JEE MAIN 2024

JANUARY SESSION

OFFICIAL QUESTION PAPERS WITH DETAILED SOLUTIONS



10 sets having 900 questions with detailed solutions of Physics, Chemistry and Mathematics

CAREERS 360

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27 Jan - Morning Shift

Physics

Q. 1 A wire of length 10 cm and radius $\sqrt{7} \times 10^{-4}\ m$ is connected across the right gap of the meter bridge. When a resistance of 4.5Ω is connected on the left gap by using a resistance box, the balance length is found to be at $60\ cm$ from the left end. If the resistivity of the wire is $R \times 10^{-7}\Omega\ m$, then the value of R is :

Option 1:
66

Option 2:
70

Option 3:
35

Option 4:
63

Correct Answer:
66

Solution:

$$\frac{4.5}{60} = \frac{R}{40}$$

$$R = 3\Omega$$

$$R = \frac{\rho L}{A} = \frac{\rho \times 0.1}{\pi\left(\sqrt{7} \times 10^{-4}\right)^2} = 3$$

$$\rho = 65.97$$

$$\rho \simeq 66$$

Hence, the answer is the option (1).

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Q. 2 A proton moving with a constant velocity passes through a region of space without any change in its velocity if $\vec{E}$ and $\vec{B}$ represents the electric and magnetic fields respectively, then the region of space may have :

$(A)\mathrm{E}=0,\ \mathrm{B}=0$

$(B)\mathrm{E}=0,\ \mathrm{B}\neq 0$

$(C)\mathrm{E}\neq 0,\ \mathrm{B}=0$

$(D)\mathrm{E}\neq 0,\ \mathrm{B}\neq 0$

Choose the most appropriate answer from the options given below ;

Option 1:
(A), (B) and (C) only

Option 2:
(A), (C) and (D) only

Option 3:
(A), (B) and (D) only

Option 4:
(B), (C) and (D) only

Correct Answer:
(A), (B) and (D) only

Solution:

$$\vec{\mathrm{F}}_{\mathrm{E}}=\mathrm{q}\vec{\mathrm{E}}$$

$$\vec{\mathrm{F}}_{\mathrm{B}}=\mathrm{q}(\vec{\mathrm{V}}\times\vec{\mathrm{B}})$$

Case (a) is correct when $\vec{E}\|\vec{B}\|\vec{V}$

Case (b) is correct when $\vec{V}\|\vec{B}$

Case (d) is correct when $\vec{\mathrm{E}}\perp\vec{\mathrm{B}}\perp\vec{\mathrm{V}}$

Hence, the answer is the option (3).

Q. 3 Given below are two statements :
Statements (I): Planck's constant and angular momentum have the same dimensions
Statements (II): Linear momentum and moment of force have the same dimensions
In light of the above statements choose the correct answer from the option given below

Option 1:
Statements (I) is true but statement II is false

Option 2:
Statements (I) is false but statement II is true

Option 3:
Both statement I and statement II are true

Option 4:
Both statement I and statement II are false

Correct Answer:
Statements (I) is true but statement II is false

Solution:

$(1)\ L = \frac{n}{2\pi}h$

Statement I is true.

(2) $p = mv \Rightarrow [p] = [MLT^{-1}]$

$\tau = rF \Rightarrow [\tau] = [ML^2T^{-2}]$

Statement II is false.

Hence, the answer is the option (1).

Q. 4 Two bodies of mass 4 g and 25 g are moving with equal kinetic energies. The ratio of the magnitude of their linear momentum is:

Option 1:
4:5

Option 2:
3:5

Option 3:
2:5

Option 4:
5:4

Correct Answer:
2:5

Solution:

$$KE = \frac{1}{2}mv^2 = \frac{p^2}{2\ m}$$

$$\frac{KE_1}{KE_2} = \frac{p_1^2}{2\ m_1} \times \frac{2mv^2}{p_2^2}$$

$$\frac{m_1}{m_2} = \frac{p_1^2}{p_2^2}$$

$$\sqrt{\frac{m_1}{m_2}} = \frac{p_1}{p_2} = \sqrt{\frac{4}{25}} = \frac{2}{5}$$

Hence, the answer is the option (3).

Q. 5 A wire of resistance R and length L is cut into 5 equal parts. If these parts are joined parallelly, then the resultant resistance will be:

Option 1:
$\frac{1}{25}R$

Option 2:
5 R

Option 3:
25 R

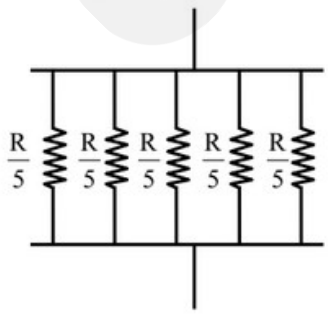
Option 4:
$\frac{1}{5}R$

Correct Answer:
$\frac{1}{25}R$

Solution:
$R_{\text{esistance}} = \frac{\rho L}{A}$

$R \propto L$

$R \rightarrow \frac{R}{5}$



Since parallel combination

So, $R_{eq} = \frac{1}{25}R$

Q. 6 A plane electromagnetic wave propagating in an x-direction is described by $E_y = (200 Vm^{-1}) \sin [1.5 \times 10^7 t - 0.05 x]$; The intensity of the wave is:
(Use $\varepsilon_0 = 8.85 \times 10^{-12} C^2\ N^{-1}\ m^{-2}$)

Option 1:
53.1 Wm^{-2}

Option 2:
106.2 Wm^{-2}

Option 3:
35.4 Wm^{-2}

Option 4:
26.6 Wm^{-2}

Correct Answer:
53.1 Wm^{-2}

Solution:

$$I = \frac{\varepsilon_0}{2} E_0^2 C$$

$$= \frac{8.85 \times 10^{-12}}{2} \times (200)^2 \times 3 \times 10^8 = 53.10 Wm^{-2}$$

Hence, the answer is the option (1).

Q. 7 The position of an ant (S in meters) moving in the Y-Z plane is given by $S = 2t^2\hat{j} + 5\hat{k}$ (where t is in second). The magnitude and direction of the velocity of the ant at t=1s will be :

Option 1:
9 m/s in z-direction

Option 2:
16 m/s in y-direction

Option 3:
4 m/s in x-direction

Option 4:
4 m/s in y-direction

Correct Answer:
4 m/s in y-direction

Solution:

$\overrightarrow{s} = 2t^2\hat{j} + 5\hat{k}$

$At\ t = 1sec$

$\overrightarrow{v} = \frac{ds}{dt} = 4\hat{j} + 0$

$\overrightarrow{v} = 4\hat{j}m/s$

Hence, the answer is the option (4).

Q. 8 Identify the physical quantity that cannot be measured using a spherometer:

Option 1:
Specific gravity of liquids

Option 2:
Radius of curvature of concave surface

Option 3:
Thickness of thin plates

Option 4:
Radius of curvature of convex surface

Correct Answer:
Specific gravity of liquids

Solution:
Spherometer not used to measure the specific gravity of liquids

Hence, the answer is the option (1).

Q. 9 The acceleration due to gravity on the surface of the earth is g. If the diameter of the earth reduces to half of its original value and mass remains constant, then acceleration due to gravity on the surface of the earth would be ;

Option 1:
4 g

Option 2:
g/4

Option 3:
2 g

Option 4:
g/2

Correct Answer:
4 g

Solution:

$$g = \frac{GM}{R^2}$$

$$if\, R \rightarrow \frac{R}{2}$$

$$g' = \frac{GM}{\left(\frac{R}{2}\right)^2} = \frac{4GM}{R^2}$$

$$\frac{g}{g'} = \frac{\frac{GM}{R^2}}{\frac{4GM}{R^2}}$$

$$g' = 4\, g$$

Hence, the answer is the option (1).

Q. 10 A train is moving with a speed of $12\ \mathrm{m/s}$ on rails which are $1.5\ \mathrm{m}$ apart. To negotiate a curve of radius $400\ \mathrm{m}$, the height by which the outer rail should be raised with respect to the inner rail is (Given, $g = 10\ \mathrm{m/s^2}$)

Option 1:
4.8 cm

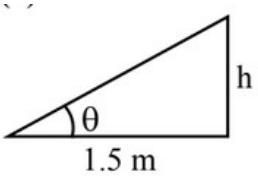
Option 2:
4.2 cm

Option 3:
6.0 cm

Option 4:
5.4 cm

Correct Answer:
5.4 cm

Solution:



$$v = \sqrt{Rg \tan\theta}$$
$$\tan\theta = \frac{(12)^2}{400 \times 10}$$
$$\frac{h}{1.5} = 3.6 \text{ cm}$$
$$h = 5.4 \text{ cm}$$

Q. 11 If the refractive index of the material of a prism is $\cot\left(\frac{A}{2}\right)$, where A is the angle of the prism, then the angle of minimum deviation will be :

Option 1:
$\pi - A$

Option 2:
$\pi - 2\ A$

Option 3:
$\frac{\pi}{2} - 2\ A$

Option 4:
$\frac{\pi}{2} - A$

Correct Answer:
$\pi - 2\ A$

Solution:

$$\mu = \cot\frac{A}{2} = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\frac{A}{2}}$$
$$\cos\frac{A}{2} = \sin\left(\frac{A+\delta_m}{2}\right)$$
$$\frac{A}{2} = 90^\circ - \left(\frac{A+\delta_m}{2}\right)$$
$$\delta_m = 180^\circ - 2\ A = \pi - 2A$$

Hence, the answer is the option (2).

Q. 12 0.08 kg air is heated at constant volume through $5^\circ C$. The specific heat of air at constant volume is $0.17 kcal/kg\ ^\circ C$ and J = 4.18 joule/cal. The change in its internal energy is approximately

Option 1:
142 J

Option 2:
298 J

Option 3:
284 J

Option 4:
318 J

Correct Answer:
284 J

Solution:

$$W = 0$$

$$Q = ms\Delta T = \Delta U = 0.08 \times 0.17 \times 10^3 \times 4.18 \times 5$$

$$= 0.28424 \times 10^3 \text{ J}$$
$$= 284 \text{ J}$$

Hence, the answer is the option (3).

Q. 13 A body of mass 1000 kg is moving horizontally with a velocity 6 m/s. If 200 kg extra mass is added, the final velocity (in m/s) is :

Option 1:
2

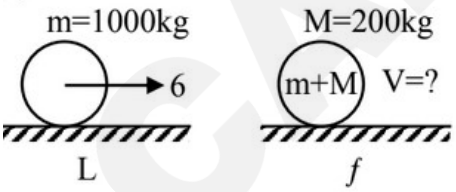
Option 2:
6

Option 3:
3

Option 4:
5

Correct Answer:
5

Solution:



From conservation of momentum

$$mV_L = (m+M)V_f$$
$$1000 \times 6 = 1200 \times V_f$$
$$\frac{1000 \times 6}{1200} = V_f$$
$$V_f = 5 \text{ m/s}$$

Q. 14 A convex lens of focal length 40 cm forms an image of an extended source of light on a photoelectric cell, A current I is produced. The lens is replaced by another convex lens having the same diameter but a focal length of 20 cm. Then photoelectric current now is :

Option 1:
I

Option 2:
I/2

Option 3:
2 I

Option 4:
4 I

Correct Answer:
I

Solution:
The no of photons reaching the photo-sensitive metal is the same in both cases. So, the photocurrent will be the same.

Hence, the answer is the option (1).

Q. 15 A rectangular loop of length 2.5 m and width 2 m is placed at 60 degrees to a magnetic field of 4 T. The loop is removed from the field in 10 s. The average emf induced in the loop during this time is :

Option 1:
+ 1 V

Option 2:
+ 2 V

Option 3:
– 2 V

Option 4:
– 1 V

Correct Answer:
+ 1 V

Solution:
$$\varepsilon_{\text{avg}} = -\frac{\Delta\phi}{\Delta t} = -\left[\frac{0. - 4 \times (2.5 \times 2)\cos 60^{\circ}}{10}\right] \text{ Volts } = +1 \text{ Volt}$$

Hence, the answer is the option (1).

Q. 16 *An electric charge* $10^{-6}\mu C$ *is placed at origin* $(0,0)$m *of* X−Y *co−ordinate system, Two points* P *and* Q *are situated at* $(\sqrt{3},\sqrt{3})$m *and* $(\sqrt{6},0)$m *respectively. The potential difference between the points* P *and* Q *willbe :*

Option 1:
$\sqrt{6}$ V

Option 2:
0 V

Option 3:
3 V

Option 4:
$\sqrt{3}$ V

Correct Answer:
0 V

Solution:

$$V = \frac{KQ}{r}$$

$For P, (\sqrt{3}, \sqrt{3}),$

$$V = \frac{KQ}{\left(\sqrt{(\sqrt{3})^2 + (\sqrt{3})^2}\right)} = \frac{KQ}{\sqrt{6}}$$

$For\ Q, (\sqrt{6}, 0),$

$$V = \frac{KQ}{\left(\sqrt{(\sqrt{6})^2}\right)} = \frac{KQ}{\sqrt{6}}$$

$Potential\ difference\ between P\ and\ Q = 0$

Hence, the answer is the option (2).

Q. 17 Given below are two statements :
Statements (I): The viscosity of gases is greater than that of liquids
Statements (II): The surface tension of a liquid decreases due to the presence of insoluble impurities.
In the light of the above statements, choose the most appropriate answer from the options given below:

Option 1:
Statement I is correct but Statement II is incorrect

Option 2:
Both statement I and Statement II are correct

Option 3:
Both statement I and statement II are incorrect

Option 4:
Statement I is incorrect but Statement II is correct

Correct Answer:
Statement I is incorrect but Statement II is correct

Solution:
The viscosity of liquid is greater than that of gases as intermolecular forces between the layer of liquid are greater than that of gases.

Surface tension can be increased or decreased depending on the type of impurity.

Hence, the answer is the option (4).

Q. 18 The radius of the third stationary orbit of the electron for Bohr's atom is R. The radius of the fourth stationary orbit will be :

Option 1:
$\frac{16}{9}\mathrm{R}$

Option 2:
$\frac{9}{16}\mathrm{R}$

Option 3:
$\frac{3}{4}\mathrm{R}$

Option 4:
$\frac{4}{3}\mathrm{R}$

Correct Answer:
$\frac{16}{9}\mathrm{R}$

Solution:
$\mathrm{r} \propto \mathrm{n}^2$

$$\frac{\mathrm{R}}{\mathrm{R}_4} = \frac{9}{16}$$

$$\mathrm{R}_4 = \frac{16\mathrm{R}}{9}$$

Hence, the answer is the option (1).

Q. 19 The average kinetic energy of a monoatomic molecule is 0.414 eV at temperature: $\left(Use\ K_B = 1.38 \times 10^{-23}\ \mathrm{J/mol - K} \right)$

Option 1:
3000 K

Option 2:
3200 K

Option 3:
1500 K

Option 4:
1600 K

Correct Answer:
3200 K

Solution:

$$\frac{3}{2} \times T \times 1.38 \times 10^{-23} = 0.414 eV$$

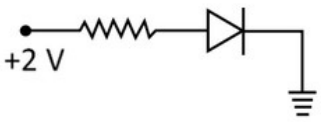
$$T = 0.414 \times 1.6 \times 10^{-19}$$

$$T = 3200\ K$$

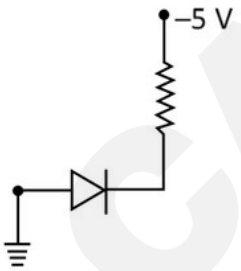
Hence, the answer is the option (2).

Q. 20 Which of the following circuits is reverse-biased?

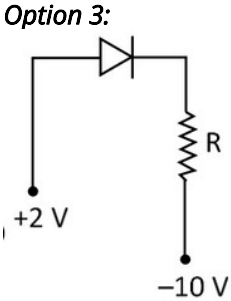
Option 1:



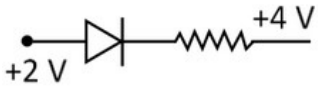
Option 2:



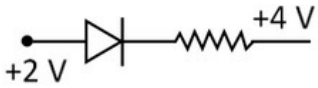
Option 3:



Option 4:



Correct Answer:



Solution:
Option (4) is RB
Because In PN, N is at high voltage.

Q. 21 Four particles each of mass $1\ \text{kg}$ are placed at four corners of a square of side $2\ \text{m}$. Moment of inertia of system about an axis perpendicular to its plane and passing through one of its vertex is ___ kgm^2.

Correct Answer:
16

Solution:
$$\text{I} = 1\ \text{kg}\left(0^2 + 2^2 + 2^2 + (2\sqrt{2})^2\right)\text{m}^2$$
$$= 16\text{kgm}^2$$

Hence, the answer is the (16).

Q. 22 A particle executes simple harmonic motion with an amplitude of $4\ \text{cm}$. At the mean position, velocity of the particle is $10\ \text{cm/s}$. The distance of the particle from the mean position when its speed becomes $5\ \text{cm/s}$ is $\sqrt{\alpha}\ \text{cm}$, where $\alpha =$

Correct Answer:
12

Solution:
$$\text{v} = \frac{\omega \text{A}}{2} = \omega\sqrt{\text{A}^2 - \text{x}^2}$$
$$\text{x} = \frac{\sqrt{3}}{2}\ \text{A} = 2\sqrt{3}\ \text{cm}$$
$$= \sqrt{12}\ \text{cm}$$

Hence, the answer is (12).

Q. 23 In a nuclear fission process, a high-mass nuclide $(\text{A} \approx 236)$ with a binding energy of 7.6 MeV/Nucleon dissociated into middle-mass nuclides A=118, having a binding energy of 8.6 MeV/Nucleon. The energy released in the process would be ___ MeV.

Correct Answer:
236

Solution:
$X \longrightarrow 2Y$

$$BE = (8.6 \times 118)2 - (236 \times 7.6) = 236MeV$$

Hence, the answer is (236)

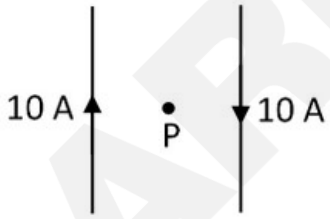
Q. 24 Two coils have a mutual inductance of 0.002 H. The current changes in the first coil according to the relation $i = i_0 sin\omega t$, where $i_0 = 5\ A$ and $\omega = 50\pi rad/s$. The maximum value of emf in the second coil is $\frac{\pi}{\alpha}V$. The value of α is _____.

Correct Answer:
2

Solution:
$$\phi_2 = Mi_2\varepsilon_2 = -\frac{d\phi_2}{dt} = -Mi_0\omega \cos\omega t$$
$$\varepsilon_2 \max = Mi_0\omega = (0.002)(5)(50\pi) = (0.5)\pi = \frac{\pi}{2}$$

Hence, the answer is (2).

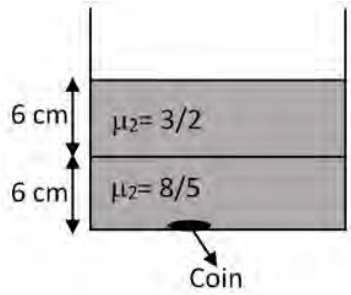
Q. 25 Two long, straight wires carry equal currents in opposite directions as shown in figure. The separation between the wires is $5.0\ cm$. The magnitude of the magnetic field at point P midway between the wires is μT.
(Given : $\mu_0 = 4\pi \times 10^{-7} TmA^{-1}$)



Correct Answer:
160

Solution:
$$B_P = 2\left[\frac{\mu_0 I}{2\pi d}\right] = \frac{\mu_0}{\pi}\frac{10}{2.5 \times 10^{-2}}$$
$$= \frac{4 \times 10^{-7} \times 10^4}{25} T = \frac{4}{25} \times 10^{-3}\ T = 160\mu T$$

Q. 26 Two immiscible liquids of refractive indices $\frac{8}{5}$ and $\frac{3}{2}$ respectively are put in a beaker as shown in the figure. The height of each column is $6\ \text{cm}$. A coin is placed at the bottom of the beaker. For near-normal vision, the apparent depth of the coin is $\frac{\alpha}{4}\ \text{cm}$. The value of α is _____.



Correct Answer:
31

Solution:
(1)

$$\text{App.Shift} = 6\left[1-\frac{1}{3/2}\right]+6\left[1-\frac{1}{8/5}\right]$$
$$= 2+\frac{9}{4} = 4.25\ \text{cm}$$

(2)

$$\text{App. depth} = 12-4.25 = 7.75\ \text{cm}$$
$$= \frac{31}{4}\ \text{cm}$$

Q. 27 If average depth of an ocean is $4000\ \text{m}$ and the bulk modulus of water is $2\times10^{9}\text{Nm}^{-2}$, then fractional compression $\frac{\Delta V}{V}$ of water at the bottom of ocean is $\alpha\times10^{-2}$. The value of α is ____. (Given, $g = 10\ \text{ms}^{-2}, \rho = 1000\ \text{kg m}^{-3}$)

Correct Answer:
2

Solution:

$$\frac{\Delta V}{V} = -\frac{\Delta P}{B} = -\frac{-\rho g h}{B}$$
$$\frac{\Delta V}{V} = \frac{10^3\times10\times4000}{2\times10^9} = 2\times10^{-2}$$

Hence, the answer is (2).

Q. 28 A particle starts from the origin at t=0 with a velocity $5\hat{i}\ \text{m/s}$ and moves in an x-y plane under the action of a force which produces a constant acceleration of $(3\hat{i}+2\hat{j})\text{m/s}^2$. If the x-coordinate of the particle at that instant is 84 m, then the speed of the particle at this time is $\sqrt{\alpha}\ \text{m/s}$. The value of α is ___.

Correct Answer:
673

Solution:

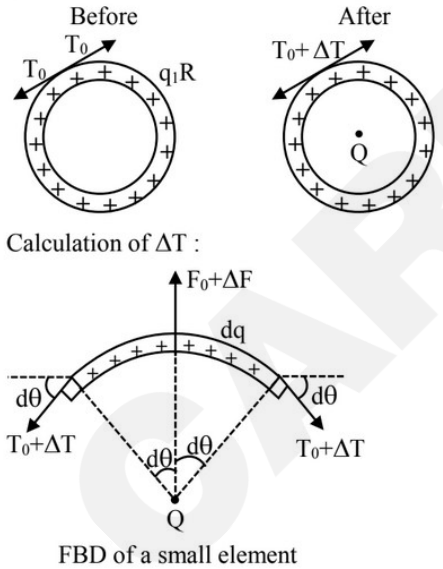
$$84 - 0 = 5t + \frac{3}{2}t^2$$
$$3t^2 + 10t - 168 = 0$$
$$t = \frac{-10 + \sqrt{100 + 4 \times 3 \times 168}}{6}$$
$$t = 6s \; \vec{v} = \vec{u} + \vec{a}t$$
$$= 5\hat{i} + (3\hat{i} + 2\hat{j})6$$
$$= 23\hat{i} + 12\hat{j} v = \sqrt{(23)^2 + (12)^2}$$
$$= \sqrt{673}\ \mathrm{m/s}$$

Hence, the answer is 673.

Q. 29 A thin metallic wire having cross sectional area of $10^{-4}\ \mathrm{m}^2$ is used to make a ring of radius $30\ \mathrm{cm}$. A positive charge of $2\pi \mathrm{C}$ is uniformly distributed over the ring, while another positive charge of $30\mathrm{pC}$ is kept at the centre of the ring. The tension in the ring is N; provided that the ring does not get deformed (neglect the influence of gravity) (given, $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ SI units)

Correct Answer:
3

Solution:



FBD of a small element

$2\ \mathrm{d}\theta$ is very small.

Arc is almost a chord

$$dq = \frac{q(2d\theta)}{2\pi}$$
$$dq = q\frac{d\theta}{\pi}$$

$\mathrm{F}_0 =$ Force applied on element dq by the $(\mathrm{q} - \mathrm{dq})$ remaining ring.

$\Delta\mathrm{F} =$ Force on dq by Q

(1) Before placing Q at centre, element was is equation.

$$\mathrm{F}_0 = 2\left[\mathrm{T}_0 \sin d\theta\right]$$

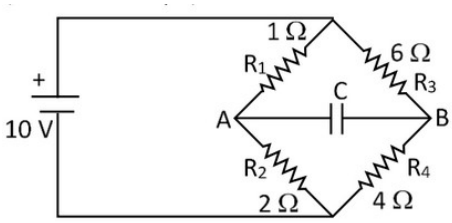
(2) After placing Q at centre, element was is equation.

$$F_0 + \Delta F = 2\,[\,T_0 + \Delta T]\sin d\theta$$

$$\frac{kQdq}{R^2} = 2 \cdot \Delta T \sin d\theta$$
$$\frac{kQ}{R^2}\frac{qd\theta}{\pi} \approx 2 \cdot \Delta T \cdot d\theta$$

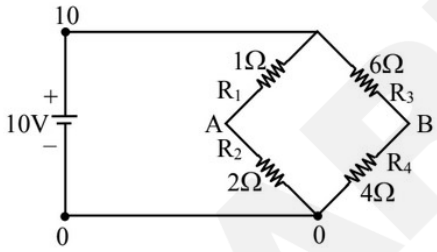
$$\Delta T = \frac{kQq}{2\pi R^2}$$
$$= \frac{9 \times 10^9 \times 2\pi \times 30 \times 10^{-12}}{2\pi(0.3)^2}$$
$$= \frac{9 \times 3}{9} = 3 \text{ Newton}$$

Q. 30 The charge accumulated on the capacitor connected in the following circuit is ____. μC. (Given $C = 150\mu F$)



Correct Answer:
400

Solution:



$$V_B = \frac{4}{4+6}10 \text{ V} = 4 \text{ V}$$
$$V_A = \frac{2}{2+1}10 \text{ V} = \frac{20}{3} \text{ V}$$
$$Q = C\,(V_A - V_B)$$
$$= 150 \times \frac{8}{3}\mu C = 400\mu C$$

Chemistry

Q. 1 A solution of two miscible liquids showing negative deviation from Raoult's law will have ?

Option 1:
decreased vapour pressure, increased boiling point

Option 2:
increased vapour pressure, decreased boiling point

Option 3:
decreased vapour pressure, decreased boiling point

Option 4:
increased vapour pressure, increased boiling point

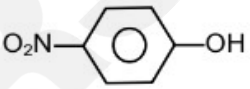
Correct Answer:
decreased vapour pressure, increased boiling point

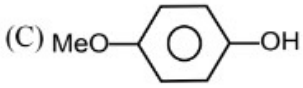
Solution:
A solution of two miscible liquids showing negative deviation from Raoult's law then vapor pressure will decrease increasing its boiling point.

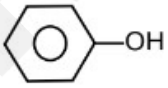
Hence, the answer is the option (1).

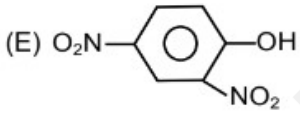
Q. 2 The ascending order of acidity of –OH group in the following compounds is :

(A) Bu – OH

(B) 

(C) 

(D) 

(E) 

Choose the correct answer from the option given below :

Option 1:
(C) < (A) < (D) < (B) < (E)

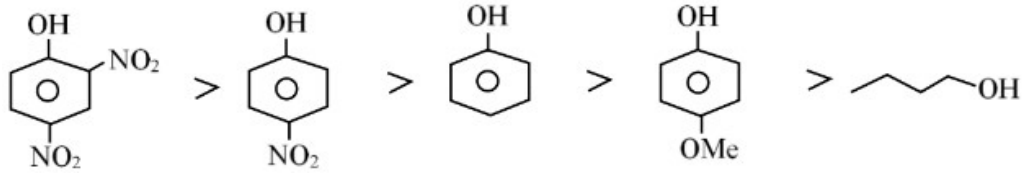
Option 2:
(C) < (D) < (B) < (A) < (E)

Option 3:
(A) < (D) < (C) < (B) < (E)

Option 4:
(A) < (C) < (D) < (B) < (E)

Correct Answer:
(A) < (C) < (D) < (B) < (E)

Solution:



Due to electron-withdrawing groups, the acidity is higher for the -OH group.

Q. 3 Yellow compound of lead chromate gets dissolved on treatment with hot NaOH solution. The product of lead formed is a :

Option 1:
Tetraanionic complex with coordination number six

Option 2:
Dianionic complex with coordination number six

Option 3:
Neutral complex with coordination number four

Option 4:
Dianionic complex with coordination number four

Correct Answer:
Dianionic complex with coordination number four

Solution:

$$PbCrO_4 + NaOH \longrightarrow Na_2[PbO_2]^{2-} + \underset{\text{Dianimic Eq. C.N. } =4}{Na_2[CrO_4]^{2-}}$$

Hence, the answer is the option(4).

Q. 4 Consider the following complex ions

$$P = [FeF_6]^{3-}$$
$$Q = [V(H_2O)_6]^{2+}$$
$$R = [Fe(H_2O)_6]^{2+}$$

The correct order of the complex ions, according to their spin only magnetic moment values in (B.M.) is :

Option 1:
R< Q< P

Option 2:
Q< R< P

Option 3:
R< P< Q

Option 4:
Q< P< R

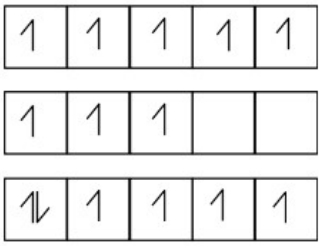
Correct Answer:
Q< R< P

Solution:

$P = [FeF_6]^{3-}$ $3d^5$

$Q = [V(H_2O)_6]^{2+}$ $3d^3$

$R = [Fe(H_2O)_6]^{2+}$ $3d^6$



Q. 5 Element not showing variable oxidation state is :

Option 1:
Chlorine

Option 2:
Iodine

Option 3:
Bromine

Option 4:
Fluorine

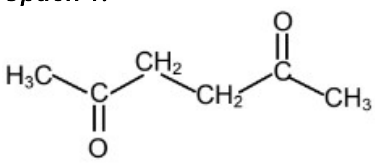
Correct Answer:
Fluorine

Solution:
In halogen, F does not exhibit a variable oxidation state due to the absence of vacant 'd' orbitals.

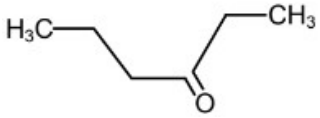
Hence, the answer is the option (4).

Q. 6 Which of the following has highly acidic hydrogen ?

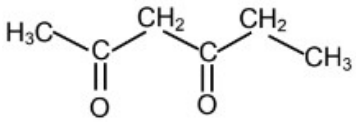
Option 1:



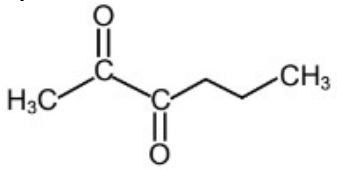
Option 2:



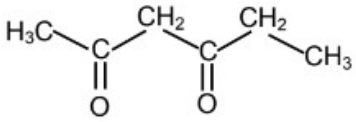
Option 3:



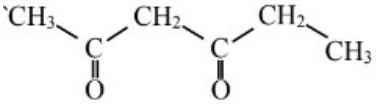
Option 4:



Correct Answer:



Solution:



Due to tautomerisation from two flanking carbonyl groups the central carbocation is very stable and enhances the loss of the proton making the molecule very acidic.

Q. 7 Which of the following electronic configuration would be associated with the highest magnetic moment?

Option 1:
$[\mathrm{Ar}]3\mathrm{d}^6$

Option 2:
$[\mathrm{Ar}]3\mathrm{d}^7$

Option 3:
$[\mathrm{Ar}]3\mathrm{d}^3$

Option 4:
$[\mathrm{Ar}]3\mathrm{d}^8$

Correct Answer:
$[\mathrm{Ar}]3\mathrm{d}^6$

Solution:
The Highest magnetic means more unpaired e^-.

$[\mathrm{Ar}]3\mathrm{d}^6$

Hence, the answer is the option (1).

Q. 8 Given below are two statement :

Statement (I) : Aqueous solution of ammonium carbonate is basic

Statement (II) : Acidic/basic nature of salt solution of a salt of weak acid and weak base depends on K_a and K_b value of acid and the base forming it.

In the light of the above statements, choose the most appropriate answer from the options given below:

Option 1:
Both Statement I and Statement II are correct

Option 2:
Statement I is correct but Statement II is incorrect

Option 3:
Statement I is incorrect but Statement II is correct

Option 4:
Both Statement I and Statement II are incorrect

Correct Answer:
Both Statement I and Statement II are correct

Solution:
Statement I – Ammonium carbonate in water dissociates into ammonium ions and carbonate ions. The carbonate ion is the conjugate base of a weak acid (carbonic acid), so it hydrolyzes in water to produce hydroxide ions, making the solution basic. Although ammonium ions are the conjugate acid of a weak base (ammonia), the basic nature of carbonate outweighs the acidic nature of ammonium in this case, resulting in an overall basic solution.
Statement II – The pH of a solution of a salt formed from a weak acid and a weak base depends on the relative strengths of the acid and base, which are quantified by their dissociation constants, (K_a) (acid dissociation constant) and (base dissociation constant). If ($K_b > K_a$), the solution will be basic; if ($K_a > K_b$), the solution will be acidic; and if ($K_a = K_b$), the solution will be approximately neutral

Hence, the answer is the option (1).

Q. 9 The correct statement regarding nucleophilic substitution reaction in a chiral alkyl halide is :

Option 1:
Racemisation occurs in S_N1 reaction and inversion occurs in S_N2 reaction.

Option 2:
Retention occurs in S_N1 reaction and inversion occurs in S_N2 reaction.

Option 3:
Racemisation occurs in S_N1 reaction and retention occurs in S_N2 reaction.

Option 4:
Racemisation occurs in both S_N1 and S_N2 reactions.

Correct Answer:
Racemisation occurs in S_N1 reaction and inversion occurs in S_N2 reaction.

Solution:
Racemization occurs in SN^1 reaction, and inversion occurs in SN^2 Reaction.

Hence, the answer is the option (1).

Q. 10 Cyclohexene  is _____ type of an organic compound.

Option 1:
Benzenoid non-aromatic

Option 2:
Benzenoid aromatic

Option 3:
Alicyclic

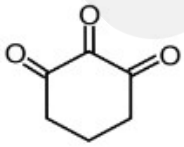
Option 4:
Acyclic

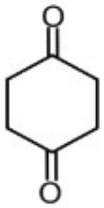
Correct Answer:
Alicyclic

Solution:
 is alicyclic compound

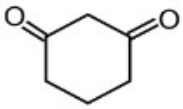
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Q. 11 Highest enol content will be shown by :

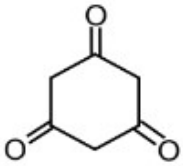
Option 1:


Option 2:


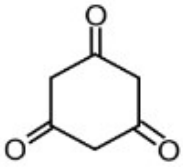
Option 3:



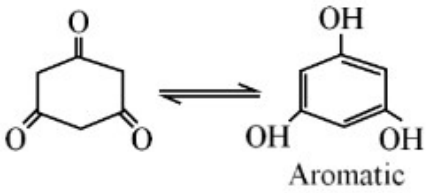
Option 4:



Correct Answer:



Solution:



Q. 12 Choose the polar molecule from the following :

Option 1:
$CHCl_3$

Option 2:
CCl_4

Option 3:
CO_2

Option 4:
$CH_2 = CH_2$

Correct Answer:
$CHCl_3$

Solution:
CCl_4, CO_2 and $CH_2 = CH_2$ are non-polar molecule due to symmetry.

Hence, the answer is the option (1).

CAREERS360

Q. 13 Given below are two statement : one is labelled as Assertion (A) and the other is labelled as Reason (R)

Assertion (A) : Melting point of Boron (2453 K) is unusually high in group 13 elements

Reason (R) : Solid Boron has very strong crystalline lattice.

In the light of the above statements, choose the most appropriate answer from the options given below:

Option 1:
Both (A) and (R) are correct but (R) is not the correct explanation of (A)

Option 2:
(A) is false but (R) is true

Option 3:
Both (A) and (R) are correct and (R) is the correct explanation of (A)

Option 4:
(A) is true but (R) is false

Correct Answer:
Both (A) and (R) are correct and (R) is the correct explanation of (A)

Solution:
Boron has an Icosahedron (strong crystalline lattice) joint covalent structure hence its M. P is very high

So, Both R & A are correct & R is the correct explanation of A.

Hence, the answer is the option (3).

Q. 14 $NaCl$ reacts with conc. H_2SO_4 and $K_2Cr_2O_7$ to give reddish fumes (B), which react with $NaOH$ to give yellow solution (C). (B) and (C) respectively are :

Option 1:
$CrO_2Cl_2, KHSO_4$

Option 2:
Na_2CrO_4, CrO_2Cl_2

Option 3:
CrO_2Cl_2, Na_2CrO_4

Option 4:
$CrO_2Cl_2, Na_2Cr_2O_7$

Correct Answer:
CrO_2Cl_2, Na_2CrO_4

Solution:

$$NaCl + K_2Cr_2O_7 + \underset{\text{conc.}}{H_2SO_4}$$

$$\downarrow$$

$$\underset{(B)}{CrO_2Cl_2} \quad \text{Red Fames}$$

$$\downarrow \ NaOH$$

$$Na_2CrO_4 \quad \text{Yellow Solution}$$

Q. 15 Two nucleotides are joined together by a linkage known is :

Option 1:
Peptide linkage

Option 2:
Disulphide linkage

Option 3:
Phosphodiester linkage

Option 4:
Glycosidic linkage

Correct Answer:
Phosphodiester linkage

Solution:
Two nucleotides are joined together by a linkage known as a **phosphodiester bond**.

The hydroxyl group of the sugar (ribose in RNA or deoxyribose in DNA) of one nucleotide and the phosphate group of another nucleotide form this link. The nucleic acid structure is made up of an alternating backbone of sugar and phosphate groups, namely the bond between the 3' carbon atom of one nucleotide and the 5' carbon atom of the subsequent nucleotide.

Hence, the answer is the option (3).

Q. 16 The electronic configuration for Neodymium is :
[Atomic Number for Neodymium 60]

Option 1:
$[Xe]5f^{7}7\ s^{2}$

Option 2:
$[Xe]4f^{6}6\ s^{2}$

Option 3:
$[Xe]4f^{4}6\ s^{2}$

Option 4:
$[Xe]4f^{1}5\ d^{1}6\ s^{2}$

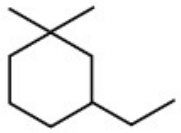
Correct Answer:
$[Xe]4f^{4}6\ s^{2}$

Solution:
Electronic configuration of 'Nd'

$[Xe]4f^{4}6\ s^{2}$

Hence, the answer is the option (3).

Q. 17 IUPAC name of following compound (P) is :



(P)

Option 1:
1-Ethyl-3,3-dimethylcyclohexane

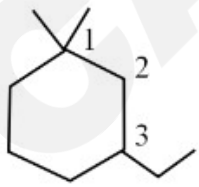
Option 2:
1-Ethyl-5,5-dimethylcyclohexane

Option 3:
1,1-Dimethyl-3-ethylcyclohexane

Option 4:
3-Ethyl-1,1-dimethylcylohexane

Correct Answer:
3-Ethyl-1,1-dimethylcylohexane

Solution:



3-Ethyl-1,1-dimethylcylohexane

Q. 18 Given below are two statement :

Statement (I) : p-nitrophenol is more acidic than m-nitrophenol and o-nitrophenol.

Statement (II) : Ethanol will give immediate turbidity with Lucas reagent.

In the light of the above statements, choose the most appropriate answer from the options given below:

Option 1:
Both Statement I and Statement II are true

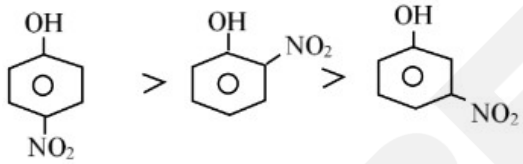
Option 2:
Statement I is false but Statement II is true

Option 3:
Statement I is true but Statement II is false

Option 4:
Both Statement I and Statement II are false

Correct Answer:
Statement I is true but Statement II is false

Solution:
Acidic strength order



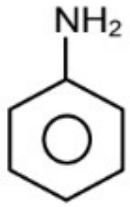
ethanol is 1°-alcohol, do not gives immediate turbidity with Lucas regent.

Q. 19 Which of the following is strongest Bronsted base ?

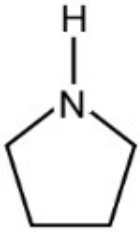
Option 1:



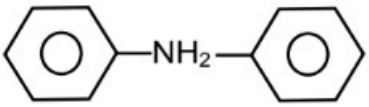
Option 2:



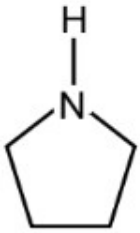
Option 3:



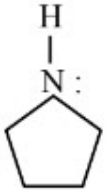
Option 4:



Correct Answer:



Solution:



Due to presence of Localised L.P., It is strongest bronsted blase.

Q. 20 Given below are two statement :

Statement (I) : The 4f and 5f – series of elements are placed separately in the periodic table to preserve the principle of classification.

Statement (II) : s-block element can be found in pure form in nature.

In the light of the above statements, choose the most appropriate answer from the options given below:

Option 1:

Statement I is false but Statement II is true

Option 2:
Both Statement I and Statement II are false

Option 3:
Both Statement I and Statement II are true

Option 4:
Statement I is true but Statement II is false

Correct Answer:
Statement I is true but Statement II is false

Solution:
s-block elements, which include alkali metals (Group 1) and alkaline earth metals (Group 2), are highly reactive and thus are not typically found in their pure (elemental) form in nature. They are usually found in compound forms, such as salts and oxides, due to their high reactivity.

Hence, the answer is the option (4).

Q. 21 Consider the following data for the given reaction

$2\mathrm{HI}_{(\mathrm{g})} \rightarrow \mathrm{H}_{2(\mathrm{g})}+\mathrm{I}_{2(\mathrm{g})}$

The order of the reaction is _____.

Correct Answer:
2

Solution:
$\mathrm{r} \alpha[\mathrm{HI}]^{\mathrm{x}}$
From (1) \& (2) data
$\frac{\left(7 \times 10^{-4}\right)}{\left(3 \times 10^{-3}\right)}=\left[\frac{\left(5 \times 10^{-3}\right)}{\left(1 \times 10^{-2}\right.}\right]^{x}$
$x=2$

Hence, the answer is (2).

Q. 22 The mass of silver (Molar mass of $\mathrm{Ag}: 108 \mathrm{gmol}^{-1}$) displaced by a quantity of electricity which displace 5600 mL of O_2 at S.T.P will be_____g.

Correct Answer:
108

Solution:

$$\text{moles of } O_2 = \frac{5600}{22400} = 0.25$$

$$\text{eq of } O_2 = 0.25 \times 4 = 1$$

$$\text{eq of Ag} = 1$$

$$\text{moles of Ag} = 1$$

$$\text{moss of Ag} = 108 \text{ g}$$

Hence, the answer is (108).

Q. 23 If three moles of an ideal gas at 300k expand isothermally from 30dm^3 to 45dm^3 against a constant opposing pressure of 80kPa, then the amount of heat transferred is_____J.

Correct Answer:
1200

Solution:
$W = -P_2 (V_2 - V_1)$

$= -80(45 - 30) = -1200 \text{ J}$

$Q = -W$

$Q = 1200 \text{ J}$

Hence, the answer is (1200).

Q. 24 The number of electrons present in all the completely filled subshells having $\text{n} = 4$ and $\text{S} = +\frac{1}{2}$ is_____.

Correct Answer:
16

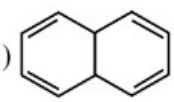
Solution:

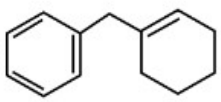
$\text{n} = 4$	4s,4p	4d	4f
$1/2\ \text{S} = 1$	1,3	5	7

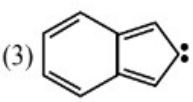
$= 16\text{e}^-$

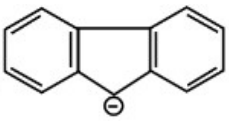
Hence, the answer is (16).

Q. 25 Among the given organic compounds, the total number of aromatic compounds is _____ .

(1) 

(2) 

(3) 

(4) 

Correct Answer:
3

Solution:
Fact

Q. 26 Sum of bond order of CO and NO^+ is_____.

Correct Answer:
6

Solution:
B.O. of $CO = 3 \quad [C \leqq O]$

B.O. of $NO^+ = 3 \quad [N \equiv O+]$

Total sum = 6

Q. 27 From the given list, the number of compounds with +4 oxidation state of Sulphur is _____ .

$SO_3, H_2SO_3, SOCl_2, SF_4, BaSO_4, H_2\ S_2O_7$

Correct Answer:
3

Solution:
$H_2SO_3 \Rightarrow +4$
$SOCl_2 \Rightarrow +4$
$SF_4 \Rightarrow +4$
$SO_3 \Rightarrow +6$
$H_2\ S_2O_7 = +6$

Hence, the answer is (3).

Q. 28 Mass of methane required to produce 22 g of CO_2 after complete combustion is_____ g.

(Given Molar mass in g $\begin{matrix} \text{mol}^{-1}\text{C} = 12.0 \\ \text{H} = 1.0 \\ \text{O} = 16.0 \end{matrix}$)

Correct Answer:
8

Solution:

$$CH_4 + 2O_2 \rightarrow \underset{22\ g}{CO_2} + 2H_2O$$

$$= \frac{1}{2} = \frac{22}{44}$$

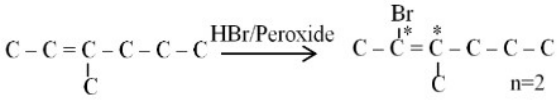
$$= \frac{1}{2} \text{ moles } \Rightarrow 8gm$$

Hence, the answer is (8).

Q. 29 3-Methylhex-2-ene on reaction with HBr in presence of peroxide forms an addition product (A). The number of possible stereoisomers for 'A' is _____ .

Correct Answer:
4

Solution:



Possible stereo isomer $= 2^2 = 4$

Q. 30 Among the following total number of meta directing functional group is_____.

(Integer based)

$-OCH_3, -NO_2, -CN, -CH_3, -NHCOCH_3, -COR, -OH, -COOH, -Cl$

Correct Answer:
4

Solution:
$-NO_2, -CN, -COR, -COOH$ are m-directing group.

Hence, the answer is (4).

Maths

Q. 1 The length of the chord of the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$, whose mid point is $\left(1, \frac{2}{5}\right)$, is equal to :

Option 1:
$\frac{\sqrt{1541}}{5}$

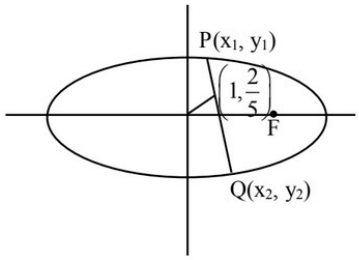
Option 2:
$\frac{\sqrt{1691}}{5}$

Option 3:
$\frac{\sqrt{1741}}{5}$

Option 4:
$\frac{\sqrt{2009}}{5}$

Correct Answer:
$\frac{\sqrt{1691}}{5}$

Solution:



$\therefore$ Mid point of chord is given as $\left(1, \frac{2}{5}\right)$

$\Rightarrow$ equation of chord is
$\mathrm{T} = \mathrm{S}_1$

$$\Rightarrow \frac{x \cdot 1}{25} + \frac{y \cdot \frac{2}{5}}{16} = \frac{1}{25} + \frac{4}{16 \times 25}$$
$$\Rightarrow \frac{x}{5} + \frac{y}{4} = \frac{1}{4} \text{ (equation of chord)}$$

Now solving this with equation of ellipse

$\Rightarrow$ Coordinate of point $\mathrm{P} = \left(\mathrm{x}_1, 2 - \frac{8\mathrm{x}_1}{5}\right)$ and Coordinate of point $\frac{x^2}{25} + \frac{1}{16}\left(2 - \frac{8x}{5}\right)^2 = 1$

$$\therefore \quad \mathrm{PQ} = \sqrt{(\mathrm{x}_1 - \mathrm{x}_2)^2 + \frac{64}{25}(\mathrm{x}_1 - \mathrm{x}_2)^2} = |\mathrm{x}_1 - \mathrm{x}_2|\sqrt{\frac{89}{25}}$$

Now for $|\mathrm{x}_1 - \mathrm{x}_2|$

$$\frac{x^2}{25} + \frac{1}{16}\left(2 - \frac{8x}{5}\right)^2 = 1$$
$$\Rightarrow \quad 4x^2 - 8x - 15 = 0$$
$$\Rightarrow \quad |x_1 - x_2| = \sqrt{19}$$

There fore, $\mathrm{PQ} = \sqrt{19} \times \sqrt{\frac{89}{25}} = \frac{\sqrt{1691}}{5}$

Q. 2 ${}^{n-1}C_r = \left(k^2 - 8\right){}^{n}C_{r+1}$ if and only if:

Option 1:
$2\sqrt{3} < \mathrm{k} \leq 3\sqrt{2}$

Option 2:
$2\sqrt{2} < \mathrm{k} \leq 3$

Option 3:
$2\sqrt{3} < \mathrm{k} < 3\sqrt{3}$

Option 4:
$2\sqrt{2} < \mathrm{k} < 2\sqrt{3}$

Correct Answer:
$2\sqrt{2} < \mathrm{k} \leq 3$

Solution:
${}^{n-1}C_r = \left(k^2 - 8\right){}^{n}C_{r+1}$

$\Rightarrow {}^{n-1}C_r = \left(k^2 - 8\right) \cdot \frac{n}{r+1} \cdot C_r$

$\Rightarrow k^2 - 8 = \frac{r+1}{n}$

$0 < \frac{\mathrm{r}+1}{\mathrm{n}} \leq 1$

$0 < \mathrm{k}^2 - 8 \leq 1$
$\Rightarrow 8 < \mathrm{k}^2 \leq 9$
$\Rightarrow [-3, -2\sqrt{2}) \cup (2\sqrt{2}, 3]$
$\Rightarrow 2\sqrt{2} < \mathrm{k} \leq 3$

Hence, the answer is the option (2).

Q. 3 If A denotes the sum of all the coefficients in the expansion of $\left(1 - 3x + 10x^2\right)^n$ and B denotes the sum of all the coefficients in the expansion of $\left(1 + x^2\right)^n$, then :

Option 1:
$A = B^3$

Option 2:
3 A =B

Option 3:
A =3 B

Option 4:
$B = A^3$

Correct Answer:
$A = B^3$

Solution:
Sum of Binomial Coefficients

$C_0 + C_1 + C_2 + C_3 + \ldots\ldots + C_n = 2^n$

or $\sum_{r=0}^{n} {}^nC_r = 2^n$

Q. 4 If $a = \lim_{x\to 0} \frac{\sqrt{1+\sqrt{1+x^4}}-\sqrt{2}}{x^4}$ and $b = \lim_{x\to 0} \frac{\sin^2 x}{\sqrt{2}-\sqrt{1+\cos x}}$, then the value of ab^3 is :

Option 1:
30

Option 2:
36

Option 3:
25

Option 4:
32

Correct Answer:
32

Solution:

$a = \lim_{x\to 0} \frac{1+\sqrt{1+x^4}-2}{x^4} \cdot \frac{1}{\sqrt{1+\sqrt{1+x_4}}+\sqrt{2}}$

$a = \lim_{x\to 0} \frac{x^4}{x^4} \cdot \frac{1}{\sqrt{1+x^4}+1} \cdot \frac{1}{\sqrt{1+\sqrt{1+x^4}}+\sqrt{2}} = \frac{1}{2} \cdot \frac{1}{2\sqrt{2}} \Rightarrow a = \frac{1}{4\sqrt{2}}$

$b = \lim_{x\to 0} \frac{\left(\frac{s^2x}{x^2}\right)\cdot x^2}{\frac{(2-1-cx)\cdot x^2}{x^2}} \cdot (\sqrt{2}+\sqrt{1+\cos x})$

$b = \frac{1}{\frac{1}{2}} \cdot 2\sqrt{2} \Rightarrow b = 4\sqrt{2}$

Now $ab^3 = \frac{1}{(4\sqrt{2})}(4\sqrt{2})^3 = 32$

Hence, the answer is the option (4).

Q. 5 Consider the function.

$$f(x) = \begin{cases} \frac{a(7x-12-x^2)}{b|x^2-7x+12|} & , x < 3 \\ 2^{\frac{\sin(x-3)}{x-[x]}} & , x > 3 \\ b & , x = 3 \end{cases}$$

where $[x]$ denotes the greatest integer less than or equal to x. If S denotes the set of all ordered pairs (a, b) such that $f(x)$ is continuous at $x = 3$, then the number of elements in S is:

Option 1:
1

Option 2:
4

Option 3:
2

Option 4:
Infinitely many

Correct Answer:
1

Solution:
$f\left(3^{+}\right)=\lim_{x\to 3^{+}} 2^{\frac{\sin(x-3)}{x-(3^{+})}}=2^{1}=2$

f(3) = b

$$f\left(3^{-}\right)=\lim_{x\to 3^{-}}\frac{-a(x-3)(x-4)}{b|(x-3)(x-4)|}$$
$$=\lim_{x\to 3^{-}}-\frac{a}{b}\frac{(x-5)(x-4)}{(x-1)(x-4)}=\frac{-a}{b}$$

$\mathrm{f}(\mathrm{x})$ is can't at $\mathrm{x}=3$
$\mathrm{f}\left(3^{-}\right)=\mathrm{f}(3)=\mathrm{f}\left(3^{+}\right)$

$2=b=\frac{-a}{b}\Rightarrow b=2, a=-4$

only one ordered pair

Hence, the answer is the option (1).

Q. 6 The number of common terms in the progressions $4, 9, 14, 19, \ldots\ldots$, up to 25^{th} term and $3, 6, 9, 12, \ldots\ldots$, up to 37^{th} term is :

Option 1:
5

Option 2:
8

Option 3:
7

Option 4:
9

Correct Answer:
7

Solution:
$4, 9, 14, 19, \ldots, 124$

$$\left.\begin{array}{l}\rightarrow \mathrm{CD}=5\\ \rightarrow \mathrm{CD}=3\end{array}\right\}\mathrm{LCM}=15$$

$3, 6, 9, 12, \ldots, 111$
$9, 24, \ldots .$

$9(\mathrm{n}-1) \cdot 15 \leq 111$

$(\mathrm{n}-1)15 \leq 102$

$(\mathrm{n}-1)15 = 90$

$\mathrm{n}-1 \Rightarrow \mathrm{n} = 7$

Hence, the answer is the option (3).

Q. 7 Let $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}, \vec{b} = 3(\hat{i} - \hat{j} + \hat{k})$. Let $\vec{c}$ be the vector such that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a} \cdot \vec{c} = 3$ Then $\vec{a} \cdot ((\vec{c} \times \vec{b}) - \vec{b} - \vec{c})$ is equal to :

Option 1:
20

Option 2:
24

Option 3:
36

Option 4:
32

Correct Answer:
24

Solution:
$\vec{a} = \hat{i} + \hat{j} + \hat{k}$
$\vec{b} = 3(\hat{i} - \hat{j} + \hat{k})$

then $= \vec{a} \cdot (\vec{c} \times \vec{b}) - \vec{a} \cdot \vec{b} - \vec{a} \cdot \vec{c}$

$= [\vec{a}\vec{c}\vec{b}] - 0 - 3 = \vec{b} \cdot \vec{b} - 3$

$= (\vec{b})^2 - 3 = 27 - 3 = 24$

Hence, the answer is the option (2).

Q. 8

Consider the matrix $f(x)=\begin{bmatrix}\cos x & -\sin x & 0\\ \sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}$

Given below are two statements :
Statement I : $f(-x)$ is the inverse of the matrix $f(x)$.
Statement II : $\mathrm{f(x)f(y)=f(x+y)}$.

In the light of the above statements, choose the correct answer from the options given below:

Option 1:
Statement I is true but Statement II is false

Option 2:
Both Statement I and Statement II are false

Option 3:
Both Statement I and Statement II are true

Option 4:
Statement I is false but Statement II is true

Correct Answer:
Both Statement I and Statement II are true

Solution:

$$f(x)=\begin{bmatrix}\cos x & -\sin x & 0\\ \sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}$$

S-I $\quad f(-x)=\begin{bmatrix}\cos x & \sin x & 0\\ -\sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}$

$$f^{-1}(x)=\begin{bmatrix}\cos x & -\sin x & 0\\ \sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}^{T}$$

$$f^{-1}(x)=\begin{bmatrix}\cos x & \sin x & 0\\ -\sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}=f(-x)$$

S-I is True

S-II $\quad f(x)f(y)=\begin{bmatrix}\cos x & -\sin x & 0\\ \sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\cos y & -\sin y & 0\\ \sin y & \cos y & 0\\ 0 & 0 & 1\end{bmatrix}$

$$= \begin{bmatrix} \cos(x+y) & -\sin(x+y) & 0 \\ \sin(x+y) & \cos(x+y) & 0 \\ 0 & 0 & 1 \end{bmatrix} = f(x+y)$$

S-II is True

Hence, the answer is the option (3).

Q. 9 If the shortest distance between the lines $\frac{x-4}{1} = \frac{y+1}{2} = \frac{z}{-3}$ and $\frac{x-\lambda}{2} = \frac{y+1}{4} = \frac{z-2}{-5}$ is $\frac{6}{\sqrt{5}}$, then the sum of all possible values of λ is:

Option 1:
10

Option 2:
8

Option 3:
5

Option 4:
7

Correct Answer:
8

Solution:

$\text{SD} = \frac{(\bar{b}-\bar{a})\cdot(\bar{p}\times\bar{q})}{|\bar{p}\times\bar{q}|} = \frac{6}{\sqrt{5}}$

$\Rightarrow \frac{((\lambda-4)\hat{i}+0\hat{j}+2\hat{k})\cdot(2\hat{i}-\hat{j}+0\hat{k})}{\sqrt{4+1}} = \frac{6}{\sqrt{5}}$

$$\bar{p}\times\bar{q} = \begin{matrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -3 \\ 2 & 4 & -5 \end{matrix} =< 2, -1, 0 >$$

$\frac{2\lambda-8}{5^2} = \frac{6}{88} \Rightarrow \frac{2\lambda-8}{\sqrt{5}} = \frac{6}{\sqrt{5}}$

$|\lambda - 4| = 1 \quad \Rightarrow 1 = 7 \text{ or } 1$

$\text{sum of } \lambda = 7 + 1 = 8$

Hence, the answer is the option (2).

Q. 10 The function $\text{f} : \mathbf{N} - \{1\} \to \mathbf{N}$; defined by $\text{f(n)} =$ the highest prime factor of n, is :

Option 1:
neither one-one nor onto

Option 2:
one-one only

Option 3:
both one-one and onto

Option 4:
onto only

Correct Answer:
neither one-one nor onto

Solution:
$\mathrm{f}: \mathrm{N}-\{1\} \rightarrow \mathrm{N}$

f(n) = the highest prime factor of n

$$\mathrm{f}(2)=2$$
$$\mathrm{f}(3)=3$$
$$\mathrm{f}(4)=2$$

$$\mathrm{f}(2)=\mathrm{f}(4)(2 \neq 4)$$
$$\Rightarrow \text{ Not one-one}$$
and a $\nexists$ any $\mathrm{n} \in$ co-domain
such that $f(n)=1$(not onto)

Hence, the answer is the option (1).

Q. 11 Let $S=\{1,2,3, \ldots, 10\}$. Suppose M is the set of all the subsets of S, then the relation $\mathrm{R}=\{(\mathrm{A}, \mathrm{B}): \mathrm{A} \cap \mathrm{B} \neq \phi ; \mathrm{A}, \mathrm{B} \in \mathrm{M}\}$ is :

Option 1:
reflexive only

Option 2:
symmetric and reflexive only

Option 3:
symmetric and transitive only

Option 4:
symmetric only

Correct Answer:
symmetric only

Solution:
S = { 1, 2, 3, ...10}
$M=P(S)$
$A, B \in M$

$\mathrm{R}=\{(\mathrm{A}, \mathrm{B}): \mathrm{A} \cap \mathrm{B} \neq \phi \mathrm{A}, \mathrm{B} \in \mathrm{M}\}$

if $A = \phi = B$ then $A \cap B = \phi$

$\phi R \phi \Rightarrow$ not Reflexive

Symmetric Yes
Not Transitive

$A = \{1, 2\} \quad B = \{2, 3, 4\} \quad C = \{4, 5, 6\}$
$\Rightarrow ARB, BRC$

But ARC

Hence, the answer is the option (4).

Q. 12 The distance, of the point $(7, -2, 11)$ from the line $\frac{x-6}{1} = \frac{y-4}{0} = \frac{z-8}{3}$ along the line $\frac{x-5}{2} = \frac{y-1}{-3} = \frac{z-5}{6}$, is

Option 1:
12

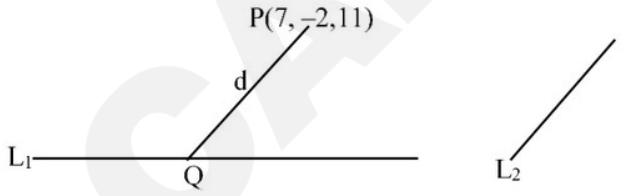
Option 2:
18

Option 3:
21

Option 4:
14

Correct Answer:
14

Solution:



$L_{PQ} : \frac{x-7}{2} = \frac{y+2}{-3} = \frac{z-11}{6} = \lambda$
$Q : (2\lambda + 7, -2 - 3\lambda, 11 + 6\lambda)$

lies on L_1

$\frac{2\lambda + 7 - 6}{1} = \frac{-2 - 3\lambda - 4}{0} = \frac{11 + 6\lambda - 8}{3}$
$\Rightarrow 3\lambda + 6 = 0 \Rightarrow \lambda = -2$

Hence

$Q = (3, 4, -1)$

$d(PQ) = \sqrt{16 + 36 + 144} = \sqrt{196} = 14$

Q. 13 If $S = \{z \in C : |z - i| = |z + i| = |z - 1|\}$, then, $n(S)$ is :

Option 1:
2

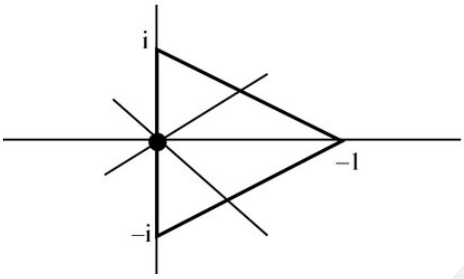
Option 2:
0

Option 3:
3

Option 4:
1

Correct Answer:
1

Solution:



Q. 14 If (a, b) be the orthocentre of the triangle whose vertices are $(1, 2), (2, 3)$ and $(3, 1)$, and $I_1 = \int_a^b x \sin(4x - x^2)dx, I_2 = \int_a^b \sin(4x - x^2)dx$, then $36\frac{I_1}{I_2}$ is equal to:

Option 1:
80

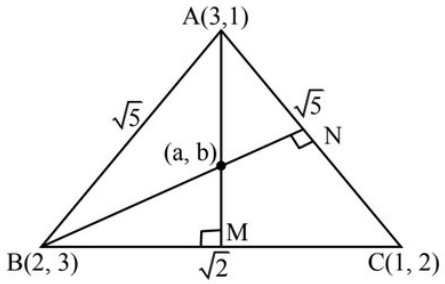
Option 2:
66

Option 3:
72

Option 4:
88

Correct Answer:
72

Solution:



$$BC : y - 3 = \frac{2-3}{1-2}(x-2)$$

$$BC : y - x = 1$$

$$\left.\begin{array}{l} AM : y + x = 4 \\ BN : y - 2x = -1 \end{array}\right\} x : \left(\frac{5}{3}, \frac{7}{3}\right)$$

$$I_1 = \int_a^b x \operatorname{Sin}\left(4x - x^2\right) dx$$

$$\downarrow K\& A$$

$$2I_1 = 4\int_a^b \sin\left(4x - x^2\right) dx$$

$$2I_1 = 4I_2$$

$$36\frac{I_1}{I_2} = 36.2 = 72$$

Q. 15 If $\int_0^1 \frac{1}{\sqrt{3+x}+\sqrt{1+x}} dx = a + b\sqrt{2} + c\sqrt{3}$, where a, b, c are rational numbers, then $2a + 3b - 4c$ is eq

Option 1:
4

Option 2:
7

Option 3:
10

Option 4:
8

Correct Answer:
8

Solution:

$\int_0^1 \frac{1}{\sqrt{3+x}+\sqrt{1+x}} dx = a + b\sqrt{x} + c\sqrt{3}$

Rationalize

$\int_0^1 \left(\frac{\sqrt{3+x}-\sqrt{1+x}}{2}\right) dx$

$= \frac{1}{2}\left[\frac{2}{3}(3+x)^{\frac{3}{2}} - \frac{2}{3}(1+x)^{3/2}\right]_0^1$

$= \frac{1}{3}[(8-2\sqrt{2}) - (3\sqrt{3}-1)]$

$= \frac{1}{3}[8-2\sqrt{2}-3\sqrt{2}+1]$

$= \frac{1}{3}[9-2\sqrt{2}-3\sqrt{3}]$

$= 3 - \frac{2}{3}\sqrt{2} - \sqrt{3}$

$= a + b\sqrt{2} + c\sqrt{3}$

$a = 3 \;\; b = -\frac{2}{3} \;\; c = -1$

$2(3) + 3\left(-\frac{2}{3}\right) - 4(-1)$

$\Rightarrow \quad 6 - 2 + 4$

$\Rightarrow \quad 8$

Hence, the answer is the option (4).

Q. 16 Let $a_1, a_2, \ldots a_{10}$ be 10 observations such that $\sum_{k=1}^{10} a_k = 50$ and $\sum_{\forall k<j} a_k \cdot a_j = 1100$. Then the standard deviation of $a_1, a_2, \ldots, a_{10}$ is equal to:

Option 1:
10

Option 2:
$\sqrt{5}$

Option 3:
$\sqrt{115}$

Option 4:
5

Correct Answer:
$\sqrt{5}$

Solution:
$\sum_{k=1}^{10} a_k = 50 \quad \Rightarrow \mu = 5$

variance $= \frac{\sum (a_i)^2}{n} - (\mu)^2$

$(a_1 + a_2 + \ldots + a_{10})^2 = \sum_{\substack{i \neq j \\ i,j=1,2\ldots,10}} \left(a_i^2 + 2\left(a_i a_j\right)\right)$

$\Rightarrow (50)^2 - 2(1100) = \Sigma {a_i}^2$

$\Rightarrow \Sigma {a_i}^2 = 2500 - 2200$

$\Rightarrow \Sigma {a_i}^2 = 300$

$$\text{variance} = \frac{300}{10} - 25$$

$$\text{variance} = \frac{50}{10} = 5$$

S.D. $= \sqrt{5}$

Hence, the answer is the option (2).

Q. 17 Let $x = x(t)$ and $y = y(t)$ be solutions of the differential equations $\frac{dx}{dt} + ax = 0$ and $\frac{dy}{dt} + by = 0$ respectively, $a, b \in \mathbf{R}$. Given that $x(0) = 2; y(0) = 1$ and $3y(1) = 2x(1)$, the value of t, for which $x(t) = y(t)$, is :

Option 1:
$\log_{\frac{2}{3}} 2$

Option 2:
$\log_4 3$

Option 3:
$\log_{\frac{4}{3}} 2$

Option 4:
$\log_3 4$

Correct Answer:
$\log_{\frac{4}{3}} 2$

Solution:
$\frac{dx}{dt} + ax = 0 \quad \& \quad \frac{dy}{dt} + by = 0$

$\ln x = -at + \lambda$
$x = k_1 e^{-at}$
$x(0) = 2$
$\Rightarrow K_1 = 2$

$\ln y = -bt + m$
$y = K_2 e^{-bt}$
$y(0) = 1$
$\Rightarrow K_2 = 1$

$3y(1) = 2x(1)$
$3e^{-b} = 2.2e^{-a}$

$e^{a-b} = \frac{4}{3}$

$x(t) = y(t)$
$2e^{-at} = e^{-bt}$
$e^{(a-b)t} = 2$

$\left(\frac{4}{3}\right)^t = 2$

$t = \log_{\frac{4}{3}} 2$

Hence, the answer is the option (3).

Q. 18 If the shortest distance of the parabola $y^2 = 4x$ from the centre of the circle $x^2 + y^2 - 4x - 16y + 64 = 0$ is d, then d^2 is equal to :

Option 1:
16

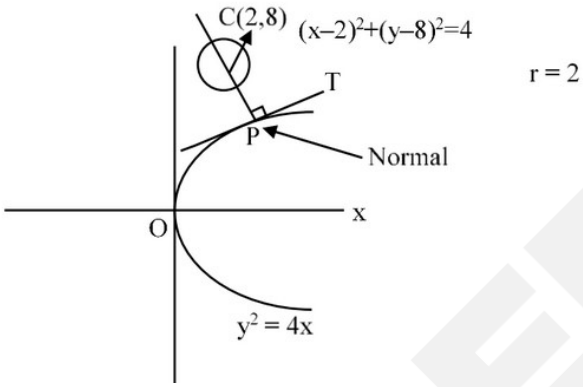
Option 2:
20

Option 3:
24

Option 4:
36

Correct Answer:
20

Solution:



(minimum distance will be along the common normal)

N and to parabola

$y = mx - 2am - am^3$

$y = mx - 2am - m^3$

$\Rightarrow 8 = 2\,\mathrm{m} - 2\,\mathrm{m} - \mathrm{m}^3$

$\Rightarrow \mathrm{m} = -2$

$\Rightarrow \mathrm{N} : \mathrm{y} = -2\mathrm{x} + 4 + 8$

$\Rightarrow y = -2x + 12$ for 'p' Solve 'N' with parabola

$y = 2x + 12$

$4x = (-2x + 12)^2$

$\Rightarrow 4x = 4x^2 + 144 - 48x$

$\Rightarrow 4x = 52x + 144 = 0$

$\Rightarrow x^2 - 13x + 36 = 0$

$$\Rightarrow x = \frac{13 \pm \sqrt{169 - 144}}{2}$$

$$\Rightarrow x = \frac{13 \pm 5}{2}, 9$$

$x = 9, 4$

$y = -6, 4$ in Ist quadrant

$\mathrm{P}(4, 4) \quad \mathrm{C}(2, 8)$

$\mathrm{d} = |\mathrm{PC}| = \sqrt{4 + 16} = \sqrt{20}$

$\mathrm{d}^2 = 20$

Q. 19 Four distinct points $(2\mathrm{k}, 3\mathrm{k}), (1, 0), (0, 1)$ and $(0, 0)$ lie on a circle for k equal to : Options

Option 1:

$\frac{1}{13}$

Option 2:

$\frac{2}{13}$

Option 3:

$\frac{5}{13}$

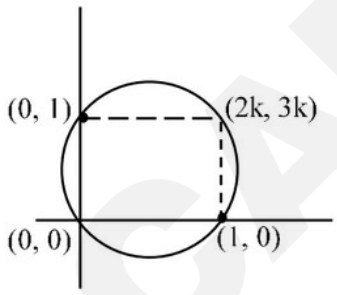
Option 4:

$\frac{3}{13}$

Correct Answer:

$\frac{5}{13}$

Solution:



$(x - 1)x + y(y - 1) = 0$

$x^2 + y^2 - x - y = 0$

Now $(2K, 3K)$ lies

$4k^2 + 9k^2 - 2k - 3k = 0$

$13k^2 - 5k = 0$

$\mathrm{k} = 0 \quad \mathrm{k} = \frac{5}{13}$

Q. 20 The portion of the line $4x + 5y = 20$ in the first quadrant is trisected by the lines L_1 and L_2 passing through the origin. The tangent of an angle between the lines L_1 and L_2 is :

Option 1:
$\frac{8}{5}$

Option 2:
$\frac{25}{41}$

Option 3:
$\frac{2}{5}$

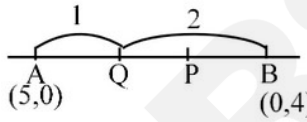
Option 4:
$\frac{30}{41}$

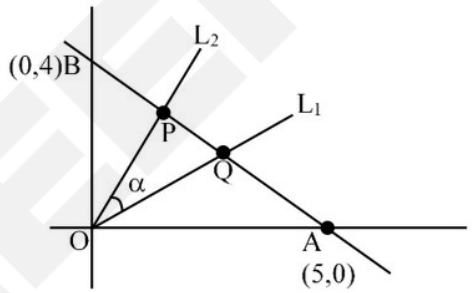
Correct Answer:
$\frac{30}{41}$

Solution:

$\mathrm{Q}: \left(\frac{10}{3}, \frac{4}{3}\right)$

$\mathrm{P}: \left(\frac{5}{3}, \frac{8}{3}\right)$





$\mathrm{m_{OP}} = \frac{8}{5}, \mathrm{M_{OQ}} = \frac{2}{5}$

$\tan\alpha = \frac{\frac{8}{5} - \frac{2}{5}}{1 + \frac{16}{25}}$

$= \frac{6.5}{41}$

$\tan\alpha = \frac{30}{41}$

Q. 21 Let the area of the region $\{(x, y) : x - 2y + 4 \geq 0, x + 2y^2 \geq 0, x + 4y^2 \leq 8, y \geq 0\}$ be $\frac{\mathrm{m}}{\mathrm{n}}$, where m and n are coprime numbers. Then m+n is equal to

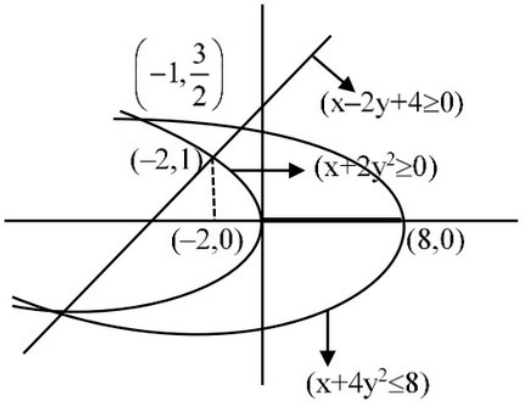
Correct Answer:
119

Solution:

$x - 2y + 4 \geq 0$

$x + 2y^2 \geq 0$

$x + 4y^2 \leq 8, y \geq 0$



$$2y^2 = -x$$
$$y = \frac{\sqrt{-x}}{\sqrt{2}}$$

$$\int_{-2}^{-1}\left(\frac{x+4}{2}\right) + \int_{-1}^{8}\frac{\sqrt{8-x}}{2} - \frac{1}{\sqrt{2}}\int_{-2}^{0}\sqrt{-x}$$

$$\Rightarrow \frac{1}{2}\left[\frac{x^2}{2}+4x\right]_{-2}^{-1} + \frac{1}{2}\left[\left(-\frac{2}{3}\right)(8-x)^{3/2}\right]_{-1}^{8} + \frac{1}{\sqrt{2}} \times \frac{2}{3}\left[(-x)^{\frac{3}{2}}\right]_{-2}^{0}$$
$$\Rightarrow \frac{1}{2}\left[\frac{1}{2}-4-(2-8)\right] - \frac{1}{3}[0-27] + \frac{\sqrt{2}}{3}[-2\sqrt{2}]$$

$$\frac{5}{4}+9-\frac{4}{3} = \frac{321}{36} = \frac{107}{12}$$
$$\frac{45+324-48}{36} = \frac{107}{12} = \frac{m}{n} \Rightarrow m+n = 107+12 = 119$$

Q. 22 If $8 = 3 + \frac{1}{4}(3+p) + \frac{1}{4^2}(3+2p) + \frac{1}{4^3}(3+3p) + \cdots\cdots\infty$, then the value of p is

Correct Answer:
9

Solution:
$$\left(3+\frac{3}{4}+\frac{3}{4^2}+\frac{3}{4^3}+\ldots\right)$$
$$+\left(\frac{P}{4}+\frac{2P}{4^2}+\frac{3P}{4^3}+\ldots\infty\right) = 8$$

$$\Rightarrow \frac{3}{1-\frac{1}{4}} + \frac{p}{4} + \frac{2p}{4^2} + \ldots\infty = 8$$
$$\Rightarrow \frac{p}{4} + \frac{2p}{4^2} + \frac{3P}{4^3} + \ldots\infty = 4 \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Let

$$\Rightarrow S = \frac{p}{4} + \frac{2p}{4^2} + \frac{3p}{4^3} \ldots \infty$$

$$\frac{S}{4} = \frac{p}{4^2} + \frac{2p}{4^3} + \ldots \infty$$

$$\frac{3S}{4} = \frac{p}{4} + \frac{p}{4^2} + \frac{p}{4^3} + \ldots \infty$$

$$\frac{3\,S}{4} = P\left(\frac{\frac{1}{4}}{1-\frac{1}{4}}\right) = \frac{p}{3}$$

Now from (1)

$$\frac{4p}{9} = 4 \Rightarrow P = 9$$

Hence, the answer is the (9).

Q. 23 Let $f(x) = x^3 + x^2 f'(1) + x f''(2) + f'''(3), x \in \mathbf{R}$. Then $f'(10)$ is equal to

Correct Answer:
202

Solution:
$f(x) = x^3 + x^2 f'(1) + x f''(2) + f''(3)$
$f'(x) = 3x^2 + 2x f'(1) + f''(2) - - - - - - - - (1)$

Put $x = 1$
$f'(1) = 3 + 2f'(1) + f''(2)$

$\Rightarrow f'(1) + f''(2) + 3 = 0 - - - - - - - - (2)$

again differentiation (1)

$f''(x) = 6x + 2f'(1)$ Put $x = 2$

$f''(2) = 12 + 2f'(1) 0 - - - - - - - - (3)$

Solving (2) & (3)
$2f'(1) - f''(2) + 12 = 0$
$2f'(1) + 2f''(2) + 6 = 0$

$3f''(2) - 6 = 0$
$f''(2) = 2$
$f'(1) = -5$

$$\begin{aligned} f'(10) &= 300 + 20(-5) + 2 \\ &= 300 - 100 + 2 \\ &= 202 \end{aligned}$$

Hence, the answer is (202).

Q. 24 The least positive integral value of α, for which the angle between the vectors $\alpha i - 2j + 2k$ and $\alpha i + 2\alpha j - 2k$ is acute, is

Correct Answer:
5

Solution:
$\vec{a} = \alpha\hat{i} - 2\hat{j} + 2k$
$\vec{b} = \alpha\hat{i} + 2\alpha\hat{j} - 2k$

$\cos\theta$ should positive for acute

$\cos\theta = \frac{\alpha^2-4\alpha-4}{\sqrt{\alpha^2+8}\sqrt{5\alpha^2+4}} \geq 0$

$= \alpha^2 - 4\alpha - 4 \geq 0$

$\Rightarrow (\alpha-2)^2 - 8 \geq 0$
$\Rightarrow (\alpha-2)^2 \geq 8$
$\Rightarrow |(\alpha-2)| \geq 2\sqrt{2}$

$(\alpha-2) \in (-\infty - 2\sqrt{2}] \cup [2\sqrt{2}, \infty]$

$\Rightarrow (-\infty, -2\sqrt{2}+2] \cup [2+2\sqrt{2}, \infty)$

$2(1+\sqrt{2})$
$\alpha = 2(1+1.141) = 2(2.414)$

= 4.828
$\Rightarrow$ least positive α is $= 5$

Hence, the answer is 5.

Q. 25 A fair die is tossed repeatedly until a six is obtained. Let X denote the number of tosses required and let $a = P(X=3), b = P(X \geq 3)$ and $c = P(X \geq 6X > 3)$. Then $\frac{b+c}{a}$ is equal to

Correct Answer:
12

Solution:
P(A) =1/ 6

$P(x=3) = 5/6 \times 5/6 \times 1/6 \Rightarrow \frac{25}{216} = a$

$P(x \geq 3) = \left(\frac{5}{6}\right)^2 \frac{1}{6} + \left(\frac{5}{6}\right)^3 (1/6) + \cdots$

$= \frac{\frac{25}{216}}{1-5/6} \Rightarrow 25/36 = b$

$P(x \geq 6/x > 3) = \frac{P(x \geq 6)}{P(x>3)}$

$= \frac{\left(\frac{5}{6}\right)^5\left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^6 \cdot \left(\frac{1}{6}\right) + \cdots}{\left(\frac{5}{6}\right)^3\left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^4\left(\frac{1}{6}\right) + \cdots}$

$= \frac{\left(\frac{5}{6}\right)^5\left(\frac{1}{6}\right)}{\left(1-\frac{5}{6}\right)} \times \frac{\left(1-\frac{5}{6}\right)}{\left(\frac{5}{6}\right)^3\left(\frac{1}{6}\right)}$

$= (5/6)^2 = \frac{25}{36} = \mathrm{C}$

$\frac{\left(\frac{50}{36}\right)}{25/216} = 12$

Hence, the answer is 12.

Q. 26 Let for a differentiable function
$f : (0, \infty) \to \mathbf{R}, f(x) - f(y) \geq \log_e \left(\frac{x}{y}\right) + x - y, \forall x, y \in (0, \infty)$
Then $\sum_{\mathrm{n}=1}^{20} \mathrm{f}'\left(\frac{1}{\mathrm{n}^2}\right)$ is equal to

Correct Answer:
2890

Solution:
$f; (0, \infty) \to R$
$f(x) - f(y) \geq \log(x) - \log y + x - y$
$f(x) - \log x - x \geq f(y) - \ln y - y$
$x \leftrightarrow y$

$f(y) - \log y - y \geq f(x) - \ln x - x$
$\Rightarrow f(x) = \ln x + x + \lambda$

$\mathrm{f}'(\mathrm{x}) = \frac{1}{\mathrm{x}} + 1$

$\sum_{\mathrm{n}=1}^{20} \left(\mathrm{n}^2 + 1\right)$

$20 + \frac{20 \times 21 \times 41}{6} = 20 + 2870 = 2890$

Hence, the answer is (2890).

Q. 27 If α satisfies the equation $\mathrm{x}^2 + \mathrm{x} + 1 = 0$ and $(1 + \alpha)^7 = \mathrm{A} + \mathrm{B}\alpha + \mathrm{C}\alpha^2,\ \mathrm{A},\ \mathrm{B}, \mathrm{C} \geq 0$, then $5(3\ \mathrm{A} - 2\ \mathrm{B} - \mathrm{C})$ is equal to _____

Correct Answer:
5

Solution:
$\alpha^2 + \alpha + 1 = 0 \left\langle {}^{\omega}_{\omega^2}\ \omega^3 = 1 \left(x^2 + x + 1 = 0\right) 0 \left\langle \left\langle {}^{\omega}_{\omega^2}\ \omega^3 = 1 \right.\right.$ let $\alpha = \omega$

$(1 + \alpha)^7 = 1 + {}^7c_1\alpha + {}^7c_2\alpha^2 + {}^7c_3\alpha^3 + {}^7c_4\alpha^4 + {}^7c_5\alpha^5 + {}^7c_6\alpha^6 + {}^7c_7\alpha^7$

$= 1 + 7\alpha + 21\alpha^2 + 35\alpha^3 + 35\alpha^4 + 24\alpha^5 + 7\alpha^6 + \alpha^9$

$\because \alpha = \omega$

$= 1 + 7\omega + 21\omega^2 + 35\omega^3 + 35\omega^4 + 21\omega^5 + 7\omega^6 + \omega^7$

$\left(\because \omega^3 = 1\right)$

$= 1 + 7\omega + 21\omega^2 + 35 + 35\omega + 21\omega + 7 + \omega$

$(\because \omega^3 = 1)$

$= 1 + 7\omega + 21\omega^2 + 35 + 35\omega + 21\omega + 7 + \omega$
$= 43 + 43\omega + 42\omega^2$

$= 43 + 43\alpha + 42\alpha^2 = A + B\alpha + C\alpha^2$
$\Rightarrow A = 43 | B = 43 | C = 42$

$5(129 - 86 - 42)$
$= 5(129 - 128) = 5$

Hence, the answer is the (5).

Q. 28 Let $A = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}, B = [B_1, B_2, B_3]$, where $\mathrm{B}_1,\ \mathrm{B}_2,\ \mathrm{B}_3$ are column matrics, and

$$\mathrm{AB}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \mathrm{AB}_2 = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}, \mathrm{AB}_3 = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

If $\alpha = |B|$ and β is the sum of all the diagonal elements of B, then $\alpha^3 + \beta^3$ is equal to

Correct Answer:
28

Solution:

$$\begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} \mathrm{B}_1 & \mathrm{B}_2 & \mathrm{B}_3 \\ \mathrm{a} & \mathrm{d} & \mathrm{g} \\ \mathrm{b} & \mathrm{e} & \mathrm{h} \\ \mathrm{c} & \mathrm{f} & \mathrm{i} \end{bmatrix}$$

$$\mathrm{AB}_1 = \begin{bmatrix} 2\mathrm{a} + \mathrm{c} \\ \mathrm{a} + \mathrm{b} \\ \mathrm{a} + \mathrm{c} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{matrix} \mathrm{a} = -\mathrm{b} \\ \mathrm{a} = -\mathrm{c} \end{matrix} \qquad \mathrm{a} = 1,\ \mathrm{b} = -1, \mathrm{c} = -1$$

$$\mathrm{AB}_2 = \begin{bmatrix} 2\,\mathrm{d} + \mathrm{f} \\ \mathrm{d} + \mathrm{e} \\ \mathrm{d} + \mathrm{f} \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$$

$d = -f$
$d = 2, f = -2$
$e = 1$

$$\mathrm{AB}_3 = \begin{bmatrix} 2\,\mathrm{g} + \mathrm{i} \\ \mathrm{g} + \mathrm{h} \\ \mathrm{g} + \mathrm{i} \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} \Rightarrow \mathrm{g} = 1 - \mathrm{i}$$

$2 - 2i + i = 3$
$i = -1$

$g = 2,\ h = 0$

$$B = \begin{bmatrix} 1 & 2 & 2 \\ -1 & 1 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

$\alpha = |B| = 1(-1) - 2(1) + 2(2+1) = 3$
$\alpha = 3, \beta = \text{trace}(B) = 1$
$\alpha^3 + \beta^3 = 3^3 + 1^3 = 28$

Hence, the answer is (28).

Q. 29 If the solution of the differential equation $(2x + 3y - 2)dx + (4x + 6y - 7)dy = 0, y(0) = 3$, is $\alpha x + \beta y + 3\log_e|2x + 3y - \gamma| = 6$, then $\alpha + 2\beta + 3\gamma$ is equal to

Correct Answer:
29

Solution:
$(2x + 3y - 2)dx + (4x + 6y - 7)dy = 0$

$\frac{dy}{dx} = -\frac{(2x+3y-2)}{(4x+6y-7)}$
$\frac{dy}{dx} = \frac{-(2x+3y-2)}{(2)(2x+3y)-7}$

$2x + 3y = v$
$2 + 3 \cdot \frac{dy}{dx} = \frac{dv}{dx}$

$$\Rightarrow \left(\frac{dv}{dx} - 2\right)\frac{1}{3} = \frac{-(v-2)}{2v-7}$$
$$\Rightarrow \frac{dv}{dx} - 2 = \frac{-3v+6}{2v-7}$$
$$\Rightarrow \frac{dv}{dx} = \frac{-3v+6}{2v-7} + 2 = \frac{-3v+6+4v-14}{2v-7}$$
$$\Rightarrow \frac{dv}{dx} = \frac{v-8}{2v-7}$$
$$\Rightarrow \left(\frac{2v-7}{v-8}\right)dv = dx$$
$$\Rightarrow \left(\frac{(v-8)+v+1}{(v-8)}\right)dv = dx$$
$$\Rightarrow dv + \left(\frac{v-8+9}{v-8}\right)dv = dx$$
$$\Rightarrow dv + dv + \left(\frac{9}{v-8}\right)dv = dx$$

$\Rightarrow \int 2dx + \int \frac{9}{v-8}dx = \int dx$

$\Rightarrow 2\text{v} + 9\log|\text{v} - 8| = \text{x} + \text{c}$
$\Rightarrow 2(2\text{x} + 3\text{y}) + 9\log|2\text{x} + 3\text{y} - 8| = \text{x} + \text{c}$

y (0) = 3

$2(0+9)+9\log|9-8| = 0+C \Rightarrow C = 18$
$\Rightarrow 9(2x+3y)+9\log|2x-3y-8| = x+18$

$\Rightarrow 4x+6y+9\log|2x+3y-8| = x+18$
$\Rightarrow 3x+6y+9\log|2x+3y-8| = 18$
$\Rightarrow x+2y+3\log|2x+3y-8| = 6$

$\Rightarrow \alpha x+\beta y+3\log|2x+3y-\gamma| = 6$

$\alpha = 1, \beta = 2, \gamma = 8$
$1+4+24 = 29$

Hence, the answer is the (29).

Q. 30 Let the set of all $a \in \mathbf{R}$ such that the equation $\cos 2x + a\sin x = 2a - 7$ has a solution be $[p, q]$ and $r = \tan 9^\circ - \tan 27^\circ - \frac{1}{\cot 63^\circ} + \tan 81^\circ$, then pqr is equal to _____

Correct Answer:
48

Solution:
$\cos 2x + a\sin x = 2a - 7$
$\Rightarrow 1 - 2\sin^2 x + 7 = a(2 - \sin x)$

$\Rightarrow \dfrac{2\left(4-\sin^2 x\right)}{2-\sin x} = a$
$\Rightarrow \dfrac{2(2-\sin x)(2+\sin x)}{2-\sin x} = a$

$\Rightarrow a = 4 + 2\sin x$
$\Rightarrow a \in [2, 6] \cong [p, q]$

$\tan 9^\circ - \tan 27^\circ - \tan 63^\circ + \tan 81^\circ$
$= \tan 9^\circ - \tan 27^\circ - \cot 27^\circ + \cot 9^\circ$

$= \frac{\sin 9^\circ}{\cos 9^\circ} + \frac{\cos 9^\circ}{\sin 9^\circ} - \left(\frac{\cos 27^\circ}{\sin 27^\circ} + \frac{\sin 27^\circ}{\cos 27^\circ}\right)$

$= \dfrac{2}{2} \times \dfrac{1}{\sin 9^\circ \cos 9^\circ} - \dfrac{2}{2}\dfrac{1}{\sin 27^\circ \cos 27^\circ}$
$= \dfrac{2}{\sin 18^\circ} - \dfrac{2}{\sin 54^\circ}$

$\dfrac{2}{\sqrt{5}-1} \times 4 - \dfrac{2}{\sqrt{5}+1} \times 4 = 8\left[\dfrac{\sqrt{5}+1}{4} - \dfrac{\sqrt{5}-1}{4}\right]$
$= 2[2] = 4 \Rightarrow \text{pqr} = 48$

Hence, the answer is (48).

27 Jan - Evening Shift

Physics

Q. 1 The equation of state of a real gas is given by $\left(\mathrm{P}+\frac{\mathrm{a}}{\mathrm{V}^2}\right)(\mathrm{V}-\mathrm{b})=\mathrm{RT}$, where P, V and T are pressure, volume and temperature respectively and R is the universal gas constant. The dimensions of $\frac{\mathrm{a}}{\mathrm{b}^2}$ is similar to that of:

Option 1:
PV

Option 2:
P

Option 3:
RT

Option 4:
R

Correct Answer:
P

Solution:

$$\left(\mathrm{P}+\frac{\mathrm{a}}{\mathrm{V}^2}\right)(\mathrm{V}-\mathrm{b})=\mathrm{RT}$$

$$\text{dimension } [\mathrm{P}]=\frac{[\mathrm{a}]}{[\mathrm{V}^2]} \Rightarrow [\mathrm{a}]=[\mathrm{P}]\left[\mathrm{V}^2\right]$$

also, $[\mathrm{V}]=[\mathrm{b}]$

So, dimension of $\frac{\mathrm{a}}{\mathrm{b}^2}=\frac{[\mathrm{a}]}{[\mathrm{b}^2]}=\frac{[\mathrm{P}][\mathrm{V}^2]}{[\mathrm{V}^2]}=[\mathrm{P}]$

Hence, the answer is the option (2).

Q. 2 Wheatstone bridge principle is used to measure the specific resistance (S_1) of a given wire, having length L, radius r. If X is the resistance of wire, then specific resistance is; $S_1=X\left(\frac{\pi r^2}{L}\right)$. If the length of the wire gets doubled then the value of the specific resistance will be:

Option 1:
$\frac{S_1}{4}$

Option 2:
$2S_1$

Option 3:
$\frac{S_1}{2}$

Option 4:
S_1

Correct Answer:
S_1

Solution:
As specific resistance only depends on the material it remains the same.

Hence, the answer is the option (4).

Q. 3 Given below are two statements; one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A): The angular speed of the moon in its orbit around the Earth is more than the angular speed of the earth in its orbit around the sun.
Reasons (R): The moon takes less time to move around the earth than the time taken by the earth to move around the sun.

In the light of the above statements, choose the most appropriate answer from the options given below:

Option 1:
(A) is correct but (R) is not correct

Option 2:
Both (A) and (R) are correct and (R) is the correct explanation of (A)

Option 3:
Both (A) and (R) are correct but (R) is not the correct explanation of (A)

Option 4:
(A) is not correct but (R) is correct

Correct Answer:
Both (A) and (R) are correct and (R) is the correct explanation of (A)

Solution:
Angular speed, $\omega = \frac{2\pi}{T}$
The period of the moon is smaller than time period of the earth
So, $\omega_{\text{moon}} > \omega_{\text{earth}}$

Hence, the answer is the option (2).

Q. 4 Given below are two statements:

Statement (I): The limiting force of static friction depends on the area of contact and is independent of materials.
Statement (II): The limiting force of kinetic friction is independent of the area of contact and depends on materials.

In the light of the above statements, choose the most appropriate answer from the options given below:

Option 1:
Statement I is correct but Statement II is incorrect

Option 2:
Statement I is incorrect but Statement II is correct

Option 3:
Both Statement I and Statement II are incorrect

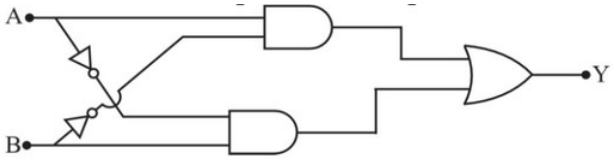
Option 4:
Both Statement I and Statement II are correct

Correct Answer:
Statement I is incorrect but Statement II is correct

Solution:
Static friction and kinetic friction depend on material and are independent of area of contact.

Hence, the answer is the option (2).

Q. 5 The truth table of the given circuit diagram is:



Option 1:

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

Option 2:

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

Option 3:

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

Option 4:

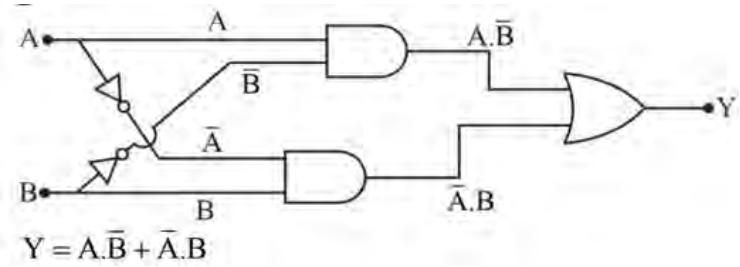
A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

Correct Answer:

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

Solution:

CAREERS360



So, the truth table is of XOR gate

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

Q. 6 A current of $200\mu A$ deflects the coil of a moving coil galvanometer through 60°. The current to cause deflection through $\frac{\pi}{10}$ radian is:

Option 1:
$30\mu A$

Option 2:
$120\mu A$

Option 3:
$60\mu A$

Option 4:
$180\mu A$

Correct Answer:
$60\mu A$

Solution:
As deflection $\phi \propto$ current I

$\Rightarrow \frac{i_2}{i_1} = \frac{\phi_2}{\phi_1}$

$i_2 = \left(\frac{180^{\circ}/10^{\circ}}{60^{\circ}}\right) 200\mu A = 60\mu A$

Hence, the answer is the option (3).

Q. 7 The atomic mass of ${}_6C^{12}$ is $12.000000u$ and that of ${}_6C^{13}$ is $13.003354u$. The required energy to remove a neutron from ${}_6C^{13}$, if mass of neutron is $1.008665u$, will be:

Option 1:
62.5 MeV

Option 2:
6.25 MeV

Option 3:
4.95 MeV

Option 4:
49.5 MeV

Correct Answer:
4.95 MeV

Solution:

$$^{13}_{6}\mathrm{C} \longrightarrow {}^{12}_{6}\mathrm{C} + \mathrm{D} + \theta$$

$$\mathrm{Q} = \left[\text{ mass of } {}^{13}_{6}\mathrm{C} - \text{ mass of } {}^{12}_{6}\mathrm{C} - \text{ mass of neutron }\right]\mathrm{c}^2$$

$$\mathrm{Q} = [13.003354 - 12 - 1.008665][931.5\mathrm{Mev}]$$

$$\mathrm{Q} = -4.95\mathrm{MeV}$$

$$\Rightarrow \text{ Energy required is } 4.95\mathrm{MeV}$$

Hence, the answer is the option (3).

Q. 8 A ball suspended by a thread swings in a vertical plane so that its magnitude of acceleration in the extreme position and lowest position are equal. The angle (θ) of thread deflection in the extreme position will be:

Option 1:
$\tan^{-1}(\sqrt{2})$

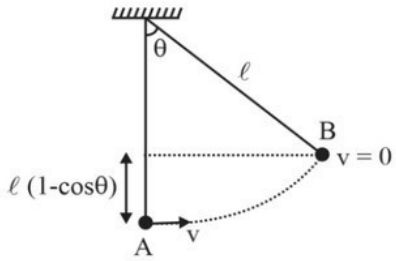
Option 2:
$2\tan^{-1}\left(\frac{1}{2}\right)$

Option 3:
$\tan^{-1}\left(\frac{1}{2}\right)$

Option 4:
$2\tan^{-1}\left(\frac{1}{5}\right)$

Correct Answer:
$2\tan^{-1}\left(\frac{1}{2}\right)$

Solution:



By energy conservation

$$\frac{1}{2}mv^2 + 0 = 0 + mg\ell(1-\cos\theta)$$
$$\Rightarrow v^2 = 2\,g\ell(1-\cos\theta)$$

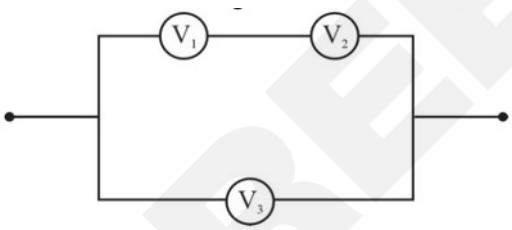
Given, $a_A = a_B$

$$\frac{v^2}{\ell} = g\sin\theta$$

$$2\,g(1-\cos\theta) = g\sin\theta \Rightarrow 2\left(2\sin^2\frac{\theta}{2}\right) = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$$

$$\tan\frac{\theta}{2} = \frac{1}{2} \Rightarrow \theta = 2\tan^{-1}\frac{1}{2}$$

Q. 9 Three voltmeters, all having different internal resistances are joined as shown in the figure. When some potential difference is applied across A and B, their readings are V_1, V_2 and V_3. Choose the correct option.



Option 1:
$V_1 = V_2$

Option 2:
$V_1 \neq V_3 - V_2$

Option 3:
$V_1 + V_2 > V_3$

Option 4:
$V_1 + V_2 = V_3$

Correct Answer:
$V_1 + V_2 = V_3$

Solution:
as $V_1 + V_2$ is parallel with V_3
So, $\mathrm{V_1 + V_2 = V_3}$

Q. 10 The total kinetic energy of 1 mole of oxygen at $27^{\circ}\mathrm{C}$ is: [Use universal gas constant $(\mathrm{R}) = 8.31\ \mathrm{J/moleK}$]

Option 1:
6845.5 J

Option 2:
5942.0 J

Option 3:
6232.5 J

Option 4:
5670.5 J

Correct Answer:
6232.5 J

Solution:
$\mathrm{Totat\ K.E.} = \frac{\mathrm{nfRT}}{2} = \frac{1\times5\times8.314\times300}{2} \approx 6235.5\ \mathrm{J}$

Hence, the answer is the option (3).

Q. 11 Given below are two statements; one is labelled as Assertion (A) and the other is labelled as Reason (R).
Assertion (A): In Vernier Calliper if a positive zero error exists, then while taking measurements, the reading
taken will be more than the actual reading.
Reasons (R): The zero error in Vernier Calliper might have happened due to a manufacturing defect or due to rough handling.

In the light of the above statements, choose the correct answer from the options given below:

Option 1:
Both (A) and (R) are correct and (R) is the correct explanation of (A)

Option 2:
Both (A) and (R) are correct but (R) is not the correct explanation of (A)

Option 3:
(A) is true but (R) is false

Option 4:
(A) is false but (R) is true

Correct Answer:
Both (A) and (R) are correct but (R) is not the correct explanation of (A)

Solution:
Actual reading = observed reading – zero error
⇒ observed reading > actual reading

Hence, the answer is the option (2).

Q. 12 Primary side of a transformer is connected to $230\ \text{V}, 50\ \text{Hz}$ supply. Turns ratio of primary to secondary winding is 10 : 1 . Load resistance connected to secondary side is 46Ω. The power consumed in it is :

Option 1:
12.5 W

Option 2:
10.0 W

Option 3:
11.5 W

Option 4:
12.0 W

Correct Answer:
11.5 W

Solution:

$$\text{As } \frac{V_2}{V_1} = \frac{N_2}{N_1} \Rightarrow \frac{V_2}{230} = \frac{1}{10} \Rightarrow V_2 = 23 \text{ Volt}$$

$$\text{Power } = \frac{V^2}{R} = \frac{(23)^2}{46} = \frac{23}{2} = 11.5 \text{ W}$$

Hence, the answer is the option (3).

Q. 13 During an adiabatic process, the pressure of a gas is found to be proportional to the cube of its absolute temperature. The ratio of $\frac{C_p}{C_v}$ for the gas is:

Option 1:
$\frac{5}{3}$

Option 2:
$\frac{3}{2}$

Option 3:
$\frac{7}{5}$

Option 4:
$\frac{9}{7}$

Correct Answer:
$\frac{3}{2}$

Solution:

$$\text{As } \frac{V_2}{V_1} = \frac{N_2}{N_1} \Rightarrow \frac{V_2}{230} = \frac{1}{10} \Rightarrow V_2 = 23 \text{ Volt}$$

$$\text{Power } = \frac{V^2}{R} = \frac{(23)^2}{46} = \frac{23}{2} = 11.5 \text{ W}$$

Hence, the answer is the option (2).

Q. 14 The threshold frequency of a metal with work function 6.63 eV is:

Option 1:
16×10^{15} Hz

Option 2:
16×10^{12} Hz

Option 3:
1.6×10^{12} Hz

Option 4:
1.6×10^{15} Hz

Correct Answer:
1.6×10^{15} Hz

Solution:
$$\text{work function } \phi = \mathrm{h}\nu_{\mathrm{th}}$$
$$v_{\mathrm{th}} = \frac{\phi}{\mathrm{h}} = \frac{6.63 \times 1.6 \times 10^{-19}}{6.63 \times 10^{-34}} = 1.6 \times 10^{15}\ \mathrm{Hz}$$

Hence, the answer is the option (4).

Q. 15 Given below are two statements; one is labelled as Assertion (A) and the other is labelled as Reason (R).
Assertion (A): The property of the body, by virtue of which it tends to regain its original shape when the external force is removed, is Elasticity.
Reasons (R): The restoring force depends upon the bonded inter-atomic and inter-molecular forces of a solid.

In the light of the above statements, choose the correct answer from the options given below:

Option 1:
(A) is false but (R) is true

Option 2:
(A) is true but (R) is false

Option 3:
Both (A) and (R) are true and (R) is the correct explanation of (A)

Option 4:
Both (A) and (R) are true but (R) is not the correct explanation of (A)

Correct Answer:
Both (A) and (R) are true and (R) is the correct explanation of (A)

Solution:
The property of matter by virtue of which a body tends to regain its original shape and size after the removal of deforming force is called elasticity.

Hence, the answer is the option (3).

Q. 16 When a polaroid sheet is rotated between two crossed polaroids then the transmitted light intensity will be maximum for a rotation of:

Option 1:
(A) is false but (R) is true

Option 2:
(A) is true but (R) is false

Option 3:
Both (A) and (R) are true and (R) is the correct explanation of (A)

Option 4:
Both (A) and (R) are true but (R) is not the correct explanation of (A)

Correct Answer:
Both (A) and (R) are true and (R) is the correct explanation of (A)

Solution:
by definition of elasticity

Q. 17 An object is placed in a medium of refractive index 3 . An electromagnetic wave of intensity $6 \times 10^8\ \mathrm{W/m^2}$ falls normally on the object and it is absorbed completely. The radiation pressure on the object would be (speed of light in free space $= 3 \times 10^8\ \mathrm{m/s}$)

Option 1:
60°

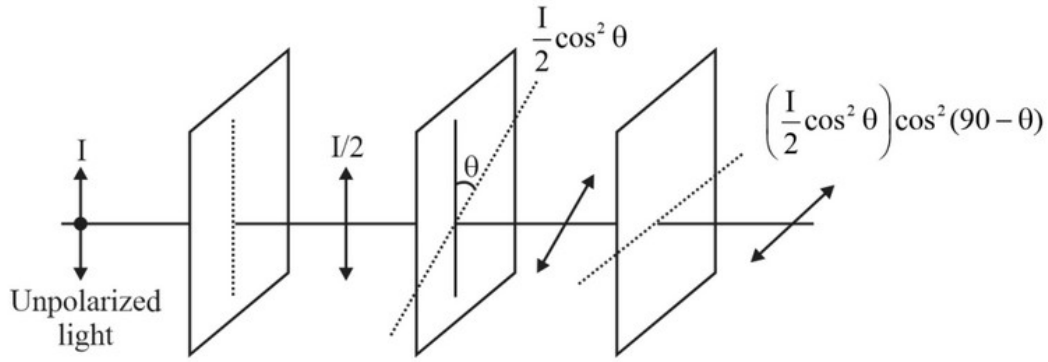
Option 2:
30°

Option 3:
90°

Option 4:
45°

Correct Answer:
45°

Solution:



find output $= \frac{I}{2}\cos^2\theta\sin^2\theta = \frac{I}{8}(\sin 2\theta)^2$
Clearly output is maximum for $\theta = 45^\circ$

Q. 18 Given below are two statements; one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A): Work done by electric field on moving a positive charge on an equipotential surface is always zero.
Reasons (R): Electric lines of forces are always perpendicular to equipotential surfaces.

In the light of the above statements, choose the most appropriate answer from the options given below:

Option 1:
Both (A) and (R) are correct and (R) is not the correct explanation of (A)

Option 2:
(A) is correct but (R) is not correct

Option 3:
(A) is not correct but (R) is correct

Option 4:
Both (A) and (R) are correct but (R) is the correct explanation of (A)

Correct Answer:
Both (A) and (R) are correct but (R) is the correct explanation of (A)

Solution:

As electric field and electric force on charge are perpendicular to the equipotential surface. So work done by electric force is zero on moving a charge on an equipotential surface.

Hence, the answer is the option (4).

Q. 19 A heavy iron bar of weight 12 kg is having its one end on the ground and the other on the shoulder of a man.
The rod makes an angle 60° with the horizontal, the weight experienced by the man is:

Option 1:
6 kg

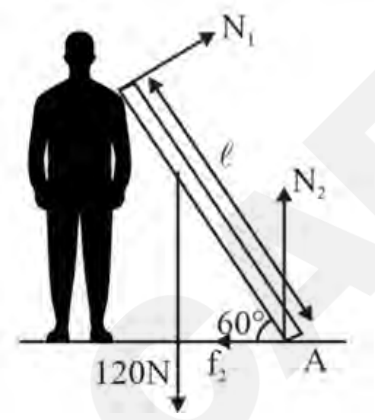
Option 2:
12 kg

Option 3:
3 kg

Option 4:
$6\sqrt{3}$ kg

Correct Answer:
3 kg

Solution:



As rod is in equilibrium, net torque on it about point A is zero
$\tau_A = 0 = (120)\frac{\ell}{2}\cos 60^\circ - N_1\ell \Rightarrow N_1 = 30\ N = 3\ kg.f$

Q. 20 A bullet is fired into a fixed target looses one third of its velocity after travelling $4\ cm$. It penetrates further $D\times 10^{-3}\ m$ before coming to rest. The value of D is:

Option 1:
2

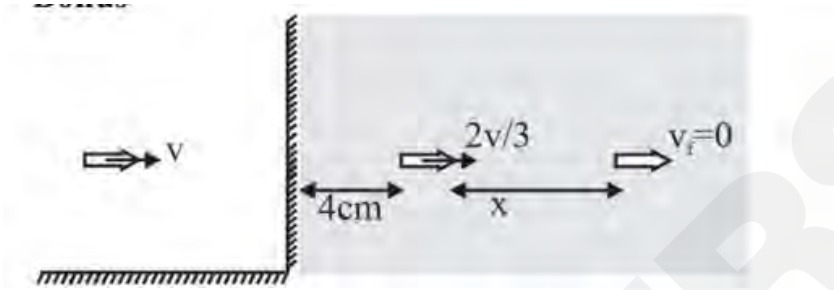
Option 2:
5

Option 3:
3

Option 4:
4

Correct Answer:
2

Solution:



Let resistive force acting on bullet remain same ⇒ retardation is uniform

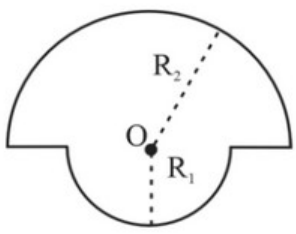
by $v^2 = u^2 + 2as \rightarrow \left(\frac{2v}{3}\right)^2 = v^2 - 2a(4\ cm) \Rightarrow 2a(4\ cm) = \frac{5v^2}{9}$

$$\text{also, } O^2 = \left(\frac{2v}{3}\right)^2 - 2ax$$

$$2ax = \frac{4v^2}{9} = 4\left[\frac{2a(4\ cm)}{5}\right]$$

$$\Rightarrow x = \frac{16}{5}\ cm = 3.2\ cm = 32 \times 10^{-3}\ m$$

Q. 21 The magnetic field at the centre of a wire loop formed by two semicircular wires of radii $R_1 = 2\pi m$ and $R_2 = 4\pi m$, carrying current $I = 4\ A$ as per figure given below is $\alpha \times 10^{-7}\ T$. The value of α is (Centre O is common for all segments)



Correct Answer:
3

Solution:

$$B = \frac{\mu_0 i}{4R_1} + \frac{\mu_0 i}{4R_2} = \frac{\mu_0 \times 4}{4}\left[\frac{1}{2\pi} + \frac{1}{4\pi}\right] = 3\left(\frac{\mu_0}{4\pi}\right)$$
$$= 3 \times 10^{-7}\ T$$
$$\Rightarrow \alpha = 3$$

Q. 22 Two charges of $-4\mu C$ and $+4\mu C$ are placed at the points $A(1, 0, 4)m$ and $B(2, -1, 5)m$ located in an electric field $\vec{E} = 0.20\hat{i}V/cm$. The magnitude of the torque acting on the dipole is $8\sqrt{\alpha} \times 10^{-5}Nm$, where $\alpha =$

Correct Answer:
2

Solution:

$$\text{dipole moment } \vec{p} = q\vec{a} = 4\mu C(\hat{i} - \hat{j} + \hat{k})$$
$$\text{Torque } = \vec{P} \times \vec{E} = 4\mu C(\hat{i} - \hat{j} + \hat{k}) \times (20\ V/m)\hat{i}$$
$$\vec{\tau} = 8 \times 10^{-5}Nm(\hat{k} + \hat{j}) \Rightarrow |\vec{\tau}| = 8\sqrt{2} \times 10^{-5}Nm$$
$$\Rightarrow \alpha = 2$$

Hence, the answer is (2).

Q. 23 A body falling under gravity covers two points A and B separated by 80 m is 2s. The distance of upper point A from the starting point is ____ m (use g = 10 ms^{-2})

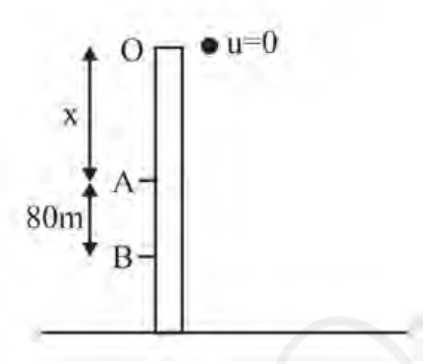
Correct Answer:
45

Solution:

by $S = ut + \frac{1}{2}at^2$

$-x = -\frac{1}{2}gt_1^2$ ($t_1 \rightarrow$ time to travel from O to A)

$\Rightarrow x = 5t_1^2$



also $-(x+80) = -\frac{1}{2}g(t_1+2)^2$

$\Rightarrow x + 80 = 5\left(t_1^2 + 4t_1 + 4\right)$

$\Rightarrow 5t_1^2 + 80 = 5t_1^2 + 20t_1 + 20 \Rightarrow 20t_1 = 60$

$\Rightarrow t_1 = 3\text{sec}$

$\Rightarrow x = 5(3)^2 = 45\ \text{m}$

Q. 24 A closed organ pipe 150 cm long gives 7 beats per second with an open organ pipe of length 350 cm, both vibrating in fundamental mode.The velocity of sound is_____m/s.

Correct Answer:
294

Solution:

$f_1 = \frac{v}{4l_1} = \frac{v}{4(1.5)} = \frac{v}{6}\ \text{Hz}$

$f_2 = \frac{v}{2(3.5)} = \frac{v}{7}$

Clearly $f_1 > f_2$ So beats $7 = f_1 - f_2 = \frac{v}{6} - \frac{v}{7} = \frac{v}{42}$

$\Rightarrow v = 42 \times 7 = 294\ \text{ms}^{-2}$

Hence, the answer is the option (1).

Q. 25 The reading of pressure metre attached with a closed pipe is $4.5 \times 10^4\ \text{N/m}^2$. On opening the valve, water starts flowing and the reading of pressure metre falls to $2.0 \times 10^4\ \text{N/m}^2$. The velocity of water is found to be $\sqrt{V}\text{m/s}$ The value of V is

Correct Answer:
50.00

Solution:

To Bernoullie's equation

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

$$4.5 \times 10^4 + 0 + 0 = 2 \times 10^4 + \frac{1}{2}(1000)v_2^2 + 0$$

$$v_2 = \sqrt{50}\ m/s$$
$$\Rightarrow v = 50$$

Hence, the answer is (50).

Q. 26 A ring and a solid sphere roll down the same inclined plane without slipping. They start from rest. The radii of both bodies are identical and the ratio of their kinetic energies is $\frac{7}{x}$, where x is

Option 1:
8

Option 2:
9

Option 3:
11

Option 4:
None of these.

Correct Answer:
None of these.

Solution:
We assume the masses of both objects are the same
If we consider total kinetic energy then both objects have the same K.E. and are equal to mgh

$$\Rightarrow \frac{7}{x} = 1 \Rightarrow x = 7$$

If we consider only translation K.E. then both have different kinetic energy
Translational K.E. $= \frac{1}{\left(1+\frac{K^2}{R^2}\right)}mgh$

$$\frac{\text{K.E.of Ring}}{\text{K.E. of Solid Sphere}} = \frac{\left(\frac{1}{1+1}\right)mgh}{\left(\frac{1}{1+\frac{2}{5}}\right)mgh} = \frac{7}{10}$$

⇒x=10

Hence, the answer is (7 or 10.00).

Hence, the answer is the option (4).

Q. 27 A parallel beam of monochromatic light of wavelength $5000\AA$ is incident normally on a single narrow slit of width $0.001\ \text{mm}$. The light is focused by convex lens on screen, placed on its focal plane. The first minima will be formed for the angle of diffraction of (degree).

Correct Answer:
30.00

Solution:
for 1^{st} minima $\sin\theta = \frac{d}{\lambda}$

$\text{Sin}\,\theta = \frac{0.001\ \text{mm}}{5000\AA} = \frac{10^{-6}\ \text{m}}{5\times10^{-7}\ \text{m}} = \frac{1}{2}$

$\theta = 30^{\circ}$

Q. 28 The electric potential at the surface of an atomic nucleus $(\text{z} = 50)$ of radius $9 \times 10^{-13}\ \text{cm}$ is $\ldots \times 10^{6}\ \text{V}$.

Correct Answer:
8.00

Solution:
We assume positive charge is uniformly distributed in the nucleus
charge $\text{q} = \text{ze} = 50 \times 1.6 \times 10^{-19} = 8 \times 10^{-18}\text{C}$

So, potential $\text{V} = \frac{\text{kq}}{\text{R}} = \frac{9\times10^{9}\times8\times10^{-18}}{9\times10^{-15}} = 8 \times 10^{6}\text{Volt}$

Hence, the answer is (8).

Q. 29 If Rydberg's constant is R, the longest wavelength of radiation in Paschen series will be $\frac{\alpha}{7R}$, where $\alpha =$

Correct Answer:
144

Solution:
Longest wavelength $\Rightarrow$ smallest energy gap of paschen $\Rightarrow n = 4$ to $n = 3$

$$\frac{1}{\lambda}=\mathrm{R}\left(\frac{1}{\mathrm{n}_1^2}-\frac{1}{\mathrm{n}_2^2}\right)=\mathrm{R}\left(\frac{1}{3^2}-\frac{1}{4^2}\right)$$

$$\frac{1}{\lambda}=\mathrm{R}\left(\frac{1}{9}-\frac{1}{16}\right)=\frac{7\mathrm{R}}{144}$$

$\lambda=\frac{144}{7\mathrm{R}} \Rightarrow \alpha=144$

Hence, the answer is the option (1).

Q. 30 A series LCR circuit with $\mathrm{L}=\frac{100}{\pi}\mathrm{mH}, \mathrm{C}=\frac{10^{-3}}{\pi}\mathrm{F}$ and $\mathrm{R}=10\Omega$, is connected across an ac source of $220\ \mathrm{V}, 50$ Hz supply. The power factor of the circuit would be

Correct Answer:
1

Solution:
$\omega=2\pi\mathrm{f}=2\pi(50)=100\pi\mathrm{rad/sec.}$

$\mathrm{X_L}=\omega\mathrm{L}=(100\pi)\left(\frac{100}{\pi}\mathrm{mH}\right)=10\Omega$

$X_c=\frac{1}{\omega C}=\frac{1}{(100\pi)\left(\frac{10^{-3}}{\pi}\right)}=10\Omega$

net reactance of circuit $X=X_L-X_C=10-10=0$

$\Rightarrow$ Circuit becomes purely resistive
$\Rightarrow$ Power factor $=\cos\phi=1$

Chemistry

Q. 1 Which of the following statements is not correct about rusting of iron?

Option 1:
Coating of iron surface by tin prevents rusting, even if the tin coating is peeling off.

Option 2:
When pH lies above 9 or 10, rusting of iron does not take place.

Option 3:
Dissolved acidic oxides $\mathrm{SO_2}, \mathrm{NO_2}$ in water act as catalyst in the process of rusting

Option 4:
Rusting of iron is envisaged as setting up of electrochemical cell on the surface of iron object.

Correct Answer:
Coating of iron surface by tin prevents rusting, even if the tin coating is peeling off.

Solution:
Rusting processes and prevention methods are crucial in many industries to protect infrastructure, machinery, and various metal products from corrosion and extend their lifespan.

Hence, the answer is the option (1).

