. **Q** and **R** on oxidation with pyridinium chlorochromate (PCC) followed by heating give **S** and **T**, respectively. Both **S** and **T** show positive iodoform test.

Complete copolymerization of 500 moles of **Q** and 500 moles of **R** gives one mole of a single acyclic copolymer **U**.

[Given, atomic mass: H =1, C = 12, O =16]

Q.14 Sum of number of oxygen atoms in **S** and **T** is _____.

"PARAGRAPH I"

An organic compound **P** with molecular formula $C_9H_{18}O_2$ decolorizes bromine water and also shows positive iodoform test. **P** on ozonolysis followed by treatment with H_2O_2 gives **Q** and **R**. While ompound **Q** shows positive iodoform test, compound **R** does not give positive iodoform test. **Q** and **R** on oxidation with pyridinium chlorochromate (PCC) followed by heating give **S** and **T**, respectively. Both **S** and **T** show positive iodoform test.

Complete copolymerization of 500 moles of **Q** and 500 moles of **R** gives one mole of a single acyclic copolymer **U**.

[Given, atomic mass: H =1, C = 12, O =16]

Q.15 The molecular weight of **U** is _____.

"PARAGRAPH II"

When potassium iodide is added to an aqueous solution of potassium ferricyanide, a reversible reaction is observed in which a complex **P** is formed. In a strong acidic medium, the equilibrium shifts completely towards **P**. Addition of zinc chloride to **P** in a slightly acidic medium results in a sparingly soluble complex **Q**.

Q.16 The number of moles of potassium iodide required to produce two moles of **P** is _____.

"PARAGRAPH II"

When potassium iodide is added to an aqueous solution of potassium ferricyanide, a reversible reaction is observed in which a complex **P** is formed. In a strong acidic medium, the equilibrium shifts completely towards **P**. Addition of zinc chloride to **P** in a slightly acidic medium results in a sparingly soluble complex **Q**.

Q.17 The number of zinc ions present in the molecular formula of **Q** is _____.

FIITJEE JEE (ADVANCED) – 2024
(PAPER-2)

ANSWER KEY

Mathematics

1.	**B**	2.	**B**	3.	**B**	4.	**D**
5.	**B, C**	6.	**A, C**	7.	**A, C**	8.	**51**
9.	**11**	10.	**1**	11.	**2**	12.	**12**
13.	**5**	14.	**20.00**	15.	**36.00**	16.	**0.00**
17.	**0.25**						

Physics

1.	**A**	2.	**A**	3.	**A**	4.	**A**
5.	**B, D**	6.	**A, B, D**	7.	**A, B**	8.	**3**
9.	**2**	10.	**3**	11.	**12**	12.	**171**
13.	**96 Pa**	14.	**601.50**	15.	**24**	16.	**0.75**
17.	**4.25**						

Chemistry

1.	**B**	2.	**D**	3.	**D**	4.	**B**
5.	**A, C, D**	6.	**C, D**	7.	**B, D**	8.	**2500**
9.	**150**	10.	**41**	11.	**142.65 or 143**	12.	**5**
13.	**12**	14.	**2 or 4**	15.	**93018**	16.	**3**
17.	**2 or 3**						

HINTS AND SOLUTIONS
Mathematics

1. $\tan\left(\sin^{-1}\frac{3}{5}-2\cos^{-1}\frac{2}{\sqrt{5}}\right) = \tan\left(\tan^{-1}\frac{3}{4}-2\tan^{-1}\frac{1}{2}\right)$

$= \tan\left(\tan^{-1}\frac{3}{4}-\tan^{-1}\frac{2\times\frac{1}{2}}{1-\left(\frac{1}{2}\right)^2}\right) = \tan\left(\tan^{-1}\frac{3}{4}-\tan^{-1}\frac{4}{3}\right)$

$= \tan\left(\tan^{-1}\left(\frac{-7}{24}\right)\right)=\frac{-7}{24}$

2. $x \ge 0, y \ge 0, y^2 \le 4x, y^2 \le 12 - 2x$

$3y+\sqrt{8}x \le 5\sqrt{8}$

Area $= \int_0^2 \sqrt{4x}dx + \frac{1}{2}\times 3\times 2\sqrt{2}$

$= \left[2\times\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^2 + 3\sqrt{2} = 2\cdot\frac{2\sqrt{2}\times 2}{3}+3\sqrt{2}$

$= 2\cdot\frac{2\sqrt{2}\times 2}{3}+3\sqrt{2} = \frac{17\sqrt{2}}{3} \Rightarrow \alpha = \frac{17}{3}$

3. $\lim_{x\to 0^+}(\sin(\sin kx)+\cos x+x)^{\frac{2}{x}} = e^6$ (it is 1^∞ form)

$\Rightarrow e^{2\lim_{x\to 0^+}\frac{\sin(\sin kx)+\cos x+x-1}{x}} = e^6$

$= e^{2\lim_{x\to 0^+}\frac{\cos(\sin kx)\times k\cos kx+\sin x+1}{1}} = e^6$ (Using L.H. Rule)

$\Rightarrow e^{2(k+1)} = e^6$

$\Rightarrow k = 2$

4. $x^2\sin\left(\frac{\pi}{x^2}\right)=0 \Rightarrow \frac{\pi}{x^2}=n\pi \Rightarrow x=\pm\frac{1}{\sqrt{n}}$