Q. 2 Identify the incorrect pair from the following:

Option 1:
Photography – AgBr

Option 2:
Polythene preparation – $\mathrm{TiCl_4, Al(CH_3)_3}$

Option 3:
Haber process - Iron

Option 4:
Wacker process – $\mathrm{PtCl_2}$

Correct Answer:
Wacker process – $\mathrm{PtCl_2}$

Solution:
Wacker process $\mathrm{PdCl_2}$ acts as catalyst unsaturated hydrocarbon $\xrightarrow{\mathrm{PdCl_2}}$ saturated hydrocarbon.

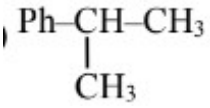
Hence, the answer is the option (4).

Q. 3 The final product A, formed in the following reaction sequence is :

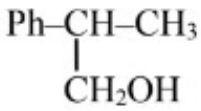
$$\mathrm{Ph{-}CH{=}CH_2} \xrightarrow[\substack{(iii)\,HBr \\ (iv)\,Mg,\,ether,\,then\,HCHO/H_3O^+}]{\substack{(i)\,BH_3 \\ (ii)\,H_2O_2,\,^{\ominus}OH}} A$$

Option 1:
$\mathrm{Ph - CH_2 - CH_2 - CH_3}$

Option 2:



Option 3:



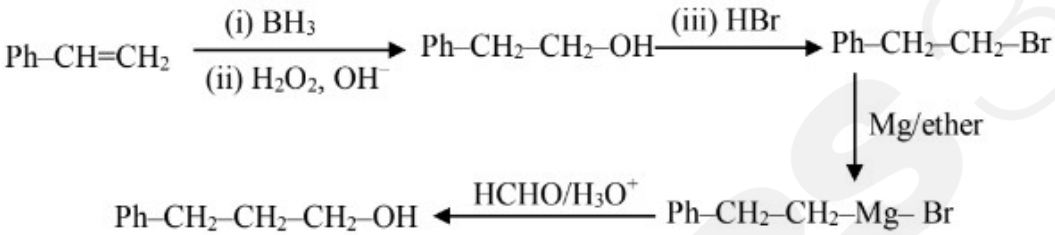
Option 4:

$Ph-CH_2-CH_2-CH_2-OH$

Correct Answer:

$Ph-CH_2-CH_2-CH_2-OH$

Solution:



Q. 4 Choose the correct option having all the elements with d^{10} electronic configuration from the following :

Option 1:

$^{27}Co, {}^{28}Ni, {}^{26}Fe, {}^{24}Cr$

Option 2:

$^{29}Cu, {}^{30}zn, {}^{48}Cd, {}^{47}Ag$

Option 3:

$^{46}Pd, {}^{28}Ni, {}^{26}Fe, {}^{24}Cr$

Option 4:

$^{28}Ni, {}^{24}Cr, {}^{26}Fe, {}^{29}Cu$

Correct Answer:

$^{29}Cu, {}^{30}zn, {}^{48}Cd, {}^{47}Ag$

Solution:

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The transition metals, or the d-block of the periodic table, contain elements having an electronic configuration of d^{10}, since the d subshell there is entirely occupied with 10 electrons as given below:

$\mathrm{Cu} = 3\mathrm{d}^{10}4\mathrm{S}^{1}$

$\mathrm{Zn} = 3\mathrm{d}^{10}4\mathrm{S}^{2}$

$\mathrm{cd} = 4\mathrm{d}^{10}5\mathrm{S}^{2}$

$\mathrm{Ag} = 4\mathrm{d}^{10}5\mathrm{S}^{1}$

Hence, the answer is the option (2).

Q. 5 Given below are two statements:

Statement (I): Oxygen being the first member of group 16 exhibits only –2 oxidation state.

Statement (II): Down the group 16 stability of +4 oxidation state decreases and +6 oxidation state increases.

In the light of the above statements, choose the most appropriate answer from the options given below:

Option 1:
Statement I is correct but Statement II is incorrect

Option 2:
Both Statement I and Statement II are correct

Option 3:
Both Statement I and Statement II are incorrect

Option 4:
Statement I is incorrect but Statement II is correct

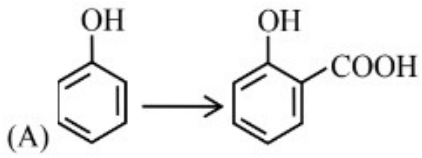
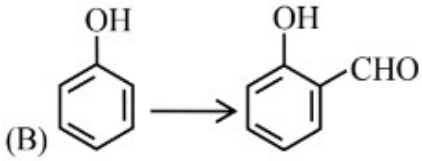
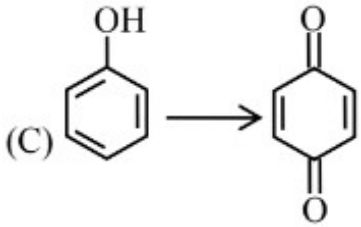
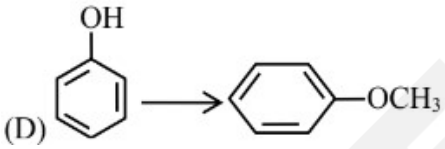
Correct Answer:
Both Statement I and Statement II are incorrect

Solution:
$\rightarrow$ O.S of oxygen are = –2, –1, 0

$\rightarrow$ down the group stability of lower O.S. increases and Higher O.S. will decrease.

Hence, the answer is the option (3).

Q. 6 Match List-I with List-II

List - I (Reaction)	List - II (Reagent(s))
 (A)	(i) $Na_2Cr_2O_7, H_2SO_4$
 (B)	(ii) (i) $NaOH$ (ii) CH_3Cl
 (C)	(iii) (i) $NaOH, CHCl_3$ (ii) $NaOH$ (iii) HCl
 (D)	(iv) (i) $NaOH$ (ii) CO_2 (iii) HCl

Choose the correct answer from the options given below:

Option 1:
(A)-(IV), (B)-(I), (C)-(III), (D)-(II)

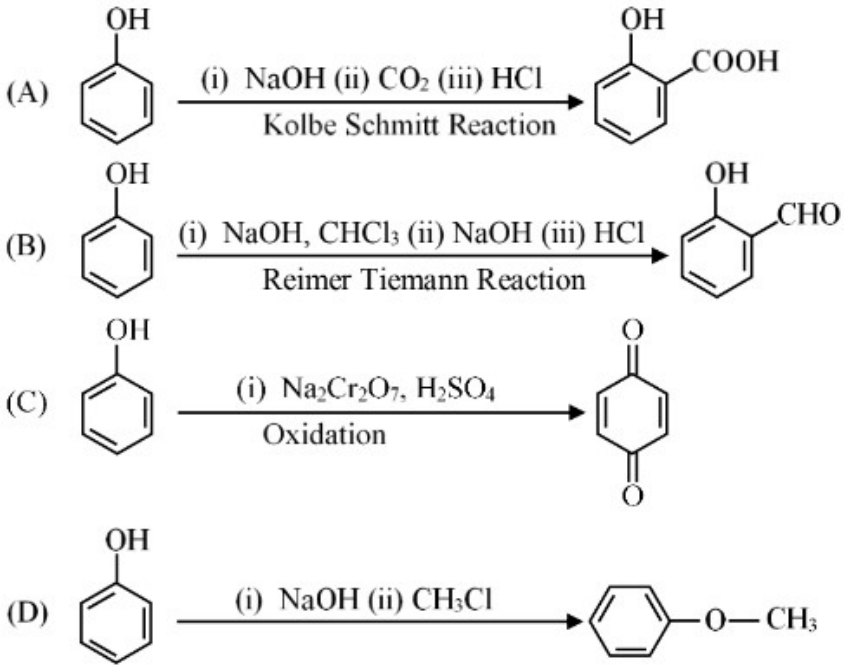
Option 2:
(A)-(II), (B)-(III), (C)-(I), (D)-(IV)

Option 3:
(A)-(II), (B)-(I), (C)-(III), (D)-(IV)

Option 4:
(A)-(IV), (B)-(III), (C)-(I), (D)-(II)

Correct Answer:
(A)-(IV), (B)-(III), (C)-(I), (D)-(II)

Solution:



Q. 7 The molecular formula of second homologue in the homologous series of mono carboxylic acid is_____.

Option 1:
$C_3H_6O_2$

Option 2:
$C_2H_4O_2$

Option 3:
CH_2O

Option 4:
$C_2H_2O_2$

Correct Answer:
$C_2H_4O_2$

Solution:
Ist homologue of Carboxylic acid = $HCOOH$
IInd homologue of Carboxylic acid = $CH_3COOH\ (C_2H_4O_2)$

Hence, the answer is the option (2).

Q. 8 Given below are two statements :

Statement (I) : In the Lanthanoids the formation Ce^{+4} is favoured by its noble gas configuration

Statement (II) : Ce^{+4} is a strong oxidant reverting to the common +3 state.

In the light of the above statements, choose the most appropriate answer from the options given below:

Option 1:
Statement I is false but Statement II is true

Option 2:
Both Statement I and Statement II are true

Option 3:
Statement I is true but Statement II is false

Option 4:
Both Statement I and Statement II are false

Correct Answer:
Both Statement I and Statement II are true

Solution:
Each of the two assertions is true on its own. The noble gas arrangement is the explanation for Ce^{+4}'s stability, as stated in Statement (I). Ce^{+4} is correctly described in Statement (II) as a potent oxidant that returns to the +3 state.

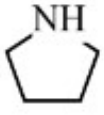
$Ce = [Xe]4f'5\ d^1 6\ s^2$
$Ce^{+4} = [Xe]4f^\circ 5\ d^\circ 6x^\circ$

Hence, the answer is the option (2).

Q. 9 Identify B formed in the reaction

$$Cl-(CH_2)_4-Cl \xrightarrow{\text{excesss } NH_3} A \xrightarrow{NaOH} B + H_2O + NaCl$$

Option 1:



Option 2:

$H_2N-(CH_2)_4-NH_2$

Option 3:

$\overset{+}{Cl}NH_3-(CH_2)_4-\overset{+}{N}H_3Cl^-$

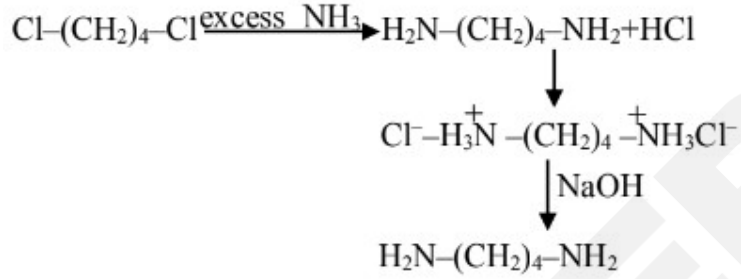
Option 4:



Correct Answer:

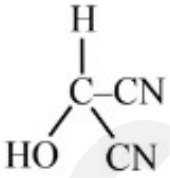
$H_2N-(CH_2)_4-NH_2$

Solution:

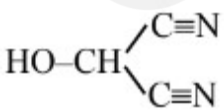


Q. 10 Bond line formula of $HOCH(CN)_2$ is :

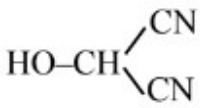
Option 1:



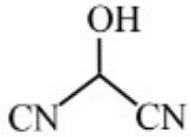
Option 2:



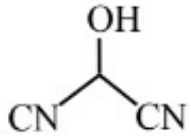
Option 3:



Option 4:

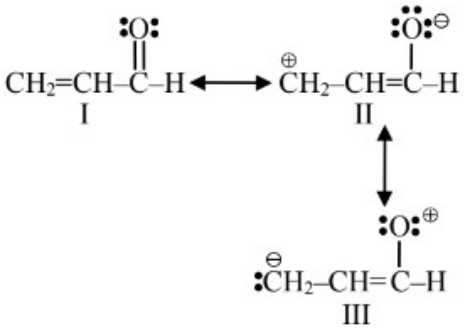


Correct Answer:



Solution:
Fact

Q. 11 The order of relative stability of the contributing structure is



Choose the correct answer from the options given below

Option 1:
I>II>III

Option 2:
II>I>III

Option 3:
I=II=III

Option 4:
III>II>I

Correct Answer:
I>II>III

Solution:
Ist structure is most stable due to complete octate and it is non polar.
IInd structure is more stable than IIIrd because –ve charge on more E.N. atom is more stable than –ve charge on less E.N. atom.

Q. 12 Which structure of protein remains intact after coagulation of egg white on boiling?

Option 1:
Primary

Option 2:
Tertiary

Option 3:
Secondary

Option 4:
Quaternary

Correct Answer:
Primary

Solution:
The coagulation of egg white on boiling is a common example of denaturation. Due to denaturation, the secondary and tertiary structures are destroyed but the primary structure remains intact.

Hence, the answer is the option (1).

Q. 13 Phenolic group can be identified by a positive :

Option 1:
Phthalein dye test

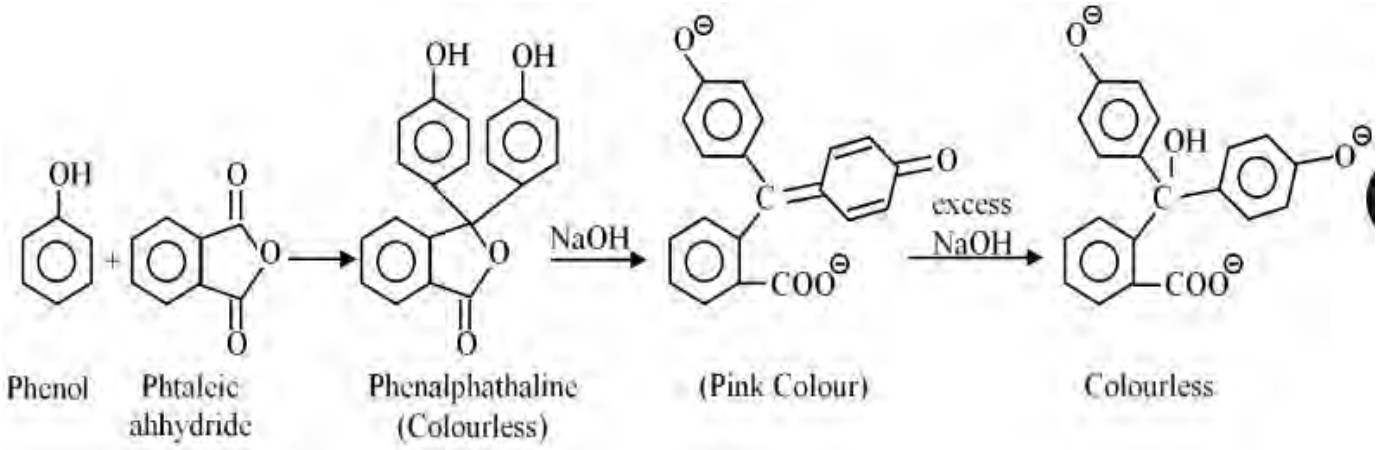
Option 2:
Lucas test

Option 3:
Tollen's test

Option 4:
Carbylamine test

Correct Answer:
Phthalein dye test

Solution:



Q. 14 The quantity which changes with temperature is:

Option 1:
Molarity

Option 2:
Mass percentage

Option 3:
Molality

Option 4:
Mole fraction

Correct Answer:
Molarity

Solution:
Molarity depends on temperature while other terms are mass dependent and temperature independent.

Hence, the answer is the option (1).

Q. 15 Identify from the following species in which d^2sp^3 hybridization is shown by central atom:

Option 1:
$[Co(NH_3)_6]^{3+}$

Option 2:
BrF_5

Option 3:
$[Pt\,(Cl_4)]^{2-}$

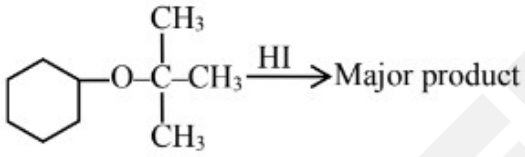
Option 4:
SF_6

Correct Answer:
BrF_5

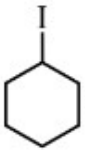
Solution:
$[Co\,(NH_3)6]^{3+} \rightarrow 3\ d^6 \& SFLd^2sp^3$
$BrFs \quad sp^3\ d^2$
$[PtCl_4]^{2-} \quad dsp^2$
$SF_6 \quad sp^3\ d^2$

Hence, the answer is the option (2).

Q. 16 Major product formed in the following reaction is a mixture of :

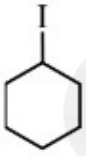


Option 1:



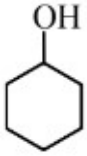
and $(CH_3)_3Cl$

Option 2:



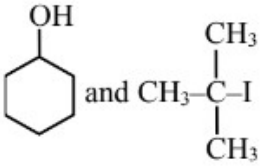
and $(CH_3)_3COH$

Option 3:

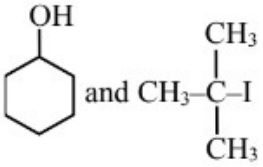


and $(CH_3)_3COH$

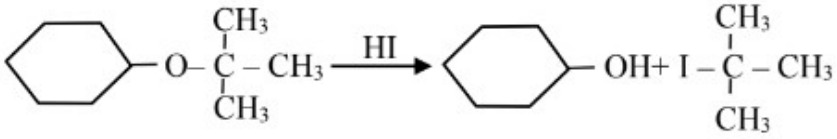
Option 4:



Correct Answer:



Solution:



Because of the formation of a tertiary carbocation.

Q. 17 Which among the following halide/s will not show S_N1 reaction

(A) $H_2C = CH - CH_2Cl$

(B) $CH_3 - CH = CH - Cl$



Choose the most appropriate answer from the options given below :

Option 1:
(A), (B) and (D) only

Option 2:
(A) and (B) only

Option 3:
(B) and (C) only

Option 4:
(B) only

Correct Answer:
(B) only

Solution:

$CH_3 - CH = CH - Cl$ has double bond character between C & Cl atom due to resonance.

Q. 18 Which of the following cannot function as an oxidising agent?

Option 1:
N^{3-}

Option 2:
SO_4^{2-}

Option 3:
BrO_3^-

Option 4:
MnO_4^-

Correct Answer:
N^{3-}

Solution:
$SO_4{}^{2-}$, BrO_3^-, MnO_4^- } acts as oxidising agent

N^{3-} cannot acts as an oxidising agent.

Hence, the answer is the option (1).

Q. 19 The technique used for purification of steam volatile water immiscible substances is :

Option 1:
Fractional distillation

Option 2:
Fractional distillation under reduced pressure

Option 3:
Distillation

Option 4:
steam distillation

Correct Answer:
steam distillation

Solution:
The steam distillation method is used to separate substances that are stem volatile and are immiscible with water. However, the impurities should not be steam volatile to purify the substance by steam distillation.

Hence, the answer is the option (4).

Q. 20 The incorrect statement regarding conformations of ethane is :

Option 1:
Ethane has infinite number of conformations.

Option 2:
The dihedralangle in staggered conformation is 60°.

Option 3:
Eclipsed conformation is the most stable conformation.

Option 4:
The conformations of ethane are inter-convertible to one-another.

Correct Answer:
Eclipsed conformation is the most stable conformation.

Solution:
Staggered conformation is most stable not eclipsed.

Hence, the answer is the option (3).

Q. 21 Volume of $3\mathrm{M}\mathrm{NaOH}$ (formula weight $40\ \mathrm{g\ mol^{-1}}$) which can be prepared from $84\ \mathrm{g}$ of NaOH is $\times 10^{-1}\mathrm{dm}^3$____$\times 10^{-1}\mathrm{dm}^3$.

Correct Answer:
7

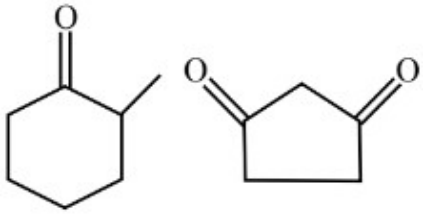
Solution:
Moles of $\mathrm{NaOH} = \frac{84}{40} = 2.1\ \mathrm{mol}$

Volume of solution $= \frac{^{\mathrm{n}}\mathrm{NaOH}}{\mathrm{m}} = \frac{2.1}{3}$

$= 0.7 = 7 \times 10^{-1}$

Hence, the answer is (7).

Q. 22 Total number of compounds with Chiral carbon atoms from following is _____.

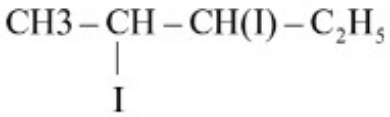


$CH_3 - CH_2 - CH(NO_2) - COOH$

$CH_3 - CH_2 - CHBr - CH_2 - CH_3$

$CH_3 - CH(I) - CH_2 - NO_2$

$CH_3 - CH_2 - CH(OH) - CH_2OH$



Option 1:
5

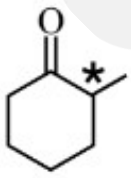
Option 2:
-

Option 3:
-

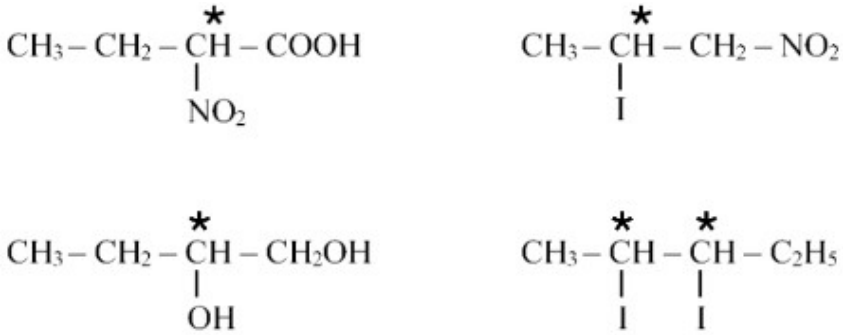
Option 4:
-

Correct Answer:
5

Solution:



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Q. 23 1 mole of PbS is oxidised by "X" moles of $\mathrm{O_3}$ to get "Y" moles of $\mathrm{O_2}$. $\mathrm{X+Y=}$_____.

Correct Answer:
8

Solution:
$\mathrm{PbS + 4O_3 \rightarrow PbSO_4 + 4O_2}$

According to the balanced equation

$X = 4Y = 4$
$X + Y = 4 + 4 = 8$

Hence, the answer is (8).

Q. 24 The hydrogen electrode is dipped in a solution of $\mathrm{pH = 3}$ at $25^{\circ}\mathrm{C}$. The potential of the electrode will be _____$\times 10^2$ V.

$\left(\frac{2.303\mathrm{RT}}{\mathrm{F}} = 0.059\ \mathrm{V}\right)$

Correct Answer:
18

Solution:
$\mathrm{pH = 3}$
$\left[\mathrm{H^+}\right] = 10^{-3}$
$\mathrm{2H^+ + 2e^- \rightarrow H_2(\,g)}$
$$\mathrm{E = E^0 - \frac{0.059}{2}\log\frac{P_{H_2}}{(H^+)^2}}$$
$$\mathrm{E} = 0 - \frac{0.059}{2} \times 6$$
$$= -0.177 = -17.7 \times 10^{-2} \approx 18 \times 10^{-2}$$

Hence, the answer is (18).

Q. 25 Time required for completion of 99.9% of a First order reaction is _____ times of half life $\left(t_{1/2}\right)$ of the reaction.

Option 1:
10

Option 2:
-

Option 3:
-

Option 4:
-

Correct Answer:
10

Solution:

$$t_{99.9}=\frac{2.303\times t_{1/2}}{0.693}\log\frac{100}{0.1}$$

$$t_{99.9}=10t_{1/2}$$

Hence, the answer is (10).

Q. 26 For a certain thermochemical reaction $\mathrm{M}\rightarrow\mathrm{N}$ at $\mathrm{T}=400\ \mathrm{K}, \Delta\mathrm{H}^{\ominus}=77.2\ \mathrm{kJ\ mol^{-1}}, \Delta\mathrm{S}=122\mathrm{JK}^{-1}$, log equilibrium constant (logK) is - _____$\times10^{-1}$

Correct Answer:
37

Solution:

$$\Delta\mathrm{G}^{0}=\Delta\mathrm{H}^{\circ}-\mathrm{T}\Delta\mathrm{S}^{\circ}$$

$$=\left(77.2\times10^{3}\right)-(400\times122)$$

$$=28400\ \mathrm{J}$$

$$\Delta\mathrm{G}^{0}=-2.303\mathrm{RT}\log\mathrm{K}$$

$$\log\mathrm{K}=3.7=37\times10^{-1}$$

Hence, the answer is (37).

Q. 27 The Spin only magnetic moment value of square planar complex $[Pt(NH_3)_2Cl(NH_2CH_3)]Cl$ is _____B.M. (Nearest integer)

(Given atomic number for Pt=78)

Correct Answer:
0

Solution:
$[Pt^{2+}(NH_3)_2Cl(NH_2CH_3)]Cl$
$Pt = 5\,d^6 6\,s^1$
$Pt^{2+} = 5\,d^8 6\,s^\circ$

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$\mu = 0.0.$

Hence, the answer is (0).

Q. 28 Total number of ions from the following the noble gas configuration is _____.

$Sr^{2+}(z = 38), Cs^{+}(z = 55), La^{2+}(z = 57), Pb^{2+}(z = 82), Yb^{2+}(z = 70)$ and $Fe^{2+}(z = 26)$

Correct Answer:
2

Solution:
Sr^{2+}
Cs^{+}

Hence, the answer is (2).

Q. 29 The number of non-polar molecules from the following is _____.

$HF, H_2O, SO_2, H_2, CO_2, CH_4, NH_3, HCl, CHCl_3, BF_3$

Correct Answer:
4

Solution:

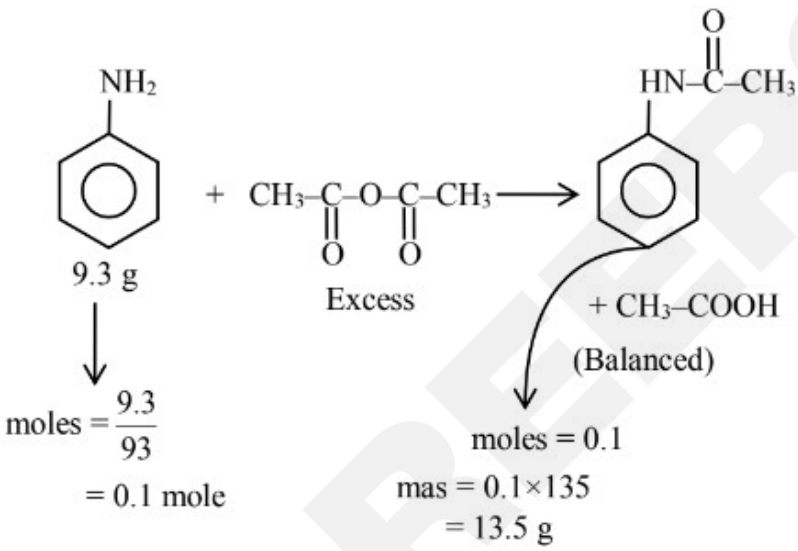
H_2
CO_2
CH_4
BF_3
Nonpolar molecules

Q. 30 9.3 g of aniline is subjected to reaction with excess of acetic anhydride to prepare acetanilide. The mass of acetanilide produced if the reaction is 100% completed is _____$\times 10^{-1}$ g.

(Given molar mass in g mol $^{-1}$ N : 14, O : 16, C : 12, H : 1)

Correct Answer:
135

Solution:



Maths

Q. 1 Considering only the principal values of inverse trigonometric functions, the number of positive real values of x satisfying $\tan^{-1}(x) + \tan^{-1}(2x) = \frac{\pi}{4}$ is :

Option 1:
more than 2

Option 2:
1

Option 3:
2

Option 4:
0

Correct Answer:
1

Solution:

$$\tan^{-1} 2x = \frac{\pi}{4} - \tan^{-1} x$$

$$2x = \frac{1-x}{1+x}$$

$$2x + 2x^2 = 1 - x$$

$$2x^2 + 3x - 1 = 0$$

$$x = \frac{-3 \pm \sqrt{9+8}}{4}$$

Case (1) $x = \frac{-3+\sqrt{17}}{4} \cong \frac{1.1}{4}$

Case (2) $x = \frac{-3-\sqrt{17}}{4} \simeq \frac{-7.1}{4}$

Hence, the answer is the option (2).

Q. 2 Consider the function $f(0,2) \to R$ defined by $f(x) = \frac{x}{2} + \frac{2}{x}$ and the function $g(x)$ defined by

$$g(x) = \begin{cases} \min\{f(t)\}, & 0 < t \leq x \text{ and } 0 < x \leq 1 \\ \frac{3}{2} + x, & 1 < x < 2 \end{cases}$$. Then,

Option 1:
g is continuous but not differentiable at $x = 1$

Option 2:
g is not continuous for all $x \in (0,2)$

Option 3:
g is neither continuous nor differentiable at $x = 1$

Option 4:
g is continuous and differentiable for all $x \in (0,2)$

Correct Answer:
g is continuous but not differentiable at $x = 1$

Solution:

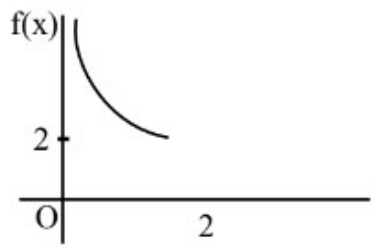
$f(0,2) \rightarrow R$

$f(x) = \frac{x}{2} + \frac{2}{x}$

$f'(x) = \frac{1}{2} - \frac{2}{x^2}$

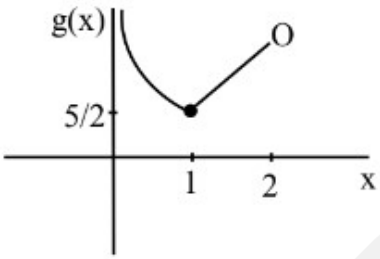
$f'(x) = \frac{x^2-4}{2x^2} = \frac{(x+2)(x-2)}{2x^2}$





decreasing function in (0, 2)

$$g(x) = \begin{cases} \min\{f(x)\}, & 0 < t \leq x \quad 0 < x \leq 1 \\ \frac{3}{2} + x & 1 < x < 2 \end{cases}$$



Q. 3 Let the image of the point $(1, 0, 7)$ in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ be the point (α, β, γ). Then which one of the following points lies on the line passing through (α, β, γ) and making angles $\frac{2\pi}{3}$ and $\frac{3\pi}{4}$ with y-axis and z-axis respectively and an acute angle with x -axis?

Option 1:

$(1, -2, 1 + \sqrt{2})$

Option 2:

$(1, 2, 1 - \sqrt{2})$

Option 3:

$(3, 4, 3 - 2\sqrt{2})$

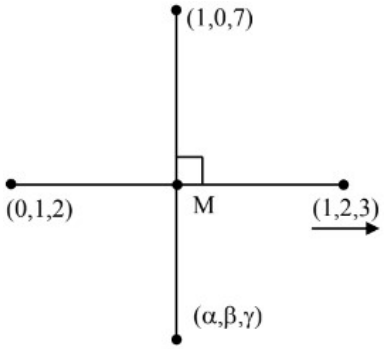
Option 4:

$(3, -4, 3+2\sqrt{2})$

Correct Answer:

$(3, 4, 3-2\sqrt{2})$

Solution:



$M\left(\frac{\alpha+1}{2}, \frac{\beta}{2}, \frac{\gamma+7}{2}\right) \rightarrow$ put in line

$\frac{\alpha+1}{2} = \frac{\frac{\beta}{2}-1}{2} = \frac{\frac{\gamma+7}{2}-2}{3}$

$\frac{\alpha+1}{2} = \frac{\beta-2}{4} = \frac{\gamma+3}{6} = \lambda$

$\alpha = \lambda - 1$
$\beta = 2\lambda + 2$
$\gamma = 3\lambda - 3$

also

$(\alpha-1)\cdot 1 + \beta \cdot 2 + (\gamma-7)\cdot 3 = 0$
$\alpha - 1 + 2\beta + 3\gamma - 21 = 0$
$\alpha + 2\beta + 3\gamma = 22$
$\lambda - 1 + 4\lambda + 4 + 9\lambda - 9 = 22$
$14\lambda = 28 \Rightarrow \lambda = 2$
$\therefore \alpha = 1, \beta = 6, \gamma = 3$

Image (1,6,3)

$\ell^2 + m^2 + n^2 = 1 \quad m = -\frac{1}{2}, n = -\frac{1}{\sqrt{2}}$

$\ell^2 + \frac{1}{4} + \frac{1}{2} = 1$

$\ell^2 + \frac{3}{4} = 1$

$\ell^2 = \frac{1}{4} \Rightarrow l = \frac{1}{2}$

$\therefore$ line

$\frac{x-1}{1/2} = \frac{y-6}{-1/2} = \frac{z-3}{-1/\sqrt{2}}$

Q. 4 Let R be the interior region between the lines $3x - y + 1 = 0$ and $x + 2y - 5 = 0$ containing the origin. The set of all values of a, for which the points $\left(a^2, a+1\right)$ lie in R, is :

Option 1:
$(-3,-1) \cup \left(-\frac{1}{3}, 1\right)$

Option 2:
$(-3,0) \cup \left(\frac{1}{3}, 1\right)$

Option 3:
$(-3,0) \cup \left(\frac{2}{3}, 1\right)$

Option 4:
$(-3,-1) \cup \left(\frac{1}{3}, 1\right)$

Correct Answer:
$(-3,0) \cup \left(\frac{1}{3}, 1\right)$

Solution:
To determine the set of all values of a for which the point $(a^2, a+1)$ lies in the region R bounded by the lines $3x - y + 1 = 0$ and $x + 2y - 5 = 0$, we first need to express these lines in the form $y = mx + c$.

Starting with the line $3x - y + 1 = 0$:

$$y = 3x + 1$$

Next, for the line $x + 2y - 5 = 0$:

$$2y = -x + 5 \implies y = -\frac{1}{2}x + 2.5$$

Now, to determine the region containing the origin, we check where the origin $(0,0)$ lies relative to these lines:

$$3(0) - 0 + 1 = 1 > 0 \quad (\text{Above the line } 3x - y + 1 = 0)$$

$$0 + 2(0) - 5 = -5 < 0 \quad (\text{Below the line } x + 2y - 5 = 0)$$

Thus, the region R containing the origin is above the line $3x - y + 1 = 0$ and below the line $x + 2y - 5 = 0$.

To find the intersection points of the lines, we solve the system:

$$3x - y + 1 = 0$$

$$x + 2y - 5 = 0$$

Express y from the first equation:

$$y = 3x + 1$$

Substitute into the second equation:

$$x + 2(3x + 1) - 5 = 0 \implies x + 6x + 2 - 5 = 0 \implies 7x - 3 = 0 \implies x = \frac{3}{7}$$

Now find y:

$$y = 3\left(\frac{3}{7}\right) + 1 = \frac{9}{7} + \frac{7}{7} = \frac{16}{7}$$

So, the lines intersect at $\left(\frac{3}{7}, \frac{16}{7}\right)$.

Next, we want the point $(a^2, a + 1)$ to be above the line $3x - y + 1 = 0$ and below the line $x + 2y - 5 = 0$. Let's substitute $(a^2, a + 1)$ into these inequalities:

1. $3x - y + 1 > 0$

$$3(a^2) - (a + 1) + 1 > 0 \implies 3a^2 - a > 0 \implies a(3a - 1) > 0$$

2. $x + 2y - 5 < 0$

$$a^2 + 2(a + 1) - 5 < 0 \implies a^2 + 2a + 2 - 5 < 0 \implies a^2 + 2a - 3 < 0$$

To solve these inequalities:

1. $a(3a - 1) > 0$:

$$a \in (-\infty, 0) \cup \left(\frac{1}{3}, \infty\right)$$

2. $a^2 + 2a - 3 < 0$:

$$(a + 3)(a - 1) < 0 \implies -3 < a < 1$$

Combining these conditions:

$$a \in (-\infty, 0) \cup \left(\frac{1}{3}, \infty\right) \quad \text{and} \quad -3 < a < 1$$

$$(-3,0)\cap\left(\frac{1}{3},1\right)=(-3,0)\cup\left(\frac{1}{3},1\right)$$

Thus, the set of all values of a for which the points $(a^2,a+1)$ lie in R is:

$(-3,0)\cup\left(\frac{1}{3},1\right)$

Hence, the answer is option (2).

Q. 5 The 20^{th} term from the end of the progression $20, 19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}, \ldots, -129\frac{1}{4}$ is:

Option 1:
-118

Option 2:
-110

Option 3:
-115

Option 4:
-100

Correct Answer:
-115

Solution:

$$\mathrm{T}_1=-129\frac{1}{4},\ \mathrm{d}=\frac{3}{4}$$

$$\mathrm{T}_{20}=-129\frac{1}{4}+19\cdot\frac{3}{4}$$

$$=-\frac{-517}{4}+\frac{57}{4}=\frac{-460}{4}=-115$$

Hence, the answer is the option (3).

Q. 6 Let $\mathrm{f}:\mathbf{R}-\left\{\frac{-1}{2}\right\}\to\mathbf{R}$ and $\mathrm{g}:\mathbf{R}-\left\{\frac{-5}{2}\right\}\to\mathbf{R}$ be defined as $\mathrm{f}(\mathrm{x})=\frac{2\mathrm{x}+3}{2\mathrm{x}+1}$ and $\mathrm{g}(\mathrm{x})=\frac{|\mathrm{x}|+1}{2\mathrm{x}+5}$. Then, the domain of the function fog is :-

Option 1:
$\mathbf{R}-\left\{-\frac{5}{2}\right\}$

Option 2:
R

Option 3:
$\mathbf{R}-\left\{-\frac{7}{4}\right\}$

Option 4:
$\mathbf{R}-\left\{-\frac{5}{2},-\frac{7}{4}\right\}$

Correct Answer:
$\mathbf{R}-\left\{-\frac{5}{2}\right\}$

Solution:

$$y=\frac{\frac{2(|x|+1)}{2x+5}+3}{2\left(\frac{|x|+1}{2x+5}\right)+1}$$

$$=\frac{2|x|+2+6x+15}{2|x|+2+2x+5}=\frac{(2|x|+6x+17)}{2|x|+2x+7}$$

And $x \neq -\frac{5}{2}$ for defining $g(x)$

Hence, the answer is the option (1).

Q. 7 For $0 < a < 1$, the value of the integral $\int_0^{\pi} \frac{dx}{1-2a\cos x+a^2}$ is:

Option 1:
$\frac{\pi^2}{\pi+a^2}$

Option 2:
$\frac{\pi^2}{\pi-a^2}$

Option 3:
$\frac{\pi}{1-a^2}$

Option 4:
$\frac{\pi}{1+a^2}$

Correct Answer:
$\frac{\pi}{1-a^2}$

Solution:

$I=\int_0^{\pi} \frac{dx}{1-2a\cos x+a^2}$ [Apply King's property and add]

$2I = \int_0^{\pi} \left(\frac{1}{1-2a\cos x+a^2} + \frac{1}{1+2a\cos x+a^2} \right) dx$

$2\mathrm{I} = 2\left(1+\mathrm{a}^2\right) \cdot 2\int_0^{\pi/2} \frac{\mathrm{dx}}{(1+\mathrm{a}^2)^2-4\mathrm{a}^2\cos^2 \mathrm{x}}$ [Queen property]

$$I = 2\left(1+a^2\right)\int_0^{\pi/2} \frac{\sec^2 x dx}{\tan^2 x(1+a^2)^2 + (1-a^2)^2}$$

$$= \frac{2}{1+a^2}\int_0^{\pi/2} \frac{\sec^2 x dx}{\tan^2 x + \left(\frac{1-a^2}{1+a^2}\right)^2}$$

$$= \frac{2}{1+a^2}\int_0^{\infty} \frac{dt}{t^2+A^2}$$

$$= \frac{2}{(1+a^2)\frac{(1-a^2)}{1+a^2}} \cdot \tan^{-1}\left(\frac{\tan x\left(1+a^2\right)}{(1-a^2)}\right)_0^{\infty} = \frac{\pi}{1-a^2}$$

Hence, the answer is the option (3).

Q. 8 Let $\mathrm{g(x)} = 3\mathrm{f}\left(\frac{\mathrm{x}}{3}\right) + \mathrm{f}(3-\mathrm{x})$ and $\mathrm{f''(x)} > 0$ for all $\mathrm{x} \in (0,3)$. If g is decreasing in $(0,\alpha)$ and increasing in $(\alpha,3)$, then 8α is:

Option 1:
24

Option 2:
0

Option 3:
18

Option 4:
20

Correct Answer:
18

Solution:
$g(x) = 3f\left(\frac{x}{3}\right) + f(3-x), \quad f''(x) > 0$

$g'(x) = f'\left(\frac{x}{3}\right) - f'(3-x) \quad \Rightarrow f'(x)$ is increasing function

Case (1) $\mathrm{g'(x)} > 0$

$f'\left(\frac{x}{3}\right) > f'(3-x) \quad \Rightarrow \frac{x}{3} > 3-x$

$\frac{4x}{3} > 3 \Rightarrow x > \frac{9}{4}$

Case (2) $g'(x) < 0$

$f'\left(\frac{x}{3}\right) < f'(3-x) \quad x < \frac{9}{4}$

$\therefore \quad \alpha = \frac{9}{4}$

$\therefore 8\alpha = 18$

Hence, the answer is the option (3).

Q. 9 If $\lim_{x\to 0} \frac{3+\alpha \sin x+\beta \cos x+\log_e(1-x)}{3\tan^2 x} = \frac{1}{3}$, then $2\alpha - \beta$ is equal to:

Option 1:
2

Option 2:
7

Option 3:
5

Option 4:
1

Correct Answer:
5

Solution:

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$$\lim_{x\to 0}\frac{3+\alpha[x]+\beta\left[1-\frac{x^2}{2}\right]+\left(-x-\frac{x^2}{2}\right)=\frac{1}{3}}{3x^2}$$

$\Rightarrow 3+\beta=0$

$\beta=-3$

$\Rightarrow \alpha-1=0$

$\alpha=1$

$$\Rightarrow \frac{\frac{-\beta}{2}\frac{-1}{2}}{3}=\frac{1}{3}$$

$\frac{-\beta-1}{6}=\frac{1}{3} \quad \Rightarrow \quad -\beta-1=2$

$-\beta=3$

$\beta=-3$

$2\alpha-\beta=2+3=5$

Hence, the answer is the option (3).

Q. 10 If α, β are the roots of the equation, $x^2-x-1=0$ and $S_n=2023\alpha^n+2024\beta^n$, then:

Option 1:
$2\,S_{12}=S_{11}+S_{10}$

Option 2:
$S_{12}=S_{11}+S_{10}$

Option 3:
$2\,S_{11}=S_{12}+S_{10}$

Option 4:
$S_{11}=S_{10}+S_{12}$

Correct Answer:
$S_{12}=S_{11}+S_{10}$

Solution:

$$S_n = 2023\alpha^n + 2024\beta^n$$

$$= 2023\left(\alpha^n + \beta^n\right) + \beta^n$$

$$X^2 - X - 1 = 0$$

$$S_{12} = 2023\alpha^{12} + 2024\beta^{12} = 2023\left(p_{12}\right) + \beta^{12}$$

$$S_{11} = 2023\alpha^{11} + 2024\beta^{11} = 2023\left(P_{11}\right) + \beta^{11}$$

$$S_{10} = 2023\alpha^{10} + 2024\beta^{10} = 2023\left(P_{10}\right) + \beta^{10}$$

$$\frac{S_{12} - \beta^{12}}{2023} = \frac{S_{11} - \beta^{11}}{2023} + \frac{S_{10} - \beta^{10}}{2023} \quad (\therefore P_{12} = P_{11} + P_{10})$$

$$S_{12} = S_{11} + S_{10} + \left(\beta^{12} - \beta^{11} - \beta^{10}\right)$$

$$\beta^{12} - \beta^{10}(\beta + 1)$$

$$\beta^{12} - \beta^{10}\beta^2$$

$$\therefore \quad S_{12} = S_{11} + S_{10}$$

Hence, the answer is the option 2.

Q. 11 Let A and B be two finite sets with m and n elements respectively. The total number of subsets of the set A is 56 more than the total number of subsets of B. Then the distance of the point P(m, n) from the point Q(–2,–3) is:

Option 1:
10

Option 2:
6

Option 3:
4

Option 4:
8

Correct Answer:
10

Solution:

$$2^m = 2^n + 56$$

$$2^m - 2^n = 56$$

$2^n\left(2^{m-n}-1\right)=56=2^3\times 7$

$\mathrm{n}=3 \quad 2^{\mathrm{m}-\mathrm{n}}=8$

$\mathrm{m}=6$

$\mathrm{P}(6,3)\, and\, \mathrm{Q}(-2,-3)$

$PQ=\sqrt{64+36}=10$

Hence, the answer is the option (1).

Q. 12

The values of α, for which $\begin{vmatrix} 1 & \frac{3}{2} & \alpha+\frac{3}{2} \\ 1 & \frac{1}{3} & \alpha+\frac{1}{3} \\ 2\alpha+3 & 3\alpha+1 & 0 \end{vmatrix}=0$, lie in the interval

Option 1:
$(-2,1)$

Option 2:
$(-3,0)$

Option 3:
$\left(-\frac{3}{2},\frac{3}{2}\right)$

Option 4:
$(0,3)$

Correct Answer:
$(-3,0)$

Solution:

$$\begin{vmatrix} 1 & 3/2 & \alpha+3/2 \\ 1 & 1/3 & \alpha+1/3 \\ 2\alpha+3 & 3\alpha+1 & 0 \end{vmatrix}=0$$

$$\begin{vmatrix} 0 & \frac{7}{6} & \frac{7}{6} \\ 1 & 1/3 & \alpha+\frac{1}{3} \\ 2\alpha+3 & 3\alpha+1 & 0 \end{vmatrix}=0$$

$$+\frac{7}{6}\left[(2\alpha+3)\left(\alpha+\frac{1}{3}\right)\right]$$

$$+\frac{7}{6}\left[(3\alpha+1)-\frac{1}{3}(2\alpha+3)\right]=0$$

$$\left(2\alpha^2+3\alpha+\frac{2\alpha}{3}+1\right)+\left(3\alpha-\frac{2\alpha}{3}\right)=0$$

$$2\alpha^2+6\alpha+1=0$$

$$\alpha=\frac{-6\pm\sqrt{36-8}}{4}$$

$$=\frac{-6\pm2\sqrt{7}}{4}=\frac{-3\pm\sqrt{7}}{2}$$

$$\frac{-3+\sqrt{7}}{2}\simeq-0.2$$

$$\frac{-3-\sqrt{7}}{2}\simeq-2.8$$

Hence, the answer is the option (2).

Q. 13 An urn contains 6 white and 9 black balls. Two successive draws of 4 balls are made without replacement. The probability, that the first draw gives all white balls and the second draw gives all black balls, is:

Option 1:
$\frac{5}{256}$

Option 2:
$\frac{5}{715}$

Option 3:
$\frac{3}{715}$

Option 4:
$\frac{3}{256}$

Correct Answer:
$\frac{3}{715}$

Solution:

$Probability = \frac{^6C_4}{^{15}C_4} \times \frac{^9C_4}{^{11}C_4} = \frac{3}{715}$

Hence, the answer is the option (3).

Q. 14 The integral $\int \frac{\left(x^8-x^2\right)dx}{\left(x^{12}+3x^6+1\right)\tan^{-1}\left(x^3+\frac{1}{x^3}\right)}$ is equal to:

Option 1:
$\log_e\left(\tan^{-1}\left(x^3+\frac{1}{x^3}\right)\right)^{1/3}+C$

Option 2:
$\log_e\left(\tan^{-1}\left(x^3+\frac{1}{x^3}\right)\right)^{1/2}+C$

Option 3:
$\log_e\left(\tan^{-1}\left(x^3+\frac{1}{x^3}\right)\right)+C$

Option 4:
$\log_e\left(\tan^{-1}\left(x^3+\frac{1}{x^3}\right)\right)^{3}+C$

Correct Answer:
$\log_e\left(\tan^{-1}\left(x^3+\frac{1}{x^3}\right)\right)^{1/3}+C$

Solution:

$$\int \frac{\left(x^8-x^2\right)dx}{\left(x^{12}+3x^6+1\right)\tan^{-1}\left(x^3+\frac{1}{x^3}\right)}$$

$$\tan^{-1}\left(x^3+\frac{1}{x^3}\right)=t$$

$$\frac{\left(3x^2-\frac{3}{x^4}\right)dx}{1+\left(x^3+\frac{1}{x^3}\right)^2}=dt$$

$$\frac{3\left(x^6-1\right)dx}{x^4\left(1+x^6+\frac{1}{x^6}+2\right)}=dt$$

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$$\frac{3\left(x^8-x^2\right)dx}{\left(x^{12}+3x^6+1\right)}=dt$$

$$=\frac{1}{3}\int\frac{dt}{t}$$

$$=\frac{1}{3}\ln|t|+c$$

$$\log_e\ \tan^{-1}\left(x^3+\frac{1}{x^3}\right)^{1/3}+c$$

Hence, the answer is the option (1).

Q. 15 $If 2\tan^2\theta-5\sec\theta=1$ has exactly 7 solutions in the interval $[0,n\pi/2]$, for the least value of n $\in N, then \sum_{k=1}^{n}\frac{k}{2^k}$ is Equal to.

Option 1:

$\frac{1}{2^{15}}\left(2^{14}-14\right)$

Option 2:

$\frac{1}{2^{14}}\left(2^{15}-15\right)$

Option 3:

$1-\frac{15}{2^{13}}$

Option 4:

$\frac{1}{2^{13}}\left(2^{14}-15\right)$

Correct Answer:

$\frac{1}{2^{13}}\left(2^{14}-15\right)$

Solution:

$$S = \frac{1}{2} + \frac{2}{2^2} + \frac{3}{2^3} \ldots\ldots + \frac{13}{2^{13}}$$

$2\tan^2\theta - 5\sec\theta - 1 = 0$

$\text{Sec}\,\theta = 3$

$\therefore n = 13$

$$\therefore \sum_{k=1}^{13} \frac{k}{2^k}$$

$$\frac{S}{2} = \frac{1}{2^2} + \frac{2}{2^3} + \ldots + \frac{12}{2^{13}} + \frac{13}{2^{14}}$$

$$\frac{S}{2} = \left(\frac{1}{2} + \frac{1}{2^2} + \ldots + \frac{1}{2^{13}}\right) - \frac{13}{2^{14}}$$

$$\frac{1}{2}\left[\frac{1-\left(\frac{1}{2}\right)^{13}}{1-\frac{1}{2}}\right] - \frac{13}{2^{14}}$$

$$S = 2 - \frac{1}{2^{12}} - \frac{13}{2^{13}}$$

$$= \frac{2^{14} - 2 - 13}{2^{13}}$$

$$= \frac{2^{14} - 15}{2^{13}}$$

Hence, the answer is the option (4).

Q. 16 The position vectors of the vertices A, B and C of a triangle are $2\hat{i} - 3\hat{j} + 3\hat{k}, 2\hat{i} + 2\hat{j} + 3\hat{k}$ and $-\hat{i} + \hat{j} + 3\hat{k}$ respectively. Let ℓ denotes the length of the angle bisector AD of $\angle BAC$ where D is on the line segment BC, then $2\ell^2$ equals:

Option 1:
49

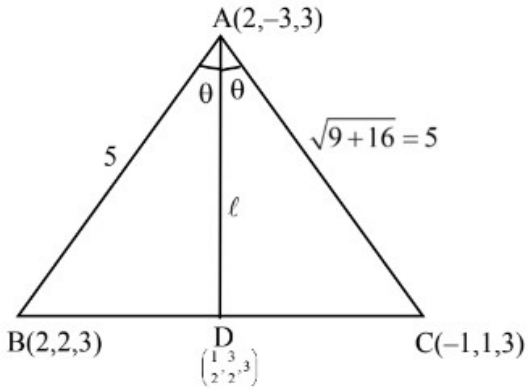
Option 2:
42

Option 3:
50

Option 4:
45

Correct Answer:
45

Solution:



$$\therefore \quad 1^2 = \frac{9}{4} + \frac{81}{4}$$

$$= \frac{90}{4}$$

$$\therefore 2\ell^2 = 2 \times \frac{90}{4} = 45$$

Q. 17 If y=y(x) is the solution curve of the differential equation $\left(x^2-4\right)dy-\left(y^2-3y\right)dx=0, x>2, y(4)=\frac{3}{2}$ and the slope of the curve is never zero, then the value of y(10) equals :

Option 1:
$\frac{3}{1+(8)^{1/4}}$

Option 2:
$\frac{3}{1+2\sqrt{2}}$

Option 3:
$\frac{3}{1-2\sqrt{2}}$

Option 4:
$\frac{3}{1-2\sqrt{2}}$

Correct Answer:
$\frac{3}{1+(8)^{1/4}}$

Solution:

$$\frac{1}{3}\ln\left|\frac{y-3}{y}\right| = \frac{1}{4}\ln\left|\frac{x-2}{x+2}\right| + C \qquad y(10) \Rightarrow \frac{1}{3}\ln\left|\frac{y-3}{y}\right| = \frac{1}{4}\ln\left|\frac{8\cdot 3}{12}\right| = \frac{1}{4}\ln 2$$

$$y(4) = \frac{3}{2} \qquad \ln\left|\left(\frac{y-3}{y}\right)\right| = \ln\left(2^{3/4}\right), y \in (0,3)$$

$$\left(x^2-4\right)dy = \left(y^2-3y\right)dx$$

$$\frac{1}{3}\ln\left|\frac{\frac{3}{2}-3}{3/2}\right| = \frac{1}{4}\ln\left|\frac{4-2}{4+2}\right| + C \qquad -1+\frac{3}{y} = 8^{1/4}$$

$$\frac{dy}{y(y-3)} = \frac{dx}{x^2-4}$$

$$C = -\frac{1}{4}\ln\frac{1}{3} = \frac{1}{4}\ln 3 \qquad 1+8^{\frac{1}{4}} = \frac{3}{y}$$

$$\therefore \frac{1}{3}\ln\left|\frac{y-3}{y}\right| = \frac{1}{4}\ln\left|\frac{x-2}{x+2}\right| .3 \qquad y = \frac{3}{1+8^{\frac{1}{4}}}$$

Hence, the answer is the option (1).

Q. 18 Let e_1 be the eccentricity of the hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$ and e_2 be the eccentricity of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a >$ b, which passes through the foci of the hyperbola. If $e_1e_2 = 1$, then the length of the chord of the ellipse parallel to the x-axis and passing through $(0, 2)$ is :

Option 1:
$4\sqrt{5}$

Option 2:
$\frac{8\sqrt{5}}{3}$

Option 3:
$\frac{10\sqrt{5}}{3}$

Option 4:
$3\sqrt{5}$

Correct Answer:
$\frac{10\sqrt{5}}{3}$

Solution:

$$\frac{x^2}{16} - \frac{y^2}{9} = 1 \qquad \text{Focii} : (5,0), (-5,0)$$

$$e_1^2 = 1 + \frac{9}{16}$$

$$e_1 = \frac{5}{4} \Rightarrow e_2 = \frac{4}{5} \Rightarrow \boxed{\frac{16}{25} = 1 - \frac{b^2}{a^2}} \Rightarrow \frac{b}{a} = \frac{3}{5}$$

put (5,0) in ellipse

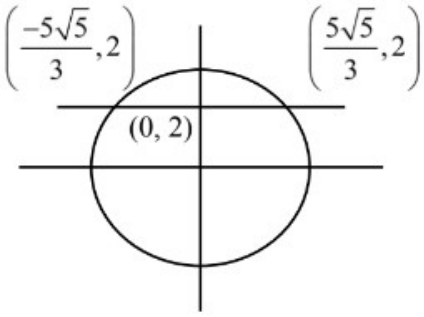
$\frac{25}{a^2} = 1 \Rightarrow a = 5 \quad \therefore b = 3$

$\frac{x^2}{25} + \frac{y^2}{9} = 1$

y = 2 put in the equation ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$

$\frac{x^2}{25} + \frac{4}{9} = 1$

$x = \pm\frac{5\sqrt{5}}{3}$



length of chord

$= \frac{10\sqrt{5}}{3}$

Q. 19 Let $\alpha = \frac{(4!)!}{(4!)^{3!}}$ and $\beta = \frac{(5!)!}{(5!)^{4!}}$ Then :

Option 1:
$\alpha \in \mathrm{N}$ and $\beta \notin \mathrm{N}$

Option 2:
$\alpha \notin \mathrm{N}$ and $\beta \in \mathrm{N}$

Option 3:
$\alpha \in \mathrm{N}$ and $\beta \in \mathrm{N}$

Option 4:
$\alpha \notin \mathrm{N}$ and $\beta \notin \mathrm{N}$

Correct Answer:
$\alpha \in \mathrm{N}$ and $\beta \in \mathrm{N}$

Solution:

$$\alpha = \frac{(24)!}{(24)^6} = \frac{2^{12+6+3+1} \cdot 3^{8+2}}{(2^3 \times 3)^6}$$

$$= \frac{2^{22} \cdot 3^{10}}{2^{18} \cdot 3^6} \quad \therefore \alpha \in \mathrm{N}$$

$$\beta = \frac{(120)!}{(120)^{24}} = \frac{2^{60+30+15+7+3+1} \cdot 3^{40+13+4+1} \cdot 5^{24+4}}{(2^3 \cdot 3 \cdot 5)^{24}}$$

$$= \frac{2^{116} \cdot 3^{58} \cdot 5^{28}}{2^{72} \cdot 3^{24} \cdot 5^{24}} \quad \therefore \beta \in \mathrm{N}$$

Hence, the answer is the option (3).

Q. 20 Let the position vectors of the vertices A, B and C of a triangle be $2\hat{i} + 2\hat{j} + \hat{k}, \hat{i} + 2\hat{j} + 2\hat{k}$ and $2\hat{i} + \hat{j} + 2\hat{k}$ respectively. Let ℓ_1, ℓ_2 and ℓ_3 be the lengths of perpendiculars drawn from the ortho center of the triangle on the sides AB, BC and CA respectively, then $\ell_1^2 + \ell_2^2 + \ell_3^2$ equals:

Option 1:
$\frac{1}{5}$

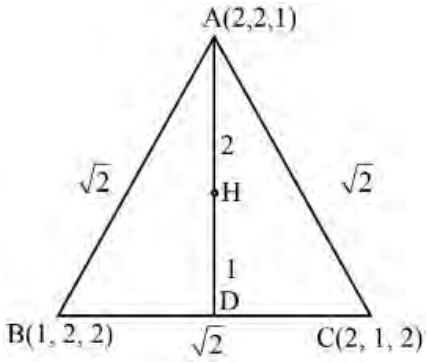
Option 2:
$\frac{1}{2}$

Option 3:
$\frac{1}{4}$

Option 4:
$\frac{1}{3}$

Correct Answer:
$\frac{1}{2}$

Solution:



Its an equilateral Δ

$\therefore$ ortho centre and centroid all are same

$$\mathrm{AD} = \sqrt{2}\sin 60^\circ = \frac{\sqrt{3}}{\sqrt{2}} \quad \therefore \mathrm{HD} = \frac{1}{\sqrt{6}} = \ell_1 = \ell_2 = \ell_3$$

$$\therefore \ell_1^2 + \ell_2^2 + \ell_3^2 = \frac{3}{6} = \frac{1}{2}$$

Q. 21 The mean and standard deviation of 15 observations were found to be 12 and 3 respectively. On rechecking, it was found that an observation was read as 10 in place of 12. If μ and σ^2 denote the mean and variance of the correct observations respectively, then $15\left(\mu + \mu^2 + \sigma^2\right)$ is equal to

Correct Answer:
2521

Solution:

$$\sum_{i=1}^{15} x_i = 12 \times 15$$

$$\text{standard deviation } = \sqrt{\frac{\sum \mathrm{x_i}^2}{15} - 144} = 3 \Rightarrow \frac{\sum \mathrm{x_1}^2}{15} = 153$$

$$\sum {x_i}^2 = 15 \times 153$$

$$(\mathrm{x_1} \ldots\ldots + \mathrm{x_{14}}) = 180 - 10 = 170$$

$$\text{correct sum } = \sum_{i=1}^{15} x_i = 182 \quad \Rightarrow \mu = \frac{182}{15}$$

$$\text{similarly } x_1^2 + \ldots\ldots + x_{14}^2 = 2295 - 100$$

$$\text{correct } \sum_{i=1}^{15} {x_1}^2 = 2339$$

$$\sigma^2 = \frac{2339}{15} - (\mu)^2$$

$$\Rightarrow \mu^2 + \sigma^2 = \frac{2339}{15}$$

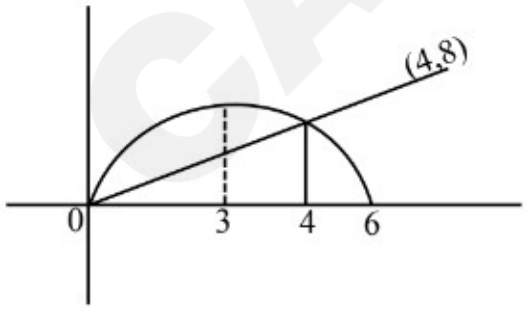
$$\therefore 15\left(\frac{182}{15} + \frac{2339}{15}\right) = 2521$$

Hence, the answer is 2521.

Q. 22 If the area of the region $\{(x, y) : 0 \leq y \leq \min\{2x, 6x - x^2\}\}$ is A, then 12 A is equal to 304

Correct Answer:
304

Solution:
$0 \leq y \leq \text{minimum}\,(2x, 6x - x^2)$



$$A = \frac{1}{2} \times 4 \times 8 + \int_4^6 (6x - x^2)dx$$

$$= 16 + \left(3x^2 - \frac{x^3}{3}\right)_4^6$$

$$= 16 + \left(3 \times 36 - \frac{216}{3}\right) - \left(3 \times 16 - \frac{64}{3}\right)$$

$$= 16 + (108 - 48) - \left(\frac{216 - 64}{3}\right)$$

$$= 16 + 60 - \frac{152}{3}$$

$\mathrm{A} = \frac{76}{3}$

$\therefore 12\ \mathrm{A} = 76 \times 4 = 304$

Q. 23 Let A be a 2×2 real matrix and I be the identity matrix of order 2 . If the roots of the equation $|A - xI| = 0$ be -1 and 3 , then the sum of the diagonal elements of the matrix A^2 is_____.

Correct Answer:
10

Solution:
$|A - x| \ | = 0$

for A^2 values of diagonals will be $(-1)^2$ and 3^2

$\therefore \mathrm{Tr}\left(\mathrm{A}^2\right) = 10$ Ans.

Hence, the answer is the (10).

Q. 24 If the sum of squares of all real values of α, for which the lines $2x - y + 3 = 0, 6x + 3y + 1 = 0$ and $\alpha x + 2y - 2 = 0$ do not form a triangle is p, then the greatest integer less than or equal to p is_____.

Correct Answer:
32

Solution:
$2x - y + 3 = 0; 6x + 3y + 1 = 0; \alpha x + 2y - 2 = 0$

$m_1 = 2, m_2 = -2, m_3 = \frac{-\alpha}{2}$

(1) $-\frac{\alpha}{2}=2$

$\alpha=-4$

$(2)\ -\frac{\alpha}{2}=-2$

$\alpha=4$

(3) lines are concurrent

$$\begin{vmatrix} 2 & -1 & 3 \\ 6 & 3 & 1 \\ \alpha & 2 & -2 \end{vmatrix}=0$$

$2(-6-2)+1(-12-\alpha)+3(12-3\alpha)=0$

$-16-12-\alpha+36-9\alpha=0$

$\alpha=\frac{4}{5}$

$\mathrm{P}=32+\frac{16}{25}$

$\therefore[\mathrm{P}]=32$

Hence, the answer is the (32).

Q. 25 The coefficient of x^{2012} in the expansion of $(1-x)^{2008}\left(1+x+x^2\right)^{2007}$ is equal to_____.

Correct Answer:
0

Solution:
$\left((1-x)^{2008}\right)\left[1+x+x^2\right]^{2007}$

$(1-x)\left[1-x^3\right]^{2007}$

$(1-x)^{2007}c_r\left(-x^3\right)^r$

$^{2007}c_r\left[(-x)^{3r}+\left(-x^{33r+1}\right]\right.$

$\therefore 3r=2012$

or $3r+1=2012$

no such r exists

$\therefore$ Coefficient is zero

Hence, the answer is the (0).

Q. 26 If the solution curve, of the differential equation $\frac{dy}{dx} = \frac{x+y-2}{x-y}$ passing through the point $(2, 1)$ is $\tan^{-1}\left(\frac{y-1}{x-1}\right) - \frac{1}{\beta}\log_e\left(\alpha + \left(\frac{y-1}{x-1}\right)^2\right) = \log_e|x-1|$, then $5\beta + \alpha$ is equal to_____.

Correct Answer:
11

Solution:

$\frac{dy}{dx} = \frac{(x-1)+(y-1)}{(x-1)-(y-1)}$

$$\frac{dY}{dX} = \frac{X+Y}{X-Y}$$

$$v + x\frac{dv}{dx} = \frac{1+v}{1-v}$$

$$\tan^{-1} v - \frac{1}{2}\ln\ 1+v^2\ = \ln x + c$$

$$\tan^{-1}\left(\frac{y-1}{x-1}\right) - \frac{1}{2}\ln\ 1 + \frac{(y-1)^2}{(x-1)^2}\ = \ln|x-1| + c$$

$$\tan^{-1} 0 - \frac{1}{2}\ln|1| = \ln 1 + c \quad \Rightarrow c = 0$$

$$\beta = 2, \quad \alpha = 1$$

$$\therefore 5\beta + \alpha = 11$$

Hence, the answer is the (11).

Q. 27 Let $f(x) = \int_0^x g(t)\log_e\left(\frac{1-t}{1+t}\right)dt$, where g is continuous odd function. If $\int_{-\pi/2}^{\pi/2}\left(f(x) + \frac{x^2\cos x}{1+e^x}\right)dx = \left(\frac{\pi}{\alpha}\right)^2 - \alpha$, then α is equal to_____.

Correct Answer:
2

Solution:
$f(x) = \int_0^x g(t) \cdot \log_e \left(\frac{1-t}{1+t}\right)$ alt

$f'(x) = g(x) \cdot \ln\left(\frac{1-x}{1+x}\right) \Rightarrow (f(x)$ will be add function)

$\therefore \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} f(x)dx + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{x^2 \cos x}{1+e^x} dx = 0$

[apply king property]

$\therefore \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{x^2 e^x \cos x dx}{1+e^x}$

$2I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x^2 \cos x dx \Rightarrow I = \int_0^{\frac{\pi}{2}} x^2 \cos x dx$

integrating by parts

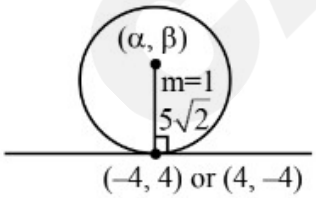
$I = \frac{\pi^2}{4} - 2$

$\therefore \alpha = 2$ Ans.

Hence, the answer is the (2).

Q. 28 Consider a circle $(x-\alpha)^2 + (y-\beta)^2 = 50$, where $\alpha, \beta > 0$. If the circle touches the line $y + x = 0$ at the point P, whose distance from the origin is $4\sqrt{2}$, then $(\alpha+\beta)^2$ is equal to_____.

Correct Answer:
100

Solution:


$\alpha = -4 + 5\sqrt{2} \times \frac{1}{\sqrt{2}}, \quad \beta = 4 + 5\sqrt{2} \times \frac{1}{\sqrt{2}}$

$(\alpha = 1, \beta = 9)$

$\therefore (\alpha+\beta)^2 = 100$ Ans.

Q. 29 The lines $\frac{x-2}{2}=\frac{y}{-2}=\frac{z-7}{16}$ and $\frac{x+3}{4}=\frac{y+2}{3}=\frac{z+2}{1}$ intersect at the point P. If the distance of P from the line $\frac{x+1}{2}=\frac{y-1}{3}=\frac{z-1}{1}$ is ℓ, then $14\ell^2$ is equal to_____

Correct Answer:
108

Solution:
Any point on $\mathrm{L}_1(2\mathrm{a}+2,-2\mathrm{a},16\mathrm{a}+7)$

Any point on $L_2(4b-3,3b-2,b-2)$

$$\begin{array}{l|l|l} 2\mathrm{a}+2=4\mathrm{b}-3 & -2\mathrm{a}=3\mathrm{b}-2 & 16\mathrm{a}+7=\mathrm{b}-2 \\ 2\mathrm{a}-4\mathrm{b}=-5 & -2\mathrm{a}-3\mathrm{b}=-2 & 16\mathrm{a}-\mathrm{b}=-9 \end{array}$$

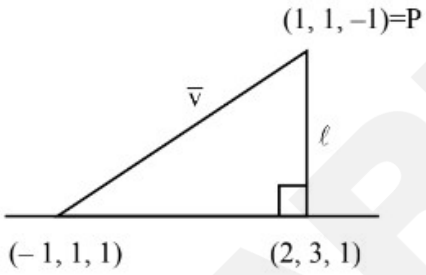
$-7b=-7$

$\mathrm{b}=\mathrm{l}$

$\mathrm{a}=-\frac{1}{2}$

$\mathrm{P}(1,1,-1)$

$\overline{\mathrm{V}}=(2,0,-2)$



$\ell=\frac{|\overline{\mathrm{v}}\times\overline{\mathrm{p}}|}{|\overline{\mathrm{p}}|}=\frac{\sqrt{\overline{108}}}{\sqrt{\overline{14}}}=\sqrt{\frac{54}{7}}$ $\quad\therefore 14\ell^2=14\times\frac{54}{7}=108$ Ans.

Q. 30 Let the complex numbers α and $\frac{1}{\bar{\alpha}}$ lie on the circles $|z-z_0|^2=4$ and $|z-z_0|^2=16$ respectively, where $z_0=1+i$. Then, the value of $100|\alpha|^2$ is

Correct Answer:
20

Solution:

$$|\alpha-(1+i)|^2=4$$

$$(\alpha-(1+i))(\bar{\alpha}-(1-i))=4$$

$$|\alpha|^2-(1+i)\bar{\alpha}-\alpha(1-i)+2=4.\ldots\ldots(1)$$

$$(1-\bar{\alpha}(1+i))(1-\alpha(1-i))=16|\alpha|^2$$

$$1-\underbrace{\bar{\alpha}(1+i)-\alpha(1-i)}_{M}+|\alpha|^2 2=16|\alpha|^2$$

$$|\alpha|^2+M=2$$

$$\frac{1+M=14|\alpha|^2}{|\alpha|^2-1=2-14|\alpha|^2}$$

$$15|\alpha|^2=3$$

Hence, the answer is the (3).

29 Jan - Morning Shift

Physics

Q. 1 The de-Broglie wavelength of an electron is the same as that of a photon. If velocity of electron is 25% of the velocity of light, then the ratio of K.E. of electron and K.E. of photon will be:

Option 1:
$\frac{1}{4}$

Option 2:
$\frac{1}{1}$

Option 3:
$\frac{8}{1}$

Option 4:
$\frac{1}{8}$

Correct Answer:
$\frac{1}{8}$

Solution:
For photon, $E = \frac{hC}{\lambda_p}$

$\lambda_p = \frac{hC}{E_p}$

for electron, $\lambda_e = \frac{h}{m_e v_e \times \frac{2}{2} \times \frac{v_e}{v_c}}$

$= \frac{h}{\frac{1}{2} m_e v_e^2 \times \frac{2}{v_e}}$

$= \frac{h \times v_e}{2k_E}$

$= \frac{h\frac{c}{4}}{2kE_e}$

$\lambda_e = \frac{hC}{8kE}$

given :- $\lambda_e = \lambda_p$

$\frac{hC}{8k_e} = \frac{hC}{E_p}$

$$\frac{1}{8}=\frac{kE_e}{E_p}$$

Hence, the answer is the option (4).

Q. 2 If the radius of curvature of the path of two particles of same mass are in the ratio $3:4$, then in order to have constant centripetal force, their velocities will be in the ratio of:

Option 1:
$1:\sqrt{3}$

Option 2:
$\sqrt{3}:1$

Option 3:
$\sqrt{3}:2$

Option 4:
$2:\sqrt{3}$

Correct Answer:
$\sqrt{3}:2$

Solution:

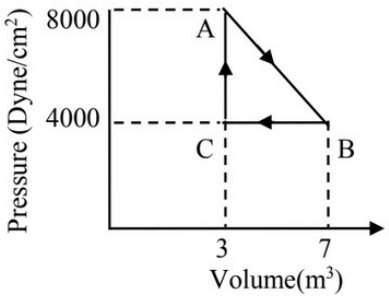
$$\frac{mv_1^2}{R_1}=\frac{mv_2^2}{R_2}$$

$$\left(\frac{v_1}{v_2}\right)^2=\frac{R_1}{R_2}$$

$$\frac{v_1}{v_2}=\left(\frac{3}{4}\right)^{\frac{1}{2}}=\frac{\sqrt{3}}{2}$$

Hence, the answer is the option (3).

Q. 3 A thermodynamic system is taken from an original state A to an intermediate state B by a linear process as shown in the figure. It's volume is then reduced to the original value from B to C by an isobaric process. The total work done by the gas from A to B and B to C would be :



Option 1:
2200 J

Option 2:
1200 J

Option 3:
33800 J

Option 4:
600 J

Correct Answer:
2200 J

Solution:
Work done from A to B
$=$ area under line AB
$=$ area of trapezium

$$= \frac{1}{2} \times (8000 + 4000) \left(\text{dyne} / \text{cm}^2 \right) \times 4\ \text{m}^3$$
$$= 12000 \times \frac{10^{-5}\ \text{N}}{10^{-4}\ \text{m}^2} \times \text{m}^3 \times 2$$
$$= 2400\ \text{J}$$

Work done from B to C
$=$ area under line BC
$= 4000 \left(\text{dyne} / \text{cm}^2 \right) \times 4\ \text{m}^3$

$$= 4000 \times \frac{10^{-5}}{10^{-4}} \times 4$$

$$= 1600\ \text{J}$$
$$\text{Total} = 2400 - 1600 = 800\ \text{J}$$

Q. 4 A body starts moving from rest with constant acceleration covers displacement S_1 in first (p-1) seconds and S_2 in the first p seconds. The displacement $S_1 + S_2$ will be made in time:

Option 1:
$\sqrt{(2p^2 - 2p + 1)}s$

Option 2:
$(2p^2 - 2p + 1)s$

Option 3:
$(2p + 1)s$

Option 4:
$(2p - 1)s$

Correct Answer:
$\sqrt{(2p^2 - 2p + 1)}s$

Solution:

$$S_1 = \frac{1}{2}a(p-1)^2 = \frac{1}{2}a\left(p^2 + 1 - 2p\right)$$

$$S_2 = \frac{1}{2}ap^2$$

$$S_1 + S_2 = \tfrac{1}{2}ap^2 + \tfrac{1}{2}ap^2 + \tfrac{a}{2} - ap$$

$$\tfrac{1}{2}at^2 = ap^2 - ap + \tfrac{a}{2}$$

$$t^2 = \tfrac{2a}{a}\left(p^2 - p + \tfrac{1}{2}\right)$$

$$t = \sqrt{2\left(p^2 - p + \frac{1}{2}\right)}$$

$$= \sqrt{2p^2 - 2p + 1}$$

Hence, the answer is the option (1).

Q. 5 Given below are two statements:

Statement I : If a capillary tube is immersed first in cold water and then in hot water, the height of capillary rise will be smaller in hot water.

Statement II : If a capillary tube is immersed first in cold water and then in hot water, the height of capillary rise will be smaller in cold water.

In the light of the above statements, choose the most appropriate from the options given below

Option 1:
Statement I is false but Statement II is true

Option 2:
Both Statement I and Statement II are false

Option 3:
Statement I is true but Statement II is false

Option 4:
Both Statement I and Statement II are true

Correct Answer:
Statement I is true but Statement II is false

Solution:

$$h = \frac{2T\cos\Theta}{\rho g r}$$

As $\mathrm{T} \uparrow, \mathrm{S} \downarrow$

as temperature increases surface tension decreases so h decreases.

Hence, the answer is the option (3).

Q. 6 Match List I with List II

LIST I		LIST II	
A	$\oint \vec{B} \cdot \vec{dl} = \mu_0 i_c + \mu_0 \varepsilon_0 \frac{d\phi_E}{dt}$	I	Gauss law for electricity
B	$\oint \vec{E} \cdot \vec{dl} = \frac{d\phi_B}{dt}$	II	Gauss law for magnetism
C	$\int \vec{E} \cdot \vec{dA} = \frac{Q}{\varepsilon_0}$	III	Faraday law
D	$\int \vec{B} \cdot \vec{dA} = 0$	IV	Ampere – Maxwell law

Choose the correct answer from the options given below :

Option 1:
A-II, B=III, C-I, D-IV

Option 2:
A-IV, B=I, C-III, D-II

Option 3:
A-IV, B=III, C-I, D-II

Option 4:
A-I, B=II, C-III, D-IV

Correct Answer:
A-IV, B=III, C-I, D-II

Solution:

$$\text{Gauss law in electrostatic} \oint \vec{E} \cdot \vec{ds} = \frac{q}{\varepsilon_0}$$

$$\text{Faradays law} \oint \vec{E} \cdot \vec{dl} = \frac{d\phi_B}{dt}$$

$$\text{Gauss law in magnetism} \oint \vec{B} \cdot \vec{dA} = 0$$

$$\text{Ampere Maxwell law} \oint \vec{B} \cdot \vec{dl} = \mu_0 i_c + \mu_0 \varepsilon_0 \frac{d\phi_E}{dt}$$

Hence, the answer is the option (3).

Q. 7 The explosive in a Hydergine bomb is a mixture of ${}_1H^2$, ${}_1H^3$ and ${}_3Li^6$ in some condensed form. The chains reaction is given by

$${}_3Li^6 + {}_0nl \rightarrow {}_2He^4 + {}_1H^3$$
$${}_1H^2 + {}_1H^3 \rightarrow {}_2He^4 + {}_0n^1$$

During the explosion the energy released is approximately

[Given; $M(Li) = 6.01690amu, M\left({}_1H^2\right) = 2.01471amu, M\left({}_2He^4\right) = 4.00388amu$, and $1amu = 931.5MeV$]

Option 1:
28. 12 MeV

Option 2:
16.48 MeV

Option 3:
12.64 MeV

Option 4:
22.22 MeV

Correct Answer:
22.22 MeV

Solution:
${}_3 L_i{}^6 + {}_0\not{n}^1 \rightarrow {}_2 He^4 + {}_\chi H^6$

$\frac{{}_1H^2 + {}_\chi H \not\rightarrow {}_2H_e{}^4 + {}_0\not{1}}{{}_3 L_1{}^6 + {}_1H^2 \rightarrow 2{}_2H_e{}^4}$

$$\Delta m = [M(L_i) + M(H)] - [2M(He)]$$
$$= [6.01690 + 2.01471] - [2 \times 4.00388]$$
$$= 8.03161 - 8.00776$$

$$\Delta m = 0.02385$$
$$E = \Delta m \times 931.5Mev$$
$$= 22.22MeV$$

Hence, the answer is the option (4).

Q. 8 Two vessels A and B are of the same size and are at same temperature A contains 1 g of hydrogen and B contains 1 g of oxygen. P_A and P_B are the pressures of the gases in A and B respectively, then $\frac{P_A}{P_B}$ is :

Option 1:
8

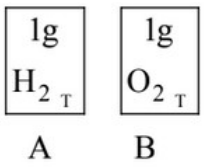
Option 2:
16

Option 3:
32

Option 4:
4

Correct Answer:
16

Solution:



$\mathrm{P}=\frac{nRT}{V}$

as $\mathrm{T}\&\ \mathrm{V}$ are same so
$\mathrm{P} \propto \mathrm{n}$

$\frac{\mathrm{P_A}}{\mathrm{P_B}}=\frac{\mathrm{n_A}}{\mathrm{n_B}}=\frac{1\,\mathrm{g}\times 32}{2\times 1\,\mathrm{g}}=16$

Q. 9 A galvanometer having coil resistance 10Ω shows a full scale deflection for a current of $3\ \mathrm{mA}$. For it to measure a current of $8\ \mathrm{A}$, the value of the shunt should be:

Option 1:
$4.85\times 10^{-3}\Omega$

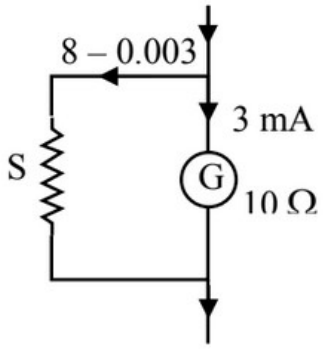
Option 2:
$3\times 10^{-3}\Omega$

Option 3:
$3.75\times 10^{-3}\Omega$

Option 4:
$2.75\times 10^{-3}\Omega$

Correct Answer:
$3.75\times 10^{-3}\Omega$

Solution:



$S \times (8 - 0.003) = 0.003 \times 10$

$S = \dfrac{0.03}{7.997}$

$= 3.75 \times 10^{-3}\Omega$

Q. 10 A convex mirror of radius of curvature $30\ \text{cm}$ forms an image that is half the size of the object. The object distance is :

Option 1:
-15 cm

Option 2:
-45 cm

Option 3:
45 cm

Option 4:
15 cm

Correct Answer:
-15 cm

Solution:

$m = \frac{f}{f-u} \quad \left(f = \frac{R}{2}\right)$

$+\dfrac{1}{2} = \dfrac{+15}{15-u}$

$15 - u = 30$

$15 - 30 = u$

$u = -15\ \text{cm}$

Hence, the answer is the option (1).

Q. 11 A biconvex lens of refractive index 1.5 has a focal length of $20\ \text{cm}$ in air. Its focal length when immersed in a liquid of refractive index 1.6 will be:

Option 1:
–16 cm

Option 2:
+ 16 cm

Option 3:
160 cm

Option 4:
– 160 cm

Correct Answer:
– 160 cm

Solution:

$$f = \frac{R}{2\left(\frac{\mu_L}{\mu_S} - 1\right)}$$

$$20 = \frac{\text{R}}{2(1.5-1)} - - - - - (1)$$

$$\text{f}' = \frac{\text{R}}{2\left(\frac{1.5}{1.6} - 1\right)} - - - - - (2)$$

$$\frac{20}{f'} = \frac{(1.5 - 1.6)/1.6}{1.5 - 1}$$

$$\frac{20}{f'} = \frac{-0.1}{1.6} \times \frac{1}{0.5} \Rightarrow f' = -160\ \text{cm}$$

Hence, the answer is the option (4).

Q. 12 The electric current through a wire varies with time as $I = I_0 + \beta t$, where $I_0 = 20\ \text{A}$ and $\beta = 3\ \text{A/s}$. The amount of electric charge crossed through a section of the wire in $20\ \text{s}$ is :

Option 1:
1600 C

Option 2:
800 C

Option 3:
80 C

Option 4:
1000 C

Correct Answer:
1000 C

Solution:
The average electric current through the area is given by :

$$i = \frac{dq}{dt}$$

$$\int_0^Q dq = \int_0^{20} i dt$$

$$= \int_0^{20} (i_0 + \beta t) dt$$

$$= \int_0^{20} (20 + 3t) dt$$

$$= 20[t]_0^{20} + 3\left[\frac{t^2}{2}\right]_0^{20}$$

$$= 400 + \frac{3}{2}\left[20^2 - 0\right]$$

$$= 400 + 600 = 1000C$$

Hence, the answer is the option (4).

Q. 13 The deflection in moving coil galvanometer falls from 25 divisions to 5 division when a shunt of 24Ω is applied. The resistance of galvanometer coil will be :

Option 1:
100Ω

Option 2:
48Ω

Option 3:
12Ω

Option 4:
96Ω

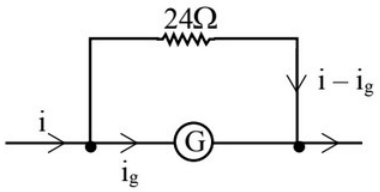
Correct Answer:
96Ω

Solution:

$i \propto \theta$

$\frac{i_g}{i} = \frac{5}{25}$

$5i_g = i$



Potential is same across galvanometer shunt

$Si_5 = i_g G$
$S(i - i_g) = i_g G$
$24(5i_g - i_g) = i_g G$
$24 \times 4i_g = i_g G$
$G = 24 \times 4 = 96$

Q. 14 A block of mass $100\ kg$ slides over a distance of $10\ m$ on a horizontal surface. If the co-efficient of friction between the surfaces is 0.4 , then the work done against friction (in J) is :

Option 1:
4000

Option 2:
3900

Option 3:
4200

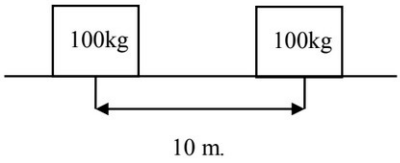
Option 4:
4500

Correct Answer:
4000

Solution:

$W.D = f_s.s$
$W.D = \mu mg(10)$
$= (0.4)(100)(10)(10)$
$W.D = 4000\ \text{J}$

W.D Against friction $= 4000\ \text{J}$



Q. 15 Two charges of 5Q and -2Q are situated at the points $(3\text{a}, 0)$ and $(-5\text{a}, 0)$ respectively. The electric flux through a sphere of radius ' 4a ' having center at origin is :

Option 1:
$\frac{3\text{Q}}{\varepsilon_0}$

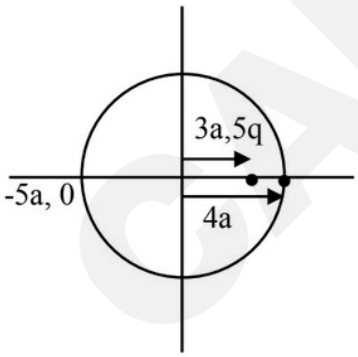
Option 2:
$\frac{5\text{Q}}{\varepsilon_0}$

Option 3:
$\frac{7\text{Q}}{\varepsilon_0}$

Option 4:
$\frac{2\text{Q}}{\varepsilon_0}$

Correct Answer:
$\frac{5\text{Q}}{\varepsilon_0}$

Solution:



gauss law,
$\phi = \frac{q_{in}}{\epsilon_0} = \frac{5q}{\epsilon_0}$

Q. 16 A capacitor of capacitance $100\mu F$ is charged to a potential of $12\ V$ and connected to a $6.4mH$ inductor to produce oscillations. The maximum current in the circuit would be :

Option 1:
1.2 A

Option 2:
1.5 A

Option 3:
3.2 A

Option 4:
2.0 A

Correct Answer:
1.5 A

Solution:

$\frac{Q_{max}}{2C} = \frac{1}{2}Li^2$

$C = 100\mu f, \quad V = 12\ V$

$L = 6.4 \times 10^{-3}$

$Q_{max} = CV$

$= \frac{(100\times10^{-6}\times12)^2}{2\times100\times10^{-6}} = \frac{1}{2} \times 6.4 \times 10^{-3} \times i^2$

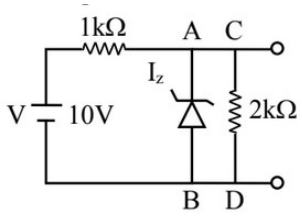
$\frac{12^2\times10^{-8-3}}{10^{-4}} = 6.4 \times 10^{-3} \times i^2$

$\sqrt{\frac{12^2 \times 10^{-1}}{6.4}} = \sqrt{2.25}$

$i = 1.5A$

Hence, the answer is the option (2).

Q. 17 In the given circuit, the breakdown voltage of the Zener diode is $3.0\ \mathrm{V}$. What is the value of $\mathrm{I_z}$?



Option 1:
7 mA

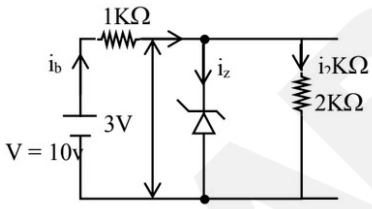
Option 2:
5.5 mA

Option 3:
10 mA

Option 4:
3.3 mA

Correct Answer:
5.5 mA

Solution:



$$\mathrm{i_{battery}} = \frac{10-3}{1000}$$
$$\mathrm{i_{battery}} = 7\ \mathrm{mA}$$
$$\mathrm{i\Omega} = \frac{\mathrm{V}}{\mathrm{R}} = \frac{3}{2000}$$
$$\mathrm{i_{2\,K\Omega}} = 1.5\ \mathrm{mA}$$
$$\mathrm{i_z} = \mathrm{i_b} - \mathrm{i_{2\,K\Omega}}$$
$$\mathrm{i_z} = (7 - 1.5) = 5.5\ \mathrm{mA}$$

Q. 18 At what distance above and below the surface of the earth a body will have same weight. (take radius of earth as R.)

Option 1:
$\frac{\mathrm{R}}{2}$

Option 2:
$\sqrt{5}\mathrm{R}-\mathrm{R}$

Option 3:
$\frac{\sqrt{3}R-R}{2}$

Option 4:
$\frac{\sqrt{5}\mathrm{R}-\mathrm{R}}{2}$

Correct Answer:
$\frac{\sqrt{5}\mathrm{R}-\mathrm{R}}{2}$

Solution:
Since the body's weight should remain constant, the value of the acceleration caused by gravity should also remain constant.Therefore,

$$\frac{\mathrm{GMm}}{(\mathrm{R}+\mathrm{x})^2}=\frac{\mathrm{GMm}(\mathrm{R}-\mathrm{x})}{\mathrm{R}^3}$$

$$R^3=(R+x)^2(R-x)$$
$$R^3=\left(R^2-x^2\right)(R+x)$$
$$x^2+Rx-R^2=0$$

$$x=\frac{-R\pm\sqrt{R+4R^2}}{2}$$

$$x=\frac{\sqrt{5}R-R}{2}$$

Hence, the answer is the option (4).

Q. 19 The resistance $\mathrm{R}=\frac{\mathrm{V}}{\mathrm{I}}$ where $\mathrm{V}=(200\pm5)\mathrm{V}$ and $\mathrm{I}=(20\pm0.2)\mathrm{A}$, the percentage error in the measurement of R is :

Option 1:
3.5%

Option 2:
5.5%

Option 3:
7%

Option 4:
3%

Correct Answer:
3.5%

Solution:
$\mathrm{V} = (200 \pm 5)$ volt, and , $\mathrm{I} = (20 \pm 0.2)\mathrm{A}$

$\%$ error $= \frac{\mathrm{dV}}{\mathrm{V}} + \frac{\mathrm{dI}}{\mathrm{I}} \quad \therefore \mathrm{R} = \frac{\mathrm{V}}{\mathrm{I}}$

$$= \left(\frac{5}{200} + \frac{0.2}{20}\right) \times 100$$
$$= 3.5\%$$

Hence, the answer is the option (1).

Q. 20 The potential energy function (in J) of a particle in a region of space is given as :

$$U = \left(2x^2 + 3y^3 + 2z\right).$$

Here x, y and z are in meter. The magnitude of x-component of force (in N) acting on the particle at point $\mathrm{P}(1, 2, 3)\mathrm{m}$ is :

Option 1:
6

Option 2:
2

Option 3:
8

Option 4:
4

Correct Answer:
4

Solution:
The potential energy function of a particle in a region of space is given as :

$$u = 2x^2 + 3y^3 + 2z$$

Now, the three-dimensional formula for potential energy only conservative fields F is given by :

$$F = -[\frac{dU}{dx}\vec{i} + \frac{dU}{dy}\vec{j} + \frac{dU}{dz}\vec{k}]$$

$$F = -\left[\frac{\partial}{\partial x}\left(2x^2\right)\hat{i} + \frac{\partial}{\partial y}\left(3y^3\right)\hat{j} + \frac{\partial}{\partial z}(2z)\hat{k}\right]$$

$\overrightarrow{F}(1,2,3) = -4(1)\hat{i} - 9(2)^2\hat{j} - 2\hat{k}$

$\overrightarrow{F}_{net} = -4\hat{i} - 36\hat{j} - 2\hat{k}$

$\overrightarrow{F}_x = 4$

Hence, the answer is the option (4).

Q. 21 When the displacement of a simple harmonic oscillator is one third of its amplitude, the ratio of total energy to the kinetic energy is $\frac{x}{8}$, where $x =$_____

Correct Answer:
9

Solution:
Total energy $= \frac{1}{2}kA^2$

$u = \frac{1}{2}K\left(\frac{A}{3}\right)^2 = \frac{KA^2}{2\times 9} = \frac{E}{9}$

$KE = E - \frac{E}{9} = \frac{8E}{9}$

$\text{Ratio, } \frac{\text{total}}{KE} = \frac{E}{\frac{8E}{9}} = \frac{9}{8}$

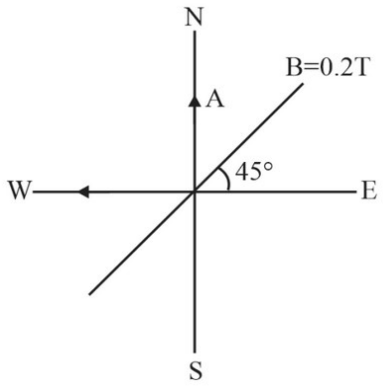
x = 9

Hence, the answer is (9).

Q. 22 A square loop of side $10\ cm$ and resistance 0.7Ω is placed vertically in east-west plane. A uniform magnetic field of $0.20\ T$ is set up across the plane in north east direction. The magnetic field is decreased to zero in $1\ s$ at a steady rate. Then, magnitude of induced emf is $\sqrt{x} \times 10^{-3}\ V$. The value of x is _____

Correct Answer:
2

Solution:



$\overrightarrow{\mathrm{A}} = (0.1)^2\hat{\mathrm{j}}$

$\overrightarrow{\mathrm{B}} = \frac{0.2}{\sqrt{2}}\hat{\mathrm{i}} + \frac{0.2}{\sqrt{2}}\hat{\mathrm{j}}$

Mag. of Emf,

$$\mathrm{e} = \frac{\Delta\phi}{\Delta \mathrm{t}} = \frac{\overrightarrow{\mathrm{B}} \cdot \overrightarrow{\mathrm{A}} - 0}{1} = \sqrt{2} \times 10^{-3}\ \mathrm{V}$$

$\mathrm{x} = 2$

Q. 23 In a test experiment on a model aeroplane in wind tunnel, the flow speeds on the upper and lower surfaces of the wings are $70\ \mathrm{ms}^{-1}$ and $65\ \mathrm{ms}^{-1}$ respectively. If the wing area is $2\ \mathrm{m}^2$, the lift of the wing is _____N. (Given density of air $= 1.2\ \mathrm{kg\ m}^{-3}$)

Correct Answer:
810

Solution:
Let v_1 and v_2 be the speed of air at the upper and the lower surface of the wing respectively. Let ρ the pressure of air at the upper and the lower surface of the wing respectively.

Bernoulli's equation applied on the upper and the lower surface of the wing is given as,

$$\mathrm{F} = \frac{1}{2}\rho\left(\mathrm{v}_1^2 - \mathrm{V}_2^2\right)\mathrm{A}$$

$$\mathrm{F} = \frac{1}{2} \times 1.2 \times \left[70^2 - 65^2\right] \times 2$$

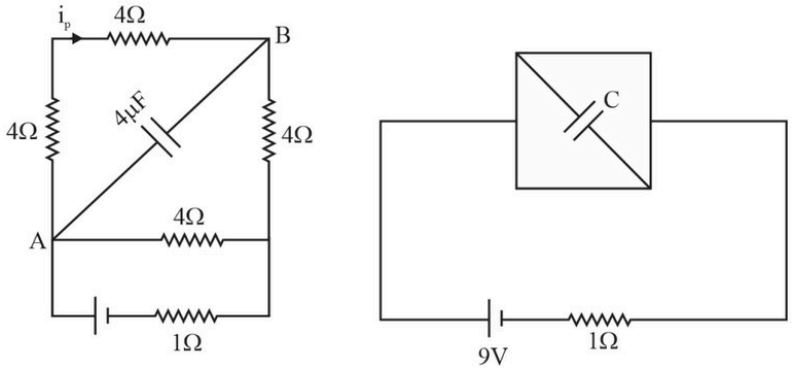
$\overrightarrow{\mathrm{F}} = 810\ \mathrm{N}$

Hence, the answer is 810 N.

Q. 24 A 16Ω wire is bend to form a square loop. A $9\ \mathrm{V}$ battery with internal resistance 1Ω is connected across one of its sides. If a $4\mu\mathrm{F}$ capacitor is connected across one of its diagonals, the energy stored by the capacitor will be $\frac{x}{2}\mu\mathrm{J}$, where $\mathrm{x} =$_____

Correct Answer:
81

Solution:



$i = \frac{V}{R_{eq}} = \frac{9}{1+\frac{12\times4}{12+4}} = \frac{9}{4}$

$i_1 = \frac{9}{4} \times \frac{4}{16} = \frac{9}{16}$

$\mathrm{V_A - V_B = i_1 \times 8 = \frac{9}{16} \times 8 = \frac{9}{2}\ V}$

$\mathrm{u = \frac{1}{2} \times 4 \times \frac{81}{4} = \frac{81}{2} uJ}$

x = 81

Q. 25 The magnetic potential due to a magnetic dipole at a point on its axis situated at a distance of $20\ \mathrm{cm}$ from its center is $1.5 \times 10^{-5}\mathrm{Tm}$. The magnetic moment of the dipole is _____$\mathrm{Am^2}$. (Given : $\frac{\mu_0}{4\pi} = 10^{-7}\mathrm{TmA^{-1}}$)

Correct Answer:
6

Solution:
The magnetic potential at a point on the axis of a magnetic dipole is given by the formula:

$\mathrm{V = \frac{\mu_0}{4\pi}\frac{M}{r^2}}$

$\Rightarrow 1.5 \times 10^{-5} = 10^{-7} \times \frac{\mathrm{M}}{(20\times10^{-2})^2}$

$\mathrm{m = \frac{1.5\times10^{-5}\times20\times20\times10^{-4}}{10^{-7}}}$

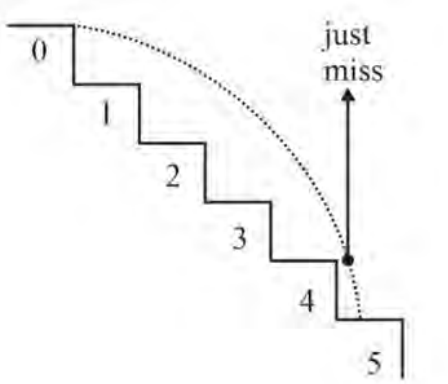
$M = 1.5 \times 4 = 6$

Hence, the answer is 6.

Q. 26 A ball rolls off the top of a stairway with horizontal velocity u. The steps are $0.1\ \mathrm{m}$ high and $0.1\ \mathrm{m}$ wide. The minimum velocity u with which that ball just hits the step 5 of the stairway will be $\sqrt{\mathrm{x}}\mathrm{ms}^{-1}$ where $\mathrm{x}=$____ [use $\mathrm{g}=10\ \mathrm{m/s^2}$.

Correct Answer:
2

Solution:



the ball needs to just cross 4 steps to just hit 5^{th} step
$\therefore$ horizontal range $(\mathrm{R})=0.4\ \mathrm{m}$
$\mathrm{R}=\mathrm{u}.\mathrm{t}$

Similarly in vertical direction

$$\mathrm{h}=\frac{1}{2}\mathrm{gt}^2$$

$$0.4=\frac{1}{2}\mathrm{gt}^2$$

$$0.4=\frac{1}{2}\ \mathrm{g}\left(\frac{0.4}{4}\right)^2$$

$$\mathrm{u}^2=2$$

$$\mathrm{u}=\sqrt{2}\ \mathrm{m/s}, \mathrm{x}=2$$

Q. 27 An electron is moving under the influence of the electric field of a uniformly charged infinite plane sheet S having surface charge density $+\sigma$. The electron at $t=0$ is at a distance of $1\ \mathrm{m}$ from S and has a speed of $1\ \mathrm{m/s}$. The maximum value of σ if the electron strikes S at $\mathrm{t}=1\ \mathrm{s}$ is $\alpha\left[\frac{\mathrm{m}\in_0}{\mathrm{e}}\right]\frac{\mathrm{C}}{\mathrm{m}^2}$, the value of α is_____

Correct Answer:
8

Solution:

$$S = ut + \frac{1}{2}at^2$$

$$-1 = 1(1) - \frac{1}{2}a(1)^2$$

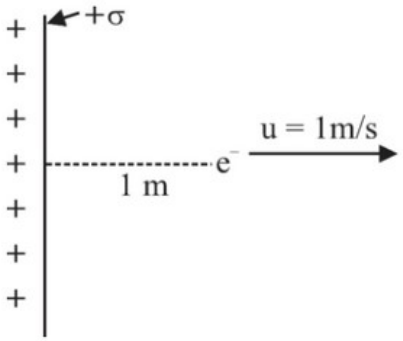
$$A = 4\ \mathrm{m/s^2}$$

$$= \frac{\mathrm{eE}}{\mathrm{m}} = 4$$

$$\frac{\mathrm{q}\sigma}{2\varepsilon_0\ \mathrm{m}} = 4$$

$$\sigma = \frac{8\varepsilon_0\ \mathrm{m}}{2}$$

$$\alpha = 8$$



Q. 28 A cylinder is rolling down on an inclined plane inclination 60°. It's acceleration during rolling down will be $\frac{x}{\sqrt{3}}\ \mathrm{m/s^2}$, where $\mathrm{x} =$_____ (use $\mathrm{g} = 10\ \mathrm{m/s^2}$). Given -

Correct Answer:
10

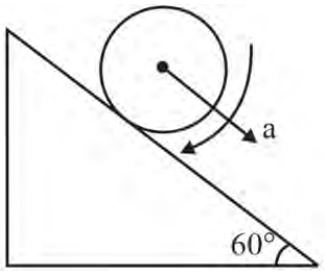
Solution:

for rolling motion

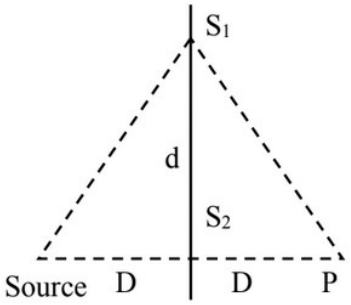
$$\mathrm{a} = \frac{\mathrm{g}\sin\theta}{1+\frac{\mathrm{I_{COM}}}{\mathrm{MR^2}}}$$

$$\mathrm{a} = \frac{\mathrm{g}\sin\theta}{1+\frac{1}{2}} = \frac{2\times10\times\frac{\sqrt{3}}{2}}{3}$$

$$= \frac{10}{\sqrt{3}} = x = 10$$



Q. 29 In a double slit experiment shown in figure, when light of wavelength $400\ \text{nm}$ is used, dark fringe is observed at P. If $\text{D} = 0.2\ \text{m}$, the minimum distance between the slits S_1 and S_2 is _____mm.



Correct Answer:
0.20

Solution:
Path diff. for minima at P

$$2\sqrt{\text{D}^2 + \text{d}^2} - 2\text{D} = \frac{\lambda}{2}$$

$$\therefore \sqrt{\text{D}^2 + \text{d}^2} - \text{D} = \frac{\lambda}{4}$$

$$\sqrt{\text{D}^2 + \text{d}^2} = \tfrac{\lambda}{4} + \text{D}$$

$$\text{D}^2 + \text{d}^2 = \text{D}^2 + \frac{\lambda^2}{16} + \frac{\lambda\text{D}}{2}$$

$$\text{d}^2 = \frac{\text{D}\lambda}{2} + \frac{\lambda^2}{16}$$

$$\text{d}^2 = \tfrac{0.2\times400\times10^{-9}}{2} + \tfrac{4\times10^{-14}}{4}$$

$$\text{d}^2 \approx 400 \times 10^{-10} \qquad \therefore \text{d} = 20 \times 10^{-5}$$

$$\text{d} = 0.20\ \text{mm}$$

Q. 30 When a hydrogen atom going $\text{n} = 2$ to $\text{n} = 1$ emits a photon, its recoil speed is $\frac{\text{x}}{5}\ \text{m/s}$. Where $\text{x} =$_____ (Use, mass of hydrogen atom $= 1.6 \times 10^{-27}\ \text{kg}$) Given -

Correct Answer:
17

Solution:
Recoil speed is given by :

$$\text{V} = \tfrac{\Delta\text{E}}{\text{mC}}$$

$$= \frac{10.2\text{eV}}{1.6 \times 10^{-27} \times 3 \times 10^{8}}$$

$$= \frac{10.2 \times 1.6 \times 10^{-19}}{1.6 \times 10^{-27} \times 3 \times 10^{8}}$$

$$= \text{V} = 3.4\ \text{m/s} = \frac{17}{5}\ \text{m/s}$$

$$\text{x} = 17$$

Hence, the answer is 17.

Chemistry

Q. 1 Given below are two statements :

Statement I : The electronegativity of group 14 elements from Si to Pb, gradually decreases.

Statement II : Group 14 contains non-metallic, metallic, as well as metalloid elements.

In the light of the above statements, choose the most appropriate from the options given below :

Option 1:
Statement I is true but Statement II is false.

Option 2:
Statement I is false but Statement II is true.

Option 3:
Both Statement I and Statement II are false.

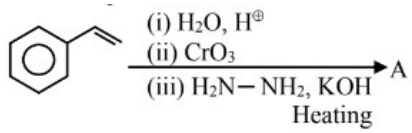
Option 4:
Both Statement I and Statement II are true.

Correct Answer:
Statement I is false but Statement II is true.

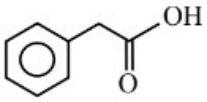
Solution:
EN of gp. $14 \rightarrow \text{C} > \text{Si} = \text{Ge} = \text{Sn} < \text{Pb}$



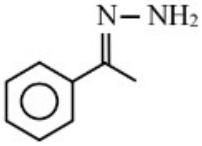
Q. 2 The final product A formed in the following multistep reaction sequence is :



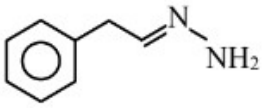
Option 1:



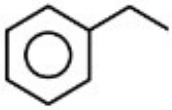
Option 2:



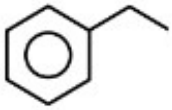
Option 3:



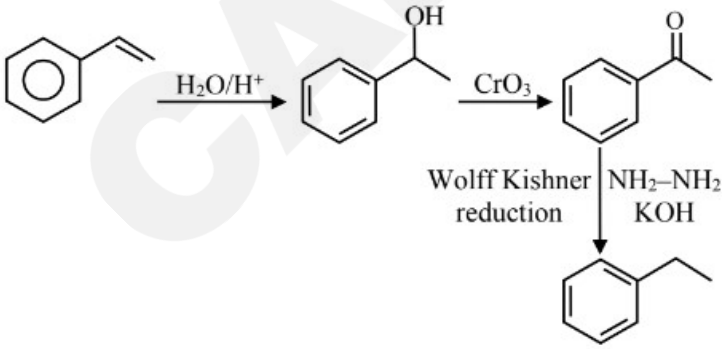
Option 4:



Correct Answer:



Solution:



Q. 3 Which of the following is not correct?

Option 1:

ΔG is zero for a reversible reaction.

Option 2:
ΔG is negative for a spontaneous reaction

Option 3:
ΔG is positive for a spontaneous reaction

Option 4:
ΔG is positive for a non-spontaneous reaction

Correct Answer:
ΔG is positive for a spontaneous reaction

Solution:
If $\Delta G < 0$ (negative), the reaction is spontaneous. - If $\Delta G > 0$ (positive), the reaction is non-spontaneous. - If $\Delta G = 0$, the system is in equilibrium.

Hence, the answer is the option (3).

Q. 4 Identify the incorrect pair from the following :

Option 1:
Carnallite $-\mathrm{KCl \cdot MgCl_2, 6H_2O}$

Option 2:
Cryolite $-\mathrm{Na_3AlF_6}$

Option 3:
Fluroapatite - $\mathrm{3Ca_3(PO_4)_2 \cdot CaF_2}$

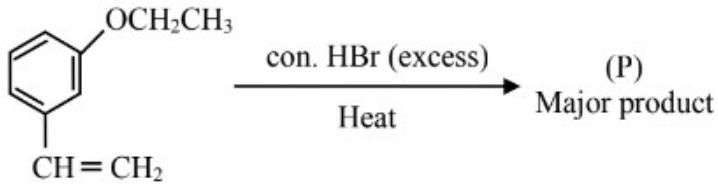
Option 4:
Fluorspar - $\mathrm{BF_3}$

Correct Answer:
Fluorspar - $\mathrm{BF_3}$

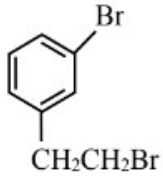
Solution:
Fluorspar, also known as fluorite, is a mineral composed of calcium fluoride (CaF_2).

Hence, the answer is the option (4).

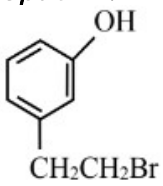
Q. 5 The major product(P) in the following reaction is :



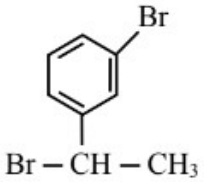
Option 1:



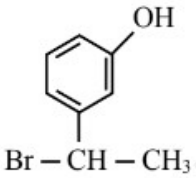
Option 2:



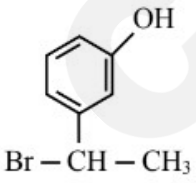
Option 3:



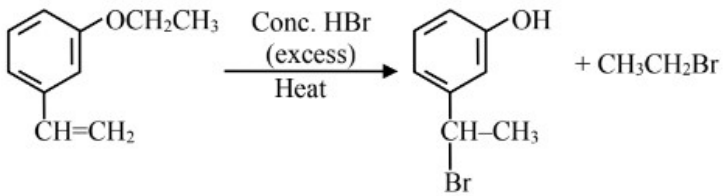
Option 4:

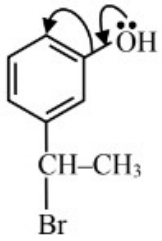


Correct Answer:



Solution:





further does not react with HBr due to double bond character between C–O bond.

Q. 6 In which one of the following metal carbonyls, CO forms a bridge between metal atoms?

Option 1:
$[Os_3(CO)_{12}]$

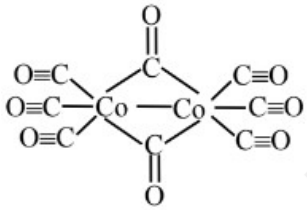
Option 2:
$[Ru_3(CO)_{12}]$

Option 3:
$[Co_2(CO)_8]$

Option 4:
$[Mn_2(CO)_{10}]$

Correct Answer:
$[Co_2(CO)_8]$

Solution:



Q. 7 Given below are two statements : one is labelled as **Assertion A** and the other is labelled as **Reason R** :

Assertion A : Aryl halides connot be prepared by replacement of hydroxyl group of phenol by halogen atom.

Reason R : Phenols react with halogen acids violently.

In the light of the above statements, choose the most appropriate from the options given below :

Option 1:
A is false but R is true

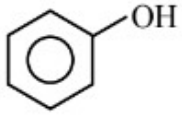
Option 2:
Both A and R are true and R is the correct explanation of A

Option 3:
A is true but R is false

Option 4:
Both A and R are true but R is NOT the correct explanation of A

Correct Answer:
A is true but R is false

Solution:

$\rightarrow$  does not give substitution $\mathrm{rx^n}$ due to double bond character in $\mathrm{C-O}$ bond.

$\rightarrow$ Phenol does not react with HX violently.

Q. 8 In alkaline medium, $\mathrm{MnO_4^-}$ oxidises $\mathrm{I^-}$ to :

Option 1:
$\mathrm{IO^-}$

Option 2:
$\mathrm{IO_4^-}$

Option 3:
$\mathrm{I_2}$

Option 4:
$\mathrm{IO_3^-}$

Correct Answer:
$\mathrm{IO_3^-}$

Solution:
$\mathrm{I^- + 2MnO_4^- + H_2O \rightarrow IO_3^- + 2MnO_2 + 2OH^-}$

Hence, the answer is the option(4).

Q. 9 $\mathrm{KMnO_4}$ decomposes on heating at $513\ \mathrm{K}$ to form $\mathrm{O_2}$ along with :

Option 1:
$\mathrm{Mn\&KO_2}$

Option 2:
$K_2MnO_4 \& MnO_2$

Option 3:
$K_2MnO_4 \& Mn$

Option 4:
$MnO_2 \& K_2O_2$

Correct Answer:
$K_2MnO_4 \& MnO_2$

Solution:

$$2KMnO_4 \xrightarrow{513\ K} K_2MnO_4 + MnO_2 + O_2$$

Hence, the answer is the option (2).

Q. 10 The difference in energy between the actual structure and the lowest energy resonance structure for the given compound is :

Option 1:
electromeric energy

Option 2:
ionization energy

Option 3:
hyperconjugation energy

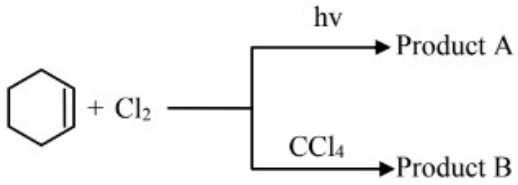
Option 4:
resonance energy

Correct Answer:
resonance energy

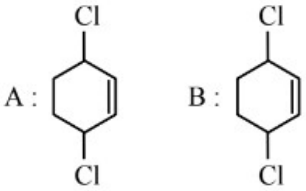
Solution:
The difference in energy between the actual structure and the lowest energy resonance structure for the given compound is called resonance energy.

Hence, the answer is the option (4).

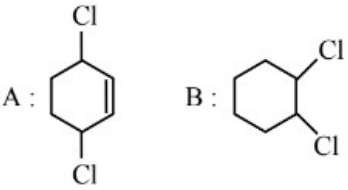
Q. 11 Identify product A and product B :



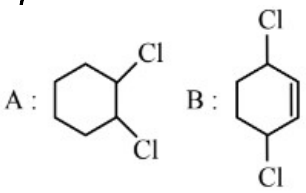
Option 1:



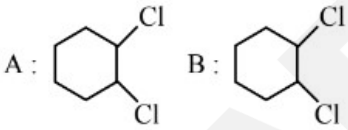
Option 2:



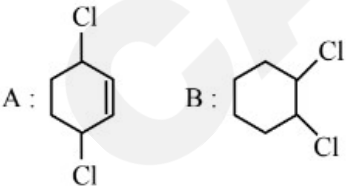
Option 3:



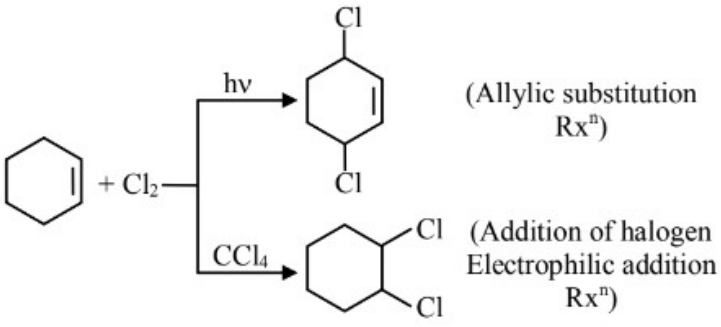
Option 4:



Correct Answer:



Solution:



Q. 12 Type of amino acids obtained by hydrolysis of proteins :

Option 1:
γ

Option 2:
δ

Option 3:
β

Option 4:
α

Correct Answer:
α

Solution:
$\rightarrow$ Natural proteins are polymers of α-amino acids.

Hence, the answer is the option(4).

Q. 13 Match List I with List II

List I (Substances)		**List II (Element Present)**	
A.	Ziegler catalyst	I.	Rhodium
B.	Blood Pigment	II.	Cobalt
C.	Wilkinson Catalyst	III.	Iron
D.	Vitamin B_{12}	IV.	Titanium

Choose the correct answer from the options given below :

Option 1:
A-II, B-IV, C-I, D-III

Option 2:
A-IV, B-III, C-I, D-II

Option 3:
A-II, B-III, C-IV, D-I

Option 4:
A-III, B-II, C-IV, D-I

Correct Answer:
A-IV, B-III, C-I, D-II

Solution:
(1) Wilkinson catalyst $= [\mathrm{Rh(PPh_3)_3Cl}]$
(2) Zigler catalyst $= \mathrm{TiCl_4 - AlR_3}$

Hence, the answer is option (2).

Q. 14 Appearance of blood red colour, on treatment of the sodium fusion extract of an organic compound with $\mathrm{FeSO_4}$ in presence of concentrated $\mathrm{H_2SO_4}$ indicates the presence of element/s :

Option 1:
N and S

Option 2:
N

Option 3:
S

Option 4:
Br

Correct Answer:
N and S

Solution:
If an organic compound contains N and S, then sodium fusion extract has NaCNS, which gives blood red colour with $\mathrm{Fe^{3+}}$ ion

$$\mathrm{NaCNS + FeCl_3 \rightarrow \underset{\text{blood Red}}{Fe(SCN)_3}}$$

Hence, the answer is the option (1).

Q. 15 In chromyl chloride test for confirmation of Cl^- ion, a yellow solution is obtained. Acidification of the solution and addition of amyl alcohol and $10\% H_2O_2$ turns organic layer blue indicating formation of chromium pentoxide. The oxidation state of chromium in that is :

Option 1:
+5

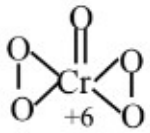
Option 2:
+3

Option 3:
+10

Option 4:
+6

Correct Answer:
+6

Solution:
CrO_5 has two peroxide linkage



Q. 16 Chlorine undergoes disproportionation in alkaline medium as shown below :

$$aCl_{2(g)} + bOH^-_{(aq)} \rightarrow cClO^-_{(aq)} + dCl^-_{(aq)} + eH_2O_{(l)}$$

The value of a, b, c and d in a balanced redox reaction are respectively :

Option 1:
2, 2, 1 and 3

Option 2:
1, 2, 1 and 1

Option 3:
2, 4, 1 and 3

Option 4:
3, 4, 4 and 2

Correct Answer:
1, 2, 1 and 1

Solution:
$Cl_2 \rightarrow 2Cl^- + 2e^- (\text{Reduction half Reaction})$
$2H_2O + Cl_2 \rightarrow 2ClO^- + 4H^+ + 2e^- (\text{oxidation half})$
$2Cl_2 + 2H_2O \rightarrow 2Cl^- + 2ClO^- + 4H^+$
$Cl_2 + 2OH^- \rightarrow Cl^- + ClO^- + H_2O$

Hence, the answer is the option (2).

Q. 17 The correct set of four quantum numbers for the valence electron of rubidium atom (Z = 37) is :

Option 1:
$5, 1, 0, +\frac{1}{2}$

Option 2:
$5, 1, 1, +\frac{1}{2}$

Option 3:
$5, 0, 1, +\frac{1}{2}$

Option 4:
$5, 0, 0, +\frac{1}{2}$

Correct Answer:
$5, 0, 0, +\frac{1}{2}$

Solution:
1^{st} group element.

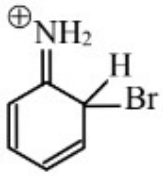
valence e^- configuration $= 5\ s^1$

$n = 5, \ell = 0,\ m = 0,\ s = +1/2$

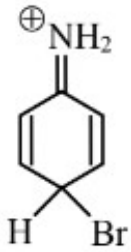
Hence, the answer is the option (4).