$\frac{1}{\pi^2}<\frac{1}{\sqrt{n}}<\frac{1}{\pi} \Rightarrow \pi^2 < n < \pi^4$

f(x) = 0 has more than 25 solutions

5. $\lim_{x\to\infty}\left(\frac{\sin(x^2)(\log_e x)^\alpha \sin\left(\frac{1}{x^2}\right)}{x^{\alpha\beta}\left(\log_e(1+x)\right)^\beta}\right)=0$

$$\Rightarrow \lim_{x\to\infty}\left(\frac{\sin(x^2)(\log_e x)^{\alpha}\dfrac{\sin\left(\frac{1}{x^2}\right)}{\frac{1}{x^2}}}{x^{(\alpha\beta+2)}\left(\log_e(1+x)\right)^{\beta}}\right)=0$$

$$\Rightarrow \lim_{x\to\infty}\left(\frac{\sin(x^2)(\log_e x)^{\alpha}}{x^{(\alpha\beta+2)}\left(\log_e(1+x)\right)^{\beta}}\right)=0$$

$$\Rightarrow \lim_{x\to\infty}\left(\frac{\sin(x^2)(\log_e x)^{\alpha}}{x^{(\alpha\beta+2)}\left((\log_e x)\left(1+\frac{\log_e\left(1+\frac{1}{x}\right)}{\log_e x}\right)\right)^{\beta}}\right)=0$$

$$\Rightarrow \lim_{x\to\infty}\left(\frac{\sin(x^2)(\log_e x)^{\alpha-\beta}}{x^{(\alpha\beta+2)}}\right)=0 \Rightarrow \alpha\beta+2>0 \Rightarrow (\alpha,\beta)=(-1,1),(1,-1)$$

6. Equation of line passing through P(1, 3, 2) parallel to the line $\frac{x-2}{1}=\frac{y-4}{2}=\frac{z-6}{1}$ is

$\frac{x-1}{1}=\frac{y-3}{2}=\frac{z-2}{1} \Rightarrow$ Point Q is (2, 5, 3)

Line passing through Q(2, 5, 3) and perpendicular to the plane $L_1 : x - y + 3z = 6$ is

$\frac{x-2}{1}=\frac{y-5}{-1}=\frac{z-3}{3} \Rightarrow$ Point R is (1, 6, 0) $\Rightarrow$ Length PQ is $\sqrt{6}$

$\Rightarrow$ Centroid of triangle PQR is $\left(\frac{4}{3},\frac{14}{3},\frac{5}{3}\right)$

$\Rightarrow$ Perimeter of triangle PQR is $\sqrt{6}+\sqrt{11}+\sqrt{13}$

7. Let OA_1 be $y = mx + c$

$\Rightarrow 0 = -4m + c$ or $c = 4m$

Also, $2 = mc$

$\therefore m = \pm\frac{1}{\sqrt{2}}$, $c = \pm 2\sqrt{2}$

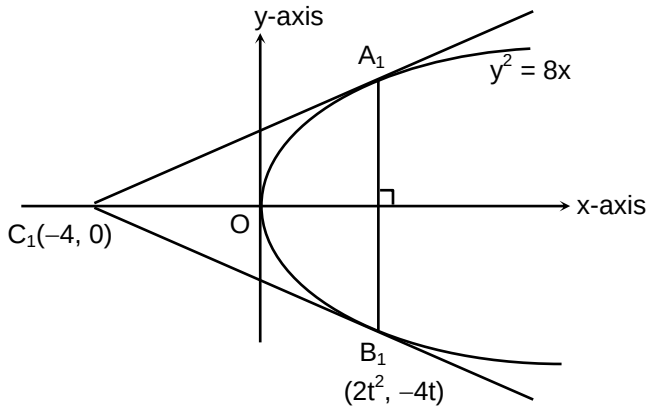
$\frac{1}{t} = m$

$\Rightarrow A_1(4,4\sqrt{2})$ and $B_1(4,-4\sqrt{2})$

Orthocentre is (0, 0)

$A_1B_1 = 8\sqrt{2}$

$OA_1 = 4\sqrt{3}$



8. $f(x) = kx$, $g(x) = a^x$

$f\left(-\frac{3}{5}\right)=12$, $g\left(-\frac{1}{3}\right)=2$

$-\frac{3k}{3}=12$, $a^{-\frac{1}{3}}=2$

$k = -20$, $\frac{1}{a}=8 \Rightarrow a=\frac{1}{8}$

$f(x) = -20x$, $g(x) = \left(\frac{1}{8}\right)^x$, $g(0) = 1$

$f\left(\frac{1}{4}\right) = -20 \times \frac{1}{4} = -5,\ g(-2) = \left(\frac{1}{8}\right)^{-2} = 64$

$f\left(\frac{1}{4}\right) + g(-2) - 8 = -5 + 64 - 8 = 51$

$\left[f\left(\frac{1}{4}\right) + g(-2) - 8\right] = 51$

9. $P\left(\frac{B_3}{W_1 \cap G_2}\right) = P\frac{(B_3 \cap W_1 \cap G_2)}{P(W_1 \cap G_2)} = \frac{2}{9}$

$\Rightarrow \frac{\frac{3}{N} \times \frac{N-9}{N-1} \times \frac{6}{N-2}}{\frac{3}{N} \times \frac{6}{N-1}} = \frac{2}{9}$

$\Rightarrow \frac{3 \times (N-9) \times 6}{(N-2) \times 18} = \frac{2}{9}$

$9N - 81 = 2N - 4$; $7N = 77$ $\therefore$ $N = 11$

10. $f(x) = \frac{(\sin x + 2)(x^{2023} + 2024x + 2025)}{e^{\pi x}(x^2 - x + 3)} = 0$

$\because$ $\sin x + 2 > 0$ $\forall$ $x \in R$

$e^{\pi x} > 0$ $\forall$ $x \in R$

$x^2 - x + 3 > 0$ $\forall$ $x \in R$

Now, let $g(x) = x^{2023} + 2024x + 2025$

$\because$ $g'(x) > 0$ $\forall$ $x \in R$ (strictly increasing)

$\therefore$ Number of solution of $f(x) = 0$ is 1

Solution is $(x = -1)$

11. $\vec{p} = 2\hat{i} + \hat{j} + 3\hat{k}$, $\vec{q} = \hat{i} - \hat{j} + \hat{k}$

$$\vec{p} \times \vec{q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 3 \\ 1 & -1 & 1 \end{vmatrix}$$

$\vec{p} \times \vec{q} = 4\hat{i} + \hat{j} - 3\hat{k}$

$2\vec{p} + \vec{q} = 5\hat{i} + \hat{j} + 7\hat{k}$

$\vec{p} - 2\vec{q} = 0\hat{i} + 3\hat{j} + \hat{k}$

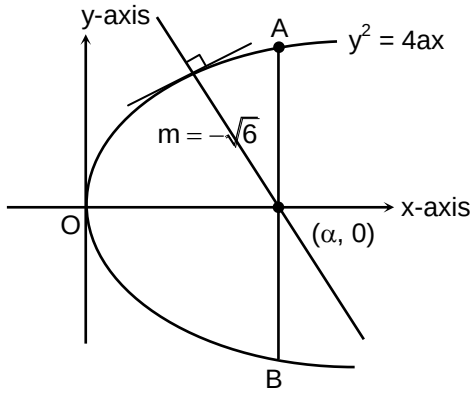
Now, $15\hat{i} + 10\hat{j} + 6\hat{k} = \alpha(2\vec{p} + \vec{q}) + \beta(\vec{p} - 2\vec{q}) + \gamma(\vec{p} \times \vec{q})$

$15\hat{i} + 10\hat{j} + 6\hat{k} = (5\alpha + 4\gamma)\hat{i} + (\alpha + 3\beta + \gamma)\hat{j} + (7\alpha + \beta - 3\gamma)\hat{k}$

$\Rightarrow 5\alpha + 4\gamma = 15$; $\alpha + 3\beta + \gamma = 10$; $7\alpha + \beta - 3\gamma = 6$

$\Rightarrow \gamma = 2$

12. Rotating the axis by an angle of 90° in clock-wise direction then slope of normal becomes $-\sqrt{6}$ and point (0, $-\alpha$) becomes (α, 0) and parabola being $y^2 = 4ax$



$\Rightarrow$ Equation of normal $y = mx - 2am - am^3$ passes through (α, 0)

$\Rightarrow 0 = -\sqrt{6}\alpha + 2\sqrt{6}a + 6\sqrt{6}a$

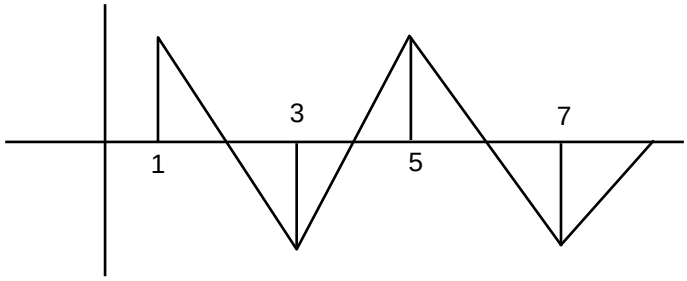
$\Rightarrow \alpha = 8a = at^2 \Rightarrow t = \pm 2\sqrt{2}$

$\Rightarrow AB = 4at = 8\sqrt{2}a$

Now $AB^2 = 64 \times 2 \times a^2$

$\Rightarrow \frac{r}{s} = \frac{4a}{128a^2} = \frac{1}{16} \Rightarrow a = \frac{1}{2} \Rightarrow 24a = 12$

13. $$f(x) = \begin{cases} 2 & x = 1 \\ 4 - 2x & 1 < x < 3 \\ -2 & x = 3 \\ 2x - 8 & 3 < x < 5 \\ 2 & x = 5 \\ 12 - 2x & 5 < x < 7 \\ -2 & x = 7 \\ 2x - 16 & 7 < x < 8 \end{cases}$$



$$g(x) = \int_1^x f(t)dt = 0$$

$\Rightarrow x = 3, 5, 7 \Rightarrow \alpha = 3$

$$\beta = \lim_{x \to 1^+} \frac{g(x)}{47} = \lim_{x \to 1^+} \frac{\int_1^x f(t)dt = 0}{x - 1}$$

$$= \lim_{x \to 1^+} \frac{f(x)}{1} = f(1) = 2$$

$\alpha + \beta = 3 + 2 = 5$

14. Total elements in relation R_0 such that $|a - b| \geq 2$ are 20

{(1, 3), (1, 4) (1, 5) (1, 6) (2, 4) (2, 5) (2, 6), (3, 1) (3, 5), (3, 6) (4, 1) (4, 2) (4, 6), (5, 1) (5, 2) (5, 3), (6, 1) (6, 2) (6, 3), (6, 4)}