Q. 18 The arenium ion which is not involved in the bromination of Aniline is _____.

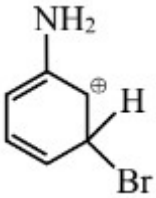
Option 1:



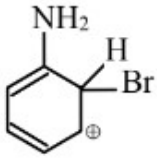
Option 2:



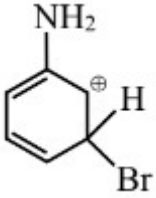
Option 3:



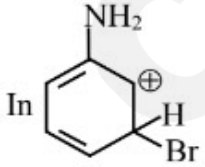
Option 4:



Correct Answer:



Solution:



,positive charge is not stabilized by $+\mathrm{M}$ effect of $-\ddot{\mathrm{N}}\mathrm{H}_2$ so this arenium ion is not formed during bromination of aniline.

Q. 19 Given below are two statements : one is labelled as **Assertion A** and the other is labelled as **Reason R** :

Assertion A : The first ionisation enthalpy decreases across a period.

Reason R : The increasing nuclear charge outweighs the shielding across the period.

In the light of the above statements, choose the most appropriate from the options given below :

Option 1:
A is false but R is true

Option 2:
A is ture but R is false

Option 3:
Both A and R are true and R is the correct explanation of A

Option 4:
Both A and R are true but R is NOT the correct explanation of A

Correct Answer:
A is false but R is true

Solution:
Along the period shielding increases in less amount than nuclear charge.

Hence, the answer is the option (1).

Q. 20 The interaction between π bond and lone pair of electrons present on an adjacent atom is responsible for :

Option 1:
Resonance effect

Option 2:
Electromeric effect

Option 3:
Inductive effect

Option 4:
Hyperconjugation

Correct Answer:
Resonance effect

Solution:
$\rightarrow$ The interaction between π bond and lone pair of electron present on adjacent atom is responsible for resonance effect.



Q. 21 The osmotic pressure of a dilute solution is 7×10^5 Pa at 273 K. Osmotic pressure of the same solution at 283 K is_____$\times 10^4 \mathrm{Nm}^{-2}$.

Correct Answer:
73

Solution:
$\pi = \mathrm{CRT}$

$$\frac{\pi_1}{\pi_2} = \frac{\mathrm{T}_1}{\mathrm{T}_2}$$

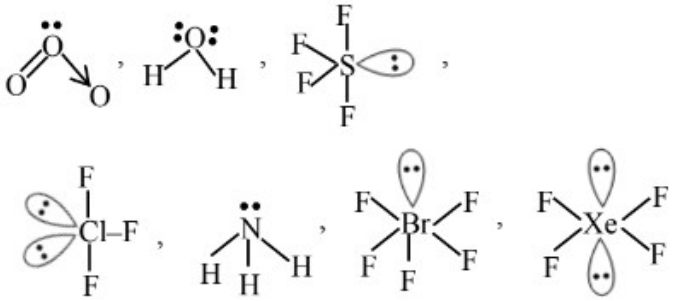
$$\Rightarrow \pi_2 = \frac{7 \times 10^5 \times 283}{273}$$

$$= 72.5 \times 10^4 \approx 73 \times 10^4$$

Hence, the answer is (73).

Q. 22 Number of compounds with one lone pair of electrons on central atom amongst following is_____ $\mathrm{O_3}, \mathrm{H_2O}, \mathrm{SF_4}, \mathrm{ClF_3}, \mathrm{NH_3}, \mathrm{BrF_5}, \mathrm{XeF_4}$

Correct Answer:
4

Solution:


$\mathrm{O_3}, \mathrm{SF_4}, \mathrm{NH_3}$ and $\mathrm{BrF_5}$ contain $1\mathrm{lp}$ only at central atom.

Q. 23 Number of compounds among the following which contain sulphur as heteroatom is _____.

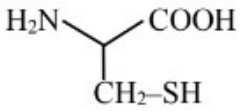
Furan, Thiophene, Pyridine, Pyrrole, Cysteine, Tyrosine

Correct Answer:
2

Solution:



Trioptene



Cystein

Q. 24 The mass of zinc produced by the electrolysis of zinc sulphate solution with a steady current of $0.015\ \mathrm{A}$ for 15 minutes is _____$\times 10^{-4}$ g.

(Atomic mass of zinc = 65.4 amu)

Correct Answer:
46

Solution:
Mass of Zinc

deposited = ZIt

$$= \frac{65.4/2}{96500} \times 0.015 \times 15 \times 60$$

$$= 4.57 \times 10^{-3}$$

$$= 45.7 \times 10^{4}$$

Hence, the answer is (1).

Q. 25 For the reaction $\mathrm{N_2O_{4(g)}} \rightleftarrows 2\mathrm{NO_{2(g)}}, \mathrm{K_p} = 0.492\ \mathrm{atm}$ at $300\ \mathrm{K}$. $\mathrm{K_c}$ for the reaction at same temperature is_____$\times 10^{-2}$

(Given : $\mathrm{R} = 0.082\ \mathrm{L\ atm\ mol^{-1}\ K^{-1}}$)

Correct Answer:
2

Solution:

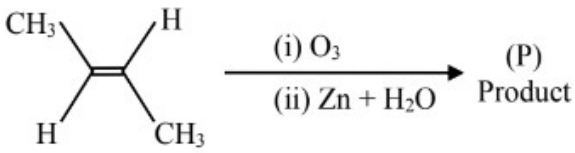
$K_P = K_C(RT)^{\Delta n_g}$

$0.492 = K_C(0.082 \times 300)^1$

$K_C = 2 \times 10^{-2}$

Hence, the answer is (2).

Q. 26

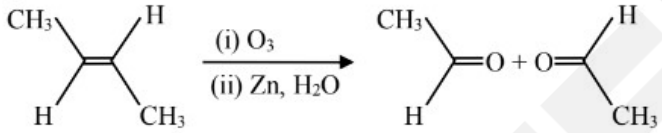


Consider the given reaction. The total number of oxygen atom/s present per molecule of the product (P) is_____.

Correct Answer:

1

Solution:



Q. 27 The number of species from the following which are paramagnetic and with bond order equal to one is_____. $H_2, He_2^+, O_2^+, {N_2}^{2-}, {O_2}^{2-}, F_2, {Ne_2}^+, B_2$

Correct Answer:

1

Solution:

$B_2 = \sigma 1\,s^2 \sigma^* 1\,s^2 \sigma 2\,s^2 \sigma^* 2\,s^2 \pi 2p^1 = \pi 2p^1$

$BO = \frac{6-4}{2} = 1$

Paramagnetic due to the presence of two unpaired electrons.

Hence, the answer is (1).

Q. 28 A solution of H_2SO_4 is $31.4\% H_2SO_4$ by mass and has a density of $1.25\ g/mL$. The molarity of the H_2SO_4 solution is_____ M (nearest integer).

[Given molar mass of $H_2SO_4 = 98\ g\ mol^{-1}$]

Correct Answer:
4

Solution:

$$M = \%\frac{W}{W} \times d \times \frac{10}{M_{solute}}$$

$$= 31.4 \times 1.25 \times \frac{10}{98}$$

$$= 4M$$

Hence, the answer is (4).

Q. 29 For a reaction taking place in three steps at same temperature, overall rate constant $K = \frac{K_1 K_2}{K_3}$. If Ea_1, Ea_2 and Ea_3 are 40,50 and $60\ kJ/mol$ respectively, the overall Ea is_____ kJ/mol.

Correct Answer:
30

Solution:

$$K = \frac{K_1 K_2}{K_3}$$

$$= \frac{A_1 e^{-Ea_1/RT} A_2 e^{-Ea_2/RT}}{A_3 e^{-Ea_3/RT}}$$

$$= \frac{A_1 A_2}{A_3} \cdot e^{-1/RT(Ea_1+Ea_2-Ea_3)}$$

overall $Ea = Ea_1 + Ea_2 - Ea_3$

$$= 40 + 50 - 60$$

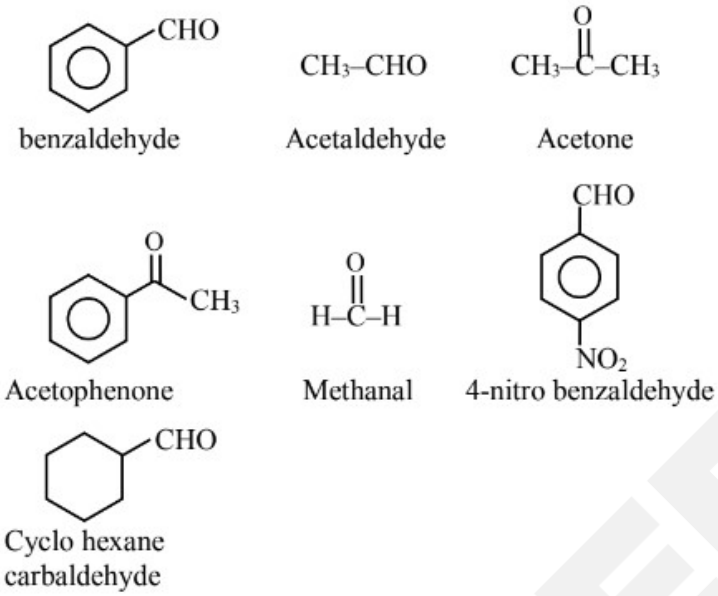
$$= 30 KJ/mol$$

Hence, the answer is (30).

Q. 30 From the compounds given below, number of compounds which give positive Fehling's test is _____.
Benzaldehyde, Acetaldehyde, Acetone, Acetophenone, Methanal, 4-nitrobenzaldehyde, cyclohexane carbaldehyde.

Correct Answer:
3

Solution:



→ Aliphatic aldehydes give positive fehling test. So acetaldehyde, methanol \& cyclohexane carbaldehyde give positive fehling test.

Maths

Q. 1 A fair die is thrown until 2 appears. Then the probability, that 2 appears in even number of throws, is

Option 1:
$\frac{6}{11}$

Option 2:
$\frac{1}{6}$

Option 3:
$\frac{5}{11}$

Option 4:
$\frac{5}{6}$

Correct Answer:
$\frac{5}{11}$

Solution:
A = 2 appears on dice

$$P(A) = \frac{1}{6}$$
$$P(\overline{A}) = \frac{5}{6}$$
$$P(\overline{A}A + \overline{AAAA}A + \overline{AAAAAA}A + \cdots)$$
$$= \frac{5}{6} \times \frac{1}{6} + \left(\frac{5}{6}\right)^3 \frac{1}{6} + \left(\frac{5}{6}\right)^5 \frac{1}{6} + \cdots$$
$$= \frac{5/36}{1 - \frac{25}{36}} = \frac{5}{11}$$

Hence, the answer is the option (3).

Q. 2 If in a G.P. of 64 terms, the sum of all the terms is 7 times the sum of the odd terms of the G.P, then the common ratio of the G.P. is equal to

Option 1:
4

Option 2:
7

Option 3:
6

Option 4:
5

Correct Answer:
6

Solution:

$$S_{64} = 7\left(a + ar^2 + ar^4 + \cdots + ar^{62}\right)$$

$$\Rightarrow \quad \frac{\mathrm{a}\left(\mathrm{r}^{64}-1\right)}{\mathrm{r}-1} = \frac{7\cdot \mathrm{a}\left(\left(\mathrm{r}^2\right)^{32}-1\right)}{\mathrm{r}^2-1}$$

$$\Rightarrow \quad \frac{1}{r-1} = \frac{7}{(r-1)(r+1)} \Rightarrow r+1=7$$

$\mathrm{r} = 6$

Hence, the answer is the option (3).

Q. 3 If $f(x) = \begin{cases} 2+2x, & -1 \le x < 0 \\ 1-\frac{x}{3}, & 0 \le x \le 3 \end{cases}; g(x) = \begin{cases} -x, & -3 \le x \le 0 \\ x, & 0 < x \le 1 \end{cases}$, then range of $(f \circ g)(x)$ is

Option 1:
$[0, 3)$

Option 2:
$[0, 1]$

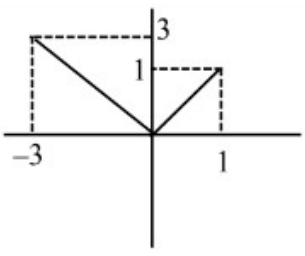
Option 3:
$[0, 1)$

Option 4:
$(0, 1]$

Correct Answer:
$[0, 1]$

Solution:

$$g(x) = \begin{cases} -x & , -3 \le x \le 0 \\ x & , 0 < x \le 1 \end{cases}$$



$$\text{fog}\,(x)=\begin{cases}2+2g(x), & -1\le g(x)<0\\ 1-\frac{g(x)}{3}, & 0\le g(x)\le 3\end{cases}$$

$$=\begin{cases}1-\frac{(-x)}{3}; & -3\le x\le 0\\ 1-\frac{x}{3}; & 0<x\le 1\end{cases}=\begin{cases}1+\frac{x}{3} & -3\le x\le 0\\ 1-\frac{x}{3} & ;0<x\le 1\end{cases}$$

range of $g(x)=[0,1]$

Q. 4 In an A.P., the sixth term $a_6=2$. If the product $a_1a_4a_5$ is the greatest, then the common difference of the A.P. is equal to

Option 1:
$\frac{2}{3}$

Option 2:
$\frac{8}{5}$

Option 3:
$\frac{5}{8}$

Option 4:
$\frac{3}{2}$

Correct Answer:
$\frac{8}{5}$

Solution:
$a_6=2$

$P=a_1a_4a_5$

$=(a_6-5d)\,(a_6-2d)\,(a_6-d)$

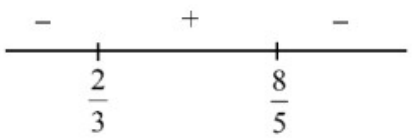
$=(2-5d)(2-2d)(2-d)$

$=2\left(2+5d^2-7d\right)(2-d)$

$= 2\left[4 + 10d^2 - 14d - 2d - 5d^3 + 7d^2\right]$

$= 2\left[-5d^3 + 17d^2 - 16d + 4\right]$

$\frac{dP}{dd} = 2\left[-15d^2 + 34d - 16\right]$



at $\mathrm{d} = \frac{8}{5}\mathrm{P}$ is maximum

Q. 5 Consider the function $\mathrm{f}: \left[\frac{1}{2}, 1\right] \rightarrow \mathbb{R}$ defined by $\mathrm{f}(\mathrm{x}) = 4\sqrt{2}\mathrm{x}^3 - 3\sqrt{2}\mathrm{x} - 1$.

Consider the statements.

(I) The curve $y = f(x)$ intersects the x-axis exactly at one point.
(II) The curve $y = f(x)$ intersects the x-axis at $x = \cos\frac{\pi}{12}$.

Then

Option 1:
Both (I) and (II) are correct.

Option 2:
Both (I) and (II) are incorrect.

Option 3:
only (I) is correct

Option 4:
Only (II) is correct.

Correct Answer:
Both (I) and (II) are correct.

Solution:

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$f(x) = 4\sqrt{2}x^3 - 3\sqrt{2}x - 1$

$f'(x) = 12\sqrt{2}x^2 - 3\sqrt{2}$

$f'(x) \leq 0 \text{ for } x \in \left[\frac{1}{2}, 1\right] \Rightarrow f(x) \downarrow$

$f(1) = \sqrt{2} > 0$

$f\left(\frac{1}{2}\right) = \frac{1}{\sqrt{2}} - \frac{3}{\sqrt{2}} - 1$

$= -\sqrt{2} - 1 < 0$

y = f(x) intersects x-axis exactly at one point
Statement I is correct

Let $\cos\alpha = x$

$\sqrt{2}\cos 3\alpha - 1 = 0$

$\cos 3\alpha = \frac{1}{\sqrt{2}}$

$3\alpha = \frac{\pi}{4} \Rightarrow \alpha = \frac{\pi}{12}$

Statement II is correct

Hence, the answer is the option (1).

Q. 6 Let A be a square matrix such that $AA^T = I$. Then $\frac{1}{2} A\left[\left(A + A^T\right)^2 + \left(A - A^T\right)^2\right]$ is equal to

Option 1:
$A^2 + I$

Option 2:
$A^2 + A^T$

Option 3:
$A^3 + I$

Option 4:
$A^3 + A^T$

Correct Answer:
$A^3 + A^T$

Solution:
$\frac{1}{2} A\left[A^2 + (A^T)^2 + 2AA + A^2 + (A^T)^2 - 2AA\right.$

$= A\left[A^2 + (A^T)^2\right]$

$= A^3 + (AA$

$= A^3 + A^T$

Hence, the answer is the option (4).

Q. 7 Let R be a relation on $Z \times Z$ defined by $(a, b)R(c, d)$ if and only if $ad - bc$ is divisible by 5 .The R is

Option 1:
Reflexive and transitive but not symmetric

Option 2:
Reflexive, symmetric and transitive

Option 3:
Reflexive and symmetric but not transitive

Option 4:
Reflexive but neither symmetric nor transitive

Correct Answer:
Reflexive and symmetric but not transitive

Solution:
$(a, b)R(c, d) \Rightarrow ad - bc = 5\lambda$

$\Rightarrow \quad (bc - ad) = -5\lambda$

$\Rightarrow \quad cb - da = -5\lambda$

$\Rightarrow \quad (c, d)R(a, b) \rightarrow$ Symmetric

$(a, b)R(a, b) \Rightarrow ab - ab = 0$

$\Rightarrow$ Reflexive

let $(a,b)=(6,1),(c,d)=(0,0),(e,f)=1,7$

$ad-bc=0$ and $cf-de=0$

but af-be $=42-1=41$ is

Not multiple of 5
R not transitive

Hence, the answer is the option (3).

Q. 8 Suppose $f(x)=\frac{(2^x+2^{-x})\tan x\sqrt{\tan^{-1}(x^2-x+1)}}{(7x^2+3x+1)^3}$. Then the value of $f'(0)$ is equal to

Option 1:
π

Option 2:
$\frac{\pi}{2}$

Option 3:
0

Option 4:
$\sqrt{\pi}$

Correct Answer:
$\sqrt{\pi}$

Solution:

$$\text{Where } g(x)=\frac{(2^x+2^{-x})\sqrt{\tan^{-1}(x^2-x+1)}}{(7x^2+3x+1)^2}$$

$$f'(x)=g'(x)\tan x+g(x)\sec^2 x$$

$$f'(0)=g'(0)\times 0+g(0)\times 1$$

$$=g(0)=\frac{2\times\sqrt{\pi/4}}{1}=\sqrt{\pi}$$

Hence, the answer is the option (4).

Q. 9 For $x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$, If $y(x)=\int\frac{\operatorname{cosec}x+\sin x}{\operatorname{cosec}x\sec x+\tan x\sin^2 x}dx$, and $\lim_{x\to\left(\frac{\pi}{2}\right)^-}y(x)=0$ then $y\left(\frac{\pi}{4}\right)$ is equal to

Option 1:

$-\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$

Option 2:

$\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$

Option 3:

$\frac{1}{\sqrt{2}}\tan^{-1}\left(-\frac{1}{2}\right)$

Option 4:

$\frac{1}{2}\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$

Correct Answer:

$\frac{1}{\sqrt{2}}\tan^{-1}\left(-\frac{1}{2}\right)$

Solution:

$y(x)=\int\frac{\left(1+\sin^2 x\right)\cos x}{1+\sin^4 x}dx$

Put $\sin x = t$

$$=\int\frac{1+t^2}{t^4+1}dt=\int\frac{\frac{1}{t^2}+1}{\left(t-\frac{1}{t}\right)^2+(\sqrt{2})^2}dt$$

$$=\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{t-\frac{1}{t}}{\sqrt{2}}\right)+c$$

$$=\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{t-\frac{1}{t}}{\sqrt{2}}\right)+c$$

$x=\frac{\pi}{2}, t=1 \quad \Rightarrow c=0$

$$y\left(\frac{\pi}{4}\right)=\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{-1}{2}\right)$$

Hence, the answer is the option (3).

Q. 10 A function $y=f(x)$ satisfies $f(x)\sin 2x+\sin x-\left(1+\cos^2 x\right)f'(x)=0$ with condition $f(0)=0$. Then, $f\left(\frac{\pi}{2}\right)$ is equal to

Option 1:
1

Option 2:
2

Option 3:
0

Option 4:
-1

Correct Answer:
1

Solution:

$$y\sin 2x+\sin x=\left(1+\cos^2 x\right)\frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx}-\left(\frac{\sin 2x}{1+\cos^2 x}\right)y=\frac{\sin x}{1+\cos^2 x}$$

Linear D.E.

$$\text{I.F.}=\text{e}^{\left(\int \frac{-\sin 2x}{1+\cos^2 x}dx\right)}=1+\cos^2 x$$

$$\text{I.F.}=\text{e}^{\left(\int \frac{-\sin 2x}{1+\cos^2 x}dx\right.}=1+\cos^2 x$$

$$y\left(1+\cos^2 x\right)=\int \sin x dx$$

$$\Rightarrow y\left(1+\cos^2 x\right)=-\cos x+c$$

$$f(0)=0 \quad \Rightarrow c=1$$

$$\text{put } x=\frac{\pi}{2}$$

$$y(1+0)=0+1 \quad \Rightarrow y=1$$

Hence, the answer is the option (1).

Q. 11 In a $\triangle ABC$, suppose $y=x$ is the equation of the bisector of the angle B and the equation of the side AC is $2\,\text{x}-\text{y}=2$. If $2\text{AB}=\text{BC}$ and the points A and B are respectively $(4,6)$ and (α,β), then $\alpha+2\beta$ is equal to

Option 1:
42

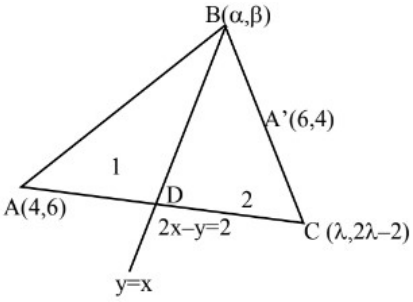
Option 2:
45

Option 3:
39

Option 4:
48

Correct Answer:
42

Solution:



$$\frac{AB}{BC}=\frac{AD}{DC}=\frac{1}{2}$$

$$D\left(\frac{\lambda+8}{3},\frac{2\lambda-2+12}{3}\right) \text{ lies on } y=x$$

$$\frac{\lambda+8}{3}=\frac{2\lambda+10}{3}$$

$$\Rightarrow \lambda=-2 \quad \Rightarrow c(-2,-6)$$

Image of A w.r.t y = x lies on BC
m(BC) = m (A'C)

$$\Rightarrow \frac{\alpha+6}{\alpha+2}=\frac{4+6}{6+2}\Rightarrow \alpha=14=\beta$$

$$\alpha=2\beta=42$$

Q. 12 If $\alpha, -\frac{\pi}{2}<\alpha<\frac{\pi}{2}$ is the solution of $4\cos\theta+5\sin\theta=1$, then the value of $\tan\alpha$ is

Option 1:
$\frac{\sqrt{10}-10}{6}$

Option 2:
$\frac{\sqrt{10}-10}{12}$

Option 3:
$\frac{10-\sqrt{10}}{12}$

Option 4:
$\frac{10-\sqrt{10}}{6}$

Correct Answer:
$\frac{\sqrt{10}-10}{12}$

Solution:
$4+5\tan\theta=\sec\theta$

Squaring : $24\tan^2\theta+40\tan\theta+15=0$

$\tan\theta=\frac{-10\pm\sqrt{10}}{12}$

and $\tan\theta=-\left(\frac{10+\sqrt{10}}{12}\right)$ is Rejected

Hence, the answer is the option (2).

Q. 13 $\lim_{x\to\frac{\pi}{2}}\left(\frac{1}{\left(x-\frac{\pi}{2}\right)^2}\int_{x^3}^{\left(\frac{\pi}{2}\right)^3}\cos\left(t^{\frac{1}{3}}\right)dt\right)$ is equal to

Option 1:
$\frac{3\pi}{8}$

Option 2:
$\frac{3\pi^2}{4}$

Option 3:
$\frac{3\pi}{4}$

Option 4:
$\frac{3\pi^2}{8}$

Correct Answer:
$\frac{3\pi^2}{8}$

Solution:

$$\lim_{x\to\frac{\pi}{2}} \frac{\int_{x^3}^{\left(\frac{\pi}{2}\right)^3} \cos\left(t^{\frac{1}{3}}\right)}{\left(x-\frac{p}{2}\right)^2} = \frac{0}{0} \text{ form}$$

Apply leibniz

$\lim_{x\to\frac{\pi}{2}} \frac{\cos\left(\frac{\pi}{2}\right)\times 0-\cos(x)\cdot 3x^2}{2\left(x-\frac{\pi}{2}\right)}$

$x \to \frac{\pi}{2}+t \qquad \Rightarrow t\to 0$

$$\lim_{t\to 0} \frac{-\cos\left(\frac{\pi}{2}+t\right)3\left(\frac{\pi}{2}\right)^2}{2t}$$

$$\lim_{t\to 0} \frac{3\pi^2 \sin t}{8t} = \frac{3\pi^2}{8}$$

Q. 14 Let $\left(5, \frac{a}{4}\right)$ be the circumcenter of a triangle with vertices $A(a,-2), B(a,6)$ and $C\left(\frac{a}{4},-2\right)$. Let α denote the circumradius, β denote the area and γ denote the perimeter of the triangle. Then $\alpha+\beta+\gamma$ is

Option 1:
30

Option 2:
53

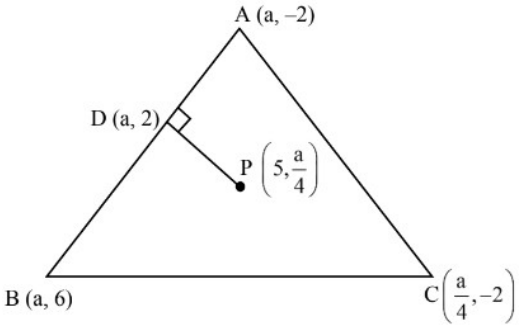
Option 3:
62

Option 4:
60

Correct Answer:
53

Solution:
D is mid point of AB
AB is perpendicular to PD

$2=\frac{a}{4} \Rightarrow a=8$



A (8, –2), B (8, 6), C (2,–2), P(5,2)

$\alpha = \mathrm{AP} = \sqrt{9+16} = 5$

$\mathrm{AB} = 8, \mathrm{BC} = 10, \mathrm{AC} = 6$

$\text{Area } = \beta = \frac{1}{2}(8)(6) = 24$

$\text{Perimeter } = \lambda = 24$

$\alpha + \beta + \lambda = 53$

Q. 15 Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \alpha & \beta \\ 0 & \beta & \alpha \end{bmatrix}$ and $|2A|^{\beta} = 2^{21}$ where $\alpha, \beta \in Z$, Then a value of α is

Option 1:
5

Option 2:
9

Option 3:
3

Option 4:
17

Correct Answer:
5

Solution:

$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \alpha & \beta \\ 0 & \beta & \alpha \end{bmatrix}$

$|\mathrm{A}| = \alpha^2 - \beta^2$
$|2\ \mathrm{A}|^3 = 2^{21}$

$\Rightarrow 2^9 |\ \mathrm{A}|^3 = 2^{21}$
$\Rightarrow |\mathrm{A}|^3 = 2^{12}$
$|\ \mathrm{A}| = 2^4 \quad \Rightarrow \quad \alpha^2 - \beta^2 = 16$

$(\alpha, \beta) = (5, 3)$
$\alpha = 5$

Hence, the answer is the option (1).

Q. 16 Let PQR be a triangle with $R(-1, 4, 2)$. Suppose $M(2, 1, 2)$ is the mid point of PQ. The distance of the centroid of $\triangle \mathrm{PQR}$ from the point of intersection of the lines $\frac{x-2}{0} = \frac{y}{2} = \frac{z+3}{-1}$ and $\frac{x-1}{1} = \frac{y+3}{-3} = \frac{z+1}{1}$ is

Option 1:
$\sqrt{99}$

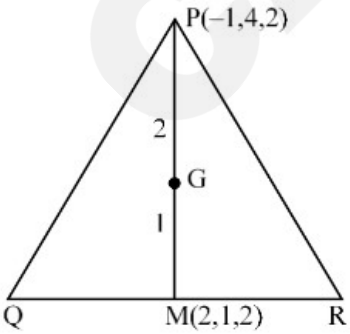
Option 2:
9

Option 3:
$\sqrt{69}$

Option 4:
69

Correct Answer:
$\sqrt{69}$

Solution:



$$G\left(\frac{4-1}{3}, \frac{2+4}{3}, \frac{4+2}{3}\right) \equiv G(1,2,2)$$

$$\frac{x-2}{0} = \frac{y}{2} = \frac{z+3}{-1} = \lambda \Rightarrow (2, 2\lambda, -\lambda - 3$$

$$\frac{x-1}{1} = \frac{y+3}{-3} = \frac{z+1}{1} = \mu \Rightarrow (\mu+1, -3\mu-3, \mu-1)$$

$\mu + 1 = 2 \quad \Rightarrow \mu = 1$

Point of interaction $P(2, -6, 0)$

$PG = \sqrt{1+64+4} = \sqrt{69}$

Q. 17 Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three non-zero vectors such that $\vec{b}$ and $\vec{c}$ are non-collinear. If $\vec{a} + 5\vec{b}$ is collinear with $\vec{c}, \vec{b} + 6\vec{c}$ is collinear with $\vec{a}$ and $\vec{a} + \alpha\vec{b} + \beta\vec{c} = \vec{0}$, then $\alpha + \beta$ is equal to

Option 1:
-25

Option 2:
35

Option 3:
-30

Option 4:
30

Correct Answer:
35

Solution:

$\vec{a} + 5\vec{b} = \lambda\vec{c} \qquad \ldots(i)$

$\vec{b} + 6\vec{c} = \mu\vec{a} \qquad \ldots(ii)$

(i) –5 x (ii)

$\Rightarrow \vec{a} - 30\vec{c} = \lambda\vec{c} - 5\mu\vec{a}$

$\lambda = -30, \mu = \frac{-1}{5}$

From (i)

$\vec{a}+5\vec{b}+30\vec{c}=0 \Rightarrow \alpha+\beta=35$

Hence, the answer is the option (2).

Q. 18 If $z=\frac{1}{2}-2i$ is such that $|z+1|=\alpha z+\beta(1+i), i=\sqrt{-1}$ and $\alpha, \beta \in \mathbb{R}$, then $\alpha+\beta$ is equal to

Option 1:
-1

Option 2:
-4

Option 3:
2

Option 4:
3

Correct Answer:
3

Solution:

$Z+1=\frac{3}{2}-2i$

$|Z+1|=\alpha z+\beta(i+1)$

$\Rightarrow \frac{5}{2}=\alpha\left(\frac{1}{2}-2i\right)+\beta(i+1)$

$\frac{\alpha}{2}+\beta=\frac{5}{2} \quad \ldots(i)$

$-2\alpha+\beta=0 \quad \ldots(ii)$

(i) – (ii)

$\frac{5\alpha}{2}=\frac{5}{2} \Rightarrow \alpha=1, \beta=2$

$\alpha+\beta=3$

Hence, the answer is the option (4).

Q. 19 Let O be the origin and the position vectors of A and B be $2\hat{i}+2\hat{j}+\hat{k}$ and $2\hat{i}+4\hat{j}+4\hat{k}$ respectively. If the internal bisector of $\angle AOB$ meets the line AB at C, then the length of OC is

Option 1:
$\frac{3}{2}\sqrt{34}$

Option 2:
$\frac{3}{2}\sqrt{31}$

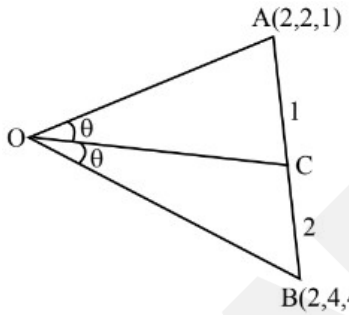
Option 3:
$\frac{2}{3}\sqrt{34}$

Option 4:
$\frac{2}{3}\sqrt{31}$

Correct Answer:
$\frac{2}{3}\sqrt{34}$

Solution:

$|\overrightarrow{OA}| = \sqrt{4+4+1} = 3$



$|\overrightarrow{OB}| = \sqrt{4+16+16} = 6$

$AC : CB = 3 : 6 = 1 : 2$

$c\left(\frac{4+2}{3}, \frac{4+4}{3}, \frac{2+4}{3}\right)$

$c\left(2, \frac{8}{3}, 2\right)$

$|\overrightarrow{OC}| = \sqrt{4+\frac{64}{9}+4} = \frac{\sqrt{136}}{3} = \frac{2}{3}\sqrt{34}$

Q. 20 If the value of the integral $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left(\frac{x^2\cos x}{1+\pi^x}+\frac{1+\sin^2 x}{1+e^{\sin x^{2023}}}\right)dx=\frac{\pi}{4}(\pi+a)-2$, then the value of a is

Option 1:
2

Option 2:
$-\frac{3}{2}$

Option 3:
$\frac{3}{2}$

Option 4:
3

Correct Answer:
3

Solution:
$I=\int_{-\pi/2}^{\pi/2}\left(\frac{x^2\cos x}{1+\pi^x}+\frac{1+\sin^2 x}{1+e^{\sin x^{2023}}}\right)dx$

Apply King's property

$I=\int_{-\pi/2}^{\pi/2}\left(\frac{x^2\cos x}{1+\pi^{-x}}+\frac{1+\sin^2 x}{1+e^{\sin(-x)^{2023}}}\right)dx$

On adding, we get

$2I=\int_{-\pi/2}^{\pi/2}\left(x^2\cos x+1+\sin^2 x\right)dx$

On solving

$I=\frac{\pi^2}{4}+\frac{3\pi}{4}-2$

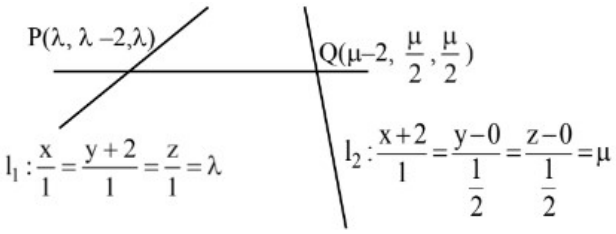
$a=3$

Hence, the answer is the option (4).

Q. 21 A line with direction ratios 2, 1,2 meets the lines $x=y+2=z$ and $x+2=2y=2z$ respectively at the points P and Q. If the length of the perpendicular from the point $(1,2,12)$ to the line PQ is l, then l^2 is_____.

Correct Answer:
65

Solution:



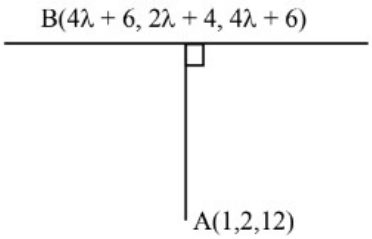
$\frac{\mu-2-\lambda}{2} = \frac{\frac{\mu}{2}-\lambda+2}{1} = \frac{\frac{\mu}{2}-\lambda}{2}$

$\Rightarrow \mu - 2 - \lambda = \mu - 2\lambda + 4$ and $\mu - 2\lambda + 4 = \frac{\mu}{2} - \lambda$

$\lambda = 6 \qquad \frac{\mu}{2} = \lambda - 4$

$\Rightarrow \quad \mu = 4$

P(6, 4, 6) , Q (2, 2, 2)



$\frac{x-6}{4} = \frac{y-4}{2} = \frac{z-6}{4} = \lambda$

$\mathrm{AB} \perp \mathrm{PQ}$

$\langle 4\lambda + 5, 2\lambda + 2, 4\lambda - 6\rangle \cdot \langle 4, 2, 4\rangle = 0$

$\Rightarrow 36\lambda = 0$

$\lambda = 0$

B (6, 4, 6)

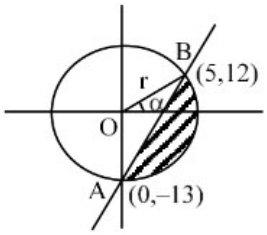
$\mathrm{AB} = \sqrt{25 + 4 + 36} = \sqrt{65} = 1$

$l^2 = 65$

Q. 22 The area (in sq. units) of the part of the circle $x^2 + y^2 = 169$ which is below the line $5x - y = 13$ is $\frac{\pi\alpha}{2\beta} - \frac{65}{2} + \frac{\alpha}{\beta}\sin^{-1}\left(\frac{12}{13}\right)$, where α, β are coprime numbers. Then $\alpha + \beta$ is equal to_____.

Correct Answer:
171

Solution:



$\mathrm{r}\sin\alpha = 12$

$\mathrm{r}\cos\alpha = 5$

$\tan\alpha = \frac{12}{5}$

Shaded area $= \text{area}(\text{arc } AOB) - \text{area}(\triangle \text{AOB})$

$$= \frac{1}{2}r^2\left(\frac{\pi}{2} + \alpha\right) - \frac{1}{2}r^2\sin\left(\frac{\pi}{2} + \alpha\right)$$

$$= \frac{169\pi}{4} + \frac{169\alpha}{2} - \frac{169}{2}\cos\alpha$$

$$= \frac{169\pi}{4} + \frac{169}{2}\sin^{-1}\left(\frac{12}{13}\right) - \frac{169}{2} \times \frac{5}{13}$$

$$= \frac{169\pi}{4} - \frac{65}{2} + \frac{169}{2}\sin^{-1}\left(\frac{12}{13}\right)$$

$\alpha = 169, \beta = 2$

$\alpha + \beta = 171$

Q. 23 If the solution curve $y = y(x)$ of the differential equation $\left(1 + y^2\right)\left(1 + \log_e x\right)dx + xdy = 0, x > 0$ passes through the point $(1, 1)$ and $y(e) = \frac{\alpha - \tan\left(\frac{3}{2}\right)}{\beta + \tan\left(\frac{3}{2}\right)}$, then $\alpha + 2\beta$ is_____.

Correct Answer:
3

Solution:

$$\int \left(\frac{1}{x}+\frac{\ln x}{x}\right)dx+\int \frac{dy}{1+y^2}=0$$

$$\ln x+\frac{(\ln x)^2}{2}+\tan^{-1}y=C$$

$$\text{Put } x=1, y=1$$

$$\therefore C=\frac{\pi}{4}$$

$$\Rightarrow \ln x+\frac{(\ln x)^2}{2}+\tan^{-1}y=\frac{\pi}{4}$$

Put $x = e$

$$\Rightarrow y=\tan\left(\frac{\pi}{4}-\frac{3}{2}\right)=\frac{1-\tan\frac{3}{2}}{1+\tan\frac{3}{2}}$$

$$\therefore \alpha=1, \beta=1$$

$$\Rightarrow \alpha+2\beta=3$$

Hence, the answer is (3).

Q. 24 If the mean and variance of the data $65, 68, 58, 44, 48, 45, 60, \alpha, \beta, 60$ where $\alpha > \beta$, are 56 and 66.2 respectively, then $\alpha^2+\beta^2$ is equal to

---

Correct Answer:
6344

Solution:

$$\sigma^2=66.2$$

$$\Rightarrow 66.2=\frac{\sum xi^2}{10}-(56)^2$$

$$\Rightarrow 25678+\alpha^2+\beta^2=32022$$

$$\Rightarrow \alpha^2+\beta^2=6344$$

Hence, the answer is (6344).

Q. 25 If $\frac{{}^{11}C_1}{2}+\frac{{}^{11}C_2}{3}+\ldots+\frac{{}^{11}C_9}{10}=\frac{n}{m}$ with $\gcd(n,m)=1$, then $n+m$ is equal to_____.

Correct Answer:
2041

Solution:
$(1+x)^{11}={}^{11}C_0+11C_1x+{}^{11}C_2X^2+\cdots+{}^{11}C_{11}x^{11}$

$$\int_0^1(1+x)^{11}dx=\int_0^1({}^{11}c_0+{}^{11}c_1x+{}^{11}cX^2\ldots+{}^{11}c_{11}x^{11})dx$$

$$\frac{2^{12}}{12}-\frac{1}{12}={}^{11}c_0+\frac{{}^{11}c_1}{2}+\frac{{}^{11}c_2}{3}+\cdots+\frac{{}^{11}c_9}{10}$$

$$+\frac{{}^{11}c_{10}}{11}+\frac{{}^{11}c_{11}}{12}$$

$$\frac{2^{12}}{12}-\frac{1}{12}=1+1+\frac{1}{12}+S$$

$$\Rightarrow S=\frac{2^{12}}{12}-\frac{2}{12}-2$$

$$=\frac{2^{12}-26}{12}=\frac{2^{11}-13}{6}=\frac{n}{m}$$

n+m=2048-13+6=2041

Hence, the answer is (2041).

Q. 26 If the points of intersection of two distinct conics $x^2+y^2=4b$ and $\frac{x^2}{16}+\frac{y^2}{b^2}=1$ lie on the curve $y^2=3x^2$, then $3\sqrt{3}$ times the area of the rectangle formed by the intersection points is_____.

Correct Answer:
432

Solution:
Putting $y^2=2x^2$ in both the conics

We get $x^2=b$ and $\frac{b}{16}+\frac{3}{b}=1$

$\Rightarrow b=4,12$ ($b=4$ is rejected because curves coincide)

$\therefore \quad b = 12$

Hence points of intersection are

$(\pm\sqrt{12}, \pm 6) \quad \Rightarrow \text{ area of rectangle } = 12 \times 2\sqrt{12} = 48\sqrt{3}$

$\text{answer } = (3\sqrt{3})(48\sqrt{3}) = 432$

Hence, the answer is (432).

Q. 27 Let α, β be the roots of the equation $x^2 - x + 2 = 0$ with $\text{Im}(\alpha) > \text{Im}(\beta)$. Then $\alpha^6 + \alpha^4 + \beta^4 - 5\alpha^2$ is equal to

Correct Answer:
13

Solution:
$\alpha^6 + \alpha^4 + \beta^4 - 5\alpha^2$

$= \alpha^4(\alpha - 2) + \alpha^4 - 5\alpha^2 + (\beta - 2)^2$

$= \alpha^5 - \alpha^4 - 5\alpha^2 + \beta^2 - 4\beta + 4$

$= \alpha^3(\alpha - 2) - \alpha^4 - 5\alpha^2 + \beta - 2 - 4\beta + 4$

$= -2\alpha^3 - 5\alpha^2 - 3\beta + 2$

$= -2\alpha(\alpha - 2) - 5\alpha^2 - 3\beta + 2$

$= -7\alpha^2 + 4\alpha - 3\beta + 2$

$= -7(\alpha - 2) + 4\alpha - 3\beta + 2$

$= -3\alpha - 3\beta + 16 = -3(1) + 16 = 13$

Hence, the answer is (13).

Q. 28 Equations of two diameters of a circle are $2x - 3y = 5$ and $3x - 4y = 7$. The line joining the points $\left(-\frac{22}{7}, -4\right)$ and $\left(-\frac{1}{7}, 3\right)$ intersects the circle at only one point $\text{P}(\alpha, \beta)$. Then, $17\beta - \alpha$ is equal to_____.

Correct Answer:
2

Solution:
$2x - 3y = 5 \quad .....(i)$

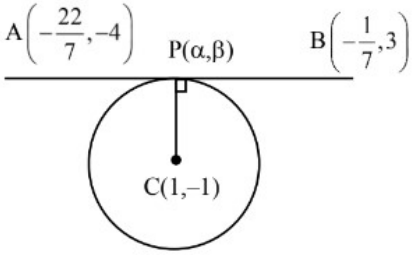
$3x - 4y = 7 \quad .....(ii)$

(i) × 3 – (ii) × 2

$\Rightarrow -y = 1 \quad \Rightarrow y = -1$

x=1

(1,-1) is centre of triangle



Equation AB:7x-3y+10=0 ...(i)

Equation CP:3x+7y+4=0 ...(ii)

Solving (i) and (ii)

$\alpha = \frac{-41}{29}, \beta = \frac{1}{29}$

$17\beta - \alpha = \frac{17 + 41}{29} = 2$

Q. 29 All the letters of the word "GTWENTY" are written in all possible ways with or without meaning and these words are written as in a dictionary. The serial number of the word "GTWENTY" is

Correct Answer:
553

Solution:

$E\ldots\ldots = \frac{6!}{2!} = 360$

$GE\ldots\ldots = \frac{5!}{2!} = 60$

$GN.... = \frac{5!}{2!} = 60$

$GTE.... = 4! = 24$

$GTN... = 4! = 24$

$GTT\ldots = 4! = 24$

GTWENTY =1

total $360 + 60 + 60 + 24 + 24 + 24 + 1 = 553$

Hence, the answer is (553).

Q. 30 Let $f(x) = 2^x - x^2, x \in \mathbb{R}$. If m and n are respectively the number of points at which the curves $y = f(x)$ and $y = f'(x)$ intersect the x-axis, then the value of $m + n$ is_____.

Correct Answer:
5

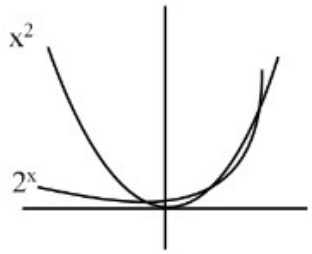
Solution:
$f(x) = 2^x - x^2$

$f(x) = 0 \quad \Rightarrow 2^x = x^2$

$m = 3$

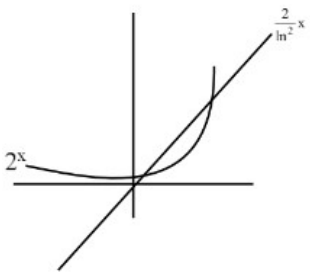
$f'(x) = 2^x \ln 2 - 2x = 0$

$2^x = \frac{2}{\ln 2}x$



n = 2

m + n = 5



29 Jan - Evening Shift

Physics

Q. 1 If the distance between object and its two times magnified virtual image produced by a curved mirror is $15\ \text{cm}$, the focal length of the mirror must be:

Option 1:
10 / 3 cm

Option 2:
–12 cm

Option 3:
–10 cm

Option 4:
15 cm

Correct Answer:
–10 cm

Solution:

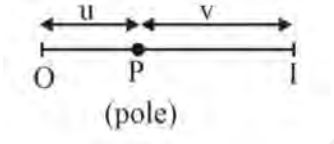


Image is virtual. So $m = -\frac{v}{u}$ is positive.

$\Rightarrow m = +2$

$\Rightarrow -\frac{v}{-u} = 2 \Rightarrow v = 2u$

Also distance between object and image is 15cm.

i.e., $3u = 15\ \text{cm} \Rightarrow u = 5\ \text{cm}$

$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$\Rightarrow \frac{1}{10} + \frac{1}{-5} = \frac{1}{f}$

$\Rightarrow \frac{1}{f} = -\frac{1}{10}$

$\Rightarrow f = -10\ \text{cm}$

Q. 2 If the distance between object and its two times magnified virtual image produced by a curved mirror is $15\ \mathrm{cm}$, the focal length of the mirror must be:

Option 1:
$\left(\frac{\mathrm{R}_1}{\mathrm{R}_2}\right)$

Option 2:
$\left(\frac{\mathrm{R}_2}{\mathrm{R}_1}\right)$

Option 3:
$\left(\frac{\mathrm{R}_2}{\mathrm{R}_1}\right)^2$

Option 4:
$\left(\frac{\mathrm{R}_1}{\mathrm{R}_2}\right)^2$

Correct Answer:
$\left(\frac{\mathrm{R}_1}{\mathrm{R}_2}\right)^2$

Solution:
The radius of curvature in the charged magnetic field is given by -

$$R = \frac{mv}{qB} = \frac{\sqrt{2mKE}}{qB}$$
$$R \propto \sqrt{m}$$
$$\therefore \frac{R_1}{R_2} = \sqrt{\frac{m_1}{m_2}} \Rightarrow \frac{m_1}{m_2} = \frac{R_1^2}{R_2^2}$$

Hence, the given answer is the option(4).

Q. 3 The temperature of a gas having 2.0×10^{25} molecules per cubic meter at $1.38\ \mathrm{atm}$ (Given, $\mathrm{k} = 1.38 \times 10^{-23}\mathrm{JK}^{-1}$) is :

Option 1:
300 K

Option 2:
500 K

Option 3:
100 K

Option 4:
200 K

Correct Answer:
500 K

Solution:
With the ideal gas relationship,

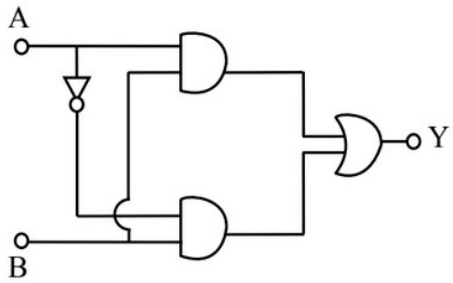
$$PV = nRT$$
$$\mathrm{P} = 1.38\ \mathrm{atm} = 1.38 \times 10^5\ \mathrm{Pa}$$

After putting the given values, we get-

$$\Rightarrow T = \frac{PV}{nR} = \frac{PV}{\frac{N}{N_A}R} = \frac{PV}{NK} = \frac{1.38\times10^5\times1}{2\times10^{25}\times1.38\times10^{-23}} = 500\ \mathrm{K}$$

Hence, the answer is the option (2).

Q. 4 The truth table for this given circuit is:



Option 1:

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

Option 2:

A	B	Y
0	0	1
0	1	0
1	0	1
1	1	0

Option 3:

A	B	Y
0	0	0
0	1	1
1	0	0
1	1	1

Option 4:

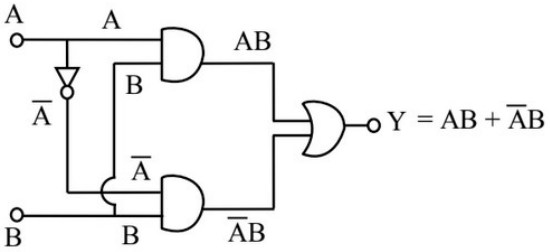
A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

CAREERS360

Correct Answer:

A	B	Y
0	0	0
0	1	1
1	0	0
1	1	1

Solution:



A	B	AB	$\overline{A}B$	Y
0	0	0	0	0
0	1	0	1	1
1	0	0	0	0
1	1	1	0	1

Q. 5 In an a.c. circuit, voltage and current are given by:
$\mathrm{V} = 100\sin(100\mathrm{t})\mathrm{V}$ and

$I = 100\sin\left(100t + \frac{\pi}{3}\right)\mathrm{mA}$ respectively

The average power dissipated in one cycle is:

Option 1:
10 W

Option 2:
2.5 W

Option 3:
25 W

Option 4:
6 W

Correct Answer:
2.5 W

Solution:
$P_{\text{avg}} = V_{rms} \times I_{rms} \times \cos\phi$

$\Rightarrow P_{avg} = \frac{100}{\sqrt{2}} \times \frac{100\times10^{-3}}{\sqrt{2}} \times \frac{1}{2} = 2.5\ \text{W}$

Hence, the answer is the option (2).

Q. 6 A stone of mass $900\ \text{g}$ is tied to a string and moved in a vertical circle of radius $1\ \text{m}$ making 10rpm. The tension in the string, when the stone is at the lowest point is (if $\pi^2 = 9.8$ and $\text{g} = 9.8\ \text{m/s}^2$):

Option 1:
17.8 N

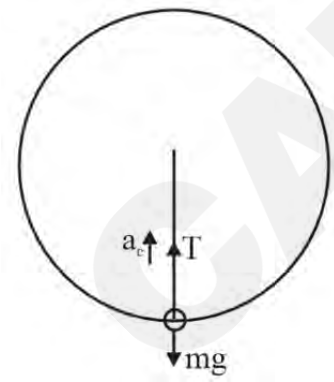
Option 2:
8.82 N

Option 3:
97 N

Option 4:
9.8 N

Correct Answer:
9.8 N

Solution:



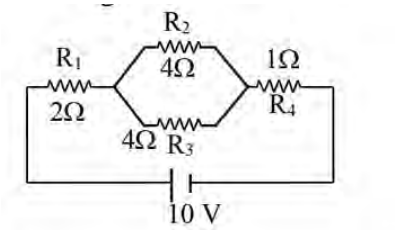
At lowermost point,

$T - mg = m\omega^2 r$

$\Rightarrow T = mg + m\omega^2 r = mg + m \times 4\pi^2 f^2 r = mg\left(1 + 4f^2 r\right)$

$\Rightarrow T = mg\left(1 + 4 \times \frac{100}{3600} \times 1\right) = 0.9 \times 9.8 \times \left(1 + \frac{1}{9}\right) = 9.8\ \text{N}$

Q. 7 In the given circuit, the current in resistance R_3 is:



Option 1:
2.5 A

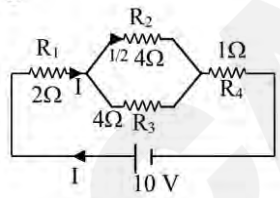
Option 2:
1 A

Option 3:
1.5 A

Option 4:
2 A

Correct Answer:
1 A

Solution:



$\mathrm{R_{eq}} = 5\Omega$
$\mathrm{I} = 2\ \mathrm{A}$
$\mathrm{I_1} = \mathrm{I}/2 = 1\ \mathrm{A}$

Q. 8 A particle is moving in a straight line. The variation of position ' x ' as a function of time ' t ' is given as $x = \left(t^3 - 6t^2 + 20t + 15\right)m$. The velocity of the body, when its acceleration becomes zero, is:

Option 1:
6 m/s

Option 2:
10 m/s

Option 3:
8 m/s

Option 4:
4 m/s

Correct Answer:
8 m/s

Solution:
$v = \frac{dx}{dt} = 3t^2 - 12t + 20$
$a = 6t - 12$

According to qn. We have to find the velocity at the instant when acceleration is zero.
So, a = 0
$\Rightarrow 6t - 12 = 0$
$\Rightarrow t = 2 \text{ sec}$

Now velocity at t=2sec will be $v = \left(3 \times 2^2 - 12 \times 2 + 20\right)\text{m/s} = 8 \text{ m/s}$

Hence, the answer is the option (3).

Q. 9 In Young's double-slit experiment, light from two identical sources is superimposed on a screen. The path difference between the two lights reaching a point on the screen is $7\lambda/4$. The ratio of the intensity of fringe at this point with respect to the maximum intensity of the fringe is :

Option 1:
$\frac{1}{4}$

Option 2:
$\frac{1}{3}$

Option 3:
$\frac{3}{4}$

Option 4:
$\frac{1}{2}$

Correct Answer:
$\frac{1}{2}$

Solution:

$\Delta x = \frac{7\lambda}{4}$

$\therefore \Delta\phi = \frac{2\pi}{\lambda}\Delta x = \frac{7\pi}{2}$

Now intensity at given point is $I = 4I_0\cos^2\frac{\Delta\phi}{2} = 4I_0\cos^2\frac{7\pi}{4} = 2I_0$

$\therefore \frac{I}{4I_0} = \frac{1}{2}$

Hence, the answer is the option (4).

Q. 10 Two sources of light emit with a power of 200 W. The ratio of the number of photons of visible light emitted by each source having wavelengths 300 nm and 500 nm respectively, will be:

Option 1:
3: 5

Option 2:
1: 5

Option 3:
5: 3

Option 4:
1: 3

Correct Answer:
3: 5

Solution:
By using the conditions of photoelectric effect :

$n = \frac{pt}{\frac{hc}{\lambda}} = \frac{pt\lambda}{hc}$

$\frac{n_1}{n_2} = \frac{\lambda_1}{\lambda_2} = \frac{3}{5}$

Hence, the answer is the option (1).

Q. 11 A small liquid drop of radius R is divided into 27 identical liquid drops. If the surface tension is T, then the work done in the process will be:

Option 1:
$4\pi\mathrm{R}^2\,\mathrm{T}$

Option 2:
$8\pi\mathrm{R}^2\,\mathrm{T}$

Option 3:
$3\pi \mathrm{R}^2\ \mathrm{T}$

Option 4:
$\frac{1}{8}\pi \mathrm{R}^2\ \mathrm{T}$

Correct Answer:
$8\pi \mathrm{R}^2\ \mathrm{T}$

Solution:
$V_{\text{initial}} = V_{\text{final}}$

$\Rightarrow \frac{4}{3}\pi R^3 = 27 \times \frac{4}{3}\pi r^3$
$\Rightarrow R^3 = 27r^3$

$\Rightarrow r = \frac{R}{3}$

Now, $\Delta W = E_f - E_i = 4\pi r^2 \times 27 \times T - 4\pi R^2 \times T = 27 \times 4\pi \times \frac{R^2}{9} T - 4\pi R^2 \times T$
$\Delta W = 8\pi R^2 T$

Hence, the answer is the option (2).

Q. 12 The bob of a pendulum was released from a horizontal position. The length of the pendulum is $10\ \mathrm{m}$. If it dissipates 10% of its initial energy against air resistance. the speed with which the bob arrives at the lowest point is:

[Use, $\mathrm{g} : 10\ \mathrm{ms}^{-2}$]

Option 1:
$6\sqrt{5}\ \mathrm{m/s}^{-1}$

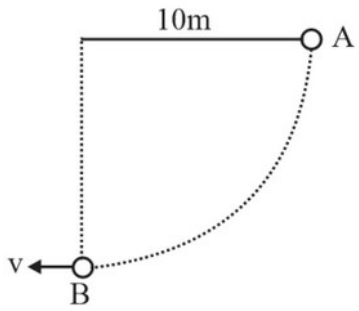
Option 2:
$5\sqrt{5}\ \mathrm{ms}^{-1}$

Option 3:
$2\sqrt{5}\ \mathrm{ms}^{-1}$

Option 4:
$5\sqrt{6}\ \mathrm{ms}^{-1}$

Correct Answer:
$6\sqrt{5}\ \mathrm{m/s}^{-1}$

Solution:



$E_A - \text{loss} = E_B$

$mgl - 0.1mgl = \frac{1}{2}mv^2$

$\Rightarrow 0.9mgl = \frac{1}{2}mv^2$

$\Rightarrow v = \sqrt{0.9 \times 2 \times 10 \times 10}\ \text{m/s}$

$\Rightarrow v = 6\sqrt{5}\ \text{ms}^{-1}$

Q. 13 A physical quantity Q is found to depend on quantities a, b, c by the relation $Q = \frac{a^4b^3}{c^2}$. The percentage error in a, b and c are 3%,4% and 5 % respectively. Then, the percentage error in Q is:

Option 1:
66%

Option 2:
14%

Option 3:
34%

Option 4:
43%

Correct Answer:
34%

Solution:
From log method, $\frac{\Delta Q}{Q} = \frac{4\Delta a}{a} + \frac{3\Delta b}{b} + \frac{2\Delta c}{c}$

$\%$ error in $Q = 4 \times 3 + 3 \times 4 + 2 \times 5 = 34$

Hence, the answer is the option (3).

Q. 14 A plane electromagnetic wave of frequency 35MHz travels in free space along the X-direction. At a particular point (in space and time) $\vec{E} = 9.6\hat{j}\text{V/m}$. The value of the magnetic field at this point is :

Option 1:
$9.6\hat{\mathrm{j}}\mathrm{T}$

Option 2:
$3.2 \times 10^{-8}\hat{\mathrm{i}}\mathrm{T}$

Option 3:
$3.2 \times 10^{-8}\hat{\mathrm{k}}\mathrm{T}$

Option 4:
$9.6 \times 10^{-8}\hat{\mathrm{k}}\mathrm{T}$

Correct Answer:
$3.2 \times 10^{-8}\hat{\mathrm{k}}\mathrm{T}$

Solution:
From Maxwell's equation with the magnitude of the electric and the magnetic fields in an electromagnetic wave are related as -

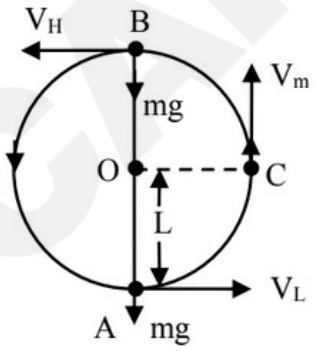
$$\mathrm{E}_0 = \mathrm{CB}_0$$
$$B_0 = \frac{E_0}{C} = \frac{9.6}{3 \times 10^8} = 3.2 \times 10^{-8}$$

The Direction of the velocity of the EM Wave is along $\vec{E} \times \vec{B}$.

So, according to this $\vec{B} = 3.2 \times 10^{-8}\hat{k}T$

Hence, the answer is the option (3).

Q. 15 A bob of mass ' m ' is suspended by a light string of length ' L '. It is imparted a minimum horizontal velocity at the lowest point A such that it just completes half circle reaching the top most position B. The ratio of kinetic energies $\frac{(\mathrm{K.E})_\mathrm{A}}{(\mathrm{K.E})_\mathrm{B}}$ is:



Option 1:
3:2

Option 2:
5:1

Option 3:
2:5

Option 4:
1:5

Correct Answer:
5:1

Solution:
$V_L = \sqrt{5gL}$
$V_H = \sqrt{gL}$

$\frac{KE_A}{KE_B} = \frac{\frac{1}{2}mv_A^2}{\frac{1}{2}mv_B^2} = \frac{5}{1}$

Q. 16 Given below are two statements:
Statement I : Most of the mass of the atom and all its positive charge are concentrated in a tiny nucleus and the electrons revolve around it, is Rutherford's model.
Statement II : An atom is a spherical cloud of positive charges with electrons embedded in it, which is a special case of Rutherford's model.
In the light of the above statements, choose the most appropriate from the options given below

Option 1:
Statement I is true but Statement II is false

Option 2:
Both Statement I and Statement II are true

Option 3:
Both statement I and statement II are false

Option 4:
Statement I is false but Statement II is true

Correct Answer:
Statement I is true but Statement II is false

Solution:
According to Rutherford's atomic model, most of the mass of the atom and all its positive charge is concentrated in a tiny nucleus & electrons revolve around it. According to Thomson's atomic model, the atom is a spherical cloud of positive charge with electrons embedded in it.

Hence, the answer is the option (1).

Q. 17 N moles of a polyatomic gas (f = 6) must be mixed with two moles of a monoatomic gas so that the mixture behaves as a diatomic gas. The value of N is:

Option 1:
4

Option 2:
3

Option 3:
6

Option 4:
2

Correct Answer:
4

Solution:
The Degree of freedom of mixture is given as:

$\gamma_{\text{mix}} = \frac{n_1 f_1 + n_2 f_2}{n_1 + n_2} = \frac{6N + 2 \times 3}{N + 2}$

The degree of freedom of diatomic is 5.

$$\text{So, } 5 = \frac{6N + 6}{N + 2}$$
$$\Rightarrow 5N + 10 = 6N + 6$$
$$\Rightarrow N = 4$$

Hence, the answer is the option (1).

Q. 18 A planet takes 200 days to complete one revolution around the Sun. If the distance of the planet from the Sun is reduced to one-fourth of the original distance, how many days will it take to complete one revolution:

Option 1:
20

Option 2:
25

Option 3:
50

Option 4:
100

Correct Answer:
25

Solution:
According to Kepler's law :

Time period is given by $T = 2\pi r\sqrt{\frac{r}{GM}} = \frac{2\pi}{\sqrt{GM}}r^{3/2}$

$\Rightarrow \frac{T_1}{T_2} = \left(\frac{r_1}{r_2}\right)^{3/2}$

$\Rightarrow \frac{200}{T_2} = \left(\frac{r_1}{\frac{r_1}{4}}\right)^{\frac{3}{2}} = 8$

$\Rightarrow T_2 = \frac{200}{8} = 25$ days

Hence, the answer is the option (2).

Q. 19 An electric field is given by $(6\hat{i} + 5\hat{j} + 3\hat{k})N/C$. The electric flux through a surface area $30\hat{i}m^2$ lying in YZ-plane (in SI unit) is:

Option 1:
180

Option 2:
90

Option 3:
150

Option 4:
60

Correct Answer:
180

Solution:
Given: Electric field $\vec{E} = (6\hat{i} + 5\hat{j} + 3\hat{k})\,\mathrm{N/C}$ Surface area $\vec{A} = 30\hat{i}\,\mathrm{m}^2$ (in the YZ-plane) The normal vector to the surface area in the YZ-plane is $\hat{i}$. The electric flux Φ_E through the surface area is given by the dot product of the electric field vector and the surface area normal vector:

$$\Phi_E = \vec{E} \cdot \vec{A}$$

$$\Phi_E = (6\hat{i} + 5\hat{j} + 3\hat{k}) \cdot (30\hat{i})$$

$$\Phi_E = (6 \times 30)\,\mathrm{N/C}$$

$$\Phi_E = 180\,\mathrm{N \cdot m^2/C}$$

So, the electric flux through the surface area lying in the YZ-plane is $180\,\mathrm{N \cdot m^2/C}$.

Hence, the answer is the option (1).

Q. 20 A wire of length L and radius r is clamped at one end. If its other end is pulled by a force F, its length increases by l. If the radius of the wire and the applied force both are reduced to half of their original values keeping the original length constant, the increase in length will become:

Option 1:
3 times

Option 2:
4 times

Option 3:
$\frac{3}{2}$ times

Option 4:
2 times

Correct Answer:
2 times

Solution:

$$Y = \frac{FL}{A\Delta l}$$

$$\Rightarrow \Delta l = \frac{FL}{AY}$$

According to the question, L and Y are constant.

$$\text{So, } \frac{\Delta l_1}{\Delta l_2} = \frac{F_1}{\pi r_1^2} \times \frac{\pi r_2^2}{F_2} = \frac{F_1}{F_1/2} \times \frac{\frac{r_1^2}{4}}{r_1^2} = \frac{2}{4} = \frac{1}{2}$$

$$\Rightarrow \Delta l_2 = 2\Delta l_1$$

Hence, the answer is the option(4).

Q. 21 In a single slit diffraction pattern, a light of wavelength 6000Å is used. The distance between the first and third minima in the diffraction pattern is found to be $3\ \text{mm}$ when the screen in placed $50\ \text{cm}$ away from slits. The width of the slit is _____$\times 10^{-4}\ \text{m}$.

Correct Answer:
2

Solution:
\text { For } 3^{\mathrm{rd}} \text { Minima; } \sin \theta \approx \theta \approx \tan \theta=\frac{y_3}{D}=\frac{3 \lambda}{a}

$$\Rightarrow y_3 = \frac{3\lambda D}{a}$$

Similarly, $y_1 = \frac{\lambda D}{a}$

Now, according to qn. $y_3 - y_1 = 3\ \text{mm}$

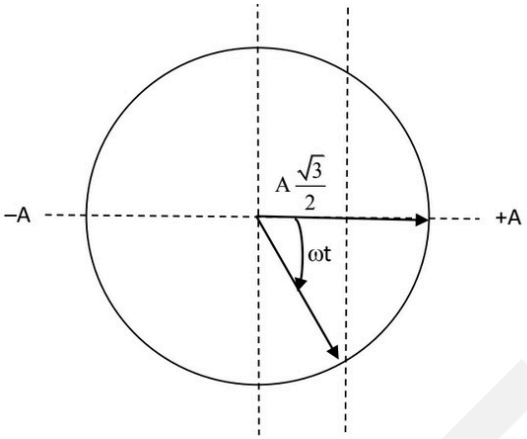
$\Rightarrow \frac{2\lambda D}{a} = 3\ \text{mm}$

$\Rightarrow a = \frac{2\times6000\times10^{-10}\times50\times10^{-2}}{3\times10^{-3}} = 20\times10^{-5}\ \text{m} = 2\times10^{-4}\ \text{m}$

Q. 22 A simple harmonic oscillator has an amplitude A and time period 6π second. Assuming the oscillation starts from its mean position, the time required by it to travel from $x = A$ to $x = \frac{\sqrt{3}}{2}$ A will be $\frac{\pi}{x}s$, where $\text{x} =$_____

Correct Answer:
2

Solution:



$\omega t = \frac{\pi}{6}$
$\Rightarrow \frac{2\pi}{T}t = \frac{\pi}{6}$
$\Rightarrow t = \frac{\pi}{2} = \frac{\pi}{x}$
$x = 2$

Q. 23 A particle is moving in a circle of radius $50\ \text{cm}$ in such a way that at any instant the normal and tangential components of it's acceleration are equal. If its speed at $t = 0$ is $4\ \text{m/s}$, the time taken to complete the first revolution will be $\frac{1}{\alpha}\left[1 - \text{e}^{-2\pi}\right]\text{s}$, where $\alpha =$

Correct Answer:
8

Solution:
According to question, $\text{a}_\text{t} = \text{a}_\text{c}$

$$\frac{dv}{dt} = \frac{v^2}{r} = \frac{v^2}{0.5} = 2v^2$$

$$\Rightarrow \frac{dv}{v^2} = 2dt$$

$$\Rightarrow \int_4^v \frac{dv}{v^2} = \int_0^t 2dt$$

$$\Rightarrow \left[-\frac{1}{v}\right]_4^v = 2t$$

$$\Rightarrow -\frac{1}{v} + \frac{1}{4} = 2t$$

$$\Rightarrow \frac{1}{v} = 0.25 - 2t$$

$$\Rightarrow v = \frac{1}{0.25 - 2t}$$

$$\Rightarrow \frac{dl}{dt} = \frac{1}{0.25 - 2t}$$

$$\Rightarrow \int_0^{2\pi r} dl = \int_0^T \frac{1}{0.25-2t} dt$$

$$\Rightarrow \frac{2\pi}{2} = \frac{1}{2} \ln \frac{0.25}{025-2t}$$

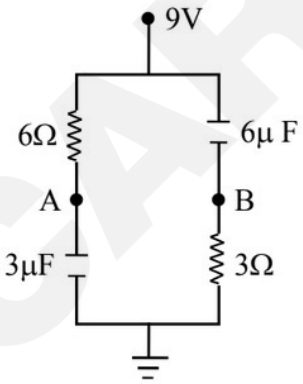
$$\Rightarrow \frac{0.25}{0.25-2t} = e^{2\pi}$$

$$\Rightarrow t = \frac{1}{8}\left(1 - e^{-2\pi}\right)$$

Equating from the given equation, $\alpha = 8$

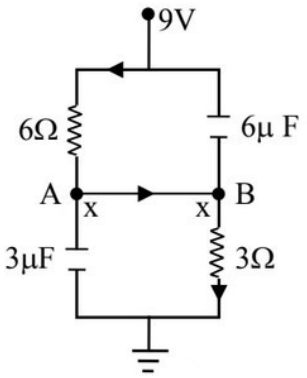
Hence, the answer is 8.

Q. 24 In the given figure, the charge stored in $6\mu\text{F}$ capacitor, when points A and B are joined by a connecting wire is μC.



Correct Answer:
36

Solution:



$V_{6\Omega} = 6\ \text{V}$
$V_{3\Omega} = 3\ \text{V}$

Now, Potential across $6\mu F$ capacitor and 6Ω resistor will be same.

$Q = CV = 6 \times 10^{-6} \times 6 = 36\mu C$

Q. 25 Two metallic wires P and Q have the same volume and are made up of the same material. If their area of cross sections is in the ratio $4:1$ and force F_1 is applied to P, an extension of Δl is produced. The force which is required to produce the same extension in Q is F_2. The value of $\frac{F_1}{F_2}$ is

Correct Answer:
16

Solution:
According to the question, wires have the same volume.

$\therefore V_1 = V_2$
$\Rightarrow A_1 l_1 = A_2 l_2$
$\Rightarrow \frac{A_1}{A_2} = \frac{l_2}{l_1} = \frac{4}{1}$

Now,

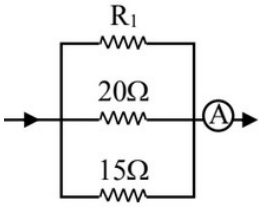
$Y = \frac{FL}{A\Delta l}$
$\Rightarrow \Delta l = \frac{FL}{AY}$
$\Delta l \propto \frac{FL}{A}$

And both wires have the same Δl

$\therefore \frac{F_1 l_1}{A_1} = \frac{F_2 l_2}{A_2}$
$\frac{F_1}{F_2} = 16$

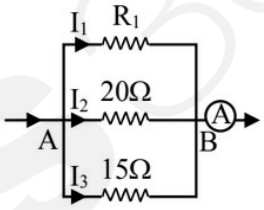
Hence, the answer is 16.

Q. 26 In the given circuit, the current flowing through the resistance 20Ω is $0.3\ \mathrm{A}$, while the ammeter reads $0.9\ \mathrm{A}$. The value of R_1 is____ Ω.



Correct Answer:
30

Solution:
$I_2 = 0.3\ \mathrm{A}$
$V_{AB} = 0.3 \times 20 = 6\ \mathrm{V}$

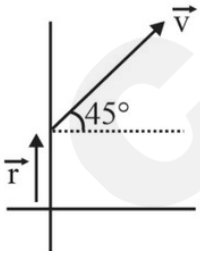


$$I_3 = \frac{6}{15} = 0.4A$$
$$\therefore I_1 = 0.9 - 0.4 - 0.3 = 0.2A$$
$$R_1 = \frac{V_{AB}}{I_1} = \frac{6}{0.2} = 30\Omega$$

Q. 27 A body of mass $5\ \mathrm{kg}$ moving with a uniform speed $3\sqrt{2}\ \mathrm{ms}^{-1}$ in $X-Y$ plane along the line $y = x + 4$. The angular momentum of the particle about the origin will be $\mathrm{kgm}^2\ \mathrm{s}^{-1}$.

Correct Answer:
60

Solution:



Take component of $\vec{v}$ perpendicular to $\vec{r}$.

$$L = 5 \times 4 \times 3 = 60\ \mathrm{kg\ m^2\ s^{-1}}$$

Q. 28 A charge of $4.0\mu C$ is moving with a velocity of $4.0 \times 10^6\ \text{ms}^{-1}$ along the positive y-axis under a magnetic field $\vec{B}$ of strength $(2\hat{k})T$. The force acting on the charge is $xi\hat{N}$. The value of x is _____.

Correct Answer:
32

Solution:
Magnetism/Force on moving charge :

$$\vec{F} = q(\vec{v} \times \vec{B})$$
$$F = qvB\sin 90$$
$$= 4 \times 10^{-6} \times 4 \times 10^6 \times 2$$
$$= 32N$$

Hence, the answer is (32).

Q. 29 A horizontal straight wire $5\ \text{m}$ long extending from east to west falling freely at right angle to horizontal component of earths magnetic field $0.60 \times 10^{-4}\text{Wbm}^{-2}$. The instantaneous value of emf induced in the wire when its velocity is $10\ \text{ms}^{-1}$ is _____$\times 10^{-3}\ \text{V}$

Correct Answer:
3

Solution:
The Motional EMF is given by -

$$\varepsilon = Bvl = 0.60 \times 10^{-4} \times 10 \times 5 = 3 \times 10^{-3}\ \text{V}$$

Hence, the answer is 3.

Q. 30 Hydrogen atom is bombarded with electrons accelerated through a potential difference of V, which causes excitation of hydrogen atoms. If the experiment is being performed at $\text{T} = 0\ \text{K}$, the minimum potential difference needed to observe any Balmer series lines in the emission spectra will be $\frac{\alpha}{10}\ \text{V}$,
where $\alpha =$ _____

Correct Answer:
121

Solution:
At.,the energy of electronic levels in hydrogen atom is determined by the equation:

$$\Delta E = 13.6\left(1-\frac{1}{9}\right)\text{eV}$$
$$\Delta E = 12.08\text{eV} \approx 12.1\text{eV}$$

$$\Delta E = \frac{121}{10}\text{eV}$$

Hence, the answer is 121.

Chemistry

Q. 1 Which of the following acts as a strong reducing agent? (Atomic number: $\text{Ce}=58, \text{Eu}=63, \text{Gd}=64, \text{Lu}=71$)

Option 1:
Lu^{3+}

Option 2:
Ce^{4+}

Option 3:
Gd^{3+}

Option 4:
Eu^{2+}

Correct Answer:
Eu^{2+}

Solution:
Stable OS of Lanthanides is +3

$$\text{Eu}^{2+} \longrightarrow \text{Eu}^{3+} + \text{e}-$$

R.A

Hence, the answer is the option (4).

Q. 2 Which of the following statements are correct about Zn, Cd and Hg?

A. They exhibit high enthalpy of atomization as the d-subshell is full.

B. Zn and Cd do not show variable oxidation state while Hg shows +I and +II

C. Compounds of Zn,Cd and Hg are paramagnetic in nature

D.Zn,Cd and Hg are called soft metals.

Choose the most appropriate from the options given below:

Option 1:
B, C only

Option 2:
A, D only

Option 3:
C. D only

Option 4:
B, D only

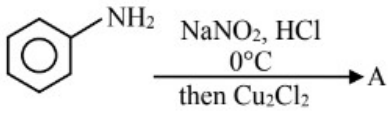
Correct Answer:
B, D only

Solution:
Zinc (Zn), cadmium (Cd), and mercury (Hg) all have completely filled d-subshells in their neutral, ground state configurations. This stability contributes to their high enthalpies of atomization.

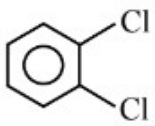
Zinc (Zn), cadmium (Cd), and mercury (Hg) are categorized as "soft metals." This classification is based on their relatively low melting points and malleability.

Hence, the answer is the option (4).

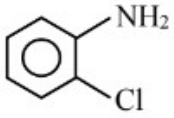
Q. 3 The product A formed in the following reaction is



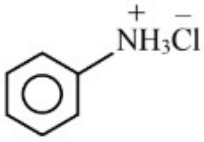
Option 1:



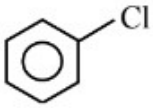
Option 2:



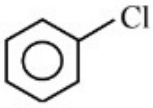
Option 3:



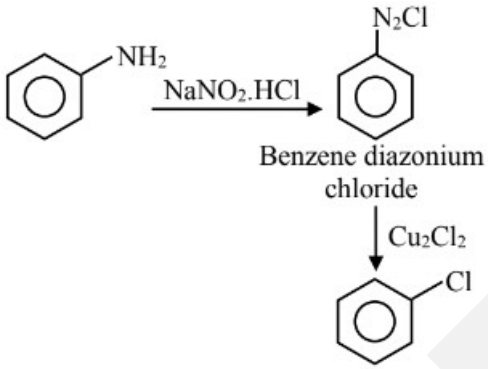
Option 4:



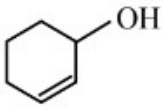
Correct Answer:



Solution:



Q. 4

According to IUPAC system, the compound  is named as :

Option 1:

Cyclohex-1-en-3-ol

Option 2:

Cyclohex-2-en-1-ol

Option 3:

Cyclohex-1-en-2-ol

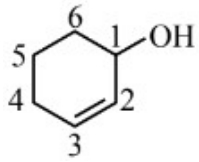
Option 4:

1-Hydroxyhex-2-ene

Correct Answer:

Cyclohex-2-en-1-ol

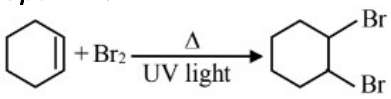
Solution:



Cyclohex-2-en-1-ol

Q. 5 Which of the following reaction is correct?

Option 1:



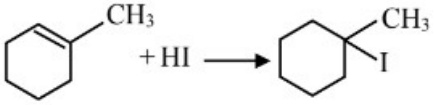
Option 2:

$$C_2H_5CONH_2 + Br_2 + NaOH \rightarrow C_2H_5CH_2NH_2 + Na_2CO_3 + NaBr + H_2O$$

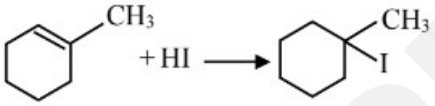
Option 3:

$$CH_3CH_2CH_2NH_2 \xrightarrow[H_2O]{HNO_2,0^\circ C} CH_3CH_2OH + N_2 + HCl$$

Option 4:

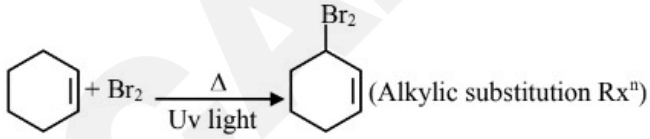


Correct Answer:



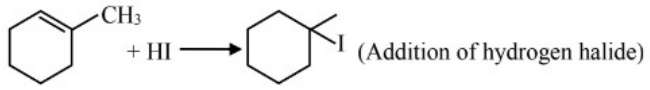
Solution:

Correct reaction are :

1. 

2. $C_2H_5CONH_2 + Br_2 + NaOH \rightarrow C_2H_5NH_2 + Na_2CO_3 + NaBr + H_2O$ (Hoffmann Bromamide Rx^n)

3. $CH_3CH_2CH_2NH_2 \xrightarrow[H_2O]{HNO_2 0^\circ C} CH_3CH_2CH_2 - OH + N_2 + HCl$

4. 

Q. 6 The element having the highest first ionization enthalpy is

Option 1:
C

Option 2:
Al

Option 3:
Si

Option 4:
N

Correct Answer:
N

Solution:
Periodic property ionisation enthalpy, easy
due to half filled configuration and small size, I.E. of N is maximum.

$$\underset{\text{Si}}{\overset{\text{V}}{\text{Al} < \text{C} >}}\ \text{N}$$

Al < Si < C < N

Q. 7 Given below are two statements:

Statement I : Fluorine has most negative electron gain enthalpy in its group.

Statement II : Oxygen has least negative electron gain enthalpy in its group.

In the light of the above statements, choose the most appropriate from the options given below

Option 1:
Statement I is false but Statement II is true

Option 2:
Both Statement I and Statement II are true

Option 3:
Both Statement I and Statement II are false

Option 4:
Statement I is true but Statement II is false

Correct Answer:
Statement I is false but Statement II is true

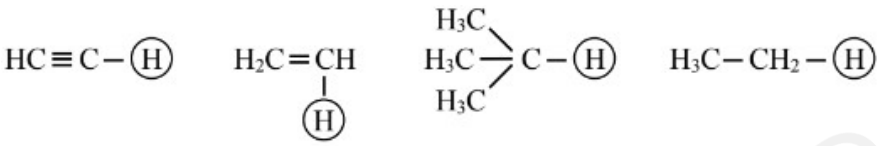
Solution:

Statement I: Fluorine is in Group 17 (the halogens). Within the halogen group, the electron gain enthalpy becomes less negative as we move down the group. This is due to the increasing atomic size and electron-electron repulsion in larger atoms. Thus, fluorine, being the smallest and the topmost element in this group, has the most negative electron gain enthalpy.

Statement II: Oxygen is in Group 16 (the chalcogens). Typically, in Group 16, the electron gain enthalpy becomes less negative down the group due to similar reasons of increasing atomic size and electron-electron repulsion.

Hence, the answer is the option (1).

Q. 8 The ascending acidity order of the following H atoms is :



Option 1:
A < B < C < D

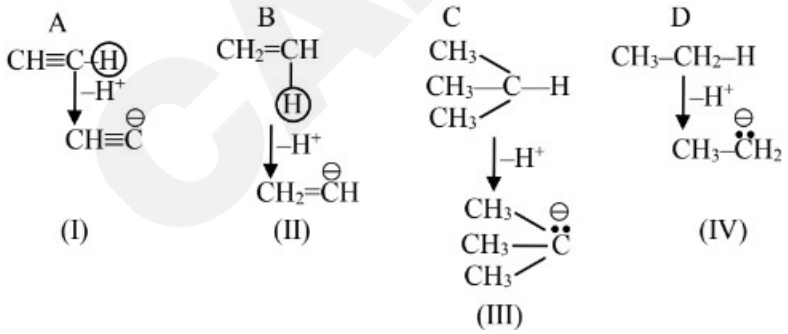
Option 2:
D < C < B < A

Option 3:
A < B < D < C

Option 4:
C < D < B < A

Correct Answer:
C < D < B < A

Solution:



Stability of conjugate base : I > II > IV > III
Acidic strength ∝ Stability of Conjugate base
So order of acidic strength is A > B > D > C

Q. 9 Match List I with List II

List I (Bio Polymer)		List II (Monomer)	
A.	Starch	I.	Nucleotide
B.	Cellulose	II.	α-glucose
C.	Nucleic acid	III.	β-glucose
D.	Protein	IV.	α-amino acid

Choose the correct answer from the options given below :

Option 1:
A-II, B-III, C-I, D-IV

Option 2:
A-II, B-I, C-III, D-IV

Option 3:
A-IV, B-II, C-I, D-III

Option 4:
A-I, B-III, C-IV, D-II

Correct Answer:
A-II, B-III, C-I, D-IV

Solution:

- Starch is a polymer of α-Glucose
- Cellulose is a polymer of β—-Glucose
- Proteins are polymer of α-amino acids.
- When nucleoside is linked to phosphoric acid, a nucleotide is formed.

Hence, the answer is option (1).

Q. 10 The correct IUPAC name of $\mathrm{K_2MnO_4}$ is

Option 1:
Potassium tetraoxidomanganese (VI)

Option 2:
Potassium tetraoxidomanganate (VI)

Option 3:
Dipotassium tetraoxidomanganate (VII)

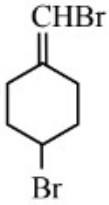
Option 4:
Potassium tetraoxopermanganate (VI)

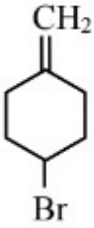
Correct Answer:
Potassium tetraoxidomanganate (VI)

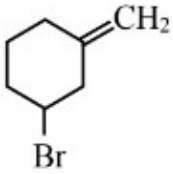
Solution:
Thus, the IUPAC name of K_2MnO_4 is potassium manganate.

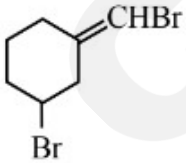
Hence, the answer is the option (2).

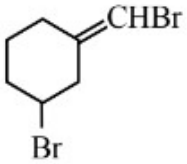
Q. 11 Which one of the following will show geometrical isomerism?

Option 1:


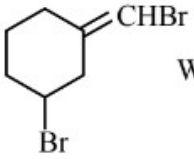
Option 2:


Option 3:


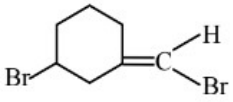
Option 4:


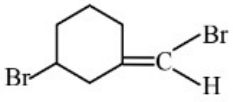
Correct Answer:


Solution:



Will show G.I.





Q. 12 Anomalous behavior of oxygen is due to its

Option 1:
small size and high electronegativity

Option 2:
small size and low electronegativity

Option 3:
large size and high electronegativity

Option 4:
large size and low electronegativity

Correct Answer:
small size and high electronegativity

Solution:
Due to the small size and high E.N. of oxygen atoms.

Hence, the answer is the option (1).

Q. 13 Alkyl halide is converted into alkyl isocyanide by reaction with

Option 1:
$NaCN$

Option 2:
$AgCN$

Option 3:
NH_4CN

Option 4:
KCN

Correct Answer:
$AgCN$

Solution:

$$\mathrm{R-X+AgCN\rightarrow \underset{\text{Alkylisocyanide}}{R-NC}+AgX}$$

Hence, the answer is the option (2).

Q. 14 A reagent which gives brilliant red precipitate with Nickel ions in basic medium is

Option 1:
dimethyl glyoxime

Option 2:
meta-dinitrobenzene

Option 3:
sodium nitroprusside

Option 4:
neutral $\mathrm{FeCl_3}$

Correct Answer:
dimethyl glyoxime

Solution:
Fact (Test of $\mathrm{Ni^{2+}}$ ion)

$$\mathrm{Ni^{2+}+dmg\xrightarrow{base}\underset{(RedPPt)}{[Ni(dmg)_2]}}$$

Hence, the answer is the option (1).

Q. 15 On passing a gas, 'X', through Nessler's regent, a brown precipitate is obtained. The gas 'X' is

Option 1:
$\mathrm{Cl_2}$

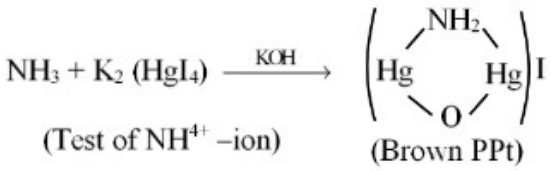
Option 2:
$\mathrm{H_2\,S}$

Option 3:
$\mathrm{CO_2}$

Option 4:
$\mathrm{NH_3}$

Correct Answer:
$\mathrm{NH_3}$

Solution:



Q. 16 Phenol treated with chloroform in presence of sodium hydroxide, which further hydrolyzed in presence of an acid results

Option 1:
Benzene-1,2-diol

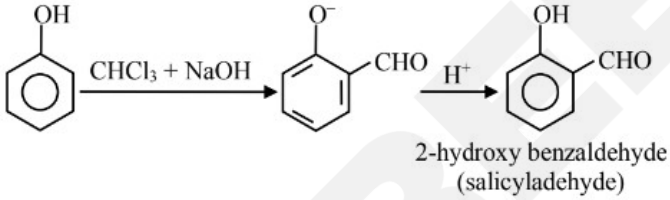
Option 2:
Benzene-1,3-diol

Option 3:
2-Hydroxybenzaldehyde

Option 4:
Salicylic acid

Correct Answer:
2-Hydroxybenzaldehyde

Solution:



It is Reimer Tiemann reaction

Q. 17 Match List I with List II :

List I (Spectral Series for Hydrogen)		List II (Spectral Region/Higher Energy State)	
A.	Lyman	I.	Infrared region
B.	Balmer	II.	UV region
C.	Paschen	III.	Infrared region
D.	Pfund	IV.	Visible region

Choose the correct answer from the options given below :

Option 1:
A-I, B-III, C-II, D-IV

Option 2:
A-II, B-III, C-I, D-IV

Option 3:
A-II, B-IV, C-III, D-I

Option 4:
A-I, B-II, C-III, D-IV

Correct Answer:
A-II, B-III, C-I, D-IV

Solution:
Lyman : - UV region
Balmer : - visible region
Paschen, pfund: - Infrared region

Hence, the answer is the option (2).

Q. 18 Match List I with List II :

	List I (Compound)		**List II (pK p_a value)**
A.	Ethanol	I.	10.0
B.	Phenol	II.	15.9
C.	m-Nitrophenol	III.	7.1
D.	p-Nitrophenol	IV.	8.3

Choose the correct answer from the options given below :

Option 1:
A-I, B-II, C-III, D-IV

Option 2:
A-II, B-I, C-IV, D-III

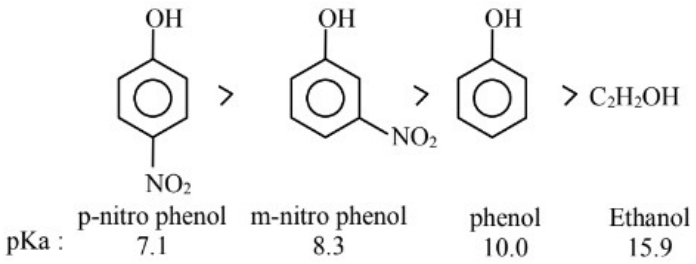
Option 3:
A-III, B-IV, C-I, D-II

Option 4:
A-IV, B-I, C-II, D-III

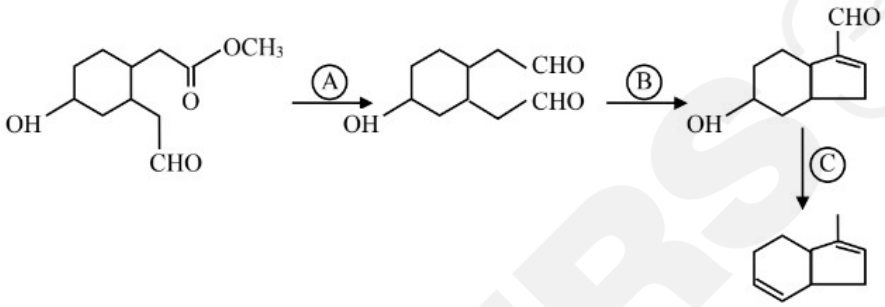
Correct Answer:
A-II, B-I, C-IV, D-III

Solution:
Acidic strength $\alpha \frac{1}{\text{pKa value}}$

order of acidic strength :



Q. 19 Identify the reagents used for the following conversion :



Option 1:
$\mathrm{A = LiAlH_4,\ B = NaOH_{(aq)}, C = NH_2 - NH_2/KOH}$, ethylene glycol

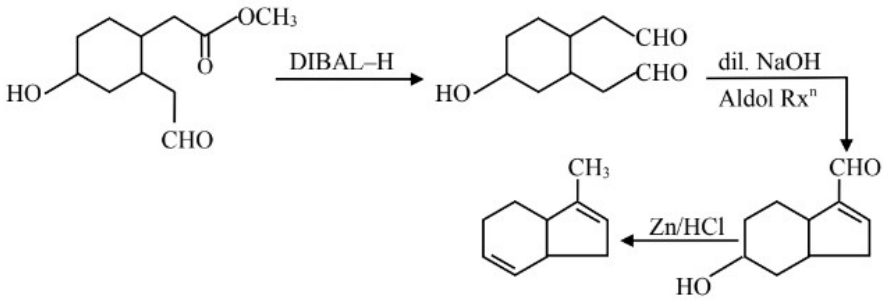
Option 2:
$\mathrm{A =}$ DIBAL-H, $\mathrm{B = NaOH_{(aq)}, C = NH_2 - NH_2/KOH}$, ethylene glycol

Option 3:
$\mathrm{A =}$ DIBAL-H, $\mathrm{B = NaOH_{(alc)}, C = Zn/HCl}$

Option 4:
$\mathrm{A = LiAlH_4,\ B = NaOH_{(alc),}, C = Zn/HCl}$

Correct Answer:
$\mathrm{A =}$ DIBAL-H, $\mathrm{B = NaOH_{(alc)}, C = Zn/HCl}$

Solution:



Q. 20 Chromatographic technique/s based on the principle of differential adsorption is/are

A. Column chromatography

B. Thin layer chromatography

C. Paper chromatography

Choose the most appropriate answer from the options given below:

Option 1:
B only

Option 2:
A only

Option 3:
A & B only

Option 4:
C only

Correct Answer:
A & B only

Solution:
Column chromatography and thin-layer chromatography techniques are based on differential adsorption.

Hence, the answer is the option (3).

Q. 21 The total number of anti bonding molecular orbitals, formed from 2s and 2p atomic orbitals in a diatomic molecule is _____.

Correct Answer:
4

Solution:
Antibonding orbital is:

$\sigma 2\ s^*, \sigma 2p^*, \pi 2p^*, \pi^* 2p$

Hence, the answer is (4).

Q. 22 Standard enthalpy of vapourisation for CCl_4 is $30.5\ kJ\ mol^{-1}$. Heat required for vapourisation of $284\ g$ of CCl_4 at constant temperature is_____ kJ.

(Given molar mass in $gmol^{-1}$; $C = 12, Cl = 35.5$)

Correct Answer:
56.22

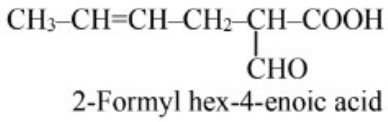
Solution:
$\Delta H = n\Delta H_{vap} = \frac{284}{154} \times 30.5 = 56.24\ K$

Hence, the answer is (56.22).

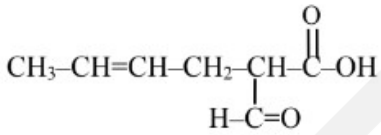
Q. 23 The total number of 'Sigma' and 'Pi' bonds in 2-formylhex-4-enoic acid is ____.

Correct Answer:
22

Solution:



2-Formyl hex-4-enoic acid



no. of σ-bonds -19 π bonds -3

sum of σ bonds $\&\pi$ bonds $= 22$

Q. 24 If 50 mL of 0.5M oxalic acid is required to neutralise 25 mL of NaOH solution, the amount of NaOH in 50 mL of given NaOH solution is _____ g.

Correct Answer:
4

Solution:
eq of $NaOH$ = eq of oxalic acid

$25 \times M = 0.5 \times 50 \times 2$

$M = 2\ mol\ L^{-1}$

weight NaOH in $50\mathrm{ml} = 2 \times 50 \times 40 \times 10^{-3}\mathrm{gm} = 4\ \mathrm{g}$

Hence, the answer is (4).

Q. 25 The oxidation number of iron in the compound formed during brown ring test for $\mathrm{NO_3^-}$ ion is_____.

Correct Answer:
1

Solution:
$\left[\overset{+1}{\mathrm{Fe}}\ (\mathrm{H_2O})_5\mathrm{No}\right]\mathrm{SO_4}$

Hence, the answer is (1).

Q. 26 The half-life of radioisotope bromine - 82 is 36 hours. The fraction which remains after one day is_____ $\times 10^{-2}$.

(Given antilog 0.2006 = 1.587)

Correct Answer:
63

Solution:
$$\mathrm{n} = \text{ no. of half life } = \frac{24}{36} = \frac{2}{3}$$

$$\text{Amount left } = \frac{\mathrm{N_0}}{2^{2/3}} = 63 \times 10^{-2}\ \mathrm{N_0}$$

$$\text{Fraction left } = 63 \times 10^{-2}$$

Hence, the answer is (63).

Q. 27 A constant current was passed through a solution of $\mathrm{AuCl_4^-}$ ion between gold electrodes. After a period of 10.0 minutes, the increase in mass of cathode was 1.31 g. The total charge passed through the solution is_____$\times 10^{-2}$ F.

(Given atomic mass of Au =197)

Correct Answer:
2

Solution:

$Au^{+3} + 3e^- \rightarrow Au(s) = 1.314gm$

$\frac{1.314}{197} = 0.00667 \text{ mol}$

for 1 mol gold $\rightarrow 3$ F

for 0.00667 mole $\rightarrow 3 \times 0.00667 \text{ F} = 0.02 = 2 \times 10^{-2} \text{ F}$

Hence, the answer is (2).

Q. 28 Molality of $0.8M H_2SO_4$ solution (density 1.06 g cm^{-3}) is_____ $\times 10^{-3}$ m.

Correct Answer:
815

Solution:

$$m = \frac{1000M}{1000\,d - MM_{solute}}$$

$$= \frac{1000 \times 0.8}{(1000 \times 1.06) - (0.8 \times 98)} = 814.9 \times 10^{-3} = 815 \times 10^{-3}$$

Hence, the answer is (815).

Q. 29 The total number of molecules with zero dipole moment among $CH_4, BF_3, H_2O, HF, NH_3, CO_2$ and SO_2 is_____

Correct Answer:
3

Solution:
D.M. of CH_4, BF_3 and CO_2 is zero.

Hence, the answer is (3).

Q. 30 The following concentrations were observed at 500 K for the formation of NH_3 from N_2 and H_2. At equilibrium; $[N_2] = 2 \times 10^{-2}$M, $[H_2] = 3 \times 10^{-2}$M and $[NH_3] = 1.5 \times 10^{-2}$M. Equilibrium constant for the reaction is_____.

Correct Answer:
417

Solution:

$N_2 + 3H_2 \rightarrow 2NH_3$

$$K = \frac{[NH_3]}{[N_2][H_2]^3} = \frac{\left[1.5 \times 10^{-2}\right]^2}{\left[2 \times 10^{-2}\right]\left[3 \times 10^{-2}\right]^3} = 0.04166 \times 10^4 = 416.6 \approx 417$$

Hence, the answer is (417).

Maths

Q. 1 The function $f(x) = 2x + 3(x)^{\frac{2}{3}}, x \in \mathbb{R}$, has

Option 1:
exactly one point of local minima and no point of local maxima

Option 2:
exactly one point of local maxima and no point of local minima

Option 3:
exactly one point of local maxima and exactly one point of local minima

Option 4:
exactly two points of local maxima and exactly one point of local minima

Correct Answer:
exactly one point of local minima and no point of local maxima

Solution:
$f(x) = 2x + 3(x)^{\frac{2}{3}}$

$f'(x) = 2 + 2x^{\frac{-1}{3}}$

$= 2\left(1 + \frac{1}{x^{\frac{1}{3}}}\right)$

$= 2\left(\frac{x^{\frac{1}{3}} + 1}{x^{\frac{1}{3}}}\right)$



So, maxima (M) at x = –1 & minima(m) at x = 0

Q. 2 Let A be the point of intersection of the lines 3x + 2y = 14, 5x – y = 6 and B be the point of intersection of the lines 4x + 3y = 8, 6x + y = 5. The distance of the point P(5, –2) from the line AB is

Option 1:
$\frac{13}{2}$

Option 2:
8

Option 3:
$\frac{5}{2}$

Option 4:
6

Correct Answer:
6

Solution:
Solving lines $\mathrm{L_1(3x+2y=14)}$ and $\mathrm{L_2(5x-y=6)}$ to get $\mathrm{A(2,4)}$ and solving lines $\mathrm{L_3(4x+3y=8)}$ and $\mathrm{L_4(6x+y=5)}$ to get $\mathrm{B\left(\frac{1}{2},2\right)}$.

Finding Eqn. of AB : 4x – 3y + 4 = 0
Calculate distance PM

$\Rightarrow \frac{4(5)-3(-2)+4}{5} = 6$

Hence, the answer is the option (4).

Q. 3 If the mean and variance of five observations are $\frac{24}{5}$ and $\frac{194}{25}$ respectively and the mean of the first four observations is $\frac{7}{2}$, then the variance of the first four observations in equal to

Option 1:
$\frac{4}{5}$

Option 2:
$\frac{4}{5}$

Option 3:
$\frac{5}{4}$

Option 4:
$\frac{105}{4}$

Correct Answer:
$\frac{5}{4}$

Solution:
$\bar{X}=\frac{24}{5}; \sigma^2=\frac{194}{25}$

Let the first four observations be x_1, x_2, x_3, x_4

Here, $\frac{x_1+x_2+x_3+x_4+x_5}{5}=\frac{24}{5}$

Also, $\frac{x_1+x_2+x_3+x_4}{4}=\frac{7}{2}$

$\Rightarrow x_1+x_2+x_3+x_4=14$

Now from eqn -1

$x_5=10$

Now, $\sigma^2=\frac{194}{25}$

$\frac{x_1^2+x_2^2+x_3^2+x_4^2+x_5^2}{5}-\frac{576}{25}=\frac{194}{25}$
$\Rightarrow x_1^2+x_2^2+x_3^2+x_4^2=54$

Now, the variance of the first 4 observations

$\text{Var}=\frac{\sum_{i=1}^{4} x_i^2}{4}-\left(\frac{\sum_{i=1}^{4} x_i}{4}\right)^2=\frac{54}{4}-\frac{49}{4}=\frac{5}{4}$

Hence, the answer is option (3).

Q. 4 The distance of the point $(2,3)$ from the line $2x-3y+28=0$, measured parallel to the line $\sqrt{3}x-y+1=0$, is equal to

Option 1:
$4\sqrt{2}$

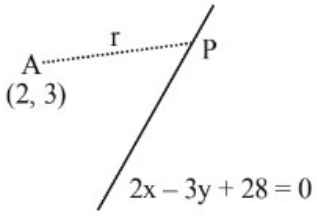
Option 2:
$6\sqrt{3}$

Option 3:
$3+4\sqrt{2}$

Option 4:
$4+6\sqrt{3}$

Correct Answer:
$4+6\sqrt{3}$

Solution:



Writing P in terms of parametric co-ordinates $2+r\cos\theta, 3+r\sin\theta$ as $\tan\theta=\sqrt{3}$

$\mathrm{P}\left(2+\frac{\mathrm{r}}{2}, 3+\frac{\sqrt{3}\mathrm{r}}{2}\right)$

P must satisfy 2x – 3y + 28 = 0

So, $2\left(2+\frac{r}{2}\right)-3\left(3+\frac{\sqrt{3}r}{2}\right)+28=0$

We find $r=4+6\sqrt{3}$

Q. 5 If $\log_e a, \log_e b, \log_e c$ are in an A.P. and $\log_e a-\log_e 2b, \log_e 2b-\log_e 3c, \log_e 3c-\log_e a$ are also in an A.P, then $a:b:c$ is equal to

Option 1:
9 : 6 : 4

Option 2:
16 : 4 : 1

Option 3:
25 : 10 : 4

Option 4:
6 : 3 : 2

Correct Answer:
9 : 6 : 4

Solution:
$\log_e a, \log_e b, \log_e c$ are in A.P.

$\therefore \mathrm{b}^2=\mathrm{ac}$

Also

$\log_c\left(\frac{a}{2b}\right), \log_c\left(\frac{2b}{3c}\right), \log_c\left(\frac{3c}{a}\right)$ are in A.P.

$$\left(\frac{2b}{3c}\right)^2 = \frac{a}{2b} \times \frac{3c}{a}$$

$$\frac{b}{c} = \frac{3}{2}$$

Putting in eq. (i) $b^2 = a \times \frac{2b}{3}$

$\frac{a}{b} = \frac{3}{2}$

$a : b : c = 9 : 6 : 4$

Hence, the answer is the option (1).

Q. 6 Let $A = \begin{bmatrix} 2 & 1 & 2 \\ 6 & 2 & 11 \\ 3 & 3 & 2 \end{bmatrix}$ and $P = \begin{bmatrix} 1 & 2 & 0 \\ 5 & 0 & 2 \\ 7 & 1 & 5 \end{bmatrix}$. The sum of the prime factors of $P^{-1}AP - 2I$ is equal to

Option 1:
26

Option 2:
27

Option 3:
66

Option 4:
23

Correct Answer:
26

Solution:

$$\begin{aligned} P^{-1}AP-2I &= P^{-1}AP-2P^{-1}P \\ &= P^{-1}(A-2I)P \\ &= P^{-1}|A-2I||P| \\ &= |A-2I| \\ &= \begin{vmatrix} 0 & 1 & 2 \\ 6 & 0 & 11 \\ 3 & 3 & 0 \end{vmatrix} = 69 \end{aligned}$$

So, the Prime factor of 69 is 3 & 23

So, sum = 26

Hence, the answer is the option (1).

Q. 7 If each term of a geometric progression $a_1, a_2, a_3, \ldots$ with $a_1 = \frac{1}{8}$ and $a_2 \neq a_1$, is the arithmetic mean of the next two terms and $S_n = a_1 + a_2 + \ldots + a_n$, then $S_{20} - S_{18}$ is equal to

Option 1:
2^{15}

Option 2:
-2^{18}

Option 3:
2^{18}

Option 4:
-2^{15}

Correct Answer:
-2^{15}

Solution:
Let r' th term of the GP be a^{n-1}.

Given,

$2a_r = a_{r+1} + a_{r+2}$

$2ar^{n-1} = ar^n + ar^{n+1}$

$\frac{2}{r} = 1 + r$

$r^2 + r - 2 = 0$

Hence, we get, $r = -2$(as $r \neq 1$)

So, $S_{20} - S_{18} =$ (Sum upto 20 terms) $-$ (Sum upto 18 terms) $= T_{19} + T_{20}$

$T_{19} + T_{20} = ar^{18}(1 + r)$

Putting the values $a = \frac{1}{8}$ and $r = -2$;

we get $T_{19} + T_{20} = -2^{15}$

Hence, the answer is the option (4).

Q. 8 An integer is chosen at random from the integers $1, 2, 3, \ldots, 50$. The probability that the chosen integer is a multiple of atleast one of 4,6 and 7 is

Option 1:
$\frac{8}{25}$

Option 2:
$\frac{21}{50}$

Option 3:
$\frac{9}{50}$

Option 4:
$\frac{14}{25}$

Correct Answer:
$\frac{21}{50}$

Solution:
Given set $= \{1, 2, 3, \ldots\ldots\ldots 50\}$
$P(A) =$ Probability that number is multiple of 4
$P(B) =$ Probability that number is multiple of 6
$P(C) =$ Probability that number is multiple of 7
Now,

$P(A) = \frac{12}{50}, P(B) = \frac{8}{50}, P(C) = \frac{7}{50}$

again

$\mathrm{P(A \cap B)} = \frac{4}{50}, \mathrm{P(B \cap C)} = \frac{1}{50}, \mathrm{P(A \cap C)} = \frac{1}{50}$
$\mathrm{P(A \cap B \cap C)} = 0$

Thus

$P(A \cup B \cup C) = \frac{12}{50} + \frac{8}{50} + \frac{7}{50} - \frac{4}{50} - \frac{1}{50} - \frac{1}{50} + 0 = \frac{21}{50}$

Hence, the answer is the option (2).

Q. 9 Let $\mathrm{P}(3, 2, 3), \mathrm{Q}(4, 6, 2)$ and $\mathrm{R}(7, 3, 2)$ be the vertices of $\triangle \mathrm{PQR}$. Then, the angle $\angle \mathrm{QPR}$ is

Option 1:
$\frac{\pi}{6}$

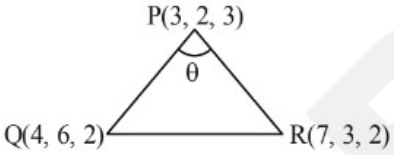
Option 2:
$\cos^{-1}\left(\frac{7}{18}\right)$

Option 3:
$\cos^{-1}\left(\frac{1}{18}\right)$

Option 4:
$\frac{\pi}{3}$

Correct Answer:
$\frac{\pi}{3}$

Solution:



Direction ratio of PR = (4, 1, –1)
Direction ratio of PQ = (1, 4, –1)

Now, $\cos\theta = \frac{4+4+1}{\sqrt{18}\cdot\sqrt{18}} \Rightarrow \theta = \frac{\pi}{3}$

Q. 10 Let r and θ respectively be the modulus and amplitude of the complex number $z = 2 - i\left(2\tan\frac{5\pi}{8}\right)$, then (r, θ) is equal to

Option 1:
$\left(2\sec\frac{3\pi}{8}, \frac{3\pi}{8}\right)$

Option 2:
$\left(2\sec\frac{3\pi}{8}, \frac{5\pi}{8}\right)$

Option 3:
$\left(2\sec\frac{5\pi}{8},\frac{3\pi}{8}\right)$

Option 4:
$\left(2\sec\frac{11\pi}{8},\frac{11\pi}{8}\right)$

Correct Answer:
$\left(2\sec\frac{3\pi}{8},\frac{3\pi}{8}\right)$

Solution:

$$z=2-i\left(2\tan\frac{5\pi}{8}\right)=x+iy(\text{ let })$$

$$r=\sqrt{x^2+y^2}\&\theta=\tan^{-1}\frac{y}{x}$$

$$r=\sqrt{(2)^2+\left(2\tan\frac{5\pi}{8}\right)^2}$$

$$=\ 2\sec\frac{5\pi}{8}\ =\ 2\sec\left(\pi-\frac{3\pi}{8}\right)$$

$$=2\sec\frac{3\pi}{8}$$

$$\&\quad\theta=\tan^{-1}\left(\frac{-2\tan\frac{5\pi}{8}}{2}\right)$$

$$=\tan^{-1}\left(\tan\left(\pi-\frac{5\pi}{8}\right)\right)=\frac{3\pi}{8}$$

Hence, the answer is the option (1)

Q. 11 If $\sin\left(\frac{y}{x}\right)=\log_e|x|+\frac{\alpha}{2}$ is the solution of the differential equation $x\cos\left(\frac{y}{x}\right)\frac{dy}{dx}=y\cos\left(\frac{y}{x}\right)+x$ and $y(1)=\frac{\pi}{3}$, then α^2 is equal to

Option 1:
3

Option 2:
12

Option 3:
4

Option 4:
9

Correct Answer:
3

Solution:
Differential equation :–

$$x\cos\frac{y}{x}\frac{dy}{dx}=y\cos\frac{y}{x}+x$$

$$\cos\frac{y}{x}\left[x\frac{dy}{dx}-y\right]=x$$

Divide both sides by x^2

$$\cos\frac{y}{x}\left(\frac{x\frac{dy}{dx}-y}{x^2}\right)=\frac{1}{x}$$

$$\text{Let } \frac{y}{x}=t$$

$$\cos t\left(\frac{dt}{dx}\right)=\frac{1}{x}$$

$$\cos t dt=\frac{1}{x}dx$$

Integrating both sides

$$\sin t=\ln|x|+c$$

$$\sin\frac{y}{x}=\ln|x|+c$$

Using $y(1)=\frac{\pi}{3}$, we get $c=\frac{\sqrt{3}}{2}$

So, $\alpha=\sqrt{3}\Rightarrow\alpha^2=3$

Hence, the answer is the option (1).

Q. 12 Let $y=\log_e\left(\frac{1-x^2}{1+x^2}\right),-1<x<1$. Then at $x=\frac{1}{2}$, the value of $225\left(y'-y''\right)$ is equal to

Option 1:
732

Option 2:
746

Option 3:
742

Option 4:
736

Correct Answer:
736

Solution:

$$y = \log_e\left(\frac{1-x^2}{1+x^2}\right)$$

$$\frac{dy}{dx} = y' = \frac{-4x}{1-x^4}$$

Again,

$$\frac{d^2y}{dx^2} = y'' = \frac{-4\left(1+3x^4\right)}{\left(1-x^4\right)^2}$$

Again

$$y' - y'' = \frac{-4x}{1-x^4} + \frac{4\left(1+3x^4\right)}{\left(1-x^4\right)^2}$$

$$\text{at } x = \frac{1}{2}$$

$$y' - y'' = \frac{736}{225}$$

$$\text{Thus } 225\left(y' - y''\right) = 225 \times \frac{736}{225} = 736$$

Hence, the answer is the option (4).

Q. 13 If $\int \frac{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x}{\sqrt{\sin^3 x \cos^3 x \sin(x-\theta)}}dx = A\sqrt{\cos\theta \tan x - \sin\theta} + B\sqrt{\cos\theta - \sin\theta \cot x} + C$, where C is the integration constant, then AB is equal to

Option 1:
$4\,\text{cosec}(2\theta)$

Option 2:
$4\sec\theta$

Option 3:
$2\sec\theta$

Option 4:
$8\,\text{cosec}(2\theta)$

Correct Answer:
$8\,\text{cosec}(2\theta)$

Solution:

$$\int \frac{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x}{\sqrt{\sin^3 x \cos^3 x \sin(x-\theta)}}dx$$

$$I = \int \frac{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x}{\sqrt{\sin^3 x \cos^3 x(\sin x\cos\theta - \cos x \sin\theta)}}dx$$

$$= \int \frac{\sin^{\frac{3}{2}} x}{\sin^{\frac{3}{2}} x \cos^2 x\sqrt{\tan x\cos\theta - \sin\theta}}dx + \int \frac{\cos^{\frac{3}{2}} x}{\sin^2 x\cos^{\frac{3}{2}} x\sqrt{\cos\theta - \cot x\sin\theta}}dx$$

$$= \int \frac{\sec^2 x}{\sqrt{\tan x\cos\theta - \sin\theta}}dx + \int \frac{\text{cosec}\, x}{\sqrt{\cos\theta - \cot x\sin\theta}}dx$$

$\mathrm{I} = \mathrm{I}_1 + \mathrm{I}_2 \quad \ldots\ldots \{\text{Let}\}$

For I_1, let $\tan \mathrm{x}\cos\theta - \sin\theta = \mathrm{t}^2$

$\sec^2 x dx = \frac{2tdt}{\cos\theta}$

For I_2, let $\cos\theta - \cot \mathrm{x}\sin\theta = \mathrm{z}^2$

$\text{cosec}^2\, x dx = \frac{2zdz}{\sin\theta}$

$I = I_1 + I_2$

$$= \int \frac{2tdt}{\cos\theta t} + \int \frac{2zdz}{\sin\theta z}$$

$$= \frac{2t}{\cos\theta} + \frac{2z}{\sin\theta}$$

$$= 2\sec\theta\sqrt{\tan x\cos\theta - \sin\theta} + 2\,\text{cosec}\,\theta\sqrt{\cos\theta - \cot x\sin\theta}$$

Comparing

$AB = 8\,\text{cosec}\, 2\theta$

Hence, the answer is the option (4).

Q. 14 Let $\mathrm{x} = \frac{\mathrm{m}}{\mathrm{n}}$ (m, n are co-prime natural numbers) be a solution of the equation $\cos\left(2\sin^{-1}\mathrm{x}\right) = \frac{1}{9}$ and let $\alpha, \beta(\alpha > \beta)$ be the roots of the equation $m^2 - nx - m + n = 0$. Then the point (α, β) lies on the line

Option 1:
$3x + 2y = 2$

Option 2:
$5x - 8y = -9$

Option 3:
$3x - 2y = -2$

Option 4:
$5x + 8y = 9$

Correct Answer:
$5x + 8y = 9$

Solution:

$$\text{Assume } \sin^{-1} x = \theta$$

$$\cos(2\theta) = \frac{1}{9}$$

$$\sin\theta = \pm\frac{2}{3}$$

as m and n are co-prime natural numbers,

$x = \frac{2}{3}$

i.e. $m = 2, n = 3$

So, the quadratic equation becomes $2x^2 - 3x + 1 = 0$ whose roots are $\alpha = 1, \beta = \frac{1}{2}$

$\left(1, \frac{1}{2}\right)$ lies on $5x + 8y = 9$

Hence, the answer is the option (4).

Q. 15 Number of ways of arranging 8 identical books into 4 identical shelves where any number of shelves may remain empty is equal to

Option 1:
18

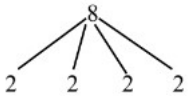
Option 2:
16

Option 3:
12

Option 4:
15

Correct Answer:
15

Solution:



2	2	2	2
2	2	3	1
2	3	3	0
8	0	0	0
7	1	0	0
6	2	0	0
5	3	0	0
4	4	0	0

6	1	1	0
5	2	1	0
4	3	1	0
4	2	2	0
3	3	1	1
2	1	1	4
1	1	1	5

Q. 16 If R is the smallest equivalence relation on the set $\{1,2,3,4\}$ such that $\{(1,2),(1,3)\} \subset \mathrm{R}$, then the number of elements in R is_____.

Option 1:
10

Option 2:
12

Option 3:
8

Option 4:
15

Correct Answer:
10

Solution:
Given set $\{1,2,3,4\}$

Minimum order pairs are

$(1,1),(2,2),(3,3),(4,4),(3,1),(2,1),(2,3),(3,2),(1,2),(1,3)$

Thus no. of elements $= 10$

Hence, the answer is the option (1).

Q. 17 Let a unit vector $\hat{\mathrm{u}} = x\hat{\mathrm{i}} + \mathrm{y}\hat{\mathrm{j}} + \mathrm{z}\hat{\mathrm{k}}$ make angles $\frac{\pi}{2}$, $\frac{\pi}{3}$ and $\frac{2\pi}{3}$ with the vectors $\frac{1}{\sqrt{2}}\hat{\mathrm{i}} + \frac{1}{\sqrt{2}}\hat{\mathrm{k}}$, $\frac{1}{\sqrt{2}}\hat{\mathrm{j}} + \frac{1}{\sqrt{2}}\hat{\mathrm{k}}$ and $\frac{1}{\sqrt{2}}\hat{\mathrm{i}} + \frac{1}{\sqrt{2}}\hat{\mathrm{j}}$ respectively. If $\overrightarrow{\mathrm{v}} = \frac{1}{\sqrt{2}}\hat{\mathrm{i}} + \frac{1}{\sqrt{2}}\hat{\mathrm{j}} + \frac{1}{\sqrt{2}}\hat{\mathrm{k}}$, then $|\hat{\mathrm{u}} - \overrightarrow{\mathrm{v}}|^2$ is equal to

Option 1:
$\frac{11}{2}$

Option 2:
$\frac{5}{2}$

Option 3:
9

Option 4:
7

Correct Answer:
$\frac{5}{2}$

Solution:
Unit vector $\hat{u} = x\hat{i} + y\hat{j} + z\hat{k}$

$$\overrightarrow{p_1} = \frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{k}, \overrightarrow{p_2} = \frac{1}{\sqrt{2}}\hat{j} + \frac{1}{\sqrt{2}}\hat{k}$$

$$\overrightarrow{p_3} = \frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}$$

Now angle between $\hat{u}$ and $\overrightarrow{p_1} = \frac{\pi}{2}$

$$\hat{u} \cdot \overrightarrow{p_1} = 0 \Rightarrow \frac{x}{\sqrt{2}} + \frac{z}{\sqrt{2}} = 0$$

$$\Rightarrow x + z = 0 \quad \ldots(i)$$

Angle between $\hat{u}$ and $\overrightarrow{p_2} = \frac{\pi}{3}$

$$\hat{u} \cdot \overrightarrow{p_2} = |\hat{u}| \cdot \overrightarrow{p_2} \cos\frac{\pi}{3}$$

$$\Rightarrow \frac{y}{\sqrt{2}} + \frac{z}{\sqrt{2}} = \frac{1}{2} \Rightarrow y + z = \frac{1}{\sqrt{2}} \quad \ldots..(ii)$$

Angle between uand $\overrightarrow{p_3} = \frac{2\pi}{3}$

$$\hat{u} \cdot \overrightarrow{p_3} = |\hat{u}| \cdot \overrightarrow{p_3} \cos\frac{2\pi}{3}$$

$$\Rightarrow \frac{x}{\sqrt{2}} + \frac{4}{\sqrt{2}} = \frac{-1}{2} \Rightarrow x + y = \frac{-1}{\sqrt{2}} \quad \ldots.(iii)$$

from equation (i), (ii) and (iii) we get

$$x = \frac{-1}{\sqrt{2}} \quad y = 0 \quad z = \frac{1}{\sqrt{2}}$$

$$\text{Thus } \hat{u} - \vec{v} = \frac{-1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{k} - \frac{1}{\sqrt{2}}\hat{i} - \frac{1}{\sqrt{2}}\hat{j} - \frac{1}{\sqrt{2}}\hat{k}$$

$$\hat{u} - \vec{v} = \frac{-2}{\sqrt{2}}\hat{i} - \frac{1}{\sqrt{2}}\hat{j}$$

$$\therefore |\hat{u} - \vec{v}|^2 = \left(\sqrt{\frac{4}{2} + \frac{1}{2}}\right)^2 = \frac{5}{2}$$

Hence, the answer is the option (2).