But X is set of all subsets of R_0 which have exactly 6 elements

$n(X) = {}^{20}C_6$; ${}^{20}C_6 = {}^{m}C_6 \Rightarrow m = 20$

15. $n(Y) = 0$, as no relation R have only one element as its range

$n(Z) = 4 \times 3 \times 3 \times 3 \times 3 \times 4 = 1296$

as $n(Y) + n(Z) = k^2 = 1296 \therefore k = 36$

16. $$I_1 = 2\int_0^{\frac{\pi}{2}} f(x) \cdot g(x)dx = 2\int_0^{\frac{\pi}{2}} \sin^2 x \cdot g(x)dx$$

$$I_1 = 2\int_0^{\frac{\pi}{2}} \cos^2 x \cdot g(x)dx$$

$$2I_1 = 2\int_0^{\frac{\pi}{2}} g(x)dx$$

$$I_1 - \int_0^{\frac{\pi}{2}} g(x)dx = 0$$

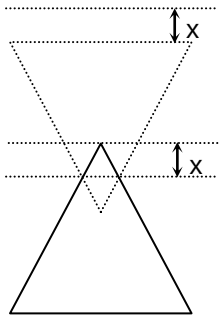
17. $$I = \frac{16}{\pi^3}\int_0^{\frac{\pi}{2}} f(x)g(x)dx = \frac{8}{\pi^3}\int_0^{\frac{\pi}{2}} g(x)dx$$

$$I = \frac{8}{\pi^3}\int_0^{\frac{\pi}{2}} \sqrt{\left(\frac{\pi}{4}\right)^2 - \left(\frac{\pi}{4} - x\right)^2}\, dx$$

$$I = \frac{1}{4}$$

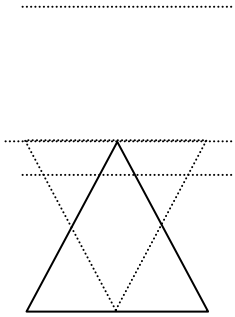
Physics

1. **A**



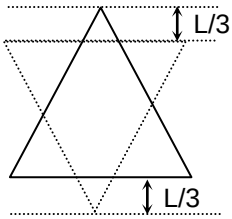
$\varepsilon = -\frac{Bv2x}{\sqrt{3}}$

Figure -1



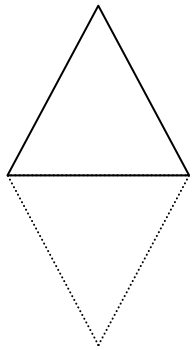
$\varepsilon = -\frac{Bv2x}{\sqrt{3}}$ where x = L

Figure -2



$\varepsilon = 0$, where $x = \frac{4L}{3}$

Figure -3



$\varepsilon = BvL$, where x = 2L

Figure -4

2. **A**

$$\frac{GMm}{r_0^2} = m\omega^2 r_0$$

$$\omega = \sqrt{\frac{GM}{r_0^3}}$$

$$T_0^2 = 4\pi^2\left(\frac{r_0^3}{GM}\right) \quad \ldots(1)$$

$$F = -\frac{dv_c(r)}{dr}$$

$$F = \frac{3m\alpha}{r^4}$$

Net force

$$\frac{GMm}{r_0^2} - \frac{3m\alpha}{r_0^4} = m\omega^2 r_0$$

$$\omega^2 = \frac{GM}{r_0^3} - \frac{3\alpha}{r_0^5}$$

We know $\omega = \frac{2\pi}{T_1}$

$$\frac{4\pi^2}{T_1 2} = \frac{GM}{r_0^3} - \frac{3\alpha}{r_0^5}$$

$$T_1^2 = \frac{4\pi^2}{\frac{GM}{r_0^3} - \frac{3\alpha}{r_0^5}} \quad \ldots(2)$$

From equation (1) and (2)

$$\frac{T_1^2 - T_0^2}{T_1^2} = \frac{3\alpha}{GMr_0^2}.$$

3. **A**

$$\frac{(K\alpha)_{z=46}}{\lambda_{cutoff}} = 2 \quad \ldots.(1)$$

$$\frac{(K\alpha)_{Z=41}}{\lambda_{cutoff}} = x \quad \ldots.(2)$$

Dividing both equation

$$\frac{1/(46-1)^2}{1/(41-1)^2} = \frac{2}{x}$$

x = 2.53

4. **A**

At equilibrium

$$(mg\sin\theta)r = \mu B_0 \cos\theta$$

$$(mg\sin\theta)r = (I\pi r^2)B_0 \cos\theta$$

$$\tan\theta = \frac{\pi r I B_0}{mg}$$

5. **B, D**

$$\sigma = \frac{Q}{4\pi R^2} \quad \ldots(1)$$

$$E = \frac{1}{4\pi\varepsilon_0}.\frac{Q}{r^2} = \frac{\sigma R^2}{\varepsilon_0 r^2}$$

Now $\vec{\tau} = \vec{p}_0 \times \vec{E}$

$$I\alpha = \frac{\sigma R^2}{\varepsilon_0 r^2} p_0 \theta \quad [\theta \to \text{small}]$$

$$\alpha = \frac{\sigma R^2}{\varepsilon_0 r^2} p_0 \theta$$

So, $\omega = \sqrt{\frac{\sigma R^2}{\varepsilon_0 r^2}}$

6. **A, B, D**

(A) $W_{ext} + W_g + W_B = \Delta K$ ($\because$ WD by viscous force = 0)

W_{ext} + mgh – ρvgh = 0

$W_{ext} = \rho_\ell vgh - mgh = gh\,(\rho_g v - m)$

$= 10 \times 0.7\left[10^3 \times \frac{4}{3}\pi(1.5\times10^{-2})^3 - \pi\times10^{-3}\right] = 0.77\,J$

(B) $W_g + W_B = \frac{1}{2}mv^2 - 0$

$0.77 = \frac{1}{2}\left(\frac{22}{7}\times10^{-3}\right)v^2 \Rightarrow v = 7$ m/s

(C) $H = \frac{v^2}{2g} = \frac{(7)^2}{2\times10} = 2.45\,m$

(D) Ratio $= \left|\frac{\rho_\ell Vg - Mg}{6\pi\eta rv}\right| = \frac{500}{9}J$

7. **A, B**

$x = 2R = \frac{2mv}{qB} = \sqrt{\frac{8mV}{qB^2}}$

$x_1 - x_0 = 14\times4 - 4 = 52$ cm

Minimum width w for $A_m = 196 \Rightarrow R = 14 \times 2 = 28$ cm

8. **3**

Percentage error in Volume

$\frac{\Delta V}{V}\times100 = 2\frac{\Delta d}{d}\times100 + \frac{\Delta h}{h}\times100 \qquad \left(\frac{\Delta d}{d} = \frac{\Delta h}{h} = \frac{1}{100}\right)$

$\Rightarrow \frac{\Delta V}{V}\times100 = 3\%$

9. **2**

$v_0 \sin 45^0 = v_1 \sin 45^0$ $\Rightarrow v_1 = v_0$ … (I case)

$v_0 \sin 60^0 = v_2 \sin 30^0$ $\Rightarrow v_2 = v_0\sqrt{3}$ … (II case)

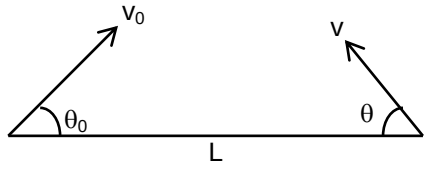
$T_1 = \frac{L}{v_1\cos45^0 + v_0\cos45^0} = \frac{L}{v_0(\sqrt{2})}$ $\Rightarrow T_1^2 = \frac{L^2}{2v_0^2} = v_0\sqrt{3}$

$T_2 = \frac{L}{v_2\cos30^0 + v_0\cos60^0}$

$= \frac{L}{(v_0\sqrt{3}).\frac{\sqrt{3}}{2} + \frac{v_0}{2}} = \frac{2L}{4v_0} = \frac{L}{2v_0}$

$\Rightarrow T_2^2 = \frac{L^2}{4v_0^2}$

$\left(\frac{T_1}{T_2}\right)^2 = 2$



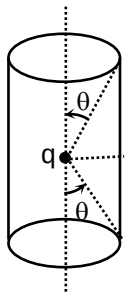
10. **3**

$$\phi_1 = \frac{q}{\varepsilon_0} - \frac{q}{\varepsilon_0}\left(\frac{1-\cos 30^\circ}{2}\right)\times 2 = \frac{q}{\varepsilon_0}\left[1-1+\frac{\sqrt{3}}{2}\right] = \frac{\sqrt{3}}{2}\frac{q}{\varepsilon_0}$$

$$\phi_2 = \frac{q}{\varepsilon_0} - \frac{q}{\varepsilon_0}\left(\frac{1-\cos 60^\circ}{2}\right)\times 2 = \frac{1}{2}\frac{q}{\varepsilon_0}$$

$$\frac{\phi_2}{\phi_1} = \frac{1/2}{\sqrt{3}/2} = \frac{1}{\sqrt{3}}$$

$$\phi_2 = \frac{\phi_1}{\sqrt{3}} \Rightarrow N = 3$$



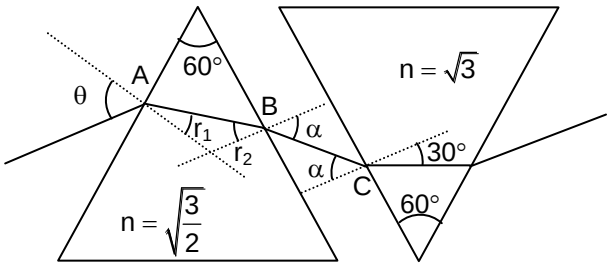
11. **12**

At C, $1\sin\alpha = \sqrt{3}\sin 30^\circ \Rightarrow \alpha = 60^\circ$

At B, $\sqrt{\frac{3}{2}}\sin r_2 = 1\sin\alpha \Rightarrow r_2 = 45^\circ$

At A, $1\sin\theta = \sqrt{\frac{3}{2}}\sin 15^\circ$

$$\Rightarrow \theta = \sin^{-1}\left[\sqrt{\frac{3}{2}}\sin\left(\frac{\pi}{12}\right)\right]$$

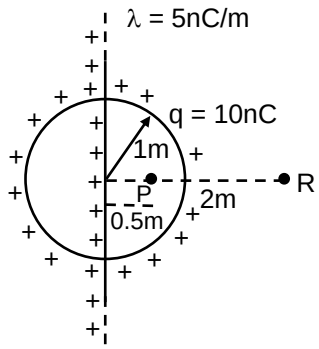


12. **171**

$$(V_P - V_R)_{\text{line charge}} = 2k\lambda \ln\frac{r_R}{r_P} = 126\,V$$

$$(V_P - V_R)_{\text{sphere}} = kq\left(\frac{1}{1}-\frac{1}{r_R}\right) = \frac{kq}{2} = 45\,V$$

$$V_P - V_R = 126 + 45 = 171\,V.$$



13. **96 Pa**

Since 144 Pa is negligible compared to P_0. Therefore

$$P_0\frac{4}{3}\pi r_1^3 = \frac{8P_0}{27}\left(\frac{4}{3}\pi r_2^3\right)$$

$$\Rightarrow r_2 = \frac{3}{2}r_1$$

$$\Rightarrow \Delta P' = \frac{4T}{r_2} = \frac{4T}{\frac{3}{2}r_1} = \frac{\Delta P_0 \times 2}{3} = 96\ \text{Pa}$$