Q. 18 Let $\overrightarrow{OA} = \vec{a}, \overrightarrow{OB} = 12\vec{a} + 4\vec{b}$ and $\overrightarrow{OC} = \vec{b}$, where O is the origin. If S is the parallelogram with adjacent sides OA and OC, then $\frac{\text{area of the quadrilateral OABC}}{\text{area of S}}$ is equal to_____.

Option 1:
6

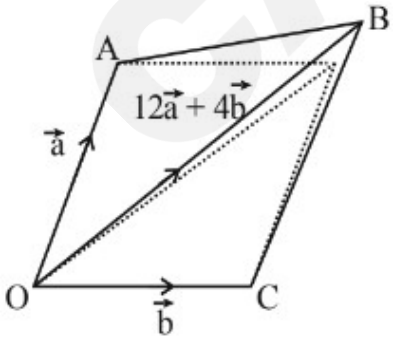
Option 2:
10

Option 3:
7

Option 4:
8

Correct Answer:
8

Solution:



$\text{Area of parallelogram, } S = |\vec{a} \times \vec{b}|$

$\text{Area of quadrilateral} = \text{Area } (\triangle \text{OAB}) + \text{Area}(\triangle \text{OBC})$

$= \frac{1}{2}\{|\overrightarrow{a} \times (12\overrightarrow{a} + 4\overrightarrow{b})| + |\overrightarrow{b} \times (12\overrightarrow{a} + 4\overrightarrow{b})|\}$

$= 8|(\overrightarrow{a} \times \overrightarrow{b})|$

Ratio $= \frac{8|(\vec{a}\times\vec{b})|}{|(\vec{a}\times\vec{b})|} = 8$

Q. 19 The function $f(x) = \frac{x}{x^2-6x-16}, x \in \mathbb{R} - \{-2, 8\}$

Option 1:
decreases in $(-2, 8)$ and increases in $(-\infty, -2) \cup (8, \infty)$

Option 2:
decreases in $(-\infty, -2) \cup (-2, 8) \cup (8, \infty)$

Option 3:
decreases in $(-\infty, -2)$ and increases in $(8, \infty)$

Option 4:
increases in $(-\infty, -2) \cup (-2, 8) \cup (8, \infty)$

Correct Answer:
decreases in $(-\infty, -2) \cup (-2, 8) \cup (8, \infty)$

Solution:
$f(x) = \frac{x}{x^2-6x-16}$

Now,

$$\mathrm{f'(x)} = \frac{-\left(\mathrm{x^2} + 16\right)}{\left(\mathrm{x^2} - 6\mathrm{x} - 16\right)^2}$$

$\mathrm{f'(x)} < 0$

Thus f(x) is decreasing in

$(-\infty, -2) \cup (-2, 8) \cup (8, \infty)$

Hence, the answer is the option (2).

Q. 20 The sum of the solutions $x \in \mathbb{R}$ of the equation $\frac{3\cos 2x + \cos^3 2x}{\cos^6 x - \sin^6 x} = x^3 - x^2 + 6$ is

Option 1:
0

Option 2:
1

Option 3:
-1

Option 4:
3

Correct Answer:
-1

Solution:

$\frac{3\cos 2x+\cos^3 2x}{\cos^6 x-\sin^6 x}=x^3-x^2+6$

$\Rightarrow \frac{\cos 2x\left(3+\cos^2 2x\right)}{\cos 2x\left(1-\sin^2 x\cos^2 x\right)}=x^3-x^2+6$

$\Rightarrow \frac{4\left(3+\cos^2 2x\right)}{\left(4-\sin^2 2x\right)}=x^3-x^2+6$

$\Rightarrow \frac{4\left(3+\cos^2 2x\right)}{\left(3+\cos^2 2x\right)}=x^3-x^2+6$

$x^3-x^2+2=0$

so, sum of solution = –1

Hence, the answer is the option (3).

Q. 21 Let α,β be the roots of the equation $x^2-\sqrt{6}x+3=0$ such that $\mathrm{Im}(\alpha)>\mathrm{Im}(\beta)$. Let a, b be integers not divisible by 3 and n be a natural number such that $\frac{\alpha^{99}}{\beta}+\alpha^{98}=3^n(a+ib), i=\sqrt{-1}$. Then $n+a+b$ is equal to_____.

Correct Answer:
49

Solution:

$x^2 - \sqrt{6}x + 3 = 0 \begin{matrix} -\alpha \\ \searrow \beta \end{matrix}$

$x = \frac{\sqrt{6} \pm i\sqrt{6}}{2} = \frac{\sqrt{6}}{2}(1 \pm i)$

$\alpha = \sqrt{3}\left(e^{i\frac{\pi}{4}}\right), \beta = \sqrt{3}\left(e^{-i\frac{\pi}{4}}\right)$

$\therefore \frac{\alpha^{99}}{\beta} + \alpha^{98} = \alpha^{98}\left(\frac{\alpha}{\beta} + 1\right)$

$= \frac{\alpha^{98}(\alpha + \beta)}{\beta} = 3^{49}\left(e^{i99\frac{\pi}{4}}\right) \times \sqrt{2}$

$= 3^{49}(-1 + i)$

$= 3^n(a + ib)$

$\therefore n = 49, a = -1, b = 1$

$\therefore n + a + b = 49 - 1 + 1 = 49$

Hence, the answer is (49).

Q. 22 Let the slope of the line $45x + 5y + 3 = 0$ be $27r_1 + \frac{9r_2}{2}$ for some $r_1, r_2 \in R$. Then $\lim_{x\to 3}\left(\int_3^x \frac{8t^2}{\frac{3r_2x}{2} - r_2x^2 - r_1x^3 - 3x}dt\right)$ is equal to_____.

Correct Answer:
12

Solution:
According to the question,

$27r_1 + \frac{9r_2}{2} = -9 \Rightarrow 6r_1 + r_2 = -2$

$\lim_{x\to 3} \frac{\int_3^x 8t^2 dt}{\frac{3r_2x}{2} - r_2x^2 - r_1x^3 - 3x}$

$= \lim_{x\to 3} \frac{8x^2}{\frac{3r_2^2}{2} - 2r_2x - 3r_1x^2 - 3}$ (using LH' Rule)

$$= \lim_{x \to 3} \frac{72}{\frac{3r_2}{2} - 6r_2 - 27r_1 - 3}$$

$$= \lim_{x \to 3} \frac{144}{3r_2 - 12r_2 - 54r_1 - 6}$$

$$= \lim_{x \to 3} \frac{144}{-9\left(r_2 + 6r_1\right) - 6} = \lim_{x \to 3} \frac{144}{18 - 6} = 12$$

Hence, the answer is (12).

Q. 23 Let $\mathrm{P}(\alpha, \beta)$ be a point on the parabola $\mathrm{y}^2 = 4\mathrm{x}$. If P also lies on the chord of the parabola $\mathrm{x}^2 = 8\mathrm{y}$ whose mid point is $\left(1, \frac{5}{4}\right)$, then $(\alpha - 28)(\beta - 8)$ is equal to_____.

Correct Answer:
192

Solution:
Parabola is $x^2 = 8y$

Chord with mid point $(\mathrm{x}_1, \mathrm{y}_1)$ is $\mathrm{T} = \mathrm{S}_1$

$$\therefore \mathrm{x}\mathrm{x}_1 - 4\left(\mathrm{y}_1\mathrm{y}_1\right) = \mathrm{x}_1^2 - 8\mathrm{y}_1$$

$$\therefore (\mathrm{x}_1, \mathrm{y}_1) = \left(1, \frac{5}{4}\right)$$

$$\Rightarrow \mathrm{x} - 4\left(\mathrm{y} + \frac{5}{4}\right) = 1 - 8 \times \frac{5}{4} = -9$$

$\therefore x - 4y + 4 = 0$

(α, β) lies on (i) \& also on $y^2 = 4x$

$\therefore \alpha - 4\beta + 4 = 0$

$\& \beta^2 = 4\alpha \ldots\ldots$ (iii)

Solving (ii) & (iii)

$\beta^2 = 4(4\beta - 4) \Rightarrow \beta^2 - 16\beta + 16 = 0$

$\therefore \beta = 8 \pm 4\sqrt{3}$ and $\alpha = 4\beta - 4 = 28 \pm 16\sqrt{3}$

$\therefore (\alpha, \beta) = (28 + 16\sqrt{3}, 8 + 4\sqrt{3}) \& (28 - 16\sqrt{3}, 8 - 4\sqrt{3})$

$\therefore |(\alpha - 28)(\beta - 8)| = |16\sqrt{3} \times 4\sqrt{3}| = 192$

Hence, the answer is 192.

Q. 24 Let O be the origin, and M and N be the points on the lines $\frac{x-5}{4} = \frac{y-4}{1} = \frac{z-5}{3}$ and $\frac{x+8}{12} = \frac{y+2}{5} = \frac{z+11}{9}$ respectively such that MN is the shortest distance between the given lines. Then $\overrightarrow{OM} \cdot \overrightarrow{ON}$ is equal to_____.

Correct Answer:
9

Solution:

$$L_1 : \frac{x-5}{4} = \frac{y-4}{1} = \frac{z-5}{3} = \lambda \quad drs(4,1,3) = b_1$$

$$M(4\lambda+5, \lambda+4, 3\lambda+5)$$

$$L_2 : \frac{x+8}{12} = \frac{y+2}{5} = \frac{z+11}{9} = \mu$$

$$N(12\mu-8, 5\mu-2, 9\mu-11)$$

$$MN = (4\lambda - 12\mu + 13, \lambda - 5\mu + 6, 3\lambda - 9\mu + 16) \ldots (1)$$

Now,

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 1 & 3 \\ 12 & 5 & 9 \end{vmatrix} = -6\hat{i} + 8\hat{k} \qquad \ldots . (2)$$

Equation (1) and (2)

$$\therefore \frac{4\lambda-12\mu+13}{-6} = \frac{\lambda-5\mu+6}{0} = \frac{3\lambda-9\mu+16}{8}$$

I and II

$$\lambda - 5\mu + 6 = 0 \qquad \ldots . (3)$$

I and III

$$\lambda - 3\mu + 4 = 0 \qquad \ldots . (4)$$

Solve (3) and (4) we get

$\lambda = -1, \mu = 1$

$\therefore \mathrm{M}(1, 3, 2)$
$\mathrm{N}(4, 3, -2)$

$\therefore \quad \overrightarrow{\mathrm{OM}} \cdot \overrightarrow{\mathrm{ON}} = 4 + 9 - 4 = 9$

Hence, the answer is (9).

Q. 25 Let for any three distinct consecutive terms a, b, c of an A.P, the lines $ax + by + c = 0$ be concurrent at the point P and $\mathrm{Q}(\alpha, \beta)$ be a point such that the system of equations

$x + y + z = 6$

$2x + 5y + \alpha z = \beta$ and

$x + 2y + 3z = 4$, has infinitely many solutions. Then $(PQ)^2$ is equal to_____.

Correct Answer:
113

Solution:
$\because$ a, b, c and in A.P

$\Rightarrow 2b = a + c \Rightarrow a - 2b + c = 0$\therefore \mathrm{ax}+\mathrm{by}+\mathrm{c}$

passes through fixed point $(1, -2)$

therefore {P}=(1,-2)

For infinite solution,

$D = D_1 = D_2 = D_3 = 0$

$$D: \begin{vmatrix} 1 & 1 & 1 \\ 2 & 5 & \alpha \\ 1 & 2 & 3 \end{vmatrix} = 0$$

$\Rightarrow \alpha = 8$

$$D_1: \begin{vmatrix} 6 & 1 & 1 \\ \beta & 5 & \alpha \\ 4 & 2 & 3 \end{vmatrix} = 0 \Rightarrow \beta = 6$$

$\therefore \mathrm{Q} = (8, 6)$

$\therefore \mathrm{PQ}^2 = 113$

Hence, the answer is 113.

Q. 26 Remainder when $64^{32^{32}}$ is divided by 9 is equal to_____

Correct Answer:
1

Solution:
Let $32^{32} = \mathrm{t}$

$64^{32^{32}} = (1+63)^t$

$= 1 + t \cdot 63 + {}^tC_2 63^2 + \ldots. + 63^t$

$= 1 + 9\left[7\mathrm{t} + {}^tC_2 \cdot 9 \cdot 7^2 + \ldots. + 7^t \cdot 9^{t-1}\right]$

$= 1 + 9\mathrm{k}$

Hence remainder = 1

Hence, the answer is (1).

Q. 27 Let $f(x) = \sqrt{\lim_{r\to x}\left\{\frac{2r^2[(f(r))^2 - f(x)f(r)]}{r^2-x^2} - r^3 e^{\frac{f(r)}{r}}\right\}}$ be differentiable in $(-\infty, 0) \cup (0, \infty)$ and $f(1) = 1$. Then the value of ea, such that $f(a) = 0$, is equal to_____.

Correct Answer:
2

Solution:
f(1)=1, f(a) = 0

$$f^2(x) = \mathrm{Lim}_{r\to x}\left(\frac{2r^2\left(f^2(r) - f(x)f(r)\right)}{r^2 - x^2} - r^3 e^{\frac{f(r)}{r}}\right)$$

$$= \mathrm{Lim}_{r\to x}\left(\frac{2r^2 f(r)}{r+x}\frac{(f(r) - f(x))}{r-x} - r^3 e^{\frac{f(r)}{r}}\right)$$

$$f^2(x) = \frac{2x^2 f(x)}{2x} f'(x) - x^3 e^{\frac{f(x)}{x}}$$

$$y^2 = xy\frac{dy}{dx} - x^3 e^{\frac{y}{x}}$$

$$\frac{y}{x} = \frac{dy}{dx} - \frac{x^2}{y} e^{\frac{y}{x}}$$

Put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$v = v + x\frac{dv}{dx} - \frac{x}{v}e^v$

$\frac{dv}{dx} = \frac{e^v}{v} \Rightarrow e^{-v}vdv = dx$

Integrating both sides

$\mathrm{e^v(x + c) + 1 + v = 0}$

$\mathrm{f(1) = 1 \Rightarrow x = 1, y = 1}$

$\Rightarrow \mathrm{c} = -1 - \dfrac{2}{\mathrm{e}}$

$e^v\left(-1 - \frac{2}{e} + x\right) + 1 + v = 0$

$e^{\frac{y}{x}}\left(-1 - \frac{2}{e} + x\right) + 1 + \frac{y}{x} = 0$

$x = a, y = 0 \Rightarrow a = \frac{2}{e}$

$ae = 2$

Hence, the answer is (2).

Q. 28 Let the set $C = \{(x, y) \mid x^2 - 2^y = 2023, x, y \in \mathbb{N}\}$. Then $\sum_{(x,y)=C}(x + y)$ is equal to_____.

Correct Answer:
46

Solution:
$x^2 - 2^y = 2023$

$\Rightarrow x = 45, y = 1$ (for $y \geq 2, 2^y + 2023$ will be of the form $4m + 3$)

$\sum_{(x,y)\in C}(x + y) = 46$

Hence, the answer is 46.

Q. 29 If $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sqrt{1-\sin 2x}dx = \alpha + \beta\sqrt{2} + \gamma\sqrt{3}$, where α, β and γ are rational numbers, then $3\alpha + 4\beta - \gamma$ is equal to_____.

Correct Answer:
6

Solution:

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sqrt{1-\sin 2x}dx$$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} |\sin x - \cos x|dx$$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} (\cos x - \sin x)dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} (\sin x - \cos x)dx$$

$$= -1 + 2\sqrt{2} - \sqrt{3}$$

$$= \alpha + \beta\sqrt{2} + \gamma\sqrt{3}$$

$$\alpha = -1, \beta = 2, \gamma = -1$$

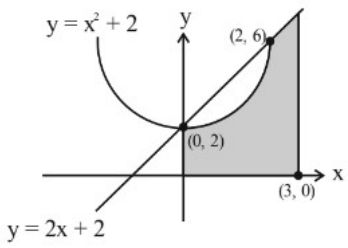
$$3\alpha + 4\beta - \gamma = 6$$

Hence, the answer is (6).

Q. 30 Let the area of the region $\{(x, y) : 0 \leq x \leq 3, 0 \leq y \leq \min\{x^2 + 2, 2x + 2\}\}$ be A. Then $12\ A$ is equal to_____.

Correct Answer:
164

Solution:



$$A = \int_0^2 (x^2 + 2)dx + \int_2^3 (2x + 2)dx$$

$$A = \frac{41}{3}$$

$$12A = 41 \times 4 = 164$$

CAREERS360

30 Jan - Morning Shift

Physics

Q. 1 Match List - I with List - II.

List – I	List – II
(A) Coefficient of viscosity	(I) $\left[\mathrm{ML}^2\ \mathrm{T}^{-2}\right]$
(B) Surface tension	(II) $\left[\mathrm{ML}^2\ \mathrm{T}^{-1}\right]$
(C) Angular momentum	(III) $\left[\mathrm{ML}^{-1}\ \mathrm{T}^{-1}\right]$
(D) Rotational kinetic energy	(IV) $\left[\mathrm{ML}^0\ \mathrm{T}^{-2}\right]$

Choose the correct answer from the options given below :

Option 1:
(A)-(II), (B)-(I), (C)-(IV), (D)-(III)

Option 2:
(A)-(I), (B)-(II), (C)-(III), (D)-(IV)

Option 3:
(A)-(III), (B)-(IV), (C)-(II), (D)-(I)

Option 4:
(A)-(IV), (B)-(III), (C)-(II), (D)-(I)

Correct Answer:
(A)-(III), (B)-(IV), (C)-(II), (D)-(I)

Solution:
(A) Coefficient of viscosity (η)

$\mathrm{F} = 6\pi\eta\mathrm{rv}$

$\left[\mathrm{M}^1\ \mathrm{L}^1\ \mathrm{T}^{-2}\right] = [\eta]\ \left[\mathrm{M}^0\ \mathrm{L}^1\ \mathrm{T}^0\right]\ \left[\mathrm{M}^0\ \mathrm{L}^1\ \mathrm{T}^{-1}\right]$

$[\eta] = \frac{[M^1 L^L T^{-2}]}{[M^0 L^1 T^0][M^0 L^1 T^{-1}]} = [M^1 L^{-1} T^{-1}] \rightarrow$ (III)

(B) Surface Tension $(T) = \frac{F}{\ell} = \frac{[M^1 L^1 T^{-2}]}{[M^0 L^1 T^0]} = [M^1 L^0 T^{-2}] \rightarrow (IV)$

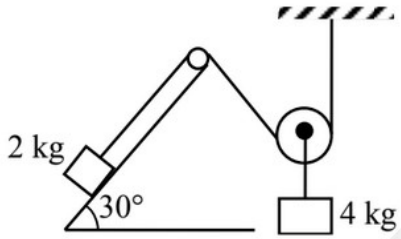
(C) Angular momentum (L) = mvr

$[M^1 L^0 T^0][M^0 L^1 T^{-1}][M^0 L^1 T^0]$
$= [M^1 L^2 T^{-1}] \rightarrow (II)$

(D) Rotational kinetic energy $[M^1 L^2 T^{-2}] \rightarrow (I)$

Hence, the answer is the option(3).

Q. 2 All surfaces shown in figure are assumed to be frictionless and the pulleys and the string are light. The acceleration of the block of mass $2\ kg$ is :



Option 1:
g

Option 2:
$\frac{g}{3}$

Option 3:
$\frac{g}{2}$

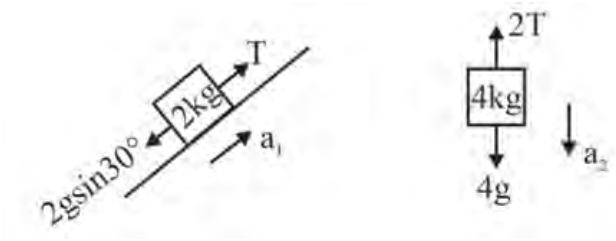
Option 4:
$\frac{g}{4}$

Correct Answer:
$\frac{g}{3}$

Solution:
By constrained motion

$-a_1 + a_2 + a_2 = 0$

$a_1 - 2a_2$ or $a_2 = \frac{a_1}{2}$



$T - 2\,g \sin 30^\circ = 2a_1 \quad 4\,g - 2\,T = 4a_2$

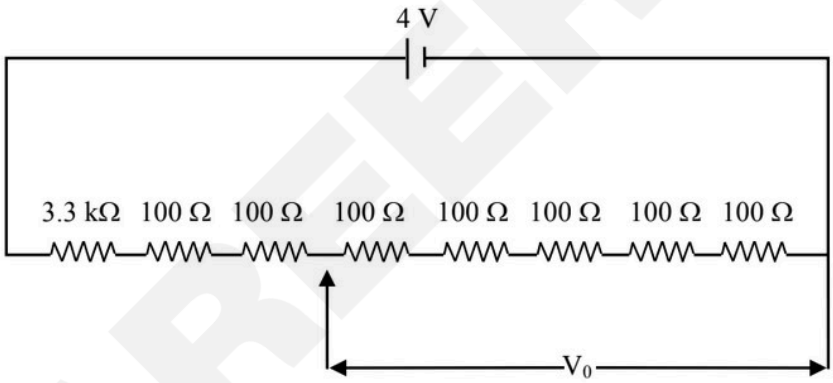
$\Rightarrow T - g = 2a_1 \quad \ldots.(1) \Rightarrow 4\,g - 2\,T = 4\left[\frac{a_1}{2}\right]$

$\Rightarrow 2\,g - T = a_1$

adding equation (1) and (2)

$g = 3a_1 \Rightarrow a_1 = \frac{g}{3}$

Q. 3 A potential divider circuit is shown in figure. The output voltage V_0 is :



Option 1:
4 V

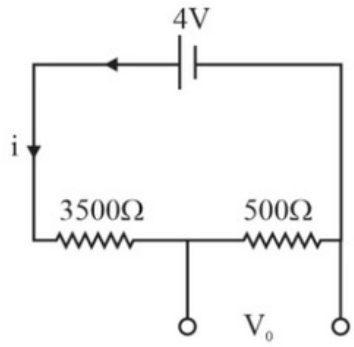
Option 2:
2 mV

Option 3:
0.5 V

Option 4:
12 m V

Correct Answer:
0.5 V

Solution:



$i = \frac{4V}{4000\Omega} = \frac{1}{1000} A$

$V_0 = (i)\,(500\Omega)$

$= \frac{1}{1000} \times 500 = 0.5$ volt

Q. 4 Young's modules of material of a wire of length ' L ' and cross-sectional area A is Y. If the length of the wire is doubled and cross-sectional area is halved then Young's modules will be:

Option 1:
$\frac{Y}{4}$

Option 2:
4 Y

Option 3:
Y

Option 4:
2Y

Correct Answer:
Y

Solution:
From the given relation-

$$Y = \frac{longitudinal\ stress}{longitudinal\ strain} = \frac{F/A}{\Delta L/L} = \frac{Fl}{A\Delta L}$$

Young's modulus(Y) remains the same for a given material, it does not depend on the dimensions of the rod. So, If the length of the wire is doubled and the cross-sectional area is halved then Young's modules will same as Y.

Hence, the answer is the option(3).

Q. 5 The work function of a substance is 3.0 eV. The longest wavelength of light that can cause the emission of photoelectrons from this substance is approximately;

Option 1:
215 nm

Option 2:
414 nm

Option 3:
400 nm

Option 4:
200 nm

Correct Answer:
414 nm

Solution:
Work function = 3eV

$\frac{hc}{\lambda_{Th}} = 3eV$ or $\lambda_{Th} = \frac{12420}{3}$ (Å)

= 414 (nm) Approx

Q. 6 The ratio of the magnitude of the kinetic energy to the potential energy of an electron in the 5^{th} excited state of a hydrogen atom is :

Option 1:
4

Option 2:
$\frac{1}{4}$

Option 3:
$\frac{1}{2}$

Option 4:
1

Correct Answer:
$\frac{1}{2}$

Solution:
In Bohr's model,

The energy of an electron in the n^{th} orbit-

Potential energy:

$$U = k\frac{(Ze)(-e)}{r_n} = -\frac{kZe^2}{r_n}$$

Kinetic energy:

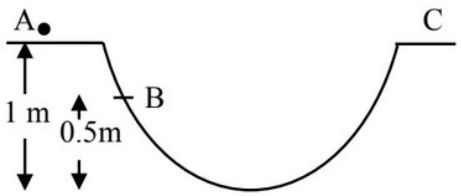
$$K = \frac{mv^2}{r_n} = \frac{k(Ze)(e)}{r_n^2}$$

$$K = \frac{kZe^2}{2r_n} = \frac{|U|}{2}$$

$$\frac{|K|}{|U|} = \frac{1}{2}$$

Hence, the answer is the option (3).

Q. 7 A particle is placed at the point A of a frictionless track ABC as shown in figure. It is gently pushed towards right. The speed of the particle when it reaches the point B is : (Take $g = 10\ \mathrm{m/s^2}$).



Option 1:
20 m/s

Option 2:
$\sqrt{10}\ \mathrm{m/s}$

Option 3:
$2\sqrt{10}\ \mathrm{m/s}$

Option 4:
10 m/s

Correct Answer:
$\sqrt{10}\ \mathrm{m/s}$

Solution:
by energy conservation

$$|\Delta \mathrm{PE}| = |\Delta \mathrm{KE}|$$

$$\mathrm{mg}\left(\mathrm{h_2} - \mathrm{h_1}\right) = \left(\tfrac{1}{2}\mathrm{mv^2} - 0\right)$$

$$\mathrm{mg}[0.5] = \tfrac{1}{2}\mathrm{mv^2}$$

Q. 8 The electric field of an electromagnetic wave in free space is represented as $\overrightarrow{\mathrm{E}} = \mathrm{E_0}\cos(\omega \mathrm{t} - \mathrm{kz})\hat{\mathrm{i}}$. The corresponding magnetic induction vector will be :

Option 1:

$\overrightarrow{\mathrm{B}}=\mathrm{E}_0\mathrm{C}\cos(\omega\mathrm{t}-\mathrm{kz})\hat{\mathrm{j}}$

Option 2:

$\vec{B}=\frac{E_0}{C}\cos(\omega t-kz)\hat{j}$

Option 3:

$\vec{B}=E_0C\cos(\omega t+kz)\hat{j}$

Option 4:

$\vec{B}=\frac{E_0}{C}\cos(\omega t+kz)\hat{j}$

Correct Answer:

$\vec{B}=\frac{E_0}{C}\cos(\omega t-kz)\hat{j}$

Solution:

Given -

Electric filed is

$\overrightarrow{\mathrm{E}}=\mathrm{E}_0\cos(\omega\mathrm{t}-\mathrm{kz})\hat{\mathrm{i}}$

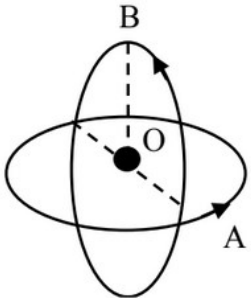
The magnitude of the electric and the magnetic fields in an electromagnetic wave are related as -

$\mathrm{E}_0=\mathrm{B}_0\mathrm{c},\Rightarrow\left(\mathrm{B}_0=\frac{\mathrm{E}_0}{\mathrm{c}}\right)$

therefore $\vec{B}=\frac{E_0}{C}\cos(\omega t-kz)\hat{j}$

Hence, the answer is the option (1).

Q. 9 Two insulated circular loop A and B of radius 'a' carrying a current of 'I' in the anti clockwise direction as shown in the figure. The magnitude of the magnetic induction at the centre will be:



Option 1:
$\frac{\sqrt{2}\mu_0 I}{a}$

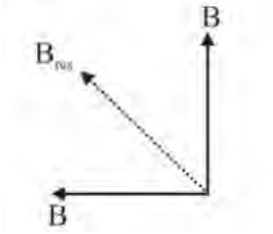
Option 2:
$\frac{\mu_0 I}{2a}$

Option 3:
$\frac{\mu_0 I}{\sqrt{2a}}$

Option 4:
$\frac{2\mu_0 I}{a}$

Correct Answer:
$\frac{\mu_0 I}{\sqrt{2a}}$

Solution:



$B = \frac{\mu_0 I}{2a};$

$B_{res} = \sqrt{2}\, B \qquad (resultant)$

$= \sqrt{2} \cdot \frac{\mu_0 I}{2a}$

$= \frac{\mu_0 I}{\sqrt{2a}}$

Q. 10 The diffraction pattern of a light of wavelength 400 nm diffracting from a slit of width 0.2 mm is focused on the focal plane of a convex lens of focal length 100 cm. The width of the 1st secondary maxima will be :

Option 1:
2 mm

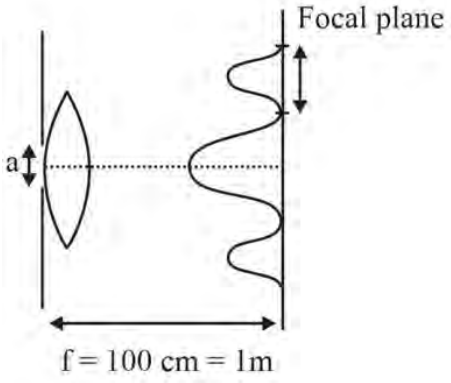
Option 2:
2 cm

Option 3:
2 cm

Option 4:
0.2 mm

Correct Answer:
2 mm

Solution:



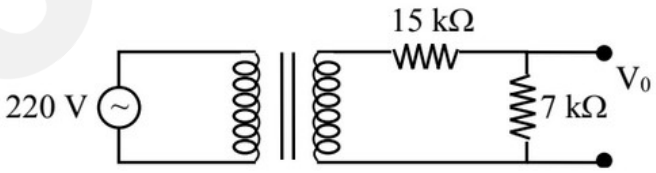
width of secondary maxima $= \frac{\lambda f}{a}$

$= \frac{400 \times 10^{-9} \times 1}{0.2 \times 10^{-3}} = \frac{2 \times 10^{-7}}{10^{-4}}$

$= 2 \times 10^{-3}\ \text{m}$

$= 2\ \text{mm}$

Q. 11 Primary coil of a transformer is connected to $220\ \text{V}$ ac. Primary and secondary turns of the transforms are 100 and 10 respectively. Secondary coil of transformer is connected to two series resistances shown in figure. The output voltage (V_0) is :



Option 1:
7 V

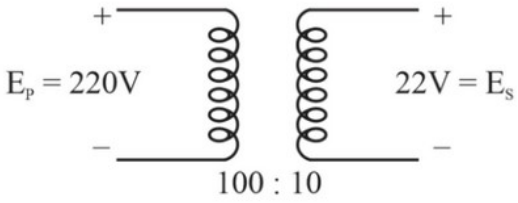
Option 2:
15 V

Option 3:
44 V

Option 4:
22 V

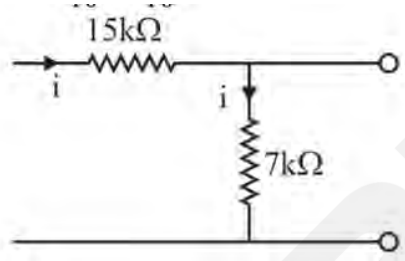
Correct Answer:
7 V

Solution:



$$\frac{E_P}{E_S} = \frac{N_P}{N_S} = \frac{100}{10} = 10$$

$$E_S = \frac{E_P}{10} = \frac{220}{10} = 22V$$



$$V_0 = (i)(7k\Omega)$$
$$= \frac{22}{(15+7)k\Omega} \times 7k\Omega$$
$$V_0 = 7\ V$$

Q. 12 The gravitational potential at a point above the surface of the earth is $-5.12 \times 10^7\ J/kg$ and the acceleration due to gravity at that point is $6.4\ m/s^2$. Assume that the mean radius of the earth is $6400\ km$. The height of this point above the earth's surface is :

Option 1:
1600 km

Option 2:
540 km

Option 3:
1200 km

Option 4:
1000 km

Correct Answer:
1600 km

Solution:
Let r be the distance of the given point from the centre of the earth. Then Gravitational potential

$$V_g = \frac{-GM}{r}$$

r=R+h

$$V_g = \frac{-GM}{(R+h)} = -5.12 \times 10^7 \text{ J/kg}$$

$$g = \frac{GM}{(R+h)^2} = 6.4 \text{ m/s}^2$$

$$\frac{V_g}{g} = -(R+h)$$

$$\Rightarrow \frac{-5.12 \times 10^7}{6.4} = -(R+h)$$

$$R + h = 0.8 \times 10^7$$

$$6400 \text{ km} + h = 8000 \text{ km}$$

$$h = 1600 \text{ km}$$

Hence, the answer is the option (1).

Q. 13 An electric toaster has resistance of 60Ω at room temperature (27°C). The toaster is connected to a 220 V supply. If the current flowing through it reaches 2.75 A, the temperature attained by toaster is around :
(if $\alpha = 2 \times 10^{-4}/^\circ\text{C}$)

Option 1:
694 °C

Option 2:
1235 °C

Option 3:
1694 °C

Option 4:
1667 °C

Correct Answer:
1694 °C

Solution:
Given -

$i = 2.75 \text{ A}. \text{ V } = 220 \text{ V}$
$R_{\mathrm{T}} = \frac{V}{i} = \frac{220}{2.75}\Omega = 80\Omega$

$R_0 = 60\Omega$
$R_{\mathrm{T}} = R_0 \left[1 + \alpha \Delta T\right]$

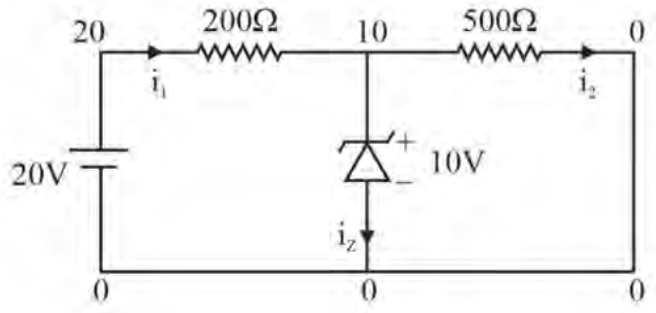
Putting all values in the above equation, we get

$80 = 60 \left[1 + 2 \times 10^{-4} \Delta \mathrm{T}\right]$

$\Delta \mathrm{T} = 1666.67$
$\mathrm{T} - 27 = 1666.67$
$\mathrm{T} = 1693.66 = 1694^{\circ}\mathrm{C}$

Hence, the answer is the option (3).

Q. 14 A Zener diode of breakdown voltage $10\ \mathrm{V}$ is used as a voltage regulator as shown in the figure. The current through the Zener diode is:



Option 1:
50 mA

Option 2:
0

Option 3:
30 mA

Option 4:
20 mA

Correct Answer:
30 mA

Solution:

$$i_1 = \frac{20-10}{200} = \frac{1}{20}$$

$$i_2 = \frac{10-0}{500} = \frac{1}{50}$$

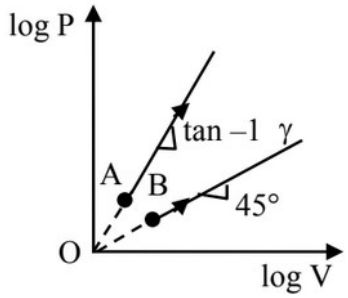
$$i_z = i_1 - i_2$$

$$= \frac{1}{20} - \frac{1}{50}$$

$$= \frac{5-2}{100}$$

$$= \frac{3}{100}\ \mathrm{A} = 30\ \mathrm{mA}$$

Q. 15 Two thermodynamical processes are shown in the figure. The molar heat capacity for process A and B are $\mathrm{C_A}$ and $\mathrm{C_B}$. The molar heat capacity at constant pressure and constant volume are represented by $\mathrm{C_P}$ and $\mathrm{C_V}$, respectively. Choose the correct statement.



Option 1:
$C_B = \infty, C_A = 0$

Option 2:
$\mathrm{C_A} = 0$ and $\mathrm{C_B} = \infty$

Option 3:
$\mathrm{C_P} > \mathrm{C_V} > \mathrm{C_A} = \mathrm{C_B}$

Option 4:
$\mathrm{C_A} > \mathrm{C_P} > \mathrm{C_V}$

Correct Answer:
$\mathrm{C_A} > \mathrm{C_P} > \mathrm{C_V}$

Solution:
Bonus

Q. 16 The electrostatic potential due to an electric dipole at a distance ' r ' varies as :

Option 1:
r

Option 2:
$\frac{1}{\mathrm{r^2}}$

Option 3:
$\frac{1}{r^3}$

Option 4:
$\frac{1}{r}$

Correct Answer:
$\frac{1}{r^2}$

Solution:
Electric potential due to an electric dipole

$$V = \frac{1}{4\pi\varepsilon_0}\frac{P}{r^2}$$

$$V \propto \frac{1}{r^2}$$

Hence, the answer is the option (2).

Q. 17 A spherical body of mass $100\ g$ is dropped from a height of $10\ m$ from the ground. After hitting the ground, the body rebounds to a height of $5\ m$. The impulse of force imparted by the ground to the body is given by :

(given, $g = 9.8\ m/s^2$)

Option 1:
4.32 kg ms^{-1}

Option 2:
43.2 kg ms^{-1}

Option 3:
23.9 kg ms^{-1}

Option 4:
2.39 kg ms^{-1}

Correct Answer:
2.39 kg ms^{-1}

Solution:

$m = 100\ g = 0.1\ kg,\ h_1 = 10\ m,\ h_2 = 5\ m$

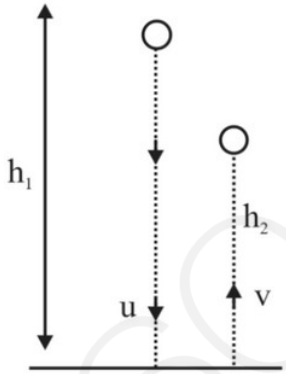
$u = \sqrt{2gh_1} = \sqrt{20\ g} = \sqrt{2 \times 98}\ m/s$
$v = \sqrt{2gh_2} = \sqrt{10\ g} = \sqrt{98}\ m/s$

Impuse $= \Delta\vec{P} = \vec{P_f} - \vec{p_i}$

$= mv - (-mu)$
$= m(v + u)$

$= 0.1[\sqrt{98} + \sqrt{2 \times 98}]$
$= 0.1\sqrt{98}[1 + \sqrt{2}]$

$\approx 2.39\ kg\ m/s$



Q. 18 A particle of mass m is projected with a velocity ' u ' making an angle of 30° with the horizontal. The magnitude of angular momentum of the projectile about the point of projection when the particle is at its maximum height h is :

Option 1:
$\frac{\sqrt{3}}{16}\frac{mu^3}{g}$

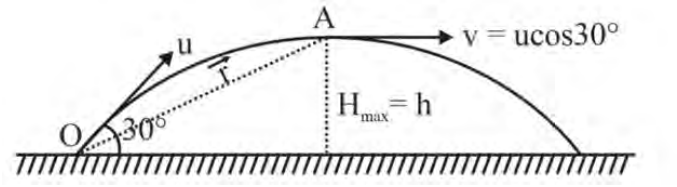
Option 2:
$\frac{\sqrt{3}}{2}\frac{mu^3}{g}$

Option 3:
$\frac{mu^3}{\sqrt{2}\,g}$

Option 4:
Zero

Correct Answer:
$\frac{\sqrt{3}}{16}\frac{mu^3}{g}$

Solution:



velocity of particle at maximum height

$$v = u\cos 30^\circ = \frac{u\sqrt{3}}{2}$$

maximum height

$$h = \frac{u^2 \sin^2 30^\circ}{2g} = \frac{u^2}{8g}$$

magnitude of angular momentum

$$|\vec{L}| = hmv$$

$$= \left(\frac{u^2}{8g}\right) m \left(\frac{u\sqrt{3}}{2}\right)$$

$$= \frac{\sqrt{3}mu^3}{16g}$$

Q. 19 At which temperature the r.m.s. velocity of a hydrogen molecule equal to that of an oxygen molecule at $47^\circ C$?

Option 1:
80 K

Option 2:
–73 K

Option 3:
4 K

Option 4:
20 K

Correct Answer:
20 K

Solution:

$$V_{rms} = \sqrt{\frac{3RT}{M}}$$

$$\Rightarrow (V_{rms})_{H_2} = (V_{rms})_{O_2}$$

$$\Rightarrow \sqrt{\frac{3RT_1}{M_1}} = \sqrt{\frac{3RT_2}{M_2}}$$

$$\Rightarrow \frac{T_1}{M_1} = \frac{T_2}{M_2}$$

$$\Rightarrow T_1 = \frac{M_1}{M_2} T_2$$

The molar mass of hydrogen $M_1 = 2$ and The molar mass of oxygen $M_2 = 32$

$$T_1 = \frac{2}{32}(273 + 47)$$

$$= \frac{2}{32} \times 320 = 20\ K$$

Hence, the answer is the option (4).

Q. 20 A series L.R circuit connected with an ac source $E = (25 \sin 1000t)V$ has a power factor of $\frac{1}{\sqrt{2}}$. If the source of emf is changed to $E = (20 \sin 2000t)V$, the new power factor of the circuit will be :

Option 1:
$\frac{1}{\sqrt{2}}$

Option 2:
$\frac{1}{\sqrt{3}}$

Option 3:
$\frac{1}{\sqrt{5}}$

Option 4:
$\frac{1}{\sqrt{7}}$

Correct Answer:
$\frac{1}{\sqrt{5}}$

Solution:
For the first source power factor

$$\cos\phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + \omega^2 L^2}}$$

$$\Rightarrow \frac{1}{\sqrt{2}} = \frac{R}{\sqrt{R^2 + \omega^2 L^2}}$$

$$\Rightarrow R^2 + \omega^2 L^2 = 2R^2$$

$$\Rightarrow R = \omega L$$

for the second source

$$\omega' = 2000 = 2\omega$$

Power Factor $= \frac{R}{\sqrt{R^2+\omega'^2 L^2}}$

$$= \frac{\omega L}{\sqrt{\omega^2 L^2 + 4\omega^2 L^2}}$$

$$= \frac{\omega L}{\sqrt{5\omega^2 L^2}}$$

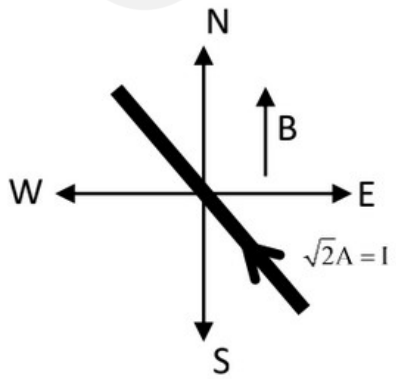
$$= \frac{1}{\sqrt{5}}$$

Hence, the answer is the option (3).

Q. 21 The horizontal component of earth's magnetic field at a place is 3.5×10^{-5} T. A very long straight conductor carrying current of $\sqrt{2}$ A in the direction from South east to North West is placed. The force per unit length experienced by the conductor is _____10^{-6} N/m.

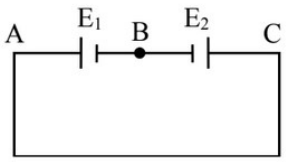
Correct Answer:
35

Solution:



$$F = I\ell B \sin\theta$$
$$\frac{F}{\ell} = IB \sin\theta$$
$$= \sqrt{2} \times 3.5 \times 10^{-5} \times \sin 45^\circ$$
$$= \sqrt{2} \times 3.5 \times 10^{-5} \times \frac{1}{\sqrt{2}}$$
$$= 3.5 \times 10^{-5}$$
$$= 35 \times 10^{-6}\ \mathrm{N/m}$$

Q. 22 Two cells are connected in opposition as shown. Cell E_1 is of $8\ V$ emf and 2Ω internal resistance; the cell E_2 is of $2\ V$ emf and 4Ω internal resistance. The terminal potential difference of cell E_2 is _____V.

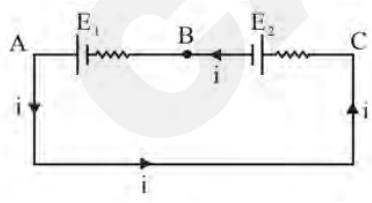


Correct Answer:
6

Solution:
$\varepsilon_{eq} = E_1 - E_2 = (8-2)V = 6V$
$r_{eq} = r_1 + r_2 = (2+4)\Omega = 6\Omega$

Current in the loop,

$i = \frac{\varepsilon_{eq}}{r_{eq}} = \frac{6\ V}{6\Omega} = 1\ A$



Terminal potential difference of cell $E_2 \Rightarrow E_2 + ir_2$

$\Rightarrow [2 + (1)(4)]V$
$\Rightarrow 6\ V$

Q. 23 An electron of a hydrogen atom in an excited state has energy $\mathrm{E_n = -0.85eV}$. The maximum number of allowed transitions to lower energy level is_____

Correct Answer:
6

Solution:
The energy of the n[th] level of hydrogen atom (z=1) is given as

$$E_n = -\frac{z^2 13.6}{n^2}eV = -\frac{13.6}{n^2}eV \qquad (\because z = 1)$$

$$-0.85 = \frac{-13.6}{\mathrm{n^2}}$$

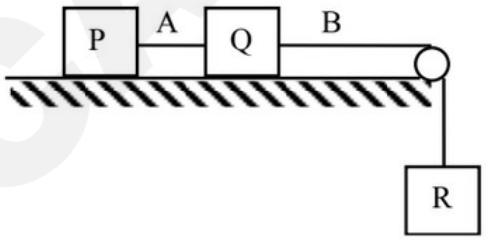
$$\Rightarrow \mathrm{n^2} = \frac{-13.6}{0.85} = 16$$

n = 4
maximum number of transitions

$= \frac{4\times3}{2} = 6$

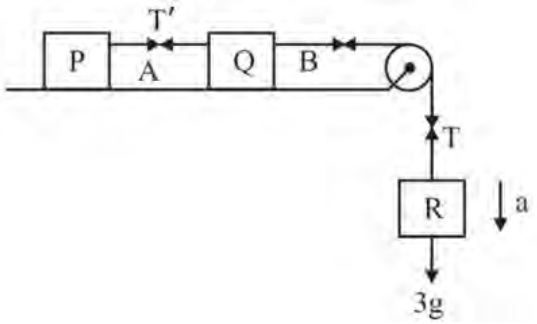
Hence, the answer is (6).

Q. 24 Each of three blocks P, Q and R shown in figure has a mass of $3\ \mathrm{kg}$. Each of the wires A and B has crosssectional area $0.005\ \mathrm{cm^2}$ and Young's modulus $2 \times 10^{11}\ \mathrm{N\ m^{-2}}$. Neglecting friction, the longitudinal strain on wire B _____is $\times 10^{-4}$. (Take $g = 10\ \mathrm{m/s^2}$)



Correct Answer:
2

Solution:



$3g = (3 + 3 + 3)a$

$\Rightarrow \mathrm{a} = \frac{3}{9}\,\mathrm{g} = \frac{1}{3}\,\mathrm{g}$

Now, 3g – T = 3a

$\Rightarrow \mathrm{T} = 3(\mathrm{g} - \mathrm{a}) = 3\left(\mathrm{g} - \frac{\mathrm{g}}{3}\right) = 2\,\mathrm{g} = 20\ \mathrm{N}$

$\mathrm{Y} = \frac{\text{longitudinal stress}}{\text{longitudinal strain}}$

longitudinal strain $= \frac{\text{longitudinal stress}}{\mathrm{Y}}$

$= \frac{T}{AY}$

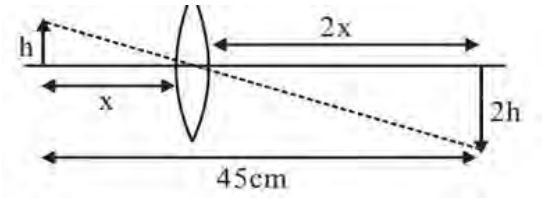
$= \frac{20}{5 \times 10^{-7} \times 2 \times 10^{11}}$

$= 2 \times 10^{-4}$

Q. 25 The distance between object and its two times magnified real image as produced by a convex lens is $45\ \mathrm{cm}$. The focal length of the lens used is_____ cm.

Correct Answer:
10

Solution:



$3\mathrm{x} = 45\ \mathrm{cm} \quad \mathrm{u} = -15\ \mathrm{cm}$

$\mathrm{x} = 15\ \mathrm{cm} \quad \mathrm{v} = +30\ \mathrm{cm}$

By lens formula

$\frac{1}{\mathrm{v}}-\frac{1}{\mathrm{u}}=\frac{1}{f}$

$\frac{1}{30}-\frac{1}{-15}=\frac{1}{f}$

$\frac{1+2}{30}=\frac{1}{f}$

$f=10\ \mathrm{cm}$

Q. 26 The displacement and the increase in the velocity of a moving particle in the time interval of t to $(t+1)s$ are $125\ \mathrm{m}$ and $50\ \mathrm{m/s}$, respectively. The distance travelled by the particle in $(\mathrm{t}+2)^{\mathrm{th}}\mathrm{s}$ is _____m.

Correct Answer:
175

Solution:
$\mathrm{a}=\frac{\Delta \mathrm{V}}{\Delta \mathrm{t}}=\frac{50}{1}=50\ \mathrm{m/s^2}$

We know Displacement in the n^{th} second is-

$$S_n=u+\frac{a}{2}(2n-1)$$

displacement in $(t+1)^{th}$ second

$S_1 = 125$ m

$$\Rightarrow 125=u+\frac{50}{2}[2(\mathrm{t}+1)-1]$$

$$\Rightarrow \mathrm{u}=125-25[2\mathrm{t}+1]-----(1)$$

displacement in $(t+2)^{th}$ second

$$\mathrm{S}_2=\mathrm{u}+\frac{50}{2}[2(\mathrm{t}+2)-1]$$
$$=\mathrm{u}+25[2\mathrm{t}+3]-------(2)$$

using equation (1) in equation (2)

$S_2 = 125 - 25(2t+1) + 25(2t+3)$
$= 125 - 25 + 75$
$= 175\ \text{m}$

Hence, the answer is 175.

Q. 27 A capacitor of capacitance C and potential V has energy E. It is connected to another capacitor of capacitance $2\mathrm{C}$ and potential $2\ \mathrm{V}$. Then the loss of energy is $\frac{\mathrm{x}}{3}\mathrm{E}$, where x is_____

Correct Answer:
2

Solution:
The initial potential energy of the system

$$\Rightarrow \frac{1}{2}\mathrm{CV}^2 + \frac{1}{2}(2\mathrm{C})(2\ \mathrm{V})^2$$
$$\mathrm{U_i} = \mathrm{E} + 8\mathrm{E} = 9\mathrm{E}$$

Common potential differences across capacitors, finally

$$\Rightarrow \mathrm{V}_0 = \frac{\mathrm{C}_1\ \mathrm{V}_1 + \mathrm{C}_2\ \mathrm{V}_2}{\mathrm{C}_1 + \mathrm{C}_2} = \frac{\mathrm{CV} + 4\mathrm{CV}}{\mathrm{C} + 2\mathrm{C}} = \frac{5\ \mathrm{V}}{3}$$

The final potential energy of the system

$$\frac{1}{2}\mathrm{CV}_0^2 + \frac{1}{2}(2\mathrm{C})\mathrm{V}_0^2$$

$$\mathrm{U}_f = \frac{3\mathrm{C}}{2}\ \mathrm{V}_0^2 = \frac{3\mathrm{C}}{2}\left(\frac{5\ \mathrm{V}}{3}\right)^2 = \frac{1}{2}\mathrm{CV}^2\left(\frac{25}{3}\right) = \frac{25}{3}\mathrm{E}$$

Loss of energy

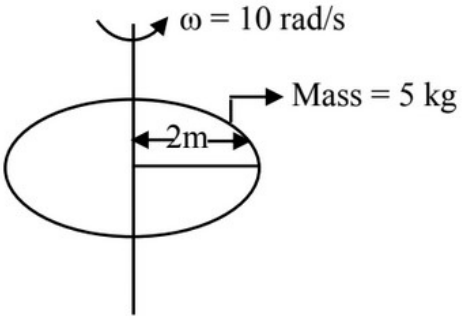
$$\Rightarrow \mathrm{U_i} - \mathrm{U}_f = 9\mathrm{E} - \frac{25\mathrm{E}}{3} = \frac{(27-25)\mathrm{E}}{3} = \frac{2\mathrm{E}}{3}$$

$$\Rightarrow \frac{\mathrm{xE}}{3} = \frac{2\mathrm{E}}{3}$$

$$\Rightarrow \mathrm{x} = 2$$

Hence, the answer is (2).

Q. 28



Consider a Disc of mass $5\ \mathrm{kg}$, radius $2\ \mathrm{m}$, rotating with angular velocity of $10\mathrm{rad/s}$ about an axis perpendicular to the plane of rotation. An identical disc is kept gently over the rotating disc along the same axis. The energy dissipated so that both the discs continue to rotate together without slipping is J.

Correct Answer:
250

Solution:
Net torque about axis of rotation is zero

$$\Rightarrow L_i = L_f$$
$$\Rightarrow I_i\omega_i = I_f\omega_f$$
$$\Rightarrow \left(\frac{5\times 2^2}{2}\right)\times 10 = \left(\frac{10\times 2^2}{2}\right)\omega_f$$

$$5 = \omega_f$$
$$\Rightarrow \omega_f = 5$$
$$\Delta E = E_i - E_f$$

$$= \tfrac{1}{2}I_i\omega_i^2 - \tfrac{1}{2}I_f\omega_f^2$$

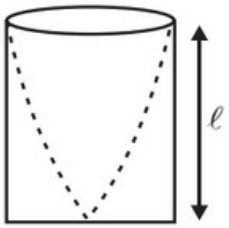
$$= \tfrac{1}{2}\left(\tfrac{5\times 2^2}{2}\right)\times 10^2 - \tfrac{1}{2}\left(\tfrac{10\times 2^2}{2}\right)5^2$$

= 500 – 250 = 250 J

Q. 29 In a closed organ pipe, the frequency of fundamental note is $30\ \text{Hz}$. A certain amount of water is now poured in the organ pipe so that the fundamental frequency is increased to $110\ \text{Hz}$. If the organ pipe has a cross-sectional area of $2\ \text{cm}^2$, the amount of water poured in the organ tube is _____ g. (Take speed of sound in air is $330\ \text{m/s}$)

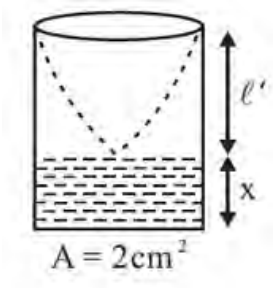
Correct Answer:
400

Solution:



Fundamental frequency $= \frac{v}{4\ell} = 30\ \text{Hz}$

$\frac{330}{4\ell} = 30 \Rightarrow \ell = \frac{11}{4}\ \text{m}$



New fundamental frequency $= \frac{v}{4\ell'} = 110\ \text{Hz}$

$\frac{330}{4(\ell')} = 110 \Rightarrow \ell' = \frac{3}{4}\ \text{m}$

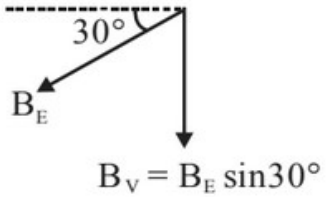
$x = \ell - \ell' = \left(\frac{11}{4} - \frac{3}{4}\right) \times 100\ \text{cm} \Rightarrow 200\ \text{cm}$

$$\begin{aligned}\text{mass of water} &= \rho \times V \\ &= 1\ \text{g/cm}^3 \times \left(2\ \text{cm}^2 \times 200\ \text{cm}\right) \\ &= 400\ \text{g}\end{aligned}$$

Q. 30 A ceiling fan having 3 blades of length $80\ \text{cm}$ each is rotating with an angular velocity of 1200rpm. The magnetic field of earth in that region is 0.5G and angle of dip is 30°. The emf induced across the blades is $\text{N}\pi \times 10^{-5}\ \text{V}$. The value of N is_____

Correct Answer:
32

Solution:



$$= \left[\left(0.5 \times 10^{-4}\right) \times \frac{1}{2}\right] \text{T}$$

$$\text{B} = 0.25 \times 10^{-4}\ \text{T}$$

$$\omega = 1200\text{rpm} = 1200 \times \frac{2\pi\text{rad}}{60\text{sec}}$$

$$40\pi\text{rad/s}$$

emf induced in rotating blades $= \frac{1}{2}\ \text{B}\omega\ell^2$

$$= \frac{1}{2} \cdot \left(0.25 \times 10^{-4}\right) \cdot (40\pi) \cdot (0.8)^2\ \text{V}$$

$$= 3.2\pi \times 10^{-4}\ \text{V} = 32\pi \times 10^{-5}\ \text{V}$$

Chemistry

Q. 1 Given below are two statements :

Statements I : The gas liberated on warming a salt with dil H_2SO_4, turns a piece of paper dipped in lead acetate into black, it is a confirmatory test for sulphide ion.

Statements II : In statement-I the colour of paper turns black because of formation of lead sulphite.

In the light of the above statements, choose the most appropriate from the options given below :

Option 1:
Both Statement I and Statement II are false

Option 2:
Statement I is false but Statement II is true

Option 3:
Statement I is true but Statement II is false

Option 4:
Both Statement I and Statement II are true

Correct Answer:
Statement I is true but Statement II is false

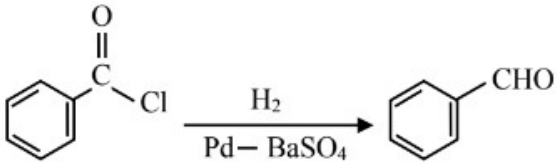
Solution:

$$Na_2S + H_2SO_4 \longrightarrow Na_2SO_4 + H_2S$$

$$(CH_3COO)_2Pb + H_2S \longrightarrow \underset{\text{Black leadsulphide}}{PbS} + 2CH_3COOH$$

Hence, the answer is the option (3).

Q. 2



This reduction reaction is known as :

Option 1:
Rosenmund reduction

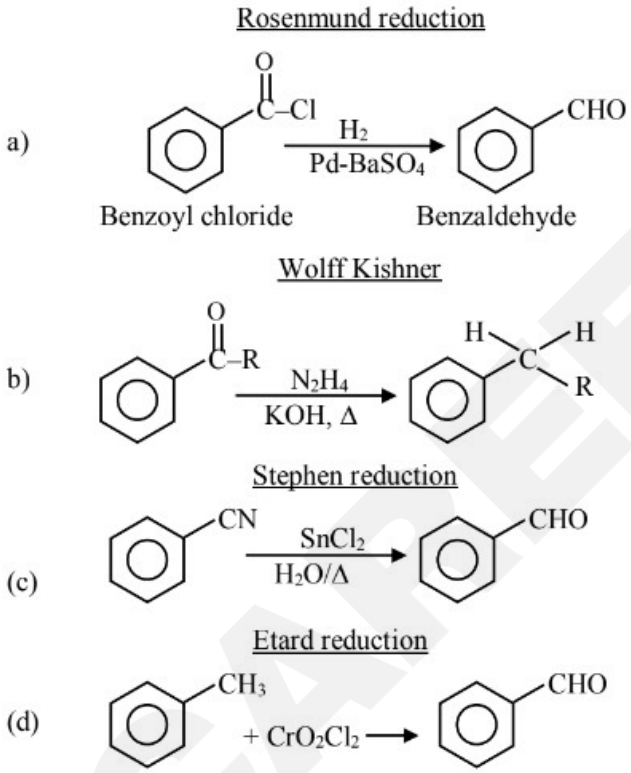
Option 2:
Wolff-Kishner reduction

Option 3:
Stephen reduction

Option 4:
Etard reduction

Correct Answer:
Rosenmund reduction

Solution:



Q. 3 Sugar which does not give reddish brown precipitate with Fehling's reagent is :

Option 1:
Sucrose

Option 2:
Lactose

Option 3:
Glucose

Option 4:
Maltose

Correct Answer:
Sucrose

Solution:
Sucrose is a non-reducing sugar and does not contain a free aldehyde group; thus does not give reddish brown ppt with Fehling reagent.

Hence, the answer is the option (1).

Q. 4 Below are two statements: one is labelled as Assertion (A), and the other is labelled as Reason (R).

Assertion (A) : There is a considerable increase in covalent radius from N to P. However from As to Bi only a small increase in covalent radius is observed.

Reason (R) : Covalent and ionic radii in a particular oxidation state increases down the group.

In the light of the above statements, choose the most appropriate answer from the options given below :

Option 1:
(A) is false but (R) is true

Option 2:
Both (A) and (R) are true but (R) is not the correct explanation of (A)

Option 3:
(A) is true but (R) is false

Option 4:
Both (A) and (R) are true and (R) is the correct explanation of (A)

Correct Answer:
Both (A) and (R) are true but (R) is not the correct explanation of (A)

Solution:

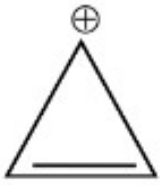
Statement – I : It is accurate to say that the covalent radius increases noticeably from nitrogen (N) to phosphorus (P). This is because an element's atomic size increases as it moves down a group in the periodic table since each subsequent element has an extra electron shell. Nevertheless, the rise in covalent radius is somewhat lesser from bismuth (Bi) to arsenic (As). The outer electrons are drawn closer to the nucleus due to the effective nuclear charge rising due to inadequate shielding by d and f electrons.

Statement – II This is a true general statement. As we move down a group in the periodic table, the covalent and ionic radii do increase because of the addition of extra electron shells.

Hence, the answer is the option (2).

Q. 5 Which of the following molecule/species is most stable?

Option 1:



Option 2:



Option 3:



Option 4:



Correct Answer:



Solution:
Stability order Aromatic > Non Aromatic > Anti aromatic

(1) 
$2\pi\ e^-$
Huckel rule
aromatic

(2) 
$4\pi\ e^-$
Anti
aromatic

(3) 
Carbene

(4) 
Non
aromatic

Q. 6 Diamagnetic Lanthanoid ions are :

Option 1:
$Nd^{3+}\&Eu^{3+}$

Option 2:
$La^{3+}\&Ce^{4+}$

Option 3:
$Nd^{3+}\&Ce^{4+}$

Option 4:
$Lu^{3+}\&Eu^{3+}$

Correct Answer:
$La^{3+}\&Ce^{4+}$

Solution:
$Ce \rightarrow [Xe]4f^{1}5\ d^{1}6\ s^{2}$

$Ce^{4+} \Rightarrow$ Diamagnetic

$La \rightarrow [Xe]4f^{0}5\ d^{1}6\ s^{2}$

$La^{3+} \Rightarrow$ Diamagnetic

Hence, the answer is the option (2).

Q. 7 Aluminium chloride in acidified aqueous solution forms an ion having geometry :

Option 1:
Octahedral

Option 2:
Square planar

Option 3:
Tetrahedral

Option 4:
Trigonal bipyramidal

Correct Answer:
Octahedral

Solution:
$AlCl_3$ in acidified aqueous solution forms octahedral geometry $[Al(H_2O)_6]^{3+}$.

Hence, the answer is the option (1).

Q. 8 Given below are two statements :

Statement (I) : The orbitals having same energy are called as degenerate orbitals.

Statement (II) : In hydrogen atom, 3p and 3d orbitals are not degenerate orbitals.

In the light of the above statements, choose the most appropriate answer from the options given below :

Option 1:
Statement I is true but Statement II is false

Option 2:
Both Statement I and Statement II are true

Option 3:
Both Statement I and Statement II are false

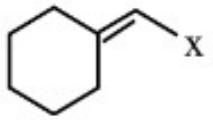
Option 4:
Statement I is false but Statement II is true

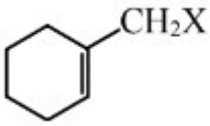
Correct Answer:
Statement I is true but Statement II is false

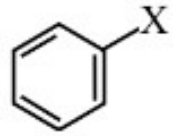
Solution:
For single electron species, the energy depends only on principal quantum number(n), so statement –II is false.

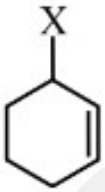
Hence, the answer is the option (1).

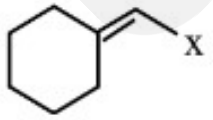
Q. 9 Example of vinylic halide is :

Option 1:


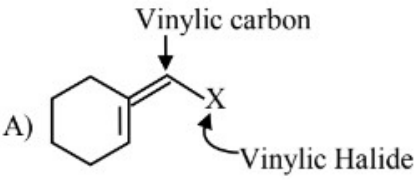
Option 2:


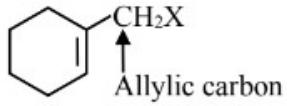
Option 3:


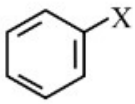
Option 4:


Correct Answer:


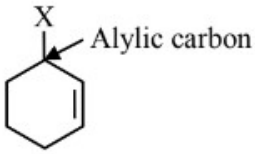
Solution:

A) 

B) 

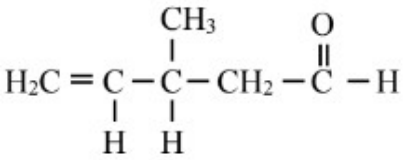
C) 

Aryl halide

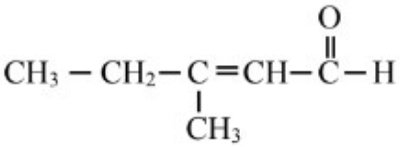
D) 

Q. 10 Structure of 4-Methylpent-2-enal is :

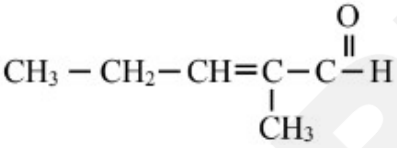
Option 1:



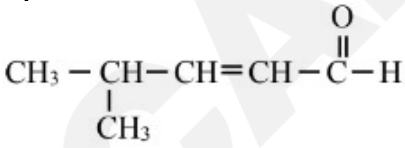
Option 2:



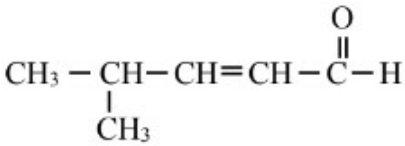
Option 3:



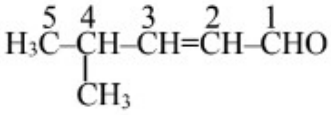
Option 4:



Correct Answer:



Solution:


4-Methyl pent-2-enal

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Q. 11 Match List – I with List – II.

List-I Molecule	List - II Shape
(A) BrF_5	(I) T-Shape
(B) H_2O	(II) See Saw
(C) ClF_3	(III) Bent
(D) SF_4	(IV) Square pyramidal

Choose the correct answer from the options given below :

Option 1:
(A) – (I), (B) – (II), (C) – (IV), (D) – (III)

Option 2:
(A) – (II), (B) – (I), (C) – (III), (D) – (IV)

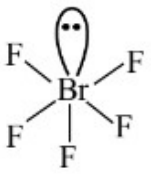
Option 3:
(A) – (III), (B) – (IV), (C) – (I), (D) – (II)

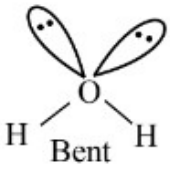
Option 4:
(A) – (IV), (B) – (III), (C) – (I), (D) – (II)

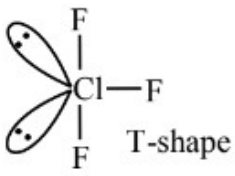
Correct Answer:
(A) – (IV), (B) – (III), (C) – (I), (D) – (II)

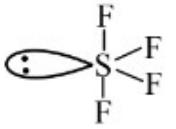
Solution:

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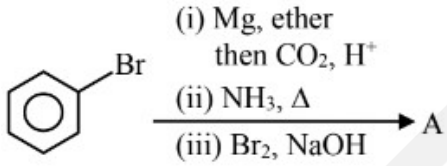
$BrF_5 \Rightarrow$  [Square pyramidal]

$H_2O \Rightarrow$  Bent

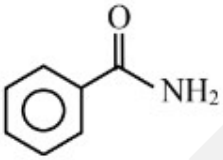
$ClF_3 \Rightarrow$  T-shape

$SF_4 \Rightarrow$  [Sea–saw]

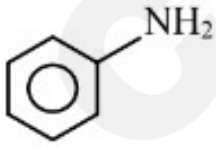
Q. 12 The final product A, formed in the following multistep reaction sequence is :



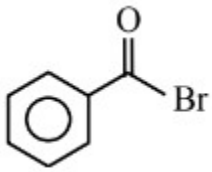
Option 1:



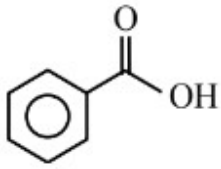
Option 2:



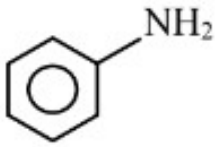
Option 3:



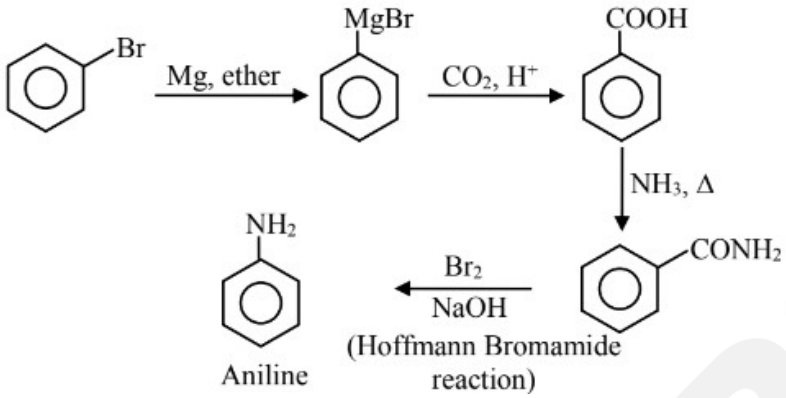
Option 4:



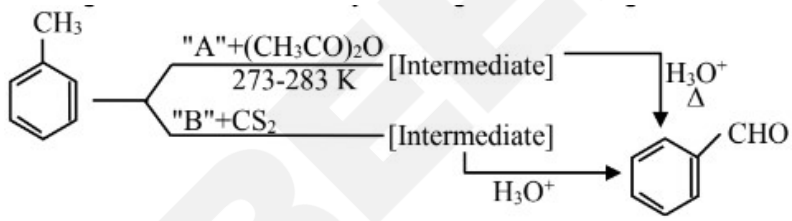
Correct Answer:



Solution:



Q. 13 In the given reactions, identify the reagent A and reagent B.



Option 1:

$A - CrO_3 \quad B - CrO_3$

Option 2:

$A - CrO_3 \quad B - CrO_2Cl_2$

Option 3:

$A - CrO_2Cl_2 \quad B - CrO_2Cl_2$

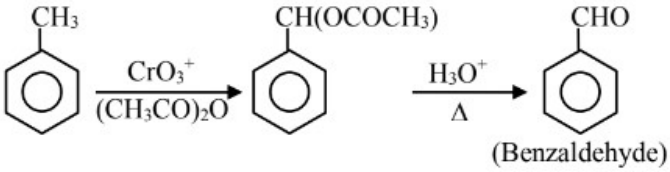
Option 4:

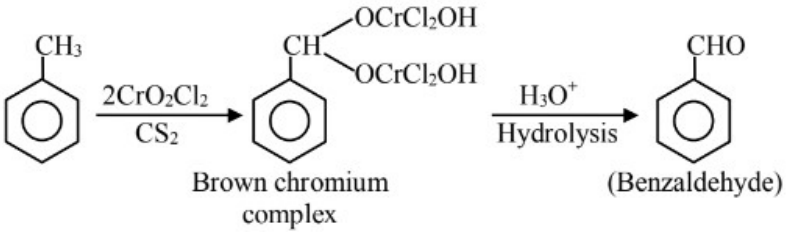
$A - CrO_2Cl_2 \quad B - CrO_3$

Correct Answer:

$A - CrO_3 \quad B - CrO_2Cl_2$

Solution:





Q. 14 Given below are two statements : one is labelled as Assertion (A) and the other is labelled Reason (R).

Assertion (A) : $CH_2 = CH - CH_2 - Cl$ is an example of allyl halide.

Reason (R) : Allyl halides are the compounds in which the halogen atom is attached to sp^2 hybridized carbo atom.

In the light of the above statements, choose the most appropriate answer from the options given below :

Option 1:
(A) is true but (R) is false

Option 2:
Both (A) and (R) are true but (R) is not the correct explanation of (A)

Option 3:
(A) is false but (R) is true

Option 4:
Both (A) and (R) are true and (R) is the correct explanation of (A)

Correct Answer:
(A) is true but (R) is false

Solution:
$C = C - C - Cl$ is an example of allyl halide.
Allyl halide - Halogen atom is bonded to sp^3 hybridised carbon atom adjacent to $C = C$ to an allyic carbon.

Hence, the answer is the option (1).

Q. 15 What happens to freezing point of benzene when small quantity of naphthalene is added to benzene?

Option 1:
Increases

Option 2:
Remains unchanged

Option 3:
First decreases and then increases

Option 4:
Decreases

Correct Answer:
Decreases

Solution:
When a small amount of naphthalene is added to benzene, depression in the freezing point takes place and the freezing point of benzene decreases.

Hence, the answer is the option (4).

Q. 16 Match List I with List II :

List I (Species)		**List II (Electronic distribution)**	
A.	Cr^{+2}	I.	$3\,d^8$
B.	Mn^{+}	II.	$3\,d^3 4\,s^1$
C.	Ni^{+2}	III.	$3\,d^4$
D.	$V+$	IV.	$3\,d^5 4\,s^1$

Choose the correct answer from the options given below :

Option 1:
(A) – (I), (B) – (II), (C) – (III), (D) – (IV)

Option 2:
(A) – (III), (B) – (IV), (C) – (I), (D) – (II)

Option 3:
(A) – (IV), (B) – (III), (C) – (I), (D) – (II)

Option 4:
(A) – (II), (B) – (I), (C) – (IV), (D) – (III)

Correct Answer:
(A) – (III), (B) – (IV), (C) – (I), (D) – (II)

Solution:

$$Cr \rightarrow [Ar]3\,d^5 4\,s^1, Cr^{2+} \Rightarrow [Ar]3\,d^4$$
$$Mn \rightarrow [Ar]3\,d^5 4\,s^2, Mn^{+} \Rightarrow [Ar]3\,d^5 4\,s^1$$
$$Ni \rightarrow [Ar]3\,d^8 4\,s^2, Ni^{2+} \Rightarrow [Ar]3\,d^8$$
$$V \rightarrow [Ar]3\,d^3 4\,s^2,\ V^{+} \Rightarrow [Ar]3\,d^3 4\,s^1$$

Hence, the answer is the option (2).

Q. 17 Compound A formed in the following reaction reacts with B gives the product C. Find out A and B.

$$CH_3-C\equiv CH + Na \longrightarrow A \xrightarrow{B} \underset{(C)}{CH_3-C\equiv C-CH_2-\underset{\displaystyle CH_3}{\underset{|}{CH_2}}} + NaBr$$

Option 1:
$A = CH_3-C\equiv \overset{-}{C}\overset{+}{Na},\ B = CH_3-CH_2-CH_2-Br$

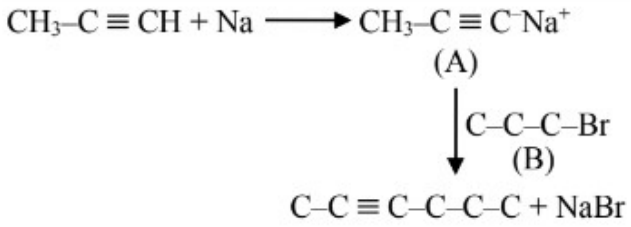
Option 2:
$A = CH_3-CH=CH_2, B = CH_3-CH_2-CH_2-Br$

Option 3:
$A = CH_3-CH_2-CH_3, B = CH_3-C\equiv CH$

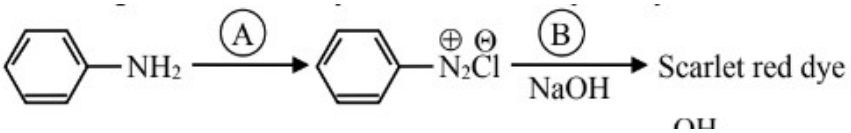
Option 4:
$A = CH_3-C\equiv \overset{-}{C}\overset{+}{Na},\ B = CH_3-CH_2-CH_3$

Correct Answer:
$A = CH_3-C\equiv \overset{-}{C}\overset{+}{Na},\ B = CH_3-CH_2-CH_2-Br$

Solution:

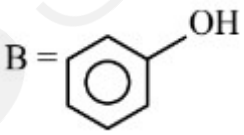


Q. 18 Following is a confirmatory test for aromatic primary amines. Identify reagent (A) and (B).



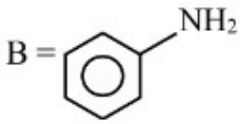
Option 1:

$A = HNO_3/H_2SO_4$

B = 

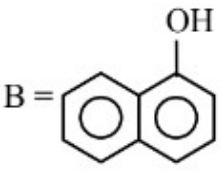
Option 2:

$A = NaNO_2 + HCl, 0 - 5^\circ C$

B = 

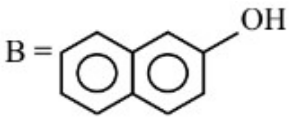
Option 3:

$A = NaNO_2 + HCl, 0 - 5^\circ C$

B = 

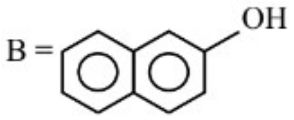
Option 4:

$A = NaNO_2 + HCl, 0 - 5^\circ C$

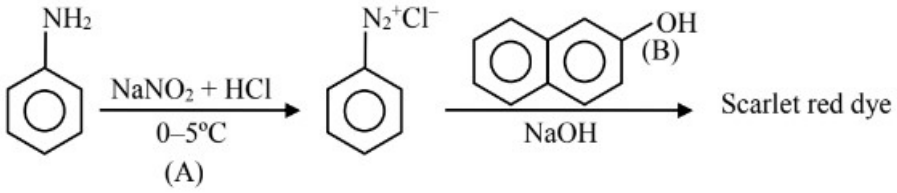
B = 

Correct Answer:

$A = NaNO_2 + HCl, 0 - 5^\circ C$

B = 

Solution:



Q. 19 The Lassiagne's extract is boiled with dil HNO_3 before testing for halogens because,

Option 1:
$AgCN$ is soluble in HNO_3

Option 2:
Silver halides are soluble in HNO_3

Option 3:
Ag_2S is soluble in HNO_3

Option 4:
Na_2S and $NaCN$ are decomposed by HNO_3

Correct Answer:
Na_2S and $NaCN$ are decomposed by HNO_3

Solution:
If Nitrogen or Sulphur is present in the compound, the sodium Fusion extract is boiled with concentrate HNO_3 to decompose sulphide or cyanide of sodium during lassiagne's test.

Hence, the answer is the option (4).

Q. 20 Choose the correct statements from the following :

(A) Ethane-1, 2-diamine is a chelating ligand.

(B) Metallic aluminium is produced by electrolysis of aluminium oxide in presence of cryolite.

(C) Cyanide ion is used as ligand for leaching of silver.

(D) Phosphine act as a ligand in Wilkinson catalyst.

(E) The stability constants of Ca^{2+} and Mg^{2+} are similar with EDTA complexes.

Choose the correct answer from the options given below :

Option 1:
(B), (C), (E) only

Option 2:
(C), (D), (E) only

Option 3:
(A), (B), (C) only

Option 4:
(A), (D), (E) only

Correct Answer:
(A), (B), (C) only

Solution:
The stability constants (formation constants) of EDTA complexes with Ca^{2+} and Mg^{2+} are not similar. Typically, the stability constant of the EDTA complex with Ca^{2+} is lower than that with Mg^{2+}, meaning the Mg^{2+}-EDTA complex is more stable than the Ca^{2+}-EDTA complex.

Given these evaluations, the correct statements are:

(A), (B), (C).

Hence, the answer is the option (3).

Q. 21 The rate of First order reaction is $0.04\ \text{mol}\ \text{L}^{-1}\ \text{s}^{-1}$ at 10 minutes and $0.03\ \text{mol}\ \text{L}^{-1}\ \text{s}^{-1}$ at 20 minutes after initiatation. Half life of the reaction is minutes. (Given $\log 2 = 0.3010, \log 3 = 0.4771$)

Correct Answer:
24

Solution:

$\text{r}_1 = \text{K}\,[\text{A}_1]$

$0.04 = \text{K}\,[\text{A}_1]$

$\text{r}_2 = \text{K}\,[\text{A}_2]$

$0.03 = \text{K}\,[\text{A}_2]$

$$\frac{\text{A}_1}{\text{A}_2} = \frac{0.04}{0.03} = \frac{4}{3}$$

let A_1 be the initial and A_2 be the final concentration after a given gap of $10\ \text{min}$

$$\text{then K} = \frac{2.303}{\text{t}} \log\left[\frac{\text{A}_1}{\text{A}_2}\right]$$

$$\text{K} = \frac{2.303}{10} \log\frac{4}{3} = 2.855 \times 10^{-2}\ \text{min}^{-1}$$

$$\because \text{t}_{1/2} = \frac{0.693}{\text{k}} = \frac{0.693}{2.855 \times 10^{-2}} = 24.08$$

$= 24\ \text{min}.$

Hence, the answer is (24).

Q. 22 The pH at which $\text{Mg(OH)}_2\ [\text{K}_{\text{sp}}] = 1 \times 10^{-11}]$ begins to precipitate from a solution containing $0.10\text{M}\text{Mg}^{2+}$ ions is_____.

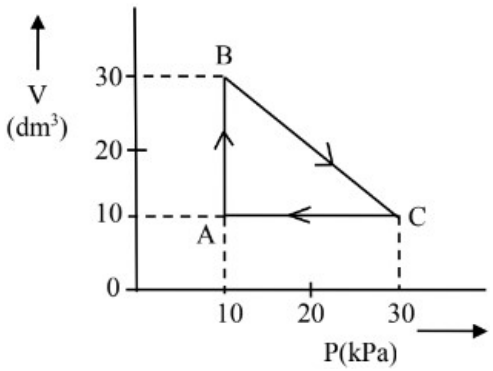
Correct Answer:
9

Solution:

$$\left[Mg^{2+}\right]\left[OH^{-}\right]^{2}=K_{sp}$$
$$[0.1]\left[OH^{-}\right]^{2}=1\times10^{-11}$$
$$\left[OH^{-}\right]=10^{-5}M$$
$$P^{OH}=-\log\left(10^{-5}\right)=5$$
$$pH=14-P^{OH}$$
$$pH=14-5=9$$

Hence, the answer is (9).

Q. 23



An ideal gas undergoes a cyclic transformation starting from the point A and coming back to the same point by
tracing the path$A\rightarrow B\rightarrow C\rightarrow A$ as shown in the diagram above. The total work done in the process is
_____ J.

Correct Answer:
200

Solution:
$W=$ Area of $\triangle ABC$

$$=\frac{1}{2}(30-10)(30-10)$$

$$=\frac{1}{2}\times20\times20$$

$$=\frac{400}{2}=200\ J$$

Q. 24 If IUPAC name of an element is "Unununnium" then the element belongs to n^{th} group of Periodic table. The value of n is_____.

Correct Answer:
11

Solution:
Atomic number $\rightarrow 111$

Group No. $\rightarrow 11^{\text{th}}$

Hence, the answer is (11).

Q. 25 The total number of molecular orbitals formed from 2s and 2p atomic orbitals of a diatomic molecule is ____.

Correct Answer:
8

Solution:
Two molecular orbital $\sigma 2\ \text{s}$ and $\sigma * 2\ \text{s}$

six molecular orbital $\sigma 2\text{pz}, \sigma^{*} 2\text{pz} \pi 2\text{px}, \pi^{*} 2\text{px}, \pi 2\text{py}, \pi^{*} 2\text{py}$

Total $= 2 + 6 = 8$

Hence, the answer is (8).

Q. 26 On a thin layer chromatographic plate, an organic compound moved by $3.5\ \text{cm}$, while the solvent moved by 5 cm. The retardation factor of the organic compound is_____ $\times 10^{-1}$.

Correct Answer:
7

Solution:

$$R_f(\text{ retardation factor }) = \frac{\text{Distance travelled by solute}}{\text{Distance travelled by solvent}} \longrightarrow$$

$$= \frac{3.5}{5} = 0.7 = 7 \times 10^{-1}$$

Hence, the answer is (7).

Q. 27 The compound formed by the reaction of ethanal with semicarbazine contains _____ number of nitrogen atoms.

Correct Answer:
3

Solution:

$$\underset{\text{Ethanal}}{CH_3 - CHO} + H_2N - \underset{\text{Semicarbazide}}{NHCONH_2} \xrightarrow{H^+} \underset{\text{(Semicarbazone derivative)}}{CH_3 - CH = N - NH_2CONH_2} + H_2O$$

Hence, the answer is (3).

Q. 28 $0.05\ \text{cm}$ thick coating of silver is deposited on a plate of $0.05\ \text{m}^2$ area. The number of silver atoms deposited on plate are_____$\times 10^{23}$. (At mass $\text{Ag} = 108,\ \text{d} = 7.9\ \text{g cm}^{-3}$)

Correct Answer:
11

Solution:
Volume of Ag deposited $= 0.05 \times 10^4\ \text{cm}^2 \times 0.05\ \text{cm} = 25\ \text{cm}^3$

Mass of Ag deposited $= \text{V} \times \text{d}$

Number of Ag atom $= \frac{\text{V} \times \text{d}}{108} \times \text{N}_\text{A}$

$= \frac{25 \times 7.9 \times 6.023 \times 10^{23}}{108}$

$= 1.1 \times 10^{24}$

$= 11 \times 10^{23}$

Hence, the answer is (11).

Q. 29 $2MnO_4^- + bI^- + cH_2O \rightarrow xI_2 + yMnO_2 + z\overline{O}H$

If the above equation is balanced with integer coefficients, the value of z is_____.

Correct Answer:
8

Solution:

$$\overset{+7}{2Mn}o_4^- + b\overset{-1}{I^-} + cH_2O \rightarrow x\overset{0}{I_2} + y\overset{+4}{MnO_2} + zOH^-$$

$n_r = 3 \quad n_f = 1$

$2MnO_4^- + 6I^- + 4H_2O \longrightarrow 3I_2 + 2MnO_2 + 8OH^-$

Q. 30 The mass of sodium acetate (CH_3COONa) required to prepare $250\ mL$ of $0.35M$ aqueous solution is_____ g. (Molar mass of CH_3COONa is $82.02\ g\ mol^{-1}$)

Correct Answer:
7

Solution:
$0.35M \Rightarrow 1000ml$ Solution contains 0.35 mole of CH_3COONa

then $250ml$ solution. contains $\frac{0.35}{1000} \times 250$ mole of CH_3COONa

$= 0.0875$ moles

Mass of $CH_3COONa = 0.0875 \times 82.02$

$= 7.17gm$

$= 7gm$

Hence, the answer is (7).

Maths

Q. 1 A line passing through the point $A(9, 0)$ makes an angle of 30° with the positive direction of x-axis. If this line is rotated about A through an angle of 15° in the clockwise direction, then its equation in the new position is:

Option 1:
$\frac{y}{\sqrt{3}-2} + x = 9$

Option 2:
$\frac{x}{\sqrt{3}-2} + y = 9$

Option 3:
$\frac{x}{\sqrt{3}+2} + y = 9$

Option 4:
$\frac{y}{\sqrt{3}+2} + x = 9$

Correct Answer:
$\frac{y}{\sqrt{3}-2} + x = 9$

Solution:
$y - 0 = \tan 15^\circ (x - 9)$

$\Rightarrow y = (2 - \sqrt{3})(x - 9)$

$\Rightarrow \frac{y}{2 - \sqrt{3}} = x - 9$

$\Rightarrow x + \frac{y}{\sqrt{3} - 2} = 9$

Hence, the answer is the option (1).

Q. 2 Let S_n denote the sum of first n terms of an arithmetic progression. If $S_{20} = 790$ and $S_{10} = 145$, then $\mathrm{S}_{15} - \mathrm{S}_5$ is:

Option 1:
395

Option 2:
390

Option 3:
405

Option 4:
410

Correct Answer:
395

Solution:

$\mathrm{S}_{20} = 790$

$\Rightarrow 10[2\mathrm{a} + 19\ \mathrm{d}] = 790$

$\Rightarrow 2\mathrm{a} + 19\ \mathrm{d} = 79 \ldots (1)$

$\mathrm{S}_{10} = 145$

$\Rightarrow 5[2\mathrm{a} + 9\ \mathrm{d}] = 145$

$\Rightarrow 2\mathrm{a} + 9\ \mathrm{d} = 29 \ldots (2)$

By (1) and (2)

10d = 50

d = 5

$\Rightarrow a = -8$

$$S_{15} = \frac{15}{2}[-16 + 14 \times 5]$$

$$S_{15} = \frac{15}{2}[-16 + 54]$$

$$= \frac{15}{2} \times 54 = 405$$

$$S_5 = 10$$

$$S_{15} - S_5 = 405 - 10 = 395$$

Hence, the answer is option (1).

Q. 3 If $z = x + iy, xy \neq 0$, satisfies the equation $z^2 + i\bar{z} = 0$, then z^2 is equal to:

Option 1:
9

Option 2:
1

Option 3:
4

Option 4:
$\frac{1}{4}$

Correct Answer:
1

Solution:

CAREERS360

$z = x + iy$

$z^2 = -i\bar{z}$

$\Rightarrow z^2 = |-i\bar{z}|$

$\Rightarrow |z|^2 = |-i||\bar{z}|$

$\Rightarrow |z|^2 = |z| \Rightarrow |z| = 0$

$|z| = 1$

$\Rightarrow |z|^2 = 1$

$Z^2 = 1$

Hence, the answer is the option (2).

Q. 4 Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ be two vectors such that $|\vec{a}| = 1, \vec{a} \cdot \vec{b} = 2$ and $|\vec{b}| = 4$. If $\vec{c} = 2(\vec{a} \times \vec{b}) - 3\vec{b}$, then the angle between $\vec{b}$ and $\vec{c}$ is equal to:

Option 1:
$\cos^{-1}\left(\frac{2}{\sqrt{3}}\right)$

Option 2:
$\cos^{-1}\left(-\frac{1}{\sqrt{3}}\right)$

Option 3:
$\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

Option 4:
$\cos^{-1}\left(\frac{2}{3}\right)$

Correct Answer:
$\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

Solution:

$$\vec{c} = 2(\vec{a} \times \vec{b}) - 3\vec{b}$$

$$\vec{b} \cdot \vec{c} = 2(\vec{a} \times \vec{b}) \cdot \vec{b} - 3\vec{b} \cdot \vec{b}$$

$$\Rightarrow |\vec{b}||\vec{c}| \cos\theta = 0 - 3|\vec{b}|^2$$

$$|\vec{c}| \cos\theta = -12$$

$$|\vec{c}|^2 = 4|\vec{a} \times \vec{b}|^2 + 9|\vec{b}|^2$$

$$= 4\left(|\vec{a}|^2|\vec{b}|^2 - (\vec{a} \cdot \vec{b})^2\right) + 144$$

$$= 4(16 - 4) + 144$$

$$|\vec{c}|^2 = 48 + 144 = 192$$

$$|\vec{c}| = 8\sqrt{3}$$

$$\cos\theta = \frac{-12}{8\sqrt{3}} = \frac{-3}{2\sqrt{3}}$$

$$\cos\theta = \frac{-\sqrt{3}}{2}$$

$$\theta = \cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)$$

Hence, the answer is the option (3).

Q. 5 The maximum area of a triangle whose one vertex is at $(0, 0)$ and the other two vertices lie on the curve $y = -2x^2 + 54$ at points (x, y) and $(-x, y)$, where $y > 0$, is:

Option 1:
88

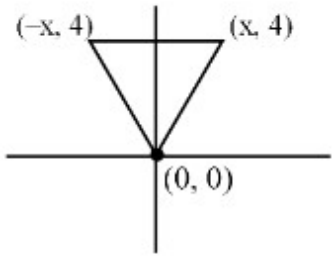
Option 2:
122

Option 3:
92

Option 4:
108

Correct Answer:
108

Solution:



$A = xy$

$A = x\left(-2x^2 + 54\right)$

$A = -2x^3 + 54x$

$\frac{dA}{dx} = -6x^2 + 54 = 0$

$x = 3 \quad x = -3$

$A_{\max} = 3(-2 \times 9 + 54)$

$3(54 - 18) = 108$

Q. 6 The value of $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{n^3}{(n^2+k^2)(n^2+3k^2)}$ is:

Option 1:
$\frac{(2\sqrt{3}+3)\pi}{24}$

Option 2:
$\frac{13\pi}{8(4\sqrt{3}+3)}$

Option 3:
$\frac{13(2\sqrt{3}-3)\pi}{8}$

Option 4:
$\frac{\pi}{8(2\sqrt{3}+3)}$

Correct Answer:
$\frac{13\pi}{8(4\sqrt{3}+3)}$

Solution:

$$\lim_{n\to\infty}\sum_{k=1}^{n}\frac{n^3}{\left(n^2+k^2\right)\left(n^2+3k^2\right)}$$

$$\Rightarrow \lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}\frac{1}{\left(1+\frac{k^2}{n^2}\right)\left(1+\frac{3k^2}{n^2}\right)}$$

$$\Rightarrow \int_0^1\frac{dx}{\left(1+x^2\right)\left(1+3x^2\right)}=\frac{1}{2}\int_0^1\frac{3\left(1+x^2\right)-\left(1+3x^2\right)}{\left(1+x^2\right)\left(1+3x^2\right)}dx$$

$$\Rightarrow \frac{1}{2}\left[\int_0^1\frac{3dx}{1+3x^2}-\int_0^1\frac{dx}{1+x^2}\right]=\frac{13\pi}{8(4\sqrt{3}+3)}$$

Hence, the answer is the option (2).

Q. 7 Let $g:\mathbf{R}\to\mathbf{R}$ be a non constant twice differentiable function such that $g'\left(\frac{1}{2}\right)=g'\left(\frac{3}{2}\right)$. If a real valued function f is defined as $f(x)=\frac{1}{2}[g(x)+g(2-x)]$, then

Option 1:
$f''(x)=0$ for atleast two x in $(0,2)$

Option 2:
$f''(x)=0$ for exactly one x in $(0,1)$

Option 3:
$f''(x)=0$ for no x in $(0,1)$

Option 4:
$f'\left(\frac{3}{2}\right)+f'\left(\frac{1}{2}\right)=1$

Correct Answer:
$f''(x)=0$ for atleast two x in $(0,2)$

Solution:
$g'\left(\frac{1}{2}\right)=g'\left(\frac{3}{2}\right)$

$f(x)=\frac{1}{2}(g(x)+g(2-x))$

$f'(x)=\frac{1}{2}\left(g'(x)-g'(2-x)\right)$

Now $f'\left(\frac{1}{2}\right)=\frac{1}{2}\left(g'\left(\frac{1}{2}\right)-g'\left(\frac{3}{2}\right)\right)=0$

$f'(1)=\frac{1}{2}(g'(1)-g'(1))=0$

$f'\left(\frac{3}{2}\right)=\frac{1}{2}\left(g'\left(\frac{3}{2}\right)-g'\left(\frac{1}{2}\right)\right)=0$

$\Rightarrow f''(x)=0$ for at least two x in $(0,2)$

Hence, the answer is the option (1).

Q. 8 The area (in square units) of the region bounded by the parabola $y^2=4(x-2)$ and the line $y=2x-8$, is:

Option 1:
8

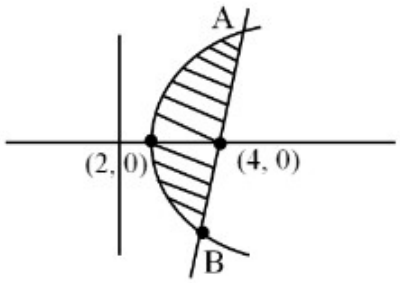
Option 2:
9

Option 3:
6

Option 4:
7

Correct Answer:
9

Solution:



$y^2 = 4x - 8$

$y = 2x - 8$

$\Rightarrow y^2 = 2(y + 8) - 8$

$\Rightarrow y^2 - 2y - 8 = 0$

$y = 4 \quad y = -2$

$$\int_{-2}^{4} \left(\frac{y+8}{2} - \frac{y^2+8}{4} \right) dy$$

$$\Rightarrow \int_{-2}^{4} \frac{y}{2} dy - \int_{-2}^{4} \frac{y^2}{4} dy + \int_{-2}^{4} 2dy$$

$$\Rightarrow \left[\frac{y^2}{4} \right]_{-2}^{4} - \frac{1}{4} \left[\frac{y^3}{3} \right]_{-2}^{4} + 2[y]_{-2}^{4} = 9$$

Q. 9 Let $y = y(x)$ be the solution of the differential equation $\sec x dy + \{2(1-x)\tan x + x(2-x)\}dx = 0$ such that $y(0) = 2$. Then $y(2)$ is equal to:

Option 1:
2

Option 2:
$2[1 - \sin(2)]$

Option 3:
$2\{\sin(2) + 1\}$

Option 4:
1

Correct Answer:
2

Solution:

$$dy = \frac{2(x-1)\tan x + x(x-2)}{\sec x}dx$$

$$\Rightarrow \int dy = \int 2(x-1)\sin x + \left(x^2-2x\right)\cos x$$

$$\Rightarrow y = 2[(x-1)(-\cos x) - (1)(-\sin x)] + \left[\left(x^2-2x\right)(\sin x) - (2x-2)(-\cos x) + (2)(-\sin x)\right] + c$$

$$y(0) = 2[1] + [-2] + C$$

$$y(0) = 2$$

$$C = 2$$

$$y(2) = 2(1+\sin 2) - 2\sin 2 = 2$$

Hence, the answer is option (1).

Q. 10 Let (α, β, γ) be the foot of perpendicular from the point $(1, 2, 3)$ on the line $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3}$. Then $19(\alpha+\beta+\gamma)$ is equal to:

Option 1:
102

Option 2:
101

Option 3:
99

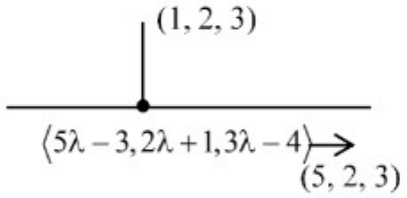
Option 4:
100

Correct Answer:
101

Solution:

$\overrightarrow{r} = \langle -3, 1, -4 \rangle + \lambda \langle 5, 2, 3 \rangle$

Foot of $\perp^{r}$ $\langle 5\lambda - 3, 2\lambda + 1, 3\lambda - 4 \rangle$



$\langle 5\lambda - 4, 2\lambda - 1, 3\lambda - 7 \rangle \cdot \langle 5, 2, 3 \rangle = 0$

$\Rightarrow 25\lambda - 20 + 4\lambda - 2 + 9\lambda - 21 = 0$

$38\lambda = 43$

$\lambda = \frac{43}{38}$

$\left\langle \frac{101}{38}, \frac{124}{38}, \frac{-23}{38} \right\rangle \Rightarrow 101$

Q. 11 Two integers x and y are chosen with replacement from the set $\{0, 1, 2, 3, \ldots, 10\}$. Then the probability that $|x - y| > 5$, is:

Option 1:
$\frac{30}{121}$

Option 2:
$\frac{62}{121}$

Option 3:
$\frac{60}{121}$

Option 4:
$\frac{31}{121}$

Correct Answer:
$\frac{30}{121}$

Solution:

$|x - y| > 5$

$\Rightarrow x = 0 \quad y = 6, 7, 8, 9, 10$

$\Rightarrow x = 1 \quad y = 789, 10$

$\Rightarrow x = 2 \quad y = 8, 9, 10$

$\Rightarrow x = 3 \quad y = 9, 10$

$\Rightarrow x = 4 \quad y = 10$

$P(A) = \frac{30}{121}$

Hence, the answer is the option (1).

Q. 12 If the domain of the function $f(x) = \cos^{-1}\left(\frac{2-|x|}{4}\right) + \{\log_e(3-x)\}^{-1}$ is $[-\alpha, \beta) - \{\gamma\}$, then $\alpha + \beta + \gamma$ is equal to:

Option 1:
12

Option 2:
9

Option 3:
11

Option 4:
8

Correct Answer:
11

Solution:

$$f(x) = \cos^{-1}\left(\frac{2-|x|}{4}\right) + \frac{1}{\ln(3-x)}$$

$$f(x) = -1 \leq \frac{2-|x|}{4} \leq 1 \quad x < 3 - \{2\}$$

$$\Rightarrow -6 \leq x \leq 6 \quad x \in [-6, 3) - \{2\}$$

$$\alpha + \beta + \gamma = 11$$

Hence, the answer is the option (3).

Q. 13 Consider the system of linear equations $x+y+z=4\mu, x+2y+2\lambda z=10\mu, x+3y+4\lambda^2 z=\mu^2+15$, where $\lambda, \mu \in \mathbf{R}$. Which one of the following statements is NOT correct?

Option 1:
The system has unique solution if $\lambda \neq \frac{1}{2}$ and $\mu \neq 1, 15$

Option 2:
The system is inconsistent if $\lambda = \frac{1}{2}$ and $\mu \neq 1$

Option 3:
The system has infinite number of solutions if $\lambda = \frac{1}{2}$ and $\mu = 15$

Option 4:
The system is consistent if $\lambda \neq \frac{1}{2}$

Correct Answer:
The system is inconsistent if $\lambda = \frac{1}{2}$ and $\mu \neq 1$

Solution:

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 2\lambda \\ 1 & 3 & 4\lambda^2 \end{vmatrix} = (2\lambda-1)^2$$

For unique solution $\Delta \neq 0 \Rightarrow \lambda \neq \frac{1}{2}$

If $\lambda = \frac{1}{2}$, $\begin{vmatrix} 4\mu & 1 & 1 \\ 10\mu & 2 & 1 \\ \mu^2+15 & 3 & 1 \end{vmatrix} = 0$

$(\mu-15)(\mu-1)=0$ Infinite solution.

Hence, the answer is the option (2).

Q. 14 If the circles $(x+1)^2+(y+2)^2=r^2$ and $x^2+y^2-4x-4y+4=0$ intersect at exactly two distinct points, then

Option 1:
$5 < r < 9$

Option 2:
$0 < r < 7$

Option 3:
$3 < r < 7$

Option 4:
$\frac{1}{2} < r < 7$

Correct Answer:
$3 < r < 7$

Solution:
for intereaction.

$|r_1 - r_2| < C_1C_2 < r_1 + r_2$

$|r - 2| < 5 < r + 2$

$r > 3 \quad \ldots(1)$

$-5 < r - 2 < 5$

$-3 < r < 7 \quad \ldots(2)$

By (1) and (2)

$3 < r < 7$

Hence, the answer is the option (3).

Q. 15 If the length of the minor axis of an ellipse is equal to half of the distance between the foci, then the eccentricity of the ellipse is:

Option 1:
$\frac{\sqrt{5}}{3}$

Option 2:
$\frac{\sqrt{3}}{2}$

Option 3:
$\frac{1}{\sqrt{3}}$

Option 4:

$\frac{2}{\sqrt{5}}$

Correct Answer:

$\frac{2}{\sqrt{5}}$

Solution:

$2\,b = ae$

$\frac{b}{a} = \frac{e}{2}$

$e^2 = 1 - \frac{b^2}{a^2}$

$\Rightarrow \frac{5e^2}{4} = 1 \Rightarrow e = \frac{2}{\sqrt{5}}$

Hence, the answer is the option (4).

Q. 16 Let M denote the median of the following frequency distribution

Class	0-4	4-8	8-12	12-16	16-20
Frequency	3	9	10	8	6

Then 20M is equal to :

Option 1:

416

Option 2:

104

Option 3:

52

Option 4:

208

Correct Answer:

208

Solution:

$$M = \ell + \left(\frac{\frac{N}{2} - cf}{f}\right) \times h$$

$$M = 8 + \left(\frac{18 - 12}{10}\right) \times 4$$

$$M = 8 + \frac{24}{10} = 10.4$$

$$20M = 208$$

Hence, the answer is the option (4).

Q. 17 If $f(x) = \begin{vmatrix} 2\cos^4 x & 2\sin^4 x & 3 + \sin^2 2x \\ 3 + 2\cos^4 x & 2\sin^4 x & \sin^2 2x \\ 2\cos^4 x & 3 + 2\sin^4 x & \sin^2 2x \end{vmatrix}$, then $\frac{1}{5}f'(0) =$ is equal to:

Option 1:
0

Option 2:
1

Option 3:
2

Option 4:
6

Correct Answer:
0

Solution:

$$f(x) = \begin{vmatrix} 2\cos^4 x & 2\sin^4 x & 3 + \sin^2 2x \\ 3 + 2\cos^4 x & 2\sin^4 x & \sin^2 2x \\ 2\cos^4 x & 3 + 2\sin^4 x & \sin^2 2x \end{vmatrix}$$

$f'(0) = 0 + 0 + 0 = 0$

$f'(0) = 0$

$\frac{1}{5} \cdot f'(0) = 0$

Hence, the answer is the option (1).

Q. 18 Let $A(2, 3, 5)$ and $C(-3, 4, -2)$ be opposite vertices of a parallelogram $ABCD$. If the diagonal $\overrightarrow{BD} = \hat{i} + 2\hat{j} + 3\hat{k}$, then the area of the parallelogram is equal to:

Option 1:
$\frac{1}{2}\sqrt{410}$

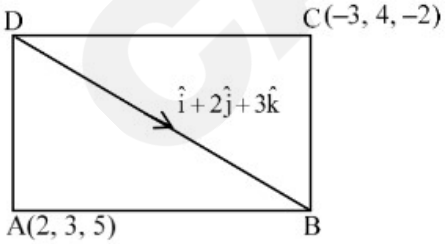
Option 2:
$\frac{1}{2}\sqrt{474}$

Option 3:
$\frac{1}{2}\sqrt{586}$

Option 4:
$\frac{1}{2}\sqrt{306}$

Correct Answer:
$\frac{1}{2}\sqrt{474}$

Solution:



$\overrightarrow{CA} = 5\hat{i} - \hat{j} + 7\hat{k} = \overrightarrow{d_1}$

$\overrightarrow{BD} = \hat{i} + 2\hat{j} + 3\hat{k} = \overrightarrow{d_2}$

$\frac{1}{2}\left|\overrightarrow{d_1} \times \overrightarrow{d_2}\right| = \frac{1}{2}\sqrt{474}$

Q. 19 If $2\sin^3 x + \sin 2x \cos x + 4\sin x - 4 = 0$ has exactly 3 solutions in the interval $\left[0, \frac{n\pi}{2}\right], n \in N$, then the roots of the equation $x^2 + nx + (n-3) = 0$ belong to:

Option 1:
$(0, \infty)$

Option 2:
$(-\infty, 0)$

Option 3:
$\left(-\frac{\sqrt{17}}{2}, \frac{\sqrt{17}}{2}\right)$

Option 4:
Z

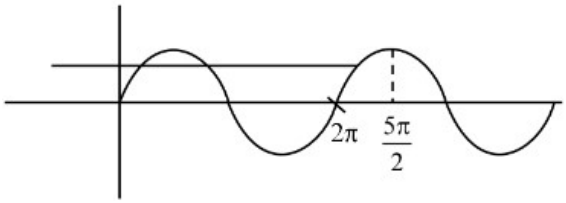
Correct Answer:
$(-\infty, 0)$

Solution:
$2\sin^3 x + 2\sin x \cos^2 x + 4\sin x - 4 = 0$

$\Rightarrow 2\sin^3 x + 2\sin x\left(1 - \sin^2 x\right) + 4\sin x - 4 = 0$

$\Rightarrow 2\sin^3 x + 6\sin x - 2\sin^3 x - 4 = 0$

$\sin x = \frac{2}{3}$



$\left[0, \frac{5\pi}{2}\right]$

$x^2+5x+2=0$

$\alpha,\beta=\frac{-5\pm\sqrt{17}}{2}$ both negative hence $(-\infty,0)$

Q. 20 Let $f:\left[-\frac{\pi}{2},\frac{\pi}{2}\right]\to \mathbf{R}$ be a differentiable function such that $f(0)=\frac{1}{2}$. If the $\lim_{x\to 0}\frac{x\int_0^x f(t)dt}{e^{x^2}-1}=\alpha$, then $8\alpha^2$ is equal to:

Option 1:
16

Option 2:
2

Option 3:
1

Option 4:
4

Correct Answer:
2

Solution:

$$\lim_{x\to 0}\frac{x\int_0^x f(t)dt}{e^{x^2}-1}$$

$$\Rightarrow \lim_{x\to 0}\frac{\int_0^x f(t)dt+xf(x)}{2xe^{x^2}}$$

$$\Rightarrow \lim_{x\to 0}\frac{f(x)+f(x)+xf'(x)}{2e^{x^2}+4x^2e^{x^2}}=\frac{2f(0)}{2}=\frac{1}{2}=\alpha$$

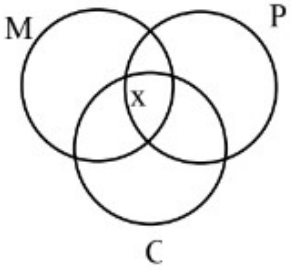
$$8\times\frac{1}{4}=2$$

Hence, the answer is the option (2).

Q. 21 A group of 40 students appeared in an examination of 3 subjects - Mathematics, Physics and Chemistry. It was found that all students passed in atleast one of the subjects, 20 students passed in Mathematics, 25 students passed in Physics, 16 students passed in Chemistry, atmost 11 students passed in both Mathematics and Physics, atmost 15 students passed in both Physics and Chemistry, atmost 15 students passed in both Mathematics and Chemistry. The maximum number of students passed in all the three subjects is_____

Correct Answer:
10

Solution:

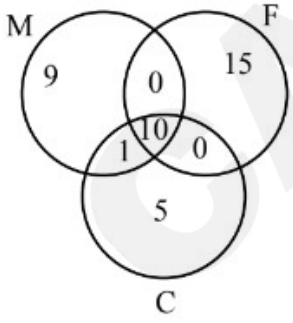


$11 - x \geq 0$

$x \leq 11$

$x = 11$ does not satisfy the data.

$x = 10$



Hence maximum number of students passed in all the three subjects is 10.

Q. 22 If d_1 is the shortest distance between the lines $x+1=2y=-12z, x=y+2=6z-6$ and d_2 is the shortest distance between the lines $\frac{x-1}{2}=\frac{y+8}{-7}=\frac{z-4}{5}, \frac{x-1}{2}=\frac{y-2}{1}=\frac{z-6}{-3}$, then the value of $\frac{32\sqrt{3}d_1}{d_2}$ is :

Correct Answer:
16

Solution:

$$L_1: \frac{x+1}{1}=\frac{y}{\frac{1}{2}}=\frac{z}{-1/12} \quad L_2: \frac{x}{1}=\frac{y+2}{1}=\frac{z-1}{1/6}$$

$$d_1=\frac{\left(\vec{a}_2-\vec{a}_1\right)\cdot\left(\vec{b}_1\times\vec{b}_2\right)}{\left(\vec{b}_1\times\vec{b}_2\right)}=2$$

Similarly $d_2=\frac{12}{\sqrt{3}}$

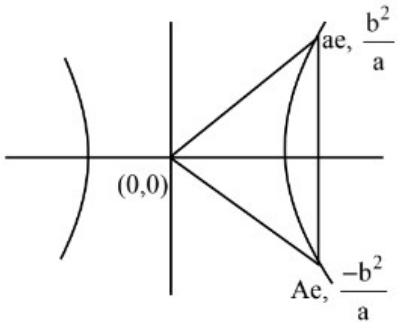
$$\frac{d_1 32\sqrt{3}}{d_2}=\frac{2\times32\times\sqrt{3}\times\sqrt{3}}{12}=16$$

Hence, the answer is the option (1).

Q. 23 Let the latus ractum of the hyperbola $\frac{x^2}{9}-\frac{y^2}{b^2}=1$ subtend an angle of $\frac{\pi}{3}$ at the centre of the hyperbola. If b^2 is equal to $\frac{1}{m}(1+\sqrt{n})A$, where 1 and m are co-prime numbers, then $1^2+m^2+n^2$ is equal to_____.

Correct Answer:
182

Solution:



$$\tan 30^\circ = \frac{\frac{b^2}{a}}{ae} = \frac{b^2}{a^2 e} = \frac{1}{\sqrt{3}}$$

$$e = \frac{\sqrt{3}b^2}{a^2}$$

$$e^2 = 1 + \frac{b^2}{9} \Rightarrow 1 + \frac{b^2}{9} = \frac{3b^4}{81}$$

$$b^4 = 3b^2 + 27$$

$$b^4 - 3b^2 - 27 = 0$$

$$b^2 = \frac{3}{2}(1 + \sqrt{13})$$

$$\ell = 3$$

$$\mathrm{m} = 2$$

$$\mathrm{n} = 13$$

$$\ell^2 + \mathrm{m}^2 + \mathrm{n}^2 = 182$$

Q. 24 Let $A = \{1, 2, 3, \ldots, 7\}$ and let $P(A)$ denote the power set of A. If the number of functions $f : A \to P(A)$ such that $\mathrm{a} \in \mathrm{f(a)}, \forall \mathrm{a} \in \mathrm{A}$ is $\mathrm{m}^\mathrm{n}, \mathrm{m}$ and $\mathrm{n} \in \mathrm{N}$ and m is least, then $\mathrm{m} + \mathrm{n}$ is equal to_____.

Correct Answer:
44

Solution:
$f : A \to P(A) \quad a \in f(x)$

a will connect with subset which contain a hence total $\left(2^6\right)^7 = 2^{42}$

2+42=44

Hence, the answer is 44.

Q. 25 The value of $9\int_0^9\left[\sqrt{\frac{10x}{x+1}}\right]dx$, where $[t]$ denotes the greatest integer less than or equal to t, is_____.

Correct Answer:
155

Solution:

$$9\times\int_0^9\left[\sqrt{\frac{10x}{x+1}}\right]dx$$

$$\Rightarrow 9\times\left(\int_0^{1/9}0\cdot dx+\int_{1/9}^{2/3}1\cdot dx+\int_{2/3}^{9}2\cdot dx\right)$$

$$\Rightarrow 9\times\left(\left[\frac{2}{3}-\frac{1}{9}\right]+2\left[9-\frac{2}{3}\right]\right)$$

$$\Rightarrow 9\times\left(\frac{6-1}{9}+2\cdot\frac{25}{3}\right)=9\left(\frac{5}{9}+\frac{50}{3}\right)=9\left(\frac{5+150}{9}\right)=155$$

Hence, the answer is (155).

Q. 26 Number of integral terms in the expansion of $\left\{7^{\left(\frac{1}{2}\right)}+11^{\left(\frac{1}{6}\right)}\right\}^{824}$ is equal to_____.

Correct Answer:
138

Solution:

$$T_{r+1}={}^{824}C_r(7)^{\frac{824-r}{2}}(11)^{\frac{r}{6}}$$

$$\Rightarrow {}^{824}C_r(7)^{412-\frac{r}{2}}(11)^{\frac{r}{6}}$$

$$r=0,6,12,\ldots\ldots 822$$

total 138

Hence the answer is 138.

Q. 27 Let $y = y(x)$ be the solution of the differential equation $(1-x^2)dy = \left[xy + (x^3+2)\sqrt{3(1-x^2)}\right]dx\ -1 < x < 1, y(0) = 0$. If $y\left(\frac{1}{2}\right) = \frac{m}{n}$, m and n are co-prime numbers, then $m + n$ is equal to_____.

Correct Answer:
97

Solution:

$$\frac{dy}{dx} = \frac{xy}{1-x^2} + \frac{(x^3+2)\sqrt{3(1-x^2)}}{1-x^2}$$

$$\frac{dy}{dx} - \frac{x}{1-x^2}y = \frac{(x^3+2)\sqrt{3}}{\sqrt{1-x^2}}$$

$$e^{\int \frac{-x}{1-x^2}dx} \quad 1-x^2 = t$$

$$-2xdx = dt$$

$$e^{\frac{1}{2}\int \frac{dt}{t}} = e^{\frac{1}{2}\ln(1-x^2)} = \sqrt{1-x^2}$$

$$y \cdot \sqrt{1-x^2} = \sqrt{3}\int x^3 + 2dx$$

$$y \cdot \sqrt{1-x^2} = \sqrt{3}\left(\frac{x^4}{4} + 2x\right) + C$$

$$C = 0$$

$$y\sqrt{1-x^2} = \sqrt{3}\left(\frac{x^4}{4} + 2x\right)$$

$$y\frac{\sqrt{3}}{2} = \sqrt{3}\left(\frac{1}{64} + 1\right)$$

$$y = \frac{65}{32}$$

$$m + n = 65 + 32 = 97$$

Hence, the answer is the (97).

Q. 28 Let $\alpha, \beta \in \mathbf{N}$ be roots of the equation $x^2 - 70x + \lambda = 0$, where $\frac{\lambda}{2}, \frac{\lambda}{3} \notin \mathbf{N}$. If λ assumes the minimum possible value, then $\frac{(\sqrt{\alpha-1}+\sqrt{\beta-1})(\lambda+35)}{|\alpha-\beta|}$ is equal to:

Correct Answer:
60

Solution:
$\alpha + \beta = 70 \quad \alpha\beta = \lambda$

$\frac{\lambda}{2}, \frac{\lambda}{3} \notin \mathrm{N}$

$\alpha = 5 \quad \beta = 65$

$$\frac{(\sqrt{\alpha-1}+\sqrt{\beta-1})(\lambda+35)}{|\alpha-\beta|}$$

$$\Rightarrow \frac{(2+8)(360)}{60} = 60$$

Hence, the answer is the (60).