14. **601.50**

$$\Delta y = \frac{8\lambda D}{(0.8-0.04)\times 10^{-3}} - \frac{8\lambda D}{(0.8+0.04)\times 10^{-3}} = 601.5\ \mu\text{m}.$$

15. **24**

$$y = \frac{8\times 6000\times 10^{-10}\times 1}{(0.8+0.04\sin\omega t)\times 10^{-3}}$$

$$y = \frac{48\times 10^{-4}}{(0.8+0.04\sin\omega t)}$$

$$\left|\frac{dy}{dt}\right| = +\frac{48\times 10^{-4}}{(0.8+0.04\sin\omega t)^2}(0.04\omega\cos\omega t)$$

$t = 0$, $v = v_{max}$

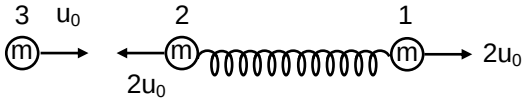
$$v_{max} = \frac{48\times 10^{-4}\times 0.04\times 0.08}{(0.8)^2} = 24\times 10^{-6}\,m/s = 24\ \mu m/s$$

16. **0.75**

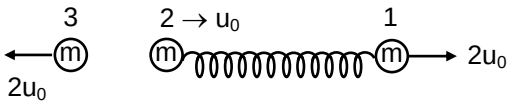
If collision occurs at $t_0 = 0$

Then, $v_1(t) = a\omega = 2u_0$

$v_2(t) = -a\omega = -2u_0$



Before collision



After collision

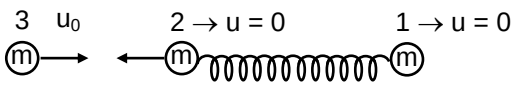
$$V_{CM} = \frac{mu_0 + 2mu_0}{m+m} = \frac{3u_0}{2}$$

$$V_{CM} = \frac{3}{4}a\omega$$

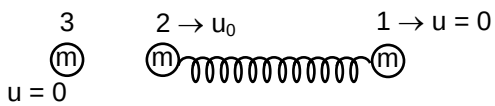
$$\Rightarrow \quad \frac{V_{CM}}{(a\omega)} = \frac{3}{4} = 0.75$$

17. **4.25**

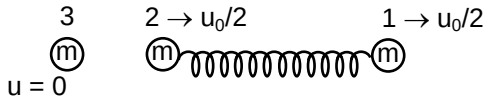
If collision occurs at $t_0 = \frac{\pi}{2\omega}$



Before collision



After collision



Final Situation

From conservation of mechanical energy

$$\frac{1}{2}K(2a)^2+\frac{1}{2}mu_0^2=\frac{1}{2}K(2b)^2+\frac{1}{2}(2m)\left(\frac{u_0}{2}\right)^2$$

Here $K=\frac{m\omega^2}{2}$ and $u_0=\frac{a\omega}{2}$

Solving we get $\frac{4b^2}{a^2}=\frac{17}{4}=4.25$

Chemistry

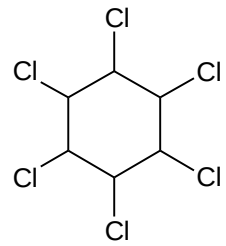
1. $K.E. = +13.6\frac{z^2}{n^2}eV$

(A) n = 1 z = 1	K. E. = +13.6 eV
(B) n = 1 z = 2	K. E. = +54.4 eV
(C) n = 2 z = 2	K. E. = +13.6 eV
(D) n = 2 z = 3	K. E. = +30.6 eV

2. $\mathbf{M_xY_2O_4}$

$+2\times\frac{x}{3}+3\times\frac{2x}{3}=+2$

$x = 0.75$

3. L – Glucose $\xrightarrow[\text{ii) } Cr_2O_3,\ 775\ K,\ 10-20\ atm]{\text{i) HI, }\Delta}$ (P) $\xrightarrow[UV]{Cl_2(\text{excess})}$ 

4. On fluorination of PCl_5 in polar organic solvent ionic isomers are formed, i.e.

$[PCl_4]^+[PCl_4F_2]^- \rightarrow (\text{Colourless crystal})$

$[PCl_4]^+[PF_6]^- \rightarrow (\text{White crystal})$

5. At anode

$$N_2H_4 + 4OH^- \longrightarrow N_2 + 4H_2O + 4e^-$$

At cathode

$$O_2 + 2H_2O + 4e^- \longrightarrow 4OH^-$$

Overall reaction $N_2H_4 + O_2 \longrightarrow N_2 + 2H_2O$

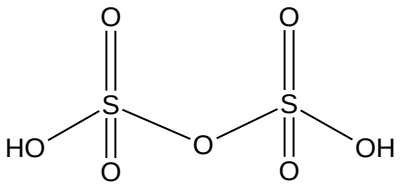
$N_2H_4 + 2O_2 \longrightarrow 2NO + 2H_2O$

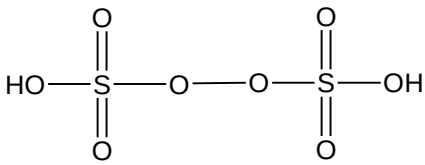
$N_2H_4 + 3O_2 \longrightarrow 2NO_2 + 2H_2O$

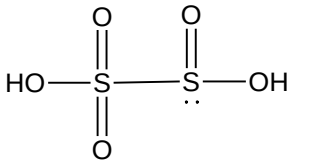
6. Reagent

(C) i) $NaBH_4$; ii) $SnCl_2/HCl$; iii) H_3O^+; iv) Lindlar's catalyst, H_2;