Q. 29 If the function

$$f(x) = \begin{cases} \frac{1}{|x|}, & |x| \geqslant 2 \\ a^2 + 2b, & |x| < 2 \end{cases}$$

Is differentiable on R, then 48(a+b) is equal to _____.

Correct Answer:
15

Solution:
$$f(x) = \begin{cases} -\frac{1}{x} & -\infty < x \leq -2 \\ \mathrm{ax}^2 + 2b & -2 < x < 2 \\ \frac{1}{x} & 2 \leq x < \infty \end{cases}$$

$$\Rightarrow \quad \frac{1}{2} = 4\mathrm{a} + 2\,\mathrm{b} \qquad \ldots\ldots(1)$$

$$\Rightarrow \quad 4\mathrm{a} = -\frac{1}{4}$$

$$\mathrm{a} = \frac{-1}{16} \Rightarrow \frac{1}{2} = -\frac{1}{4} + 2\,\mathrm{b}$$

$$\Rightarrow 2\,\mathrm{b} = \frac{1}{2} + \frac{1}{4}$$

$$\Rightarrow \mathrm{b} = \frac{3}{8}$$

$48(a+b) = 28\left(\frac{-1}{16} + \frac{3}{8}\right) = 15$

Hence, the answer is the (15).

Q. 30 Let $\alpha = 1^2 + 4^2 + 8^2 + 13^2 + 19^2 + 26^2 + \ldots$ upto 10 terms and $\beta = \sum_{n=1}^{10} n^4$. If $4\alpha - \beta = 55k + 40$, then k is equal to_____.

Correct Answer:
353

Solution:
$\alpha = 1^2 + 4^2 + 8^2 + \cdots 10$ terms

$$\alpha = \frac{1}{4}\sum_{r=1}^{10}\left(r^2 + 3r - 2\right)^2$$

$$\alpha = \frac{1}{4}\left[\sum_{r=1}^{10} r^4 + 9r^2 + 4 - 4r^2 - 12r + 6r^3\right]$$

$$4\alpha = \sum_{r=1}^{10} r^4 + 6\sum_{r=1}^{10} r^3 + 5\sum_{r=1}^{10} r^2 - 12\sum_{r=1}^{10} r + \sum_{r=1}^{10} 4$$

$$4\alpha - \beta = 6\left(\frac{r(r+1)}{2}\right)^2 + 5r\frac{(r+1)(2r+1)}{6} - 12\frac{r(r+1)}{2} + 40$$

$$= 6\left(\frac{10 \times 11}{2}\right)^2 + 5 \times \frac{10 \times 11 \times 21}{6} - 55 \times 12 + 40$$

$$\Rightarrow \quad 6 \times 55 \times 55 + 55 \times 35 - 55 \times 12 + 40$$

$$\Rightarrow \quad 55(330 + 35 - 12) + 40$$

$$55(353) + 40$$

$$\mathrm{k} = 353$$

Hence, the answer is k=353.

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30 Jan - Evening Shift

Physics

Q. 1 If 50 Vernier divisions are equal to 49 main scale divisions of a travelling microscope and the smallest reading of the main scale is 0.5 mm, the Vernier constant of travelling microscope is

Option 1:
0.1 mm

Option 2:
0.1 cm

Option 3:
0.01 cm

Option 4:
0.01 mm

Correct Answer:
0.01 mm

Solution:

$$50\text{VSD} = 49\text{MSD}$$

$$1\text{VSD} = \frac{49}{50}\text{MSD}$$

Vernier constant $=$ L.C.

$$\text{L.C.} = 1\text{MSD} - 1\text{VSD}$$

$$= 1\text{MSD} - \frac{49}{50}\text{MSD}$$

$$\text{L.C.} = \frac{\text{MSD}}{50}$$

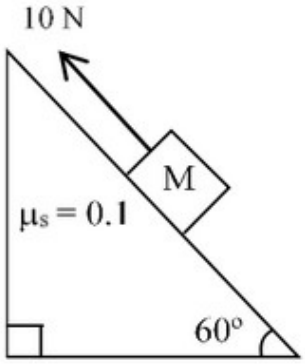
$1\text{MSD} = 0.5\ \text{mm}$ (given)

So

$$\text{L.C.} = \frac{0.5\ \text{mm}}{50} = 0.01\ \text{mm}$$

Hence, the answer is the option (4).

Q. 2 A block of mass 1 kg is pushed up a surface inclined to horizontal at an angle of 60o by a force of 10 N parallel to the inclined surface as shown in figure. When the block is pushed up by 10 m along inclined surface, the work done against frictional force is:
[g = 10 m/s^2]



Option 1:
$5\sqrt{3}$ J

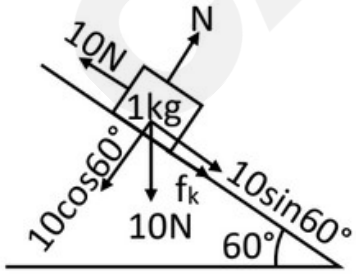
Option 2:
5 J

Option 3:
5×10^3 J

Option 4:
10 J

Correct Answer:
5 J

Solution:



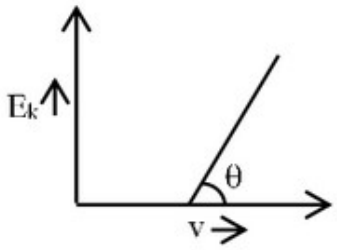
$f_k = \mu_k N$

$= \mu_k mg \cos 60^\circ$

$f_k = 0.1 \times 10 \times \frac{1}{2} = 0.5$ N

$$= -(0.5) \times 10$$
$$W_{friction} = -5\ J$$

Q. 3 For the photoelectric effect, the maximum kinetic energy (E_k) of the photoelectrons is plotted against the frequency (v) of the incident photons as shown in figure. The slope of the graph gives



Option 1:
Ratio of Planck's constant to electric charge

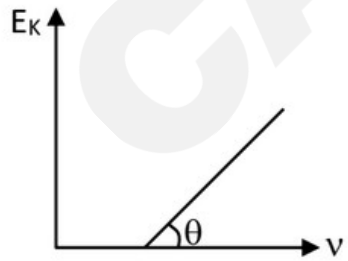
Option 2:
Work function of the metal

Option 3:
Charge of electron

Option 4:
Planck's constant

Correct Answer:
Planck's constant

Solution:

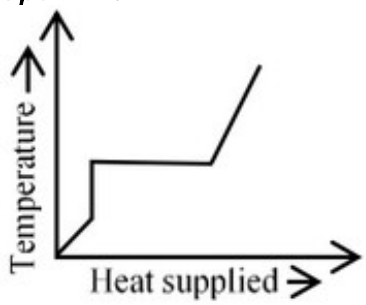


We know that
$$E = K.E. + \phi_0$$
$$h\nu = K.E. + \phi_0$$
$$K.E = h\nu - \phi_0$$

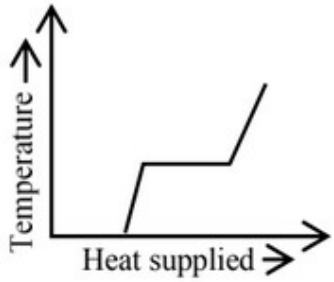
Slope of kinetic energy V_S frequency curve will be $\rightarrow$ Slope $= h$

Q. 4 A block of ice at $-10^{\circ}\mathrm{C}$ is slowly heated and converted to steam at $100^{\circ}\mathrm{C}$. Which of the following curves represent the phenomenon qualitatively:

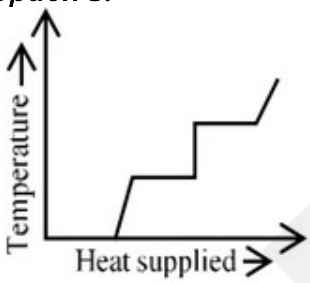
Option 1:



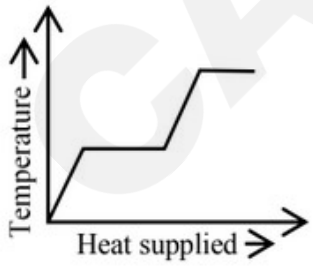
Option 2:



Option 3:

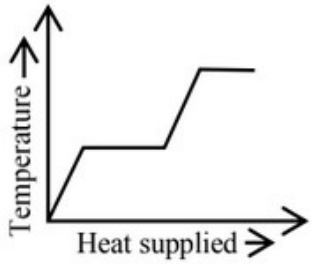


Option 4:



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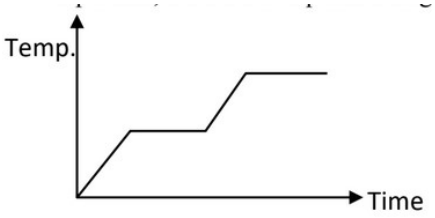
Correct Answer:



Solution:

$-10^{\circ}\mathrm{Cice} \xrightarrow[\text{change}]{\text{Temp.}} 0^{\circ}\mathrm{Cice} \xrightarrow[\text{change}]{\text{Temp.}} 0^{\circ}\mathrm{C}$ water $\xrightarrow[\text{change}]{\text{Temp.}} 100^{\circ}\mathrm{C}$ water $\xrightarrow{\text{phase change}} 100^{\circ}\mathrm{C}$ steam

In this process, there are two phase change and two temperature change.



Q. 5 In a nuclear fission reaction of an isotope of mass M, three similar daughter nuclei of same mass are formed. The speed of a daughter nuclei in terms of mass defect $\Delta \mathrm{M}$ will be :

Option 1:

$\sqrt{\frac{2c\Delta M}{M}}$

Option 2:

$\frac{\Delta Mc^2}{3}$

Option 3:

$c\sqrt{\frac{2\Delta M}{M}}$

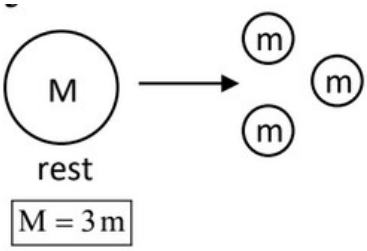
Option 4:

$c\sqrt{\frac{3\Delta M}{M}}$

Correct Answer:

$c\sqrt{\frac{2\Delta M}{M}}$

Solution:



Δ K.E. $=$ energy released due to fission

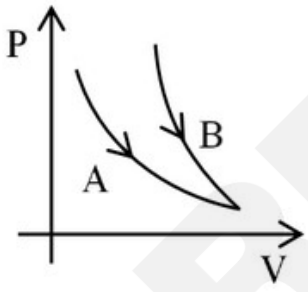
$$3\left(\frac{1}{2}\mathrm{mv}^2\right) = \Delta MC^2$$

$$\frac{3}{2}\mathrm{mv}^2 = \Delta MC^2$$

$$v^2 = \frac{2\Delta MC^2}{M}$$

$$v = \sqrt{\frac{2\Delta MC^2}{M}} = \sqrt[c]{\frac{2\Delta M}{M}}$$

Q. 6 Choose the correct statement for processes A & B shown in figure.



Option 1:
$\mathrm{PV}^{\gamma} = \mathrm{k}$ for process B and $\mathrm{PV} = \mathrm{k}$ for process A.

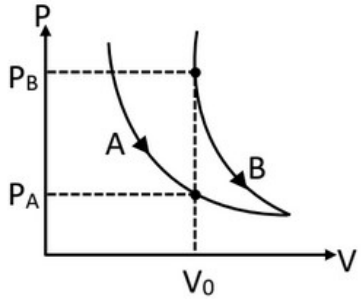
Option 2:
$\mathrm{PV} = \mathrm{k}$ for process B and A.

Option 3:
$\frac{\mathrm{P}^{\gamma-1}}{\mathrm{T}^{\gamma}} = \mathrm{k}$ for process B and $\mathrm{T} = \mathrm{k}$ for process A.

Option 4:
$\frac{T^{\gamma}}{P^{\gamma-1}} = k$ for process A and $PV = k$ for process B.

Correct Answer:
$\mathrm{PV}^{\gamma}=\mathrm{k}$ for process B and $\mathrm{PV}=\mathrm{k}$ for process A.

Solution:



Slope of isothermal $<$ Slope of adiabatic
(Slope $)_{\mathrm{A}}<$ (Slope $_{\mathrm{B}}$
Process $\mathrm{B} \rightarrow$ adiabatic $\rightarrow \mathrm{PV}^{\gamma}=\mathrm{K}$
Process $\mathrm{A} \rightarrow$ isothermal $\rightarrow \mathrm{PV}=\mathrm{K}$

Q. 7 An electron revolving in n^{th} Bohr orbit has magnetic moment μ_n. If $\mu_n \propto n^x$, the value of x is

Option 1:
2

Option 2:
1

Option 3:
3

Option 4:
0

Correct Answer:
1

Solution:
Magnetic moment $(\mu)=IA$

$$=\frac{\mathrm{e}}{\mathrm{T}} \cdot \pi \mathrm{r}^2$$

$$=\frac{\mathrm{e}}{\left(\frac{2 \pi \mathrm{r}}{\mathrm{V}}\right)} \cdot \pi \mathrm{r}^2=\frac{\mathrm{erV}}{2}$$

We know that

$$r \propto n^2$$

and $V \propto \frac{1}{n}$

so $\mu \propto rV$

$$\mu \propto \frac{n^2}{n} \Rightarrow \mu \propto n \quad \Rightarrow x = 1$$

Hence, the answer is the option (2).

Q. 8 An alternating voltage $V(t) = 220 \sin 100\pi t$ volt is applied to a purely resistive load of 50Ω. The time taken for the current to rise from half of the peak value to the peak value is:

Option 1:
5 ms

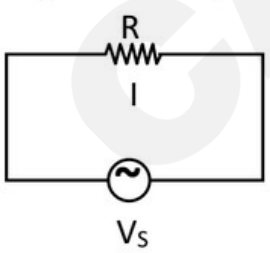
Option 2:
3.3 ms

Option 3:
7.2 ms

Option 4:
2.2 ms

Correct Answer:
3.3 ms

Solution:
$V(t) = 220 \sin(100\pi t)$



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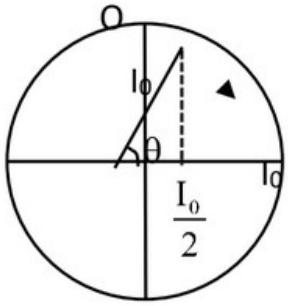
$$I = \frac{V_S}{R} = \frac{220}{50}\sin(100\pi t)$$

$$I = \frac{22}{5}\sin(100\pi t)$$

Using phasor

$$I = \frac{\pi/3}{100\pi}$$

$$\mathrm{t} = \frac{1}{300}\mathrm{sec} = \frac{10}{3}\mathrm{msec}$$
$$\mathrm{t} = 3.3\mathrm{msec}$$



$$\cos\theta = \frac{\frac{I_0}{2}}{I_0}$$

$$\cos\theta = \frac{1}{2}$$

$$\theta = 60^\circ$$
$$\theta = \frac{\pi}{3}$$

Q. 9 A block of mass m is placed on a surface having vertical crossection given by $y = x^2/4$. If coefficient of friction is 0.5 , the maximum height above the ground at which block can be placed without slipping is:

Option 1:
1/4 m

Option 2:
1/2 m

Option 3:
1/6 m

Option 4:
1/3 m

Correct Answer:
1/4 m

Solution:

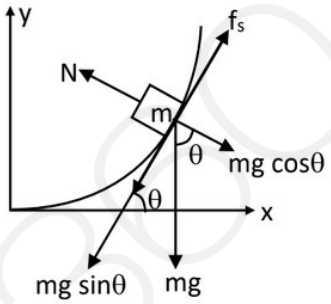
$$f_s = mg\sin\theta - - - - - - - - - - - - - (1)$$
$$N = mg\cos\theta - - - - - - - - - - - - - (2)$$

Now

$$f_s \le \mu N$$
$$mg\sin\theta \le \mu mg\cos\theta$$
$$\tan\theta \le \mu - - - - - - - - - - - - - (3)$$

Acc. to question

$$y = \frac{x^2}{4}$$
$$\tan\theta = \frac{dy}{dx} = \frac{1}{4}\cdot 2x = \frac{x}{2}$$



Put in (3)

$$\frac{x}{2} \le \mu$$
$$x \le 2\mu$$
$$x \le 2(0.5)$$
$$x \le 1$$

So $y \le \frac{1^2}{4} \Rightarrow y_{max} = \frac{1}{4} = 0.25$

Q. 10 If the total energy transferred to a surface in time t is 6.48×10^5 J, then the magnitude of the total momentum delivered to this surface for complete absorption will be:

Option 1:
2.46×10^{-3} kg m/s

Option 2:
2.16×10^{-3} kg m/s

Option 3:
1.58×10^{-3} kg m/s

Option 4:
4.32×10^{-3} kg m/s

Correct Answer:
$2.16 \times 10^{-3}\ \mathrm{kg\ m/s}$

Solution:
Energy transferred in time ' t ' $= E = 6.48 \times 10^{5}\ \mathrm{J}$

Momentum delivered $= \frac{\mathrm{E}}{\mathrm{C}}$

$$= \frac{6.48 \times 10^{5}}{3 \times 10^{8}}$$
$$= 2.16 \times 10^{-3}\ \mathrm{kg\ m/sec}$$

Hence, the answer is the option (2).

Q. 11 A beam of unpolarised light of intensity I_0 is passed through a polaroid A and then through another polaroid B which is oriented so that its principal plane makes an angle of 45° relative to that of A. The intensity of emergent light is:

Option 1:
$\mathrm{I}_0/4$

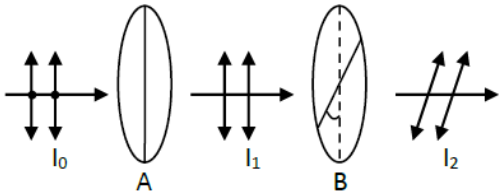
Option 2:
I_0

Option 3:
$\mathrm{I}_0/2$

Option 4:
$\mathrm{I}_0/8$

Correct Answer:
$\mathrm{I}_0/4$

Solution:



$I_1 = \frac{I_0}{2}$

and

$I_2 = I_2 \cos^2\theta$

$\& = \frac{I_0}{2}\cos^2 45^\circ$

$I_2 = \frac{I_0}{4}$

Q. 12 The escape velocity of a body from Earth is 11.2 km/s. If the radius of a planet is one third the radius of Earth and mass is one-sixth that of Earth, the escape velocity from the planet is :

Option 1:
11.2 km/s

Option 2:
8.4 km/s

Option 3:
4.2 km/s

Option 4:
7.9 km/s

Correct Answer:
7.9 km/s

Solution:

$V_e = \sqrt{\frac{2GM}{R}}$ for earth

If the radius of a planet is one third the radius of Earth and mass is one-sixth that of Earth, the escape velocity from the planet is :

$V_e^1 = \sqrt{\frac{2G(M/6)}{(R/3)}} = \sqrt{\frac{GM}{R}}$

$\sqrt{2}\, V_e^1 = V_e$

$V_e^1 = \frac{V_e}{\sqrt{2}} = \frac{11.2}{\sqrt{2}} = (11.2) \times 0.7\ \text{km/sec}$

$V_e^1 = 7.84\ km/sec \approx 7.9\ km/sec$
Hence, the answer is the option (4).

Q. 13 A particle of charge ' $-q$ ' and mass ' m ' moves in a circle of radius ' r ' around an infinitely long line charge of linear charge density ' $+\lambda$ '. Then time period will be given as:
(Consider k as Coulomb's constant)

Option 1:
$T^2 = \frac{4\pi^2 m}{2k\lambda q} r^3$

Option 2:
$T = 2\pi r\sqrt{\frac{m}{2k\lambda q}}$

Option 3:
$T = \frac{1}{2\pi r}\sqrt{\frac{m}{2k\lambda q}}$

Option 4:
$T = \frac{1}{2\pi}\sqrt{\frac{2k\lambda q}{m}}$

Correct Answer:
$T = 2\pi r\sqrt{\frac{m}{2k\lambda q}}$

Solution:
$F_e = \frac{mv^2}{r}$

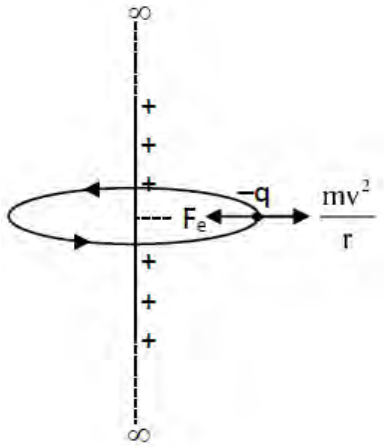
$qE = \frac{mv^2}{r}$

$q = \left(\frac{2K\lambda}{r}\right) = \frac{mv^2}{r}$

$v = \sqrt{\frac{2Kq\lambda}{m}}$

$T = \frac{2\pi r}{v} = \frac{2\pi r}{\sqrt{\frac{2Kq\lambda}{m}}}$

$$T = 2\pi r\sqrt{\frac{m}{2Kq\lambda}}$$



Q. 14 If mass is written as $m = kc^P G^{-1/2} h^{1/2}$ then the value of P will be : (Constants have their usual meaning with k a dimensionless constant)

Option 1:
$1/2$

Option 2:
$1/3$

Option 3:
2

Option 4:
$-1/3$

Correct Answer:
$1/2$

Solution:
$m = KC^P G^{-1/2} h^{1/2}$.....**(1)**

$$G = \frac{Fr^2}{m_1 \cdot m_2} = \frac{MLT^{-2} \cdot L^2}{M^2} = \left[M^{-1} L^3 T^{-2}\right]$$

The dimensional of Planck's constant h is **[ML^2T^{-1}]**.

The dimensional of the speed of light C is **[LT^{-1}]**.

The dimensional of mass constant m is **[M]**.

Now using (1)

$$[\mathrm{M}]=\left[\mathrm{LT}^{-1}\right]^{\mathrm{P}}\left[\mathrm{M}^{-1}\,\mathrm{L}^{3}\,\mathrm{T}^{-2}\right]^{-1/2}\left[\mathrm{ML}^{2}\,\mathrm{T}^{-1}\right]^{1/2}$$

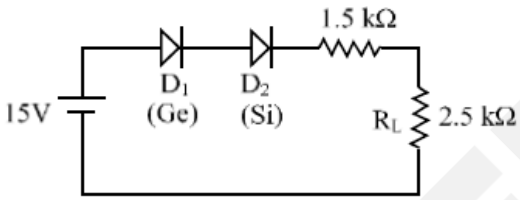
$$\mathrm{M}=\mathrm{ML}^{(\mathrm{P}-3/2+1)}\mathrm{T}^{(-\mathrm{P}+1-1/2)}$$

Compare the dimension of T from L.H.S and R.H.S then

$$-\mathrm{P}+\tfrac{1}{2}=0\Rightarrow\mathrm{P}=\tfrac{1}{2}$$

Hence, the answer is the option (1).

Q. 15 In the given circuit, the voltage across load resistance $(\mathrm{R_L})$ is:



Option 1:
8.75 V

Option 2:
9.00 V

Option 3:
8.50 V

Option 4:
14.00 V

Correct Answer:
8.75 V

Solution:

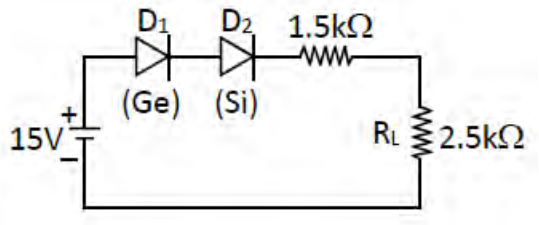
$$(\mathrm{V_B})_{\mathrm{Si}}=0.7\ \mathrm{V}$$

$$(\mathrm{V_B})_{\mathrm{Ge}}=0.3\ \mathrm{V}$$

Voltage drop across both diode $= 0.7 + 0.3 = 1\ \text{V}$

So remaining 14 volt will drop across $R_L \Rightarrow$

$\text{V}_\text{L} = \frac{2.5}{(1.5+2.5)} \times 14 = \frac{2.5}{4} \times 14\text{volt} = 8.75\text{volt}$



Q. 16 If three moles of monoatomic gas $\left(\gamma = \frac{5}{3}\right)$ is mixed with two moles of a diatomic gas $\left(\gamma = \frac{7}{5}\right)$, the value of adiabatic exponent γ for the mixture is

Option 1:
1.75

Option 2:
1.40

Option 3:
1.52

Option 4:
1.35

Correct Answer:
1.52

Solution:

$\text{Monoatomic gas} \longrightarrow 3\ \text{mole}$

$f_1 = 3, n_1 = 3$

$\text{Diatomic gas} \longrightarrow 2\ \text{mole}$

$f_2 = 5, n_1 = 2$

$\gamma_{\text{mix}} = 1 + \frac{2}{\text{f}_{\text{min}}} \quad \ldots(1)$

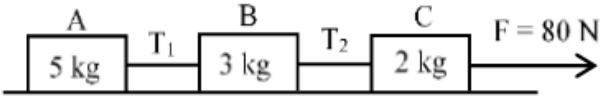
$f_{mix}=\frac{n_1f_1+n_2f_2}{n_1+n_2}=\frac{3(3)+2(5)}{5}=\frac{19}{5}$

$\gamma_{mix}=1+\frac{2}{19/5}=1+\frac{10}{19}=\frac{29}{19}$

$\gamma_{mix}=1.52$

Hence, the answer is the option (3).

Q. 17 Three blocks A, B and C are pulled on a horizontal smooth surface by a force of 80 N as shown in figure



The tensions T_1 and T_2 in the string are respectively:

Option 1:
40 N, 64 N

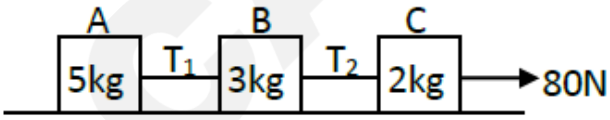
Option 2:
60 N, 80 N

Option 3:
88 N, 96 N

Option 4:
80 N, 100 N

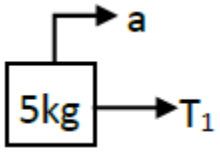
Correct Answer:
40 N, 64 N

Solution:



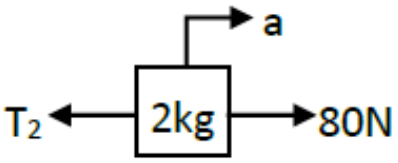
$\frac{80}{10}=8\,m/s^2$

F.B.D of 5 kg



$T_1=5(a)$

$T_1 = 5(8) = 40N$



$80 - T_2 = 2(8)$

Q. 18 When a potential difference V is applied across a wire of resistance R, it dissipates energy at a rate W. If the wire is cut into two halves and these halves are connected mutually parallel across the same supply, the energy dissipation rate will become:

Option 1:
$\frac{1}{4}$W

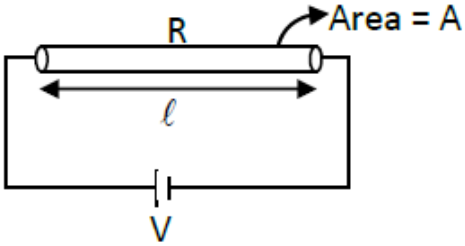
Option 2:
$\frac{1}{2}$W

Option 3:
2W

Option 4:
4W

Correct Answer:
4W

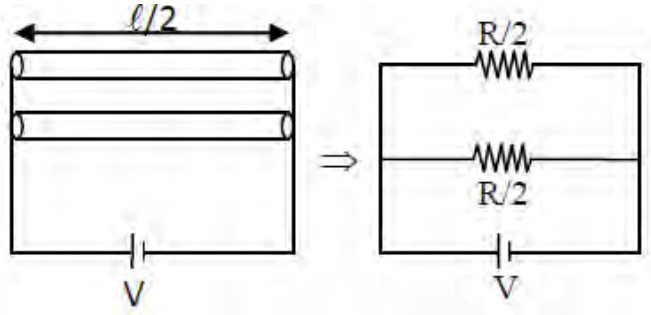
Solution:



$$w = \frac{V^2}{R} \quad .....(1)$$

And $R = \frac{\rho \ell}{A}$

Now



$$R_{eq} = \frac{R}{4}$$

$$W' = \frac{V^2}{\frac{R}{4}} = \frac{4V^2}{R} = w' = 4W$$

Q. 19 Match List I with List II

List I		List II	
A.	Gauss's law of magnetostatics	I.	$\oint \vec{E} \cdot \vec{da} = \frac{1}{\varepsilon_o} \int \rho dV$
B.	Faraday's law of electro-magnetic induction	II.	$\oint \vec{B} \cdot \vec{da} = 0$
C.	Ampere's law	III.	$\int \vec{E} \cdot \vec{dl} = \frac{-d}{dt} \int \vec{B} \cdot \vec{da}$
D.	Gauss's law of electrostatics	IV.	$\oint \vec{B} \cdot \vec{dl} = \mu_0 I$

Choose the correct answer from the options given below:

Option 1:
A-I, B-III, C-IV, D-II

Option 2:
A-III, B-IV, C-I, D-II

Option 3:
A-IV, B-II, C-III, D-I

Option 4:
A-II, B-III, C-IV, D-I

Correct Answer:
A-II, B-III, C-IV, D-I

Solution:
By Maxwell equations-

(A) Gauss's law of magnetostatics $\rightarrow \oint \vec{B} \cdot d\vec{a} = 0$ (II)

(B) Faraday's law $\rightarrow \int \vec{E} \cdot \overrightarrow{dt} = -\frac{\mathrm{d}}{\mathrm{dt}} \int \overrightarrow{\mathrm{B}} \cdot \mathrm{da}$ (III)

(C) Ampere's law $\rightarrow \oint \vec{B} \cdot d\vec{t} = \mu_0$ I (IV)

(A) Gauss's law of electrostatic $\rightarrow \oint \vec{E} \cdot d\vec{a} = \frac{1}{\varepsilon_0} \int \rho \mathrm{dV}$ (I)

Hence, the answer is the option (4).

Q. 20 Projectiles A and B are thrown at angles of 45° and 60° with vertical respectively from top of a $400\ \mathrm{m}$ high tower. If their ranges and times of flight are same, the ratio of their speeds of projection $v_A : v_B$ is:
[Take $\mathrm{g} = 10\ \mathrm{ms}^{-2}$]

Option 1:
$1 : \sqrt{3}$

Option 2:
$\sqrt{2} : 1$

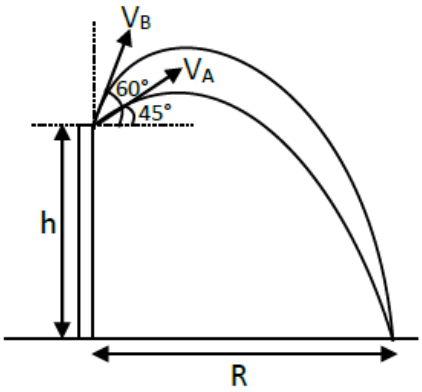
Option 3:
$1 : 2$

Option 4:
$1 : \sqrt{2}$

Correct Answer:

$1 : \sqrt{2}$

Solution:



$\mathrm{R_A} = \mathrm{R_B}$

$(\mathrm{V_A}\cos 45^\circ)\mathrm{T} = (\mathrm{V_B}\cos 60^\circ)\mathrm{T}$

$\frac{\mathrm{V_A}}{\sqrt{2}} = \frac{\mathrm{V_B}}{2}$

$\frac{\mathrm{V_A}}{\mathrm{V_B}} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$

Q. 21 A power transmission line feeds input power at $2.3\mathrm{kV}$ to a step down transformer with its primary winding having 3000 turns. The output power is delivered at $230\ \mathrm{V}$ by the transformer. The current in the primary of the transformer is $5\ \mathrm{A}$ and its efficiency | is 90%. The winding of transformer is made of copper. The output current of transformer is_____ A.

Correct Answer:

45

Solution:

Efficiency $\eta = \frac{\mathrm{P_0}}{\mathrm{P_i}} \times 100\% = 90\%$

$0.9\mathrm{P_i} = \mathrm{P_0}$

$\mathrm{P_i} = \mathrm{V_p} i_p$

$\mathrm{P_i} = 2.3 \times 10^3(5) = 11.5 \times 10^3\ \mathrm{W}$

$P_0 = V_s i_s$

$P_0 = (230)i_s$

$(0.9)\left(11.5 \times 10^3\right) = 230 i_s$

$i_s = \frac{(0.9)(11.5 \times 10^3)}{230} = 0.045 \times 10^3$

$i_s = 45\ A$

Hence, the answer is (45).

Q. 22 A big drop is formed by coalescing 1000 small identical drops of water. If E_1 is the total surface energy of 1000 small drops of water and E_2 is the surface energy of a single big drop of water, then $E_1 : E_2$ is x: 1 where x_____.

Correct Answer:
10

Solution:
Let radius of small drops $= r$

Let radius of big drops $= R$

Here volume will remain conserve (volume conservation)

$\therefore (1000)V_S = (1)V_b \rightarrow$ volume of big drop

Volume of small drop

$\Rightarrow 10^3\left(\frac{4}{3}\pi r^3\right) = \frac{4}{3}\pi R^3$

$R = 10r$

We know, surface tension $= \frac{\text{Surface energy}}{\text{Area}}$

$\therefore$ Surface energy $=$ (Surface tension)(Area)

$\Rightarrow \epsilon_1 = T \times 4\pi r^2 \times 10^3$

$\epsilon_2 = T \times 4\pi R^2$

$\therefore \quad \frac{\epsilon_1}{\epsilon_2} = \frac{(10^3)(r^2)}{R^2} = \frac{(10^3)r^2}{(10r)^2} = \frac{10}{1}$

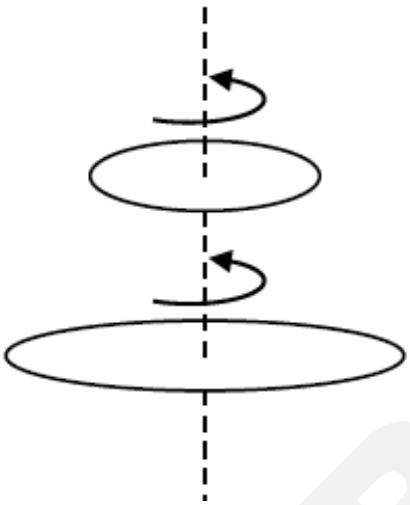
$\frac{\epsilon_1}{\epsilon_2} = \frac{10}{1} = \frac{x}{1}, \therefore \quad x = 10$

Hence, the answer is (10).

Q. 23 Two discs of moment of inertia $I_1 = 4\ kg\ m^2$ and $I_2 = 2\ kg\ m^2$, about their central axes \& normal to their planes, rotating with angular speeds $10 rad/s \& 4 rad/s$ respectively are brought into contact face to face with their axes of rotation coincident. The loss in kinetic energy of the system in the process is _____J.

Correct Answer:
24

Solution:



Here angular momentum will remain conserve

$L = I\omega$

$L_i = L_f$

$4 \times 10 + 2 \times 4 = (6)\omega$

$48 = 6\omega$

$\omega = 8 rad/sec$

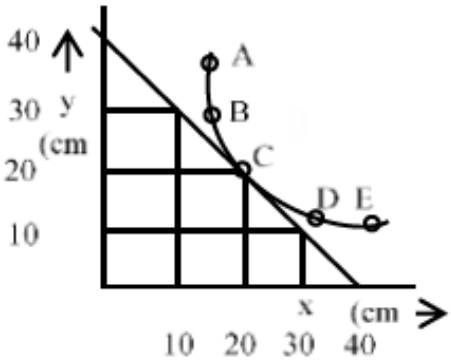
$\Delta KE = KE_f = KE_i$

$\Delta K = \frac{1}{2}(6)(8)^2 - \left[\left(\frac{1}{2}4 \times 10^2\right) + \left(\frac{1}{2}(2) \times (4)^2\right)\right]$

$\Delta K = 192 - 216$

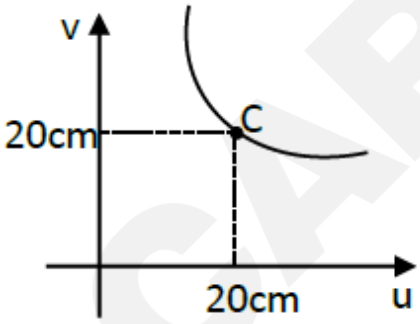
$\Delta K = -24\ J$

Q. 24 In an experiment to measure the focal length (f) of a convex lens, the magnitude of object distance (x) and the image distance (y) are measured with reference to the focal point of the lens. The y - x plot is shown in figure.
The focal length of the lens is _____ cm.



Correct Answer:
20

Solution:



Here, $u = 20\ cm$

From the focal point, $v = 20\ cm$

$xy = f^2 \rightarrow$ Newton's formula

$20 \times 20 = f^2$

$ f=20

Q. 25 A vector has magnitude same as that of $\vec{A} = 3\hat{i} + 4\hat{j}$ and is parallel to $\vec{B} = 4\hat{i} + 3\hat{j}$. The x and y components of this vector in first quadrant are x and 3 respectively where x =_____.

Correct Answer:
4

Solution:
Given

$\overrightarrow{\mathrm{A}} = 3\mathrm{i} + 4\mathrm{j}, \overrightarrow{\mathrm{B}} = 4\mathrm{i} + 3\mathrm{j},$

$|\mathrm{A}| = \sqrt{3^2 + 4^2} = 5, \hat{\mathrm{B}} = \frac{4\mathrm{i}+3\mathrm{j}}{5}, \frac{\overrightarrow{\mathrm{B}}}{|\mathrm{B}|} = \hat{\mathrm{B}}$

Now, as per question,

$\vec{C} = |A|\hat{B}$, mag of $\vec{A}$ and direction of B

$=(5)\frac{(4i+3j)}{5} = 4i + 3j,$

Therefore, x= 4

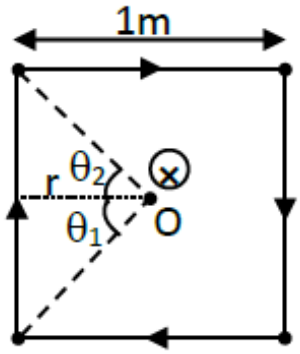
Hence, the answer is (4).

Q. 26 The current of 5A flows in a square loop of sides $1\ \mathrm{m}$ is placed in air. The magnetic field at the centre of the loop is $\mathrm{X}\sqrt{2} \times 10^{-7}\ \mathrm{T}$. The value of X is_____.

Correct Answer:
40

Solution:
$\mathrm{i} = 5A$

Square of side = 1m



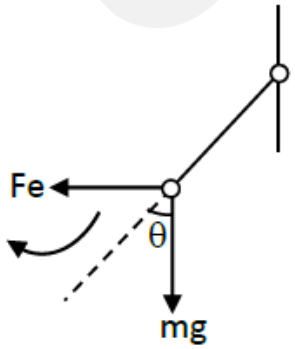
Magnetic field at centre will be into the plane (X), due to all four wires

$$\vec{B}_{\text{net}} = 4\,B_1$$
$$B_1 = \text{ Magnetic field due to one side,}$$
$$B_1 = \frac{\mu \cdot i}{4\pi r}(\sin\theta_1 + \sin\theta_2),\ \text{Here } \theta_1 = \theta_2 = 45^\circ$$
$$\&r = \frac{1}{2}\ \text{m}$$
$$B_1 = \frac{\mu \cdot (5)}{4\pi(1/2}\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)$$
$$B_{\text{net}} = 4\,B_1 = 40\sqrt{2} \times 10^{-2}\ \text{J}$$
$$\therefore x = 40$$

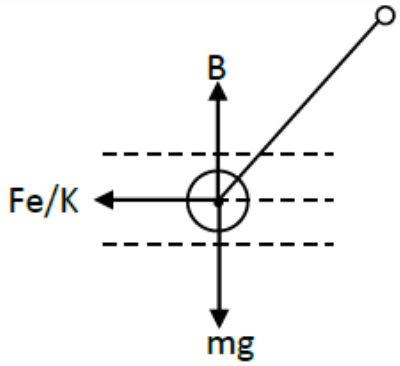
Q. 27 Two identical charged spheres are suspended by strings of equal lengths. The strings make an angle of 37° with each other. When suspended in a liquid of density $0.7\ \text{g/cm}^3$, the angle remains same. If density of material of the sphere is $1.4\ \text{g/cm}^3$, the dielectric constant of the liquid is $\left(\tan 37^\circ = \frac{3}{4}\right)$

Correct Answer:
2

Solution:
$\tan\theta = \frac{F_e}{mg}$ .....(1)



$\tan\theta = \frac{F_e/K}{(mg-B)}$ .....(2)



$\frac{F_e}{K(mg-B)} = \frac{F_e}{mg}$

$K(mg - B) = mg$

$K(\sigma_m Vg - \rho_\ell Vg) = \sigma_m Vg$

$Vkg[1.4 - 0.7] = 1.4\ kg$

$ \mathrm{~K}=\frac{1.4}{0.7}=2$

Q. 28 A simple pendulum is placed at a place where its distance from the earth's surface is equal to the radius of the earth. If the length of the string is $4\ m$, then the time period of small oscillations will be s. [take $g = \pi^2\ ms^{-2}$]

Correct Answer:
8

Solution:
$T = 2\pi\sqrt{\frac{\ell}{g_{eff}}},$

$g = \frac{GM}{R^2},\ \text{at a height, h}, g = \frac{GM}{(R+H)^2}$

$g_{eff} = g\left(\frac{R}{R+h}\right)^2,\ g_{eff} = g\left(\frac{R}{2R}\right)^2$

$g_{eff} = \frac{g}{4}$

$\therefore T = 2\pi\sqrt{\frac{\ell}{g_{eff}}}$

$= 2\pi\sqrt{\frac{4\times4}{g}} = \frac{(2\pi)(4)}{\sqrt{g}},\ \text{Given, } g = \pi^2$

$T = 8sec$
Hence, the answer is (8).

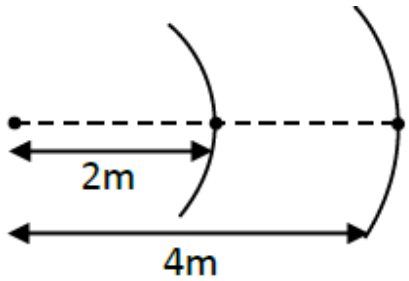
Q. 29 A point source is emitting sound waves of intensity $16 \times 10^{-8} \mathrm{Wm}^{-2}$ at the origin. The difference in intensity (magnitude only) at two points located at a distances of $2\ \mathrm{m}$ and $4\ \mathrm{m}$ from the origin respectively will be $\times 10^{-8} \mathrm{Wm}^{-2}$.

Correct Answer:
3

Solution:
We know for point source, $I \propto \frac{1}{r^2} = \frac{K}{4\pi r^2}$

$\mathrm{r}_1 = 2\ \mathrm{m}, \mathrm{r}_2 = 4\ \mathrm{m} = 2\mathrm{r}_1$

$\mathrm{I}_0 = 16 \times 10^{-8}\ \mathrm{W/m}^2$



at 2m,

$\mathrm{I}_1 = \frac{\mathrm{I}_0}{(2)^2} = \frac{16 \times 10^{-6}}{4} \quad \mathrm{I}_1 = 4 \times 10^{-8}$

$\&\mathrm{I}_2 = \frac{\mathrm{I}_0}{4^2} = \frac{16 \times 10^{-8}}{16} = 10^{-8}$

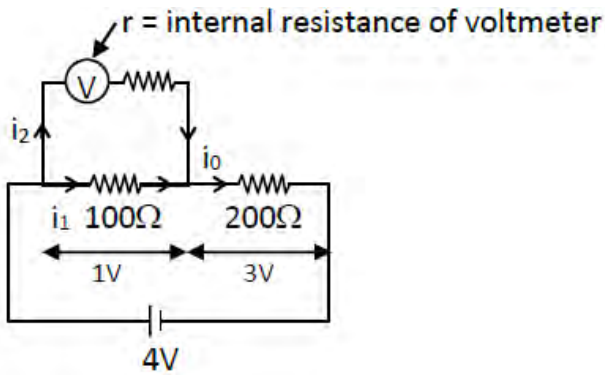
$|\mathrm{I}_2 - \mathrm{I}_1| = 3 \times 10^{-8}\ \mathrm{W/m}^2, \mathrm{x} = 3$

Q. 30 Two resistance of 100Ω and 200Ω are connected in series with a battery of $4\ \mathrm{V}$ and negligible internal resistance. A voltmeter is used to measure voltage across 100Ω resistance, which gives reading as $1\ \mathrm{V}$. The resistance of voltmeter must be_____ Ω.

Correct Answer:
200

Solution:
Given potential across Voltmeter $= 1\ \mathrm{V}$

$\therefore$ Current in $100\Omega = \frac{1}{100} A = i_1$



Now, Current in $200\ \mathrm{N} = \frac{3}{100}\ \mathrm{A} \Rightarrow$ as total potential $= 4\ \mathrm{V}_1 \& 1\ \mathrm{V}$ is drop across 100Ω.

Let current in voltmeter $= \mathrm{i}$

$\therefore$ According to K.1, law, $\mathrm{i}_1 + \mathrm{i}_2 = \mathrm{i}_0$

$\mathrm{i}_2 + \frac{1}{100} = \frac{3}{200}$

$\mathrm{i}_2 + \frac{3}{200} - \frac{1}{100} = \frac{1}{200}\ \mathrm{A}_1$

$\therefore$ 1V given across voltmeter of current $\frac{1}{200}$,

$\mathrm{V} = \mathrm{IR}_1 \Rightarrow 1 = \frac{1}{200}(\mathrm{r}), \Rightarrow \mathrm{r} = 200\Omega$

Chemistry

Q. 1 Which among the following purification methods is based on the principle of "Solubility" in two different solvents?

Option 1:
Column Chromatography

Option 2:
Sublimation

Option 3:
Distillation

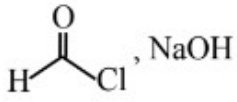
Option 4:
Differential Extraction

Correct Answer:
Differential Extraction

Solution:
Different layers are formed, which can be separated in the funnel.

Hence, the answer is the option (4).

Q. 2 Salicylaldehyde is synthesized from phenol, when reacted with

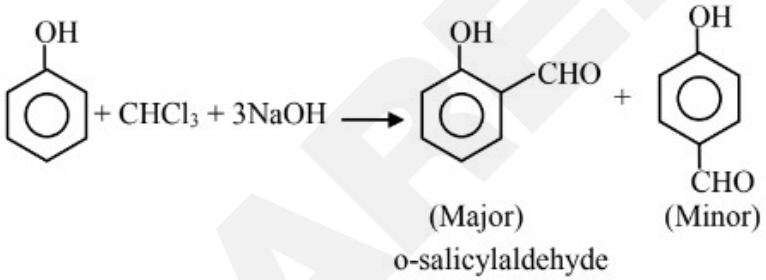
Option 1:


Option 2:
$CO_2, NaOH$

Option 3:
$CCl_4, NaOH$

Option 4:
$HCCl_3, NaOH$

Correct Answer:
$HCCl_3, NaOH$

Solution:


Reimer Tiemann Reaction

Q. 3 Given below are two statements:

Statement - I : High concentration of strong nucleophilic reagent with secondary alkyl halides which do not have bulky substituents will follow S_N2 mechanism.

Statement - II : A secondary alkyl halide when treated with a large excess of ethanol follows S_N1 mechanism.

In the light of the above statements, choose the most appropriate from the options given below:

Option 1:
Statement I is true but Statement II is false.

Option 2:
Statement I is false but Statement II is true.

Option 3:
Both Statement I and Statement II are false.

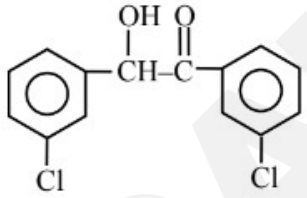
Option 4:
Both Statement I and Statement II are true.

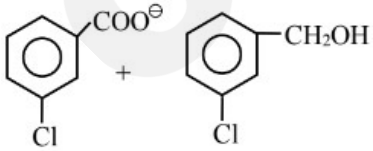
Correct Answer:
Both Statement I and Statement II are true.

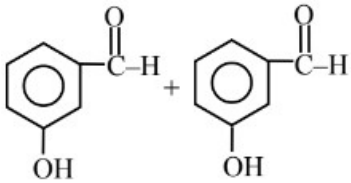
Solution:
Statement 1 : High $\mathrm{conc^n}$. of strong nu$^-$ reagent with 2° Alkyl Halide which do not have bulky substituents will follow $\mathrm{S_N}2$ mechanism.
Hence, statement I is true.
Statement 2: 2° Alkyl Halide reacts with excess of ethanol undergo $\mathrm{S_N}1$ reaction.

Hence, the answer is the option (4).

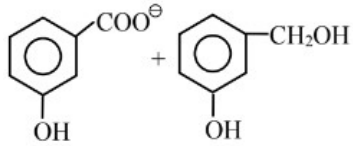
Q. 4 m-chlorobenzaldehyde on treatment with $50\%\mathrm{KOH}$ solution yields

Option 1:


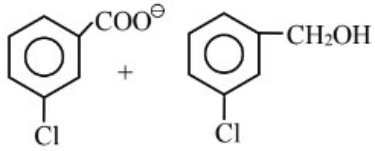
Option 2:


Option 3:


Option 4:

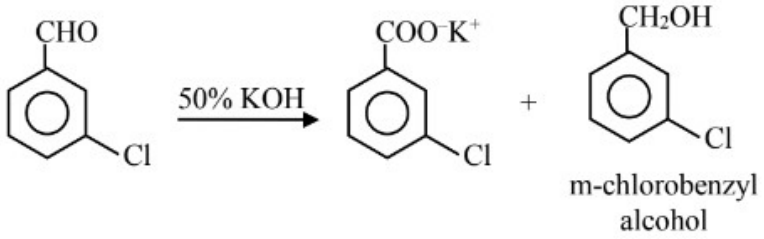


Correct Answer:



Solution:

Cannizaro reaction (Disproportionation reaction)



Q. 5 Given below are two statements: One is labelled as Assertion A and the other is labelled as Reason R:

Assertion A: H_2Te is more acidic than H_2S.

Reason R: Bond dissociation enthalpy of H_2Te is lower than H_2S.

In the light of the above statements, choose the most appropriate from the options given below:

Option 1:

Both A and R are true but R is NOT the correct explanation of A.

Option 2:

Both A and R are true and R is the correct explanation of A.

Option 3:

A is false but R is true.

Option 4:

A is true but R is false.

Correct Answer:

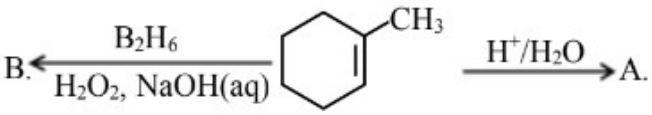
Both A and R are true and R is the correct explanation of A.

Solution:

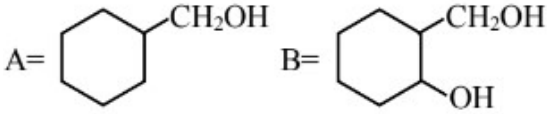
Due to lower bond dissociation energy of H_2Te it ionizes to give H^+ more easily as compare to H_2S

Hence, the answer is the option (2).

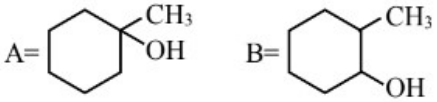
Q. 6 Products A and B formed in the following set of reactions are



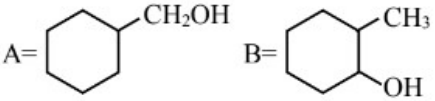
Option 1:



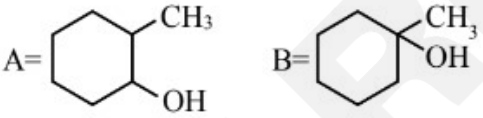
Option 2:



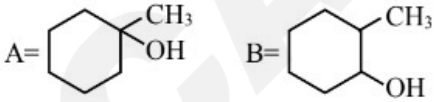
Option 3:



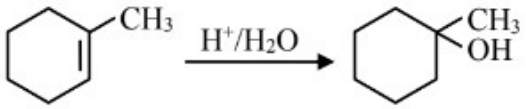
Option 4:

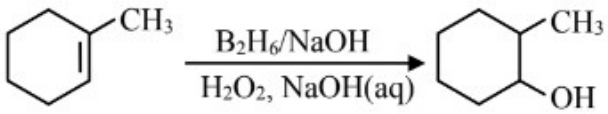


Correct Answer:



Solution:





Q. 7 IUPAC name of following compound is :



Option 1:
2-Aminopentanenitrile

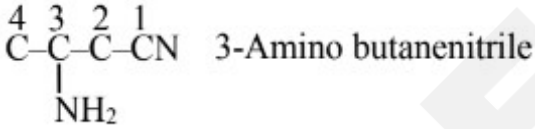
Option 2:
2-Aminobutanenitrile

Option 3:
3-Aminobutanenitrile

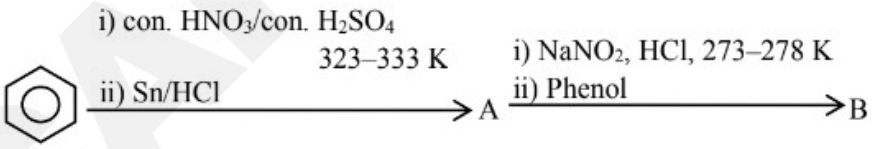
Option 4:
3-Aminopropanenitrile

Correct Answer:
3-Aminobutanenitrile

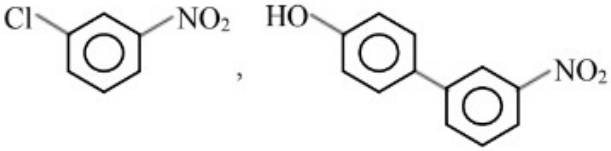
Solution:



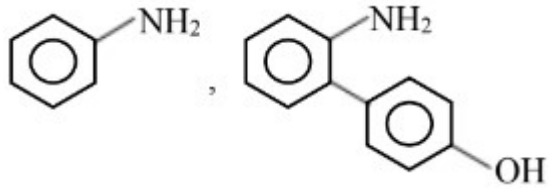
Q. 8 The products A and B formed in the following reaction scheme are respectively



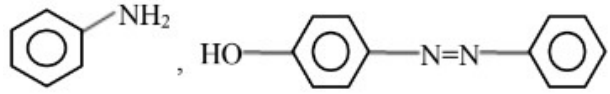
Option 1:



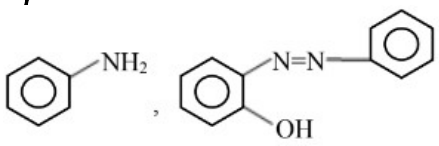
Option 2:



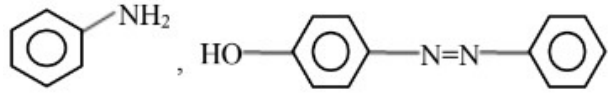
Option 3:



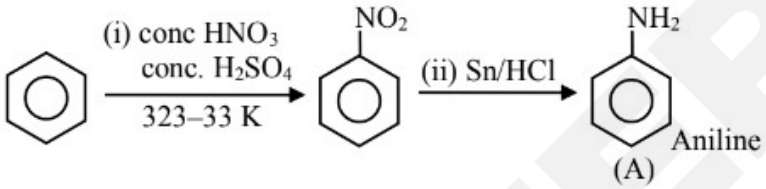
Option 4:

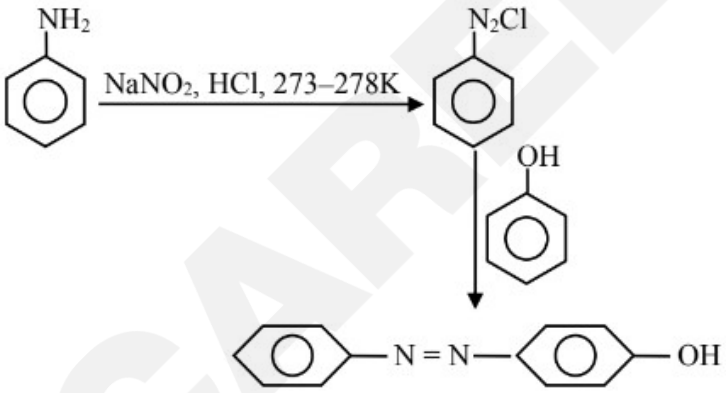


Correct Answer:



Solution:





Q. 9 The molecule / ion with square pyramidal shape is

Option 1:

$[Ni(CN)_4]^{2-}$

Option 2:

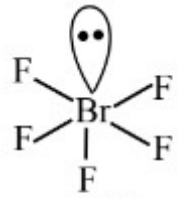
PCl_5

Option 3:
BrF_5

Option 4:
PF_5

Correct Answer:
BrF_5

Solution:



sp^3d^2
square pyramidal

Q. 10 The orange colour of $K_2Cr_2O_7$ and purple colour of $KMnO_4$ is due to

Option 1:
Charge transfer transition in both.

Option 2:
$d \rightarrow d$ transitions in $KMnO_4$ and charge transfer transitions in $K_2Cr_2O_7$

Option 3:
$d \rightarrow d$ transitions in $K_2Cr_2O_7$ and charge transfer transitions in $KMnO_4$.

Option 4:
$d \rightarrow d$ transitions in both

Correct Answer:
Charge transfer transition in both.

Solution:
Orange colour of $K_2Cr_2O_7$ and purple colour of $KMnO_4$ is due to ligand to metal charge transfer (LMCT).

Hence, the answer is the option (1).

Q. 11 Alkaline oxidative fusion of MnO_2 gives "A" which on electrolytic oxidation in alkaline solution produces B. A and B respectively are

Option 1:
Mn_2O_7 and MnO_4^-

Option 2:
MnO_4^{2-} and MnO_4^-

Option 3:
Mn_2O_3 and MnO_4^{2-}

Option 4:
MnO_4^{2-} and Mn_2O_7

Correct Answer:
MnO_4^{2-} and MnO_4^-

Solution:
Alkaline oxidative fusion of MnO_2 gives MnO_4^{2-}

$$2MnO_2 + 4OH + O_2 \rightarrow 2MnO_4^{2-} + 2H_2O$$

electrolytic oxidation of MnO_4^{2-} in alkaline solution gives MnO_4^-

$$MnO_4^{2-} \rightarrow MnO_4^- + e^-.$$

Hence, the answer is the option (2).

Q. 12 If a substance ' A ' dissolves in solution of a mixture of ' B ' and ' C' with their respective number of moles as n_A, n_B and n_C. Mole fraction of C in the solution is

Option 1:
$\frac{n_C}{n_A \times n_B \times n_C}$

Option 2:
$\frac{n_C}{n_A + n_B + n_C}$

Option 3:
$\frac{n_C}{n_A - n_B - n_C}$

Option 4:
$\frac{n_B}{n_A + n_B}$

Correct Answer:

$\frac{n_C}{n_A+n_B+n_C}$

Solution:

mole fraction of $C\ (X_c) = \frac{n_C}{n_A+n_B+n_C}$

Hence, the answer is the option (2).

Q. 13 Given below are two statements:

Statement – I : Along the period, the chemical reactivity of the elements gradually increases from group 1 to group 18 .

Statement – II : The nature of oxides formed by group 1 elements is basic while that of group 17 elements is acidic.

In the light of the above statements, choose the most appropriate from the options given below:

Option 1:

Both Statement I and Statement II are True

Option 2:

Statement I is True But Statement II is False

Option 3:

Statement I is False but statement II is true

Option 4:

Both Statement I and Statement II are False

Correct Answer:

Statement I is False but statement II is true

Solution:

Chemical reactivity along the period decreases

So, statement I is wrong

Group I elements form basic nature oxide, while group 17 forms acidic nature oxide.

Hence, the answer is the option (3).

Q. 14 The coordination geometry around the manganese in decacarbonyldimanganese (0) is

Option 1:
Octahedral

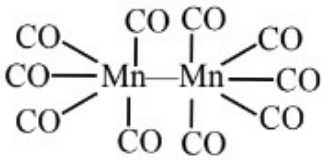
Option 2:
Trigonal bipyramidal

Option 3:
Square pyramidal

Option 4:
Square planar

Correct Answer:
Octahedral

Solution:



Octahedral around Mn

Q. 15 Given below are two statements:

Statement - I : Since Fluorine is more electronegative than nitrogen, the net dipole moment of $\mathrm{NF_3}$ is greater than $\mathrm{NH_3}$.

Statement - II : In $\mathrm{NH_3}$, the orbital dipole due to lone pair and the dipole moment of NH bonds are in opposite direction, but in $\mathrm{NF_3}$ the orbital dipole due to lone pair and dipole moments of $\mathrm{N-F}$ bonds are in same direction.

In the light of the above statements, choose the most appropriate from the options given below:

Option 1:
Statement I is true but Statement II is false.

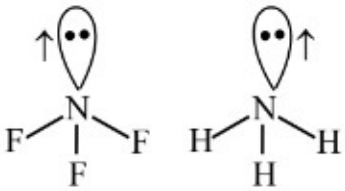
Option 2:
Both Statement I and Statement II are false.

Option 3:
Both Statement I and Statement II are true.

Option 4:
Statement I is false but Statement II is true.

Correct Answer:
Both Statement I and Statement II are false.

Solution:



Both statement are false.

Q. 16 The correct stability order of carbocations is

Option 1:
$(CH_3)_3C^+ > CH_3-\overset{+}{C}H_2 > (CH_3)_2\overset{+}{C}H > \overset{+}{C}H_3$

Option 2:
$\overset{+}{C}H_3 > (CH_3)_2\overset{+}{C}H > CH_3-\overset{+}{C}H_2 > (CH_3)_3\overset{+}{C}$

Option 3:
$(CH_3)_3\overset{+}{C} > (CH_3)_2\overset{+}{C}H > CH_3-\overset{+}{C}H_2 > \overset{+}{C}H_3$

Option 4:
$\overset{+}{C}H_3 > CH_3-\overset{+}{C}H_2 > CH_3-\underset{CH_3}{\overset{+}{C}H} > (CH_3)_3C^+$

Correct Answer:
$(CH_3)_3\overset{+}{C} > (CH_3)_2\overset{+}{C}H > CH_3-\overset{+}{C}H_2 > \overset{+}{C}H_3$

Solution:
Stability order carbocation

$CH_3-\underset{CH_3}{\overset{+}{\underset{|}{C}}}-CH_3 > CH_3-\overset{+}{C}H-CH_3 > \overset{+}{C}H-CH_3 > \overset{+}{C}H_3$

$(9\alpha - H) \qquad (6\alpha - H) \qquad (3\alpha - H)$

Q. 17 The solution from the following with highest depression in freezing point/lowest freezing point is

Option 1:
180 g of acetic acid dissolved in water

Option 2:
180 g of acetic acid dissolved in benzene

Option 3:
180 g of benzoic acid dissolved in benzene

Option 4:
180 g of glucose dissolved in water

Correct Answer:
180 g of acetic acid dissolved in water

Solution:
The highest depression in freezing point (Colligative Properties) is inversely proportional to the experimentally molar mass of non-volatile solute
* on dissociation exp. Molar mass dec.
* On association exp. Molar Mass Inc.

Hence, the answer is the option (1).

Q. 18 A and B formed in the following reactions are:

$$\mathrm{CrO_2Cl_2 + 4NaOH \rightarrow A + 2NaCl + 2H_2O}$$

$$\mathrm{A + 2HCl + 2H_2O_2 \rightarrow B + 3H_2O}$$

Option 1:
$\mathrm{A = Na_2CrO_4,\ B = CrO_5}$

Option 2:
$\mathrm{A = Na_2Cr_2O_4,\ B = CrO_4}$

Option 3:
$\mathrm{A = Na_2Cr_2O_7,\ B = CrO_3}$

Option 4:
$A = Na_2Cr_2O_7,\ B = CrO_5$

Correct Answer:
$A = Na_2CrO_4,\ B = CrO_5$

Solution:

$$CrO_2Cl_2 + 4NaOH \rightarrow Na_2CrO_4 + 2NaCl + 2H_2O$$
(A)

$$Na_2CrO_4 + 2HCl + 2H_2O_2 \rightarrow CrO_5 + 3H_2O + 2NaCl$$
(B)

Hence, the answer is the option (1).

Q. 19 Choose the correct statements about the hydrides of group 15 elements.
A. The stability of the hydrides decreases in the order
$NH_3 > PH_3 > AsH_3 > SbH_3 > BiH_3$.
B. The reducing ability of the hydride increases in the order
$NH_3 < PH_3 < AsH_3 < SbH_3 < BiH_3$.
C. Among the hydrides, NH_3 is strong reducing agent while BiH_3 is mild reducing agent.
D. The basicity of the hydrides increases in the order
$NH_3 < PH_3 < AsH_3 < SbH_3 < BiH_3$.

Choose the most appropriate from the options given below:

Option 1:
B and C only

Option 2:
C and D only

Option 3:
A and B only

Option 4:
A and D only

Correct Answer:
A and B only

Solution:

NH_3	M-H bond length ↑
PH_3	Thermal stability ↓
ASH_3	Reducing nature ↑
SbH_3	
BiH_3	

Q. 20 Reduction potential of ions are given below:

ClO_4^-	IO_4^-	BrO_4^-
$E^\circ = 1.19\ V$	$E^\circ = 1.65\ V$	$E^\circ = 1.74\ V$

The correct order of their oxidizing power is.

Option 1:
$ClO_4^- > IO_4^- > BrO_4^-$

Option 2:
$BrO_4^- > IO_4^- > ClO_4^-$

Option 3:
$BrO_4^- > ClO_4^- > IO_4^-$

Option 4:
$IO_4^- > BrO_4^- > ClO_4^-$

Correct Answer:
$BrO_4^- > IO_4^- > ClO_4^-$

Solution:
Oxidizing power $\propto$ Reduction potential

	ClO_4^-	IO_4^-	BrO_4^-
E^o_{red}	1.19 V	1.65 V	1.74 V

order of standard reduction potential

$E^\circ ClO_4^- < IO_4^- < BrO_4^-$

Thus, the order of oxidizing power

$ClO_4^- < IO_4^- < BrO_4^-$

Hence, the answer is the option (2).

Q. 21 Number of complexes which show optical isomerism among the following is

$$\text{cis} - [Cr(ox)_2Cl_2]^{3-}, [Co(en)_3]^{3+}, \text{cis} - [Pt(en)_2Cl_2]^{2+}, \text{cis} - [Co(en)_2Cl_2]^{+}$$
$$\text{trans} - [Pt(en)_2Cl_2]^{2+}, \text{trans} - [Cr(ox)_2Cl_2]^{3-}$$

Correct Answer:
4

Solution:
Those complex which has no COS and POS are optically active Cis-
$[Cr(ox)_2Cl_2]^{3}, [Co(en)_3]^{3+}, \text{Cis} - [Pt(en)_2Cl_2]^{+2}, [Co(en)_2Cl_2]$

Hence, the answer is (4).

Q. 22 NO_2 required for a reaction is produced by decomposition of N_2O_5 in CCl_4 as by equation

$$2\,N_2O_{5(g)} \rightarrow 4NO_{2(g)} + O_{2(g)}$$

The initial concetration. of N_2O_5 is $3 mol L^{-1}$ and it is $2.75 mol L^{-1}$ after 30 minutes.
The rate of formation of NO_2 is $x \times 10^{-3} mol L^{-1}\ min^{-1}$, value of x is (nearest integer)_____.

Correct Answer:
17

Solution:

$$\frac{1}{2}\frac{d[N_2O_5]}{dt} = -\frac{1}{4}\frac{d[NO_2]}{dt}$$

$$\frac{d[NO_2]}{dt} = -2\frac{d[N_2O_5]}{dt}$$

$$\frac{d[NO_2]}{dt} = -2\frac{(3-2.75)}{30}$$

$$\frac{d[NO_2]}{dt} = \frac{2\times 0.25}{30}$$

$$= 1.667 \times 10^{-2}$$

$$= 16.67 \times 10^{-3}$$

$$x \approx 17 \times 10^{-3}$$

Hence, the answer is (17).

Q. 23 Two reactions are given below:

$$2Fe_{(s)} + \frac{3}{2}O_{2(g)} \rightarrow Fe_2O_{3(s)}, \Delta H^\circ = -822\ kJ/mol$$

$$C_{(s)} + \frac{1}{2}O_{2(g)} \rightarrow CO_{(g)}, \Delta H^\circ = -110\ kJ/mol$$

Then enthalpy change for following reaction
$3C_{(s)} + Fe_2O_{3(s)} \rightarrow 2Fe_{(s)} + 3CO_{(g)}$ is_____ kJ/mol

Correct Answer:
492

Solution:
By Hess law

$2Fe + \frac{3}{2}O_2 \rightarrow Fe_2O_3 \quad \Delta H^o = -822KJ \ldots\ldots\ldots (1)$

$C + \frac{1}{2}O_2 \rightarrow CO \quad \Delta H^\circ = -110KJ/mol \ldots\ldots (2)$

$Fe_2O_3 \rightarrow 2Fe + 3/2O_2 \quad \Delta H^\circ = 822KJ \ldots\ldots (3)$

eq. (2) multiply by 3

$3C + \frac{3}{2}O_2 \rightarrow 3CO \quad \Delta H^\circ = -330KJ \ldots\ldots (4)$

eq. (3)+(4)

$3C + Fe_2O_3 \rightarrow 2Fe + 3CO \quad \Delta H = 492KJ/mol$

Hence, the answer is (492).

Q. 24 The total number of correct statements, regarding the nucleic acids is

A. RNA is regarded as the reserve of genetic information

B. DNA molecule self-duplicates during cell division

C. DNA synthesizes proteins in the cell

D. The message for the synthesis of particular proteins is present in DNA

E. Identical DNA strands are transferred to daughter cells.

Correct Answer:

3

Solution:

A	B	C	D	E
(False)	(True)	(False)	(True)	(True)

Hence, the answer is (3).

Q. 25 The pH of an aqueous solution containing $1M$ benzoic acid $(pK_a = 4.20)$ and $1M$ sodium benzoate is 4.5 The volume of benzoic acid solution in $300\ mL$ of this buffer solution is_____ mL.

(given : $\log 2 = 0.3$)

Correct Answer:
100

Solution:

$$\mathrm{pH} = \mathrm{pK_a} + \log \frac{(\mathrm{V_{salt}})}{(\mathrm{V_{acid}})}$$

$$4.5 = 4.2 + \log \frac{(\mathrm{V_{salt}})}{(\mathrm{V_{acid}})}$$

$$\log \frac{(\mathrm{V_{salt}})}{(\mathrm{V_{acid}})} = 0.3$$

$$\frac{(\mathrm{V_{salt}})}{(\mathrm{V_{acid}})} = 2 \qquad \ldots\ldots (Eq.1)$$

$$\mathrm{V_{salt}} + \mathrm{V_{acid}} = 300\mathrm{ml} \qquad \ldots\ldots (Eq.2)$$

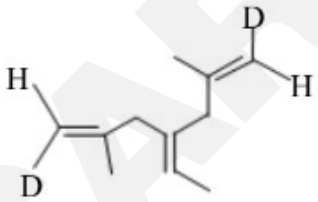
By Eq.(1) and (2)

$$\mathrm{V_{salt}} = 200\mathrm{ml}$$

$$\mathrm{V_{acid}} = 100\mathrm{ml}$$

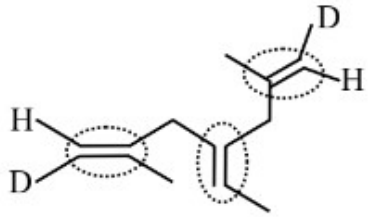
Hence, the answer is (100).

Q. 26 Number. of geometrical isomers possible for the given structure is/are _____.



Correct Answer:
-

Solution:



(E, E) (E, Z) (Z, Z) (Z, E)

Q. 27 Total number of species from the following which can undergo disproportionation reaction is _____.

$H_2O_2, ClO_3^-, P_4, Cl_2, Ag, Cu^{+1}, F_2, NO_2, K^+$

Correct Answer:
6

Solution:
The intermediate oxidation state of an element can undergo a disproportionation reaction

$H_2O_2, ClO_3^-, P_4, Cl_2, Cu^+, NO_2$

Hence, the answer is (6).

Q. 28 Number of metal ions characterized by flame test among the following is _____ .

$H_2O_2, ClO_3^-, P_4, Cl_2, Cu^+, NO_2$

Correct Answer:
4

Solution:
Metal ions $\left(Ca^{2+}, Sr^{2+}, Ba^{2+}Cu^{2+}\right)$

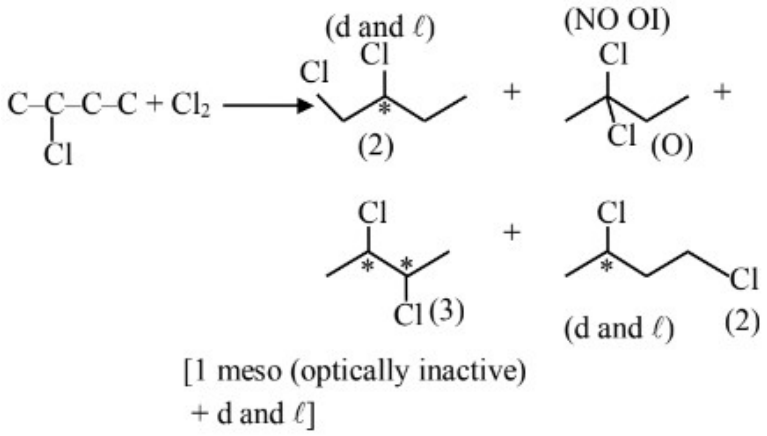
responds to flame test.

Hence, the answer is (4).

Q. 29 2-chlorobutane $+Cl_2 \rightarrow C_4H_8Cl_2$ (isomers)
Total number of optically active isomers shown by $C_4H_8Cl_2$, obtained in the above reaction is_____.

Correct Answer:
3

Solution:



Q. 30 Number of spectral lines obtained in He^{+}spectra, when an electron makes transition from fifth excited state to first excited state will be

Correct Answer:
10

Solution:
Fifth excited state $n_2 = 6$

first excited state $n_1 = 2$

Number of spectra line $\frac{[(n_2-n_1)][(n_2-n_1)+1]}{2}$

$$= \frac{(6-2)(6-2+1)}{2}$$

$$= \frac{4 \times 5}{2} = \frac{20}{2} = 10$$

Hence, the answer is (10).

Maths

Q. 1 Consider the system of linear equations $x+y+z=5, x+2y+\lambda^2 z=9, x+3y+\lambda z=\mu$ where $\lambda, \mu \in \mathbb{R}$. Then, which of the following statement is NOT correct?

Option 1:
System has infinite number of solutions if $\lambda=1$ and $\mu=13$

Option 2:
System is inconsistent if $\lambda=1$ and $\mu \neq 13$

Option 3:
System is consistent if $\lambda \neq 1$ and $\mu=13$

Option 4:
System has unique solution if $\lambda \neq 1$ and $\mu \neq 13$

Correct Answer:
System has unique solution if $\lambda \neq 1$ and $\mu \neq 13$

Solution:

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & \lambda^2 \\ 1 & 3 & \lambda \end{vmatrix} = 0$$

$\Rightarrow 2\lambda^2-\lambda-1=0$

$\lambda=1,-\frac{1}{2}$

$$\begin{vmatrix} 1 & 1 & 5 \\ 2 & \lambda^2 & 9 \\ 3 & \lambda & \mu \end{vmatrix} = 0 \Rightarrow \mu=13$$

Infinite solution $\lambda=1 \& \mu=13$
For unique sol^{n} $\lambda \neq 1$
For no sol^{n} $\lambda=1 \& \mu \neq 13$
If $\lambda \neq 1$ and $\mu \neq 13$

Considering the case when $\lambda=-\frac{1}{2}$ and $\mu \neq 13$ this will generate no solution case.

Hence, the answer is the option (4).

Q. 2 For $\alpha, \beta \in (0, \pi/2)$, let $3\sin(\alpha+\beta) = 2\sin(\alpha-\beta)$ and a real number k be such that $\tan\alpha = \mathrm{k}\tan\beta$. Then, the value of k is equal to

Option 1:
$-2/3$

Option 2:
–5

Option 3:
2/3

Option 4:
5

Correct Answer:
–5

Solution:
$3\sin\alpha\cos\beta + 3\sin\beta\cos\alpha$
$= 2\sin\alpha\cos\beta - 2\sin\beta\cos\alpha$
$5\sin\beta\cos\alpha = -\sin\alpha\cos\beta$

$$\tan\beta = -\frac{1}{5}\tan\alpha$$
$$\tan\alpha = -5\tan\beta$$

Hence, the answer is the option (2).

Q. 3 Let $A(\alpha, 0)$ and $B(0, \beta)$ be the points on the line $5x + 7y = 50$. Let the point P divide the line segment AB internally in the ratio $7:3$. Let $3x - 25 = 0$ be a directrix of the ellipse $E: \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the corresponding focus be S. If from S, the perpendicular on the x-axis passes through P, then the length of the latus rectum of E is equal to,

Option 1:
$\frac{25}{3}$

Option 2:
$\frac{32}{9}$

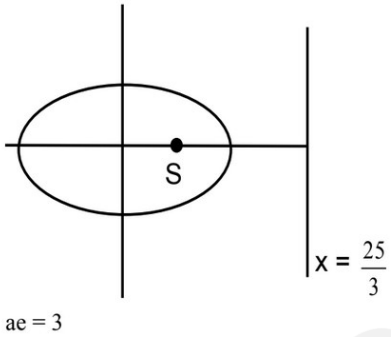
Option 3:
$\frac{25}{9}$

Option 4:
$\frac{32}{5}$

Correct Answer:
$\frac{32}{5}$

Solution:

$$\left.\begin{matrix}\mathrm{A}=(10,0)\\ \mathrm{B}=\left(0,\frac{50}{7}\right)\end{matrix}\right\}\mathrm{P}=(3,5)$$



$$\mathrm{ae}=3$$

$$\frac{\mathrm{a}}{\mathrm{e}}=\frac{25}{3}$$

$$a=5$$

$$b=4$$

Length of LR $=\frac{2b^2}{a}=\frac{32}{5}$

Q. 4 Let $\vec{a}=\hat{i}+\alpha\hat{j}+\beta\hat{k},\alpha,\beta\in\mathbb{R}$. Let a vector $\vec{b}$ be such that the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{4}$ and $|\vec{b}|^2=6$. If $\vec{a}\cdot\vec{b}=3\sqrt{2}$, then the value of $\left(\alpha^2+\beta^2\right)|\vec{a}\times\vec{b}|^2$ is equal to

Option 1:
90

Option 2:
75

Option 3:
95

Option 4:
85

Correct Answer:
90

Solution:

$|\vec{b}|^2 = 6; |\vec{a}||\vec{b}| \cos\theta = 3\sqrt{2}$

$|\vec{a}|^2|\vec{b}|^2 \cos^2\theta = 18$

$|\vec{a}|^2 = 6$

Also $1 + \alpha^2 + \beta^2 = 6$

$\alpha^2 + \beta^2 = 5$

to find

$\left(\alpha^2 + \beta^2\right)|\vec{a}|^2|\vec{b}|^2 \sin^2\theta$

$= (5)(6)(6)\left(\frac{1}{2}\right)$

$= 90$

Hence, the answer is the option (1).

Q. 5 Let $f(x) = (x+3)^2(x-2)^3, x \in [-4, 4]$. If M and m are the maximum and minimum values of f, respectively in $[-4, 4]$, then the value of $M - m$ is

Option 1:
600

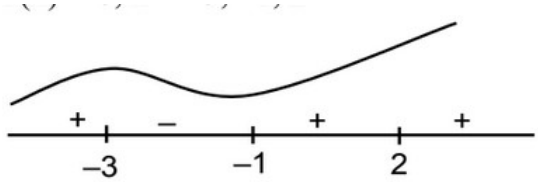
Option 2:
392

Option 3:
608

Option 4:
108

Correct Answer:
108

Solution:
$f'(x) = (x+3)^2 \cdot 3(x-2)^2 + (x-2)^3 2(x+3)$
$= 5(x+3)(x-2)^2(x+1)$
$f'(x) = 0, x = -3, -1, 2$



$f(-4) = -216$
$f(-3) = 0, f(4) = 49 \times 8 = 392$
$M = 392, m = -216$
$M - m = 392 + 216 = 608$

Q. 6 Let a and b be be two distinct positive real numbers. Let 11^{th} term of a GP, whose first term is a and third term is b, is equal to p^{th} term of another GP, whose first term is a and fifth term is b. Then p is equal to

Option 1:
20

Option 2:
25

Option 3:
21

Option 4:
24

Correct Answer:
21

Solution:
$1^{st}\ GP \Rightarrow t_1 = a, t_3 = b = ar^2 \Rightarrow r^2 = \frac{b}{a}$

$t_{11} = ar^{10} = a\left(r^2\right)^5 = a \cdot \left(\frac{b}{a}\right)^5$

$2^{nd}\ GP \Rightarrow T_1 = a, T_5 = b = ar^4$

$\Rightarrow r^4 = \left(\frac{b}{a}\right) \Rightarrow r = \left(\frac{b}{a}\right)^{1/4}$

$T_p = ar^{p-1} = a\left(\frac{b}{a}\right)^{\frac{p-1}{4}}$

$t_{11} = T_p \Rightarrow a\left(\frac{b}{a}\right)^5 = a\left(\frac{b}{a}\right)^{\frac{p-1}{4}}$

$\Rightarrow 5 = \frac{p-1}{4} \Rightarrow p = 21$

Hence, the answer is the option (3).

Q. 7 If $x^2 - y^2 + 2hxy + 2gx + 2fy + c = 0$ is the locus of a point, which moves such that it is always equidistant from the lines $x + 2y + 7 = 0$ and $2x - y + 8 = 0$, then the value of $g + c + h - f$ equals

Option 1:
14

Option 2:
6

Option 3:
8

Option 4:
29

Correct Answer:
14

Solution:
Locus of point P(x, y) whose distance from Gives.

$X + 2y + 7 = 0 \& 2x - y + 8 = 0$ are equal is $\frac{x+2y+7}{\sqrt{5}} = \pm\frac{2x-y+8}{\sqrt{5}}$

$(x + 2y + 7)^2 - (2x - y + 8)^2 = 0$

Combined equation of lines

$(x - 3y + 1)(3x + y + 15) = 0$
$3x^2 - 3y^2 - 8xy + 18x - 44y + 15 = 0$

$x^2 - y^2 - \frac{8}{3}xy + 6x - \frac{44}{3}y + 5 = 0$

$\mathrm{h}=\frac{4}{3},\ \mathrm{g}=3, \mathrm{f}=-\frac{22}{3}, \mathrm{c}=5$

$g+c+h-f=3+5-\frac{4}{3}+\frac{22}{3}=8+6=14$

Hence, the answer is the option (1).

Q. 8 Let $\vec{a}$ and $\vec{b}$ be two vectors such that $|\vec{b}|=1$ and $|\vec{b}\times\vec{a}|=2$. Then $|(\vec{b}\times\vec{a})-\vec{b}|^2$ is equal to

Option 1:
3

Option 2:
5

Option 3:
1

Option 4:
4

Correct Answer:
5

Solution:
$|\vec{b}|=1 \& |\vec{b}\times\vec{a}|=2$
$(\vec{b}\times\vec{a})\cdot\vec{b}=\vec{b}\cdot(\vec{b}\times\vec{a})=0$
$|(\vec{b}\times\vec{a})-\vec{b}|^2=|\vec{b}\times\vec{a}|^2+|\vec{b}|^2$
$=4+1=5$

Hence, the answer is the option (2).

Q. 9 Let $y=f(x)$ be a thrice differentiable function in $(-5,5)$. Let the tangents to the curve $y=f(x)$ at $(1,f(1))$ and $(3,\mathrm{f}(3))$ make angles $\pi/6$ and $\pi/4$, respectively with positive x-axis. If

Option 1:
–14

Option 2:
26

Option 3:
–16

Option 4:
36

Correct Answer:
26

Solution:

$y = f(x) \Rightarrow \frac{dy}{dx} = f'(x)$

$\left.\frac{\mathrm{dy}}{\mathrm{dx}}\right)_{(1,\mathrm{f}(1))} = \mathrm{f}'(1) = \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}} \Rightarrow \mathrm{f}'(1) = \frac{1}{\sqrt{3}}$

$\left.\frac{\mathrm{dy}}{\mathrm{dx}}\right)_{(3,\mathrm{f}(3))} = \mathrm{f}'(3) = \tan\frac{\pi}{4} = 1 \Rightarrow \mathrm{f}'(3) = 1$

$27\int_1^3 \left((f'(t))^2 + 1\right) f''(t)dt = \alpha + \beta\sqrt{3}$

$I = \int_1^3 \left((f'(t))^2 + 1\right) f''(t)dt$

$f'(t) = z \Rightarrow f''(t)dt = dz$
$z = f'(3) = 1$

$z = f'(1) = \frac{1}{\sqrt{3}}$

$I = \int_{1/\sqrt{3}}^{1} (z^2+1)dz = \left(\frac{z^3}{3} + z\right)_{1/\sqrt{3}}^{1}$
$\left(\frac{1}{3}+1\right) - \left(\frac{1}{3}\cdot\frac{1}{3\sqrt{3}} + \frac{1}{\sqrt{3}}\right)$

$= \frac{4}{3} - \frac{10}{9\sqrt{3}} = \frac{4}{3} - \frac{10}{27}\sqrt{3}$
$\alpha + \beta\sqrt{3} = 27\left(\frac{4}{3} - \frac{10}{27}\sqrt{3}\right) = 36 - 10\sqrt{3}$

$\alpha = 36, \beta = -10$
$\alpha + \beta = 36 - 10 = 26$

Hence, the answer is the option (2).

Q. 10 Let P be a point on the hyperbola $\mathrm{H}: \frac{\mathrm{x}^2}{9}-\frac{\mathrm{y}^2}{4}=1$,in the first quadrant such that the area of triangle formed by P and the two foci of H is $2\sqrt{13}$. Then, the square of the distance of P from the origin is

Option 1:
18

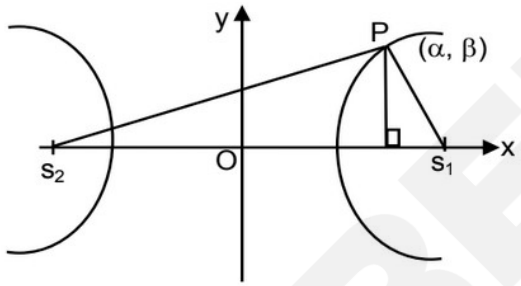
Option 2:
26

Option 3:
22

Option 4:
20

Correct Answer:
22

Solution:



$\frac{x^2}{9}-\frac{y^2}{4}=1$

$a^2=9, b^2=4$

$b^2=a^2\left(e^2-1\right) \Rightarrow e^2=1+\frac{b^2}{a^2}$

$\mathrm{e}^2=1+\frac{4}{9}=\frac{13}{9}$

$\mathrm{e}=\frac{\sqrt{13}}{3} \Rightarrow \mathrm{S}_1 \mathrm{S}_2=2 \mathrm{ae}=2 \times 3 \times \sqrt{\frac{13}{3}}=2\sqrt{13}$

Area of $\Delta \mathrm{PS}_1 \mathrm{S}_2=\frac{1}{2} \times \beta \times \mathrm{S}_1 \mathrm{S}_2=2\sqrt{13}$

$\frac{\alpha^2}{9}-\frac{\beta^2}{4}=1 \Rightarrow \frac{\alpha^2}{9}-1=1 \Rightarrow \alpha^2=18 \Rightarrow \alpha=3\sqrt{2}$

Distance of P from origin $=\sqrt{\alpha^2+\beta^2}$

$=\sqrt{18+4}=\sqrt{22}$

Then, the square of distance of P from the origin is 22 .

Q. 11 Bag A contains 3 white, 7 red balls and Bag B contains 3 white, 2 red balls. One bag is selected at random and a ball is drawn from it. The probability of drawing the ball from the bag A, if the ball drawn is white, is

Option 1:
$1/4$

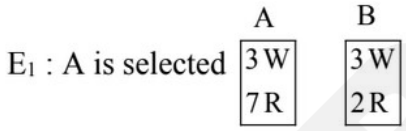
Option 2:
$1/9$

Option 3:
$1/3$

Option 4:
$3/10$

Correct Answer:
$1/4$

Solution:



$E_2 : B$ is selected
E : white ball is drawn

$$P\left(E_1/E\right) = \frac{P(E)\cdot P(E/E_1)}{P(E_1)\cdot P(E/E_1)+P(E_2)\cdot P(E/E_2)} = \frac{\frac{1}{2}\times\frac{3}{10}}{\frac{1}{2}\times\frac{3}{10}+\frac{1}{2}\times\frac{3}{5}}$$

$$= \frac{3}{3+6} = \frac{1}{3}$$

Q. 12 Let $\mathrm{f}:\mathbb{R}\to\mathbb{R}$ be defined as $\mathrm{f(x)} = \mathrm{ae^{2x}} + \mathrm{be^{x}} + \mathrm{cx}$. If $\mathrm{f}(0) = -1, \mathrm{f}'\left(\log_{\mathrm{e}} 2\right) = 21$ and $\int_0^{\log_{\mathrm{e}} 4}(\mathrm{f(x)} - \mathrm{cx})\mathrm{dx} = \frac{39}{2}$ then the value of $|a+b+c|$ equals

Option 1:
16

Option 2:
10

Option 3:
12

Option 4:
8

Correct Answer:
8

Solution:

$f(x) = ae^{2x} + be^{x} + cx \quad f(0) = -1$

$a + b = -1$

$f'(x) = 2ae^{2x} + be^{x} + c \quad f'(\ln 2) = 21$

$8a + 2b + c = 21$

$\int_0^{\ln 4} \left(ae^{2x} + be^{x}\right)dx = \frac{39}{2}$

$\left[\frac{ae^{2x}}{2} + be^{x}\right]_0^{\ln 4} = \frac{39}{2} \Rightarrow 8a + 4b - \frac{a}{2} - b = \frac{39}{2}$

$15a + 6b = 39$

$15a - 6a - 6 = 39$

$9a = 45 \Rightarrow a = 5$

$b = -6$

$c = 21 - 40 + 12 = -7$

$a + b + c - 8$

$|a + b + c| = 8$

Hence, the answer is the option (4).

Q. 13 Let $L_1 : \vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \lambda(\hat{i} - \hat{j} + 2\hat{k}), \lambda \in \mathbb{R}\$,$

$L_2 : \vec{r} = (\hat{j} - \hat{k}) + \mu(3\hat{i} + \hat{j} + pk), \mu \in \mathbb{R}$, and $L_3 : \vec{r} = \delta(\ell\hat{i} + m\hat{j} + nk), \delta \in \mathbb{R}$

be three lines such that L_1 is perpendicular to L_2 and L_3 is perpendicular to both L_1 and L_2. Then, the point which lies on L_3 is

Option 1:
(-1,7,4)

Option 2:
(-1,-7,4)

Option 3:
(1,7,-4)

Option 4:
(1,-7,4)

Correct Answer:
(-1,7,4)

Solution:
$L_1 \perp L_2 \quad L_3 \perp L_1,\ L_2$

$3 - 1 + 2P = 0$
$P = -1$

$$\begin{matrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 2 \\ 3 & 1 & -1 \end{matrix} = -\hat{i} + 7\hat{j} + 4\hat{k}$$

$\therefore (-\delta, 7\delta, 4\delta)$ will lie on L_3
For $\delta = 1$ the point will be $(-1, 7, 4)$

Hence, the answer is the option (1).

Q. 14 Let a and b be real constants such that the function f defined by

$$f(x) = \begin{cases} x^2 + 3x + a & , x \leq 1 \\ bx + 2 & , x > 1 \end{cases}$$ be differentiable be differentiable on $\mathbb{R}$.

Then, the value of $\int_{-2}^{2} f(x)dx$ equals

Option 1:
15/6

Option 2:
19/6

Option 3:
21

Option 4:
17

Correct Answer:
17

Solution:

f is continuous $f'(x)=\begin{Bmatrix}2x+3 & x<1\\ b & x>1\end{Bmatrix}$

$\therefore 4+a=b+2$

$a=b-2$ f is differentiable

$\therefore \mathrm{b}=5$
$\therefore \mathrm{a}=3$

$\int_{-2}^{1}\left(x^2+3x+3\right)dx+\int_{1}^{2}(5x+2)dx$

$\left[\frac{x^3}{3}+\frac{3x^2}{2}+3x\right]_{-2}^{1}+\left[\frac{5x^2}{2}+2x\right]_{1}^{2}$

$\left(\frac{1}{3}+\frac{3}{2}+3\right)-\left(\frac{-8}{3}+6-6\right)+\left(10+4-\frac{5}{2}-2\right)$

$=6+\frac{3}{2}+12-\frac{5}{2}=17$

Hence, the answer is the option (4).

Q. 15 Let $\mathrm{f}:\mathbb{R}-\{0\}\to\mathbb{R}$ be a function satisfying $f\left(\frac{x}{y}\right)=\frac{f(x)}{f(y)}$ for all $x, y, f(y)\neq 0$. If $f'(1)=2024$, then

Option 1:
$xf'(x)-2024f(x)=0$

Option 2:
$xf'(x)+2024f(x)=0$

Option 3:
$xf'(x)+f(x)=2024$

Option 4:
$xf'(x)-2023f(x)=0$

Correct Answer:
$xf'(x) - 2024f(x) = 0$

Solution:
$f\left(\frac{x}{y}\right) = \frac{f(x)}{f(y)} \quad f'(1) = 2024$

$$f(1) = 1$$

Partially differentiating w.r.t. x

$$f'\left(\frac{x}{y}\right) \cdot \frac{1}{y} = \frac{1}{f(y)} f'(x)$$
$$f'(1) \cdot \frac{1}{x} = \frac{f'(x)}{f(x)}$$

$y \to x$
$2024f(x) = xf'(x) \Rightarrow xf'(x) - 2024f(x) = 0$

Hence, the answer is the option (1).

Q. 16 If z is a complex number, then the number of common roots of the equations $z^{1985} + z^{100} + 1 = 0$ and $z^3 + 2z^2 + 2z + 1 = 0$, is equal to

Option 1:
1

Option 2:
2

Option 3:
0

Option 4:
3

Correct Answer:
2

Solution:
$z^{1985} + z^{100} + 1 = 0 \& z^3 + 2z^2 + 2z + 1 = 0$
$(z+1)\left(z^2 - z + 1\right) + 2z(z+1) = 0$
$(z+1)\left(z^2 + z + 1\right) = 0$

$\Rightarrow \quad z = -1, z = w, w^2$

Now putting $z = -1$ not satisfy

Now put $z = w$

$\Rightarrow \quad w^{1985} + w^{100} + 1$

$\Rightarrow \quad w^2 + w + 1 = 0$

$\text{Also, } z = w^2$

$\Rightarrow \quad w^{3970} + w^{200} + 1$

$\Rightarrow \quad w + w^2 + 1 = 0$

Thus, there are two common roots.

Hence, the answer is the option (2).

Q. 17 Suppose $2-p, p, 2-\alpha, \alpha$ are the coefficients of four consecutive terms in the expansion of $(1+x)^n$. Then the value of $p^2 - \alpha^2 + 6\alpha + 2p$ equals

Option 1:
4

Option 2:
10

Option 3:
8

Option 4:
6

Correct Answer:
4

Solution:
$2 - p, p, 2 - \alpha, \alpha$

Binomial coefficients are

${}^nC_r, {}^nC_{r+1}, {}^nC_{r+2}, {}^nC_{r+3}$ respectively

$\Rightarrow \quad {}^nC_r + {}^nC_r + 1 = 2$

$\Rightarrow \quad {}^{n+1}C_{r+1} = 2$ - - - - - - - - - - - - - - (1)

Also, ${}^nC_{r+2} + {}^nC_{r+3} = 2$

$\Rightarrow \quad {}^{n+1}C_{r+3} = 2$ ----------(2)

From (1) and (2)

${}^{n+1}C_{r+1} = {}^{n+1}C_{r+3}$

$\Rightarrow \quad 2r + 4 = n + 1$

$n = 2r + 3$

${}^{2r+4}C_{r+1} = 2$

Data Inconsistent

Q. 18 If the domain of the function $f(x) = \log_e\left(\frac{2x+3}{4x^2+x-3}\right) + \cos^{-1}\left(\frac{2x-1}{x+2}\right)$ is $(\alpha, \beta]$, then the value of $5\beta - 4\alpha$ is equal to

Option 1:
10

Option 2:
12

Option 3:
11

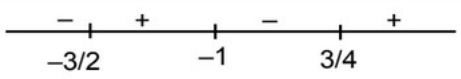
Option 4:
9

Correct Answer:
12

Solution:

$$\frac{2x+3}{4x^2+x-3} > 0 \text{ and } -1 \leq \frac{2x+3}{x+3} \leq 1$$

$$\frac{2x+3}{(4x-3)(x+1)} > 0 \frac{3x+1}{x+2} \geq 0 \& \frac{x-3}{x+2} \leq 0$$



$(-\infty,-2)\cup\left[\frac{-1}{3},\infty\right)---------(1)$

$(-2,3]---------(2)$

$\left[\frac{-1}{3},3\right]---------(3)$

$(1)\cap(2)\cap(3)$

$\left(\frac{3}{4},3\right]$

$\alpha=\frac{3}{4}\beta=3$

$5\beta-4\alpha=15-3=12$

Q. 19 Let $f:\mathbb{R}\to\mathbb{R}$ be a function defined by $f(x)=\frac{x}{(1+x^4)^{1/4}}$, and $g(x)=f(f(f(f(x))))$. Then, $18\int_0^{\sqrt{2\sqrt{5}}}x^2\,g(x)dx$ is equal to

Option 1:
33

Option 2:
36

Option 3:
42

Option 4:
39

Correct Answer:
39

Solution:
$f(x)=\frac{x}{(1+x^4)^{1/4}}$

$f\circ f(x)=\frac{f(x)}{(1+f(x)^4)^{1/4}}=\frac{\frac{x}{(1+x^4)^{1/4}}}{\left(1+\frac{x^4}{1+x^4}\right)^{1/4}}=\frac{x}{(1+2x^4)^{1/4}}$

$f(f(f(f(x)))) = \frac{x}{(1+4x^4)^{1/4}}$

$18\int_0^{\sqrt{2\sqrt{5}}} \frac{x^3}{(1+4x^4)^{1/4}}dx \quad \left[\because \text{ Let } 1+4x^4 = t^4 \right.$
$\left. 16x^3dx = 4t^3dt\right]$

$\frac{18}{4}\int_1^3 \frac{t^3dt}{t}$

$= \frac{9}{2}\left(\frac{t^3}{3}\right)_1^3$

$= \frac{3}{2}[26] = 39$

Hence, the answer is the option (4).

Q. 20

Let $R = \begin{pmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{pmatrix}$

be a non-zero 3×3 matrix, where $x\sin\theta = y\sin\left(\theta+\frac{2\pi}{3}\right) = z\sin\left(\theta+\frac{4\pi}{3}\right) \neq 0, \theta \in (0, 2\pi)$

For a square matrix M, let trace (M) denote the sum of all the diagonal entries of M. Then, among the statements:

(I) Trace (R) = 0
(II) If three (adj(adj(R)) = 0, then R has exactly one non–zero entry.

Option 1:
Both (I) and (II) are true

Option 2:
Neither (I) nor (II) is true

Option 3:
Only (II) is true

Option 4:
Only (I) is true

Correct Answer:
Neither (I) nor (II) is true

Solution:
$x\sin\theta = y\sin\left(\theta+\frac{2\pi}{3}\right) = z\sin\left(\theta+\frac{4\pi}{3}\right) \neq 0$

$\Rightarrow \mathrm{x}, \mathrm{y}, \mathrm{z} \neq 0$

Also,

$\sin\theta + \sin\left(\theta + \frac{2\pi}{3}\right) + \sin\left(\theta + \frac{4\pi}{3}\right) = 0 \forall \theta \in \mathrm{R}$

$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0$

$\Rightarrow xy + yz + zx = 0$

(1)

$\mathrm{Trace(R)} = \mathrm{x} + \mathrm{y} + \mathrm{z}$

If $x + y + z = 0$ and $xy + yz + zx = 0$

$\Rightarrow x = y = z = 0$

Statement (I) is False

(2)

$\mathrm{Adj(Adj}(R)) = |R|R$

$\mathrm{Trace(Adj(Adj}(R)))$

$= xyz(x + y + z) \neq 0$

Statement (II) is also False

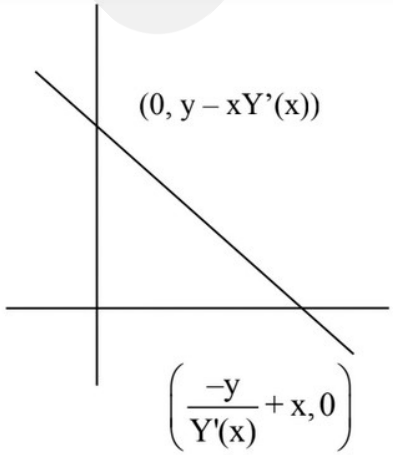
Hence, the answer is the option (2).

Q. 21 Let Y = Y(X) be a curve lying in the first quadrant such that the area enclosed by the line Y y Y (x)(X x) and the co-ordinate axes, where (x, y) is any point on the curve, is always $\frac{-y^2}{2Y'(x)} + 1, Y'(x) \neq 0$. If $Y(1) = 1$ then $12\mathrm{Y}(2)$ equals

Correct Answer:

20

Solution:



$A = \frac{1}{2}\left(\frac{-y}{Y'(x)} + x\right)(y - xY/x) = \frac{-y^2}{2Y'(x)} + 1$

$\Rightarrow \left(-y + xY'(x)\right)\left(y - xY'(x)\right) = -y^2 + 2Y'(x)$

$-y^2 + xyY'(x) + xyY'(x) - x^2\left[Y'(x)\right]^2 = -y^2 + 2Y'(x)$

$2xy - x^2Y'(x) = 2$

$\frac{dy}{dx} = \frac{2xy-2}{x^2}$

$\frac{dy}{dx} - \frac{2}{x}y = \frac{-2}{x^2}$

I.F. $= e^{-2\ln x} = \frac{1}{x^2}$

$y \cdot \frac{1}{x^2} = \frac{2}{3}x^{-3} + c$

Put $x = 1, y = 1$

$1 = \frac{2}{3} + c = c = \frac{1}{3}$

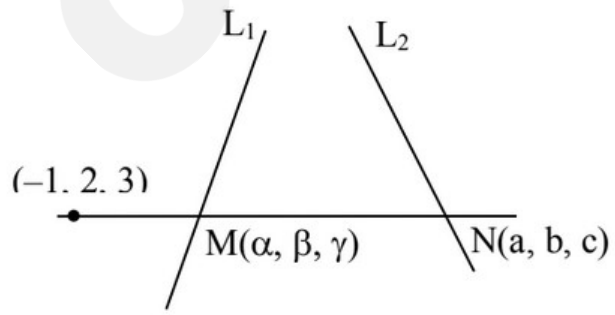
$Y = \frac{2}{3} \cdot \frac{1}{X} + \frac{1}{3}X^2$

$\Rightarrow 12Y(2) = \frac{5}{3} \times 12 = 20$

Q. 22 Let a line passing through the point $(-1, 2, 3)$ intersect the lines $L_1 : \frac{x-1}{3} = \frac{y-2}{2} = \frac{z+1}{-2}$ at $M(\alpha, \beta, \gamma)$ and $L_2 : \frac{x+2}{-3} = \frac{y-2}{-2} = \frac{z-1}{4}$ at N(a, b, c). Then, the value of $\frac{(\alpha+\beta+\gamma)^2}{(a+b+c)^2}$ equals

Correct Answer:
196

Solution:
$M(3\lambda + 1, 2\lambda + 2, -2\lambda - 1) \therefore \alpha + \beta + \gamma = 3\lambda + 2$



$N(-3\mu - 2, -2\mu + 2, 4\mu + 1) \therefore a + b + c = -\mu + 1$

$\frac{3\lambda+2}{-3\mu-1} = \frac{2\lambda}{-2\mu} = \frac{-2\lambda-4}{4\mu-2}$

$2\mu = \lambda$
$2\lambda\mu - \lambda = \lambda\mu + 2\mu$

$\lambda\mu = \lambda + 2\mu$
$\Rightarrow \mu = 2\lambda$
$\Rightarrow \mu = 2(\lambda \neq 0)$

$\therefore \lambda = 4$
$\alpha + \beta + \gamma = 14$
$a + b + c = -1$

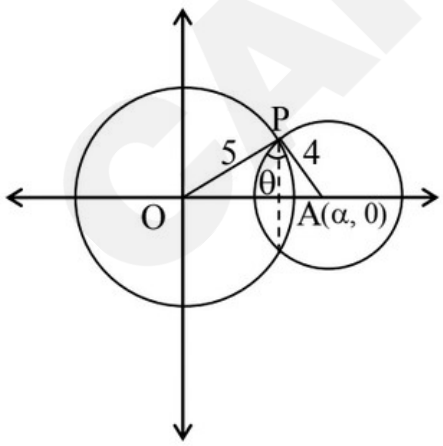
$\frac{(\alpha+\beta+\gamma)^2}{(a+b+c)^2} = 196$

Q. 23 Consider two circles $C_1 : x^2 + y^2 = 25$ and $C_2 : (x - \alpha)^2 + y^2 = 16$ where $\alpha \in (5, 9)$ Let the angle between the two radii (one to each circle) drawn from one of the intersection points of C_1 and C_2 be $\sin^{-1}\left(\frac{\sqrt{63}}{8}\right)$. If the length of common chord of C_1 and C_2 is β, then the value of $(\alpha\beta)^2$ equals _____

Correct Answer:
-

Solution:
$C_1 : x^2 + y^2 = 25, C_2 : (x - \alpha)^2 + y^2 = 16$

$5 < \alpha < 9$



$\theta = \sin^{-1}\left(\frac{\sqrt{63}}{8}\right)$

$\sin\theta = \frac{\sqrt{63}}{8}$

Area of $\Delta OAP = \frac{1}{2} \times \alpha\left(\frac{\beta}{2}\right) = \frac{1}{2} \times 5 \times 4\sin\theta$

$$\Rightarrow \alpha\beta = 40 \times \frac{\sqrt{63}}{8}$$

$$\alpha\beta = 5 \times \sqrt{63}$$

$$(\alpha\beta)^2 = 25 \times 63 = 1575$$

Q. 24 Let $\alpha = \sum_{k=0}^{n}\left(\frac{({}^{n}C_{k})^2}{k+1}\right)$ and $\beta = \sum_{k=0}^{n-1}\left(\frac{{}^{n}C_{k}\,{}^{n}C_{k+1}}{k+2}\right)$. If $5\alpha = 6\beta$, then n equals

Correct Answer:
10

Solution:

$\alpha = \sum_{k=0}^{n} \frac{{}^{n}C_{k}\cdot{}^{n}C_{k}}{k+1} \cdot \frac{n+1}{n+1}$

$= \frac{1}{n+1}\sum_{k=0}^{n} {}^{n+1}C_{k+1} \cdot {}^{n}C_{n-k}$

$\alpha = \frac{1}{n+1} \cdot {}^{2n+1}C_{n+1}$

$\beta = \sum_{k=0}^{n-1} {}^{n}C_{k} \cdot \frac{{}^{n}C_{k+1}}{k+2}\frac{n+1}{n+1}$

$\frac{1}{n+1}\sum_{k=0}^{n-1} C_{n-k} \cdot {}^{n+1}C_{k+2}$

$= \frac{1}{n+1} \cdot {}^{2n+1}C_{n+2}$

$\frac{\beta}{\alpha} = \frac{{}^{2n+1}C_{n+2}}{{}^{2n+1}C_{n+1}} = \frac{2n+1-(n+2)+1}{n+2}$

$$\frac{\beta}{\alpha} = \frac{n}{n+2} = \frac{5}{6}$$

$n = 10$

Hence, the answer is 10.

Q. 25 Let S_n be the sum to n-terms of an arithmetic progression $3, 7, 11, \ldots\ldots$ If $40 < \left(\frac{6}{n(n+1)} \sum_{k=1}^{n} S_k\right) < 42$, then n equals

Correct Answer:
9

Solution:
$S_n = 3 + 7 + 11 + \ldots\ldots n \text{ terms}$
$= \frac{n}{2}(6 + (n-1)4) = 3n + 2n^2 - 2n$
$= 2n^2 + n$

$\sum_{k=1}^{n} S_k = 2\sum_{k=1}^{n} K^2 + \sum_{k=1}^{n} K$

$= 2 \cdot \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}$
$= \frac{n(n+1)(4n+5)}{6}$

$\Rightarrow 40 < \frac{6}{n(n+1)} \sum_{k=1}^{n} S_k < 42$

$40 < 4n + 5 < 42$
$35 < 4n < 37$

n=9

Hence, the answer is 9.

Q. 26 In an examination of a Mathematics paper, there are 20 questions of equal marks and the question paper is divided into three sections: A, B and C. A student is required to attempt a total of 15 questions taking at least 4 questions from each section. If section A has 8 questions, section B has 6 questions and section C has 6 questions, then the total number of ways a student can select 15 questions is

Correct Answer:
11376

Solution:
If 4 questions from each section are selected. The remaining 3 questions can be selected either in (1,1,1 or (3,0,0) or (2,1,0)

$\therefore$ Total ways $=$

$^8c_5 \cdot {}^6c_5 \cdot {}^6c_5 + {}^8c_6 {}^6c_5 \cdot {}^6c_4 \times 2 + {}^8c_5 \cdot {}^6c_6 \cdot {}^6c_4 \times 2 + {}^8c_4 \cdot {}^6c_6 \cdot {}^6c_5 \times 2 + {}^8c_7 \cdot {}^6c_4 \cdot {}^6c_4$

$= 56 \cdot 6 \cdot 6 + 28 \cdot 6 \cdot 15 \cdot 2 + 56 \cdot 15 \cdot 2 + 70 \cdot 6 \cdot 2 + 8.15.15 = 11376$

Hence, the answer is 11376.

Q. 27 The number of symmetric relations defined on the set {1, 2, 3, 4} which are not reflexive is _____.

Solution:

Total number of relation both symmetric and reflexive $= 2^{\frac{n^2-n}{2}}$

Total number of symmetric relation $= 2^{\left(\frac{n^2+n}{2}\right)}$

⇒ Then number of symmetric relation which are not reflexive

$\Rightarrow 2^{\frac{n(n+1)}{2}} - 2^{\frac{n(n-1)}{2}} \quad (n = 5)$

$\Rightarrow 1024 - 64$
$= 960$

Q. 28 The number of real solutions of the equation $x\left(x^2 + 3|x| + 5|x-1| + 6|x-2|\right) = 0$ is

Correct Answer:

1

Solution:

$x = 0$ and $x^2 + 3|x| + 5|x-1| + 6|x-2| = 0$

Here all terms are +ve except at x = 0

So there is no value of x
Satisfies this equation

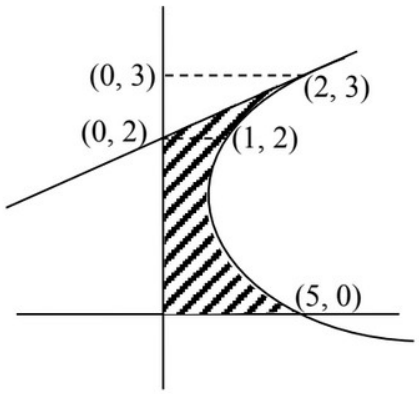
The only solution is x = 0.
Number of the solution is 1.

Hence, the answer is 1.

Q. 29 The area of the region enclosed by the parabola $(y-2)^2 = x-1$, the line $x-2y+4=0$ and the positive coordinate axes is _____

Correct Answer:
5

Solution:
Solving the equations $(y-2)^2 = x-1$ and $x-2y+4=0$



x = 2(y – 2)

$\frac{x^2}{4} = x-1$

$x^2 - 4x + 4 = 0$
$(x-2)^2 = 0$

x=2

Exclose area (w.r.t. y-axis) $= \int_0^3 x\,dy -$ Area of Δ.

$= \int_0^3 \left((y-2)^2+1\right)dy - \frac{1}{2} \times 1 \times 2$

$= \int_0^3 \left(y^2 - 4y + 5\right)dy - 1$

$= \left[\frac{y^3}{3} - 2y^2 + 5y\right]_0^3 - 1$

= 9 – 18 + 15 – 1 = 5

Q. 30 The variance σ^2 of the data

X_i	0	1	5	6	10	12	17
f_i	3	2	3	2	6	3	3

is_____

Correct Answer:
29

Solution:

X_i	f_i	f_ix_i	$f_ix_i^2$
0	3	0	0
1	2	2	2
5	3	15	75
6	2	12	72
10	6	16	600
12	3	36	432
17	3	51	867
	$\Sigma f_i = 22$		$\Sigma f_ix_i^2 = 2048$

$\therefore \Sigma f_ix_i = 176$

$$\text{So } \bar{x} = \frac{\sum f_ix_i}{\sum f_i} = \frac{176}{22} = 8$$

$$\text{for } \sigma^2 = \frac{1}{N}\sum f_ix_i^2 - (\bar{x})^2$$

$$= 93.090964 - 64$$
$$= 29.0909$$

Hence, the answer is the (29).

31 Jan - Morning Shift

Physics

Q. 1 The refractive index of a prism with apex angle A is $\cot \mathrm{A}/2$. The angle of minimum deviation is :

Option 1:
$\delta_m = 180^\circ - 4\ \mathrm{A}$

Option 2:
$\delta_m = 180^\circ - 3\ \mathrm{A}$

Option 3:
$\delta_m = 180^\circ - 2\ \mathrm{A}$

Option 4:
$\delta_m = 180^\circ - \mathrm{A}$

Correct Answer:
$\delta_m = 180^\circ - 2\ \mathrm{A}$

Solution:

$\mu = \cot \frac{A}{2} = \frac{\sin\left(\frac{A+\delta_{min}}{2}\right)}{\sin \frac{A}{2}}$

$\cos \frac{A}{2} = \sin\left(\frac{A+\delta_{min}}{2}\right)$

$\frac{A}{2} = 90^\circ - \left(\frac{A+\delta_{min}}{2}\right)$

$\delta_{min} = 180^\circ - 2A$

Hence, the answer is the option (3).

Q. 2 A coin is placed on a disc. The coefficient of friction between the coin and the disc is μ. if the distance of the coin from the center of the disc is r, the maximum angular velocity which can be given to the disc, so that the coin does not slip away, is :

Option 1:
$\frac{\mu}{\sqrt{\mathrm{rg}}}$

Option 2:
$\sqrt{\frac{\mu g}{r}}$

Option 3:
$\frac{\mu g}{r}$

Option 4:
$\sqrt{\frac{\mathrm{r}}{\mu \mathrm{g}}}$

Correct Answer:
$\sqrt{\frac{\mu g}{r}}$

Solution:
$F_{\mathrm{c}}=\mathrm{m} \omega^2 \mathrm{r}\ F_f=\mu \mathrm{N}=\mu \mathrm{mg}$

Condition for no slipping-

$F_f \geq F_{\mathrm{c}}$

$\mu \mathrm{mg} \geq \mathrm{m} \omega^2 \mathrm{r}$

$\omega_{max}=\sqrt{\frac{\mu \mathrm{g}}{\mathrm{r}}}$

Hence, the answer is the option (2).

Q. 3 The relation between time ' t ' and distance ' x ' is $t=\alpha x^2+\beta x$, where α and β are constants. The relation between acceleration (a) and velocity (v) is :

Option 1:
$\mathrm{a}=-2 \alpha v^3$

Option 2:
$a=-4 \alpha v^4$

Option 3:
$\mathrm{a}=-3 \alpha v^2$

Option 4:
$a=-5 \alpha v^5$

Correct Answer:
$\mathrm{a}=-2 \alpha v^3$

Solution:
Given-

$t = \alpha x^2 + \beta x$

Differentiating with respect to time

$1 = 2\alpha x v + \beta v$

$v = \frac{1}{2\alpha x + \beta}$

Again differentiating with respect to time

$a = \frac{vdv}{dx} = v\left(\frac{-2\alpha}{(2\alpha x+\beta)^2}\right)$

$a = -2\alpha v^3$

Hence, the answer is the option (1).

Q. 4 The parameter that remains the same for molecules of all gases at a given temperature is :

Option 1:
kinetic energy

Option 2:
mass

Option 3:
speed

Option 4:
momentum

Correct Answer:
kinetic energy

Q. 5 When a metal surface is illuminated by light of wavelength λ, the stopping potential is $8\ \mathrm{V}$. When the same surface is illuminated by light of wavelength 3λ, stopping potential is $2\ \mathrm{V}$. The threshold wavelength for this surface is :

Option 1:
9λ

Option 2:
3λ

Option 3:
4.5λ

Option 4:
5λ

Correct Answer:
9λ

Solution:
We know stoppping potential

$$eV_s = \frac{hc}{\lambda} - \frac{hc}{\lambda_0}$$

$8\text{eV} = \frac{\text{hc}}{\lambda} - \frac{\text{hc}}{\lambda_0}$.....(1)

(2) $2\text{eV} = \frac{\text{hc}}{3\lambda} - \frac{\text{hc}}{\lambda_0}$.....(2)

Dividing (1) by (2)

$4 = \frac{\frac{1}{\lambda} - \frac{1}{\lambda_0}}{\frac{1}{3\lambda} - \frac{1}{\lambda_0}}$

$\frac{1}{3\lambda} = \frac{3}{\lambda_0}$

$\lambda_0 = 9\lambda$

Hence, the answer is the option (1).

Q. 6 A coil is places perpendicular to a magnetic field of 5000 T. When the field is changed to 3000 T in 2s, an induced emf of 22 V is produced in the coil. If the diameter of the coil is 0.02 m, then the number of turns in the coil is :

Option 1:
70

Option 2:
140

Option 3:
35

Option 4:
7

Correct Answer:
70

Solution:
We know The magnitude of induced emf -

$$\varepsilon = NA\cos\Theta\left(\frac{dB}{dt}\right)$$

A coil is placed perpendicular to a magnetic field so $\Theta = 0^\circ$

$$\varepsilon = NA\left(\frac{dB}{dt}\right)$$

$$22 = \frac{(5000-3000)\mathrm{N}\pi(0.01)^2}{2}$$

N = 70 turns

Hence, the answer is the option (1).