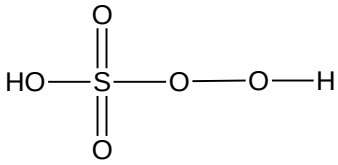
(D) i) Lindlar's catalyst, H_2; ii) $NaBH_4$; iii) $SnCl_2/HCl$; iv) H_3O^+

7.  $H_2S_2O_7$

 $H_2S_2O_8$



$H_2S_2O_5$



H_2SO_5

8. Number of moles of $NaOH = 40\times1\times10^{-3} = 4\times10^{-2}$

Number of moles of Acetic acid $= 100\times0.5\times10^{-3} = 5\times10^{-2}$

$\therefore$ Number of moles of acetic acid adsorbed on charcoal $= 5\times10^{-2} - 4\times10^{-2} = 1\times10^{-2}$

$\therefore$ Number of molecules of acetic acid adsorbed $= 1\times10^{-2}\times N_A$

$= 6\times10^{21}$

Total surface area adsorbed by one molecule of acetic acid $= \dfrac{1.5\times10^2}{6\times10^{21}} = 2500\times10^{-23}$

$\therefore P = 2500$

9. $\left(\Delta T_b\right)_1 = i_1k_f \times \dfrac{W_2}{\dfrac{M_x}{\left(\dfrac{W_1}{1000}\right)}}$

$\left(\Delta T_b\right)_2 = i_2k_f \times \dfrac{W_2}{\dfrac{M_y}{\left(\dfrac{W_1}{1000}\right)}}$

$\dfrac{\left(\Delta T_b\right)_1}{\left(\Delta T_b\right)_2} = \dfrac{i_1}{i_2}\times\dfrac{M_y}{M_x} = 1.2\times\dfrac{1}{0.8} = 1.5$

$\left(\Delta T_b\right)_1 = 1.5\left(\Delta T_b\right)_2$

Ans. 150

10. Seven A, T and Six G, C pairs are there.

Energy required $= 7\times2\times1+6\times3\times1.5 = 41$ kcal

11. Mass = Number of moles × Molar mass

$\dfrac{(\text{Mass})P_b}{(\text{Mass})U} = \dfrac{n_{Pb}\times206}{n_U\times238} = \dfrac{7}{1}$

$\dfrac{n_{Pb}}{n_U} = \dfrac{238\times7}{206}$

$\ell n\left(1+\dfrac{n_{Pb}}{n_U}\right) = \lambda t$

$\ell n\left(1+\dfrac{238\times7}{206}\right) = \dfrac{\ell n2}{t_{1/2}}\times t$

$$\ell n\left(\frac{206+1666}{206}\right)=\frac{\ell n2}{t_{1/2}}\times t$$

$$\ell n(9)=\frac{\ell n2}{t_{1/2}}\times t$$

$$t=\frac{\ell n9}{\ell n2}\times t_{1/2}$$

$$t=\frac{2\times 0.4771}{0.3010}\times 4.5\times 10^{9}$$

$$t=14.265\times 10^{9}=142.65\times 10^{8}\text{ years}$$

$$P=142.65$$

12. $\left[Co(CN)_4\right]^{4-}\rightarrow$ Tetrahedral

$\left[Co(CO)_3(NO)\right]\rightarrow$ Tetrahedral

$XeF_4\rightarrow$ Square planar

$\left[PCl_4\right]^{+}\rightarrow$ Tetrahedral

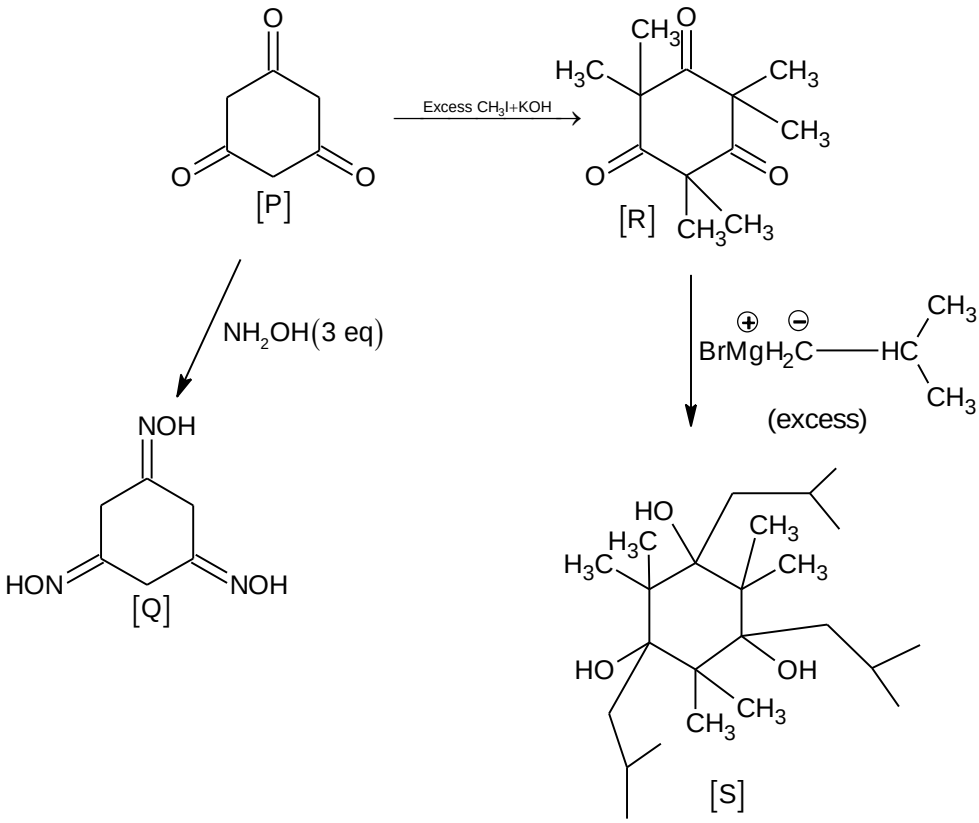
$\left[PdCl_4\right]^{2-}\rightarrow$ Square planar

$\left[ICl_4\right]^{-}\rightarrow$ Square planar

$\left[Cu(CN)_4\right]^{3-}\rightarrow$ Tetrahedral

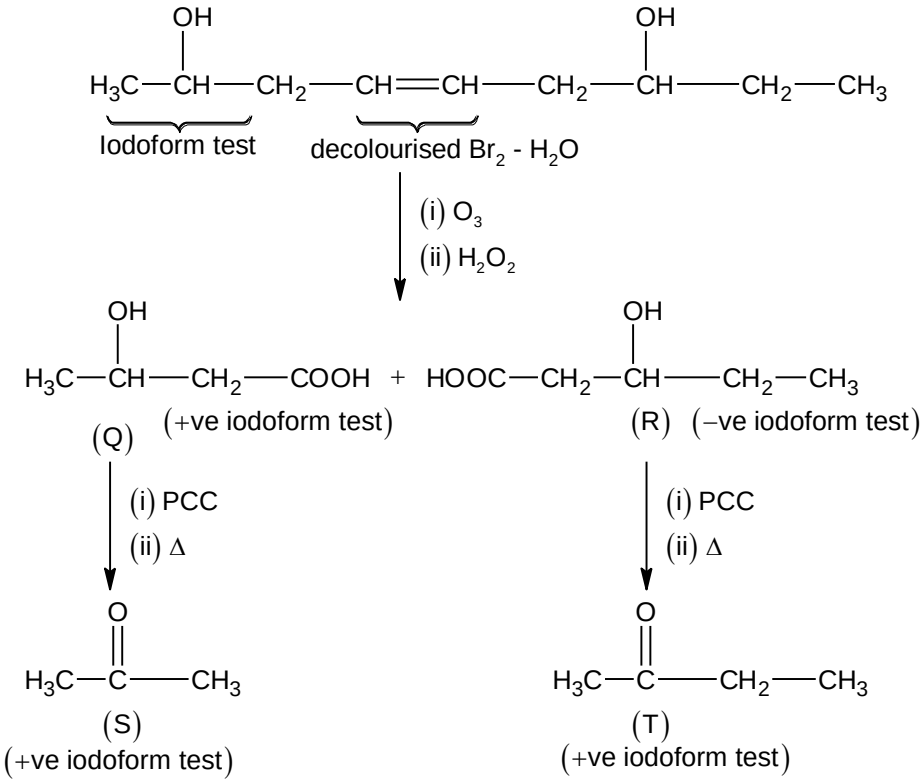
$P_4\rightarrow$ Tetrahedral

13.



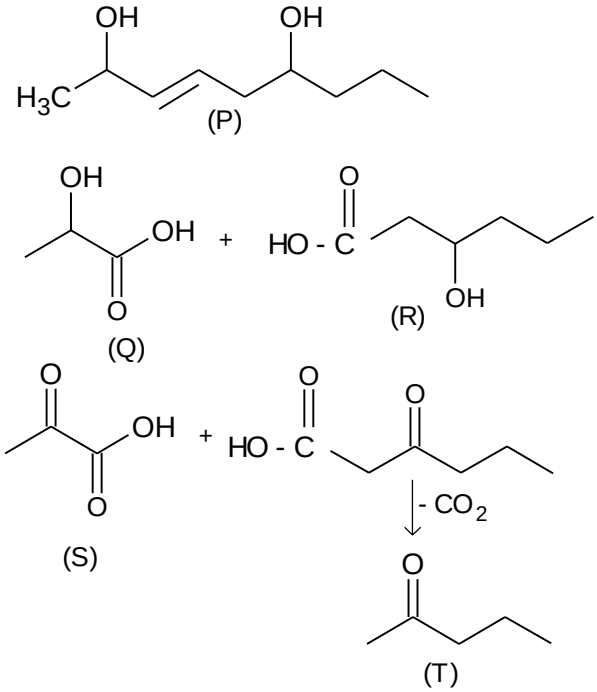
∴ Number of methyl group in product (S) = 12

14.



Sum of oxygen atom in S and T = 1 + 1 = 2

Alternative Solution



Sum of O atom = 3 + 1 = 4

15. $Q + R \longrightarrow \text{Polymer}(U)$

Since moles of U = 1.

$$\text{Moles} = \frac{\text{Weight}}{\text{Molar mass}}$$

Weight of 'U' = Weight of (Q + R) – Weight of water
Weight of 'Q' = 500 × 104 = 52,000 g
Weight of 'R' = 500 × 118 = 59,000 g
Weight of 'H_2O' = 999 × 18 = 17982 g
Weight of polymer = 52000 + 59000 – 17982
= 93018 g/mol

16. $3KI + 2K_3\left[Fe(CN)_6\right] \longrightarrow 2K_4\left[Fe(CN)_6\right] + KI_3$

(P)

17. $3ZnCl_2 + 2K_4\left[Fe(CN)_6\right] \longrightarrow K_2Zn_3\left[Fe(CN)_6\right]_2$

Excess White ppt. (Q)

$2ZnCl_2 + K_4\left[Fe(CN)_6\right] \longrightarrow Zn_2\left[Fe(CN)_6\right] + 4KCl$

White ppt. (Q)