Q. 7 The fundamental frequency of a closed organ pipe is equal to the first overtone frequency of an open organ pipe. If length of the open pipe is 60 cm, the length of the closed pipe will be :

Option 1:
30 cm

Option 2:
45 cm

Option 3:
60 cm

Option 4:
15 cm

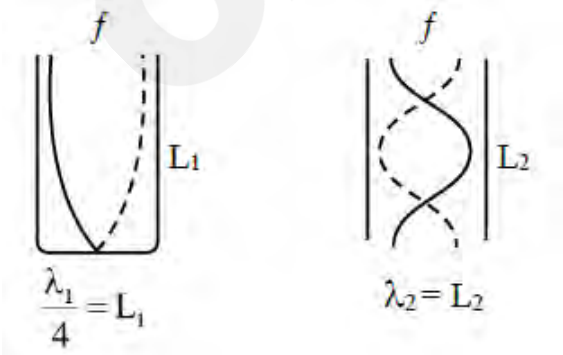
Correct Answer:
15 cm

Solution:

$\Rightarrow \quad \frac{\mathrm{C}}{\lambda_1} = \frac{\mathrm{C}}{\lambda_2}$

$\frac{1}{4\,\mathrm{L}_1} = \frac{1}{\mathrm{L}_2}$

$\mathrm{L}_1 = \frac{\mathrm{L}_2}{4} = 15\ \mathrm{cm}$



Q. 8 Two conductors have the same resistances at $0^{\circ}\mathrm{C}$ but their temperature coefficients of resistance are α_1 and α_2. The respective temperature coefficients for their series and parallel combinations are :

Option 1:
$\alpha_1+\alpha_2, \frac{\alpha_1+\alpha_2}{2}$

Option 2:
$\frac{\alpha_1+\alpha_2}{2}, \frac{\alpha_1+\alpha_2}{2}$

Option 3:
$\frac{\alpha_1+\alpha_2}{2}, \alpha_1+\alpha_2$

Option 4:
$\alpha_1+\alpha_2, \frac{\alpha_1\alpha_2}{\alpha_1+\alpha_2}$

Correct Answer:
$\frac{\alpha_1+\alpha_2}{2}, \frac{\alpha_1+\alpha_2}{2}$

Solution:
$\mathrm{R_1=R_2=R_0}$ at $0^{\circ}\mathrm{C}$

(1) For series:

$$\mathrm{R_1+R_2=R_S}$$

$$\frac{\mathrm{dR_1}}{\mathrm{dT}}+\frac{\mathrm{dR_2}}{\mathrm{dT}}=\frac{\mathrm{dR_S}}{\mathrm{dT}}$$

$$\mathrm{R_1\alpha_1+R_2\alpha_2=Rs_S\alpha_S}$$

$$\mathrm{R_0\left[\alpha_1+\alpha_2\right]=2R_0\alpha_S}$$

$$\alpha_S=\frac{\alpha_1+\alpha_2}{2}$$

(2) For parallel

$$\frac{1}{\mathrm{R_1}}+\frac{1}{\mathrm{R_2}}=\frac{1}{\mathrm{R_P}}$$

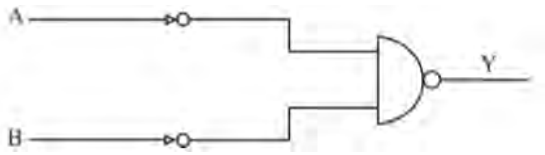
$$-\frac{1}{\mathrm{R_1^2}}\frac{\mathrm{dR_1}}{\mathrm{dT}}-\frac{1}{\mathrm{R_2^2}}\frac{\mathrm{dR_2}}{\mathrm{dT}}=-\frac{1}{\mathrm{R_P^2}}\frac{\mathrm{dR_P}}{\mathrm{dT}}$$

$$\frac{1}{\mathrm{R_0^2}}\alpha_1\mathrm{R_0}+\frac{1}{\mathrm{R_0^2}}\alpha_2\mathrm{R_0}=\frac{\alpha_P\frac{\mathrm{R_0}}{2}}{\left(\frac{\mathrm{R_0}}{2}\right)^2}$$

$$\frac{\alpha_1+\alpha_2}{2}=\alpha_P$$

Hence, the answer is the option (2).

Q. 9 Identify the logic operation performed by the given circuit.



Option 1:
AND

Option 2:
NOR

Option 3:
NAND

Option 4:
OR

Correct Answer:
OR

Solution:

A	B	Y
0	0	0
1	0	1
0	1	1
1	1	1

OR Gate

Q. 10 In a plane EM wave, the electric field oscillates sinusoidally at a frequency of $5 \times 10^{10}\ \mathrm{Hz}$ and an amplitude of $50 \mathrm{Vm}^{-1}$. The total average energy density of the electromagnetic field of the wave is : [Use $\varepsilon_0 = 8.85 \times 10^{-12} \mathrm{C}^2/\mathrm{Nm}^2$]

Option 1:
$1.106 \times 10^{-8} \mathrm{Jm}^{-3}$

Option 2:
$2.212 \times 10^{-10} \mathrm{Jm}^{-3}$

Option 3:
$4.425 \times 10^{-8} \mathrm{Jm}^{-3}$

Option 4:
$2.212 \times 10^{-8} \mathrm{Jm}^{-3}$

Correct Answer:
$1.106 \times 10^{-8} \mathrm{Jm}^{-3}$

Solution:
$\frac{\mathrm{dU}}{\mathrm{dV}} = \frac{1}{2}\varepsilon_0 \mathrm{E}_0^2$

$= \frac{8.85\times10^{-12}}{2}[50]^2$

$= 1.106 \times 10^{-8}\ \mathrm{J/m^3}$

Hence, the answer is the option (1).

Q. 11 If the wavelength of the first member of the Lyman series of hydrogen is λ. The wavelength of the second member will be

Option 1:
$\frac{5}{27}\lambda$

Option 2:
$\frac{27}{32}\lambda$

Option 3:
$\frac{32}{27}\lambda$

Option 4:
$\frac{27}{5}\lambda$

Correct Answer:
$\frac{27}{32}\lambda$

Solution:
$\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$

For Lyman series $\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n^2}\right), n = 2, 3, 4, \ldots$

$\frac{1}{\lambda} = \mathrm{R}\left[\frac{1}{1} - \frac{1}{4}\right]$

$\frac{1}{\lambda_2} = \mathrm{R}\left[\frac{1}{1} - \frac{1}{9}\right]$

$\frac{\lambda_2}{\lambda} = \frac{\frac{3}{4}}{\frac{8}{9}}$

$\lambda_2 = \frac{27}{32}\lambda$

Hence, the answer is the option (2).

Q. 12 Four identical particles of mass m are kept at the four corners of a square. If the gravitational force exerted on one of the masses by the other masses is $\left(\frac{2\sqrt{2}+1}{32}\right)\frac{\mathrm{Gm}^2}{\mathrm{L}^2}$, the length of the sides of the square is

Option 1:
$2\,\mathrm{L}$

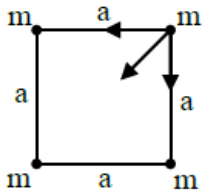
Option 2:
$4\,\mathrm{L}$

Option 3:
$3\,\mathrm{L}$

Option 4:
L

Correct Answer:
$4\,\mathrm{L}$

Solution:



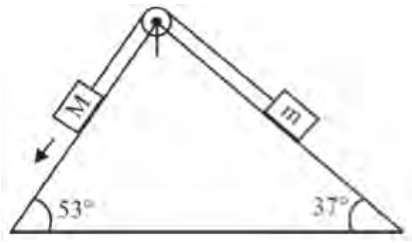
Net force $= \frac{\mathrm{Gm}^2}{\mathrm{a}^2}\left[\sqrt{2}+\frac{1}{2}\right] = \frac{\mathrm{Gm}^2}{\mathrm{a}^2}\left[\frac{2\sqrt{2}+1}{2}\right]$

Comparing them:

$2\mathrm{a}^2 = 32\,\mathrm{L}^2$

$\mathrm{a} = 4\,\mathrm{L}$

Q. 13 In the given arrangement of a doubly inclined plane two blocks of masses M and m are placed. The blocks are connected by a light string passing over an ideal pulley as shown. The coefficient of friction between the surface of the plane and the blocks is 0.25 . The value of m, for which $\mathrm{M}=10\ \mathrm{kg}$ will move down with an acceleration of $2\ \mathrm{m/s^2}$, is : $\left(\text{take } g=10\ \mathrm{m/s^2} \text{ and } \tan 37^\circ=3/4\right)$



Option 1:
6.5 kg

Option 2:
4.5 kg

Option 3:
2.25 kg

Option 4:
9 kg

Correct Answer:
4.5 kg

Solution:

$$\mathrm{a}=\frac{\mathrm{Mg}\sin 53^\circ-\mu\mathrm{Mg}\cos 53^\circ-\mathrm{mg}\sin 37^\circ-\mu\mathrm{mg}\cos 37^\circ}{10+\mathrm{m}}$$

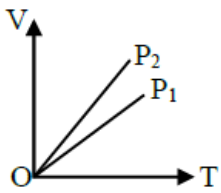
$$2=\frac{100\times\frac{4}{5}-\frac{1}{4}\times100\times\frac{3}{5}-\mathrm{mg}\left[\frac{3}{5}+\frac{1}{4}\times\frac{4}{5}\right]}{10+\mathrm{m}}$$

$$20+2\ \mathrm{m}=80-15-10\ \mathrm{m}\frac{4}{5}$$

$$10\ \mathrm{m}=45$$

$$\mathrm{M}=4.5\ \mathrm{kg}$$

Q. 14 The given figure represents two isobaric processes for the same mass of an ideal gas, then



Option 1:
$P_1 > P_2$

Option 2:
$P_2 > P_1$

Option 3:
$P_1 = P_2$

Option 4:
$P_2 \geq P_1$

Correct Answer:
$P_1 > P_2$

Solution:
$V = \frac{nR}{P}T$

$\text{Slope}_2 > \text{Slope}_1$

$\frac{nR}{P_2} > \frac{nR}{P_1}$

$P_1 > P_2$

Q. 15 If the percentage errors in measuring the length and the diameter of a wire are 0.1% each. The percentage error in measuring its resistance will be :

Option 1:
0.2%

Option 2:
0.144%

Option 3:
0.3%

Option 4:
0.1%

Correct Answer:
0.3%

Solution:

We know Resistance $R = \frac{\rho l}{A}$

where l =length , d= daimeter

$\mathrm{R} = 4\rho\frac{\ell}{\pi \mathrm{d}^2}$

$\frac{\Delta \mathrm{R}}{\mathrm{R}} = \frac{\Delta \ell}{\ell} + \frac{2\Delta \mathrm{d}}{\mathrm{d}}$

$\frac{\Delta \mathrm{R}}{\mathrm{R}}\% = [0.1 + 2(0.1)]\% = 0.3\%$

Hence, the answer is the option (3).

Q. 16 Two charges q and $3\mathrm{q}$ are separated by a distance ' r ' in air. At a distance x from charge q, the resultant electric field is zero. The value of x is :

Option 1:
$\frac{r}{(1+\sqrt{3})}$

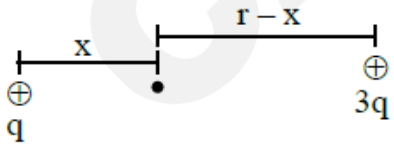
Option 2:
$\frac{(1+\sqrt{3})}{r}$

Option 3:
$\mathrm{r}(1+\sqrt{3})$

Option 4:
$\frac{r}{3(1+\sqrt{3})}$

Correct Answer:
$\frac{r}{(1+\sqrt{3})}$

Solution:



$E_q = E_{3q}$

$\frac{Kq}{x^2} = \frac{K3q}{(r-x)^2}$

$\frac{r}{x} - 1 = \sqrt{3}$

$x = \frac{r}{\sqrt{3}+1}$

Q. 17 A force is represented by $F = ax^2 + bt^{\frac{1}{2}}$ where $x =$ distance and $t =$ time. The dimensions of b^2/a are :

Option 1:
$\left[\mathrm{ML^2\ T^{-3}}\right]$

Option 2:
$\left[\mathrm{ML^3\ T^{-3}}\right]$

Option 3:
$\left[\mathrm{ML^{-1}\ T^{-1}}\right]$

Option 4:
$\left[\mathrm{MLT^{-2}}\right]$

Correct Answer:
$\left[\mathrm{ML^3\ T^{-3}}\right]$

Solution:

$[\mathrm{F}] = \left[\mathrm{ax^2}\right] + \left[\mathrm{b}\sqrt{\mathrm{t}}\right]$

(1) $\mathrm{MLT^{-2}} = \mathrm{aL^2}$

$\mathrm{a} = \mathrm{ML^{-1}\ T^{-2}}$

(2) $\mathrm{MLT^{-2}} = [\mathrm{b}]\mathrm{T}^{\frac{1}{2}}$

$[\mathrm{b}] = \mathrm{MLT}^{-\frac{5}{2}}$

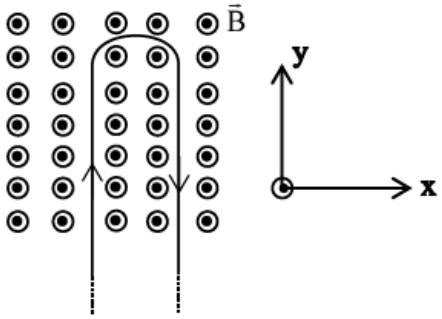
$\left[\ \mathrm{b^2}\right] = \mathrm{M^2\ L^2\ T^{-5}}$

(3) $\left[\frac{\mathrm{b^2}}{\mathrm{a}}\right] = \left[\frac{\mathrm{M^2\ L^2\ T^{-5}}}{\mathrm{ML^{-1}\ T^{-2}}}\right]$

$= \mathrm{ML^3\ T^{-3}}$

Hence, the answer is the option (2).

Q. 18 A rigid wire consists of a semicircular portion of radius R and two straight sections. The wire is partially immersed in a perpendicular magnetic field $B = B_0\hat{j}$ as shown in figure. The magnetic force on the wire if it has a current i is :



Option 1:
$-2\mathrm{iBR}\hat{\mathrm{j}}$

Option 2:
$2\mathrm{iBR}\hat{\mathrm{j}}$

Option 3:
$-\mathrm{iBR}\hat{\mathrm{j}}$

Option 4:
$\mathrm{i}BR\hat{j}$

Correct Answer:
$-2\mathrm{iBR}\hat{\mathrm{j}}$

Solution:
If $\vec{B} = B_0\hat{j}$ (according to question)

$\overrightarrow{\mathrm{F}} = \overrightarrow{\mathrm{L}}_{\mathrm{eff}} \times \overrightarrow{\mathrm{B}}$

$= \mathrm{i}(2\mathrm{R}\hat{\mathrm{i}}) \times \left(\mathrm{B}_0\hat{\mathrm{j}}\right)$

$= 2\mathrm{iRB}_0\hat{\mathrm{k}}$

(No answer matching)

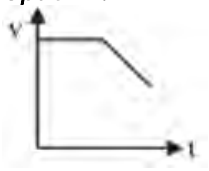
If $\overrightarrow{\mathrm{B}} = \mathrm{B}_0\hat{\mathrm{k}}$ (according to diagram)

$= -2\mathrm{iRB}\hat{\mathrm{j}}$

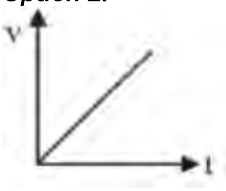
(Best fit)

Q. 19 A small steel ball is dropped into a long cylinder containing glycerine. Which one of the following is the correct representation of the velocity time graph for the transit of the ball?

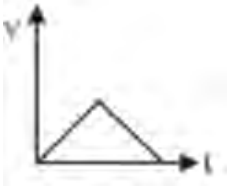
Option 1:



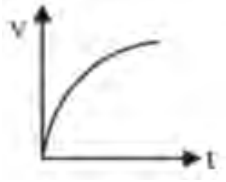
Option 2:



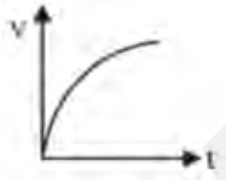
Option 3:



Option 4:



Correct Answer:



Solution:
Due to stokes law, ball will achieve terminal speed after a long time.

$v = \frac{Mg-B}{6\pi\eta R}\left[1 - e^{-kt}\right]$

Q. 20 An artillery piece of mass M_1 fires a shell of mass M_2 horizontally. Instantaneously after the firing, the ratio of kinetic energy of the artillery and that of the shell is :

Option 1:
$\mathrm{M}_1/\left(\mathrm{M}_1 + \mathrm{M}_2\right)$

Option 2:
$M_2/\left(M_1+M_2\right)$

Option 3:
$\frac{M_1}{M_2}$

Option 4:
$\frac{M_2}{M_1}$

Correct Answer:
$\frac{M_2}{M_1}$

Solution:



(1) COLM: $M_1v_1=M_2v_2$

(2) $\frac{KE_A}{KE_S}=\frac{\frac{M_1v_1^2}{2}}{\frac{M_2v_2^2}{2}}=\frac{\frac{(M_1v_1)^2}{2M_1}}{\frac{(M_2v_2)^2}{2M_2}}=\frac{M_2}{M_1}$

Q. 21 The depth below the surface of the sea to which a rubber ball be taken so as to decrease its volume by 0.02% is_____ m. $(\text{Take density of seawater }=10^3 kgm^{-3}$, Bulk modulus of rubber $=9\times10^8 Nm^{-2}$, and $g=10\ ms^{-2}$)

Correct Answer:
18

Solution:
We know Bulk modulus, B-

$B=\frac{-P}{\frac{dV}{V}}$

pressure $P=\rho gh$

So, $B=\frac{-\rho gh}{\frac{dV}{V}}$

$\left(9\times10^8\right)\left(\frac{-0.02}{100}\right)=-\rho gh$

$h=\frac{18\times10^{+4}}{10^3\times10}=18\ metre$

Hence, the answer is (18).

Q. 22 An electron moves through a uniform magnetic field $\vec{B}=B_0\hat{i}+2B_0\hat{j}T$. At a particular instant of time the velocity of electron is $\overrightarrow{u}=3\hat{i}+5\hat{j}m/s$. If the magnetic force acting on electron is $\overrightarrow{F}=5e\hat{k}N$, where e is the charge of electron, then the value of B_0 is _____T.

Correct Answer:
5

Solution:
$\vec{F} = q(\vec{v} \times \vec{B})$

$\overrightarrow{F} = (e)[3\hat{i} + 5\hat{j}] \times \left[B_0\hat{i} + 2\,B_0\hat{j}\right]$

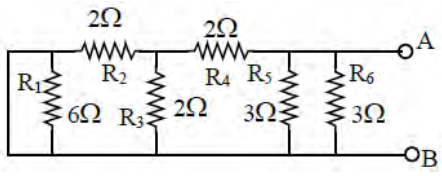
$5e\hat{k} = eB_0[6\hat{k} - 5\hat{k}]$

$5e = e\,B_0$

$B_0 = 5T$

Hence, the answer is (5).

Q. 23 Equivalent resistance of the following network is _____ Ω



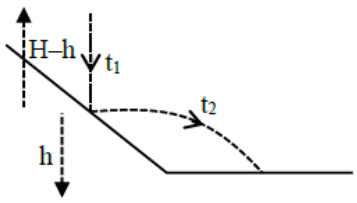
Correct Answer:
1

Solution:
Simple series & parallel combination

$R_{AB} = 1\Omega$

Q. 24 A body starts falling freely from height H hits an inclined plane in its path at height h. As a result of this perfectly elastic impact, the direction of the velocity of the body becomes horizontal. The value of $\frac{H}{h}$ for which the body will take the maximum time to reach the ground is_____.

Correct Answer:
2

Solution:


(1) $T = t_1 + t_2$

$= \sqrt{\frac{2(H-h)}{g}} + \sqrt{\frac{2h}{g}} = \sqrt{\frac{2}{g}}[\sqrt{H-h} + \sqrt{h}]$

(2) $\frac{dT}{dh} = 0$ for maximum T

$\frac{1}{2\sqrt{H-h}}(-1) + \frac{1}{2\sqrt{h}} = 0$

$\sqrt{h} = \sqrt{H-h}$

$\frac{H}{h} = 2$

Q. 25 A parallel plate capacitor with plate separation 5mm is charged up by a battery. It is found that on introducing a dielectric sheet of thickness 2mm, while keeping the battery connections intact, the capacitor draws 25% more charge from the battery than before. The dielectric constant of the sheet is _____.

Correct Answer:
2

Solution:
Without dielectric -

$$C = \frac{\epsilon_0 A}{d}$$

With dielectric of k

$$C' = \frac{\epsilon_0 A}{d - t + \frac{t}{k}}$$

Capacitance has increased by 25%

1.25 C=C'

$\frac{5}{4}\frac{A\varepsilon_0}{5} = \frac{A\varepsilon_0}{\left(\frac{3}{1}+\frac{2}{k}\right)}$

$3 + \frac{2}{k} = 4$

$k = 2$

Hence, the answer is (1).

Q. 26 Two waves of intensity ratio 1: 9 cross each other at a point. The resultant intensities at that point, when (a) waves are incoherent is $I_1(b)$ Waves are coherent is I_2 and differ in phase by 60°. If $\frac{I_1}{I_2} = \frac{10}{x}$ then x=_____.

Correct Answer:
13

Solution:
(a) For incoherent:

$I_1 = I_0 + 9I_0 = 10I_0$

(b) For coherent

$I_2 = I_0 + 9I_0 + 2\sqrt{9I_0I_0}\cos 60^\circ$

$= 13I_0\frac{I_1}{I_2}$

$= \frac{10}{13} \Rightarrow x = 13$

Hence, the answer is (13).

Q. 27 The mass defect in a particular reaction is 0.4 g. the amount of energy liberated is $n \times 10^7 kWh$, where n=_____.

(speed of light $= 3 \times 10^8$ m/s)

Correct Answer:
1

Solution:
$\Delta E = \Delta mc^2$

$\Delta E = \left(0.4 \times 10^{-3}\right)\left(9 \times 10^{16}\right)J$

$= 36 \times 10^{12}$ J

Now

$n \times 10^7 \times 10^3 \times \frac{J}{S} \times 3600$ s $= 36 \times 10^{12}$ J

$n = 1$
Hence, the answer is (1).

Q. 28 A solid circular disc of mass 50kg rolls along a horizontal floor so that its center of mass has a speed of 0.4 m/s.The absolute value of work done on the disc to stop it is _____ J.

Correct Answer:
6

Solution:
Work = change in KE

Using work energy theorem

W= $\Delta KE = 0 - (\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2)$

w=$\Delta KE = 0 - \frac{1}{2}mv^2(1 + \frac{K^2}{R^2})$

$W = 0 - \left(1 + \frac{1}{2}\right)\frac{1}{2}(50)(0.4)^2$

$W = -\frac{3}{4} \times 50 \times 0.4 \times 0.4$

$W = -6\ J$

$|W| = 6\ J$

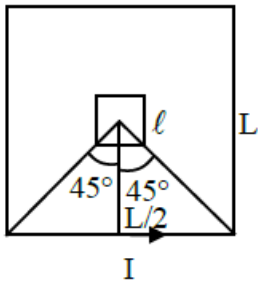
Absolute work = +6J

Hence, the answer is (6)

Q. 29 A small square loop of wire of side l is placed inside a large square loop of wire of side $L\ (L = l^2)$. The loops are coplanar and their centers coincide. The value of the mutual inductance of the system is $\sqrt{x} \times 10^{-7}H$, where x =_____.

Correct Answer:
128

Solution:



(1) MF due to larger square loop at centre

$B = 4\left[\frac{\mu_0 I}{4\pi(\frac{L}{2})}[\sin 45^\circ + \sin 45^\circ]\right]$

$B = 2\sqrt{2}\frac{\mu_0 I}{\pi L}$

(2) M flux in smaller coil $= B \cdot \ell^2$

$\phi = 2\sqrt{2}\frac{\mu_0 I}{\pi L}\ell^2$

(3) $M = \frac{\phi}{I} = 2\sqrt{2}\frac{\mu_0}{\pi}\frac{\ell^2}{L}$

$M = 2\sqrt{2} \times 4 \times 10^{-7}H = \sqrt{128} \times 10^{-7}\ h$

$x = 128$

Q. 30 A particle performs simple harmonic motion with amplitude A. Its speed is increased to three times at an instant when its displacement is $\frac{2A}{3}$. The new amplitude of motion is $\frac{nA}{3}$. The value of n is _____.

Correct Answer:
7

Solution:
Total Energy = kinetic energy + potential energy

$\frac{k}{2} A^2 = \frac{mv^2}{2} + \frac{k}{2}x^2$

(1) $\frac{k}{2}A^2 = \frac{mv^2}{2} + \frac{k}{2}\left(\frac{2A}{3}\right)^2$ [Just before]

(2) $\frac{k}{2}(A_f)^2 = \frac{mv^2}{2}(3v)^2 + \frac{k}{2}\left(\frac{2A}{3}\right)^2$ [Just after]

$\frac{k}{2}(A_f)^2 = 9\left(\frac{k}{2}A^2\left[1-\frac{4}{9}\right]\right) + \frac{k}{2}\left(\frac{4}{9}A^2\right)$

$A_f^2 = 9\left(A^2\frac{5}{9}\right) + \frac{4}{9}A^2$

$A_f^2 = \frac{49}{9}A^2$

$A_f = \frac{7}{3}A$

$n = 7$

Hence, the answer is (7).

Chemistry

Q. 1 Given below are two statements:

Statement I: Noble gases have very high boiling points.

Statement II: Noble gases are monoatomic gases. They are held together by strong dispersion forces. Because of this they are liquefied at very low temperature. Hence, they have very high boiling points.

In the light of the above statements, choose the correct answer from the options given below:

Option 1:
Statement I is false but statement II is true.

Option 2:
Both statement I and statement II are true.

Option 3:
Statement I is true but statement II is false.

Option 4:
Both statement I and statement II are false.

Correct Answer:
Both statement I and statement II are false.

Solution:
Fact $\rightarrow$ both statements are false.

Hence, the answer is the option (4).

Q. 2 The product (C) in the below mentioned reaction is:

$$CH_3-CH_2-CH_2-Br \xrightarrow[\Delta]{KOH(\text{ alc })} A \xrightarrow{HBr} B \xrightarrow[KOH(aq.)]{\Delta} C$$

Option 1:
Propene

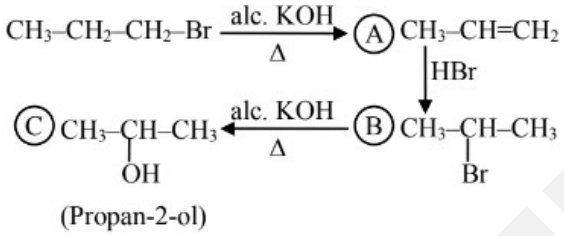
Option 2:
Propan-2-ol

Option 3:
Propyne

Option 4:
Propan-1-ol

Correct Answer:
Propan-2-ol

Solution:



Q. 3 Identify the mixture that shows positive deviations from Raoult's Law

Option 1:
$CHCl_3 + (CH_3)_2CO$

Option 2:
$CHCl_3 + C_6H_6$

Option 3:
$(CH_3)_2CO + C_6H_5NH_2$

Option 4:
$(CH_3)_2CO + CS_2$

Correct Answer:
$(CH_3)_2CO + CS_2$

Solution:
Acetone and CS_2 is an example of solutions showing positive deviation from Raoult's law Since acetone $-CS_2$ attractions are weaker than acetone-acetone attractions.

Hence, the answer is the option (4).

Q. 4 Given below are two statements:

Statement I: IUPAC name of $HO-CH_2-(CH_2)_3-CH_2-COCH_3$ is 7-hydroxyheptan-2-one.

Statement II: 2-oxoheptan-7-ol is the correct IUPAC name for above compound.

In the light of the above statements, choose the most appropriate answer from the options given below:

Option 1:
Statement I is correct but statement II is incorrect.

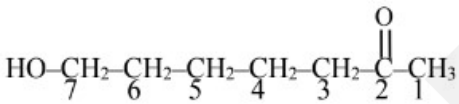
Option 2:
Statement I is incorrect but statement II is correct.

Option 3:
Both Statement I and statement II are correct.

Option 4:
Both statement I and statement II are incorrect.

Correct Answer:
Statement I is correct but statement II is incorrect.

Solution:
Statement-I :



Correct IUPAC name will be : 7- hydroxypentan-2-one.

Q. 5 A species having carbon with sextet of electrons and can act as electrophile is called -

Option 1:
pentavalent carbon

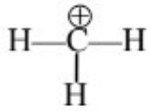
Option 2:
carbon free radical

Option 3:
carbocation

Option 4:
carbanion

Correct Answer:
carbocation

Solution:

 ; The species which have sextet of electron and act as an electrophile called as carbocation.

Q. 6 Match List I with List II

List I		List II	
A.	Glucose/ $/NaHCO_3/\Delta$	I.	Gluconic Acid
B.	Glucose $/HNO_3$	II.	No reaction
C.	Glucose/HI/ Δ	III.	n-hexane
D.	Glucose/Bromine water	IV.	Saccharic Acid

Choose the correct answer from the options given below:

Option 1:
A-IV,B-I, C-III, D-II

Option 2:
A-I, B-IV, C-III, D-II

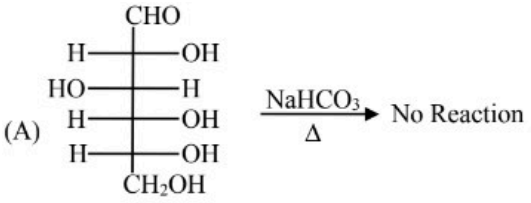
Option 3:
A-II, B-IV, C-III, D-I

Option 4:
A-III, B-II, C-I, D-IV

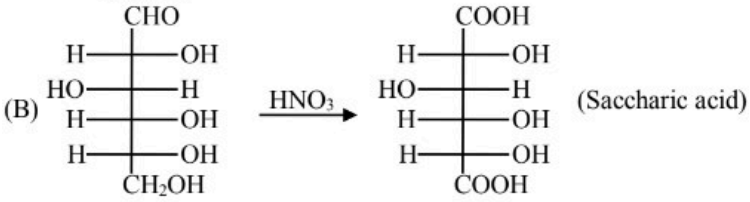
Correct Answer:
A-II, B-IV, C-III, D-I

Solution:

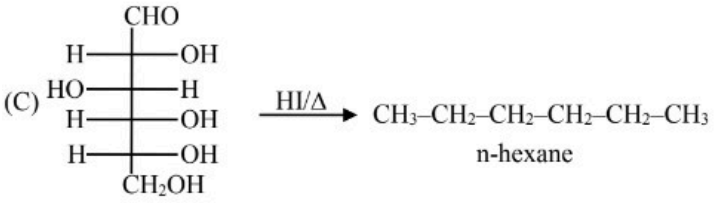
CAREERS360

(A) 

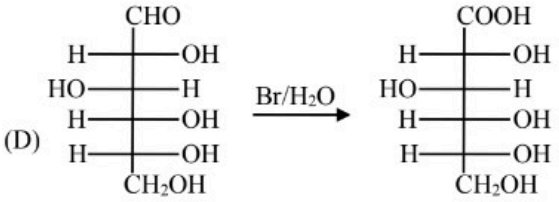
Glucose

(B) 

(Saccharic acid)

(C) 

n-hexane

(D) 

Gluconic aicd

Q. 7 Identify the factor from the following that does not affect electrolytic conductance of a solution.

Option 1:
The nature of solvent used

Option 2:
Concentration of the electrolyte.

Option 3:
The nature of the electrode used

Option 4:
The nature of the electrolyte added.

Correct Answer:
The nature of the electrode used

Solution:
The conductivity of the electrolytic cell is affected by the concentration of electrolyte, the nature of the electrolyte and the temperature.

Hence, the answer is the option (3).

Q. 8 The correct statements from following are:

A. The strength of anionic ligands can be explained by crystal field theory.

B. Valence bond theory does not give a quantitative interpretation of kinetic stability of coordination compounds.

C. The hybridization involved in formation of $[Ni(CN)_4]^{2-}$ complex is dsp^2.

D. The number of possible isomer(s) of cis- $[PtCl_2(en)_2]^{2+}$ is one

Choose the correct answer from the options given below:

Option 1:
B, C only

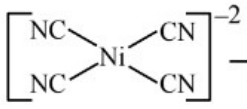
Option 2:
A, C only

Option 3:
A, D only

Option 4:
B, D only

Correct Answer:
B, C only

Solution:
Option B $\rightarrow$ Fact

(C)  → C.N. = 4
SFL thus
Hybridisation is dsp^2

Q. 9 The correct sequence of electron gain enthalpy of the elements listed below is -

A. Ar B. Br C. F D. S

Choose the most appropriate from the options given below:

Option 1:
A > D > B > C

Option 2:
D > C > B > A

Option 3:
A > D > C > B

Option 4:
C > B > D > A

Correct Answer:
A > D > B > C

Solution:
Order of electron affinity $\mathrm{F} > \mathrm{Br} > \mathrm{S} > \mathrm{Ar}$, order of e^{-}gain enthalpy is reverse of e^{-}affinity thus
$\mathrm{F} < \mathrm{Br} < \mathrm{S} < \mathrm{Ar}$

Hence, the answer is the option (1).

Q. 10 Given below are two statements: One is labelled as Assertion A and the other is labelled as Reason R:

Assertion A: pKa value of phenol is 10.0 while that of ethanol is 15.9

Reason R : Ethanol is stronger acid than phenol.

In the light of the above statements, choose the correct answer from the options given below:

Option 1:
A is true but R is false.

Option 2:
Both A and R are true but R is NOT the correct explanation of A.

Option 3:
Both A and R are true and R is the correct explanation of A.

Option 4:
A is false bur R is true.

Correct Answer:
A is true but R is false.

Solution:
More the acidic character, more will be K_a value or p^{Ka} value will be less. So Assertion is true, but reason is wrong.

Hence, the answer is the option (1).

Q. 11 Consider the oxides of group 14 elements
$\mathrm{SiO_2}, \mathrm{GeO_2}, \mathrm{SnO_2}, \mathrm{PbO_2}, \mathrm{CO}$ and GeO. The amphoteric oxides are

Option 1:
$\mathrm{GeO}, \mathrm{GeO_2}$

Option 2:
$\mathrm{SiO_2}, \mathrm{GeO_2}$

Option 3:
SnO_2, CO

Option 4:
SnO_2, PbO_2

Correct Answer:
SnO_2, PbO_2

Solution:
In group, 14^{th} the oxides of $Tin(Sn)$ and Pb are amphoteric oxide in nature. $Thus\ SnO_2\ and\ PbO_2$ amphoteric oxides.

Hence, the answer is the option (4).

Q. 12 Given below are two statements: One is labelled as Assertion A and the other is labelled as Reason R:

Assertion A: Alcohols react both as nucleophiles and electrophiles.

Reason R: Alcohols react with active metals such as sodium, potassium and aluminum to yield corresponding alkoxides and liberate hydrogen.

In the light of the above statements, choose the correct answer from the options given below:

Option 1:
Both A and R are true bur R is NOT the correct explanation of A.

Option 2:
A is true but R is false

Option 3:
Both A and R are true and R is the correct explanation of A.

Option 4:
A is false but R is true.

Correct Answer:
Both A and R are true bur R is NOT the correct explanation of A.

Solution:
Assertion is true, alcohol reacts both as nucleophiles & electrophiles.

$$R-OH+Metal(M)\longrightarrow \underset{\text{Alkoxide}}{R}-O^-M^+ + \frac{1}{2}H_2$$

Both given reason is wrong.

Hence, the answer is the option (1).

Q. 13 The compound that is white in color is

Option 1:
ammonium sulphide

Option 2:
lead sulphate

Option 3:
ammonium arsinomolybdate

Option 4:
lead iodide

Correct Answer:
lead sulphate

Solution:
The colour of lead sulphate is white
Ammonium sulphide $\rightarrow$ yellow
Ammonium arsinomolybdate $\rightarrow$ yellow
Lead iodide $\rightarrow$ yellow

Hence, the answer is the option (2).

Q. 14 Integrated rate law equation for a first order gas phase reaction is given by (where P_i is initial pressure and P_t is total pressure at time t)

Option 1:
$\mathrm{k}=\frac{2.303}{\mathrm{t}} \times \log \frac{\left(2 \mathrm{P}_{\mathrm{i}}-\mathrm{P}_{\mathrm{t}}\right)}{\mathrm{P}_{\mathrm{i}}}$

Option 2:
$\mathrm{k}=\frac{2.303}{\mathrm{t}} \times \log \frac{\mathrm{P}_{\mathrm{i}}}{\left(2 \mathrm{P}_{\mathrm{i}}-\mathrm{P}_{\mathrm{t}}\right)}$

Option 3:
$\mathrm{k}=\frac{2.303}{\mathrm{t}} \times \log \frac{2 \mathrm{P}_{\mathrm{i}}}{\left(2 \mathrm{P}_{\mathrm{i}}-\mathrm{P}_{\mathrm{t}}\right)}$

Option 4:
$\mathrm{k}=\frac{2.303}{\mathrm{t}} \times \log \frac{\mathrm{P}_{\mathrm{i}}}{\left(2 \mathrm{P}_{\mathrm{i}}-\mathrm{P}_{\mathrm{t}}\right)}$

Correct Answer:
$\mathrm{k}=\frac{2.303}{\mathrm{t}} \times \log \frac{\mathrm{P}_{\mathrm{i}}}{\left(2 \mathrm{P}_{\mathrm{i}}-\mathrm{P}_{\mathrm{t}}\right)}$

Solution:

	$\mathrm{A(g)} \rightarrow$	$\mathrm{B(g)}+$	$\mathrm{C(g)}$
$\mathrm{t}=0$	$\mathrm{P_i}$	0	0
$\mathrm{t}=\mathrm{t}$	$(\mathrm{p_i}-\mathrm{x})$	x	x

where P_i is the initial pressure

$$\mathrm{P_t = P_i - X + x + x}$$
$$\mathrm{P_t = P_i + x}$$
$$\mathrm{x = P_t - P_i}$$

$$K = \frac{2.303}{t}\log\frac{P_i}{P_i - x}$$

$$K = \frac{2.303}{t}\log\frac{P_i}{P_i - (P_t - P_i)}$$

$$K = \frac{2.303}{t}\log\frac{P_i}{2P_i - P_t}$$

Hence, the answer is the option (4).

Q. 15 The linear combination of atomic orbitals to form molecular orbitals takes place only when the combining atomic orbitals.

A. have the same energ

B. have the minimum overlap

C. have same symmetry about the molecular axis

D. have different symmetry about the molecular axis

Choose the most appropriate from the options given below:

Option 1:
A, B, C only

Option 2:
A and C only

Option 3:
B and D only

Option 4:
B, C, D only

Correct Answer:
A and C only

Solution:
In the LCAO method.

The energy of atomic orbital is nearly the same and also has the same symmetry about the molecular axis.

Hence, the answer is the option (2).

Q. 16 Adsorption' principle is used for which of the following purification method ?

Option 1:
Chromatography

Option 2:
Sublimation

Option 3:
Extraction

Option 4:
Distillation

Correct Answer:
Chromatography

Solution:
Chromatography is a versatile and widely used technique for separating and analyzing mixtures of substances. It works based on the principle of differential adsorption of compounds between a stationary phase and a mobile phase.

Hence, the answer is the option (1).

Q. 17 For the given reaction, choose the correct expression of KC from the following:-

$$Fe^{3+}_{(aq)} + SCN^{-}_{(aq)} \rightleftharpoons [FeSCN]^{2+}_{(aq)}$$

Option 1:
$K_C = \frac{[FeSCN^{2+}]^2}{[Fe^{3+}][SCN^-]}$

Option 2:
$K_C = \frac{[FeSCN^{2+}]}{[Fe^{3+}]^2[SCN^-]^2}$

Option 3:
$K_C = \frac{[Fe^{3+}][SCN^-]}{[FeSCN^{2+}]}$

Option 4:
$K_C = \frac{[FeSCN^{2+}]}{[Fe^{3+}][SCN^-]}$

Correct Answer:
$K_C = \frac{[FeSCN^{2+}]}{[Fe^{3+}][SCN^-]}$

Solution:

$$Fe^{3+}(aq) + SCN^{-}(aq) \rightleftharpoons [FeSCN]^{2+}(aq)$$

$$K_C = \frac{[FeSCN^{2+}]}{[Fe^{3+}]\,[SCN^{-}]}$$

Hence, the answer is the option (4).

Q. 18 Identify correct statements from below:
(A) The chromate ion is square planar.
(B) Dichromates are generally prepared from chromates.
(C) The green mangante ion is diamagnetic.
(D) Dark green colored K_2MnO_4 disproportionates in a neutral or acidic medium to give permanganate.
(E) With increasing oxidation number of transition metal, ionic character of the oxides decreases.
Choose the correct answer from the options given below:

Option 1:
B, C, D only

Option 2:
A, D, E only

Option 3:
A, B, C only

Option 4:
B, D, E only

Correct Answer:
B, D, E only

Solution:
B, D, E are correct.
Dichromates are generally prepared from chromates. The process involves the acidification of a chromate solution.

The dark green coloured potassium manganate (K_2MnO_4) disproportionates in a neutral or acidic medium to give permanganate ($KMnO_4$) and manganese dioxide (MnO_2). This process is known as disproportionation because the manganate ion (MnO_4^{2-}) both oxidizes and reduces itself.

As the oxidation number of a transition metal increases, the nature of its oxides shifts from being more ionic to more covalent. This is due to the increased electronegativity and polarizing power of the metal ion in higher oxidation states, which enhances covalent bonding interactions with oxygen.

Hence, the answer is the option (4).

Q. 19 Match List I with List II

List I (Technique)		List II (Application)	
A.	Distillation	I.	Separation of glycerol from spent-lye
B.	Fractional distillation	II.	Aniline – Water mixture
C.	Steam distillation	III.	Separation of crude oil fraction
D.	Distillation under reduced pressure	IV.	Chloroform – Aniline

Choose the correct answer from the options given below:

Option 1:
A-IV,B-I, C-II, D-III

Option 2:
A-IV, B-III, C-II, D-I

Option 3:
A-I, B-II, C-IV, D-III

Option 4:
A-II, B-III, C-I, D-IV

Correct Answer:
A-IV, B-III, C-II, D-I

Solution:
It is a fact base.

- Chloroform-aniline can be separated by distillation.
- Aniline water can be separated by steam distillation.
- Glycerol from spent lye can be separated by distillation under reduced pressure.
- Crude oil fraction can be separated by fractional distillation.

Hence, the answer is the option (2).

Q. 20 The metals that are employed in the battery industries are

A. Fe

B. Mn

C. Ni

D. Cr

E. Cd

Choose the correct answer from the options given below:

Option 1:
A, B, C and D only

Option 2:
B, C and E only

Option 3:
B, D and E only

Option 4:
A, B, C, D and E

Correct Answer:
B, C and E only

Solution:
The metals commonly employed in the battery industry are essential for various types of batteries due to their electrochemical properties. Some of the most important metals include:

1. **Lithium (Li)**

- **Usage**: Lithium-ion batteries
- **Applications**: Portable electronics (smartphones, laptops), electric vehicles (EVs), renewable energy storage systems.

2. **Lead (Pb)**

- **Usage**: Lead-acid batteries
- **Applications**: Automotive batteries (car batteries), backup power supplies, and energy storage for solar and wind power systems.

3. **Nickel (Ni)**

- **Usage**: Nickel-cadmium (NiCd) and nickel-metal hydride (NiMH) batteries
- **Applications**: Power tools, medical equipment, hybrid vehicles, and consumer electronics.

4. **Cadmium (Cd)**

- **Usage**: Nickel-cadmium (NiCd) batteries
- **Applications**: Power tools, emergency lighting, and medical equipment (although its use is declining due to toxicity and environmental concerns).

5. **Cobalt (Co)**

- **Usage**: Lithium-ion batteries (as part of the cathode material, e.g., lithium cobalt oxide)
- **Applications**: Portable electronics, electric vehicles, and energy storage systems.

6. **Manganese (Mn)**

- **Usage**: Lithium manganese oxide ($LiMn_2O_4$) batteries
- **Applications**: Power tools, hybrid and electric vehicles, and consumer electronics.

Hence, the answer is the option (2).

Q. 21 Consider the following reaction at $298\ \mathrm{K} \cdot \frac{3}{2}\mathrm{O}_{2(\mathrm{g})} \rightleftharpoons \mathrm{O}_3(\mathrm{g}) \cdot \mathrm{K_P} = 2.47 \times 10^{-29}$

$\Delta_r G^\circ$ for the reaction is_____ kJ. (Given $\mathrm{R} = 8.314 \mathrm{JK}^{-1}\ \mathrm{mol}^{-1}$)

Correct Answer:
71.1 $\mathrm{KJ/mol.\ K}$

Solution:
$\frac{3}{2}\mathrm{O}_2(\mathrm{g}) \rightleftharpoons \mathrm{O}_3(\mathrm{g})$

$\mathrm{K_P} = 2.47 \times 10^{-29}$
$\Delta \mathrm{G}^\circ = -\mathrm{RT}\log_{10}\ \mathrm{K_p}$
$= -8.314 \times 298 \times \log_{10}\left(2.47 \times 10^{-29}\right)$
$= -8.314 \times 298 \times (\log_{10}(2.47) - 29)$
$= -8.314 \times 298 \times (0.30 - 29)$
$\Rightarrow -8.314 \times 298 \times (-28.7)$
$\Rightarrow 71106\ \mathrm{J/mol.\ K}$
$\Rightarrow 71.1\mathrm{KJ/mol.\ K}$

Hence, the answer is (71.1).

Q. 22 Molar mass of the salt from $\mathrm{NaBr}, \mathrm{NaNO_3}, \mathrm{KI}$ and $\mathrm{CaF_2}$ which does not evolve coloured vapours on heating with concentrated $\mathrm{H_2SO_4}$ is_____ gmol^{-1}.
(Molar mass in gmol^{-1} : $\mathrm{Na}: 23,\ \mathrm{N}: 14,\ \mathrm{K}: 39, \mathrm{O}: 16, \mathrm{Br}: 80, \mathrm{I}: 127,\ \mathrm{F}: 19, \mathrm{Ca}: 40$)

Correct Answer:
78

Solution:

$$2\mathrm{NaBr} + 2\underset{\text{Conc.}}{\mathrm{H_2SO_4}} \xrightarrow{\Delta} \underset{\text{Reddish Brown Vap.}}{\mathrm{Br_2}} + \mathrm{SO_2} + 2\mathrm{H_2O} + \mathrm{Na_2SO_4}$$

$$\mathrm{NaNO_3} + \underset{\text{Conc.}}{\mathrm{H_2SO_4}} \xrightarrow{\Delta} \underset{\text{Reddish brown}}{\mathrm{NO_2}}$$

$$2\mathrm{KI} + \underset{\text{Conc.}}{\mathrm{H_2SO_4}} \xrightarrow{\Delta} \underset{\text{Violet Vap.}}{\mathrm{I_2}} + \mathrm{SO_2} + 2\mathrm{H_2O} + \mathrm{Na_2SO_4}$$

$$\mathrm{CaF_2} + \underset{\text{Conc}}{\mathrm{H_2SO_4}} \xrightarrow{\Delta} \mathrm{CaSO_4} + \underset{\text{Colourless}}{2\mathrm{HF}}$$

Molar mass of $\mathrm{CaF_2} = 78$

Q. 23

$$\mathrm{CH_3CH_2Br + NaOH} \begin{cases} \xrightarrow{\mathrm{C_2H_5OH}} \text{Product A} \\ \xrightarrow{\mathrm{H_2O}} \text{Product B} \end{cases}$$

The total number of hydrogen atoms in product A and product B is _____ .

Correct Answer:
10

Solution:

$$CH_3-CH_2-Br+NaOH \xrightarrow{C_2H_5OH} \underset{(A)}{CH_2=CH_2}$$

$$CH_3-CH_2-Br \xrightarrow[H_2O]{NaOH} \underset{(B)}{CH_3-CH_2-OH}$$

The total no of hydrogen atom in product A & product B is 10.

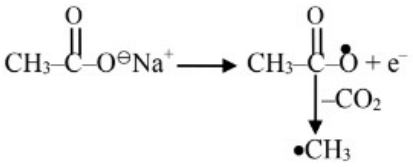
Q. 24 Number of alkanes obtained on electrolysis of a mixture of CH_3COONa and C_2H_5COONa is_____.

Correct Answer:
3

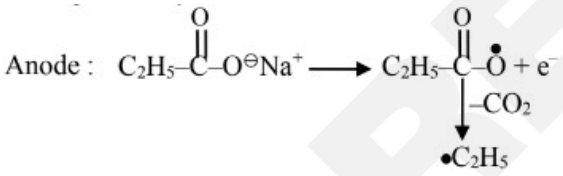
Solution:

$$CH_3COONa + C_2H_5COONa \xrightarrow{\text{electrolysis}}$$

During electrolysis of CH_3COONa



During electrolysis of C_2H_5COONa



$$\text{Now;} \quad \dot{C}H_3 + \dot{C}H_3 \longrightarrow CH_3-CH_3$$

$$\dot{C}_2H_5 + \dot{C}_2H_5 \longrightarrow CH_3-CH_2-CH_2-CH_3$$

$$\dot{C}H_3 + CH_3-\dot{C}H_2 \longrightarrow CH_3-CH_2-CH_3$$

Total (3) product is obtain.

Q. 25 The number of species from the following in which the central atom uses sp^3 hybrid orbitals in its bonding is_____.

$NH_3, SO_2, SiO_2, BeCl_2, CO_2, H_2O, CH_4, BF_3$

Correct Answer:
4

Solution:
NH_3, SiO_2, H_2O and CH_4

Hence, the answer is (4).

Q. 26 Number of moles of methane required to produce $22\ g CO_2(\ g)$ after combustion is $x \times 10^{-2}$ moles. The value of x is_____.

Correct Answer:
50

Solution:
$$CH_4 + 2O_2 \rightarrow \underset{22gm}{CO_2} + \quad 2H_2O$$

$$\frac{22}{44} = 0.5 \text{ mole}$$

1 mole $CO_2 \rightarrow 1$ mole CH_4

0.5 mole $CO_2 \rightarrow 0.5$ mole CH_4

Hence, the answer is (50).

Q. 27 The 'Spin only' Magnetic moment for $[Ni(NH_3)_6]^{2+}$ is_____$\times 10^{-1} BM$.

(given = Atomic number of Ni: 28)

Correct Answer:
28

Solution:
$Ni^{+2} \Rightarrow d8 \Rightarrow \sqrt{n(n+2)}$ $n =$ unpected e^-

n=

$\boxed{\uparrow\downarrow}\boxed{\uparrow\downarrow}\boxed{\uparrow\downarrow}\boxed{1}\boxed{1} \Rightarrow \sqrt{2 \times 4} = \sqrt{8} = 2.8 = 28 \times 10^{-1} BM$

Q. 28 The ionization energy of sodium in $kJmol^{-1}$, if electromagnetic radiation of wavelength $242\ nm$ is just sufficient to ionize sodium atom is_____

Correct Answer:
492.7

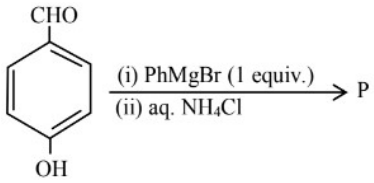
Solution:

$$E = \frac{hc}{\lambda} = \frac{6.6 \times 10^{-34} \times 3 \times 10^{8}}{242 \times 10^{-9}}$$

$\Rightarrow 8.2 \times 10^{-19}$ Joule/ atom
$\Rightarrow 8 \times 10^{-19}$ Joule/ atom
$E = 8 \times 10^{-19} \times 6.022 \times 10^{23}$ atom /mole
$E = 492709$ J/mole
$E = 492.7$ kJ/mole

Hence, the answer is (492.7).

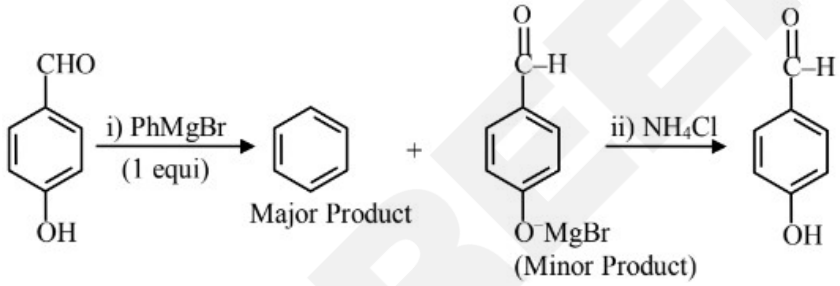
Q. 29 The product of the following reaction is P.



The number of hydroxyl groups present in the product P is _____ .

Correct Answer:
0

Solution:



So the total number of hydroxyl group present in major product will be zero.

Q. 30 One Faraday of electricity liberates $x \times 10^{-1}$ gram atom of copper from copper sulphate. x is_____.

Correct Answer:
5

Solution:
$Cu^{2+} + 2e^{-} \rightarrow Cu$
1 Faraday $\rightarrow$ 1 gm eq. Cu

$$\text{moles of Cu} \equiv \text{ gram atom of Cu} = \frac{\text{gramequivalent}}{n_f} = \frac{1}{2} = 0.5$$

Hence, the answer is (0.5).

Maths

Q. 1 $\lim_{x\to 0} \frac{e^{2|\sin x|}-2|\sin x|-1}{x^2}$

Option 1:
is equal to 1

Option 2:
does not exist

Option 3:
is equal to -1

Option 4:
is equal to 2

Correct Answer:
is equal to 2

Solution:

$$\lim_{x\to 0} \frac{e^{2|\sin x|}-2|\sin x|-1}{x^2}$$

$$e^{2|\sin x|} = 1+2|\sin x| + \frac{4|\sin x|^2}{2} \ldots\ldots$$

$$= \lim_{x\to 0} \frac{1+2|\sin x| + 2|\sin x|^2 \ldots -2|\sin x|-1}{x^2}$$

$$= \lim_{x\to 0} \frac{2|\sin x|^2}{x^2} = \lim_{x\to 0} \frac{2\sin^2 x}{x^2} = 2$$

Hence, the answer is the option (4).

Q. 2 Let $\vec{a} = 3\hat{i}+\hat{j}-2\hat{k}, \vec{b} = 4\hat{i}+\hat{j}+7\hat{k}$ and $\vec{c} = \hat{i}-3\hat{j}+4\hat{k}$ be three vectors. If a vectors $\vec{p}$ satisfies $\vec{p}\times\vec{b} = \vec{c}\times\vec{b}$ and $\vec{p}\cdot\vec{a} = 0$, then $\vec{p}\cdot(\hat{i}-\hat{j}-\hat{k})$ is equal to

Option 1:
24

Option 2:
36

Option 3:
32

Option 4:
28

Correct Answer:
32

Solution:

$\vec{p} \times \vec{b} = \vec{c} \times \vec{b}$

$(\vec{p} - \vec{c}) \times \vec{b} = 0$

$\vec{p} - \vec{c} = \lambda \vec{b}$

$\vec{p} = \vec{c} + \lambda \vec{b}$

$\overrightarrow{\mathrm{p}} \cdot \overrightarrow{\mathrm{a}} = \overrightarrow{\mathrm{c}} \cdot \overrightarrow{\mathrm{a}} + \lambda(\overrightarrow{\mathrm{b}} \cdot \overrightarrow{\mathrm{a}}) \Rightarrow \overrightarrow{\mathrm{c}} \cdot \overrightarrow{\mathrm{a}} = -\lambda(\overrightarrow{\mathrm{b}} \cdot \overrightarrow{\mathrm{a}}) (\because \overrightarrow{\mathrm{p}} \cdot \overrightarrow{\mathrm{a}}) = 0$

$\lambda = -\dfrac{\overrightarrow{\mathrm{c}} \cdot \overrightarrow{\mathrm{a}}}{\overrightarrow{\mathrm{b}} \cdot \overrightarrow{\mathrm{a}}}$

$\lambda = -8$

$\overrightarrow{\mathrm{p}} = \overrightarrow{\mathrm{c}} - 8\overrightarrow{\mathrm{b}}$

$\vec{p} = (\hat{i} - 3\hat{j} + 4k) - 8(4\hat{i} + \hat{j} + 7k)$

$\vec{p} = -31\hat{i} - 11\hat{j} - 52k$

$\vec{p} = -(31\hat{i} + 11\hat{j} + 52k)$

$\text{Now } \vec{p} \cdot (\hat{i} - \hat{j} - k) = -(31\hat{i} + 11\hat{j} + 52k)(\hat{i} - \hat{j} - k)$

$= -(31 - 11 - 52)$

$= 32$

Hence, the answer is the option (3).

Q. 3 If $f(x) = \begin{vmatrix} x^3 & 2x^2+1 & 1+3x \\ 3x^2+2 & 2x & x^3+6 \\ x^3-x & 4 & x^2-2 \end{vmatrix}$ for all $x \in \mathbb{R}$, then $2f(0) + f'(0)$ is equal to

Option 1:
18

Option 2:
24

Option 3:
48

Option 4:
42

Correct Answer:
42

Solution:

$$f(x)=\begin{matrix} x^3 & 2x^2+1 & 1+3x \\ 3x^2+2 & 2x & x^3+6 \\ x^3-x & 4 & x^2-2 \end{matrix}$$

$$f'(x)=\begin{matrix} 3x^2 & 4x & 3 \\ 3x^2+2 & 2x & x^3+6 \\ x^3-x & 4 & x^2-2 \end{matrix}+\begin{matrix} x^3 & 2x^2+1 & 1+3x \\ 6x & 2 & 3x^2 \\ x^3-x & 4 & x^2-2 \end{matrix}+\begin{matrix} x^3 & 2x^2+1 & 1+3x \\ 3x^2+2 & 2x & x^3+6 \\ 3x^2-1 & 0 & 2x \end{matrix}$$

$$2f(0)+f'(0)=2\begin{matrix} 0 & 1 & 1 \\ 2 & 0 & 6 \\ 0 & 4 & -2 \end{matrix}+\begin{matrix} 0 & 0 & 3 \\ 2 & 0 & 6 \\ 0 & 4 & -2 \end{matrix}+\begin{matrix} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 4 & -2 \end{matrix}+\begin{matrix} 0 & 1 & 1 \\ 2 & 0 & 6 \\ -1 & 0 & 0 \end{matrix}$$

$$=24+24+0-6$$

$$=42$$

Hence, the answer is the option (4).

Q. 4 Let $y=y(x)$ be the solution of the differential equation $\frac{dy}{dx}=\frac{(\tan x)+y}{\sin x(\sec x-\sin x\tan x)}, x\in\left(0,\frac{\pi}{2}\right)$ satisfying the condition $y\left(\frac{\pi}{4}\right)=2$. Then, $y\left(\frac{\pi}{3}\right)$ is

Option 1:
$\frac{\sqrt{3}}{2}(2+\log_e 3)$

Option 2:
$\sqrt{3}\left(2+\log_e\sqrt{3}\right)$

Option 3:
$\sqrt{3}\,(2+\log_e 3)$

Option 4:
$\sqrt{3}\,(1+2\log_e 3)$

Correct Answer:
$\sqrt{3}\left(2+\log_e\sqrt{3}\right)$

Solution:

$$\frac{dy}{dx} = \frac{\tan x + y}{\sin x(\sec x - \sin x \tan x)}$$

$$\frac{dy}{dx} = \frac{\sin x + y\cos x}{\sin x \cos x\left(\frac{1}{\cos x} - \frac{\sin^2 x}{\cos x}\right)}$$

$$\frac{dy}{dx} = \frac{\sin x + y\cos x}{\sin x\left(1 - \sin^2 x\right)}$$

$$\frac{dy}{dx} = \sec^2 x + y(2\,\text{cosec}\,2x)$$

$$\frac{dy}{dx} + (-2\,\text{cosec}\,2x)y = \sec^2 x$$

Compare with $\frac{dy}{dx} + P(x)y = Q(x)$

$$\text{I.F} = e^{\int p(x)dx} = e^{\int -2\,\text{cosesec}\,2xdx}$$

Put $2x = t$

$$dx = \frac{dt}{2}$$

$$= e^{-\int 2\,\text{cosect}\,\frac{dt}{2}} = e^{-\int \text{cosec}\,tdt} = e^{-\ln\tan\frac{t}{2}}$$

$$= e^{-\ln|\tan x|} = \frac{1}{|\tan x|}$$

Required solution $y(IF) = \int Q(IF)dx + c$

$$\frac{y}{|\tan x|} = \int \frac{\sec^2 x}{|\tan x|}dx + c == \ln|\tan x| + c$$

$$y = |\tan x|(\ln|\tan x| + c)$$

$$\text{Put } x = \frac{\pi}{4} \Rightarrow y = 2 \Rightarrow 2 = |1|(\ln|1| + c)$$

$$c = 2$$

put $c = 2$ in equation (1)

then $\text{y} = |\tan \text{x}|(\ln|\tan \text{x}| + 2)$

Now $\text{y}\left(\frac{\pi}{3}\right) = \sqrt{3}(\ln\sqrt{3} + 2)$

Hence, the answer is the option (2).

Q. 5 The area of the region $\left\{(x, y) : y^2 \le 4x, x < 4, \frac{xy(x-1)(x-2)}{(x-3)(x-4)} > 0, x \ne 3\right\}$ is

Option 1:
$\frac{8}{3}$

Option 2:
$\frac{64}{3}$

Option 3:
$\frac{16}{3}$

Option 4:
$\frac{32}{3}$

Correct Answer:
$\frac{32}{3}$

Solution:

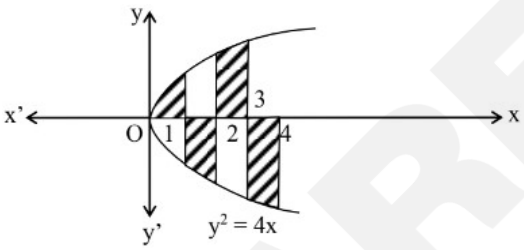
$$\left\{(x,y); y^2 \leq 4x, x < 4, \frac{xy(x-1)(x-2)}{(x-3)(x-4)} > 0, x \neq 3\right\}$$

$$y^2 \leq 4x, x < 4, \frac{xy(x-1)(x-2)}{(x-3)(x-4)} > 0$$

Case (i) $y > 0 \Rightarrow \frac{x(x-1)(x-2)}{(x-3)(x-4)} > 0$

$x \in (1,2) \cup (3,4)$

$$\text{Area } = \int_0^1 2\sqrt{x}dx + \int_1^2 2\sqrt{x}dx + \int_2^3 2\sqrt{x}dx + \int_3^4 2\sqrt{x}dx$$



$$= \frac{4}{3}\left(\left[x^{\frac{3}{2}}\right]_0^1 + \left[x^{\frac{3}{2}}\right]_1^2 + \left[x^{\frac{3}{2}}\right]_2^3 + \left(x^{\frac{3}{2}}\right)_3^4\right)$$

$$A = \frac{4}{3}(1 + 2\sqrt{2} - 1 + 3\sqrt{3} - 2\sqrt{2} + 4\sqrt{4} - 3\sqrt{3}) = \frac{4}{3} \times 8 = \frac{32}{3}$$

Q. 6 The sum of the series $\frac{1}{1-3\cdot1^2+1^4} + \frac{2}{1-3\cdot2^2+2^4} + \frac{3}{1-3\cdot3^2+3^4} + \ldots$ up to 10 -terms is

Option 1:
$\frac{45}{109}$

Option 2:
$-\frac{55}{109}$

Option 3:
$-\frac{45}{109}$

Option 4:
$\frac{55}{109}$

Correct Answer:
$-\frac{55}{109}$

Solution:

$$S=\sum_{r=1}^{10}\frac{r}{1-3r^2+r^4}$$

$$S=\sum_{r=1}^{10}\frac{r}{r^4-2r^2+1-r^2}$$

$$=\sum_{r=1}^{10}\frac{r}{\left(r^2-1\right)^2-r^2}$$

$$=\sum_{r=1}^{10}\frac{r}{\left(r^2-1+r\right)\left(r^2-1-r\right)}$$

$$=\frac{1}{2}\sum_{r=1}^{10}\frac{\left(\left(r^2-1\right)+r\right)-\left(\left(r^2-1\right)-r\right)}{\left(\left(r^2-1\right)+r\right)\left(\left(r^2-1\right)-r\right)}$$

$$=\frac{1}{2}\sum_{r=1}^{10}\left(\frac{1}{r^2-1-r}-\frac{1}{r^2-1+r}\right)$$

$$=\frac{1}{2}\left(\left(-\frac{1}{1}-\frac{1}{1}\right)+\left(\frac{1}{1}-\frac{1}{5}\right)+\ldots.+\left(-\frac{1}{109}\right)\right)$$

$$=\frac{1}{2}\left(-1-\frac{1}{109}\right)=\frac{1}{2}\left(\frac{-110}{109}\right)$$

$$S=-\frac{55}{109}$$

Hence, the answer is the option (2).

Q. 7 If the foci of a hyperbola are same as that of the ellipse $\frac{x^2}{9}+\frac{y^2}{25}=1$ and the eccentricity of the hyperbola is $\frac{15}{8}$ times the eccentricity of the ellipse, then the smaller focal distance of the point $\left(\sqrt{2},\frac{14}{3}\sqrt{\frac{2}{5}}\right)$ on the hyperbola, is equal to

Option 1:
$7\sqrt{\frac{2}{5}}+\frac{8}{3}$

Option 2:

$7\sqrt{\frac{2}{5}}-\frac{8}{3}$

Option 3:

$14\sqrt{\frac{2}{5}}-\frac{4}{3}$

Option 4:

$14\sqrt{\frac{2}{5}}-\frac{16}{3}$

Correct Answer:

$7\sqrt{\frac{2}{5}}-\frac{8}{3}$

Solution:

$$\frac{x^2}{9}+\frac{y^2}{25}=1$$

$$a=3, b=5$$

$$e=\sqrt{1-\frac{9}{25}}=\frac{4}{5}$$

$$\text{foci } =(0, \pm \text{ be })=(0, \pm 4)$$

Eccentricity of hyperbola $\mathrm{e_H}=\frac{4}{5} \times \frac{15}{8}=\frac{3}{2}$

Let the equation of hyperbola be

$$\frac{\mathrm{x}^2}{\mathrm{A}^2}-\frac{\mathrm{y}^2}{\mathrm{B}^2}=-1$$

According to equation $\mathrm{Be_H}=4$

$$\therefore \mathrm{B}=\frac{8}{3}$$

$$\because \mathrm{A}^2=\mathrm{B}^2\left(\mathrm{e}_{\mathrm{H}}^2-1\right)$$

$$=\frac{64}{9}\left(\frac{9}{4}-1\right) \Rightarrow \mathrm{A}^2=\frac{80}{9}$$

$$\therefore \frac{x^2}{\frac{80}{9}}-\frac{y^2}{\frac{64}{9}}=-1$$

$$\text{Directrix } y=\pm\frac{B}{e_H}=\pm\frac{16}{9}$$

$$PS=ePM$$

$$=\frac{3}{2} \frac{14}{3}\sqrt{\frac{2}{5}}-\frac{16}{9}$$

$$=7\sqrt{\frac{2}{5}}-\frac{8}{3}$$

Hence, the answer is the option (2).

Q. 8 If the system of linear equations

$$x - 2y + z = -4$$
$$2x + \alpha y + 3z = 5$$
$$3x - y + \beta z = 3$$

has infinitely many solutions, then $12\alpha + 13\beta$ is equal to

Option 1:
54

Option 2:
60

Option 3:
64

Option 4:
58

Correct Answer:
58

Solution:
$x - 2y + z = -4$

$2x + \alpha y + 3z = 5$

$3x - y + \beta z = 3$

For infinite solution $\mathrm{D} = 0, \mathrm{D}_1 = 0, \mathrm{D}_2 = 0 \& \mathrm{D}_3 = 0$

$$\mathrm{D} = 0 \Rightarrow \begin{vmatrix} 1 & -2 & 1 \\ 2 & \alpha & 3 \\ 3 & -1 & \beta \end{vmatrix} = 0$$

$\alpha\beta - 3\alpha + 4\beta = 17 \quad \ldots(1)$

$$D_1 = \begin{vmatrix} -4 & -2 & 1 \\ 5 & \alpha & 3 \\ 3 & -1 & \beta \end{vmatrix} = 0$$

$$D_2 = \begin{vmatrix} 1 & -4 & 1 \\ 2 & 5 & 3 \\ 3 & 3 & \beta \end{vmatrix} = 0$$

$= (5\beta - 9) + 4(2\beta - 9) + 1(6 - 15) = 0$

$13\beta = 54 \Rightarrow \beta = \dfrac{54}{13}$

Put in (1)

$$\frac{54}{13}\alpha - 3\alpha + 4 \times \frac{54}{13} = 17 \Rightarrow \alpha = \frac{1}{3}$$

$$12\alpha + 13\beta = 12 \times \frac{1}{3} + 13 \times \frac{54}{13} = 58$$

Hence, the answer is the option (4).

Q. 9 The solution curve of the differential equation $y\frac{dx}{dy} = x\left(\log_e x - \log_e y + 1\right), x > 0, y > 0$ passing through the point $(e, 1)$ is

Option 1:
$\log_e \frac{y}{x} = y^2$

Option 2:
$\log_e \frac{y}{x} = x$

Option 3:
$2 \log_e \frac{x}{y} = y + 1$

Option 4:
$\log_e \frac{x}{y} = y$

Correct Answer:
$\log_e \frac{x}{y} = y$

Solution:

$$\frac{dx}{dy} = \frac{x}{y}(\log x - \log y + 1)$$

$$\frac{dx}{dy} = \frac{x}{y}\left(\log_e\left(\frac{x}{y}\right) + 1\right)$$

$$\text{Put } \frac{x}{y} = v \Rightarrow x = vy$$

$$\frac{dx}{dy} = v + y \cdot \frac{dv}{dy}$$

$$v + y\frac{dv}{dy} = v(\ln v + 1)$$

$$v + y\frac{dv}{dy} = v \ln u + v$$

$$y\frac{dv}{dy} = v \ln v$$

$$\frac{1}{v \ln v} dv = \frac{dy}{y}$$

$$= \int \frac{dv}{v \ln v} = \int \frac{dy}{y}$$

$$\ln |\ln v| = \ln y + \ln c$$

$$\ln |\ln v| = \ln cy$$
$$cy = |\ln v|$$

$$\text{cy} = \ln \frac{x}{y}$$
$$\text{put } x = e \& y = 1$$
$$c = \ln \frac{e}{1} \Rightarrow c = 1$$

$$\text{put c} = 1 \text{ in eq. } (1)$$

$$y = \ln \frac{x}{y}$$

Hence, the answer is the option (4).

Q. 10 Three rotten apples are accidently mixed with fifteen good apples. Assuming the random variable x to be the number of rotten apples in a draw of two apples, the variance of x is

Option 1:
$\frac{47}{153}$

Option 2:
$\frac{37}{153}$

Option 3:
$\frac{57}{153}$

Option 4:
$\frac{40}{153}$

Correct Answer:
$\frac{40}{153}$

Solution:
3 rotten apples

15 good apples

Let X is the number of rotten apples

$$P(X=0)=\frac{^{15}C_2}{^{18}C_2}=\frac{105}{153}$$

$$P(X=1)=\frac{^{3}C_1\times{}^{15}C_1}{^{18}C_2}=\frac{45}{153}$$

$$P(X=2)=\frac{^{3}C_2}{^{18}C_2}=\frac{3}{153}$$

$$E(X)=0\times\frac{105}{153}+1\times\frac{45}{153}+2\times\frac{3}{153}=\frac{51}{153}=\frac{1}{3}$$

$$V_{ar}(X)=E\left(X^2\right)-(E(X))^2$$

$$=0\times\frac{105}{153}\cdot+1\times\frac{45}{153}+4\times\frac{3}{153}-\frac{1}{9}$$

$$=\frac{57}{153}-\frac{1}{9}=\frac{40}{153}$$

Hence, the answer is the option (4).

Q. 11 Let a be the sum of all coefficients in the expansion of $\left(1-2x+2x^2\right)^{2023}\left(3-4x^2+2x^3\right)^{2024}$ and $b=\lim_{x\to0}\left(\frac{\int_0^x\frac{\log(1+t)}{t^{2024}+1}dt}{x^2}\right)$. If the equations $cx^2+dx+e=0$ and $2bx^2+ax+4=0$ have a common root, where $c,d,e\in\mathbb{R}$, then $d:c:e$ equals

Option 1:
$4:1:4$

Option 2:
$1:1:4$

Option 3:
$2:1:4$

Option 4:
$1:2:4$

Correct Answer:
$1:1:4$

Solution:
$\left(1-2x+2x^2\right)^{2023}\cdot\left(3-4x^2+2x^3\right)^{2024}$

Put x = 1
for getting a sum of all coefficientsroots a = 1

$$b=\lim_{x\to 0}\left(\frac{\int_0^x \frac{\ln(1+t)}{t^{2024}+1}dt}{x^2}\right)$$

$$=\lim_{x\to 0}\frac{\frac{\ln(1+x)}{x^{2024}+1}}{2x}=\frac{1}{2}$$

$$cx^2+dx+e=0$$

$$2bx^2+ax+4=0$$

Have a common root

$$cx^2+dx+e=0$$

$$x^2+x+4=0\rightarrow(D<0)$$

Hence both root are common

$$\frac{c}{1}=\frac{d}{1}=\frac{e}{4}$$

$$c:d:e=1:1:4\Rightarrow d:c:e=1:1:4$$

Hence, the answer is the option (2).

Q. 12 The distance of the point $Q(0,2,-2)$ from the line passing through the point $P(5,-4,3)$ and perpendicular to the lines $\vec{r}=(-3\hat{i}+2\hat{k})+\lambda(2\hat{i}+3\hat{j}+5\hat{k}),\lambda\in\mathbb{R}$ and $\vec{r}=(\hat{i}-2\hat{j}+\hat{k})+\mu(-\hat{i}+3\hat{j}+2\hat{k}),\mu\in\mathbb{R}$ is

Option 1:
$\sqrt{54}$

Option 2:
$\sqrt{86}$

Option 3:
$\sqrt{74}$

Option 4:
$\sqrt{20}$

Correct Answer:
$\sqrt{74}$

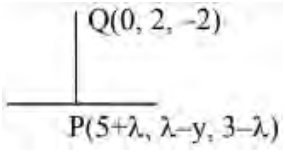
Solution:
A vector in the direction ratio of the required line can be obtained by cross product of

$$\begin{matrix}\hat{i} & \hat{j} & \mathrm{k} \\ 2 & 3 & 5 \\ -1 & 3 & 2\end{matrix} = -9\hat{i} - 9\hat{j} + 9k$$

Required line $\overrightarrow{\mathrm{r}} = (5\hat{\mathrm{i}} - 4\hat{\mathrm{j}} + 3\mathrm{k}) + \lambda(-9\hat{\mathrm{i}} - 9\hat{\mathrm{j}} + 9\hat{\mathrm{k}})$

$\overrightarrow{\mathrm{r}} = (5\hat{\mathrm{i}} - 4\hat{\mathrm{j}} + 3\mathrm{k}) + \lambda(\hat{\mathrm{i}} + \hat{\mathrm{j}} - \mathrm{k})$

Now distance of (0, 2, – 2)



Position vector of point $P = (5 + \lambda)\hat{i} + (\lambda - 4)\hat{j} + (3 - \lambda)\hat{k}$

$\overrightarrow{\mathrm{QP}} = (5 + \lambda)\hat{\mathrm{i}} + (\lambda - 6)\hat{\mathrm{j}} + (5 - \lambda)\mathrm{k}$

Accordin to question

$\overrightarrow{\mathrm{QP}} \cdot (\hat{\mathrm{i}} + \hat{\mathrm{j}} - \mathrm{k}) = 0$

$\lambda = 2$ Hence point $\mathrm{P}(7, -2, 1)$

$|\overrightarrow{\mathrm{QP}}| = \sqrt{49 + 16 + 9} = \sqrt{74}$

Q. 13 Let $\alpha, \beta, \gamma, \delta \in \mathbb{Z}$ and let $\mathrm{A}(\alpha, \beta), \mathrm{B}(1, 0), \mathrm{C}(\gamma, \delta)$ and $\mathrm{D}(1, 2)$ be the vertices of a parallelogram ABCD. If $\mathrm{AB} = \sqrt{10}$ and the points A and C lie on the line $3\mathrm{y} = 2\mathrm{x} + 1$, then $2(\alpha + \beta + \gamma + \delta)$ is equal to

Option 1:
12

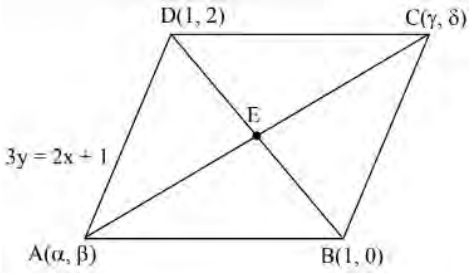
Option 2:
8

Option 3:
10

Option 4:
5

Correct Answer:
10

Solution:
E is mid point at both diagonal



$$\left(\frac{\alpha+\gamma}{2}, \frac{\beta+\delta}{2}\right) = (1,1)$$

$$\alpha+\gamma = 2 \qquad .....(i)$$

$$\beta+\delta = 2 \qquad .....(ii)$$

$$\alpha+\beta+\gamma+\delta = 4$$

$$2(\alpha+\beta+\gamma+\delta) = 8$$

Q. 14 For $\alpha, \beta, \gamma \neq 0$, if $\sin^{-1}\alpha + \sin^{-1}\beta + \sin^{-1}\gamma = \pi$ and $(\alpha+\beta+\gamma)(\alpha-\gamma+\beta) = 3\alpha\beta$, then γ equals

Option 1:
$\frac{\sqrt{3}-1}{2\sqrt{2}}$

Option 2:
$\sqrt{3}$

Option 3:
$\frac{1}{\sqrt{2}}$

Option 4:
$\frac{\sqrt{3}}{2}$

Correct Answer:
$\frac{\sqrt{3}}{2}$

Solution:

$$\text{Let } \sin^{-1}\alpha = A, \sin^{-1}\beta = B \& \sin^{-1}\gamma = C$$

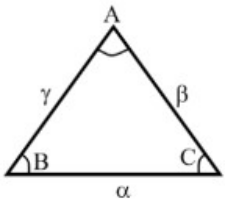
$$\alpha = \sin A, \beta = \sin B, C = \sin\gamma$$

$$(A+B+C = \pi)$$

$$\because (\alpha+\beta+\gamma)(\alpha-\gamma+\beta) = 3\alpha\beta$$

from sine law

$$\frac{\sin A}{\alpha} = \frac{\sin B}{\beta} = \frac{\sin C}{\gamma}$$



$(\alpha+\beta)^2-\gamma^2=3\alpha\beta$

$\alpha^2+\beta^2-\gamma^2=\alpha\beta$

$\frac{\alpha^2+\beta^2-\gamma^2}{2\alpha\beta}=\frac{1}{2}$

$\cos C=\frac{1}{2}\Rightarrow \sin C=\frac{\sqrt{3}}{2}$

$\gamma=\sin C=\frac{\sqrt{3}}{2}$

Q. 15 Two marbles are drawn in succession from a box containing 10 red, 30 white, 20 blue and 15 orange marbles, with replacement being made after each drawing. Then the probability, that first drawn marble is red and second drawn marble is white, is

Option 1:
$\frac{2}{25}$

Option 2:
$\frac{4}{75}$

Option 3:
$\frac{4}{25}$

Option 4:
$\frac{2}{3}$

Correct Answer:
$\frac{4}{75}$

Solution:

15 orange
20 blue
30 white
10 red

Probability of drawing first red and second white
(form multiplication theorem)

$=\frac{^{10}C_1}{^{75}C_1}\times\frac{^{30}C_1}{^{75}C_1}=\frac{4}{75}$

Q. 16 Let S be the set of positive integral values of a for which $\frac{a^2+2(a+1)x+9a+4}{x^2-8x+32} < 0, \forall x \in \mathbb{R}$. Then, the number of elements in S is:

Option 1:
∞

Option 2:
3

Option 3:
0

Option 4:
1

Correct Answer:
0

Solution:

$$\frac{ax^2+2(a+1)x+9a+4}{x^2-8x+32} < 0$$

$D = 64 - 4 \times 32 < 0$

$\&a = 1 > 0$

$\therefore x^2 - 8x + 32 > 0 \forall x \in R$

$ax^2 + 2(a+1)x + 9a + 4 < 0 \forall x \in R$

Only possible when

$\mathrm{a} < 0 \quad \& \quad \mathrm{D} < 0$

but we need the positive integral value of a.
So,
No solution

Hence, the answer is the option (3).

Q. 17 Let $g(x)$ be a linear funchion and $f(x) = \begin{cases} g(x) & x \leq 0 \\ \left(\frac{1+x}{2+x}\right)^{\frac{1}{x}} & , x > 0 \end{cases}$, is continuous at $x = 0$. If $f'(1) = f(-1)$, then the value $\mathrm{g}(3)$ is

Option 1:
$\frac{1}{3}\log_e\left(\frac{4}{9e^{1/3}}\right)$

Option 2:
$\log_e\left(\frac{4}{9e^{1/3}}\right)$

Option 3:
$\frac{1}{3}\log_e\left(\frac{4}{9}\right)+1$

Option 4:
$\log_e\left(\frac{4}{9}\right)-1$

Correct Answer:
$\log_e\left(\frac{4}{9e^{1/3}}\right)$

Solution:
Let $g(x)=ax+b$

$\because$ function is continuous at $\mathrm{x}=0$

$\lim_{x\to 0^+} f(x)=f(0)=\lim_{x\to 0^-} f(x)$

$$\lim_{x\to 0^+}\left(\frac{1+x}{2+x}\right)^{\frac{1}{x}}=b$$

$b=0$

$\therefore \mathrm{g(x)=ax}$

For $\mathrm{x}>0$

$$\mathrm{f'(x)}=\frac{1}{\mathrm{x}}\left(\frac{1+\mathrm{x}}{2+\mathrm{x}}\right)^{\frac{1}{x}-1}\frac{1}{(2+\mathrm{x})^2}+\left(\frac{1+\mathrm{x}}{2+\mathrm{x}}\right)^{\frac{1}{x}}\ln\left(\frac{1+\mathrm{x}}{2+\mathrm{x}}\right)\left(-\frac{1}{\mathrm{x}^2}\right)$$

$$\mathrm{f'(1)}=\frac{1}{9}-\frac{2}{3}\ln\frac{2}{3}$$

$$\mathrm{f'(1)=f(-1)}\Rightarrow\frac{1}{9}-\frac{2}{3}\ln\frac{2}{3}=\mathrm{g(-1)}=-\mathrm{a}$$

$$\mathrm{a}=\frac{2}{3}\ln\frac{2}{3}-\frac{1}{9}\quad\therefore \mathrm{g(3)}=2\ln\frac{2}{3}-\frac{1}{3}=\ln\left(\frac{4}{9\mathrm{e}^{1/3}}\right)$$

Hence, the answer is the option (2).

Q. 18 If one of the diameters of the circle $x^2+y^2-10x+4y+13=0$ is a chord of another circle C, whose center is the point of intersection of the lines $2x+3y=12$ and $3x-2y=5$, then the radius of the circle C is:

Option 1:
$\sqrt{20}$

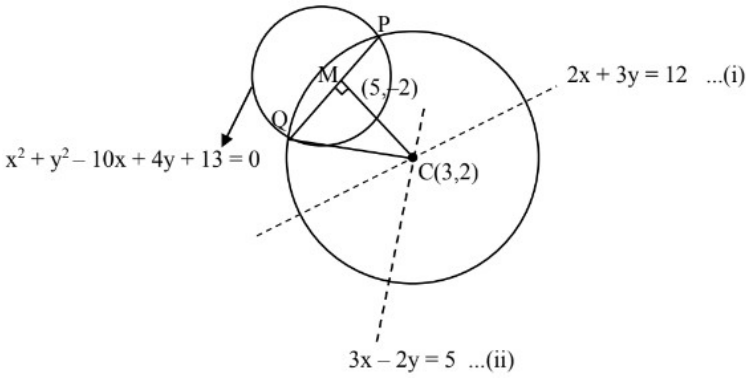
Option 2:
4

Option 3:
6

Option 4:
$3\sqrt{2}$

Correct Answer:
6

Solution:



Solve (i) and (ii) $\mathrm{C}(3,2)$
Centre of given circle is $\mathrm{M}(5,-2)$
Radius $\mathrm{PM}=\mathrm{QM}=4$

$\mathrm{CM}=\sqrt{4+16}=2\sqrt{5}$

Radius of circle $\mathrm{CP}=\mathrm{CQ}=\sqrt{16+20}=\sqrt{36}=6$

Q. 19 For $0<\mathrm{c}<\mathrm{b}<\mathrm{a}$, let $(\mathrm{a}+\mathrm{b}-2\mathrm{c})\mathrm{x}^2+(\mathrm{b}+\mathrm{c}-2\mathrm{a})\mathrm{x}+(\mathrm{c}+\mathrm{a}-2\,\mathrm{b})=0$ and $\alpha \neq 1$ be one of its root. Then, among the two statements
(I) If $\alpha \in (-1,0)$, then b cannot be the geometric mean of a and c
(II) If $\alpha \in (0,1)$, then b may be the geometric mean of a and c

Option 1:
Neither (I) nor (II) is true

Option 2:
Only (II) is true

Option 3:
Only (I) is true

Option 4:
Both (I) and (II) are true

Correct Answer:
Both (I) and (II) are true

Solution:
$f(x)=(a+b-2c)x^2+(b+c-2a)x+(c+a-2b)$

$\because f(1)=0$

$\because$ one root is a unit

$\because$ one root is unit product of roots $\alpha 1 = \frac{c+a-2b}{a+b-2c}$

If $\alpha \in (-1,0) \Rightarrow -1 < \frac{c+a-2b}{a+b-2c} < 0 \Rightarrow b+c < 2a \& b > \frac{a+c}{2}$

$\therefore$ b cannot be G.M. between a &c

If $\alpha \in (0,1) \Rightarrow 0 < \frac{c+a-2b}{a+b-2c} < 1$

$b > c$ and $b < \frac{a+c}{2}$

$\therefore$ b maybe GM between a&c

Hence, the answer is the option (4).

Q. 20 If $f(x) = \frac{4x+3}{6x-4}, x \neq \frac{2}{3}$ and (fof) (x) $g(x)$, where $g : \mathbb{R} - \left\{\frac{2}{3}\right\} \to \mathbb{R} - \left\{\frac{2}{3}\right\}$, then (gogog)(4) is equal to

Option 1:
4

Option 2:
-4

Option 3:
$-\frac{19}{20}$

Option 4:
$\frac{19}{20}$

Correct Answer:
4

Solution:

$$f(x) = \frac{4x+3}{6x-4}$$

$$g(x) = f\{f(x)\} = \frac{4f(x)+3}{6f(x)-4} = \frac{4\left(\frac{4x+3}{6x-4}\right)+3}{6\left(\frac{4x+3}{6x-4}\right)-4} = x$$

$$g(x) = x$$

Now

$$g\{g\{g(x)\}\} = x$$

$$g(g(g(4))) = 4$$

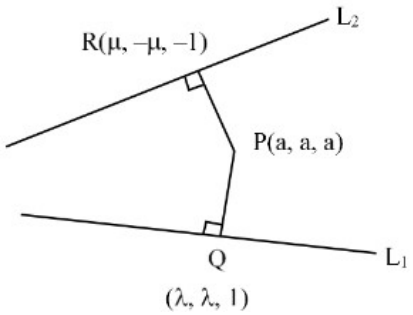
Hence, the answer is the option (1).

Q. 21 Let Q and R be the feet of perpendiculars from the point $P(a, a, a)$ on the lines $x = y, z = 1$ and $x = -\mathrm{y}, \mathrm{z} = -1$ respectively. If $\angle \mathrm{QPR}$ is a right angle, then $12\mathrm{a}^2$ is equal to_____.

Correct Answer:
12

Solution:
$\mathrm{L}_1 : \frac{\mathrm{x}}{1} = \frac{\mathrm{y}}{1} = \frac{\mathrm{z}-1}{0} = \lambda$ (let)

$L_2 : \frac{x}{1} = \frac{y}{-1} = \frac{z+1}{0} = \mu$(let)



$\mathrm{PQ} \perp \mathrm{L}_1$
$(\lambda - \mathrm{a})1 + (\lambda - \mathrm{a})1 + (1 - \mathrm{a})0 = 0$
$\lambda = \mathrm{a}$
$\mathrm{PR} \perp \mathrm{L}_2$
$1(\mu - \mathrm{a}) + 1(\mu + \mathrm{a}) + 0(-1 - \mathrm{a}) = 0$
$\mu - \mathrm{a} + \mu + \mathrm{a} = 0$
$\mu = 0$

$\therefore$ Point $Q(a, a, 1)$ and $R(0, 0, -1)$
According to question

$\angle \mathrm{QPR} = 90^\circ$

Direction ratio of $\mathrm{PR} = (\mathrm{a}, \mathrm{a}, \mathrm{a} + 1)$
Direction ratio of $\mathrm{PQ} = (0, 0, \mathrm{a} - 1)$

$a \times 0 + 0 \times a + (a + 1)(a - 1) = 0$
$a^2 = 1$
$\Rightarrow 12a^2 = 12 \times 1 = 12$

Q. 22 Let $S = (-1, \infty)$ and $f : S \to \mathbb{R}$ be defined as

$$f(x) = \int_{-1}^{x} \left(e^t - 1\right)^{11} (2t - 1)^5 (t - 2)^7 (t - 3)^{12} (2t - 10)^{61} dt,$$

Let $p =$ Sum of squares of the values of x, where $f(x)$ attains local maxima on S, and $q =$ Sum of the values of x, where $f(x)$ attains local minima on S. Then, the value of $p^2 + 2q$ is_____

Option 1:
27

Option 2:
-

Option 3:
-

Option 4:
-

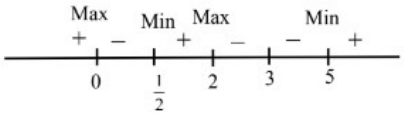
Correct Answer:
27

Solution:

$f(x)=\int_{-1}^{x}\left(e^{t}-1\right)^{11}(2t-1)^{5}(t-2)^{7}(t-3)^{12}(2t-10)^{61}\Big)dt$

Using Lebnitz

$f'(x)=(e^{x}-1)^{11}(2x-1)^{5}(x-2)^{7}(x-3)^{12}(2x-10)^{61}$



Point of maxima at x = 0,2

Point of minima at $x=\frac{1}{2},5$

$p=0^{2}+2^{2}=4$

$q=\frac{1}{2}+5=\frac{11}{2}$

$p^{2}+2q=16+11=27$

Q. 23 Let the foci and length of the latus rectum of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, a>b$ are $(\pm 5,0)$ and $\sqrt{50}$, respectively. Then, the square of the eccentricity of the hyperbola $\frac{x^2}{b^2}-\frac{y^2}{a^2b^2}=1$ equals

Correct Answer:
51

Solution:

$$\text{Foci } = (\pm 5, 0), \frac{2b^2}{a} = \sqrt{50}$$

$$ae = 5, \quad b^2 = \frac{5\sqrt{2}a}{2}$$

$$\because \quad b^2 = a^2\left(1 - e^2\right) = \frac{5\sqrt{2}a}{2}$$

$$a\left(1 - e^2\right) = \frac{5\sqrt{2}}{2}$$

$$\frac{5}{e}\left(1 - e^2\right) = \frac{5\sqrt{2}}{2}$$

$$\sqrt{2} - \sqrt{2}\mathrm{e}^2 = \mathrm{e}$$

$$\sqrt{2}\mathrm{e}^2 + \mathrm{e} - \sqrt{2} = 0$$

$$(\mathrm{e} + \sqrt{2})(\sqrt{2}\mathrm{e} - 1) = 0$$

$$\mathrm{e} = -\sqrt{2}, \frac{1}{\sqrt{2}},$$

$$\mathrm{e} = -\sqrt{2} \text{ (Rejected)}$$

$$\text{Hyperbola } \frac{\mathrm{x}^2}{\mathrm{b}^2} - \frac{\mathrm{y}^2}{\mathrm{a}^2\,\mathrm{b}^2} = 1$$

$$\mathrm{a} = 5\sqrt{2}, \ \mathrm{b} = 5$$

$$(\mathrm{e_H})^2 = 1 + \frac{\mathrm{a}^2\,\mathrm{b}^2}{\mathrm{b}^2} = 1 + \mathrm{a}^2 = 51$$

Hence, the answer is 51.

Q. 24 Let $\mathrm{A} = \{1, 2, 3, 4\}$ and $\mathrm{R} = \{(1, 2), (2, 3), (1, 4)\}$ be a relation on A. Let S be the equivalence relation on A such that $\mathrm{R} \subset \mathrm{S}$ and the number of elements in S is n. Then, the minimum value of n is_____

Correct Answer:
16

Solution:
$\mathrm{A} = \{1, 2, 3, 4\}$

$\mathrm{R} = \{(1, 2)(2, 3)(1, 4)\}$

For equivalence relation, the relation must be reflexive, symmetric & transitive

$R = \{(1,1)(2,2)(3,3)(4,4)(1,2)(2,1)(2,3)$

$(3,2)(1,4)(4,1)(1,3)(3,1)(2,4)(4,2)(4,3)(3,4)\}$

All elements are included

$\therefore$ Answer is 16

Hence, the answer is 16.

Q. 25 If the integral $525\int_0^{\frac{\pi}{2}} \sin 2x \cos^{\frac{11}{2}} x\left(1+\cos^{\frac{5}{2}} x\right)^{\frac{1}{2}} dx$ is equal to $(n\sqrt{2}-64)$, then n is equal to_____.

Correct Answer:
176

Solution:

$$I = 525\int_0^{\pi/2} \sin 2x \cos^{11/2} x\left(1+\cos^{5/2} x\right)^{\frac{1}{2}} dx$$

$\text{put } \sqrt{\cos x} = t \Rightarrow \cos x = t^2$

$-\sin x dx = 2tdt$

$\sin x dx = -2tdt$

$$I = 525\int_0^1 2(2t)\Big)t^2 t^{11}\sqrt{1+t^5}dt$$

$$= 525\times 4\int_0^1 \sqrt{1+t^5}\cdot t^{14}dt$$

$= 2100\int_0^1 \sqrt{1+t^5}\cdot t^{14}dt$

again put $\sqrt{1+t^5} = u \Rightarrow 1+t^5 = u^2$

$$5t^4dt = 2udu$$

$$I = 2100\int_1^{\sqrt{2}} u\cdot\frac{2u}{5}\cdot\left(u^2-1\right)^2 du$$

$$= 420\times 2\left(\int_1^{\sqrt{2}} u^2\left(u^4-2u^2+1\right)du\right)$$

$$= 840\int_1^{\sqrt{2}} (u^6 - 2u^4 + u^2)du$$

$$= 840\left[\frac{u^7}{7} - \frac{2u^5}{5} + \frac{u^3}{3}\right]_1^{\sqrt{2}}$$

$$= 840\left[\frac{8\sqrt{2}}{7} - \frac{8\sqrt{2}}{5} + \frac{2\sqrt{2}}{3} - \frac{1}{7} + \frac{2}{5} - \frac{1}{3}\right]$$

$$= 176\sqrt{2} - 64$$

Compare with $n\sqrt{2} - 64$

$\mathrm{n} = 176$

Hence, the answer is 176.

Q. 26 If α denotes the number of solutions of $|1-i|^x = 2^x$ and $\beta = \left(\frac{|z|}{\arg(z)}\right)$, where $z = \frac{\pi}{4}(1+i)^4\left[\frac{1-\sqrt{\pi}i}{\sqrt{\pi}+i} + \frac{\sqrt{\pi}-i}{1+\sqrt{\pi}i}\right], i = \sqrt{-1}$, then the distance of the point (α, β) from the line $4x - 3y = 7$ is_____.

Correct Answer:
3

Solution:

$$|1-i|^x = 2^x \quad \text{and } \beta = \frac{|z|}{\arg(z)}$$

$$\text{where } z = \frac{\pi}{4}(1+i)^4\left[\frac{1-\sqrt{\pi}\mathrm{i}}{\sqrt{\pi}+i} + \frac{\sqrt{\pi}-i}{1+\sqrt{\pi}i}\right]$$

$$\because |1-i|^x = 2^x \Rightarrow (\sqrt{2})^x = 2^x \Rightarrow x = 0 \Rightarrow \alpha = 1$$

$$z = \frac{\pi}{4}(1+i)^4\left[\frac{1}{i}\left(\frac{1-\sqrt{\pi}i}{1-\sqrt{\pi}i}\right) + \frac{1}{i}\left(\frac{\sqrt{\pi}-i}{\sqrt{\pi}-i}\right)\right]$$

$$= \frac{\pi}{4}(1+i)^4\left(\frac{2}{i}\right)$$

$$= \frac{\pi}{2}\left((1+i)^2\right)^2\frac{1}{i} = \frac{\pi}{2}(2i)^2\frac{1}{i}$$

$$= \frac{\pi}{2}\frac{(4i^2)}{i}$$

$$z = 2\pi i$$

$$\beta = \frac{|z|}{\arg(z)} = \frac{2\pi}{\pi/2} = 4$$

distance from point $(1, 4)$ to the line $4x - 3y = 7$

$= \frac{|4\times1-3\times4-7|}{\sqrt{9+16}} = \frac{15}{5} = 3$

Hence, the answer is 3.

Q. 27 The total number of words (with or without meaning) that can be formed out of the letters of the word 'DISTRIBUTION' taken four at a time, is equal to

Correct Answer:
3734

Solution:
DISTRIBUTION

Total letters = 12 I I I 3 S 1 U 1

T T 2 R 1 O 1

D 1 B 1 N 1

Case I: 3 alike and 1 different $= {}^{1}C_1 \times {}^{8}C_1 \times \frac{4!}{3!} = 32$

Case II: 2 alike and 2 another alike $= {}^{2}C_2 \times \frac{4!}{2!2!} = 6$

Case III: 2 alike and 2 different $= {}^{2}C_1 \times {}^{8}C_2 \times \frac{4!}{2!} = 672$

Case IV: 4 different $= {}^{9}C_4 \times 4! = 3024$

The total number of words that can be formed

= 32 + 6 + 672 + 3024 = 3734

Hence, the answer is 3734.

Q. 28 In the expansion $(1 + x)\left(1 - x^2\right)\left(1 + \frac{3}{x} + \frac{3}{x^2} + \frac{1}{x^3}\right)^5, x \neq 0$, then sum of the coefficients of x^3 and x^{-13} is equal to_____.

Correct Answer:
118

Solution:

$$(1+x)\left(1-x^2\right)\left(1+\frac{3}{x}+\frac{3}{x^2}+\frac{1}{x^3}\right)^5$$

$$(1+x)\left(1-x^2\right)\left(1+\frac{1}{x}\right)^{15}$$

$$\frac{(1+x)\left(1-x^2\right)(1+x)^{15}}{x^{15}}=\frac{(1-x)(1+x)^{17}}{x^{15}}$$

$$=\frac{(1+x)^{17}-x(1+x)^{17}}{x^{15}}$$

Coefficient of $x^3=0-1=-1$

Coefficient of x^{-13} = Coefficient of $x^2=(1+x)^{17}-x(1+x)^{17}$

$={}^{17}\mathrm{C}_2-{}^{17}\mathrm{C}_1$

$=17\times 8-17=119$

Sum of coefficient of x^3 and $x^{-13}=-1+119=118$

Hence, the answer is 118.

Q. 29 Let $\vec{a}$ and $\vec{b}$ be two vectors such that $|\vec{a}|=1, |\vec{b}|=4$, and $\vec{a}\cdot\vec{b}=2$. If $\vec{c}=(2\vec{a}\times\vec{b})-3\vec{b}$ and the angle between $\vec{b}$ and $\vec{c}$ is α, then $192\sin^2\alpha$ is equal to____.

Correct Answer:
48

Solution:
$|\vec{a}|=1, |\vec{b}|=4$ and $\vec{a}\cdot\vec{b}=2$

$\vec{b}\cdot\vec{c}=(2\vec{a}\times\vec{b})\cdot\vec{b}-3|\vec{b}|^2$

$|\vec{b}||\vec{c}|\cos\alpha=-3|\vec{b}|^2$

$|c|\cos\alpha=-12 \quad \ldots\ldots(1)$

$\because |\vec{b}|=4$

$\vec{a}\cdot\vec{b}=2$

$|\vec{a}||\vec{b}|\cos\theta=2$

$\cos\theta=\frac{1}{2}\Rightarrow\theta=\frac{\pi}{3}$

$|\vec{c}|^2 = |(2\vec{a} \times \vec{b}) - 3\vec{b}|^2$

$= |2\vec{a} \times \vec{b}|^2 + 9|\vec{b}|^2 - 6(2\vec{a} \times \vec{b}).\vec{b}$

$= 4|a|^2|\vec{b}|^2 \sin^2\theta + 9 \times 16 - 0$

$= 4 \times 1 \times 16 \times \frac{3}{4} + 144$

$= 48 + 144$

$|\vec{c}|^2 = 192$

From (1)

$|c|^2 \cos^2\alpha = 144$

$192\cos^2\alpha = 144$

$192\left(1 - \sin^2\alpha\right) = 144$

$192 - 192\sin^2\alpha = 144$

$192\sin^2\alpha = 48$

Hence, the answer is 48.

Q. 30 Let $f : \mathbb{R} \to \mathbb{R}$ be a function defined by $f(x) = \frac{4^x}{4^x+2}$ and $M = \int_{f(a)}^{f(1-a)} x\sin^4(x(1-x))dx$, $N = \int_{f(a)}^{f(1-a)} \sin^4(x(1-x))dx; a \neq \frac{1}{2}$. If $\alpha M = \beta N, \alpha, \beta \in \mathbb{N}$, then the least value of $\alpha^2 + \beta^2$ is equal to_____.

Correct Answer:
5

Solution:

$\because f(x) = \frac{4^x}{4^x+2}$

$f(1-x) = \frac{4^{1-x}}{4^{1-x}+2}$

$f(x) + f(1-x) = 1$

$\therefore f(a) + f(1-a) = 1$

$M = \int_{f(a)}^{f(1-a)} x\sin^4(x(1-x))dx$

$$M = \frac{1}{2}\int_{f(a)}^{f(1-a)} \sin^4(x(1-x))dx \quad \text{(Using elimination of } x \text{)}$$

$$M = \frac{N}{2} \Rightarrow 2M = N$$

$$\alpha M = \beta N$$

$$\alpha = 2 \& \beta = 1$$

$$\alpha^2 + \beta^2 = 4 + 1 = 5$$

Hence, the answer is 5.

CAREERS360

31 Jan - Evening Shift

Physics

Q. 1 The speed of sound in oxygen at S.T.P. will be approximately :
(given, $\mathrm{R} = 8.3\mathrm{JK}^{-1}, \gamma = 1.4$)

Option 1:
$325\ \mathrm{m/s}$

Option 2:
$310\ \mathrm{m/s}$

Option 3:
$341\ \mathrm{m/s}$

Option 4:
$333\ \mathrm{m/s}$

Correct Answer:
$310\ \mathrm{m/s}$

Solution:
Molecular weight in terms of $\mathrm{kg/mol} = 32 \times 10^{-3}\ \mathrm{kg/mol}$

$$v = \sqrt{\frac{\gamma \mathrm{RT}}{\mathrm{M}}}$$

$$v = \sqrt{\frac{1.4 \times 8.3 \times 273}{32 \times 10^{-3}}}$$

$$v = 314.85\ \mathrm{m/s}$$

Hence, the answer is the option (2).

Q. 2 Consider two physical quantities A and B related to each other as $E = \frac{B-x^2}{At}$ where E, x and t have dimensions of energy, length and time respectively. The dimension of A B is:

Option 1:
$\mathrm{L^{-2}M^{-1}\ T^{1}}$

Option 2:
$\mathrm{L^{-2}M^{1}\ T^{0}}$

Option 3:
$\mathrm{L^{0}M^{-1}\ T^{1}}$

Option 4:
$\mathrm{L^{2}M^{-1}\ T^{1}}$

Correct Answer:
$\mathrm{L^{2}M^{-1}\ T^{1}}$

Solution:
$E = \frac{B-x^2}{At}$

Dimension of B = dimension of $\mathrm{x^2} = [\mathrm{L}]^2$

Then $E = \frac{B-x^2}{\mathrm{At}}$

$\mathrm{A} = \frac{\mathrm{x^2}}{\mathrm{Et}} = \frac{\mathrm{m^2}}{\mathrm{kg \cdot m^2 \cdot sec^{-2} \cdot sec}}$

$= \frac{[\mathrm{L}]^2}{[\mathrm{ML^2\ T^{-1}}]} = [\mathrm{M^{-1}\ T^{1}}]$

Dimension of $AB = [\mathrm{M^{-1}\ T^{1}}]\ [\mathrm{L^2}] = [\mathrm{M^{-1}\ L^2\ T^{1}}]$

Hence, the answer is the option (4).

Q. 3 A small spherical ball of radius r, falling through a viscous medium of negligible density has terminal velocity ' v '. Another ball of the same mass but of radius 2 r, falling through the same viscous medium will have terminal velocity:

Option 1:
$\frac{\mathrm{v}}{2}$

Option 2:
$4\mathrm{v}$

Option 3:
$\frac{\mathrm{v}}{4}$

Option 4:
$2\mathrm{v}$

Correct Answer:
$\frac{\mathrm{v}}{2}$

Solution:
$\mathrm{V_T} = \frac{2\mathrm{r}^2}{9\eta}(\rho_0 - \rho)g \quad [\rho_0 \rightarrow$ density of material and $\rho \rightarrow$ density of fluid $]$

$\mathrm{V_T} = \frac{2}{9\eta}\mathrm{r}^2\,(\rho_0)\mathrm{g}$

Mass is same given $\mathrm{m} = \frac{4}{3}\pi\mathrm{r}^3\rho_0$

$\mathrm{V_T} = \frac{2}{9\eta}\mathrm{m}\frac{3}{4\pi\mathrm{r}^3}\cdot\mathrm{r}^2\,\mathrm{g}$

$\mathrm{V_T} = \left(\frac{2}{9\eta}\mathrm{m} \times \frac{3}{4\pi}\cdot\mathrm{g}\right)\cdot\frac{1}{\mathrm{r}}$

From here $V_T \propto \frac{1}{r}$

It radius is double then velocity becomes half $= \mathrm{V}/2$

Hence, the answer is the option (1).

Q. 4 A uniform magnetic field of $2 \times 10^{-3}\ \mathrm{T}$ acts along positive Y-direction. A rectangular loop of sides $20\ \mathrm{cm}$ and $10\ \mathrm{cm}$ with current of $5\ \mathrm{A}$ is in $\mathrm{Y}-\mathrm{Z}$ plane. The current is in anticlockwise sense with reference to negative X axis. Magnitude and direction of the torque is:

Option 1:
$2 \times 10^{-4}\ \mathrm{N}-\mathrm{m}$ along positive Y-direction

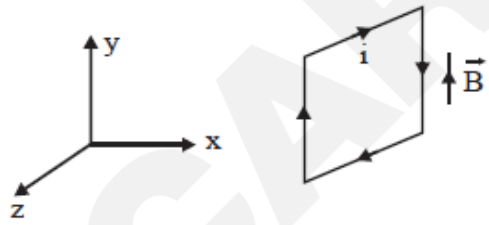
Option 2:
$2 \times 10^{-4}\ \mathrm{N}-\mathrm{m}$ along positive Z-direction

Option 3:
$2 \times 10^{-4}\ \mathrm{N}-\mathrm{m}$ along positive X-direction

Option 4:
$2 \times 10^{-4}\ \mathrm{N}-\mathrm{m}$ along negative Z-direction

Correct Answer:
$2 \times 10^{-4}\ \mathrm{N}-\mathrm{m}$ along negative Z-direction

Solution:



$$\vec{\mathrm{M}} = \vec{\mathrm{iA}}$$

$$= 5 \times 0.2 \times 0.1(-\hat{\mathrm{i}})$$

$$= 0.1(-\hat{\mathrm{i}})$$

$$\vec{\tau} = \vec{\mathrm{M}} \times \vec{\mathrm{B}}$$

$$= 0.1(-\hat{\mathrm{i}}) \times 2 \times 10^{-3}(\hat{\mathrm{j}})$$

$= 2 \times 10^{-4}(-\hat{k})\mathrm{N}-\mathrm{m}$

Q. 5 Force between two point charges q_1 and q_2 placed in vacuum at ' r ' cm apart is F. Force between them when placed in a medium having dielectric constant $\mathrm{K}=5$ at ' $\mathrm{r}/5$ ' cm apart will be:

Option 1:
$\mathrm{F}/5$

Option 2:
$25\ \mathrm{F}$

Option 3:
$F/25$

Option 4:
$5\ \mathrm{F}$

Correct Answer:
$5\ \mathrm{F}$

Solution:
$\mathrm{F}=\frac{1}{4\pi\varepsilon_0}\frac{\mathrm{q_1q_2}}{\mathrm{r^2}}$

$\text{In medium } \mathrm{F'}=\frac{1}{4\pi(\mathrm{k}\varepsilon_0)}\frac{\mathrm{q_1q_2}}{\mathrm{r'^2}}$

$\mathrm{F'}=\frac{1}{4\pi\varepsilon_0\cdot\mathrm{k}}\frac{\mathrm{q_1q_2}}{\mathrm{r^2}/25}$

$\mathrm{F'}=\frac{25\mathrm{q_1q_2}}{4\pi\varepsilon_0\mathrm{r^2}\times 5}$

$\mathrm{F'}=5\ \mathrm{F}$

Hence, the answer is the option (4).

Q. 6 The resistance per centimeter of a meter bridge wire is r, with $\mathrm{X}\Omega$ resistance in left gap. Balancing length from left end is at $40\ \mathrm{cm}$ with 25Ω resistance in right gap. Now the wire is replaced by another wire of $2\mathrm{r}$ resistance per centimeter. The new balancing length for same settings will be at

Option 1:
$10\ \mathrm{cm}$

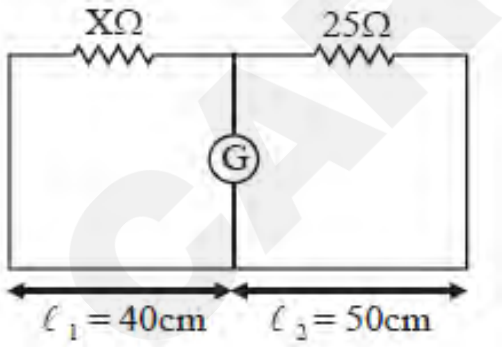
Option 2:
$80\ \mathrm{cm}$

Option 3:
$20\ \mathrm{cm}$

Option 4:
$40\ \mathrm{cm}$

Correct Answer:
$40\ \mathrm{cm}$

Solution:



$\frac{25}{\mathrm{r}\ell_1} = \frac{\mathrm{x}}{\mathrm{r}\ell_2}$.....(1)

$\frac{25}{2\mathrm{r}\ell_1'} = \frac{\mathrm{x}}{2\mathrm{r}\ell_2'}$.....(2)

using (1) & (2) $\ell_1' = \ell_2 = 40\ \mathrm{cm}$

Q. 7 The mass number of nucleus having radius equal to half of the radius of nucleus with mass number 192 is:

Option 1:
24

Option 2:
20

Option 3:
40

Option 4:
32

Correct Answer:
24

Solution:

$R_1 = \frac{R_2}{2}$

$R_0(A_1)^{1/3} = \frac{R_0}{2}(A_2)^{1/3}$

$A_1 = \frac{1}{8}A_2$

$A_1 = \frac{192}{8}$

$A_1 = 24$

Hence, the answer is the option (1).

Q. 8 In a photoelectric effect experiment a light of frequency 1.5 times the threshold frequency is made to fall on the surface of photosensitive material. Now if the frequency is halved and intensity is doubled, the number of photo electrons emitted will be:

Option 1:
Zero

Option 2:
halved

Option 3:
doubled

Option 4:
quadrupled

Correct Answer:
quadrupled

Solution:
Since $\frac{f}{2} < f_0$

So current = 0

When the frequency of incident light is halved of its original value i.e. 1.5 v_0 then it becomes less than the threshold value. In that case, no photo-electric effect takes place. No Photo electrons would be emitted.

Hence, the answer is the option (4).

Q. 9 If two vectors $\vec{A}$ and $\vec{B}$ having equal magnitude R inclined at an angle θ, then

Option 1:
$|\overrightarrow{A}-\overrightarrow{B}| = 2R\cos\left(\frac{\theta}{2}\right)$

Option 2:
$|\overrightarrow{A}+\overrightarrow{B}| = 2R\cos\left(\frac{\theta}{2}\right)$

Option 3:
$|\overrightarrow{A}-\overrightarrow{B}| = \sqrt{2}R\sin\left(\frac{\theta}{2}\right)$

Option 4:

$|\vec{A}+\vec{B}|=2R\sin\left(\frac{\theta}{2}\right)$

Correct Answer:

$|\overrightarrow{\mathrm{A}}+\overrightarrow{\mathrm{B}}|=2\mathrm{R}\cos\left(\frac{\theta}{2}\right)$

Solution:

$R'=|\vec{A}+\vec{B}|=\sqrt{R^2+R^2+2R^2\cos\theta}$

$\mathrm{a}=\mathrm{b}=\mathrm{R}$

$R'=|\vec{A}+\vec{B}|=R\sqrt{2}\sqrt{1+\cos\theta}=\sqrt{2}R\sqrt{2\cos^2\theta/2}=2R\cos\theta/2$

Hence, the answer is the option (2).

Q. 10 When unpolarized light is incident at an angle of 60° on a transparent medium from air, the reflected ray is completely polarized. The angle of refraction in the medium is:

Option 1:

45°

Option 2:

90°

Option 3:

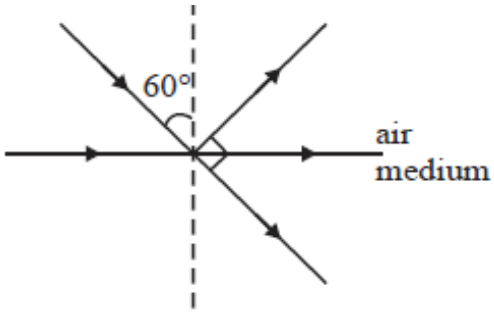
30°

Option 4:

60°

Correct Answer:
30°

Solution:
By brewster law



at complete reflection refracted ray and reflected ray are perpendicular

so $r = 30^{\circ}$

Q. 11 A gas mixture consists of 8 moles of argon and 6 moles of oxygen at temperature T. Neglecting all vibrational modes, the total internal energy of the system is:

Option 1:
27 RT

Option 2:
20 RT

Option 3:
29 RT

Option 4:
21 RT

Correct Answer:
27 RT

Solution:
$\Delta U = nC_vT$

$\Delta U = n_1Cv_1\ T + n_2Cv_2\ T$

$\Delta U = 8 \times \frac{3}{2}RT + 6 \times \frac{5}{2}RT$

$\Delta U = 12RT + 15RT$

$\Delta U = 27RT$

Hence, the answer is the option (1).

Q. 12 Given below are two statements:
Statement I: Electromagnetic waves carry energy as they travel through space and this energy is equally shared by the electric and magnetic fields.
Statement II: When electromagnetic waves strike a surface, a pressure is exerted on the surface.
In the light of the above statements, choose the most appropriate answer from the options give below:

Option 1:
Statement I is incorrect but Statement II is correct

Option 2:
Both statement I and Statement II are correct

Option 3:
Statement I is correct but statement II is incorrect

Option 4:
Both statement I and Statement II are incorrect.

Correct Answer:
Both statement I and Statement II are correct

Solution:
$\frac{1}{2}\varepsilon_0 E^2 = \frac{B^2}{2\mu_0}$

$\because E = CB \text{ and } C = \frac{1}{\mu_0 E_0}$

Hence, the answer is the option (2).

Q. 13 An AC voltage $V = 20 \sin 200\pi t$ is applied to a series LCR circuit which drives a current $I = 10 \sin\left(200\pi t + \frac{\pi}{3}\right)$.
The average power dissipated is:

Option 1:
$200\ \text{W}$

Option 2:
$50\ \text{W}$

Option 3:
$173.2\ \text{W}$

Option 4:
$21.6\ \text{W}$

Correct Answer:
$50\ \text{W}$

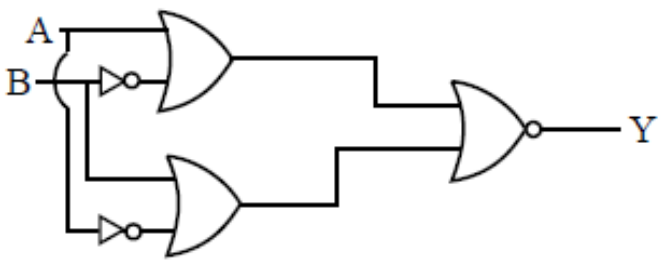
Solution:
$\langle \text{P} \rangle = \text{V}_{\text{rms}}\text{I}_{\text{rms}} \cos\phi$

$= \frac{10}{\sqrt{2}} \times \frac{20}{\sqrt{2}} \cos 60^\circ$

$= \frac{20 \times 10}{2 \times 2} = 50\ \text{W}$

Hence, the answer is the option (2).

Q. 14



The output of the given circuit diagram is:

Option 1:

A	B	Y
0	0	0
1	0	1
0	1	1
1	1	0

Option 2:

A	B	Y
0	0	0
1	0	0
0	1	0
1	1	0

Option 3:

A	B	Y
0	0	0
1	0	0
0	1	1
1	1	0

CAREERS360

Option 4:

A	B	Y
0	0	0
1	0	0
0	1	0
1	1	1

Correct Answer:

A	B	Y
0	0	0
1	0	0
0	1	0
1	1	0

Solution:

$$= \overline{(A + \overline{B}) + (B + \overline{A})}$$

$$= \overline{(A + \overline{B})}\,\overline{(B + \overline{A})}$$

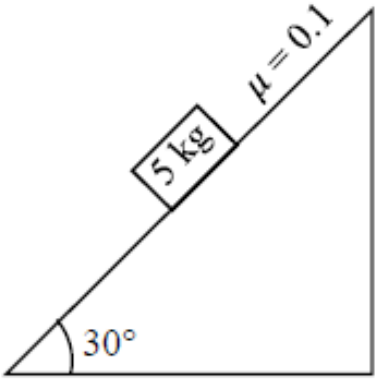
$$= (\overline{A} \cdot \overline{B}) \cdot (\overline{B} \cdot \overline{A})$$

$$= (\overline{A} \cdot B) \cdot (\overline{B} \cdot A)$$

$$= 0$$

CAREERS360

Q. 15



A block of mass $5\ \text{kg}$ is placed on a rough inclined surface as shown in the figure. If $\vec{F}_1$ is the force required to just move the block up the inclined plane and $\vec{F}_2$ is the force required to just prevent the block from sliding down, then the value of $\vec{F}_1 - \vec{F}_2$ is ; [Use $g = 10\ \text{m/s}^2$]

Option 1:
$25\sqrt{3}\ \text{N}$

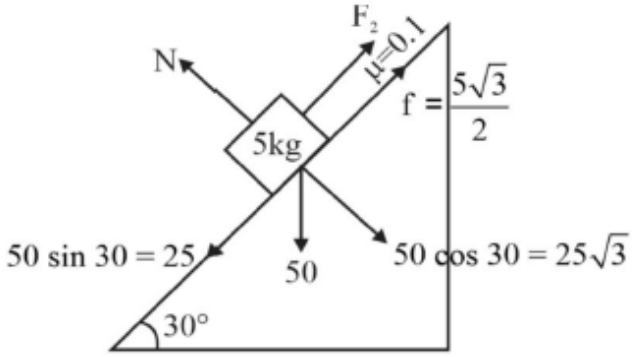
Option 2:
$\frac{5\sqrt{3}}{2}\ \text{N}$

Option 3:
$50\sqrt{3}\ \text{N}$

Option 4:
$10\ \text{N}$

Correct Answer:
$\frac{5\sqrt{3}}{2}\ \text{N}$

Solution:
Bonus
Case – I



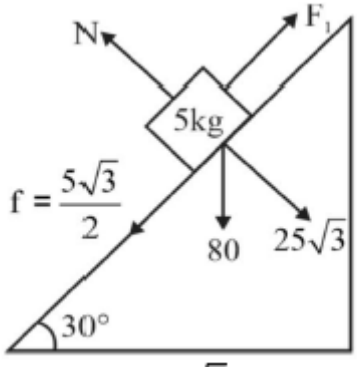
$$f_{max} = \mu N$$

$$f_{max} = 2.5\sqrt{3}$$

$$f_{max} = \frac{5\sqrt{3}}{2}$$

$$F_2 = 25 - \frac{5\sqrt{3}}{2}$$

Case - II



$$F_1 = 25 + \frac{5\sqrt{3}}{2}$$

$$|F_1| - |F_2| = \frac{10\sqrt{3}}{2}$$

$$= 5\sqrt{3}$$

Q. 16 A body of mass $2\ \text{kg}$ begins to move under the action of a time dependent force given by $\vec{F} = \left(6t + 6t^2\hat{j}\right)\text{N}$. The power developed by the force at time t is given by -

Option 1:
$\left(9t^5+6t^3\right)\mathrm{W}$

Option 2:
$\left(6t^4+9t^5\right)\mathrm{W}$

Option 3:
$\left(9t^3+6t^5\right)\mathrm{W}$

Option 4:
$\left(3t^3+6t^5\right)\mathrm{W}$

Correct Answer:
$\left(9t^3+6t^5\right)\mathrm{W}$

Solution:

$F=6t\hat{i}+6t^2\hat{j}$

$F=ma$

$F=m\frac{dv}{dt}$

$\left(6t+6t^2\right)dt=2dv$

$\frac{6t^2}{2}+\frac{6t^3}{3}=2V$

$3t^2+2t^3=2V$

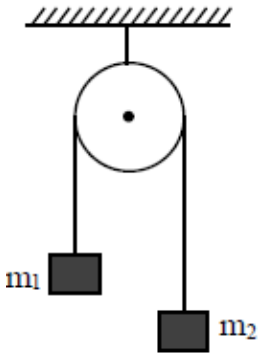
$V=\frac{3}{2}t^2\hat{i}+t^3\hat{j}$

$P=f.V$

$P=\left(9t^3+6t^5\right)W$

Hence, the answer is (3).

Q. 17 A light string passing over a smooth light fixed pulley connects two blocks of masses m_1 and m_2. If the acceleration of the system is $g/8$, then the ratio of masses is:



Option 1:
$\frac{9}{7}$

Option 2:
$\frac{5}{3}$

Option 3:
$\frac{8}{1}$

Option 4:
$\frac{4}{3}$

Correct Answer:
$\frac{9}{7}$

Solution:
$a = \frac{(m_1-m_2)g}{m_1+m_2} = \frac{g}{8}$

$8m_1 - 8m_2 = m_1 + m_2$

$7m_1 = 9m_2$

$\frac{m_1}{m_2} = \frac{9}{7}$

Q. 18 By what percentage will the illumination of the lamp decrease if the current drops by 20%?

Option 1:
56%

Option 2:
36%

Option 3:
46%

Option 4:
26%

Correct Answer:
36%

Solution:

$P_1 = i^2R$

$P_2 = (0.8\mathrm{I})^2R$

$P_2 = 0.64I^2R$

$P_2 = 0.64P_1$

$\%\text{ drop of power } = \frac{P_2-P_1}{P_1} \times 100$

$= \frac{(0.64-1)P_1}{P_1} \times 100$

$= -.36 \times 100 = 36\%\text{ drop}$

Hence, the answer is the option (2).

Q. 19 The measured value of the length of a simple pendulum is 20 cm with 2 mm accuracy. The time for 50 oscillations was measured to be 40 seconds with 1 second resolution. From these measurements, the accuracy in the measurement of acceleration due to gravity is N%. The value of N is

Option 1:
8

Option 2:
6

Option 3:
5

Option 4:
4

Correct Answer:
6

Solution:
Given the formula for the period of a pendulum:

$$T = 2\pi\sqrt{\frac{l}{g}}$$

We can rearrange this formula to solve for g:

$$g = \frac{4\pi^2 l}{T^2}$$

The relative uncertainty in g can be expressed in terms of the relative uncertainties in l and T:

$$\frac{\Delta g}{g} = \frac{\Delta l}{l} + 2\frac{\Delta T}{T}$$

Given the following values:
- Length $l = 20\,\text{cm}$ with an uncertainty of $\Delta l = 2\,\text{mm} = 0.2\,\text{cm}$
- Period T with a relative uncertainty of $\frac{\Delta T}{T} = \frac{1}{40}$

Let's substitute these into our uncertainty formula:

$$\frac{\Delta g}{g} = \frac{0.2}{20} + 2\left(\frac{1}{40}\right)$$

Calculate the individual components:

$$\frac{0.2}{20} = 0.01$$

$$2\left(\frac{1}{40}\right) = \frac{2}{40} = 0.05$$

Add these components together:

$$\frac{\Delta g}{g} = 0.01 + 0.05 = 0.06$$

Convert this to a percentage:

$$\text{Percentage change} = 0.06 \times 100 = 6\%$$

Hence, the answer is the option (2).

Q. 20 The mass of the moon is $\frac{1}{144}$ times the mass of a planet and its diameter is $\frac{1}{16}$ times the diameter of a planet. If the escape velocity on the planet is v, the escape velocity on the moon will be:

Option 1:
$\frac{v}{6}$

Option 2:
$\frac{\mathrm{V}}{12}$

Option 3:
$\frac{\mathrm{v}}{4}$

Option 4:
$\frac{\mathrm{V}}{3}$

Correct Answer:
$\frac{\mathrm{V}}{3}$

Solution:
from the escaoe velocity relation :

$$V_{\text{escape}} = \sqrt{\frac{2GM}{R}} = \frac{V_{\text{moon}}}{V_{\text{planet}}} = \frac{\sqrt{\frac{2G_m}{R_m}}}{\sqrt{\frac{2G_p}{R_p}}} = \sqrt{\frac{M_m}{M_p}}\sqrt{\frac{R_p}{R_m}}$$

$$\frac{V_{\text{moon}}}{V} = \sqrt{\frac{1}{144}}\sqrt{\frac{16}{1}}$$

$V_{\text{moon}} = \frac{4V}{12} \Rightarrow \quad V_{\text{moon}} = \frac{V}{3}$

Hence, the answer is the option (4).

Q. 21 Two circular coils P and Q of 100 turns each have same radius of πcm. The currents in P and R are $1\ \text{A}$ and $2\ \text{A}$ respectively. P and Q are placed with their planes mutually perpendicular with their centers coincide. The resultant magnetic field induction at the centre of the coils is $\sqrt{\text{x}}\text{mT}$, where x=_____.$\left[\text{Use } \mu_0 = 4\pi \times 10^{-7}\text{TmA}^{-1}\right]$

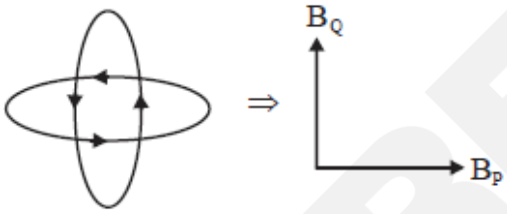
Correct Answer:
20

Solution:

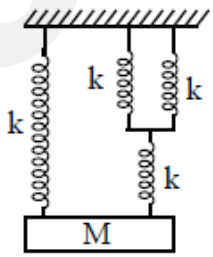
$\text{B}_\text{p} = \frac{\mu_0 \text{Ni}_1}{2\text{r}} = \frac{\mu_0 \times 1 \times 100}{2\pi} = 2 \times 10^{-3}\ \text{T}$

$\text{B}_\text{Q} = \frac{\mu_0 \text{Ni}_2}{2\text{r}} = \frac{\mu_0 \times 2 \times 100}{2\pi} = 4 \times 10^{-3}\ \text{T}$

$\text{B}_\text{net} = \sqrt{\text{B}_\text{p}^2 + \text{B}_\text{Q}^2} = \sqrt{20}\text{MT} \Rightarrow \text{x} = 20$



Q. 22 The time period of simple harmonic motion of mass M in the given figure is $\pi\sqrt{\frac{\alpha M}{5k}}$, where the value of α is _____.



Correct Answer:
12

Solution:

$K_{eq} = \frac{2K}{3} + K = \frac{5K}{3}$

$T(\text{ Time period }) = 2\pi\sqrt{\frac{M}{K_{eq}}} = \pi\sqrt{\frac{12\,m}{5\,K}}$

$a = 12$

Q. 23 Light from a point source in air falls on a convex curved surface of radius 20 cm and refractive index 1.5. If the source is located at 100 cm from the convex surface, the image will be formed at _____ cm from the object.

Correct Answer:

200

Solution:

By using Len's maker formula, we get-

$\frac{\mu_2}{V} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

$\frac{1.5}{V} - \left(\frac{1}{-100}\right) = \frac{1.5-1}{20}$

$\frac{1.5}{V} + \frac{1}{100} = \frac{1}{40}$

$\frac{1.5}{V} = \frac{1}{40} - \frac{1}{100}$

$\frac{1.5}{V} = \frac{5-2}{200}$

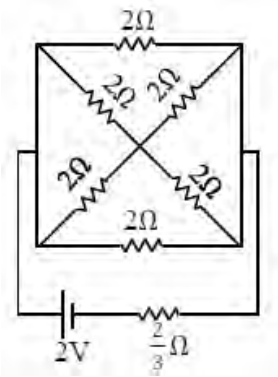
$V = \frac{200 \times 1.5}{3}$

$V = 100\ cm$

so distance from object $= 200\ cm$

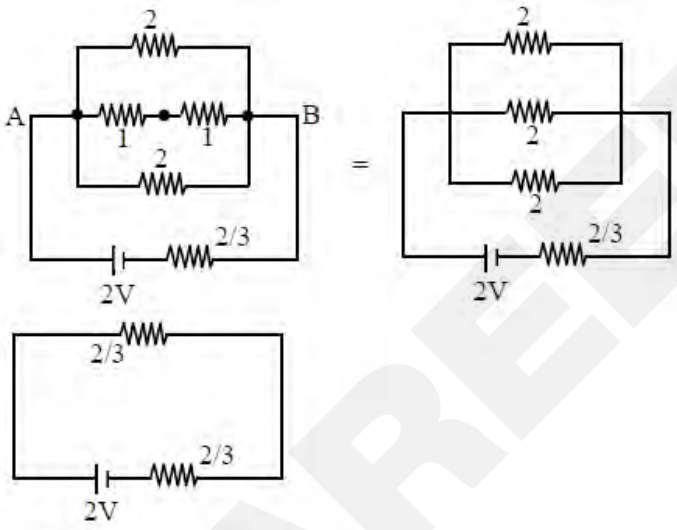
Hence, the answer is 200cm.

Q. 24 In the following circuit, the battery has an emf of $2\ \mathrm{V}$ and an internal resistance of $\frac{2}{3}\Omega$. The power consumption in the entire circuit is _____W.



Correct Answer:
3

Solution:



$\Rightarrow \mathrm{R_{eq}} = \frac{4}{3}$

$\mathrm{P} = \frac{\mathrm{V}^2}{\mathrm{R}} = \frac{(2)^2}{4/3} = \frac{4}{4/3}$

$\mathrm{P} = 3\ \mathrm{W}$

Q. 25 The magnetic flux ϕ (in weber) linked with a closed circuit of resistance 8Ω varies with time (in seconds) as $\phi = 5t^2 - 36t + 1$. The induced current in the circuit at $t = 2$ s is_____ A.

Correct Answer:
2

Solution:
According to Faraday's Law,

$\varepsilon = -\frac{\mathrm{d}\phi}{\mathrm{dt}}$

$= \frac{-\mathrm{d}\phi}{\mathrm{dt}}\left(5\mathrm{t}^2 - 36\mathrm{t} + 1\right)$

$\varepsilon = -[10\mathrm{t} - 36]$

Now at $\mathrm{t} = 2\mathrm{sec}$

$\varepsilon = -[10 \times 2 - 36]$

$\mathrm{q} = -[20 - 36]$

$\varepsilon = 16$

Now $i = \frac{\varepsilon}{R}$

$\mathrm{i} = \frac{16}{8}$

$\Rightarrow \mathrm{i} = 2\ \mathrm{A}$

Hence, the answer is 2A.

Q. 26 A nucleus has mass number A_1 and volume V_1. Another nucleus has mass number A_2 and volume V_2. If relation between mass number is $A_2 = 4A_1$. Then $\frac{V_2}{V_1}$ =_____.

Correct Answer:
4

Solution:
The volume of the nucleus is given as :

$\mathrm{V} = \frac{4}{3}\pi\mathrm{R}^3$

$\mathrm{R} = \mathrm{R}_0(\ \mathrm{A})^{\frac{1}{3}}$

$\mathrm{V} = \frac{4}{3}\pi\mathrm{R}_0^3\ \mathrm{A}$

$\frac{V_2}{V_1} = \frac{A_2}{A_1} = 4$

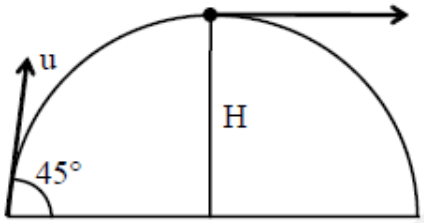
Hence, the answer is 4.

Q. 27 A body of mass ' m ' is projected with a speed ' u ' making an angle of 45° with the ground. The angular momentum of the body about the point of projection at the highest point it expressed as $\frac{\sqrt{2}mu^3}{Xg}$. The value of ' X ' is_____.

Correct Answer:
8

Solution:
$H = \frac{u^2 \sin^2\theta}{2g} = \frac{u^2}{4g}$

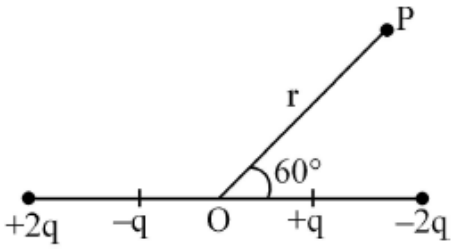
Angular momentum $= mvH$



$= m \cdot \frac{u}{\sqrt{2}} \cdot \frac{u^2}{4g} = \frac{mu^3}{4\sqrt{2}g} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}mu^3}{8g}$

$x = 8$

Q. 28 The distance between charges $+q$ and $-q$ is $2l$ and between $+2q$ and $-2q$ is $4l$. The electrostatic potential at point P at a distance r from center O is $-\alpha\left[\frac{ql}{r^2}\right] \times 10^9$ V, where the value of α is _____.... (Use $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 Nm^2C^{-2}$)



Correct Answer:
27

Solution:
due to q

$$V_q = \frac{kP\cos\theta}{r^2} = \frac{K(q\cdot 2\ell)\times\frac{1}{2}}{r^2}$$

$$V_q = \frac{Kq\ell}{r^2} \ldots (i)$$

Due to 2q

$$V_{2q} = \frac{K[2q(4\ell)]\cos 120^\circ}{r^2} = \frac{8Kq\ell}{r^2}\cdot\left(-\frac{1}{2}\right)$$

$$V_{2q} = \frac{-4Kq\ell}{r^2} \ldots (ii)$$

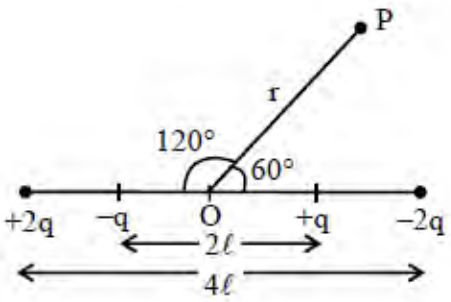
Using (i) and (ii)

$$V_{net} = \frac{Kq\ell}{r^2} - \frac{4Kq\ell}{r^2} = \frac{-3Kq\ell}{r^2}$$

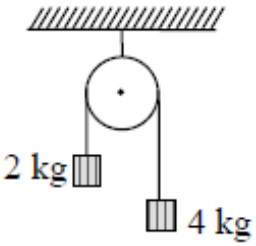
$$V_{net} = -3\left[\frac{q\ell}{r^2}\right] \times 9 \times 10^9$$

$$= -27\left[\frac{q\ell}{r^2}\right] \times 10^9 r$$

$$\alpha = 27$$



Q. 29 Two blocks of mass $2\ \text{kg}$ and $4\ \text{kg}$ are connected by a metal wire going over a smooth pulley as shown in figure. The radius of wire is $4.0 \times 10^{-5}\ \text{m}$ and Young's modulus of the metal is $2.0 \times 10^{11}\ \text{N/m}^2$. The longitudinal strain developed in the wire is $\frac{1}{\alpha\pi}$. The value of α is _____.[Use $= 10\ \text{m/s}^2$]



Correct Answer:
12

Solution:
$\text{r} = 4 \times 10^{-5}$

$\text{Y} = 2 \times 10^{11}$

$\text{g} = 10$

$40 - \text{T} = 4\text{a}$

$\text{T} - 20 = 2\text{a}$

$\overline{20 = 6\text{a}}$

$\text{a} = \frac{10}{3}\ \text{m/sec}^2$

And

$40 - T = 4a$

$40 - 4a = T$

$T = 40 - 4 \times \frac{10}{3}$

$T = 40 - \frac{40}{3}$

$T = \frac{120-40}{3}$

$T = \frac{80}{3}\ N$

$Y = \frac{F/A}{\frac{\Delta \ell}{\ell}}$

$\frac{\Delta \ell}{\ell} = \frac{F}{AY}$

$\frac{\Delta \ell}{\ell} = \frac{80/3}{2 \times 10^{11} \pi r^2}$

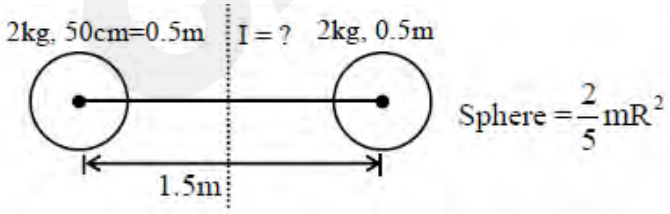
$\frac{\Delta \ell}{\ell} = \frac{80}{3 \times 2 \times 10^{11} \times \pi \times (4 \times 10^{-5})^2}$

$\frac{\Delta \ell}{\ell} = \frac{1}{12\pi} = 12$

Q. 30 Two identical spheres each of mass $2\ kg$ and radius $50\ cm$ are fixed at the ends of a light rod so that the separation between the centers is $150\ cm$. Then, moment of inertia of the system about an axis perpendicular to the rod and passing through its middle point is $\frac{x}{20}\ kg\ m^2$, where the value of x is

Correct Answer:
53

Solution:



$I = I_1 + I_2$

$I = \left(\frac{2}{5}mR^2 + md^2\right) \times 2$

$I = \left[\frac{2}{5} \times 2 \times (0.5)^2 + 2 \times (0.75)^2\right] \times 2$

$I = \left[\frac{2}{5} \times 2 \times 0.25 + 2 \times 0.5625\right] \times 2$

$I = \left[\frac{1}{5} + 1.125\right] \times 2$

$I = [0.2 + 1.125] \times 2$

$I = 2.65$

$\Rightarrow I = \frac{265}{100}$

$I = \frac{53}{20} \Rightarrow x = 53$

Chemistry

Q. 1 Choose the correct statements from the following

(A) $\mathrm{Mn_2O_7}$ is an oil at room temperature
(B) $\mathrm{V_2O_4}$ reacts with acid to give $\mathrm{VO_2}^{2+}$
(C) CrO is a basic oxide
(D) $\mathrm{V_2O_5}$ does not react with acid

Choose the correct answer from the options given below:

Option 1:
A, B and D only

Option 2:
B and C only

Option 3:
A, B and C only

Option 4:
A and C only

Correct Answer:
A and C only

Solution:
Fact

Mn_2O_7 is an oily liquid at room temperature, known for its strong oxidizing properties and high reactivity.
CrO is a basic oxide due to its ability to react with acids to form salts and water, which is a characteristic behaviour of basic oxides. This property distinguishes it from other oxides of chromium, such as Cr_2O_3 (amphoteric) and CrO_3 (acidic).

Hence, the answer is the option (4).

Q. 2 Given below are two statements:

Statement I : S_8 solid undergoes disproportionation reaction under alkaline conditions to form $S^{2-}$and $S_2O_3{}^{2-}$.

Statement II : ClO_4^-can undergo disproportionation reaction under acidic condition.

In the light of the above statements, choose the most appropriate answer from the options given below:

Option 1:
Statement I is incorrect but statement II is correct

Option 2:
Both statement I and statement II are incorrect

Option 3:
Statement I is correct but statement II is incorrect

Option 4:
Both statement I and statement II are correct.

Correct Answer:
Statement I is correct but statement II is incorrect

Solution:
In alkaline conditions, sulfur (S_8) undergoes a disproportionation reaction to form sulfide (S^{2-}) and thiosulfate ($S_2O_3^{-2}$). The balanced chemical equation for this reaction is:

$$[4S_8 + 16OH^- \rightarrow 8S^{2-} + 2S_2O_3^{2-} + 6H_2O]$$

Perchlorate ion (ClO_4^-) does not undergo disproportionation due to the high oxidation state of chlorine (+7) in ClO_4^-. Disproportionation is a type of redox reaction where a species undergoes simultaneous oxidation and reduction to form two different products. However, since chlorine is already in its highest possible oxidation state (+7) in ClO_4^-, it cannot be oxidized further, and it is very stable. Hence, ClO_4^- does not undergo disproportionation. In higher O.S. disproportionation cannot take place.

Hence, the answer is the option (3).

Q. 3 Given below are two statements:

Statement - I: Group 13 trivalent halides get easily hydrolyzed by water due to their covalent nature.

Statement - II: $AlCl_3$ upon hydrolysis in acidified aqueous solutions forms an octahedral $[Al\ (H_2O)_6]^{3+}$ ion.

In the light of the above statements, choose the correct answer from the options given below:

Option 1:
Both statement I statement II are false.

Option 2:
Statement I is false but Statement II is true.

Option 3:
Both Statement I and Statement II are false.

Option 4:
Statement I is true but statement II is false.

Correct Answer:
Both Statement I and Statement II are false.

Solution:
When aluminium chloride ($AlCl_3$) undergoes hydrolysis, it forms an octahedral complex with water molecules.

In this reaction, aluminium chloride reacts with water to form the hexaaquaaluminum ion $[Al(H_2O)_6]^{3+}$, which has an octahedral geometry. This complex ion consists of an aluminium ion surrounded by six water molecules, coordinated through their oxygen

atoms.

$$AlCl_3 + 6H_2O \rightarrow [Al(H_2O)_6]^{3+} + 3Cl^-$$

Hence, the answer is the option (3).

Q. 4 Match List I with List II

List I (Complex ion)		List II (Electronic configuration)	
A.	$[Cr(H_2O)_6]^{3+}$	I.	$t_{2g}^2 eg^0$
B.	$[Fe(H_2O)_6]^{3+}$	II.	$t_{2g}^3 eg^0$
C.	$[Ni(H_2O)_6]^{2+}$	III.	$t_{2g}^3 eg^2$
D.	$[V(H_2O)_6]^{3+}$	IV.	$t_{2g}^6 eg^2$

Choose the correct answer from the options given below:

Option 1:
A-III, B-II, C-IV, D-I

Option 2:
A-IV, B-I, C-II, D-III

Option 3:
A-II, B-III, C-IV, D-I

Option 4:
A-IV, B-III, C-I, D-II

Correct Answer:
A-II, B-III, C-IV, D-I

Solution:
Fact

$[Cr(H_2O)_6]^{3+} = t_2\ g^3 eg^0$

$[Fe(H_2O)_6]^{3+} = t_2\ g^3 eg^2$

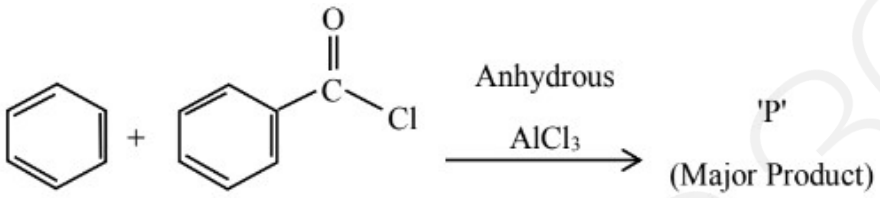
$[Ni(H_2O)_6]^{2+} = t_2\ g^6 eg^2$

$[V(H_2O)_6]^{3+} = t_2\ g^2 eg^0$

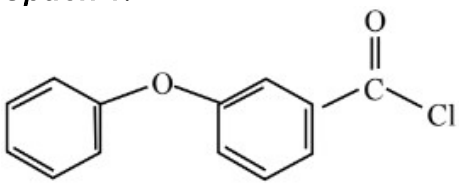
H_2O is WFL there for $\Delta_0 < P$

Hence, the answer is the option (3).

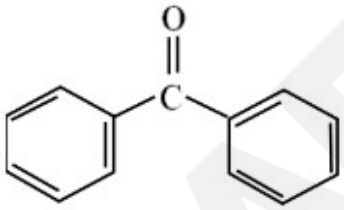
Q. 5 Identify major product 'P' formed in the following reaction.



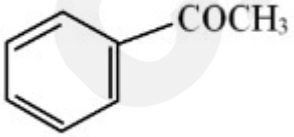
Option 1:



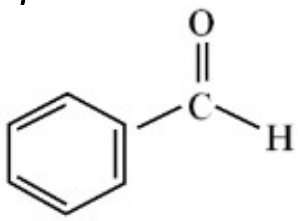
Option 2:



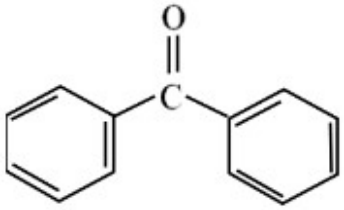
Option 3:



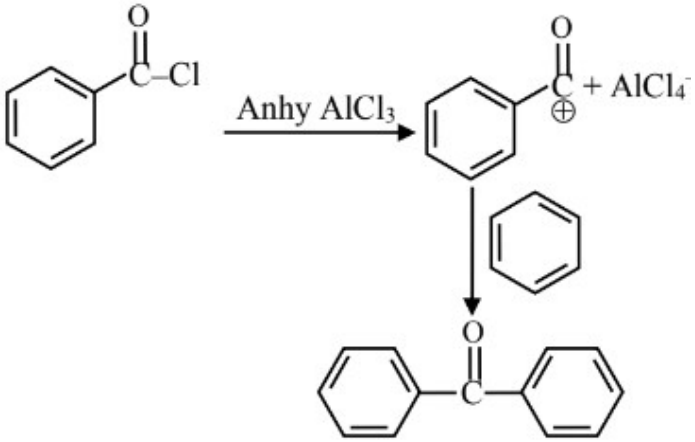
Option 4:



Correct Answer:



Solution:



It is simple friedel craft acylation reaction.

Q. 6 Select the option with correct property -

Option 1:
$[NiCl_4]^{2-}$ Diamagnetic, $[Ni(CO)_4]$ Paramagnetic

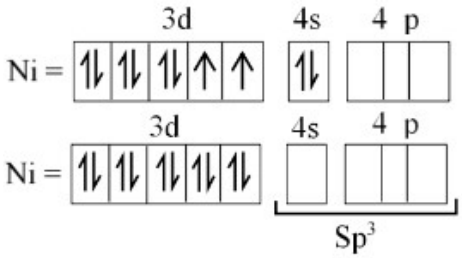
Option 2:
$[Ni(CO)_4]^{2-}$ and $[NiCl_4]^{2-}$ both Paramagnetic

Option 3:
$[Ni(CO)_4]$ and $[NiCl_4]^{2-}$ both Diamagnetic

Option 4:
$[Ni(CO)_4]$ Diamagnetic, $[NiCl_4]^{2-}$ Paramagnetic

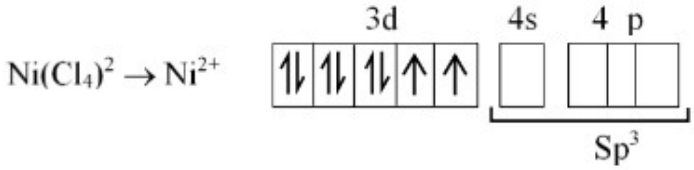
Correct Answer:
$[Ni(CO)_4]$ Diamagnetic, $[NiCl_4]^{2-}$ Paramagnetic

Solution:
$Ni\,(CO_4) \rightarrow$



$\rightarrow$ Tetrahedral

$\rightarrow \mu = 0$ (dia.)



$\rightarrow$ Tetrahedral

$\rightarrow \mu = \sqrt{8}$ (Para.)

Q. 7 Which of the following is least ionic?

Option 1:
$BaCl_2$

Option 2:
KCl

Option 3:
$CoCl_2$

Option 4:
$AgCl$

Correct Answer:
$AgCl$

Solution:
due to more polarising power of Ag^+.

The silver ion (Ag^+) has a relatively high charge density due to its smaller size and higher charge (+1) compared to ions like Na^+ or K^+. This high charge density means Ag^+ has a strong polarizing effect on the electron cloud of the chloride ion (Cl^-). This polarization

distorts the electron cloud of Cl^-, creating a more covalent character in the bond between Ag and Cl.

Hence, the answer is the option (4).

Q. 8 $A_{(g)} \rightleftharpoons B_{(g)} + \frac{C}{2}(g)$ The correct relationship between Kp, α and equilibrium pressure P is -

Option 1:

$Kp = \frac{\alpha^{\frac{3}{2}} p^{\frac{1}{2}}}{(2+\alpha)^{1/2}(1-\alpha)}$

Option 2:

$K_P = \frac{\alpha^{\frac{1}{2}} P^{\frac{3}{2}}}{(2+\alpha)^{3/2}}$

Option 3:

$K_P = \frac{\alpha^{\frac{1}{2}} P^{\frac{1}{2}}}{(2+\alpha)^{3/2}}$

Option 4:

$K_P = \frac{\alpha^{\frac{1}{2}} P^{\frac{1}{2}}}{(2+\alpha)^{1/2}}$

Correct Answer:

$Kp = \frac{\alpha^{\frac{3}{2}} p^{\frac{1}{2}}}{(2+\alpha)^{1/2}(1-\alpha)}$

Solution:

$A(g) \rightleftharpoons B(g) + \frac{1}{2}C(g)$

$t = 0$	n	$-$	$-$
$t = eq$	$n(1-\alpha)$	$n\alpha$	$n\alpha/2$

total

moles at equilibrium $\Rightarrow n - n\alpha + n\alpha + n\alpha/2 \Rightarrow n\left(1 + \frac{\alpha}{2}\right)$

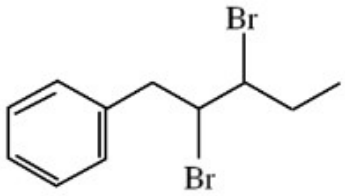
$$K_P = \frac{(P_B)(P_C)^{1/2}}{(P_A)} \Rightarrow \frac{\frac{n\alpha}{n(1+\alpha/2)} \times P\left[\frac{n\alpha}{n(1+\alpha/2)} \times P\right]^{1/2}}{\left[\frac{n(1-\alpha)}{n(1+\alpha/2)} \times P\right]}$$

$$K_p = \frac{\alpha^{3/2} P^{1/2}}{(2+\alpha)^{1/2}(1-\alpha)}$$

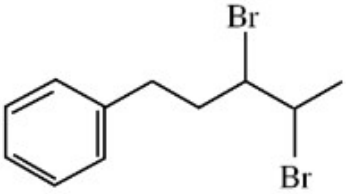
Hence, the answer is the option (1).

Q. 9 Identify structure of 2, 3-dibromo-1-phenylpentane.

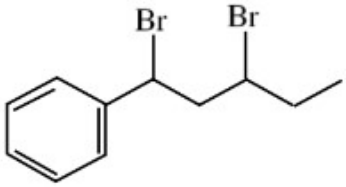
Option 1:



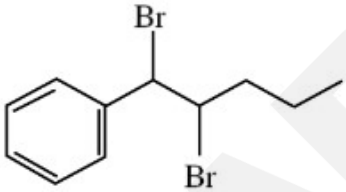
Option 2:



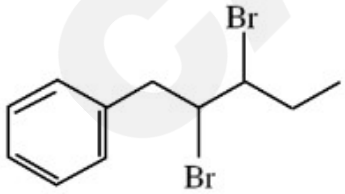
Option 3:



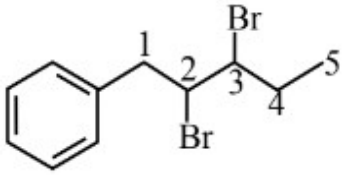
Option 4:



Correct Answer:

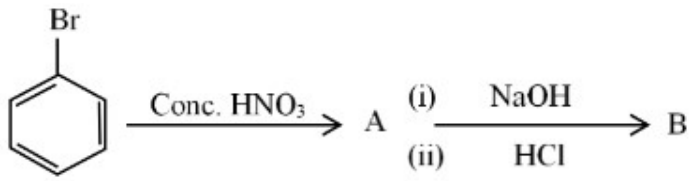


Solution:

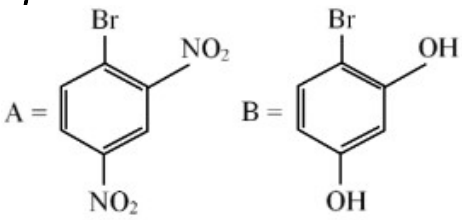


2, 3-dibromo-1-phenyl pentane

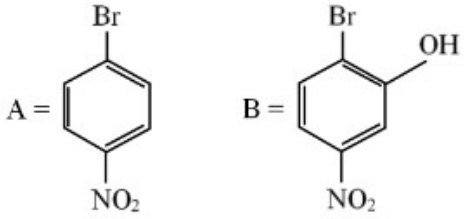
Q. 10 Identify A and B in the following reaction sequence.



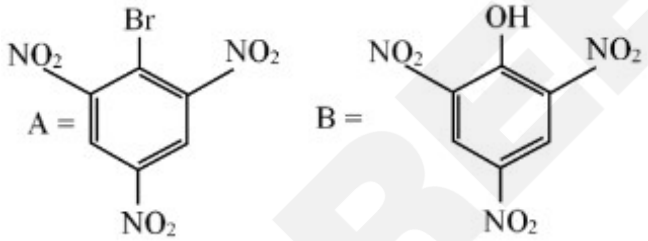
Option 1:



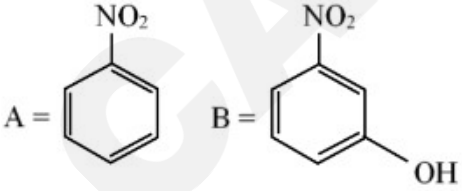
Option 2:



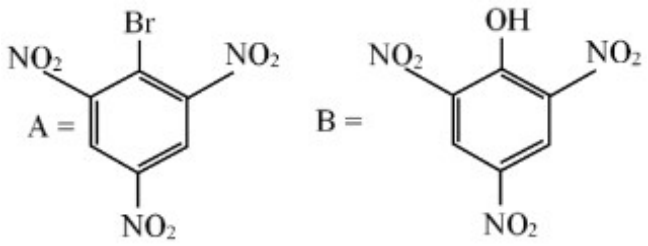
Option 3:



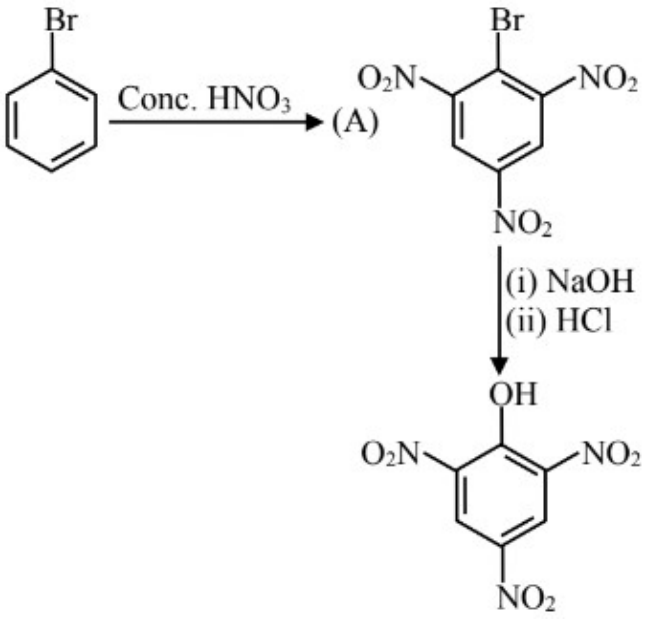
Option 4:



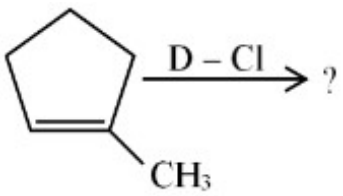
Correct Answer:



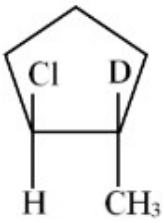
Solution:



Q. 11 Major product of the following reaction is –



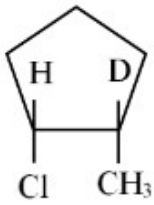
Option 1:



Option 2:

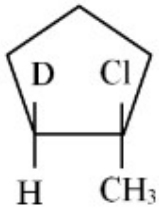


Option 3:

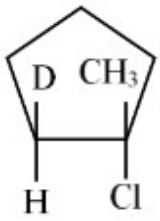


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Option 4:

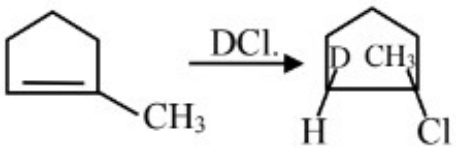


Correct Answer:

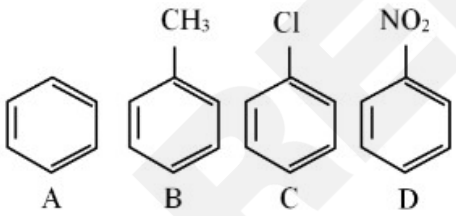


Solution:

Due to the slightly formation of non-classical carbocation, major product will be second one i.e.



Q. 12 The correct order of reactivity in electrophilic substitution reaction of the following compounds is :



Option 1:

B > A > C > D

Option 2:

A > B > C > D

Option 3:

B > C > A > D

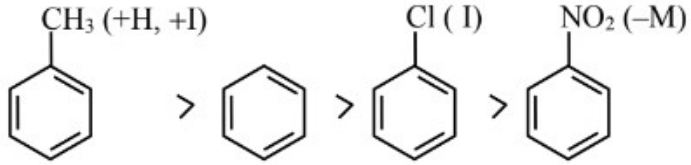
Option 4:

D > C > B > A

Correct Answer:

B > A > C > D

Solution:

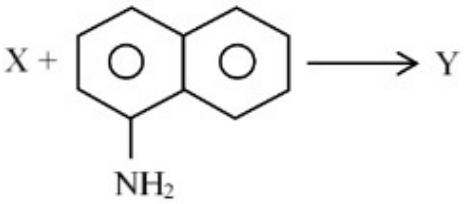


(rate of electrophilic substitution reaction)

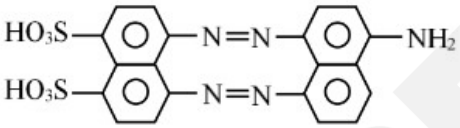
Correct order will be : B > A > C > D

Q. 13 The azo-dye (Y) formed in the following reactions is-

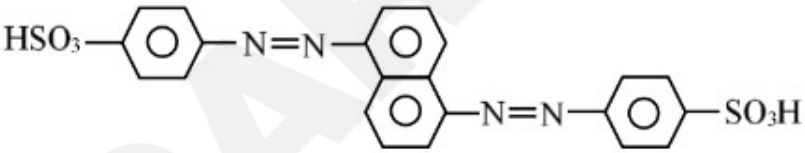
Sulphanilic acid $+\mathrm{NaNO_2} + \mathrm{CH_3COOH} \rightarrow \mathrm{X}.$



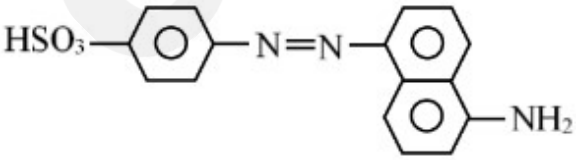
Option 1:



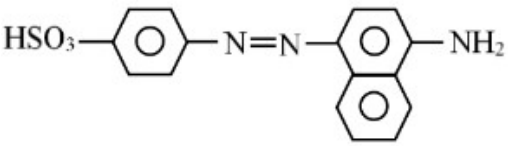
Option 2:



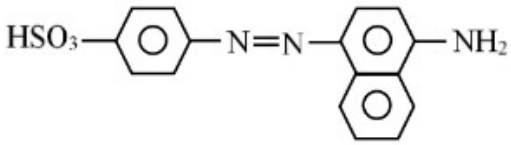
Option 3:



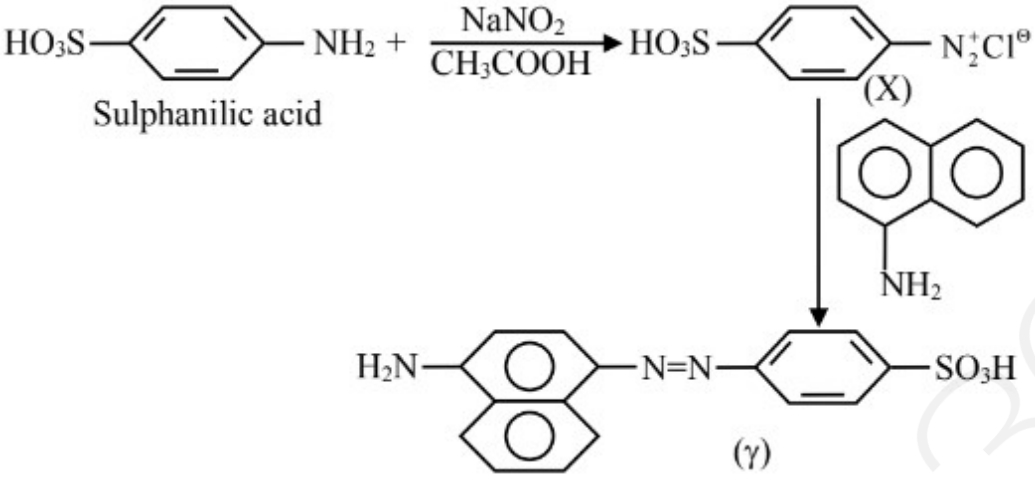
Option 4:



Correct Answer:

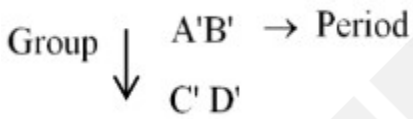


Solution:



' γ ' compound is formed by coupling reaction.

Q. 14 Consider the following elements.



Which of the following is/are true about A',B', C' and D' ?

(A) Order of atomic radii: $B' < A' < D' < C'$
(B) Order of metallic character: $B' < A' < D' < C'$
(C) Size of the element: $D' < C' < B' < A'$
(D) Order of ionic radii: $B'^{+} < A'^{+} < D'^{+} < C'^{+}$

Choose the correct answer from the options given below:

Option 1:
A and B only

Option 2:
A only

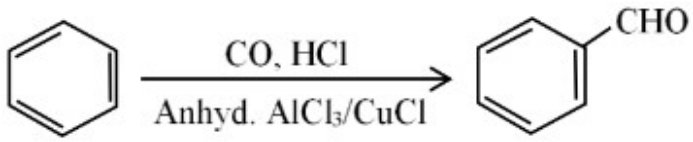
Option 3:
A, B and D only

Option 4:
B, C and D only

Correct Answer:
A, B and D only

Solution:
(1) Atomic and ionic Radius Increases down the gp but decreases along the period.
(2) Metallic character Increases down the group and decreases across the period.

Q. 15 Identify the name reaction.



Option 1:
Rosenmund Reduction

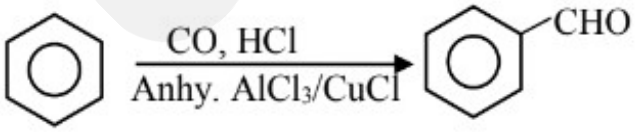
Option 2:
Gatterman – Koch Reaction

Option 3:
Stephen Reaction

Option 4:
Etard Reaction

Correct Answer:
Gatterman – Koch Reaction

Solution:



Q. 16 Given below are two statements:

Statement I: Aniline reacts with conc. H_2SO_4, followed by heating at $453-473\ K$ gives p-aminobenzene sulphonic acid, which gives blood red color in the 'Lassaigne's test'.

Statement II: In Friedel - Craft's alkylation and acylation reactions, aniline forms salt with the $AlCl_3$ catalyst.

Due to this, nitrogen of aniline aquires a positive charge and acts as deactivating group.

In the light of the above statements, choose the correct answer from the options given below:

Option 1:
Both statement I and statement II are true

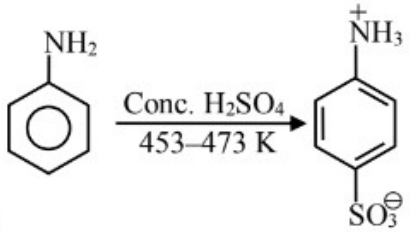
Option 2:
Statement I is false but statement II is true

Option 3:
Both statement I and statement II are false

Option 4:
Statement I is true but statement II is false

Correct Answer:
Both statement I and statement II are true

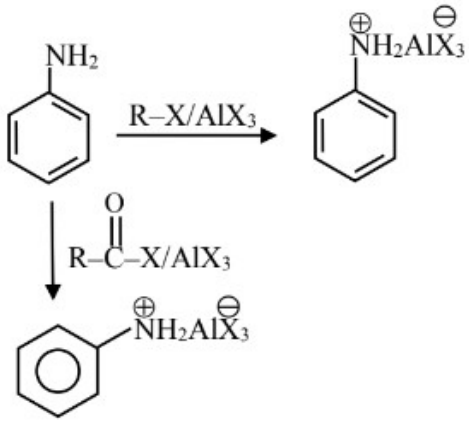
Solution:



Statement I :

(It gives blood red colour in Lassaigne's test)

statement 1st is correct.



Statement II :

Q. 17 A sample of $CaCO_3$ and $MgCO_3$ weighed 2.21 g is ignited to constant weight of 1.152 g. The composition of mixture is: (Given molar mass in $gmol^{-1} CaCO_3 : 100, MgCO_3 : 84$)

Option 1:

$1.023\ gCaCO_3 + 1.187\ gMgCO_3$

Option 2:

$1.18723\ gCaCO_3 + 1.187\ gMgCO_3$

Option 3:

$1.187\ gCaCO_3 + 1.023\ gMgCO_3$

Option 4:

$1.023\ gCaCO_3 + 1.023\ gMgCO_3$

Correct Answer:

$1.187\ gCaCO_3 + 1.023\ gMgCO_3$

Solution:

$CaCO_3 \rightarrow CaO + CO_2$

x gram

$MgCO_3 \rightarrow MgO + CO_2$

y gram

$\text{Given } \Rightarrow x + y = 2.21 \text{ g} \qquad \ldots\ldots (1)$

$\therefore 100 \text{ gram of } CaCO_3 \rightarrow 56 \text{ g} CaO$

$\text{x gram of } CaCO_3 \rightarrow \frac{56}{100} \times \text{g} CaO$

$\therefore 84 \text{ g} MgCO_3 \rightarrow 40 \text{ g} MgO$

$\text{y g } MgCO_3 \rightarrow \frac{40}{84} \times \text{y} MgO$

wt of residue $\Rightarrow \frac{56}{100}x + \frac{40y}{84} = 1.152 \ldots$ (2)

By solving equation (1) and (2)

$X = \text{wt. of } CaCO_3 = 1.187\text{gm}$
$Y = \text{wt. of } MgCO_3 = 1.023\text{gm}$

Hence, the answer is the option (3).

Q. 18 Choose the correct statements from the following :

(A) All group 16 elements form oxides of general formula EO_2 and EO_3, where $E = S, Se, Te$ and Po. Both the types of oxides are acidic in nature.
(B) TeO_2 is an oxidizing agent while SO_2 is reducing in nature.
(C) The reducing property decreases from H_2S to H_2Te down the group.
(D) The ozone molecule contains five lone pairs of electrons.

Choose the correct answer from the options given below:

Option 1:
A and B only

Option 2:
A and D only

Option 3:
A and D only

Option 4:
C and D only

Correct Answer:
A and B only

Solution:
The elements in group 16 of the periodic table, also known as the oxygen family or chalcogens, react with oxygen to form two types of oxides: dioxides (EO_2) and trioxides (EO_3). The elements in this group are sulfur (S), selenium (Se), tellurium (Te), and polonium (Po). Both dioxides and trioxides are acidic in nature, and their stability and acidic strength decrease down the group. For example, sulfur dioxide (SO_2) is a gas with greater stability and acidic strength than selenium dioxide (SeO_2), which is a solid.

TeO_2 (tellurium dioxide) can act as an oxidizing agent because tellurium in the +4 oxidation state can be further oxidized to the +6 state, forming TeO_3 (tellurium trioxide). This ability to be further oxidized characterizes it as an oxidizing agent.

SO_2 (sulfur dioxide) is well-known for its reducing properties. In its +4 oxidation state, sulfur can be oxidized to SO_3 (sulfur trioxide) in the +6 state. This tendency to lose electrons and be oxidized characterizes SO_2 as a reducing agent.

Hence, the answer is the option (1).

Q. 19 The fragrance of flowers is due to the presence of some steam volatile organic compounds called essential oils. These are generally insoluble in water at room temperature but are miscible with water vapor in vapor phase. A suitable method for the extraction of these oils from the flowers is -

Option 1:
distillation

Option 2:
steam distillation

Option 3:
distillation under reduced pressure

Option 4:
crystallization

Correct Answer:
steam distillation

Solution:
Fact Base

Steam Distillation: This technique is particularly suitable for extracting essential oils because:

- Essential oils are typically insoluble in water but can vaporize with steam.
- During steam distillation, steam is passed through the plant material containing the essential oils.
- The steam carries the volatile essential oils with it.
- When the steam and the essential oils are condensed, the essential oils can be separated from the water due to their immiscibility in liquid form.

Hence, the answer is the option (2).

Q. 20 The four quantum numbers for the electron in the outer most orbital of potassium (atomic no. 19) are

Option 1:
$\mathrm{n}=3, \ell=0, \mathrm{m}=1, \mathrm{s}=+\frac{1}{2}$

Option 2:
$\mathrm{n}=2, \ell=0, \mathrm{m}=0, \mathrm{s}=+\frac{1}{2}$

Option 3:
$\mathrm{n}=4, \ell=0, \mathrm{m}=0, \mathrm{s}=+\frac{1}{2}$

Option 4:
$\mathrm{n}=4, \ell=2, \mathrm{m}=-1, \mathrm{s}=+\frac{1}{2}$

Correct Answer:
$\mathrm{n}=4, \ell=0, \mathrm{m}=0, \mathrm{s}=+\frac{1}{2}$

Solution:
$\mathrm{K} \rightarrow \text{ atomic No. } = 19$

electronic configuration $\Rightarrow 1s^2 2s^2 2p^6 3s^2 3p^6 4s^1$

$n = 4, l = 0, m = 0, s = +l/2$

Hence, the answer is the option (3).

Q. 21 If 5 moles of an ideal gas expands from $10\ L$ to a volume of $100\ L$ at $300\ K$ under isothermal and reversible condition then work, w, is $-xJ$. The value of x is_____ (Given $R = 8.314\ J\ K^{-1}\ mol^{-1}$)

Correct Answer:
96

Solution:

$$w = -nRT \ln \frac{V_2}{V_1}$$

$$= -5 \times 8.314 \times 300 \times \ln \frac{100}{10}$$

$$= -5 \times 2.303 \times 8.314 \times \log_{10} 10$$

$$= -5 \times 2.303 \times 8.314$$

$$= -95.7\ J$$

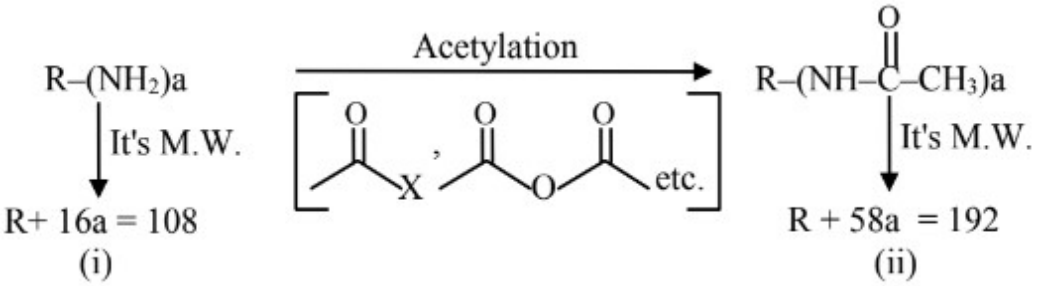
$$W = -96\ J$$

Hence, the answer is (96).

Q. 22 A compound (x) with molar mass $108\ g\ mol^{-1}$ undergoes acetylation to give product with molar mass $192\ g\ mol^{-1}$. The number of amino groups in the compound (x) is_____.

Correct Answer:
2

Solution:



By Subtracting equation (ii) from (i)

$$R + 58a - R - 16a = 192 - 108$$

$$42a = 84$$

$$[a = 2]$$

So, Number of amino group will be 2.

Q. 23 A diatomic molecule has a dipole moment of 1.2D. If the bond distance is $1\ \text{A}^\circ$, then fractional charge on each atom is_____$\times 10^{-1}$ esu. (Given $1\text{D} = 10^{-18}$ esu cm)

Correct Answer:
2.5

Solution:

$$\text{Charge} = \frac{\mu_{\text{exp.}}}{\mu_{\text{cal.}}}$$

$$\mu_{\text{exp.}} = 1.2\text{D} = 1.2 \times 10^{-18}\text{esucm}$$

$$\mu_{\text{cal.}} = 1 \times 10^{-8} \times 4.8 \times 10^{-10}\text{esucm}$$

$$\text{Fractional charge} = \frac{1.2 \times 10^{-18}}{4.8 \times 10^{-18}} = \frac{1.2}{4.8} = \frac{1}{4} = 0.25 = 2.5 \times 10^{-1}$$

Hence, the answer is (2.5).

Q. 24 The values of conductivity of some materials at $298.15\ \mathrm{K}$ in Sm^{-1} are $2.1 \times 10^{3}, 1.0 \times 10^{-16}, 1.2 \times 10, 3.91, 1.5 \times 10^{-2}, 1 \times 10^{-7}, 1.0 \times 10^{3}$. The number of conductors among the materials is_____.

Correct Answer:
4

Solution:
$1 \times 10^{-16}\mathrm{Sm}^{-1}$
$1.5 \times 10^{-2}\mathrm{Sm}^{-1}$
$1 \times 10^{-7}\mathrm{Sm}^{-1}$
} these are very low conductivity so they are not conductors

There are 4 conductors.

Hence, the answer is (4).

Q. 25 $\mathrm{r} = \mathrm{k}[\mathrm{A}]$ for a reaction, 50% of A is decomposed in 120 minutes. The time taken for 90% decomposition of A is_____ minutes.

Correct Answer:
400

Solution:

$$t_{90} = \frac{2.303}{K}\log\left(\frac{100}{100-90}\right)$$

$$\text{Given } t_{50} = \frac{0.693}{K}$$

$$K = \frac{0.693}{120}$$

$$= \frac{2.303 \times 120}{0.693} \times \log 10$$

$$= \frac{2.303 \times 120}{0.3 \times 2.303} \times 1$$

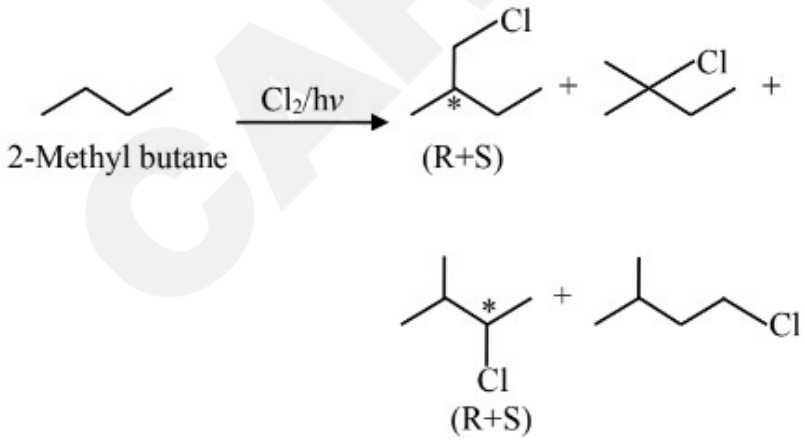
$$\Rightarrow 400 \text{ min}$$

Hence, the answer is (400).

Q. 26 Number of isomeric products formed by monocholorination of 2-methylbutane in presence of sunlight is ___ .

Correct Answer:
6

Solution:



Total no. of product = 06

Q. 27 Number of moles of H^+ ions required by 1 mole of MnO_4^- to oxidise oxalate ion to CO_2 is_____.

Correct Answer:
8

Solution:
$2MnO_4^- + 5C_2O_4^{2-} + 16H^+ \rightarrow 2Mn^{2+} + 10CO_2 + 8H_2O$

2 mole of MnO_4^- reacts with 16 mole H^+
1 mole of MnO_4^- reacts with 8 mole H^+

Hence, the answer is (8).

Q. 28 From the vitamins $A, B_1, B_6, B_{12}, C, D, E$ and K, the number of vitamins that can be stored in our body is_____.

Correct Answer:
5

Solution:
Vitamin V can be stored in our body
A,D,E,K & B12

Hence, the answer is (5).

Q. 29 In the reaction of potassium dichromate, potassium chloride and sulfuric acid (conc.), the oxidation state of the chromium in the product is (+) _____ .

Correct Answer:
6

Solution:
Non-redox reaction chromyl chloride is formed

$$K_2Cr_2O_7 + H_2SO_4 + KCl \xrightarrow{\Delta} \underset{\text{Red Vupours}}{CrO_2Cl_2}$$

Hence, the answer is (6).

Q. 30 The molarity of $1\ \mathrm{L}$ orthophosphoric acid $(\mathrm{H_3PO_4})$ having 70% purity by weight (specific gravity $1.54\ \mathrm{g\ cm^{-3}}$) is _____M. (Molar mass of $\mathrm{H_3PO_4} = 98\ \mathrm{g\ mol^{-1}}$)

Correct Answer:
11

Solution:
Mass of $\mathrm{H_3PO_4} = 1540\mathrm{gm} \times \frac{70}{100} = 154 \times 7\mathrm{gm}$

Moles of $\mathrm{H_3PO_4} = \frac{154\times7}{98}$

Molarity $= \frac{\frac{154\times7}{98}}{1} = 11\mathrm{M}$

Hence, the answer is (11).

Maths

Q. 1 Let 2^{nd} , 8^{th} and 44^{th} terms of a non-constant A.P. be respectively the 1^{st} , 2^{nd} and 3^{rd} terms of a G.P. If the first term of A.P. is 1 , then the sum of its first 20 terms is equal to-

Option 1:
970

Option 2:
980

Option 3:
960

Option 4:
990

Correct Answer:
970

Solution:

$$\begin{array}{lccc} & a_2, & a_8, & a_{44} \rightarrow \text{A.P} \\ \text{If} & \downarrow & \downarrow & \downarrow \\ & A_1 & A_2 & A_3 \rightarrow \text{G.P.} \end{array}$$

$a_2 = a + d$
$a_8 = a + 7d$ where $\{a = 1\}$
$a_{44} = a + 43d$

and $A_1, A_2, A_3 \rightarrow$ G.P

$$\frac{A_2}{A_1} = \frac{A_3}{A_2}$$

$$\frac{a + 7d}{a + d} = \frac{a + 43d}{a + 7d}\{a = 1\}$$

$$(1 + 7d)^2 = (1 + 43d)(1 + d)$$

$$1 + 49d^2 + 14d = 1 + 44d + 43d^2$$

$$6d^2 - 30d = 0$$

$$6d(d - 5) = 0$$

$$d = 0, d = 5$$

x

Sum of first 20 terms

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$S_{20} = \frac{20}{2}(2 \times 1 + 19 \times 5)$$

$$= 10(97) = 970$$

Q. 2 A coin is biased so that a head is twice a likely to occur as tail. If the coin is tossed 3 times, then the probability of getting two tails and one head is

Option 1:
$\frac{1}{27}$

Option 2:
$\frac{1}{9}$

Option 3:
$\frac{2}{27}$

Option 4:
$\frac{2}{9}$

Correct Answer:
$\frac{2}{9}$

Solution:
$\mathrm{p(H)} = 2\mathrm{p(T)}$

Let $p(T) = x$

$p(H) = 2x$

$x + 2x = 1$

$\mathrm{x} = \frac{1}{3}$

$\mathrm{p(T)} = \frac{1}{3} \quad \mathrm{p(H)} = \frac{2}{3}$

Probability of $2\ \mathrm{T\&1H}$ is

$${}^{3}\mathrm{C}_{2}\left(\frac{1}{3}\right)^{2}\left(\frac{2}{3}\right)$$

$$3\left(\frac{1}{9}\right)\left(\frac{2}{3}\right) = \frac{2}{9}$$

Hence, the answer is the option (4).

Q. 3 The area of the region enclosed by the parabolas $y = 4x - x^2$ and $3y = (x-4)^2$ is equal to-

Option 1:
4

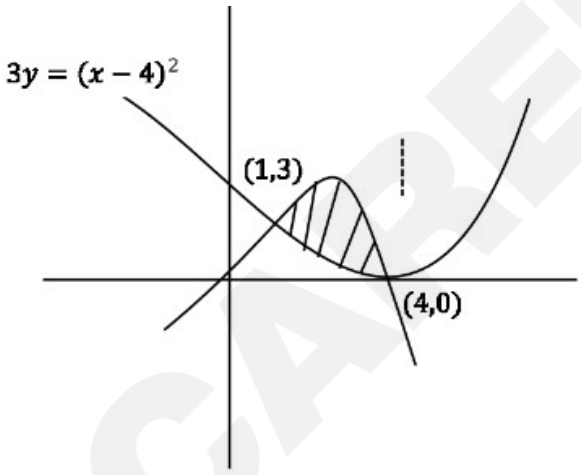
Option 2:
6

Option 3:
$\frac{32}{9}$

Option 4:
$\frac{14}{3}$

Correct Answer:
6

Solution:
The area region enclosed by the parabolas $y = 4x - x^2$ and $3y = (x-4)^2$



$3\left(4x - x^2\right) = (x-4)^2$
$12x - 3x^2 = x^2 - 8x + 16$
$4x^2 - 20x + 16 = 0$
$x^2 - 5x + 4 = 0$
$(x-1)(x-4) = 0$
$x = 1, 4$

If $x = 1$

$y = 4x - x^2 = 4 - 1 = 3$

If $x = 4, y = 0$

Area is

$$A = \int_1^4 \left((4x - x^2) - \frac{(x-4)^2}{3} \right) dx$$

$$A = \left(\frac{4x^2}{2} - \frac{x^3}{3} \right) - \frac{(x-4)^3}{9} \Bigg|_1^4$$

$$A = \left(32 - \frac{64}{3} - 0 \right) - \left(2 - \frac{1}{3} + 3 \right)$$

$$A = \frac{32}{3} - \left(5 - \frac{1}{3} \right) = 6$$

Q. 4 Let P be a parabola with vertex $(2, 3)$ and directrix $2\mathrm{x} + \mathrm{y} = 6$. Let an ellipse $\mathrm{E} : \frac{\mathrm{x}^2}{\mathrm{a}^2} + \frac{\mathrm{y}^2}{\mathrm{b}^2} = 1, \mathrm{a} > \mathrm{b}$, of eccentricity $\frac{1}{\sqrt{2}}$ pass through the focus of the parabola P. Then the square of the length of the latus rectum of E, is

Option 1:
$\frac{512}{25}$

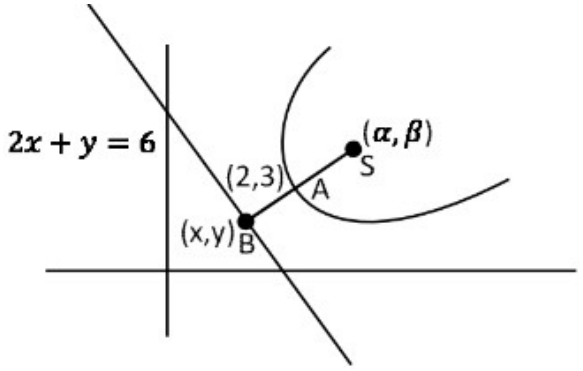
Option 2:
$\frac{385}{8}$

Option 3:
$\frac{656}{25}$

Option 4:
$\frac{347}{8}$

Correct Answer:
$\frac{656}{25}$

Solution:



$$\frac{x-x_1}{a}=\frac{y-y_1}{b}=-\left(\frac{ax_1+by_1+c}{a^2+b^2}\right)$$

$$\frac{x-2}{2}=\frac{y-3}{1}=-\left(\frac{2\times 2+3-6}{4+1}\right)$$

$$\frac{x-2}{2}=\frac{y-3}{1}=-\frac{1}{5}$$

$$x=\frac{-2}{5}+2=\frac{8}{5}$$

$$y=\frac{-1}{5}+3=\frac{14}{5}$$

$$\text{So, } \frac{\alpha+x}{2}=2$$

$$\Rightarrow \alpha+\frac{8}{5}=4$$

$$\Rightarrow \alpha=4-\frac{8}{5}=\frac{12}{5}$$

$$\frac{\beta+y}{2}=3$$

$$\beta=6-\frac{14}{5}=\frac{16}{5}$$

$(\alpha,\beta)=\left(\frac{12}{5},\frac{16}{5}\right)$ lies on the ellipse

Now ellipse

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

$$\frac{\left(\frac{12}{5}\right)^2}{a^2}+\frac{\left(\frac{16}{5}\right)^2}{b^2}=1$$

$$\text{and } \frac{144}{a^2}+\frac{256}{b^2}=25 \qquad \ldots\ldots(1)$$

eccentricity $\Rightarrow \sqrt{1-\frac{b^2}{a^2}}=\frac{1}{\sqrt{2}}$

$1-\frac{b^2}{a^2}=\frac{1}{2}$

$\frac{b^2}{a^2}=\frac{1}{2}$

$a^2=2\,b^2 \qquad \ldots.(2)$

Using eq. (2) in equation (1)

$$\frac{144}{2b^2}+\frac{256}{b^2}=25$$

$$72+256=25b^2$$

$$\frac{328}{25}=b^2$$

Length of L.R. $\Rightarrow \frac{2b^2}{a}$ Sq. of Length of L.R. $=\left(\frac{2b^2}{a}\right)^2=\frac{4b^4}{a^2}$

$$\Rightarrow \frac{4b^4}{2b^2}$$

$$2b^2 \Rightarrow 2\times\frac{328}{25}$$

$$\Rightarrow \frac{656}{25}$$

Q. 5 The shortest distance, between lines L_1 and L_2, where $L_1: \frac{x-1}{2}=\frac{y+1}{-3}=\frac{z+4}{2}$ and L_2 is the line, passing through the points $A(-4,4,3), B(-1,6,3)$ and perpendicular to the line $\frac{x-3}{-2}=\frac{y}{3}=\frac{z-1}{1}$, is

Option 1:

$\frac{42}{\sqrt{117}}$

Option 2:
$\frac{24}{\sqrt{117}}$

Option 3:
$\frac{121}{\sqrt{221}}$

Option 4:
$\frac{141}{\sqrt{221}}$

Correct Answer:
$\frac{141}{\sqrt{221}}$

Solution:

$$L_2 : \frac{x+4}{3} = \frac{y-4}{2} = \frac{z-3}{0}$$

$$L_1 : \frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+4}{2}$$

$$S \cdot D = \frac{\left(\vec{a}_2 - \vec{a}_1\right) \cdot \left(\vec{b}_1 \times \vec{b}_2\right)}{\vec{b}_1 \times \vec{b}_2}$$

$\vec{a}_1 = \hat{i} - \hat{j} - 4\hat{k}$ $\qquad$ $\vec{b}_2 = 3\hat{i} + 2\hat{j} + 0\hat{k}$

$\vec{a}_2 = -4\hat{i} + 4\hat{j} + 3\hat{k}$ $\qquad$ $\vec{b}_1 = 2\hat{i} - 3\hat{j} + 2\hat{k}$

$\vec{b}_1 \times \vec{b}_2 = -4\hat{i} + 6\hat{j} + 13\hat{k}$

$$S \cdot D = \frac{(-5\hat{i}+5\hat{j}+7\hat{k}) \cdot (-4\hat{i}+6\hat{j}+13\hat{k})}{\sqrt{221}}$$

$$= \frac{141}{\sqrt{221}} = \frac{141}{\sqrt{221}}$$

Hence, the answer is the option (4).

Q. 6 If for some m, n; ${}^{6}C_m + 2\left({}^{6}C_{m+1}\right) + {}^{6}C_{m+2} > {}^{8}C_3$ and ${}^{n-1}P_3 : {}^{n}P_4 = 1 : 8$, then ${}^{n}P_{m+1} + {}^{n+1}C_m$ is equal to

Option 1:
384

Option 2:
376

Option 3:
372

Option 4:
380

Correct Answer:
372

Solution:

$6_{C_m} + 2\,(6_{C_{m+1}}) + 6_{C_{m+2}} > 8_{C_3}$

$6_{C_m} + 6_{C_{m+1}} + 6_{C_{m-1}} + 6_{C_{m+2}} > 8_{C_3}$

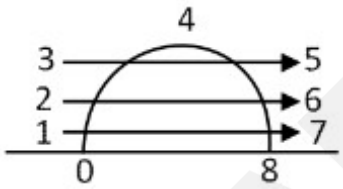
$\left\{{}^{n}C_r + {}^{n}C_{r-1} \Rightarrow {}^{n+1}C_r\right\}$

${}^{7}C_{m+1} + {}^{7}C_{m+2} > {}^{8}C_3$

${}^{8}C_{m+2} > {}^{8}C_3$

${}^{8}C_{m+2} > {}^{8}C_3$

$\Rightarrow m = 2$



$$\frac{{}^{n-1}P_3}{{}^{n}P_4} = \frac{1}{8}$$

$$\frac{(n-1)(n-2)(n-3)}{n(n-1)(n-2)(n-3)} = \frac{1}{8}$$

$n = 8$

Now

$^{8}P_{2+1} + {}^{8+1}C_{2}$

$\Rightarrow 8 \times 7 \times 6 + \frac{9 \times 8}{2}$

$\Rightarrow 336 + 36 = 372$

Q. 7 Let A be a 3×3 real matrix such that $A\begin{pmatrix}1\\0\\1\end{pmatrix} = 2\begin{pmatrix}1\\0\\1\end{pmatrix}, A\begin{pmatrix}-1\\0\\1\end{pmatrix} = 4\begin{pmatrix}-1\\0\\1\end{pmatrix}, A\begin{pmatrix}0\\1\\0\end{pmatrix} = 2\begin{pmatrix}0\\1\\0\end{pmatrix}$. Then, the system $(A-3I)\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}1\\2\\3\end{pmatrix}$ has

Option 1:
exactly two solutions

Option 2:
unique solution

Option 3:
no solution

Option 4:
infinitely many solutions

Correct Answer:
unique solution

Solution:

CAREERS360

$A \rightarrow 3 \times 3$ matrix

$$A\begin{pmatrix}1\\0\\1\end{pmatrix} = 2\begin{pmatrix}1\\0\\1\end{pmatrix}$$

$$A\begin{pmatrix}-1\\0\\1\end{pmatrix} = 4\begin{pmatrix}-1\\0\\1\end{pmatrix}$$

$$A\begin{pmatrix}0\\1\\0\end{pmatrix} = 2\begin{pmatrix}0\\1\\0\end{pmatrix}$$

Then $(A-3I)\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}1\\2\\3\end{pmatrix}$

Let

$$A = \begin{bmatrix}x_1 & y_1 & z_1\\ x_2 & y_2 & z_2\\ x_3 & y_3 & z_3\end{bmatrix}$$

Now using the given condition

(i) $$\begin{bmatrix}x_1 & y_1 & z_1\\ x_2 & y_2 & z_2\\ x_3 & y_3 & z_3\end{bmatrix}_{3\times 3}\begin{pmatrix}0\\1\\0\end{pmatrix}_{3\times 1} = \begin{pmatrix}0\\2\\0\end{pmatrix}$$

$$\begin{pmatrix}y_1\\y_2\\y_3\end{pmatrix} = \begin{pmatrix}0\\2\\0\end{pmatrix} \Rightarrow \begin{matrix}y_1 = 0\\ y_2 = 2\\ y_3 = 0\end{matrix}$$

(ii) $$\begin{bmatrix}x_1 & y_1 & z_1\\ x_2 & y_2 & z_2\\ x_3 & y_3 & z_3\end{bmatrix}_{3\times 3}\begin{pmatrix}-1\\0\\1\end{pmatrix} = \begin{pmatrix}-4\\0\\4\end{pmatrix}$$

$$-x_1 + z_1 = -4$$
$$-x_2 + z_2 = 0$$
$$-x_3 + z_3 = 4$$

(iii) $\begin{bmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{bmatrix}_{3\times 3} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 2 \end{pmatrix}$

$$x_1 + z_1 = 2$$
$$x_2 + z_2 = 0$$
$$x_3 + z_3 = 2$$

$$-x_1 + z_1 = -4$$
$$x_1 + z_1 = 2$$
$$\Rightarrow z_1 = -1$$
$$x_1 = 3$$

$$\left.\begin{aligned} x_2 + z_2 &= 0 \\ -x_2 + z_2 &= 0 \end{aligned}\right\} \Rightarrow z_2 = 0 \Rightarrow x_2 = 0$$

$$x_3 + z_3 = 2$$
$$-x_3 + z_3 = 4$$
$$z_3 = 3$$
$$x_3 = -1$$

So now

$$A = \begin{bmatrix} 3 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 3 \end{bmatrix}$$

$$A - 3I = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

$$(A - 3I)\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

$-z = 3 \Rightarrow z = -3$

$-y = 2 \Rightarrow y = -2$

$-x = 1 \Rightarrow x = -1$

$\Rightarrow$ unique Solution

Hence, the answer is the option (2).

Q. 8 Let a variable line passing through the centre of the circle $x^2 + y^2 - 16x - 4y = 0$, meet the positive co-ordinate axes at the points A and B. Then the minimum value of $\mathrm{OA} + \mathrm{OB}$, where O is the origin is equal to

Option 1:
18

Option 2:
24

Option 3:
20

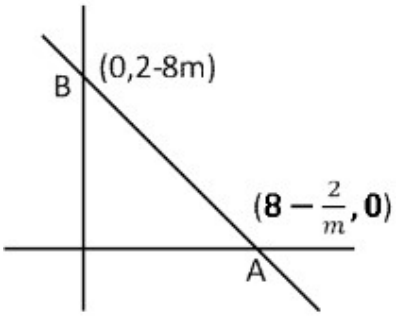
Option 4:
12

Correct Answer:
18

Solution:
certre $(8, 2)$

equation of line

$\mathrm{y} - 2 = \mathrm{m}(\mathrm{x} - 8)$



$$f(m) = OA + OB = 8 - \frac{2}{m} + 2 - 8\,m$$

$$f(m) = 10 - \frac{2}{m} - 8\,m$$

$$f'(m) = \frac{2}{m^2} - 8 = 0$$

$$\frac{2}{m^2} = 8$$

$$m^2 = \frac{1}{4}$$

$$m = \pm\frac{1}{2}$$

$$m = \frac{1}{2}\times$$

$$m = -\frac{1}{2}$$

$$f\left(-\frac{1}{2}\right) = 10 - \frac{2}{\left(-\frac{1}{2}\right)} - 8\left(-\frac{1}{2}\right)$$

$$= 10 + 4 + 4 = 18$$

Q. 9 The number of ways in which 21 identical apples can be distributed among three children such that each child gets at least 2 apples, is

Option 1:
406

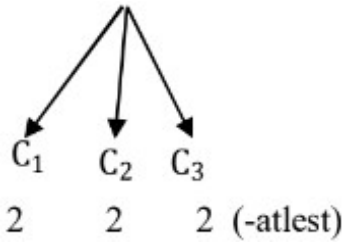
Option 2:
142

Option 3:
136

Option 4:
130

Correct Answer:
136

Solution:
21 identical apples



$21-6=15$

$\downarrow$

in 3 children

$\text{So } 15+3-1_{C_{3-1}} = {}^{17}C_2$

$\Rightarrow \frac{17\times 16}{2} = 136$

Q. 10 Let $f, g:(0,\infty)\to\mathbb{R}$ be two functions defined by $f(x)=\int_{-x}^{x}\left(|t|-t^2\right)e^{-t^2}dt$ and $g(x)=\int_0^{x^2} t^{1/2}e^{-1}dt$. Then, the value of $9\left(\left(f\sqrt{\log_e 9}\right)+g\left(\sqrt{\log_e 9}\right)\right)$ is equal to

Option 1:
10

Option 2:
9

Option 3:
8

Option 4:
6

Correct Answer:
8

Solution:

$$f(x)=\int_{-x}^{x}|t|-t^2e^{-t^2}dt$$

$$g(x)=\int_{0}^{x^2}t^{\frac{1}{2}}e^{-t}dt$$

$$f(x)=2\int_{0}^{x}(t-t^2)e^{-t^2}dt$$

$$f(x)=2\left[\int_{0}^{x}t^{-t^2}dt-\int_{0}^{x}t^2e^{-t^2}dt\right]$$

$$g(x)=\int_{0}^{x^2}\sqrt{t}e^{-t}dt \quad \sqrt{t}=y\Rightarrow\frac{dt}{2\sqrt{t}}=dy$$

$$g(x)=2\int_{0}^{x}y^2\cdot e^{-y^2}dy$$

$$f(x)+g(x)=2\left(\frac{1-e^{-x^2}}{2}\right)$$

$$=1-e^{-x^2}$$

$$\Rightarrow 9\left(\mathrm{f}\left(\sqrt{\log_e 9}\right)+\mathrm{g}\left(\sqrt{\log_e 9}\right)\right)=9\times\left(1-\frac{1}{9}\right)$$

$$=9\times\frac{8}{9}=8$$

Hence, the answer is the option (3).

Q. 11 Let $\mathrm{A}(\mathrm{a},\mathrm{b}),\mathrm{B}(3,4)$ and $\mathrm{C}(-6,-8)$ respectively denote the centroid, circumcenter and orthocenter of a triangle. Then, the distance of the point $\mathrm{P}(2\mathrm{a}+3,7\,\mathrm{b}+5)$ from the line $2\mathrm{x}+3\mathrm{y}-4=0$ measured parallel to the line $x-2y-1=0$ is

Option 1:
$\frac{\sqrt{5}}{17}$

Option 2:
$\frac{15\sqrt{5}}{7}$

Option 3:

$\frac{17\sqrt{5}}{7}$

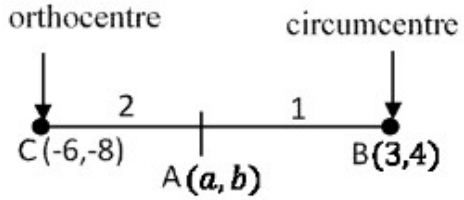
Option 4:

$\frac{17\sqrt{5}}{6}$

Correct Answer:

$\frac{17\sqrt{5}}{7}$

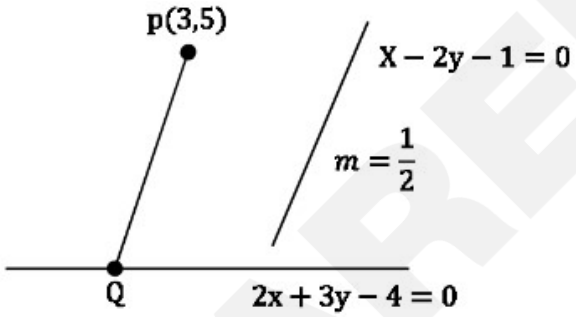
Solution:



$$a = \frac{2 \times 3 - 6}{3} = 0$$

$$b = \frac{2 \times 4 - 8}{3} = 0$$

So point $P(3, 5)$



Eq. of PQ

$$y - 5 = \frac{1}{2}(x - 3)$$

$$2y - 10 = x - 3$$

$$2y - x - 7 = 0$$

Intersection point

$2(2y - x - 7 = 0)$

$$\frac{3y + 2x - 4 = 0}{7y = 18}$$

$$y = \frac{18}{7}$$

$$2\left(\frac{18}{7}\right) - x - 7 = 0$$

$$\frac{36 - 49}{7} = x$$

$$x = \frac{-13}{7}$$

$$\text{Point } Q\left(-\frac{13}{7}, \frac{18}{7}\right)$$

$$P(3, 5), \quad Q\left(\frac{-13}{7}, \frac{18}{7}\right)$$

$$PQ = \sqrt{\left(\frac{-13}{7} - 3\right)^2 + \left(\frac{18}{7} - 5\right)^2}$$

$$PQ = \sqrt{\left(\frac{34}{7}\right)^2 + \left(\frac{17}{7}\right)^2}$$

$$PQ = \frac{\sqrt{1156 + 289}}{7} = \frac{17\sqrt{5}}{7}$$

Q. 12 Let z_1 and z_2 be two complex numbers such that $z_1 + z_2 = 5$ and $z_1^3 + z_2^3 = 20 + 15i$. Then, $z_1^4 + z_2^4$ equals-

Option 1:
$25\sqrt{3}$

Option 2:
$30\sqrt{3}$

Option 3:
$15\sqrt{15}$

Option 4:
75

Correct Answer:
75

Solution:
$z_1 + z_2 = 5$
$z_1^3 + z_2^3 = 20 + 15i$
$z_1^4 + z_2^4 = ?$
$z_1^3 + z_2^3 = (z_1 + z_2)^3 - 3z_1z_2(z_1 + z_2)$
$z_1^3 + z_2^3 = 125 - 15z_1z_2$
$20 + 15i = 125 - 15z_1z_2$
$z_1z_2 = 7 - i$

Now

$(z_1 + z_2)^2 = 5^2$
$z_1^2 + z_2^2 + 2z_1z_2 = 25$
$z_1^2 + z_2^2 = 25 - 2(7 - i)$
$= 11 + 2i$
$(z_1^2 + z_2^2)^2 = (11 + 2i)^2$
$z_1^4 + z_2^4 + 2(7 - i)^2 = 117 + 44i$
$z_1^4 + z_2^4 = 21 + 72i$

$z_1^4 + z_2^4 = \sqrt{(21)^2 + (72)^2}$
$= \sqrt{441 + 5184} = \sqrt{5625}$
$= 75$

Hence, the answer is the option (4).

Q. 13 Let (α, β, γ) be the mirror image of the point $(2, 3, 5)$ in the line $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$. Then $2\alpha + 3\beta + 4\gamma$ is equal to

Option 1:
31

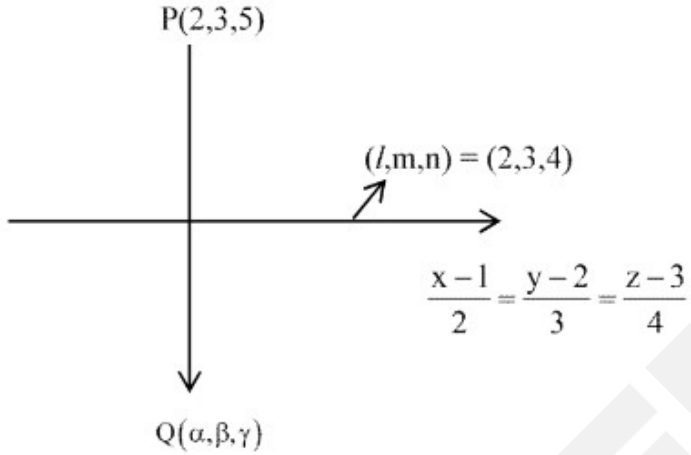
Option 2:
34

Option 3:
32

Option 4:
33

Correct Answer:
33

Solution:



Direction Ratios of PQ

$\alpha - 2, \beta - 3, \gamma - 5$

$\mathrm{PQ} \perp \text{ line}$

$2(\alpha - 2) + 3(\beta - 3) + (\gamma - 5)4 = 0$

$2\alpha - 4 + 3\beta - 9 + 4\gamma - 20 = 0$

$2\alpha + 3\beta + 4\gamma = 20 + 9 + 4 = 33$

Q. 14 The number of solutions, of the equation $e^{\sin x} - 2e^{-\sin x} = 2$, is :

Option 1:
0

Option 2:
1

Option 3:
more than 2

Option 4:
2

Correct Answer:
0

Solution:

$e^{\sin x} - 2e^{-\sin x} = 2$

$e^{\sin x} = t$

$t - \frac{2}{t} = 2$

$t^2 - 2t - 2 = 0$

$t = \frac{2 \pm \sqrt{4+8}}{2}$

$t = \frac{2 \pm 2\sqrt{3}}{2} \Rightarrow 1 \pm \sqrt{3}$

$t = 1 + \sqrt{3}, \quad t = 1 - \sqrt{3} < 0 \text{ (Rejected)}$

$t = 1 + \sqrt{3}$

$e^{\sin x} = 2.732 \quad (\because e^{\sin x} \leq 2.71 \text{ as maximum value of } \sin x = 1)$

(Rejected)

$\Rightarrow$ no Solution.

Hence, the answer is the option (1).

Q. 15 Let $f : R \to (0, \infty)$ be strictly increasing function such that $\lim_{x\to\infty} \frac{f(7x)}{f(x)} = 1$. Then, the value of $\lim_{x\to\infty} \left[\frac{f(5x)}{f(x)} - 1\right]$ is equal to

Option 1:
$\frac{7}{5}$

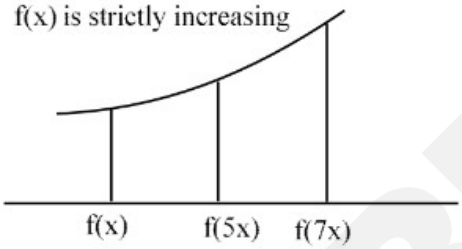
Option 2:
4

Option 3:
1

Option 4:
0

Correct Answer:
0

Solution:



$$f(x) < f(5x) < f(7x)$$

$$\frac{f(x)}{f(x)} < \frac{f(5x)}{f(x)} < \frac{f(7x)}{f(x)}$$

$$\lim_{x\to\infty} 1 < \lim_{x\to\infty} \frac{f(5x)}{f(x)} < \lim_{x\to\infty} \frac{f(7x)}{f(x)}$$

$$1 < \lim_{x\to\infty} \frac{f(5x)}{f(x)} < 1$$

$$1 - 1 < \lim_{x\to\infty} \frac{f(5x)}{f(x)} < 1 - 1$$

$$\lim_{x\to\infty} \left[\frac{f(5x)}{f(x)} - 1\right] = 0$$

Q. 16 If the function $f:(-\infty,-1)\to(a,b]$ defined by $f(x)=e^{x^3-3x+1}$ is one-one and onto, the distance of the point $P(2b+4,a+2)$ from the line $x+e^{-3}y=4$ is :

Option 1:
$\sqrt{1+e^6}$

Option 2:
$2\sqrt{1+e^6}$

Option 3:
$4\sqrt{1+e^6}$

Option 4:
$3\sqrt{1+e^6}$

Correct Answer:
$2\sqrt{1+e^6}$

Solution:
$f:(-\infty,-1)\to(a,b]$

$f(x)=e^{x^3-3x+1}$

$f'(x)=e^{x^3-3x+1}\left(3x^2-3\right)$

$f'(x)=e^{x^3-3x+1}3(x-1)(x+1)$



$(-\infty,-1]\to\uparrow \mathrm{F^n}$

$\text{So } x \to -\infty$

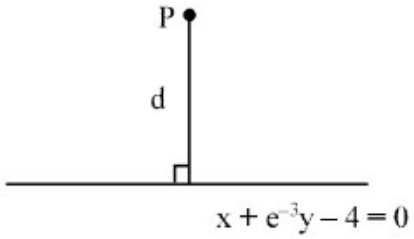
$\Rightarrow f(x) = e^{-\infty} \to 0 = a$

$x \to -1$

$\Rightarrow F(-1) = e^{-1+3+1} = e^3 = b$

$(a, b] \Rightarrow \left(0, e^3\right]$

$P\left(2e^3 + 4, 2\right)$



$$d = \frac{2e^3 + 4 + e^{-3}(2) - 4}{\sqrt{1 + (e^{-3})^2}}$$

$$d = \frac{2\left(e^3 + e^{-3}\right)}{\sqrt{\frac{e^6+1}{e^6}}}$$

$$v = \frac{\frac{2}{e^3}\left(1 + e^6\right)}{\frac{\sqrt{1+e^6}}{e^3}}$$

$$d = 2\sqrt{1 + e^6}$$

Q. 17 Let the mean and the variance of 6 observations $a, b, 68, 44, 48, 60$ be 55 and 194 , respectively. If $a > b$, then $a + 3b$ is

Option 1:
210

Option 2:
180

Option 3:
190

Option 4:
200

Correct Answer:
180

Solution:

$$\text{mean } = \frac{a+b+68+44+48+60}{6} = 55$$

$$a+b+220 = 55 \times 6 \Rightarrow 330$$

$$a+b = 110$$

$$\text{variance } = \frac{\sum x_i^2}{n} - (\bar{x})^2 = 194$$

$$\frac{a^2+b^2+(68)^2+(44)^2+(48)^2+(60)^2}{6}$$

$$-(55)^2 = 194$$

$$a^2+b^2+4624+1936+2304+3600 = 6 \times (3205+194)$$

$$a^2+b^2 = 6850$$

$$\text{Now } (a+b)^2 = (110)^2$$

$$a^2+b^2+2ab = 12100$$

$$6850+2ab = 12100$$

$$2ab = 12100-6850$$

$$2ab = 5250$$

$$ab = 2625$$

$\Rightarrow (110-b)b = 2625 \quad (\because a = 110-b)$

$b^2 - 110b + 2625 = 0$

$b = 35, 75$

$\text{but } a > b$

$a + b = 110$

$\text{So b} = 35 \text{ and a} = 75$

$\text{Now } \Rightarrow \text{a} + 3\,\text{b}$

$\Rightarrow 75 + 3(35) \Rightarrow 180$

Hence, the answer is the option (2).

Q. 18 The temperature $\mathrm{T(t)}$ of a body at time $\mathrm{t} = 0$ is $160^{\circ}\mathrm{F}$ and it decreases continuously as per the differential equation $\frac{\mathrm{dT}}{\mathrm{dt}} = -\mathrm{K(T-80)}$, where K is a positive constant. If $\mathrm{T(15)} = 120^{\circ}\mathrm{F}$, then $\mathrm{T(45)}$ is equal to

Option 1:
$85^{\circ}\mathrm{F}$

Option 2:
$95^{\circ}\mathrm{F}$

Option 3:
$90^{\circ}\mathrm{F}$

Option 4:
$80^{\circ}\mathrm{F}$

Correct Answer:
$90^{\circ}\mathrm{F}$

Solution:

$$\frac{dT}{dt} = -KT + 80\,K$$

$$\frac{dT}{dt} + KT = 80\,K$$

L.D.E. in T

$$T(I \cdot F) = \int (I \cdot F) Q(t) dt$$

$$I \cdot F = e^{\int K dt} = e^{Kt+C}$$

$$T\left(e^{Kt+C}\right) = \int e^{Kt} \cdot 80Ke^{c} dt$$

$$Te^{Kt+C} = \frac{e^{Kt}}{K} 80Ke^{C} + C$$

$$Te^{Kt} = 80e^{Kt} + C', \quad C' = \frac{C}{e^{C}}$$

$$t = 0 \Rightarrow T = 160$$

$$160 = 80 + C'$$

$$C' = 80$$

$$Te^{Kt} = 80\left(e^{Kt} + 1\right)$$

$$T = \frac{80\left(e^{Kt} + 1\right)}{e^{Kt}}$$

$$\text{at } t = 15,\ T = 120^{\circ}F$$

$$120 = 80\frac{\left(e^{15\,K} + 1\right)}{e^{15\,K}}$$

$$3e^{15\,K} = 2e^{15\,K} + 2$$

$$e^{15\,K} = 2$$

$$K = \frac{\ln 2}{15}$$

So now equation

$$T = \frac{80\left(e^{\frac{\ln 2}{15}t} + 1\right)}{\frac{\ln 2}{15}t}$$

if $t = 45$

$$T = \frac{80\left(e^{\frac{\ln 2}{15} \times 45} + 1\right)}{e^{\frac{\ln 2}{15} \times 45}}$$

$$T = \frac{80(9)}{8} = 90\,F$$

Hence, the answer is the option (3).

Q. 19 If $\alpha = \sin^{-1}(\sin(5))$ and $b = \cos^{-1}(\cos(5))$, then $a^2 + b^2$ is equal to

Option 1:
$8\pi^2 - 40\pi + 50$

Option 2:
$4\pi^2 - 20\pi + 50$

Option 3:
$4\pi^2 + 25$

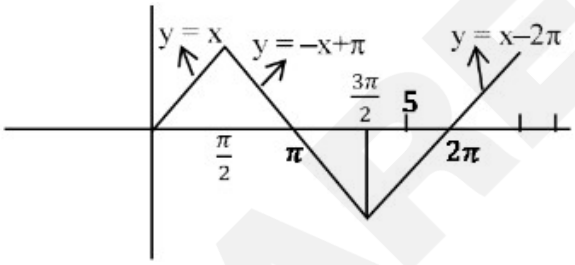
Option 4:
25

Correct Answer:
$8\pi^2 - 40\pi + 50$

Solution:
$a = \sin^{-1}(\sin(5))$

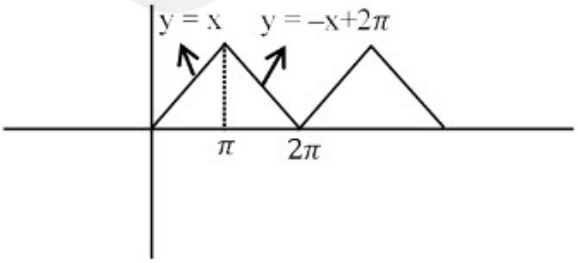
$b = \cos^{-1}(\cos(5))$

$\sin^{-1}(\sin x) =$



$a = 5 - 2\pi$

$\cos^{-1}(\cos x) =$



$b = 2\pi - 5$

Now $a^2 + b^2$

$\Rightarrow (5-2\pi)^2+(2\pi-5)^2$

$\Rightarrow 25+4\pi^2-20\pi+4\pi^2-20\pi+25$

$\Rightarrow 8\pi^2-40\pi+50$

Q. 20 Consider the function $f:(0,\infty)\to\mathbb{R}$ defined by $f(x)=e^{-|\log_e x|}$. If m and n be respectively the number of points at which f is not continuous and f is not differentiable, then $m+n$ is

Option 1:
2

Option 2:
3

Option 3:
0

Option 4:
1

Correct Answer:
1

Solution:

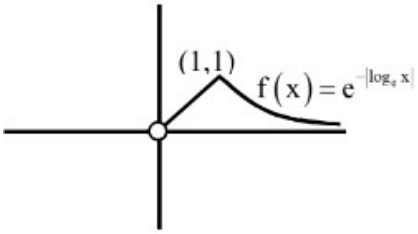
$f(x)=e^{-|\ln x|}$

$f:(0,\infty)\to R$

$f(x)=e^{-|\log_e x|}$

$$f(x)=\frac{1}{e^{|\ln x|}}=\begin{cases}\frac{1}{e^{-\ln x}}, & 0<x<1\\ \frac{1}{e^{\ln x}}, & x\geq 1\end{cases}$$

$$=\begin{cases}x, & 0<x<1\\ \frac{1}{x}, & x\geq 1\end{cases}$$



$f(x)$ is continuous in $(0,\infty) \Rightarrow m = 0$
from graph, $f(x)$ is not differentiable at 1 point at $x = 1$

$\Rightarrow n = 1$

therefore, $m + n = 1$

Q. 21 Let a, b, c be the lengths of three sides of a triangle satisfying the condition $(a^2+b^2)x^2 - 2b(a+c)x + (b^2+c^2) = 0$. If the set of all possible values of x is the interval (α, β), then $12(\alpha^2+\beta^2)$ is equal to_____.

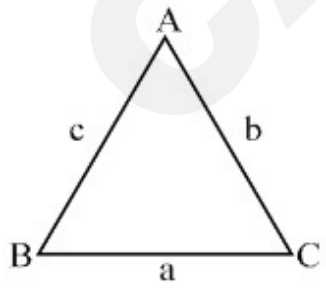
Correct Answer:
36

Solution:
$(a^2+b^2)x^2 - 2b(a+c)x + b^2 + c^2 = 0$

$(ax-b)^2 + (bx-c)^2 = 0$

$x = \frac{b}{a} \quad x = \frac{c}{b}$

Now



$|a-b| < c < (a+b)$

$b^2 = ac$

$|a-c| < b < a+c \qquad b^2 = ac \times \frac{a}{a}$

$1-\frac{c}{a} < \frac{b}{a} < 1+\frac{c}{a} \qquad \frac{b^2}{a^2} = \frac{c}{a}$

$1-\frac{c}{a} < x < 1+\frac{c}{a} \qquad x^2 = \frac{c}{a}$

$1-x^2 < x < 1+x^2$

Case I: $x < 1+x^2$

always + ve

Case II: $1-x^2 < x$

$-x < 1-x^2 < x$

Now $1-x^2 < x$

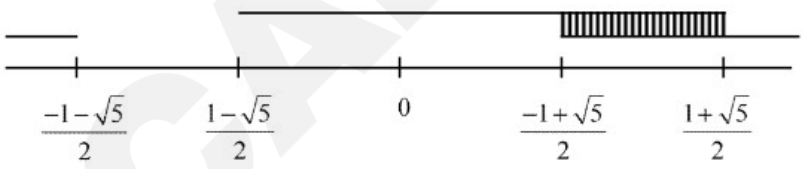
$x^2+x-1 > 0$

$x = \dfrac{-1 \pm \sqrt{5}}{2}$

$-x < 1-x^2$

Now $x^2-x-1 < 0$

$x = \dfrac{1 \pm \sqrt{5}}{2}$



$\alpha = \dfrac{-1+\sqrt{5}}{2} \quad , \quad \beta = \dfrac{1+\sqrt{5}}{2}$

Now $12\left(\left(\dfrac{-1+\sqrt{5}}{2}\right)^2 + \left(\dfrac{1+\sqrt{5}}{2}\right)^2\right)$

$= 36$

Q. 22 Let $A = \{1, 2, 3 \ldots\ldots\ldots 100\}$. Let R be a relation on A defined by $(x, y) \in R$ if and only if $2x = 3y$. Let R_1 be a symmetric relation on A such that $R \subset R_1$ and the number of elements in R_1 is n. Then, the minimum value of n is_____.

Correct Answer:
66

Solution:
$A = \{1, 2, 3 \ldots 100\}$

$R \Rightarrow 2x = 3y \Rightarrow y = \dfrac{2x}{3}$

$R = \{(3, 2), (6, 4), (9, 6) \ldots\ldots (99, 66)\}$

$n(R) = 33$

$\mathrm{R} \subset \mathrm{R}_1$

Now

$R_1 = \{(3, 2), (6, 4), (9, 6) \ldots (99, 66),$
$(2, 3), (4, 6), (6, 9) \ldots . (66, 99)\}$

$\Rightarrow$ minimum number of elements in $R_1 = 66$

Hence, the answer is the (66).

Q. 23 Let $\vec{a} = 3\hat{i} + 2\hat{j} + \hat{k}, \vec{b} = 2\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{c}$ be a vector such that $(\vec{a} + \vec{b}) \times \vec{c} = 2(\vec{a} \times \vec{b}) + 24\hat{j} - 6\hat{k}$ and $(\vec{a} - \vec{b} + \hat{i}) \cdot \vec{c} = -3$. Then $|\vec{c}|^2$ is equal to_____.

Correct Answer:
38

Solution:
$\vec{a} = 3\hat{i} + 2\hat{j} + \hat{k}$

$\vec{b} = 2\hat{i} - \hat{j} + 3\hat{k}$

Now

$(\vec{a}+\vec{b})\times\vec{c}=2(\vec{a}\times\vec{b})+24\hat{j}-6\hat{k} \quad \ldots(1)$

$(\vec{a}-\vec{b}+\hat{i})\cdot\vec{c}=-3 \quad \ldots(2)$

$\vec{a}+\vec{b}=5\hat{i}+\hat{j}+4\hat{k}$

$\vec{a}-\vec{b}=\hat{i}+3\hat{j}-2\hat{k}$

let $\vec{c}=x\hat{i}+yj+z\hat{k}$

$(5\hat{i}+\hat{j}+4\hat{k})\times(x\hat{i}+y\hat{j}+z\hat{k})=2(\vec{a}\times\vec{b})+24\hat{j}-6\hat{k}$

$(z-4y)\hat{i}+(4x-5z)\hat{j}+(5y-x)\hat{k}=14\hat{i}+10\hat{j}-20\hat{k}$

compare $z-4y=14 \quad \ldots.(1)$

$4x-5z=10 \quad \ldots.(2)$

$5y-x=-20 \quad \ldots.(3)$

Now by using .....(2)

$(\hat{i}+3\hat{j}-2\hat{k}+\hat{i})\cdot(x\hat{i}+y\hat{j}+z\hat{k})=3$

$2x+3y-2z=-3 \quad \ldots(4)$

Using (1), (2), (3), (4)

y = –3, x = 5, z = 2

$|c|^2=9+25+4=38$

Hence, the answer is the (38).

Q. 24 Let A be a 3×3 matrix and $\det(A)=2$. If $n=\det(\underbrace{\text{adj}(\text{adj}(\ldots\ldots(\text{adj}\,A)))}_{\text{2024-times}})$ then the remainder when n is divided by 9 is equal to_____.

Correct Answer:

7

Solution:

$|A| = 2$

$n = \underbrace{|adj(adj(....(adj\ A)))|}_{2024-\text{ times}}$

$= |A|^{(x-1)^{2024}}$

$\Rightarrow |A|^{2^{2024}} \Rightarrow 2^{2^{2024}}$

Now $2^{2024} \Rightarrow 2^2(2)^{2022}$

$\Rightarrow 2^2(2^3)^{674}$

$\Rightarrow 2^2(8)^{674}$

$\Rightarrow 4(9-1)^{674}$

$\Rightarrow 4(9-1)^{674} \rightarrow$ divided by 9 . Thus remainder is 4

So, $2^{2024} \rightarrow 9K + 4(K = \text{even})$

Now $\rightarrow 2^{9K+4} \Rightarrow 16.2^{9K}$

$\Rightarrow 16(2^3)^{3K}$

$\Rightarrow 16(8)^{3K}$

$\Rightarrow 16(9-1)^{3K} \rightarrow$ even

$\Rightarrow 9\alpha + 16\{\alpha \rightarrow$ integer $\}$

$\Rightarrow 9\alpha + 9 + 7$

$\Rightarrow 9(\alpha + 1) + 7$

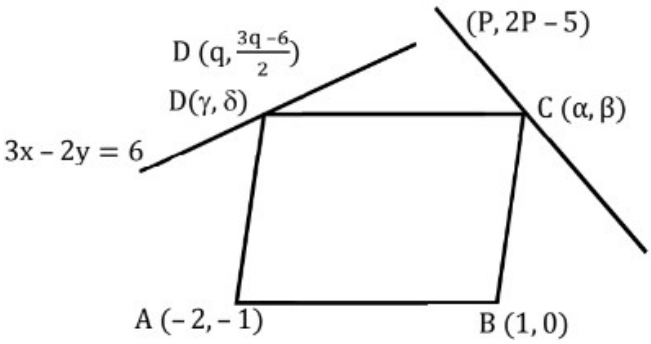
remainder $\rightarrow 7$

Hence, the answer is the (7).

Q. 25 Let $A(-2,-1), B(1,0), C(\alpha,\beta)$ and $D(\gamma,\delta)$ be the vertices of a parallelogram $ABCD$. If the point C lies on $2x - y = 5$ and the point D lies on $3x - 2y = 6$, then the value of $|\alpha + \beta + \gamma + \delta|$ is equal to_____.

Correct Answer:
32

Solution:



mid point of diagonals

$$\frac{-2+p}{2}=\frac{q+1}{2}$$

$$p-q-3=0$$

$$\frac{-1+2p-5}{2}=\frac{0+3q-6}{4}$$

$$2p-6=\frac{3q-6}{2}$$

$$4p-3q-6=0$$

$$3(p-q-3=0)$$

$$p+3=0$$

$$p=-3$$
$$q=-6$$

$$c(p,2p-5)$$
$$c(-3,-11)=(\alpha,\beta)$$

$$D(-6,-12)=(\gamma,\delta)$$

$$|\alpha+\beta+\gamma+\delta|$$

$$\Rightarrow|-3-11-6-12|\Rightarrow 32$$

Q. 26 Let the coefficient of x^r in the expansion of $(x+3)^{n-1}+(x+3)^{n-2}(x+2)+(x+3)^{n-3}(x+2)^2+.....+(x+2)^{n-1}$ be α_r. If $\sum_{r=0}^{n}\alpha_r=\beta^n-\gamma^n, \beta,\gamma\in\mathbb{N}$, then the value of $\beta^2+\gamma^2$ equals_____.

Correct Answer:
25

Solution:
$(x+3)^{n-1}+(x+3)^{n-2}(x+2)\ldots+(x+2)^{n-1}$

$$\Rightarrow (x+3)^{n-1}\left(1+\left(\frac{x+2}{x+3}\right)+\left(\frac{x+2}{x+3}\right)^2+\cdots+\left(\frac{x+2}{x+3}\right)^{n-1}\right)$$

$$\frac{1\cdot\left(\left(\frac{x+2}{x+3}\right)^n-1\right)}{\frac{x+2}{x+3}-1}\times(x+3)^{n-1}$$

$$\Rightarrow \frac{\left(\left(\frac{x+2}{x+3}\right)^n-1\right)}{\frac{x+2-x-3}{x+3}}\times(x+3)^{n-1}$$

$$\Rightarrow \left(1-\left(\frac{x+2}{x+3}\right)^n\right)(x+3)^n$$

Put $x=1$

$=4^n-3^n$

$=\beta^n-\gamma^n$

$\Rightarrow \beta=4,\gamma=3$

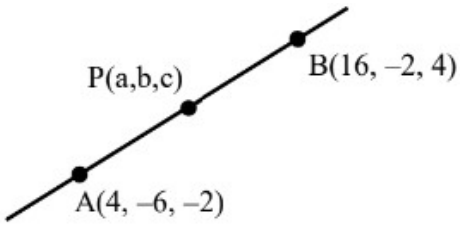
$\beta^2+\gamma^2=25$

Hence, the answer is the (25).

Q. 27 A line passes through $A(4, -6, -2)$ and $B(16, -2, 4)$. The point $P(a, b, c)$, where a, b, c are non-negative integers, on the line AB lies at a distance of 21 units, from the point A. The distance between the points $P(a, b, c)$ and $Q(4, -12, 3)$ is equal to_____.

Correct Answer:
22

Solution:



Eq. of AB

$$\frac{x-4}{12} = \frac{y+6}{4} = \frac{z+2}{6} = \alpha.$$

$$a = 12\alpha + 4$$

$$b = 4\alpha - 6$$

$$c = 6\alpha - 2$$

Distance PA = 21

$$\sqrt{(12\alpha + 4 - 4)^2 + (4\alpha - 6 + 6)^2 + (6\alpha - 2 + 2)^2} = 21$$

$$144\alpha^2 + 16\alpha^2 + 36\alpha^2 = 441$$

$$196\alpha^2 = 441$$

$$\alpha^2 = \frac{441}{196}$$

$$\alpha = \pm\frac{21}{14} \Rightarrow \pm\frac{3}{2} \quad (\text{a, b, c are non negative})$$

$\left(\alpha-\frac{8}{2}\right)(a,b,c) \rightarrow$ the

$a=12\times\frac{3}{2}+4\Rightarrow 22$

$b=4\times\frac{3}{2}-6\Rightarrow 0$

$c=6\times\frac{3}{2}-2\Rightarrow 7$

$\mathrm{P}(22,0,7), \quad \mathrm{Q}(4,-12,3)$

$\mathrm{PQ}=\sqrt{(18)^2+(12)^2+(4)^2}$

$\mathrm{PQ}=\sqrt{324+144+16}$

$\mathrm{PQ}=22$

Q. 28 Let $y=y(x)$ be the solution of the differential equation $\sec^2 x dx+\left(e^{2y}\tan^2 x+\tan x\right)dy=0, 0<x<\frac{\pi}{2}, y\left(\frac{\pi}{4}\right)=0$. If $y\left(\frac{\pi}{6}\right)=\alpha$. then $e^{8\alpha}$ is equal to_____.

Correct Answer:
9

Solution:
$\sec^2 x dx+\left(e^{2y}\tan^2 x+\tan x\right)dy=0$

$\text{Put} \quad \tan x=t$

$\sec^2 x dx=dt$

$dt+\left(e^{2y}t^2+t\right)dy=0$

$\frac{1}{t^2}\times\frac{dt}{dy}+\frac{1}{t}=-e^{2y}$

$\text{Put} \quad \frac{1}{t}=u$

$$-\frac{1}{t^2}\frac{dt}{dy}=\frac{du}{dy}$$
$$-\frac{du}{dy}+u=-e^{2y}$$

$$\frac{du}{dy}-u=e^{2y}$$

$$\text{I.f.}=e^{-\int dy}=e^{-y}$$

$$u\cdot e^{-y}=\int e^{-y}e^{2y}dy$$

$$\frac{1}{\tan xe^{y}}=e^{y}+C$$
$$x=\frac{\pi}{4},y=0$$

Now

$1=1+C \qquad C=0$
$x=\frac{\pi}{6} \qquad y=\alpha$
$\frac{\sqrt{3}}{e^{\alpha}}=e^{\alpha}$
$=e^{2\alpha}=\sqrt{3}$
$e^{8\alpha}=9$

Hence, the answer is the (9).

Q. 29 $\frac{120}{\pi^3}\int_0^{\pi}\frac{x^2\sin x\cos x}{\sin^4 x+\cos^4 x}dx$ is equal to_____.

Correct Answer:
15

Solution:

$$\frac{120}{\pi^3}\int_0^{\pi}\frac{x^2\sin x\cos x}{\sin^4 x+\cos^4 x}dx$$

$$\Rightarrow \frac{120}{\pi^3}\int_0^{\pi/2}\frac{\sin x\cos x}{\sin^4 x+\cos^4 x}\left(x^2-(\pi-x)^2\right)dx$$

$$\Rightarrow \frac{120}{\pi^3}\int_0^{\pi/2}\frac{\sin x\cdot\cos x}{\sin^4 x+\cos^4 x}\left(2\pi x-\pi^2\right)dx$$

$$\Rightarrow \frac{120}{\pi^3}\left[2\pi\int_0^{\pi/2}\frac{x\sin x\cdot\cos x}{\sin^4 x+\cos^4 x}dx-\pi^2\int_0^{\pi/2}\frac{\sin x\cdot\cos x}{\sin^4 x+\cos^4 x}dx\right]$$

$$\Rightarrow \frac{120}{\pi^3}\left[2\pi\times\frac{\pi}{4}\int_0^{\pi/2}\frac{\sin x\cdot\cos x}{\sin^4 x+\cos^4 x}dx-\pi^2\int_0^{\pi/2}\frac{\sin x\cdot\cos x}{\sin^4 x+\cos^4 x}dx\right]$$

$$\Rightarrow \frac{120}{\pi^3}\left[\frac{-\pi^2}{2}\int_0^{\pi/2}\frac{\sin x\cdot\cos x}{\sin^4 x+\cos^4 x}dx\right]$$

$$\Rightarrow \frac{120}{\pi^3}\left[\frac{-\pi^2}{2}\int_0^{\pi/2}\frac{\sin x\cdot\cos x}{1-2\sin^2 x\cos^2 x}dx\right]$$

$$\Rightarrow \frac{120}{\pi^3}\left[\frac{-\pi^2}{2}\int_0^{\pi/2}\frac{\frac{1}{2}\sin 2x}{1-\frac{1}{2}\sin^2 2x}dx\right.$$

$$\Rightarrow \frac{120}{\pi^3}\left[\frac{-\pi^2}{2}\int_0^{\pi/2}\frac{\sin 2x}{2-\sin^2 2x}dx\right]$$

$$\Rightarrow \frac{120}{\pi^3}\left[\frac{-\pi^2}{2}\int_0^{\pi/2}\frac{\sin 2x}{1+\cos^2 2x}dx\right]$$

take $\cos 2x = t, -2\sin 2x dx = dt$

$$\Rightarrow \frac{120}{\pi^3}\left[\frac{-\pi^2}{2}\int_1^{-1}\frac{-\frac{1}{2}dt}{1+t^2}\right]$$

$$\Rightarrow \frac{120}{\pi^3}\left[\frac{-\pi^2}{2}\int_{-1}^{1}\frac{\frac{1}{2}dt}{1+t^2}\right]$$

$$\Rightarrow \frac{120}{\pi^3}\times\left[\frac{-\pi^2}{4}\left(\tan^{-1}(t)\right)_{-1}^{1}\right]$$

$$\Rightarrow \frac{120}{\pi^3}\times\left[\frac{-\pi^2}{4}\times\frac{\pi}{2}\right]$$

$$\Rightarrow 15$$

Hence, the answer is the (15).

Q. 30 If $\lim_{x\to 0}\frac{ax^2e^x-b\log_e(1+x)+cxe^{-x}}{x^2\sin x}=1$, then $16\left(a^2+b^2+c^2\right)$ is equal to

Correct Answer:
81

Solution:

$$\lim_{x\to 0}\frac{ax^2e^x-b\log_e(1+x)+cxe^{-x}}{x^2\sin x}=1$$

$$\lim_{x\to 0}\frac{ax^2\left(1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}\ldots\right)-b\left(x-\frac{x^2}{2}+\frac{x^3}{3}+\ldots\right)+cx\left(1-x+\frac{x^2}{2!}\right)}{x^2\left(x-\frac{x^3}{3!}+\frac{x^5}{5!}+\ldots\right)}$$

$$\lim_{x\to 0}\frac{(-b+c)x+x^2\left(a+\frac{b}{2}-c\right)+\left(a-\frac{b}{3}+\frac{c}{2}\right)x^3+\ldots}{x^3\left(1-\frac{x^2}{3!}+\frac{x^5}{5!}+\ldots\right)}$$

$-b+c=0$ $\quad a+\frac{b}{2}-c=0$

$c=b$ $\quad a+\frac{b}{2}-b=0$

$\quad a=\frac{b}{2}$

$$a-\frac{b}{3}+\frac{c}{2}=1$$

$\frac{b}{2}-\frac{b}{3}+\frac{b}{2}=1 \quad \Rightarrow \frac{3b-2b+3b}{6}=1$

$4b=6 \qquad c=\frac{3}{2} \qquad a=\frac{b}{2}$

$b=\frac{3}{2} \qquad a=\frac{3}{2\times 2} \Rightarrow \frac{3}{4}$

$a=\frac{3}{4}$

$16\left(a^2+b^2+c^2\right)$

$16\left(\frac{9}{16}+\frac{9}{4}+\frac{9}{4}\right)$

$16\left(\frac{9+36+36}{16}\right) \Rightarrow 81$

Hence, the answer is the (81).

CAREERS360

1 Feb - Morning Shift

Physics

Q. 1 With rise in temperature, the Young's modules of elasticity:

Option 1:
changes erratically

Option 2:
decreases

Option 3:
increases

Option 4:
remains unchanged

Correct Answer:
decreases

Solution:
The temperature dependence on Young Modulas is given by :

$$\mathrm{y}=\frac{\mathrm{F/A}}{\Delta\ell/\ell}=\frac{\mathrm{F/A}}{\alpha\Delta\mathrm{T}} \quad \text{as } \Delta\mathrm{T}\uparrow,\mathrm{y}\downarrow$$

Hence, the answer is the option (2).

Q. 2 If R is the radius of the earth and the acceleration due to gravity on the surface of earth is $g=\pi^2\ \mathrm{m/s^2}$, then the length of the second's pendulum at a height $h=2R$ from the surface of earth will be, :

Option 1:
$\frac{2}{9}\ \mathrm{m}$

Option 2:
$\frac{1}{9}\ \mathrm{m}$

Option 3:
$\frac{4}{9}\ \mathrm{m}$

Option 4:
$\frac{8}{9}\ \mathrm{m}$

Correct Answer:
$\frac{1}{9}\ \mathrm{m}$

Solution:
$\mathrm{g} = \frac{\mathrm{GM}}{\mathrm{R}^2} = \pi^2$

$\mathrm{g}' = \frac{\mathrm{GM}}{9\mathrm{R}^2} = \frac{\pi^2}{9}$

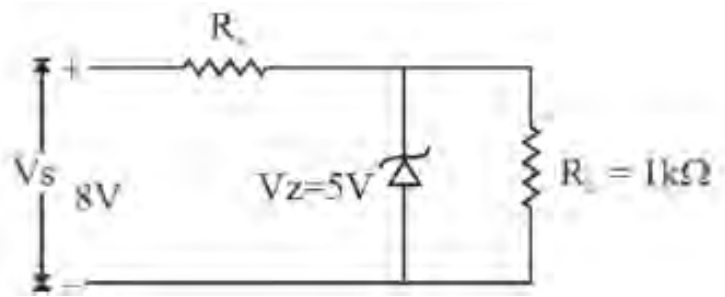
We know, $\mathrm{T} = 2\pi\sqrt{\frac{\ell}{\mathrm{g}}}$

$2 = 2\pi\sqrt{\frac{\ell \times 9}{\pi^2}}$

On solving, $\ell = \frac{1}{9}\ \mathrm{m}$

Hence, the answer is the option (2).

Q. 3 In the given circuit if the power rating of Zener diode is $10\ \mathrm{mW}$, the value of series resistance R_s to regulate the input unregulated supply is:



Option 1:
$5\mathrm{k}\Omega$

Option 2:
$10\mathrm{k}\Omega$

Option 3:
$1\mathrm{k}\Omega$

Option 4:
$10\mathrm{k}\Omega$

Correct Answer:
$1\mathrm{k}\Omega$

Solution:

$\mathrm{V_{RS}} = 3\ \mathrm{V}$

$\mathrm{I_{LR}} = \frac{5}{10^3} = 5\ \mathrm{mA}$

$I_{Z(\ \mathrm{Max.}\ )} = \frac{10}{5} = 2\ \mathrm{mA}$

$\mathrm{I_{((Min.)\ )}} = 0$

$\mathrm{I_{S\ (Max.\ )}} = 7\ \mathrm{mA}$

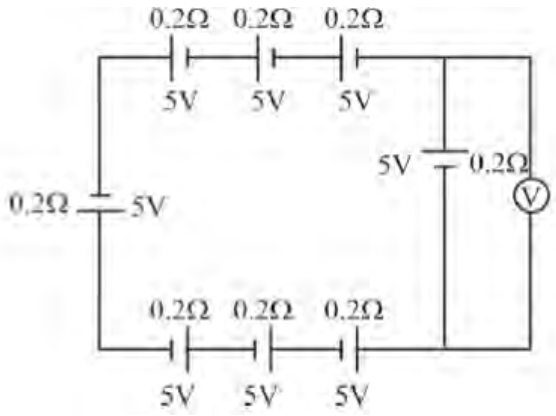
$\mathrm{I_{S(Min.)}} = 5\ \mathrm{mA}$

$\mathrm{Rs_{(Max.\ )}} = \frac{3}{5}\ \mathrm{K}\Omega$

$\mathrm{R_{S(\ Min.\ )}} = \frac{3}{7}\ \mathrm{K}\Omega$

$\frac{3}{7}\mathrm{k}\Omega < \mathrm{R_S} < \frac{3}{5}\mathrm{k}\Omega$

Q. 4 The reading in the ideal voltmeter (V) shown in the given circuit diagram is:



Option 1:
5V

Option 2:
10V

Option 3:
0V

Option 4:
3V

Correct Answer:
0V

Solution:

$i = \frac{40}{1.6}$

$i = 25A$

$\text{Reading of voltmeter } = 5 - I \times R$

$= 5 - 25 \times .2$

$ =0

CAREERS360

Q. 5 Two identical capacitors have same capacitance C. One of them is charged to the potential V and other to the potential 2V. The negative ends of both are connected together. When the positive ends are also joined together, the decrease in energy of the combined system is:

Option 1:
$\frac{1}{4}\mathrm{CV}^2$

Option 2:
$2\mathrm{CV}^2$

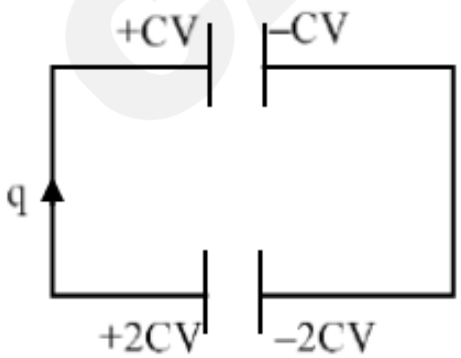
Option 3:
$\frac{1}{2}\mathrm{CV}^2$

Option 4:
$\frac{3}{4}\mathrm{CV}^2$

Correct Answer:
$\frac{1}{4}\mathrm{CV}^2$

Solution:

Initial energy $= \frac{1}{2}\mathrm{CV}^2 + \frac{1}{2}\mathrm{C} \times 4\ \mathrm{V}^2 = \frac{5\mathrm{CV}^2}{2}$



$\frac{2\mathrm{CV}-\mathrm{q}}{\mathrm{C}} = \frac{\mathrm{CV}+\mathrm{q}}{\mathrm{C}}$

$2\mathrm{CV} - \mathrm{CV} = 2\mathrm{q}$

$q = \frac{CV}{2}$

$\text{Final charge on capacitor } = \frac{3CV}{2}$

$\text{Energy }_{\text{final}} = 2 \times \left(\frac{3CV}{2}\right)^2 \times \frac{1}{2C} = \frac{9CV^2}{4}$

$\text{Decrease in energy } ^2\frac{5CV^2}{2} - \frac{9CV^2}{4} = \frac{CV^2}{4}$

Q. 6 Two moles a monoatomic gas is mixed with six moles of a diatomic gas. The molar specific heat of the mixture at constant volume is:

Option 1:
$\frac{9}{4}R$

Option 2:
$\frac{7}{4}R$

Option 3:
$\frac{3}{2}R$

Option 4:
$\frac{5}{2}R$

Correct Answer:
$\frac{9}{4}R$

Solution:
The effective heat capacity of the mixture at constant volume is given by :

$C_{V_{mis}} = \frac{n_1C_{V_1}+n_2C_{V_2}}{n_1+n_2}$

$= \frac{2\times\frac{3R}{2}+6\times\frac{5R}{2}}{8}$

$= \frac{9R}{4}$

Hence, the answer is the option (1).

Q. 7 A ball of mass 0.5 kg is attached to a string of length 50cm. The balls is rotated on a horizontal circular path about its vertical axis. The maximum tension that the string can bear is 400 N. The maximum possible value of angular velocity of the ball in rad/s is,:

Option 1:
1600

Option 2:
40

Option 3:
1000

Option 4:
20

Correct Answer:
40

Solution:
The maximum tension that the string can bear is given by -

$T = m\omega^2 R$

$400 = \frac{1}{2} \times \omega^2 \times \frac{1}{2}$

$\omega^2 = 1600$

$\omega = 40 \text{rad/sec.}$

Hence, the answer is the option (2).

Q. 8 A parallel plate capacitor has a capacitance $C = 200\text{pF}$. It is connected to $230\ \text{V}$ ac supply with an angular frequency 300rad/s. The rms value of conduction current in the circuit and displacement current in the capacitor respectively are:

Option 1:
$1.38\mu\text{A}$ and $1.38\mu\text{A}$

Option 2:
$14.3\mu\text{A}$ and $143\mu\text{A}$

Option 3:
$13.8\mu\text{A}$ and $138\mu\text{A}$

Option 4:
$13.8\mu\text{A}$ and $13.8\mu\text{A}$

Correct Answer:
$13.8\mu\text{A}$ and $13.8\mu\text{A}$

Solution:
Given data:
Capacitance $C = 200\,\text{pF}$
Voltage $V = 230\,\text{V}$
Angular frequency $\omega = 300\,\text{rad/s}$
First, convert the capacitance from picofarads to farads:

$$C = 200\,\text{pF} = 200 \times 10^{-12}\,\text{F}$$

The capacitive reactance X_C is given by:

$$X_C = \frac{1}{\omega C}$$

Substitute the values of ω and C:

$$X_C = \frac{1}{300 \times 200 \times 10^{-12}} = \frac{1}{60 \times 10^{-9}} = \frac{10^9}{60}\,\Omega = \frac{10^8}{6}\,\Omega = 1.67 \times 10^7\,\Omega$$

RMS value of conduction current
The rms value of the voltage V_{rms} is given as $230\,\text{V}$.

The rms value of the conduction current I_{rms} in the circuit can be found using Ohm's law for AC circuits:

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{X_C}$$

Substitute the values of V_{rms} and X_C:

$$I_{\text{rms}} = \frac{230}{1.67 \times 10^7}$$

$$I_{\text{rms}} \approx \frac{230}{1.67 \times 10^7} \approx 13.8 \times 10^{-6}\,\text{A} \approx 13.8\,\mu\text{A}$$

Thus, the rms value of the conduction current is $13.8\,\mu\text{A}$ and the rms value of the displacement current is also $13.8\,\mu\text{A}$.

Hence, the rms value of the conduction current and the displacement current in the capacitor are both $13.8\,\mu\text{A}$.

Hence, the answer is the option (4).

Q. 9 The pressure and volume of an ideal gas are related as $\text{PV}^{\frac{3}{2}} = \text{K}$ (Constant). The work done when the gas is taken from state $A\,(P_1, V_1, T_1)$ to state $B\,(P_2, V_2, T_2)$ is:

Option 1:
$2\,(\text{P}_1\,\text{V}_1 - \text{P}_2\,\text{V}_2)$

Option 2:
$2\left(\mathrm{P}_2\,\mathrm{V}_2-\mathrm{P}_1\,\mathrm{V}_1\right)$

Option 3:
$2\left(\sqrt{\mathrm{P}_1}\,\mathrm{V}_1-\sqrt{\mathrm{P}_2}\,\mathrm{V}_2\right)$

Option 4:
$2\left(P_2\sqrt{V_2}-P_1\sqrt{V_1}\right)$

Correct Answer:
$2\left(\mathrm{P}_1\,\mathrm{V}_1-\mathrm{P}_2\,\mathrm{V}_2\right)$

Solution:
$\mathrm{PV}^{3/2}=\mathrm{K}$

$\mathrm{WD}=\frac{\mathrm{P}_2\,\mathrm{V}_2-\mathrm{P}_1\,\mathrm{V}_1}{1-\mathrm{x}}$

$=\frac{\mathrm{P}_2\,\mathrm{V}_2-\mathrm{P}_1\,\mathrm{V}_1}{1-\frac{3}{2}}$

$=2\left(\mathrm{P}_1\,\mathrm{V}_1-\mathrm{P}_2\,\mathrm{V}_2\right)$

Hence, the answer is the option 1.

Q. 10 A galvanometer has a resistance of 50Ω and it allows a maximum current of $5\ \mathrm{mA}$. It can be converted into a voltmeter to measure up to $100\ \mathrm{V}$ by connecting in series a resistor of resistance:

Option 1:
5975Ω

Option 2:
20050Ω

Option 3:
19950Ω

Option 4:
19500Ω

Correct Answer:
19950Ω

Solution:
$\mathrm{i_g} = 5\ \mathrm{mA}$

$\mathrm{R_g} = 50\Omega$

$100 = 5 \times 10^{-3}(50 + \mathrm{S})$

$10^5 = 5(50 + \mathrm{S})$

$20000 - 50 = \mathrm{S}$

$\mathrm{S} = 19950\Omega$

Hence, the answer is the option 3.

Q. 11 The de Broglie wavelengths of a proton and an α particle are λ and 2λ respectively. The ratio of the velocities of the proton and α particle will be:

Option 1:
1:8

Option 2:
1:2

Option 3:
4:1

Option 4:
8:1

Correct Answer:
8:1

Solution:
$P = \frac{h}{\lambda}$

$mv = \frac{h}{\lambda} \quad \Rightarrow \quad v = \frac{h}{m\lambda}$

$\frac{v_p}{v_\alpha} = \frac{m_\alpha \lambda_\alpha}{m_p \lambda_p} = \frac{4\, m_p \times 2\lambda}{m_p \times \lambda} = 8:1$

Hence, the answer is the option 4.

Q. 12 10 divisions on the main scale of a Vernier calliper coincide with 11 divisions on the Vernier scale. If each division on the main scale is of 5 units, the least count of the instrument is:

Option 1:
1/2

Option 2:
10/11

Option 3:
50/11

Option 4:
5/11

Correct Answer:
5/11

Solution:
$11\text{VSD} = 10\text{MSD}$

$1\text{VSD} = \frac{10}{11}\text{MSD}$

$= \frac{50}{11}\ \text{units}$

$\text{Least count } = 1\text{MSD} - 1\text{VSD}$

$= 5 - \frac{50}{11} = \frac{5}{11} \text{ units}$

Hence, the answer is the option 4.

Q. 13 In a series LCR circuit, the capacitance is changed from C to 4C. To keep the resonance frequency unchanged, the new inductance should be:

Option 1:
reduced by $\frac{1}{4}\ \text{L}$

Option 2:
increased by $2\ \text{L}$

Option 3:
reduced by $\frac{3}{4}\ \text{L}$

Option 4:
increased to $4\ \text{L}$

Correct Answer:
reduced by $\frac{3}{4}\ \text{L}$

Solution:

$\omega = \frac{1}{\sqrt{\text{LC}}} \ldots \text{(i)}$

$\omega = \frac{1}{\sqrt{\text{L}' \times 4\text{C}}} \ldots \text{(ii)}$

on comparing (i) & (ii)

$\left(L' = \frac{L}{4}\right)$

Hence, the answer is the option 3.

Q. 14 The radius (r), length (1) and resistance (R) of a metal wire were measured in the laboratory as

$\mathrm{r} = (0.35 \pm 0.05)\mathrm{cm}$

$\mathrm{R} = (100 \pm 10)\mathrm{ohm}$

$\mathrm{l} = (15 \pm 0.2)\mathrm{cm}$

The percentage error in resistivity of the material of the wire is:

Option 1:
25.6%

Option 2:
39.9%

Option 3:
37.3%

Option 4:
35.6%

Correct Answer:
39.9%

Solution:

$R = \frac{\rho L}{A}$

$\rho = \frac{R\pi r^2}{L}$

$\frac{d\rho}{\rho} = \frac{dR}{R} + \frac{dL}{L} + 2\frac{dr}{r}$

$\frac{d\rho}{\rho} = \left(\frac{10}{100} + \frac{.2}{15} + 2 \times \frac{.05}{.35}\right) \times 100\%$

$\left(\frac{d\rho}{\rho}\right) \times 100 = 39.9\%$

Hence, the answer is the option 2.

Q. 15 The dimensional formula of angular impulse is:

Option 1:
$[\mathrm{ML}^{-2}\ \mathrm{T}^{-1}]$

Option 2:
$[\mathrm{ML}^{2}\ \mathrm{T}^{-2}]$

Option 3:
$[\mathrm{MLT}^{-1}]$

Option 4:
$[\mathrm{ML}^{2}\ \mathrm{T}^{-1}]$

Correct Answer:
$[\mathrm{ML}^{2}\ \mathrm{T}^{-1}]$

Solution:
Angular impulse = change in angular momentum.

$$[\text{Angular impulse } = [\text{Angular momentum }] = [\mathrm{mvr}]$$
$$= [ML^2T^{-1}]$$

Hence, the answer is the option 4.

Q. 16 A simple pendulum of length $1\ \mathrm{m}$ has a wooden bob of mass $1\ \mathrm{kg}$. It is struck by a bullet of mass $10^{-2}\ \mathrm{kg}$ moving with a speed of $2 \times 10^{2}\ \mathrm{ms}^{-1}$. The bullet gets embedded into the bob. The height to which the bob rises before swinging back is. (use $g = 10\ \mathrm{m/s^2}$)

Option 1:
$0.30\ \mathrm{m}$

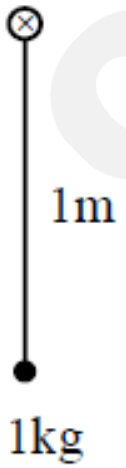
Option 2:
$0.20\ \mathrm{m}$

Option 3:
$0.35\ \mathrm{m}$

Option 4:
$0.40\ \mathrm{m}$

Correct Answer:
$0.20\ \mathrm{m}$

Solution:



$$\mathrm{P_i} = \mathrm{P_f} \Rightarrow \quad 10^{-2} \times 2 \times 10^{2} = (1 + .01)\mathrm{v}$$

$$v = \frac{2}{1.01} = 1.98 \text{ m/sec.}$$

$$\frac{1}{2} \times m_T \times v^2 = m_T g h$$

$$h = \frac{v^2}{2g} = \frac{1.98 \times 1.98}{20}$$

$$h = 0.20 \text{ m}$$

Q. 17 A particle moving in a circle of radius R with uniform speed takes time T to complete one revolution. If this particle is projected with the same speed at an angle Θ to the horizontal, the maximum height attained by it is equal to $4R$. The angle of projection Θ is then given by:

Option 1:

$$\sin^{-1}\left[\frac{2gT^2}{\pi^2 R}\right]^{\frac{1}{2}}$$

Option 2:

$$\sin^{-1}\left[\frac{\pi^2 R}{2gT^2}\right]^{\frac{1}{2}}$$

Option 3:

$$\cos^{-1}\left[\frac{2gT^2}{\pi^2 R}\right]^{\frac{1}{2}}$$

Option 4:

$$\cos^{-1}\left[\frac{\pi R}{2gT^2}\right]^{-\frac{1}{2}}$$

Correct Answer:

$$\sin^{-1}\left[\frac{2gT^2}{\pi^2 R}\right]^{\frac{1}{2}}$$

Solution:

$$T = \frac{2\pi R}{v}$$

$$\Rightarrow v = \frac{2\pi R}{T}$$

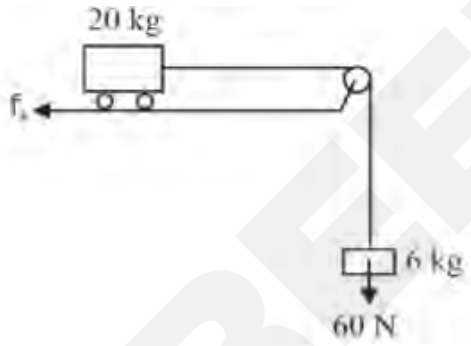
$$4R = \frac{v^2 \sin^2\theta}{2g}$$

$$\sin^2\theta = \frac{8RgT^2}{4\pi^2R^2} = \frac{2gT^2}{\pi^2R}$$

$$\theta = \sin^{-1}\left(\frac{2gT^2}{\pi^2R}\right)^{1/2}$$

Hence, the answer is the option 1.

Q. 18 Consider a block and trolley system as shown in figure. If the coefficient of kinetic friction between the trolley and the surface is 0.04 , the acceleration of the system in ms^{-2} is:
(Consider that the string is massless and unstretchable and the pulley is also massless and frictionless):



Option 1:
3

Option 2:
4

Option 3:
2

Option 4:
1.2

Correct Answer:
2

Solution:
$\mathrm{f_{max}} = .04 \times 200 = 8\ \mathrm{N}$

$\text{Acceleration } = \frac{60-8}{26} = 2\ \mathrm{m/s^2}$

Q. 19 The minimum energy required by a hydrogen atom in ground state to emit radiation in Balmer series is nearly:

Option 1:
1.5 eV

Option 2:
13.6 eV

Option 3:
1.9 eV

Option 4:
12.1 eV

Correct Answer:
12.1 eV

Solution:
Transition from $\mathrm{n} \rightarrow 1$ to $\mathrm{n} \rightarrow 3$

$\mathrm{E} = 13.6 \times Z\left[\frac{1}{n_2^2} - \frac{1}{n_1^2}\right]$

$= 13.6 \times 1 \times \left[\frac{1}{1} - \frac{1}{9}\right]$

$= 13.6 \times \frac{8}{9} = 12.1\mathrm{eV}$

Hence, the answer is the option 4.

Q. 20 A monochromatic light of wavelength 6000 $\mathring{A}$ is incident on the single slit of width 0.01 mm. If the diffraction pattern is formed at the focus of the convex lens of focal length 20 cm, the linear width of the central maximum is:

Option 1:
60 mm

Option 2:
24 mm

Option 3:
120 mm

Option 4:
12 mm

Correct Answer:
24 mm

Solution:

$$\text{Angular width} = \frac{2\lambda}{a}$$

$$\text{Linear width} = \frac{2\lambda}{a} \cdot f$$

$$= \frac{2\times6000\times10^{-10}\times20\times10^{-2}}{.01\times10^{-3}}$$

$$= 24 \times 10^{-3}\ \text{m}$$

$$= 24\ \text{mm}$$

Hence, the answer is the option 2.

Q. 21 A regular polygon of 6 sides is formed by bending a wire of length 4π meter. If an electric current of $4\pi\sqrt{3}$ A is flowing through the sides of the polygon, the magnetic field at the centre of the polygon would be $x \times 10^{-7}$ T. The value of x is_____.

Correct Answer:
72

Solution:
$B_C = \frac{\mu_0 i}{4\pi \times a\cos 30} \times 2\sin 30 \times n$

$= \frac{10^{-7} \times 4\pi\sqrt{3} \times 2 \times \tan 30 \times 6 \times 3}{2\pi}$

$= 72 \times 10^{-7} = x \times 10^{-7}$

$x = 72$
Hence, the answer is (72).

Q. 22 A rectangular loop of sides 12 cm and 5 cm with its sides parallel to the x-axis and y-axis respectively, moves with a velocity of $5\ cm/s$ in the positive x axis direction, in a space containing a variable magnetic field in the positive z direction. The field has a gradient of $10^{-3}\ T/cm$ along the negative x direction and it is decreasing with time at the rate of $10^{-3}\ T/s$. If the resistance of the loop is $6\ m\Omega$, the power dissipated by the loop as heat is_____ $\times 10^{-9}$ W.

Correct Answer:
216

Solution:
$\frac{dB}{dx} = \frac{-10^{-3}}{10^{-2}}$

$\int_B^{B_0} dB = -\int_0^{-x} 10^{-1} dx$

$B_0 - B = -\left[-\frac{x}{10}\right]$

$B = B_0 - \frac{x}{10}$

$B = B_0 - \frac{x}{10}$

Motional emf in $AB\&CD = 0$

Motional emf in $BC = B_0 \ell v = \varepsilon_1$

Motional emf in $AD = \left(B_0 - \left(\frac{-12\times10^{-2}}{10}\right)\right)\ell v$

Motional emf in $\varepsilon_2 = \left(B_0 + 12 \times 10^{-3}\right)\ell v$

$\text{Motional emf in } \varepsilon_2 = \left(B_0 + 12 \times 10^{-3}\right)\ell v$

$\varepsilon_{eq} = \varepsilon_2 - \varepsilon_1 = 12 \times 10^{-3} \times 5 \times 10^{-2} \times 5 \times 10^{-2}$

$= 300 \times 10^{-7} v$

$\varepsilon = \frac{AdB}{dt} = 60 \times 10^{-4} \times 10^{-3}$

$= 60 \times 10^{-7} v$

$\varepsilon_{net} = (300 + 60) \times 10^{-7} = 360 \times 10^{-7}$

$P = \frac{(\varepsilon_{net})^2}{R} = \frac{360\times360\times10^{-14}}{6\times10^{-3}}$

$= 21600 \times 10^{-11}$

$= 216 \times 10^{-9} w$

Hence, the answer is (216).

Q. 23 The distance between object and its 3 times magnified virtual image as produced by a convex lens is 20 cm.The focal length of the lens used is _____ cm.

Correct Answer:
15

Solution:

$m = \frac{v}{u} \quad \Rightarrow \quad v = 3u$

$\because v - u = 20$

$3u - u = 20 \quad \Rightarrow \quad u = 10 \text{ cm}, v = 30 \text{ cm}$

$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

$= -\frac{1}{30} + \frac{1}{10} = \frac{-1+3}{30} = \frac{2}{30}$

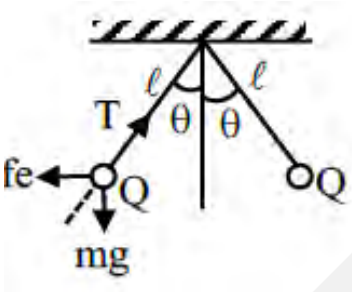
$f = 15\ cm$

Hence, the answer is (15).

Q. 24 Two identical charged spheres are suspended by strings of equal lengths. The strings make an angle θ with each other. When suspended in water the angle remains the same. If density of the material of the sphere is $1.5\ g/cc$, the dielectric constant of water will be _____(Take density of water $= 1\ g/cc$)

Correct Answer:
3

Solution:



$\tan\theta = \frac{KQ^2}{(2\ell\sin\theta)^2 \times \rho v g}$ .....(i)

$\tan\theta = \frac{KQ^2}{\varepsilon(2\ell\sin\theta)^2 \times (\rho v - \rho_w v)g}$ .....(ii)

Compare (i) & (ii)

$\varepsilon\,(\rho v - \rho_{wv} vg = \rho vg$

$\varepsilon = \frac{1.5}{1.5-1} = \frac{1.5}{.5} = 3$

$\varepsilon = 3$

Q. 25 The radius of a nucleus of mass number 64 is 4.8 fermi. Then the mass number of another nucleus having radius of 4 fermi is $\frac{1000}{\mathrm{x}}$, where x is_____.

Correct Answer:
27

Solution:
$\mathrm{R} = \mathrm{R}_0\ \mathrm{A}^{1/3}$

$4.8 = \mathrm{R}_0(64)^{1/3}$

$4.8 = \mathrm{R}_0 4$.....(i)

$4 = \mathrm{R}_0(\ \mathrm{A})^{1/3}$.....(ii)

(i) divided (ii)

$1.2 = \frac{4}{(\ \mathrm{A})^{1/3}}$

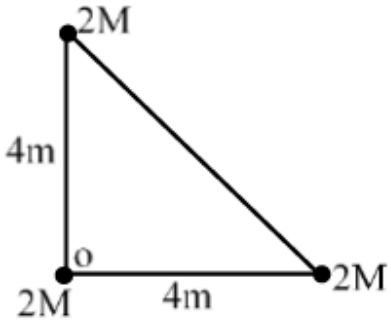
$\mathrm{A} = \frac{64}{1.728} = \frac{1000}{\mathrm{x}}$

$\mathrm{x} = \frac{1728}{64} = 27$
Hence, the answer is (27).

Q. 26 The identical spheres each of mass 2M are placed at the corners of a right angled triangle with mutually perpendicular sides equal to 4m each. Taking point of intersection of these two sides as origin, the magnitude of positon vector of the centre of mass of the system is $\frac{4\sqrt{2}}{x}$, where the value of x is_____.

Correct Answer:
3

Solution:



$$X_{COM} = \frac{2M \times 4}{6M} = \frac{4}{3}\ m$$

$$Y_{COM} = \frac{2M \times 4}{6M} = \frac{4}{3}\ m$$

$$|\vec{r}| = \sqrt{\left(\frac{4}{3}\right)^2 + \left(\frac{4}{3}\right)^2} = \frac{4}{3}\sqrt{2} = \frac{4\sqrt{2}}{x}$$

$$x = 3$$

Q. 27 A tuning fork resonates with a sonometer wire of length 1m stretched with a tension of 6N. When the tension in the wire is changed to 54N, the same tuning fork produces 12 beats per second with it. The frequency of the tuning fork is _____ Hz.

Correct Answer:
6

Solution:

$$f_1 = \frac{1}{2\times 1}\sqrt{\frac{6}{\mu}} \text{.....(i)}$$

$$f_2 = \frac{1}{2\times 1}\sqrt{\frac{54}{\mu}} \text{.....(ii)}$$

$$f_2 - f_1 = \frac{1}{2}\left[\sqrt[3]{\frac{6}{\mu}} - \sqrt{\frac{6}{\mu}}\right] = 12$$

$$2\sqrt{\frac{6}{\mu}} = 24$$

$$f_1 = \frac{1}{2} \times 12 = 6\ Hz$$

$$f_1 = 6\ Hz$$

Hence, the answer is (6).

Q. 28 A plane is in level flight at a constant speed and each of its two wings has an area of $40\ \mathrm{m}^2$. If the speed of the air is $180\ \mathrm{km/h}$ over the lower wing surface and $252\ \mathrm{km/h}$ over the upper wing surface, the mass of the plane is _____kg. (Take air density to be $1\ \mathrm{kg\ m^{-3}}$ and $\mathrm{g} = 10\ \mathrm{ms^{-2}}$)

Correct Answer:
9600

Solution:
By Bernoulli's equation

$\mathrm{P}_1 + \frac{1}{2}\rho v_1^2 = \mathrm{P}_2 + \frac{1}{2}\rho v_2^2$

$\frac{1}{2} \times \rho \times [4900 - 2500] = \frac{\mathrm{mg}}{\mathrm{A}}$

$\frac{1}{2} \times 1 \times 2400 = \frac{\mathrm{m} \times 10}{80}$

$\mathrm{m} = 9600\ \mathrm{kg}$

Hence, the answer is (9600).

Q. 29 The current in a conductor is expressed as $I = 3t^2 + 4t^3$, where I is in Ampere and t is in second. The amount of electric charge that flows through a section of the conductor during $t = 1\ \mathrm{s}$ to $t = 2\ \mathrm{s}$ is _____C.

Correct Answer:
22

Solution:
$I = 3t^2 + 4t^3$

$\int dQ = \int I dt$

$Q = \int_1^2 \left(3t^2 + 4t^3\right) dt$

$= \left[t^3 + t^4\right]_1^2$

$= (8 + 16) - (2) = 22C$

Hence, the answer is the option (1).

Q. 30 A particle is moving in one dimension (along x axis) under the action of a variable force. Its initial position was $16\ \mathrm{m}$ right of origin. The variation of its position (x) with time (t) is given as $\mathrm{x} = -3\mathrm{t}^3 + 18\mathrm{t}^2 + 16\mathrm{t}$, where x is in m and t is in s. The velocity of the particle when its acceleration becomes zero is_____$\mathrm{m/s}$.

Correct Answer:
52

Solution:
$x = -3t^3 + 18t^2 + 16t$

$v = -9t^2 + 36t + 16$

$a = -18t + 36 = 0$

$t = 2\ \mathrm{sec.}$

$\text{at } t = 2\ \mathrm{sec.}$

$v = -9 \times 4 + 72 + 16$

$= 88 - 36 = 52\ \mathrm{m/sec.}$

Hence, the answer is (52).

Chemistry

Q. 1 If one strand of a DNA has the sequence ATGCTTCA, sequence of the bases in complementary strands is:

Option 1:
CATTAGCT

Option 2:
TACGAAGT

Option 3:
GTACTTAC

Option 4:
ATGCGACT

Correct Answer:
TACGAAGT

Solution:
In DNA A bonded with T by two H-bond C bonded with G by three H-bond
ATGCTTCA complementary sequence is
TACGAAGT

Hence, the answer is the option (2).

Q. 2 Given below are two statement : one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) : Haloalkanes react with KCN to form alkyl cyanides as a main product while with AgCN form isocyanide as the main product.

Reason (R) : KCN and AgCN bot are highly ionic compounds.

In the light of the above statement, choose the most appropriate answer from the options given below :

Option 1:
(A) is correct but **(R)** is not correct

Option 2:
Both **(A)** and **(R)** are correct but **(R)** is not the correct explanation of **(A)**

Option 3:
(A) is not correct but **(R)** is correct

Option 4:
Both **(A)** and **(R)** are correct and **(R)** is the correct explanation of **(A)**

Correct Answer:
(A) is correct but **(R)** is not correct

Solution:
KCN is predominantly ionic and provides cyanide ions in solution. Although both cation and nitrogen atoms
are in a position to donate electron pairs, the attack takes place mainly through carbon atoms and not through the nitrogen atom since the C–C bond is more stable than the C–N bond. However, AgCN is mainly covalent in nature and nitrogen is free to donate electron pair forming isocyanide as the main product.

Hence, the answer is the option (3).

Q. 3 In acidic medium, $K_2Cr_2O_7$ shows oxidizing action as represented in the half reaction. $Cr_2O_7^{2-} + XH^+ + Ye^- \rightarrow 2\,A + ZH_2O$
X, Y, Z and A respectively are :

Option 1:
$8, 6, 4$ and Cr_2O_3

Option 2:
$14, 7, 6$ and Cr^{3+}

Option 3:
$8, 4, 6$ and Cr_2O_3

Option 4:
14, 6, 7 and Cr^{3+}

Correct Answer:
14, 6, 7 and Cr^{3+}

Solution:
$$Cr_2O_7^{2-} + 14H^+ + 6e^- \rightarrow 2Cr^{3+} + 7H_2O$$

Hence, the answer is the option (4).

Q. 4 Which of the following reactions are disproportionation reactions ?

(A) $Cu^+ \rightarrow Cu^{2+} + Cu$
(B) $3MnO_4^{2-} + 4H^+ \rightarrow 2MnO_4^- + MnO_2 + 2H_2O$
(C) $2KMnO_4 \rightarrow K_2MnO_4 + MnO_2 + O_2$
(D) $2MnO_4^- + 3Mn^{2+} + 2H_2O \rightarrow 5MnO_2 + 4H^+$

Choose the correct answer form the option given below :

Option 1:
(A), (B)

Option 2:
(B), (C), (D)

Option 3:
(A), (B), (C)

Option 4:
(A), (D)

Correct Answer:
(A), (B)

Solution:
When a particular oxidation state becomes less stable relative to another oxidation state, one lower, one higher it is said to undergo a disproportionation reaction.

$$Cu^{+} \longrightarrow Cu^{+2} + Cu$$
$$3MnO_4^{2-} + 4H^{+} \longrightarrow 2MnO_4^{-} + MnO_2 + 2H_2O$$

Hence, the answer is the option (1).

Q. 5 In case of isoelectronic species the size of F^{-}, Ne and $Na^{+}$is affected by:

Option 1:
Principal quantum number (n)

Option 2:
None of the factors because their size is the same

Option 3:
Electron-electron interaction in the outer orbitals

Option 4:
Nuclear charge (z)

Correct Answer:
Nuclear charge (z)

Solution:
In case of isoelectronic species size is affected by nuclear charge.

For isoelectronic species, which are atoms or ions that have the same number of electrons, the size (or ionic radius) is indeed affected by the nuclear charge.

Isoelectronic Species: Isoelectronic species have the same number of electrons but different numbers of protons. This means they have different nuclear charges.

Nuclear Charge: The number of protons in the nucleus, which creates a positive charge that attracts the negatively charged electrons.

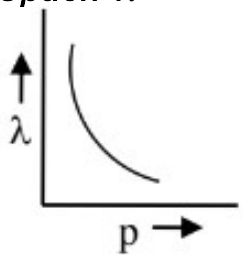
Effective Nuclear Charge: The net positive charge experienced by the electrons. It takes into account the shielding effect of inner-shell electrons.

Effect on Size: In isoelectronic species, as the nuclear charge increases, the attraction between the nucleus and the electrons increases. This pulls the electrons closer to the nucleus, resulting in a smaller ionic radius.

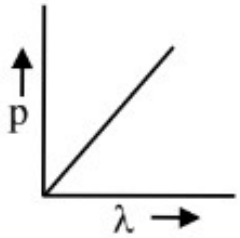
Hence, the answer is the option (4).

Q. 6 According to the wave-particle duality of matter by de-Broglie, which of the following graph plot presents most appropriate relationship between wavelength of electron (λ) and momentum of electron(p) ?

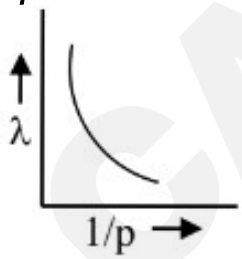
Option 1:



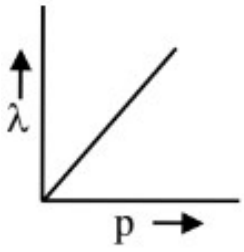
Option 2:



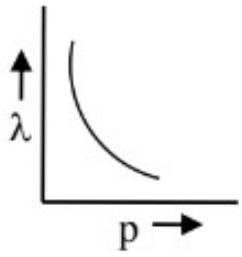
Option 3:



Option 4:



Correct Answer:



Solution:
As per de-Broglie

$\lambda = \frac{h}{p}$

hence $\lambda \propto \frac{1}{p}$

$\lambda \uparrow, P \downarrow$

So, option (1) correct

Q. 7 Given below are two statements :

Statements (I) : A solution of $[Ni(H_2O)_6]^{2+}$ is green in colour.

Statement (II) : A solution of $[Ni(CN)_4]^{2-}$ is colourless.

In the light of the above statement, choose the most appropriate answer from the options given below :

Option 1:
Both Statement I and Statement II incorrect

Option 2:
Both Statement I and Statement I are correct

Option 3:
Statement I is incorrect but Statement II correct

Option 4:
Statement I is correct but Statement II incorrect

Correct Answer:
Both Statement I and Statement I are correct

Solution:
$[Ni(H_2O)_6]^{2+} \rightarrow$ is paramagnetic and it is green colour $[Ni(CN)_2]^{2-} \rightarrow$ is diamagnetic and it is colourless.

Hence, the answer is the option (2).

Q. 8 Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) : PH_3 has low boiling point than NH_3.

Reason (R) : In liquid state NH_3 molecules are associated through vander Waal's force, but PH_3 molecules are associated through hydrogen bonding.

In the light of the above statement, choose the most appropriate answer from the options given below :

Option 1:
Both (A) and (R) are correct but (R) is not the correct explanation of (A)

Option 2:
(A) is not correct but (R) is correct

Option 3:
Both (A) and (R) are correct and (R) is not the correct explanation of (A)

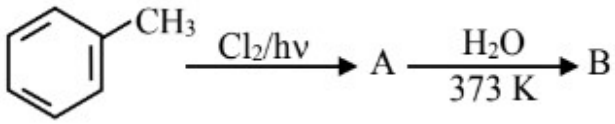
Option 4:
(A) is correct but (R) is not correct

Correct Answer:
(A) is correct but (R) is not correct

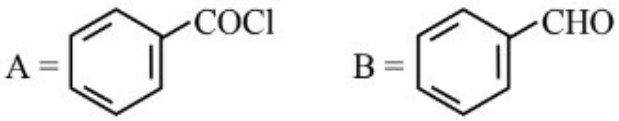
Solution:
PH_3 molecule are not associated through hydrogen bonding in liquid state So, boiling point of PH_3 is lower than NH_3.

Hence, the answer is the option (4).

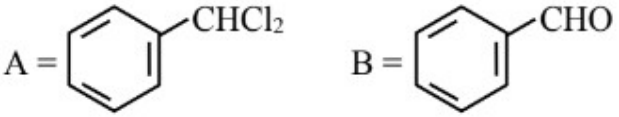
Q. 9 Identify A and B in the following sequence of reaction :



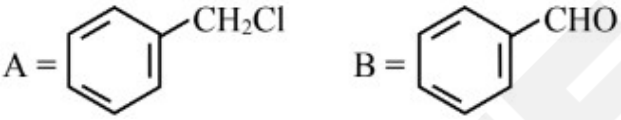
Option 1:



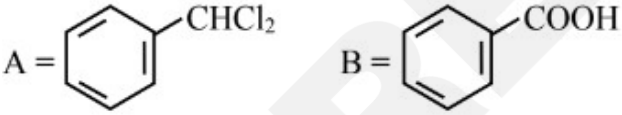
Option 2:



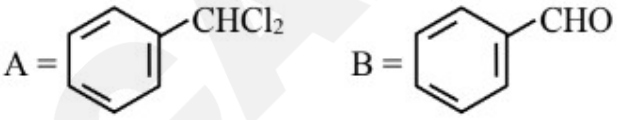
Option 3:



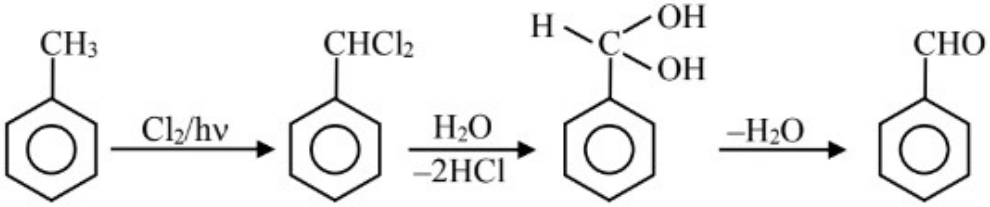
Option 4:



Correct Answer:



Solution:



Q. 10 Given below are two statements :

Statements (I) : Aminobenzene and aniline are same organic compounds.

Statement (II) : Aminobenzene and aniline are different organic compounds.

In the light of the above statement, choose the most appropriate answer from the options given below :

Option 1:
Both Statement I and Statement II correct

Option 2:
Statement I is correct but Statement II incorrect

Option 3:
Statement I is incorrect but Statement II correct

Option 4:
Both Statement I and Statement I are incorrect

Correct Answer:
Statement I is correct but Statement II incorrect

Solution:



Amino benzene or aniline or phenyl amine

$(C_6H_7\ N$ or $C_6H_5NH_2)$

Q. 11 Which of the following complex is homoleptic ?

Option 1:
$[Ni(CN)_4]^{2-}$

Option 2:
$[Ni(NH_3)_2Cl_2]$

Option 3:
$[Fe(NH_3)_4Cl_2]^{+}$

Option 4:
$[Co(NH_3)_4Cl_2]^{+}$

Correct Answer:
$[Ni(CN)_4]^{2-}$

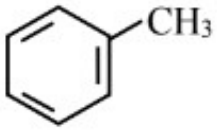
Solution:
In Homoleptic complex, all the ligand attached with central metal should be same.

$[Ni(CN)_4]^{2-}$

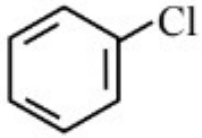
Hence, the answer is the option (1).

Q. 12 Which of the following compound will most easily be attacked by an electrophile ?

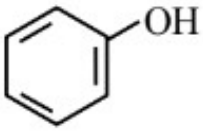
Option 1:


Option 2:


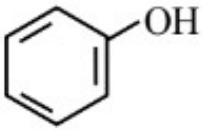
Option 3:



Option 4:



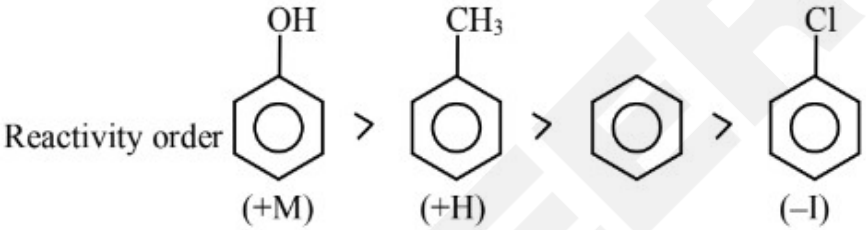
Correct Answer:



Solution:

Follow electrophilic substitution reach :-

ESR α EDG $\alpha \frac{1}{\text{EWG}}$



Q. 13 Ionic reactions with organic compounds proceed through :

(A) homolytic bond cleavage

(B) heterolytic bond cleavage

(C) free radical formation

(D) primary free radical

(E) secondary free radical

Choose the correct answer from the options given below :

Option 1:
(A) only

Option 2:
(C) only

Option 3:
(B) only

Option 4:
(D) and (E) only

Correct Answer:
(B) only

Solution:
Ionic reaction with organic compound proceeds through heterolytic bond cleavage.

In ionic reactions with organic compounds, bond cleavage usually occurs heterolytically, where one atom retains both electrons of the bond, forming ions. Exceptions may involve homolytic bond cleavage, where each atom retains one electron, common in radical reactions.

Hence, the answer is the option (3).

Q. 14 Arrange the bonds in order of increasing ionic character in the molecules. $\mathrm{LiF}, \mathrm{K_2O}, \mathrm{N_2}, \mathrm{SO_2}$ and $\mathrm{CIF_3}$.

Option 1:
$\mathrm{ClF_3} < \mathrm{N_2} < \mathrm{SO_2} < \mathrm{K_2O} < \mathrm{LiF}$

Option 2:
$\mathrm{LiF} < \mathrm{K_2O} < \mathrm{ClF_3} < \mathrm{SO_2} < \mathrm{N_2}$

Option 3:
$\mathrm{N_2} < \mathrm{SO_2} < \mathrm{ClF_3} < \mathrm{K_2O} < \mathrm{LiF}$

Option 4:

$N_2 < ClF_3 < SO_2 < K_2O < LiF$

Correct Answer:

$N_2 < SO_2 < ClF_3 < K_2O < LiF$

Solution:

$$\underset{\Delta EN\uparrow}{\xrightarrow[\text{Ionic character}\uparrow]{N_2<SO_2<ClF_3<K_2O<LiF}}$$

Q. 15 We have three aqueous solutions of NaCl labelled as 'A', 'B' and 'C' with concentration 0.1 M, 0.01 M and 0.001 M, respectively. The value of van 't Hoff factor(i) for these solutions will be in the order :

Option 1:

$i_A < i_B < i_C$

Option 2:

$i_A < i_C < i_B$

Option 3:

$i_A = i_B = i_C$

Option 4:

$i_A > i_B > i_C$

Correct Answer:

$i_A = i_B = i_C$

Solution:

NaCl is a strong electrolyte which is independent of concentration

hence $NaCl \rightarrow Na^+ + Cl^-$

$i_A = i_B = i_C$

Hence, the answer is the option (3).

Q. 16 In Kjeldahl's method for estimation of nitrogen $CuSO_4$ acts as :

Option 1:
reducing agent

Option 2:
catalytic agent

Option 3:
hydrolysis agent

Option 4:
oxidizing agent

Correct Answer:
catalytic agent

Solution:
Kjeldahl's method for estimation of nitrogen, $CuSO_4$ acts as a catalytic agent.

Hence, the answer is the option (2).

Q. 17 Given below are two statements :

Statements (I) : Potassium hydrogen phthalate is a primary standard for standardization of sodium hydroxide solutions.

Statement (II) : In this titration phenolphthalein can be used as indicator.

In the light of the above statement, choose the most appropriate answer from the options given below :

Option 1:
Both Statement I and Statement II correct

Option 2:
Statement I is correct but Statement II incorrect

Option 3:
Statement I is incorrect but Statement II correct

Option 4:
Both Statement I and Statement II are incorrect

Correct Answer:
Both Statement I and Statement II correct

Solution:
Potassium hydrogen is a primary standard for standardization of sodium hydroxide solution as it is economical and its concentration does not change with time.
Phenopthaline can act as an indicator in the acid-base range of 8.3 to 10.1.

Hence, the answer is the option (1).

Q. 18 Match List-I with List-II.

List - I (Reactions)		List - II (Reagents)	
(A)	$CH_3(CH_2)_5-\underset{\underset{O}{\|}}{C}-OC_2H_5 \longrightarrow CH_3(CH_2)_5CHO$	(I)	CH_3MgBr, H_2O
(B)	$C_6H_5COC_6H_5 \rightarrow C_6H_5CH_2C_6H_5$	(II)	Zn(Hg) and conc. HCl
(C)	$C_6H_5CHO \rightarrow C_6H_5CH(OH)CH_3$	(III)	$NaBH_4, H^+$
(D)	$CH_3COCH_2\,COOC_2H_5 \rightarrow CH_3\underset{\underset{H}{\|}}{C}(OH)\,CH_2COOC_2H_5$	(IV)	DIBAL- H, H_2O

Choose the correct answer from the options given below :

Option 1:
(A)-(III), (B)-(IV), (C)-(I), (D)-(II)

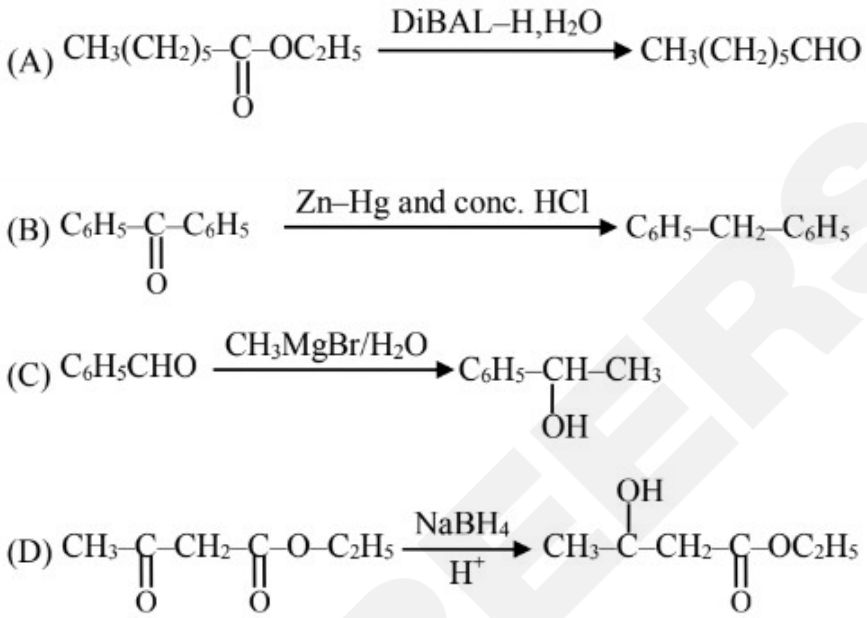
Option 2:
(A)-(IV), (B)-(II), (C)-(I), (D)-(III)

Option 3:
(A)-(IV), (B)-(II), (C)-(III), (D)-(I)

Option 4:
(A)-(III), (B)-(IV), (C)-(II), (D)-(I)

Correct Answer:
(A)-(IV), (B)-(II), (C)-(I), (D)-(III)

Solution:



Q. 19 Choose the correct option for free expansion of an ideal gas under adiabatic condition from the following :

Option 1:
$q = 0, \Delta T \neq 0, w = 0$

Option 2:
$\mathrm{q} = 0, \Delta \mathrm{T} < 0, w \neq 0$

Option 3:
$\mathrm{q} \neq 0, \Delta \mathrm{T} = 0, \mathrm{w} = 0$

Option 4:
$q = 0, \Delta T = 0, w = 0$

Correct Answer:
$q = 0, \Delta T = 0, w = 0$

Solution:
For free expansion of an ideal gas under adiabatic condition

$$q = 0, \Delta T = 0, w = 0$$

Hence, the answer is the option (4).

Q. 20 Given below are two statements :

Statement (I) : The NH_2 group in Aniline is ortho and para directing and a powerful activating group.

Statement (II) : Aniline does not undergo Friedel-Craft's reaction (alkylation and acylation). In the light of the

Above statements, choose the mot appropriate answer from the options given below.

Option 1:
Both statement I and Statement II are correct

Option 2:
Both statement I and Statement II are incorrect

Option 3:
Statement I is incorrect but statement II is correct

Option 4:
Statement I is correct but statement II is incorrect

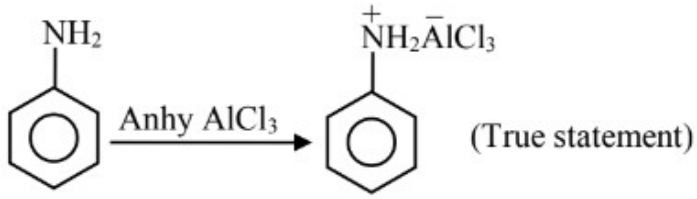
Correct Answer:
Both statement I and Statement II are correct

Solution:

Statement I : In aniline $-NH_2$ is strong activating group due to presence of lp in nitrogen. Hence aniline is ortho and para directing.

(True Statement)

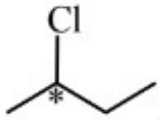
Statement II : Aniline does not undergo friedal-craft's reaction (alkylation and acylation) because anyhydrous $AlCl_3$ form salt with aniline.



Q. 21 Number of optical isomers possible for 2-chlorobutane _____.

Correct Answer:
2

Solution:

2—chlorobutane 

Chiral centre = 1
n = 1 (stereogenic centre)
Total isomers = 2^n, 2^1 =2

Q. 22 The potential for the given half cell at $298\ K$ is (-)_____ $\times 10^{-2}\ V$

$$2H^+_{(aq)} + 2e^- \rightarrow H_2(g)$$
$$[H^+] = 1M, P_{H_2} = 2\ atm$$

(Given : $2.30RT/F = 0.06\ V, \log 2 = 0.3$)

Correct Answer:
1

Solution:

1×10^{-2}

$2H^{+}_{aq} + 2e^{-} \longrightarrow H_2$ aq

$E^{0}_{cell} = E^{\circ}_{oxid.} - E^{\circ}_{red.}$

$= 0 - \frac{0.06}{2} \log \frac{P_{H_2}}{[H^{+}]^2} = -0.03 \times \log \frac{2}{1}$

$= -0.03 \times 0.3 = \frac{-0.009}{0.9 \times 10^{-2}}$ or 1×10^{-2} V

Hence, the answer is (1).

Q. 23 The number of white coloured salts, among the following is_____.

(a) $SrSO_4$
(b) $Mg(NH_4)PO_4$
(c) $BaCrO_4$
(d) $Mn(OH)_2$
(e) $PbSO_4$
(f) $PbCrO_4$
(g) $AgBr$
(h) PbI_2
(i) CaC_2O_4
(j) $[Fe(OH)_2(CH_3COO)]$

Correct Answer:
5

Solution:

$SrSO_4 \rightarrow$ White

$Mg(NH_4)PO_4 \rightarrow$ White

$BaCrO_4 \rightarrow$ Yellow

$Mn(OH)_2 \rightarrow$ White

$PbSO_4 \rightarrow$ White

$PbCrO_4 \rightarrow$ Yellow

$AgBr \rightarrow$ Pale yellow

$PbI_2 \rightarrow$ Yellow

$CaC_2O_4 \rightarrow$ White

$[Fe(OH)_2(CH_3COO)] \rightarrow$ Brown red

Hence, the answer is (5).

Q. 24 The ratio of $\frac{^{14}C}{^{12}C}$ in a piece of wood is $\frac{1}{8}$ part that of atmosphere. If half life of ^{14}C is 5730 years, the age of wood sample is_____years.

Correct Answer:
17190

Solution:

CAREERS360

$$\frac{^{14}C}{^{12}C} = \frac{1}{8}$$

$$t_{1/2}C_{14} = 5730\text{yrs.}$$

$$N = N_o e^{-\lambda t}$$

$$\frac{N}{N_o} = e^{-\lambda t} \Rightarrow \frac{1}{8} = e^{-\lambda t}$$

$$e^{\lambda t} = 8$$

$$\lambda t = \ln 8$$

$$t = \frac{3\ln^2}{\lambda} = 3t_{1/2} = 3 \times 5730 = 17190\text{yrs.}$$

Hence, the answer is (17190).

Q. 25 The number of molecules/ion/s having trigonal bipyramidal shape is_____ $PF_5, BrF_5, PCl_5, [PtCl_4]^{2-}, BF_3, Fe(CO)_5$

Correct Answer:
3

Solution:
$PF_5, PCl_5, Fe(CO)_5 \rightarrow$ Trigonal bipyramidal

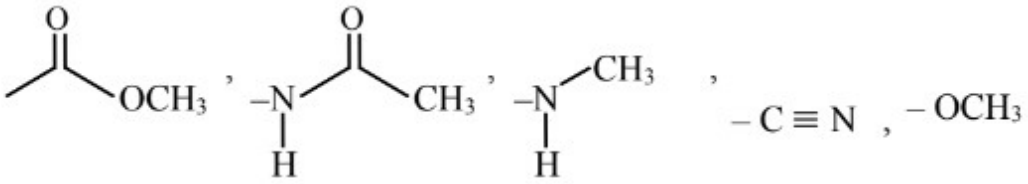
$BrF_5 \rightarrow$ square pyramidal

$[PtCl_4]^{2-} \rightarrow$ square planar

$BF_3 \rightarrow$ Trigonal planar

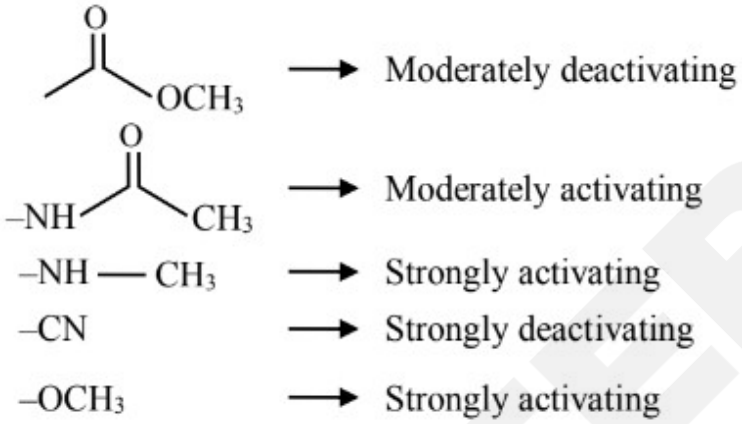
Hence, the answer is (3).

Q. 26 Total number of deactivating groups in aromatic electrophilic substitution reaction among the following is_____.



Correct Answer:
2

Solution:



Q. 27 The lowest oxidation number of an atom in a compound A_2B is -2 . The number of electrons in its valence shell is_____.

Correct Answer:
6

Solution:
$A_2B \rightarrow 2A^+ + B^{2-}$

B^{2-} has a complete octet so, the valance electron will be six.

Hence, the answer is (6).

Q. 28 Among the following oxides of p-block elements, number of oxides having amphoteric nature is_____

$Cl_2O_7, CO, PbO_2, N_2O, NO, Al_2O_3, SiO_2, N_2O_5, SnO_2$

Correct Answer:
3

Solution:
Amphoteric oxide $\rightarrow Al_2O_3, SnO_2, PbO_2$

Neutral oxide $\rightarrow CO, NO, N_2O$

Acidic oxide $\rightarrow Cl_2O_7, SiO_2, N_2O_5$

Hence, the answer is (3).

Q. 29 Consider the following reaction :

$$3PbCl_2 + 2(NH_4)_3PO_4 \rightarrow Pb_3(PO_4)_2 + 6NH_4Cl$$

If $72mmol$ of $PbCl_2$ is mixed with $50mmol$ of $(NH_4)_3PO_4$, then the amount of $Pb_3(PO_4)_2$ formed is_____ mmol (nearest integer).

Correct Answer:
24

Solution:

$$3PbCl_2 + 2(NH_4)_3PO_4 \rightarrow Pb_3(PO_4)_2 + 6NH_4Cl$$

$72mmol \qquad 50mmol$

$$\text{L.R.} \ \frac{72}{3} < \frac{50}{2}$$

$24mmol \qquad\qquad 24mmol$

Hence, the answer is (24).

Q. 30 K_a for CH_3COOH is 1.8×10^{-5} and K_b for NH_4OH is 1.8×10^{-5}. The pH of ammonium acetate solution will be _____

Correct Answer:
7

Solution:

$$pK_a = -\log K_a = -\log\left(1.8 \times 10^{-5}\right) = 4.74$$

$$pK_b = -\log K_b = -\log\left(1.8 \times 10^{-5}\right) = 4.74$$

for W.A. & W.B.

$$pH = 7 + \tfrac{1}{2}(pK_a - pK_b)$$

$$pH = 7 + \tfrac{1}{2}(4.74 - 4.74)$$

$$pH = 7$$

Hence, the answer is (7).

Maths

Q. 1 A bag contains 8 balls, whose colours are either white or black. 4 balls are drawn at random without replacement and it was found that 2 balls are white and other 2 balls are black. The probability that the bag contains equal number of white and black balls is :

Option 1:
$\frac{2}{5}$

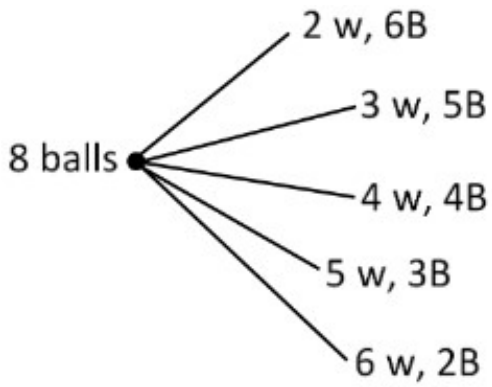
Option 2:
$\frac{2}{7}$

Option 3:
$\frac{1}{7}$

Option 4:
$\frac{1}{5}$

Correct Answer:
$\frac{2}{7}$

Solution:



Let $E = (4W, 4B)$
Then $\bar{E}$ all other cases

$$\mathrm{P(E/(2\,W, 2\,B))} = \frac{\mathrm{P(E)} \cdot \mathrm{P((2\,W, 2\,B)/E)}}{\mathrm{P(E)} \cdot \mathrm{P((2\,W, 2\,B)/E)} + \mathrm{P(\bar{E})} \cdot \mathrm{P((2\,W, 2\,B)/\bar{E})}}$$

$$\Rightarrow \frac{\frac{1}{5} \times \frac{{}^4C_2 \times {}^4C_2}{{}^8C_4}}{\frac{1}{5} \times \frac{{}^4C_2 \times {}^4C_2}{{}^8C_4} + \frac{1}{5} \times \frac{{}^2C_2 \cdot {}^6C_2}{{}^8C_4} \times 2 + \frac{1}{5} \times \frac{{}^3C_2 \times {}^5C_2}{{}^8C_4} \times 2}$$

$$\Rightarrow \frac{2}{7}$$

Q. 2 The value of the intergral $\int_0^{\pi/4} \frac{x dx}{\sin^4(2x)+\cos^4(2x)}$ equals:

Option 1:

$\frac{\sqrt{2}\pi^2}{8}$

Option 2:

$\frac{\sqrt{2}\pi^2}{16}$

Option 3:

$\frac{\sqrt{2}\pi^2}{32}$

Option 4:

$\frac{\sqrt{2}\pi^2}{64}$

Correct Answer:

$\frac{\sqrt{2}\pi^2}{32}$

Solution:

$$I = \frac{1}{2}\int_0^{\frac{\pi}{4}} \frac{2x}{\sin^4 2x + \cos^4 2x}dx, \text{ Let } 2x = t$$

$$\Rightarrow 2dx = dt$$

$$I = \frac{1}{4}\int_0^{\frac{\pi}{2}} \frac{tdt}{\sin^4 t + \cos^4 t}$$

$$I = \frac{1}{4}\int_0^{\frac{\pi}{2}} \frac{xdx}{\sin^4 x + \cos^4 x} \qquad \ldots\ldots(1)$$

Apply king's Rule

$$I = \frac{1}{4}\int_0^{\frac{\pi}{2}} \frac{\left(\frac{\pi}{2}-x\right)dx}{\cos^4 x+\sin^4 x} \qquad \ldots\ldots\ldots(2)$$

Add (1) & (2)

$$2I = \frac{1}{4}\int_0^{\frac{\pi}{2}} \frac{\left(x + \frac{\pi}{2} - x\right)dx}{\sin^4 x + \cos^4 x}$$

$$I = \frac{\pi}{16}\int_0^{\frac{\pi}{2}} \frac{dx}{\sin^4 x + \cos^4 x}$$

Let $f(x) = \frac{1}{\sin^4 x + \cos^4 x}, f\left(\frac{\pi}{2} - x\right) = f(x)$

Apply Queen's Rule,

$$I = 2 \cdot \frac{\pi}{16}\int_0^{\frac{\pi}{4}} \frac{dx}{\sin^4 + \cos^4 x}$$

$$I = \frac{\pi}{8}\int_0^{\frac{\pi}{4}} \frac{dx}{\left(\sin^2 x + \cos^2 x\right)^2 - 2\sin^2 \cos^2 x}$$

$$I = \frac{\pi}{8}\int_0^{\frac{\pi}{4}} \frac{dx}{1 - 2(\sin x \cdot \cos x)^2} = \frac{\pi}{4}\int_0^{\frac{\pi}{4}} \frac{dx}{2 - \sin^2 2x}$$

Let $2x = t \Rightarrow 2dx = dt$

$$I = \frac{\pi}{8}\int_0^{\frac{\pi}{2}} \frac{dt}{2 - \sin^2 t}$$

$$I = \frac{\pi}{8}\int_0^{\frac{\pi}{2}} \frac{\sec^2 t dt}{2\sec^2 t - \tan^2 t}$$

$$I = \frac{\pi}{8}\int_0^{\frac{\pi}{2}} \frac{\sec^2 t dt}{2 + \tan^2 t}, \text{ Let } \tan t = v$$

$$I = \frac{\pi}{8}\int_0^{\infty} \frac{dv}{(\sqrt{2})^2 + v^2}$$

$$I = \frac{\pi}{8\sqrt{2}}\left(\tan^{-1}\frac{v}{\sqrt{2}}\right)_0^{\infty}$$

$$I = \frac{\pi^2}{16\sqrt{2}} = \frac{\sqrt{2}\pi^2}{32}$$

Hence, the answer is the option (3).

Q. 3 If $A=\begin{bmatrix}\sqrt{2} & 1\\ -1 & \sqrt{2}\end{bmatrix}, B=\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}, C=ABA^T$ and $X=A^TC^2A$, then $\det X$ is equal to :

Option 1:
243

Option 2:
729

Option 3:
27

Option 4:
891

Correct Answer:
729

Solution:

$|X|=A\ \ A\cdot C^2\cdot A^T$

$\Rightarrow |A|\cdot|C|^2\cdot|A|$

$\Rightarrow |A|^2\cdot\ \ A\cdot B\cdot A^T\ ^2$

$\Rightarrow |A|^2\cdot|A|^2\cdot|B|^2\cdot|A|^2$

$\Rightarrow |A|^6\cdot|B|^2$

$\Rightarrow 3^6\cdot 1\Rightarrow 729\quad(\because |A|=3,|B|=1)$

Hence, the answer is the option (2).

Q. 4 If $\tan A=\frac{1}{\sqrt{x(x^2+x+1)}}, \tan B=\frac{\sqrt{x}}{\sqrt{x^2+x+1}}$ and $\tan C=\left(x^{-3}+x^{-2}+x^{-1}\right)^{1/2}, 0<A, B, C<\frac{\pi}{2}$, then $A+B$ is equal to:

Option 1:
C

Option 2:
$\pi-\mathrm{C}$

Option 3:
$2\pi-\mathrm{C}$

Option 4:
$\frac{\pi}{2}-\mathrm{C}$

Correct Answer:
C

Solution:

$$\tan A=\frac{1}{\sqrt{x\left(x^2+x+1\right)}}, \tan B=\frac{\sqrt{x}}{\sqrt{x^2+x+1}}$$

$$\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A \cdot \tan B}$$

$$\tan(\mathrm{A}+\mathrm{B})=\frac{\frac{1}{\sqrt{\mathrm{x}\left(\mathrm{x}^2+\mathrm{x}+1\right)}}+\frac{\sqrt{\mathrm{x}}}{\sqrt{\mathrm{x}^2+\mathrm{x}+1}}}{1-\frac{1}{\sqrt{\mathrm{x}\left(\mathrm{x}^2+\mathrm{x}+1\right)}} \cdot \frac{\sqrt{\mathrm{x}}}{\sqrt{\mathrm{x}^2+\mathrm{x}+1}}}$$

$$\tan(A+B)=\frac{1+x}{\sqrt{x}\cdot\sqrt{x^2+x+1}}\times\frac{\left(x^2+x+1\right)}{x^2+x}$$

$$\tan(A+B)=\frac{\sqrt{x^2+x+1}}{x\sqrt{x}}\ldots(1)$$

$$\text{Given }\tan C=\left(\frac{1}{x^3}+\frac{1}{x^2}+\frac{1}{x}\right)^{1/2}$$

$$\tan C=\frac{\sqrt{1+x+x^2}}{x\sqrt{x}}\ldots$$

From eq. (1) & (2)

$$\tan(\mathrm{A}+\mathrm{B})=\tan\mathrm{C}$$

or $\mathrm{A}+\mathrm{B}=\mathrm{C}$

Hence, the answer is the option (1).

Q. 5 If n is the number of ways five different employees can sit into four indistinguishable offices where any office may have any number of persons including zero, then n is equal to :

Option 1:
47

Option 2:
53

Option 3:
51

Option 4:
43

Correct Answer:
51

Solution:
5 people can be distributed in four rooms in the following ways,

$(5,0,0,0),(1,4,0,0),(2,3,0,0),(1,1,3,0),(2,2,1,0),(1,1,1,2)$

Total no. of ways respectively

$$\Rightarrow 1+\frac{5!}{1!\cdot 4!}+\frac{5!}{2!\cdot 3!}+\frac{5!}{1!\cdot 1!\cdot 3!\cdot 2!}+\frac{5!}{2!\cdot 2!\cdot 1!\cdot 2!}+\frac{5!}{(1!)^3\cdot 2!\cdot 3!}$$

$$\Rightarrow 1+5+10+10+15+10=51$$

Hence, the answer is the option (3).

Q. 6 Let $S=\{z\in C:|z-1|=1$ and $(\sqrt{2}-1)(z+\bar{z})-i(z-\bar{z})=2\sqrt{2}\}$. Let $z_1,z_2\in S$ be such that $|z_1|=\max_{Z\in S}|z|$ and $|z_2|=\min_{z\in S}|z|$. Then $\left|\sqrt{2}z_1-z_2\right|^2$ equals :

Option 1:
1

Option 2:
4

Option 3:
3

Option 4:
2

Correct Answer:
2

Solution:
Let $z=x+iy$

Then $(x-1)^2+y^2=1 \quad \ldots\ldots\ldots(1)$

$(\sqrt{2}-1)(2x)-i(2iy)=2\sqrt{2}$

$\& \Rightarrow (\sqrt{2}-1)x+y=\sqrt{2} \qquad \ldots\ldots\ldots(2)$

Solving (1) & (2) we get

Either $x=1$ or $x=\frac{1}{2-\sqrt{2}} \qquad \ldots..(3)$

On solving (3) with (2) we get

$\text{For } x=1 \Rightarrow y=1 \Rightarrow z_2=1+i$

$\& \text{ for } x=\frac{1}{2-\sqrt{2}} \Rightarrow y=\sqrt{2}-\frac{1}{\sqrt{2}} \Rightarrow z_1=\left(1+\frac{1}{\sqrt{2}}\right)+\frac{i}{\sqrt{2}}$

$\left|\sqrt{2}z_1-z_2\right|^2$

$\text{Now } = \left(\frac{1}{\sqrt{2}}+1\right)\sqrt{2}+i-(1+i)\Big|^2=(\sqrt{2})^2=2$

Hence, the answer is the option (4).

Q. 7 Let the median and the mean deviation about the median of 7 observations 170,125,230,190,210, a, b be 170 and $\frac{205}{7}$ respectively. Then the mean deviation about the mean of these 7 observations is :

Option 1:
31

Option 2:
28

Option 3:
30

Option 4:
32

Correct Answer:
30

Solution:
125, a, b, 170, 190, 210, 230

$$\sum \frac{|x_i - \bar{x}|}{n} = \text{ mean deviation about mean}$$

$$\frac{170 - 125 + 170 - a + 170 - b + 190 - 170 + 210 - 170 + 230 - 170}{7} = \frac{205}{7}$$

$$45 + 340 - a - b + 20 + 40 + 60 = 205$$

$$a + b = 300$$

$$\bar{x} = \frac{a + b + 125 + 170 + 190 + 210 + 230}{7} = \frac{1225}{7} = 175$$

Mean deviation about mean

$$= \frac{175 - 125 + 175 - a + 175 - b + 175 - 170 + 190 - 175 + 210 - 175 + 230 - 175}{7}$$

$$= \frac{50 + 350 - 300 + 5 + 15 + 35 + 55}{7} = 30$$

Hence, the answer is option (3).

Q. 8 Let $\vec{a} = -5\hat{i} + \hat{j} - 3\hat{k}, \vec{b} = \hat{i} + 2\hat{j} - 4\hat{k}$ and $\vec{c} = (((\vec{a} \times \vec{b}) \times \hat{i}) \times \hat{i}) \times \hat{i}$. Then $\vec{c} \cdot (-\hat{i} + \hat{j} + \hat{k})$ is equal to :

Option 1:
-12

Option 2:
-10

Option 3:
-13

Option 4:
-15

Correct Answer:
-12

Solution:

$$\bar{a}\times\bar{b}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\-5&1&-3\\1&2&-4\end{vmatrix}=2\hat{i}-23\hat{j}-11\hat{k}$$

$$(\bar{a}\times\bar{b})\times\hat{i}=-11\hat{j}+23\hat{k}$$

$$((\bar{a}\times\bar{b})\times\hat{i})\times\hat{i}=11\hat{k}+23\hat{j}$$

$$((((\bar{a}\times\bar{b})\times\hat{i})\times\hat{i})\times\hat{i})=11\hat{j}-23\hat{k}=\vec{c}$$

$$\vec{c}\cdot(-\hat{i}+\hat{j}+\hat{k})=11-23=-12$$

Hence, the answer is the option (1).

Q. 9 Let $S=\left\{x\in R:(\sqrt{3}+\sqrt{2})^x+(\sqrt{3}-\sqrt{2})^x=10\right\}$. Then the number of elements in S is :

Option 1:
4

Option 2:
0

Option 3:
2

Option 4:
1

Correct Answer:
0

Solution:
$(\sqrt{3}+\sqrt{2})^{x}+(\sqrt{3}+\sqrt{2})^{-x}=10$

$\left(\because \sqrt{3}+\sqrt{2}=\frac{1}{\sqrt{3}-\sqrt{2}}\right)$

$\Rightarrow t+\frac{1}{t}=10 \Rightarrow t^{2}-10t+1=0$

$\Rightarrow t=\frac{10 \pm 4\sqrt{6}}{2}=5 \pm 2\sqrt{6}=(\sqrt{3}+\sqrt{2})^{2}$

$\Rightarrow (\sqrt{3}+\sqrt{2})^{x}=(\sqrt{3}+\sqrt{2})^{2},(\sqrt{3}+\sqrt{2})^{-2}$

$\Rightarrow x=2,-2$

No of elements $\Rightarrow 2$

Hence, the answer is the option (2).

Q. 10 The area enclosed by the curves $xy+4y=16$ and $x+y=6$ is equal to :

Option 1:
$28-30\log_{e}2$

Option 2:
$30-28\log_{e}2$

Option 3:
$30-32\log_{e}2$

Option 4:
$32 - 30\log_e 2$

Correct Answer:
$30 - 32\log_e 2$

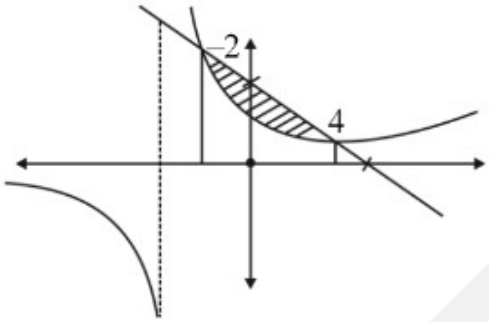
Solution:
$y(x+4) = 16 \& y = 6 - x$

solve both curves,

$(6-x)(x+4) = 16$

$6x - x^2 + 24 - 4x = 16$

$x^2 - 2x - 8 = 0 \Rightarrow x = -2, 4$



$$\int_{-2}^{4} \left((6-x) - \left(\frac{16}{x+4}\right)\right) dx$$

$\Rightarrow \left[6x - x^2 - 16\ln|x+4|\right]_{-2}^{4}$

$\Rightarrow (24 - 8 - 16\ln 8) - (-12 - 2 - 16\ln 2)$

$\Rightarrow 30 - 32\ln 2$

Q. 11 Let $f : R \to R$ and $g : R \to R$ be defined as $f(x) = \begin{cases} \log_e x, & x > 0 \\ e^{-x}, & x \leq 0 \end{cases}$ and $g(x) = \begin{cases} x, & x \geqslant 0 \\ e^x, & x < 0 \end{cases}$. Then, gof : $R \to R$ is :

Option 1:
One - one but not onto

Option 2:
neither one - one nor onto

Option 3:
onto but not one - one

Option 4:
both one - one and onto

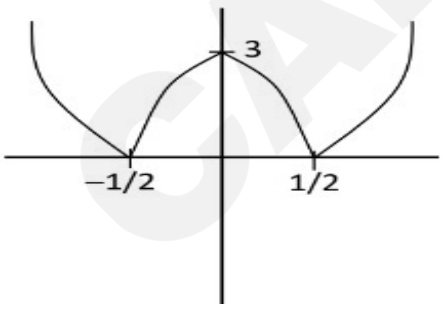
Correct Answer:
neither one - one nor onto

Solution:

$$g(f(x)) = \begin{cases} f(x), & f(x) \geq 0 \\ e^{f(x)}, & f(x) < 0 \end{cases}$$

$$g(f(x)) = \begin{cases} \ln x, & x \geq 1 \\ e^{\ln x}, & 0 < x < 1 \\ e^{-x}, & x \leq 0 \end{cases}$$

$$g(f(x)) = \begin{cases} e^{x}, & x \leq 0 \\ x, & 0 < x < 1 \\ \ln x, & x \geq 1 \end{cases}$$



g(f(x)) is clearly many-one & into

Q. 12 If the system of equations

$2x + 3y - z = 5$

$x + \alpha y + 3z = -4$

$3x - y + \beta z = 7$

has infinitely many solutions, then $13\alpha\beta$ is equal to_____.

Option 1:
1110

Option 2:
1120

Option 3:
1210

Option 4:
1220

Correct Answer:
1120

Solution:
Given a system of equations,

$2x + 3y - z = 5$
$x + \alpha y + 3z = -4$
$3x - y + \beta z = 7$

For infinite solutions,

$\Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$

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$$\Delta_2 = \begin{vmatrix} 2 & 5 & -1 \\ 1 & -4 & 3 \\ 3 & 7 & \beta \end{vmatrix} = 0 \Rightarrow \beta = \frac{-16}{13}$$

$$\Delta_3 = \begin{vmatrix} 2 & 3 & 5 \\ 1 & \alpha & -4 \\ 3 & -1 & 7 \end{vmatrix} = 0 \Rightarrow \alpha = -70$$

$\Rightarrow 13\alpha\beta = 1120$

Hence, the answer is the option (2).

Q. 13 For $0 < \theta < \pi/2$, if the eccentricity of the hyperbola $x^2 - y^2 \operatorname{cosec}^2\theta = 5$ is $\sqrt{7}$ times eccentricity of the ellipse $x^2 \operatorname{cosec}^2\theta + y^2 = 5$, then the value of θ is :

Option 1:
$\frac{\pi}{6}$

Option 2:
$\frac{5\pi}{12}$

Option 3:
$\frac{\pi}{3}$

Option 4:
$\frac{\pi}{4}$

Correct Answer:
$\frac{\pi}{3}$

Solution:

$$H: \frac{x^2}{5}-\frac{y^2}{5\sin^2\theta}=1 \Rightarrow e_H^2=1+\sin^2\theta$$

$$E: \frac{x^2}{5\sin^2\theta}+\frac{y^2}{5}=1 \Rightarrow e_E^2=1-\frac{5\sin^2\theta}{5}$$

$$\Rightarrow e_E^2=1-\sin^2\theta$$

$$\Rightarrow e_H=\sqrt{7}e_E \Rightarrow e_H^2=7e_E^2$$

$$\Rightarrow 1+\sin^2\theta=7\left(1-\sin^2\theta\right)$$

$$\Rightarrow 8\sin^2\theta=6 \Rightarrow \sin\theta=\frac{\sqrt{3}}{2},-\frac{\sqrt{3}}{2}$$

$$\Rightarrow \theta=\frac{\pi}{3}\left[\because \theta\in\left(0,\frac{\pi}{2}\right)\right]$$

Hence, the answer is the option (3).

Q. 14 Let $y=y(x)$ be the solution of the differential equation $\frac{dy}{dx}=2x(x+y)^3-x(x+y)-1, y(0)=1$ Then, $\left(\frac{1}{\sqrt{2}}+y\left(\frac{1}{\sqrt{2}}\right)\right)^2$ equals :

Option 1:
$\frac{4}{4+\sqrt{e}}$

Option 2:
$\frac{3}{3-\sqrt{e}}$

Option 3:
$\frac{2}{1+\sqrt{e}}$

Option 4:
$\frac{1}{2-\sqrt{e}}$

Correct Answer:

$\frac{1}{2-\sqrt{e}}$

Solution:

$\Rightarrow 1+\frac{dy}{dx}=2x(x+y)^2-x(x+y)$

Put $x+y=z \Rightarrow 1+\frac{dy}{dx}=\frac{dz}{dx}$

$$\Rightarrow \frac{dz}{dx}=2x\cdot z^3-xz$$

$$\Rightarrow \frac{1}{z^3}\cdot\frac{dz}{dx}=2x-\frac{x}{z^2},\ \text{put}\ \frac{-1}{z^2}=u$$

$$\Rightarrow \frac{1}{2}\frac{du}{dx}=2x+ux \quad \frac{2}{z^3}\frac{dz}{dx}=\frac{du}{dx}$$

$$\Rightarrow \frac{du}{dx}-2ux=4x\ (\text{L.D.E})$$

$$\Rightarrow I\cdot f=e^{-\int 2xdx}=e^{-x^2}$$

$$\Rightarrow ue^{-x^2}=\int 4xe^{-e^{-2}}dx$$

$$\Rightarrow -\frac{1}{z^2}\cdot e^{-x^2}=\int 4x\cdot e^{-x^2}dx$$

Let $x^2=t \Rightarrow 2xdx=dt$

$-\frac{1}{(x+y)^2}e^{-x^2}=-2e^{-t}+C$

$$-\frac{1}{(x+y)^2}=-2+Ce^{x^2}$$

$$2-\frac{1}{(x+y)^2}=C\cdot e^{x^2}$$

Put $x=0, y=1$

$$2-\frac{1}{(0+1)^2}=C\cdot e^0 \Rightarrow C=1$$

$$\Rightarrow 2-\frac{1}{(x+y)^2}=e^{x^2}$$

$$\Rightarrow (x+y)^2 = \frac{1}{2-e^{x^2}}$$

$$\text{Put } x = \frac{1}{\sqrt{2}}$$

$$\left(\frac{1}{\sqrt{2}} + y\left(\frac{1}{\sqrt{2}}\right)\right)^2 = \frac{1}{2-\sqrt{e}}$$

Hence, the answer is the option (4).

Q. 15 Let $f : R \to R$ be defined as :

$$f(x) = \begin{cases} \frac{a-b\cos 2x}{x^2}; & x < 0 \\ x^2 + cx + 2 & ; 0 \le x \le 1 \\ 2x + 1 & ; x > 1 \end{cases}$$

If f is continuous everywhere in R and m is the number of points where f is NOT differential then $m + a + b + c$ equals :

Option 1:
1

Option 2:
4

Option 3:
3

Option 4:
2

Correct Answer:
2

Solution:

$\text{At } x=0,\ \text{R.H.L } = 2$

$$\text{L.H.L } = \lim_{x\to 0}\frac{a-b\cos 2x}{x^2}$$

$$\Rightarrow \lim_{x\to 0}\frac{a-b\left(1-\frac{(2x)^2}{2!}+\frac{(2x)^4}{4!}-\ldots\infty\right)}{x^2}=2$$

$$\Rightarrow \lim_{x\to 0}\frac{(a-b)+b(2x)^2\left[\frac{1}{2!}-\frac{(2x)^2}{4!}-\ldots\infty\right]}{x^2}=2$$

$$\Rightarrow \text{ for L.H.L } = \text{ R.H.L } \Rightarrow a-b=0\Rightarrow a=b$$

$$\Rightarrow 2b=2\Rightarrow b=1$$

$$\Rightarrow a=b=1$$

At $\mathrm{x}=1, \mathrm{f}(\mathrm{x})$ is continuous

therefore

$$\mathrm{f}\left(1^{-}\right)=\mathrm{f}(1)=\mathrm{f}\left(1^{+}\right)$$

$$\mathrm{f}(1)=3+\mathrm{c}$$

$$\mathrm{f}\left(1^{+}\right)=\lim_{\mathrm{h}\to 0}(2(1+\mathrm{h})+1)$$

$$\mathrm{f}\left(1^{+}\right)=\lim_{\mathrm{h}\to 0}3+2\,\mathrm{h}=3$$

$$3+\mathrm{c}=3$$

$$\mathrm{c}=0$$

$$f(x)=\begin{cases}\frac{1-\cos 2x}{x^2}, & x<0\\ x^2+2, & 0\le x\le 1\\ 2x+1, & x>1\end{cases}$$

At $x = 1$, L.H.D $= 2$ = R.H.D

$\Rightarrow$ f is diff. at $x = 1$

At $x = 0$, R.H.D. $= 0$

L.H.D. $\Rightarrow \lim_{x\to 0} \frac{1-\cos 2x}{x^2}$

$$= \lim_{h\to 0} \frac{\frac{1-\cos 2h}{h^2} - 2}{-h}$$

$$= \lim_{h\to 0} \frac{1 - \left(1 - \frac{4h^2}{2!} + \frac{16h^4}{4!} \ldots\right) - 2h^2}{-h^3} = 0$$

$$\Rightarrow \text{ L.H.D. } = \text{ R.H.D.}$$

$$\Rightarrow \text{ fis diff at } x = 0$$

$$\Rightarrow m = 0$$

$$M + a + b + c = 0 + 1 + 1 + 0 = 2$$

Hence, the answer is the option (4).

Q. 16 Let $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a > b$ be an ellipse, whose eccentricity is $\frac{1}{\sqrt{2}}$ and the length of the latus rectum is $\sqrt{14}$, Then the square of the eccentricity of $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is:

Option 1:
3

Option 2:
$7/2$

Option 3:
$3/2$

Option 4:
$5/2$

Correct Answer:
$3/2$

Solution:

$$e(\text{ ellipse }) = \frac{1}{\sqrt{2}}$$

$$e^2 = 1 - \frac{b^2}{a^2} = \frac{1}{2}$$

$$\frac{b^2}{a^2} = \frac{1}{2}$$

L.R (ellipse) $= \frac{2b^2}{a} = \sqrt{14}$

$b^2 = \frac{\sqrt{14}a}{2}$

From eq. (1) & (2)

$$a = \sqrt{14},\ b^2 = 7$$

$$e^2(\text{ hyperbola }) = \left(1 + \frac{b^2}{a^2}\right) = \left(1 + \frac{7}{14}\right) = \frac{3}{2}$$

Hence, the answer is the option (3).

Q. 17 Let 3, a, b, c be in A.P. and 3, a – 1, b + 1, c + 9 be in G.P. Then, the arithmetic mean of a, b and c is :

Option 1:
-4

Option 2:
-1

Option 3:
13

Option 4:
11

Correct Answer:
11

Solution:
Let A.P. be $3, 3+d, 3+2d, 3+3d$

So, $3+\mathrm{d}=\mathrm{a}, 3+2\,\mathrm{d}=\mathrm{b}, 3+3\,\mathrm{d}=\mathrm{c}$

Also, Let G.P. be $3, 3\mathrm{r}, 2\mathrm{r}^2, 3\mathrm{r}^3$

So, $3\mathrm{r}=\mathrm{a}-1, 3\mathrm{r}^2=\mathrm{b}+1, 3\mathrm{r}^3=\mathrm{c}+9$

On comparing,

$3\mathrm{r}=3+\mathrm{d}-1 \Rightarrow d=3\mathrm{r}-2 \qquad \ldots..(1)$

$3\mathrm{r}^2=3+2\,\mathrm{d}+1 \Rightarrow 3\mathrm{r}^2=2\,\mathrm{d}+4 \qquad \ldots..(2)$

Solve (1) and (2), we get $\mathrm{r}=2$

$a=7, b=11, c=15$

$$\text{A.M. of } a,b,c \Rightarrow \frac{7+11+15}{3}=11$$

Hence, the answer is the option (4).

Q. 18 Let $C: x^2+y^2=4$ and $C': x^2+y^2-4\lambda x+9=0$ be two circles. If the set of all values of λ so that the circles C and C' intersect at two distinct points, is $\mathrm{R}-[\mathrm{a}, \mathrm{b}]$, then the point $(8\mathrm{a}+12, 16\,\mathrm{b}-20)$ lies on the curve:

Option 1:
$x^2+2y^2-5x+6y=3$

Option 2:
$5x^2-y=-11$

Option 3:
$x^2-4y^2=7$

Option 4:

$6x^2+y^2=42$

Correct Answer:

$6x^2+y^2=42$

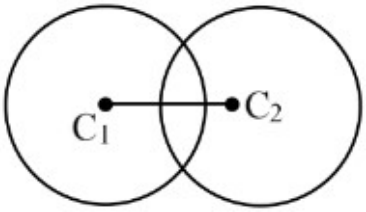
Solution:

$x^2+y^2=4, \quad x^2+y^2-4\lambda x+9=0$

$C_1(0,0), r_1=2, \quad C_2(2\lambda,0), \quad r_2=\sqrt{4\lambda^2-9}$

$|r_1-r_2|<C_1C_2<r_1+r_2$

$2-\sqrt{4\lambda^2-9}\ <|2\lambda|<2+\sqrt{4\lambda^2-9}$



$2-\sqrt{4\lambda^2-9}\ <|2\lambda|$

$4+4\lambda^2-9-4\sqrt{4\lambda^2-9}<4\lambda^2$

$4\sqrt{4\lambda^2-9}>-5$

$16\left(4\lambda^2-9\right)>25$

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$$4\lambda^2 - 9 > \frac{25}{16}$$

$$4\lambda^2 > \frac{169}{16}$$

$$\lambda^2 > \frac{169}{64}$$

$$x \in \left(-\infty, -\frac{13}{8}\right) \cup \left(\frac{13}{8}, \infty\right) \ldots (1)$$

$$|2\lambda| < 2 + \sqrt{4\lambda^2 - 9}$$

$$|2\lambda| - 2 < \sqrt{4\lambda^2 - 9}$$

$$4\lambda^2 + 4 - 8|\lambda| < 4\lambda^2 - 9$$
$$8|\lambda| > 13$$

$$\lambda \in \left(-\infty, \frac{13}{8}\right) \cup \left(\frac{13}{8}, \infty\right) \ldots (2)$$

$$\text{Also, } 4\lambda^2 - 9 \geq 0$$
$$\lambda^2 \geq \frac{9}{4}$$
$$\lambda \in \left(-\infty, -\frac{3}{2}\right) \cup \left(\frac{3}{2}, \infty\right) \ldots (3)$$

$$\text{From (1), (2), (3), } \lambda \in \mathrm{R} - \left[-\frac{13}{8}, \frac{13}{8}\right]$$
$$\therefore \mathrm{a} = \frac{-13}{8},\ \mathrm{b} = \frac{13}{8}$$

So, $8\mathrm{a} + 12 \Rightarrow 8 \times \frac{-13}{8} + 12 \Rightarrow -1$

$16\ \mathrm{b} - 20 \Rightarrow 16 \times \frac{13}{8} - 20 \Rightarrow 6$

Point is $(-1, 6)$ which satisfy $6x^2 + y^2 = 42$

Q. 19 If $5f(x)+4f\left(\frac{1}{x}\right)=x^2-2, \forall x \neq 0$ and $y=9x^2f(x)$, then y is strictly increasing in :

Option 1:
$\left(0, \frac{1}{\sqrt{5}}\right) \cup \left(\frac{1}{\sqrt{5}}, \infty\right)$

Option 2:
$\left(-\frac{1}{\sqrt{5}}, 0\right) \cup \left(\frac{1}{\sqrt{5}}, \infty\right)$

Option 3:
$\left(-\frac{1}{\sqrt{5}}, 0\right) \cup \left(0, \frac{1}{\sqrt{5}}\right)$

Option 4:
(4) $\left(-\infty, \frac{1}{\sqrt{5}}\right) \cup \left(0, \frac{1}{\sqrt{5}}\right)$

Correct Answer:
$\left(-\frac{1}{\sqrt{5}}, 0\right) \cup \left(\frac{1}{\sqrt{5}}, \infty\right)$

Solution:
$5f(x)+4f\left(\frac{1}{x}\right)=x^2-2 \quad \ldots.(1)$

Substitute $\mathrm{x} \rightarrow \frac{1}{\mathrm{x}}$

$5f\left(\frac{1}{x}\right)+4f(x)=\frac{1}{x^2}-2 \quad \ldots.(2)$

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$(1) \times 5 - (2) \times 4$

$$\Rightarrow 9f(x) = 5x^2 - 10 - \frac{4}{x^2} + 8$$

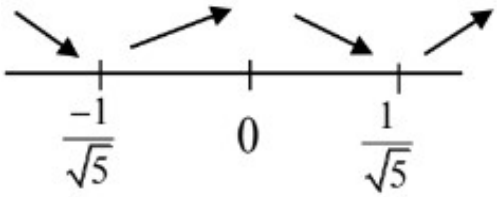
$$\Rightarrow 9f(x) = \frac{5x^2 - 2x^2 - 4}{x^2}$$

$$\Rightarrow 9x^2 f(x) = 5x^4 - 2x^2 - 4$$

$$y = 5x^4 - 2x^2 - 4$$

$$\frac{dy}{dx} = 20x^3 - 4x > 0 \text{ (Given)}$$

$$\Rightarrow 4x\left(5x^2 - 1\right) > 0$$



$\Rightarrow$ y is increasing in $\left(\frac{-1}{\sqrt{5}}, 0\right) \cup \left(\frac{1}{\sqrt{5}}, \infty\right)$

Q. 20 If the shortest distance between the lines $\frac{x-\lambda}{-2} = \frac{y-2}{1} = \frac{z-1}{1}$ and $\frac{x-\sqrt{3}}{1} = \frac{y-1}{-2} = \frac{z-2}{1}$ is 1 , then the sum of all possible values of λ is :

Option 1:
0

Option 2:
$2\sqrt{3}$

Option 3:
$3\sqrt{3}$

Option 4:

$-2\sqrt{3}$

Correct Answer:

$2\sqrt{3}$

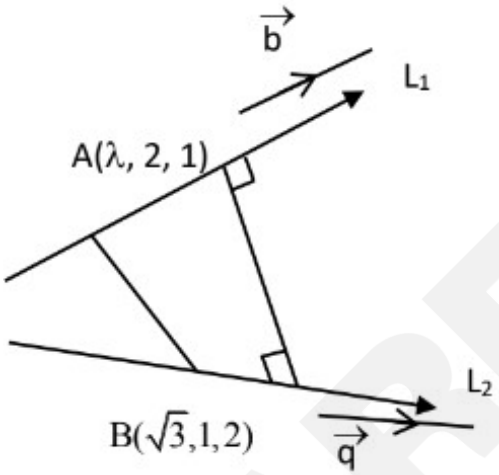
Solution:

$$\text{Let } L_1 : \frac{x-\lambda}{-2} = \frac{y-2}{1} = \frac{z-1}{1}$$

$$\text{and } L_2 : \frac{x-\sqrt{3}}{1} = \frac{y-1}{-2} = \frac{z-2}{1}$$

$$\vec{p} = -2\hat{i} + \hat{j} + k$$

$$\vec{q} = \hat{i} - 2\hat{j} + k$$



$$\overrightarrow{p} \times \overrightarrow{q} = \begin{vmatrix} \hat{i} & \hat{j} & k \\ -2 & 1 & 1 \\ 1 & -2 & 1 \end{vmatrix}$$

$$= 3\hat{i} + 3\hat{j} + 3k$$

$$\overrightarrow{AB} = (\sqrt{3} - \lambda)\hat{i} - \hat{j} + k$$

Shortest distance $\Rightarrow$ Length of projection of $\overrightarrow{AB}$ on $\overrightarrow{p} \times \overrightarrow{q}$

$$\Rightarrow \frac{\overrightarrow{AB} \cdot (\vec{p} \times \vec{q})}{|\vec{p} \times \vec{q}|} = 1$$

$$\Rightarrow \frac{3(3-\lambda) - 3 + 3}{3\sqrt{3}} = 1$$

$$\Rightarrow \lambda = 0, 2\sqrt{3}$$

$$\Rightarrow \text{ sum of values of } \lambda \Rightarrow 2\sqrt{3}$$

Q. 21 If $x = x(t)$ is the solution of the differential equation $(t+1)dx = \left(2x + (t+1)^4\right)dt, x(0) = 2$, then, $x(1)$ equals____.

Correct Answer:
14

Solution:

$$\frac{dx}{dt} = \frac{2x}{(t+1)} + (t+1)^3$$

$$\frac{dx}{dt} - \frac{2x}{t+1} = (t+1)^3 (\text{ L.D.E. })$$

$$\text{I.F. } \Rightarrow e^{-2\int \frac{dt}{t+1}} = e^{-2\ln(t+1)} = \frac{1}{(t+1)^2}$$

$$\Rightarrow x \cdot \frac{1}{(t+1)^2} = \int (t+1)^3 \cdot \frac{1}{(t+1)^2} \cdot dt$$

$$\Rightarrow \frac{x}{(t+1)^2} = \frac{(t+1)^2}{2} + C$$

$$\Rightarrow x = \frac{(t+1)^4}{2} + C(t+1)^2$$

Put $t = 0, x = 2$

$$\Rightarrow 2 = \frac{1}{2} + C \Rightarrow C = \frac{3}{2}$$

$$\Rightarrow x = \frac{(t+1)^4}{2} + \frac{3}{2}(t+1)^2$$

Put $t = 1, x = \frac{16}{2} + \frac{3}{2} \cdot 4 = 14$

Hence, the answer is the (14).

Q. 22 The number of elements in the set $S = \{(x, y, z) : x, y, z \in Z, x + 2y + 3z = 42, x, y, z \geq 0\}$ equals _____.

Correct Answer:
169

Solution:
$x + 2y + 3z = 42$

$x + 2y = 42 - 3z$

1) $z = 0 \quad x + 2y = 42 \rightarrow 22$ case

2) $z = 1 \quad x + 2y = 39 \rightarrow 20$ case

3) $z = 2 \quad x + 2y = 36 \rightarrow 19$ case

4) $z = 3 \quad x + 2y = 33 \rightarrow 17$ case

5) $z = 4 \quad x + 2y = 30 \rightarrow 16$ case

6) $z = 5 \quad x + 2y = 27 \rightarrow 14$ case

7) $z = 6 \quad x + 2y = 24 \rightarrow 13$ case

8) $z = 7 \quad x + 2y = 21 \rightarrow 11$ case

9) $z = 8 \quad x + 2y = 18 \rightarrow 10$ case

10) $z = 9 \quad x + 2y = 15 \rightarrow 8$ case

11) $z = 10 \quad x + 2y = 12 \rightarrow 7$ case

12) $z = 11 \quad x + 2y = 9 \rightarrow 5$ case

13) $z = 12 \quad x + 2y = 6 \rightarrow 4$ case

14) $z = 13 \quad x + 2y = 3 \rightarrow 2$ case

15) $z = 14 \quad x + 2y = 0 \rightarrow 1$ case

Total 169

Hence, the answer is the (169).

Q. 23 If the Coefficient of x^{30} in the expansion of $\left(1+\frac{1}{x}\right)^6\left(1+x^2\right)^7\left(1-x^3\right)^8; x \neq 0$ is α, then $|\alpha|$ equals_____.

Correct Answer:
678

Solution:
coefficient of x^{30} in

$\frac{(x+1)^6\left(1+x^2\right)^7\left(1-x^3\right)^8}{x^6}$

coefficient of x^{36} in $(1+x)^6\left(1+x^2\right)^7\left(1-x^3\right)^8$

General term

${}^6C_{r_1}\,{}^7C_{r_2}\,{}^8C_{r_3}(-1)^{r_1}x^{r_1+2r_2+3r_3}$

$r_1 + 2r_2 + 3r_3 = 36$

Case - I

r_1	r_2	r_3
0	6	8
2	5	8
4	4	8
4	3	8

$r_1 + 2r_2 = 12$ (taking $r_3 = 8$)

Case - II

r_1	r_2	r_3
1	7	7
3	6	7
5	5	7

$r_1 + 2r_2 = 15$ (taking $r_3 = 7$)

Case - III

r_1	r_2	r_3
4	7	6
6	6	6

$r_1 + 2r_2 = 18$ (taking $r_3 = 6$)

$$\text{Coefficient } = 7 + (15 \times 21) + (15 \times 35) + (35)$$

$$- (6 \times 8) - (20 \times 7 \times 8) - (6 \times 21 \times 8) + (15 \times 28)$$

$$+ (7 \times 28) = -678 = \alpha$$

$$|\alpha| = 678$$

Hence, the answer is the (678).

Q. 24 Let $3, 7, 11, 15, \ldots, 403$ and $2, 5, 8, 11, \ldots, 404$ be two arithmetic progressions. Then the sum, of the common terms in them, is equal to_____.

Correct Answer:
6699

Solution:

I^{st} A.P. : $3, 7, 11, \ldots, 403 \Rightarrow d_1 = 4$

II $^{\mathrm{nd}}$ A.P. : $2, 5, 8, \ldots, 404 \Rightarrow d_2 = 3$

Common diff. of common A.P. $\Rightarrow$

D = L.C.M. $(d_1,\ d_2) \Rightarrow 12.$

First common term $\Rightarrow a = 11$

Common A.P. : $11, 23, \ldots, \mathrm{T_n}$

$\Rightarrow \mathrm{T_n} \leq 403$

$\Rightarrow 11 + (\mathrm{n} - 1)12 \leq 403$

$\Rightarrow (\mathrm{n} - 1) \leq \dfrac{392}{12} \Rightarrow \mathrm{n} - 1 \leq 32.66$
$\Rightarrow \mathrm{n} \leq 33.66 \Rightarrow \mathrm{n} = 33$

The sum of common terms :

$$S = \frac{33}{2}[2.11 + (33 - 1)12]$$

$$S = 6699$$

Hence, the answer is the (6699).

Q. 25 Let $\{x\}$ denote the fractional part of x and
$f(x) = \frac{\cos^{-1}(1-\{x\}^2)\sin^{-1}(1-\{x\})}{\{x\}-\{x\}^3}, x \neq 0.$
If L and R respectively denotes the left hand limit and the right hand limit of $f(x)$ at $x = 0$, then $\frac{32}{\pi^2}\left(L^2 + R^2\right)$ is equal to____.

Correct Answer:
18

Solution:
Finding right had limit

$$\lim_{x\to 0^+} f(x) = \lim_{h\to 0} f(0+h)$$

$$= \lim_{h\to 0} f(h)$$

$$= \lim_{h\to 0} \frac{\cos^{-1}\left(1-h^2\right)\sin^{-1}(1-h)}{h\left(1-h^2\right)}$$

$$= \lim_{h\to 0} \frac{\cos^{-1}\left(1-h^2\right)}{h}\left(\frac{\sin^{-1}1}{1}\right)$$

Let $\cos^{-1}\left(1-h^2\right) = \theta \Rightarrow \cos\theta = 1-h^2$

$$= \frac{\pi}{2}\lim_{\theta\to 0}\frac{\theta}{\sqrt{1-\cos\theta}}$$

$$= \frac{\pi}{2}\lim_{\theta\to 0}\frac{1}{\sqrt{\frac{1-\cos\theta}{\theta^2}}}$$

$$= \frac{\pi}{2}\frac{1}{\sqrt{1/2}}$$

$$R = \frac{\pi}{\sqrt{2}}$$

$$L = \lim_{x\to 0^-} f(x)$$

$$= \lim_{h\to 0} f(-h)$$

$$= \lim_{h\to 0} \frac{\cos^{-1}\left(1-\{-h\}^2\right)\sin^{-1}(1-\{-h\})}{\{-h\}-\{-h\}^3}$$

$$= \lim_{h\to 0} \frac{\cos^{-1}\left(1-(-h+1)^2\right)\sin^{-1}(1-(-h+1))}{(-h+1)-(-h+1)^3}$$

$$= \lim_{h \to 0} \frac{\cos^{-1}\left(-h^2 + 2h\right) \sin^{-1} h}{(1-h)\left(1-(1-h)^2\right)}$$

$$= \lim_{h \to 0} \left(\frac{\pi}{2}\right) \frac{\sin^{-1} h}{(1-(1-h)^2)}$$

$$= \frac{\pi}{2} \lim_{h \to 0} \left(\frac{\sin^{-1} h}{-h^2 + 2h}\right)$$

$$= \frac{\pi}{2} \lim_{h \to 0} \left(\frac{\sin^{-1} h}{h}\right) \left(\frac{1}{-h+2}\right)$$

$$L = \frac{\pi}{4}$$

$$\frac{32}{\pi^2}\left(L^2 + R^2\right) = \frac{32}{\pi^2}\left(\frac{\pi^2}{2} + \frac{\pi^2}{16}\right)$$

$$= 16 + 2 = 18$$

Hence, the answer is the (18).

Q. 26 Let the line $L : \sqrt{2}x + y = \alpha$ pass through the point of the intersection P (in the first quadrant) of the circle $x^2 + y^2 = 3$ and the parabola $x^2 = 2y$. Let the line L touch two circles C_1 and C_2 of equal radius $2\sqrt{3}$. If the centres Q_1 and Q_2 of the circles C_1 and C_2 lie on the y-axis, then the square of the area of the triangle $\mathrm{PQ}_1\mathrm{Q}_2$ is equal to_____.

Correct Answer:
72

Solution:

$x^2 + y^2 = 3 \& x^2 = 2y$

$y^2 + 2y - 3 = 0$

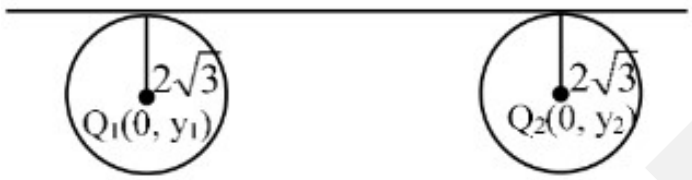
$(y-1)(y+3) = 0$

$y = 1$

$x = \sqrt{2}$

L Passes through $\mathrm{P}(\sqrt{2}, 1)$
So $(\sqrt{2})\sqrt{2} + 1 = \alpha = 3$

$\sqrt{2}x + y = 3$



$\frac{|y-3|}{\sqrt{3}} = 2\sqrt{3}$
$|y-3| = 6$
$y_1 = 9$

$$\text{Area of } \Delta PQ_1R_2 = \frac{1}{2}\begin{matrix}\sqrt{2} & 1 & 1\\ 0 & 9 & 1\\ 0 & -3 & 1\end{matrix}$$

$= 6\sqrt{2}$

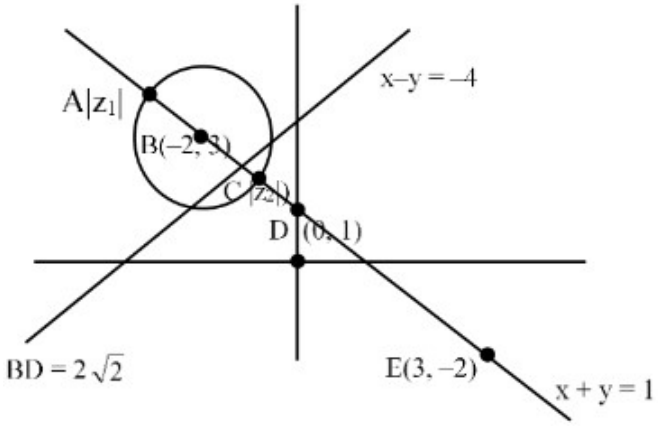
Square of are of $\triangle \mathrm{PQ}_1, \mathrm{Q}_2 = 72$

Q. 27 Let $P = \{z \in C : |z + 2 - 3i| \leq 1\}$ and $Q = \{z \in C : z(1+i) + \bar{z}(1-i) \leq -8\}$. Let in $P \cap Q, |z - 3 + 2i|$ be maximum and minimum at z_1 and z_2 respectively. If $|z_1|^2 + 2|z_2|^2 = \alpha + \beta\sqrt{2}$, where α, β are integers, then $\alpha + \beta$ equals_____.

Correct Answer:
36

Solution:
$(x+2)^2+(y-3)^2 \le 1$

$x-y \le -4$



$$A\left(0+(2\sqrt{2}+1)\left(\frac{-1}{\sqrt{2}}\right),1+(2\sqrt{2}+1)\frac{1}{\sqrt{2}}\right)$$

$$A\left(-\frac{(2\sqrt{2}+1)}{\sqrt{2}},\frac{3\sqrt{2}+1}{\sqrt{2}}\right)$$

$$C\left(\frac{-3}{2},\frac{5}{2}\right)$$

$$|Z_1|^2+2|z_2|^2=$$

$$=\frac{(2\sqrt{2}+1)^2}{2}+\frac{(3\sqrt{2}+1)^2}{2}+2\left(\frac{9}{4}+\frac{25}{4}\right)$$

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$$= \frac{28 + 10\sqrt{2} + 34}{2}$$

$$= 31 + 5\sqrt{2}$$

$\alpha = 31, \beta = 5$

$\alpha + \beta = 36$

Q. 28 If $\int_{-\pi/2}^{\pi/2} \frac{8\sqrt{2}\cos x dx}{(1+e^{\sin x})(1+\sin^4 x)} = \alpha\pi + \beta\log_e(3+2\sqrt{2})$, where α, β are integers, then $\alpha^2 + \beta^2$ equals

Correct Answer:
8

Solution:

$$I = \int_{-\pi/2}^{\pi/2} \frac{8\sqrt{2}\cos x \cdot dx}{(1+e^{\sin x})(1+\sin^4 x)} = \alpha\pi + \beta\log_e(3+2\sqrt{2})$$

$$I = \int_{-\pi/2}^{\pi/2} \frac{8\sqrt{2}\cos x \cdot dx}{(1+e^{-\sin x})(1+\sin^4 x)}$$

$$2I = \int_{-\pi/2}^{\pi/2} \frac{8\sqrt{2}\cos x\,(1+e^{\sin x})}{(e^{\sin x}+1)(1+\sin^4 x)} dx$$

$$2I = 2\int_{0}^{\pi/2} \frac{8\sqrt{2}\cdot\cos x}{1+\sin^4 x} dx$$

$\text{Put } \sin x = t$

$\therefore \cos x dx = dt$

$$I = 8\sqrt{2}\int_0^1 \frac{dt}{1+t^4}$$

$$I = 4\sqrt{2}\int_0^1 \frac{2}{1+t^4}dt$$

$$I = 4\sqrt{2}\int_0^1 \frac{(1+t^2)-(t^2-1)}{t^2\left(\frac{1}{t^2}+t^2\right)}\cdot dt$$

$$I = 4\sqrt{2}\left[\int_0^1 \frac{\left(\frac{1}{t^2}+1\right)\cdot dt}{\left(t-\frac{1}{t}\right)^2+2} - \int_0^1 \frac{\left(1-\frac{1}{t^2}\right)}{\left(t+\frac{1}{t}\right)^2-2}\cdot dt\right.$$

$\Downarrow \qquad\qquad \Downarrow$

$$\text{Put, t} - \frac{1}{\text{t}} = \text{z}; \quad \text{Put, t} + \frac{1}{\text{t}} = \text{s}$$

$$\left(1+\frac{1}{\text{t}^2}\right)\text{dt} = \text{dz}; \quad \left(1-\frac{1}{\text{t}^2}\right)\text{dt} = \text{ds}$$

$$\text{I} = 4\sqrt{2}\left[\int_{-\infty}^{0} \frac{\text{dz}}{\text{z}^2+(\sqrt{2})^2} - \int_{\infty}^{2} \frac{\text{ds}}{\text{s}^2-(\sqrt{2})^2}\right]$$

$$\text{I} = 4\sqrt{2}\left[\left(\frac{1}{\sqrt{2}}\tan^{-1}\frac{\text{z}}{\sqrt{2}}\right)_{-\infty}^{0} - \left(\frac{1}{2\sqrt{2}}\log_e\frac{\text{s}-\sqrt{2}}{\text{s}+\sqrt{2}}\right)_{\infty}^{2}\right]$$

$$\text{I} = 2\pi + 2\ln(3+2\sqrt{2})$$

$\lambda = 2, \beta = 2$

$\therefore \alpha^2 + \beta^2 = 8$

Hence, the answer is the (8).

Q. 29 Let the line of the shortest distance between the lines

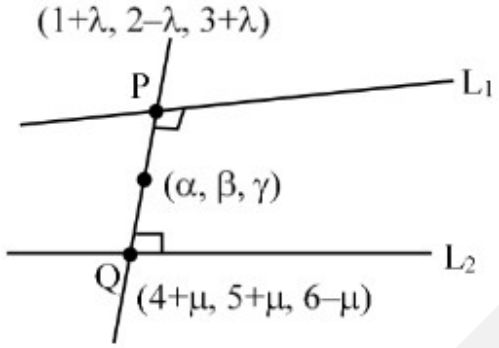
$$L_1 : \vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k}) \text{ and}$$

$$L_2 : \vec{r} = (4\hat{i} + 5\hat{j} + 6\hat{k}) + \mu(\hat{i} + \hat{j} - \hat{k})$$

intersect L_1 and L_2 at P and Q respectively. If (α, β, γ) is the mid point of the line segment PQ, then $2(\alpha + \beta + \gamma)$ is equal to_____.

Correct Answer:
21

Solution:



$\overrightarrow{PQ}$ is $\perp^r$ to L_1 & L_2

$$(3 + \mu - \lambda, 3 + \mu + \lambda, 3 + \mu - \lambda)$$

$$(3 + \mu - \lambda)(1) + (3 + \mu + \lambda)(-1) + (3 - \mu - \lambda)(1) = 0$$

$$-3\lambda - \mu + 3 = 0 \qquad \ldots\ldots (i)$$

$$(3 + \mu - \lambda)(1) + (3 + \mu + \lambda)(1) + (3 - \mu - \lambda)(-1) = 0$$

$$\lambda + 3\mu + 3 = 0 \qquad \ldots\ldots (ii)$$

$$8\mu = -12$$

$\mu = \frac{-3}{2} \quad \lambda = -\frac{3}{2}$

$\mathrm{P}\left(\frac{5}{2}, \frac{1}{2}, \frac{9}{2}\right) \quad \mathrm{Q}\left(\frac{5}{2}, \frac{7}{2}, \frac{15}{2}\right)$

$\alpha = \frac{5}{2} \quad \beta = 2 \quad \gamma = 6$

$2(\alpha + \beta + \gamma) = 21$

Q. 30 Let $A = \{1, 2, 3, \ldots, 20\}$. Let R_1 and R_2 two relation on A such that

$\mathrm{R_I} = \{(\mathrm{a}, \mathrm{b}) : \mathrm{b}$ is divisible by $\mathrm{a}\}$
$R_2 = \{(a, b) : a$ is an integral multiple of $b\}$.
Then, number of elements in $R_1 - R_2$ is equal to____.

Correct Answer:
46

Solution:
$A = \{1, 2, 3, \ldots 20\}$

$R_1 = \{(a, b) : b \text{ is divisible by } a\}$

$R_1 = \{(1, 1), (1, 2) \ldots (1, 20, (2, 2), (2, 4) \ldots . . (2, 20)$

$(3, 3), (3, 6) \ldots (3, 18), (4, 4), (4, 8) \ldots (4, 20)$

$(5, 5), (5, 10)(5, 15), (5, 20), (6, 6), (6, 12), (6, 18)$

$(7, 7), (7, 14), (8, 8), (8, 16), (9, 9), (9, 18), (10, 10)$

$(10, 20), (11, 11), (12, 12) \ldots . . . . (20, 20)\}$

Total $= 66$ ordered pairs

$R_1 \cap R_2 = \{(1,1),(2,2)(3,3)\ldots(20,20)\} \rightarrow 20$ ordered pairs

$R_1 - R_2 = R_1 - (R_1 \cap R_2) \Rightarrow 66 - 20 = 46$

Hence, the answer is the (46).

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1 Feb - Evening Shift

Physics

Q. 1 From the statements given below :
(A) The angular momentum of an electron in n^{th} orbit is an integral multiple of h.
(B) Nuclear forces do not obey inverse square law.
(C) Nuclear forces are spin dependent
(D) Nuclear forces are central and charge independent.
(E) Stability of nucleus is inversely proportional to the value of packing fraction.
Choose the correct answer from the options given below:

Option 1:
(B), (C), (D), (E) only

Option 2:
(A), (C), (D), (E) only

Option 3:
(A), (B), (C), (E) only

Option 4:
(A), (B), (C), (D) only

Correct Answer:
(A), (B), (C), (E) only

Q. 2 A body of mass $4\ \text{kg}$ experiences two forces $\vec{F}_1 = 5\hat{i} + 8\hat{j} + 7\hat{k}$ and $\vec{F}_2 = 3\hat{i} - 4\hat{j} - 3\hat{k}$. The acceleration acting on the body is:

Option 1:
$2\hat{i}+\hat{j}+\hat{k}$

Option 2:
$4\hat{i}+2\hat{j}+2\hat{k}$

Option 3:
$-2\hat{i}-\hat{j}-\hat{k}$

Option 4:
$2\hat{i}+3\hat{j}+3\hat{k}$

Correct Answer:
$2\hat{i}+\hat{j}+\hat{k}$

Solution:

$\mathrm{F_{net}}=\overrightarrow{\mathrm{F}}_1+\overrightarrow{\mathrm{F}}_2$

$=8\hat{\mathrm{i}}+4\hat{\mathrm{j}}+4\hat{\mathrm{k}}$

$\mathrm{a}=\frac{\mathrm{F_{net}}}{\mathrm{m}}=2\hat{\mathrm{i}}+\hat{\mathrm{j}}+\hat{\mathrm{k}}$

Hence, the answer is the option (1).

Q. 3 Monochromatic light of frequency $6\times10^{14}\ \mathrm{Hz}$ is produced by a laser. The power emitted is $2\times10^{-3}\ \mathrm{W}$. How many photons per second on an average, are emitted by the source? (Given $\mathrm{h}=6.63\times10^{-34}\mathrm{Js}$)

Option 1:
5×10^{15}

Option 2:
7×10^{16}

Option 3:
6×10^{15}

Option 4:
9×10^{18}

Correct Answer:
5×10^{15}

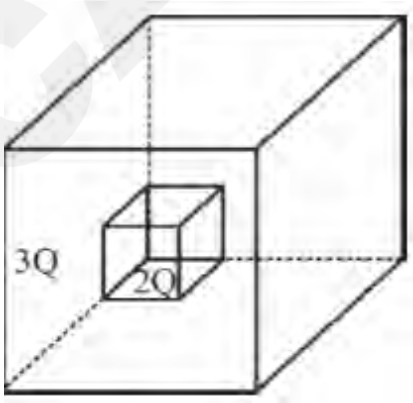
Solution:
Power $\mathrm{P} = 2 \times 10^{-3}\ \mathrm{W}$

Energy $\mathrm{E} = \mathrm{h}\nu = 6.63 \times 10^{-34} \times 6 \times 10^{14}$

$\therefore \mathrm{n} = \frac{\mathrm{P}}{\mathrm{E}} = \frac{2 \times 10^{-3}}{6.63 \times 10^{-34} \times 6 \times 10^{14}}$

$\mathrm{n} = 5 \times 10^{15}$
Hence, the answer is the option (1).

Q. 4 C_1 and C_2 are two hollow concentric cubes enclosing charges $2\mathrm{Q}$ and $3\mathrm{Q}$ respectively as shown in figure. The ratio of electric flux passing through C_1 and C_2 is:



Option 1:
3:2

Option 2:
5:2

Option 3:
2:5

Option 4:
2:3

Correct Answer:
2:5

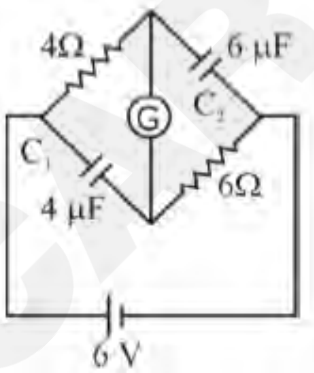
Solution:

Flux through $C_1, \phi_1 = \frac{q_{in}}{\epsilon_0} = \frac{2Q}{\epsilon_0}$

Flux through $C_2, \phi_2 = \frac{q_{in}}{\epsilon_0} = \frac{2Q+3Q}{\epsilon_0} = \frac{5Q}{\epsilon_0}$

$\frac{C_1}{C_2} = \frac{\phi_1}{\phi_2} = \frac{2Q/\epsilon_0}{5Q/\epsilon_0} = \frac{2}{5}$

Q. 5 A galvanometer (G) of 2Ω resistance is connected in the given circuit. The ratio of charge stored in C_1 and C_2 is:



Option 1:
1

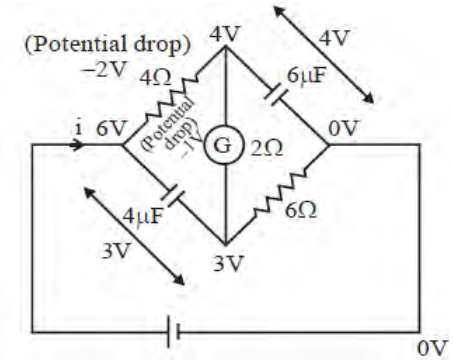
Option 2:
2/3

Option 3:
3/2

Option 4:
1/2

Correct Answer:
1/2

Solution:



Resistance $\mathrm{R} = 12\Omega$

Current $i = \frac{6}{12} = \frac{1}{2}A$

$\frac{q_{4\mu F}}{q_{6\mu F}} = \frac{4\times10^{-6}\times3}{6\times10^{-6}\times4} = \frac{1}{2}$

Q. 6 Match List-I with List-II.

List – I	List – II
(Number)	(Significant figure)
(A) 1001	(I) 3
(B) 010.1	(II) 4
(C) 100.100	(III) 5
(D) 0.0010010	(IV) 6

Choose the correct answer from the options given below:

Option 1:
(A)-(II), (B)-(I), (C)-(IV), (D)-(III)

Option 2:
(A)-(IV), (B)-(III), (C)-(I), (D)-(II)

Option 3:
(A)-(I), (B)-(II), (C)-(III), (D)-(IV)

Option 4:
(A)-(III), (B)-(IV), (C)-(II), (D)-(I)

Correct Answer:
(A)-(II), (B)-(I), (C)-(IV), (D)-(III)

Solution:
(A) $1001 \rightarrow 4$
(B) $010.1 \rightarrow 3$
(C) $100.100 \rightarrow 6$
(D) $0.0010010 \rightarrow 5$

Hence, the answer is the option (1).

Q. 7 A big drop is formed by coalescing 1000 small droplets of water. The surface energy will become:

Option 1:
(1/100)th

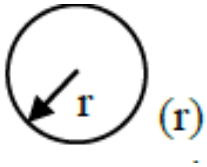
Option 2:
(1/10)th

Option 3:
100 times

Option 4:
10 times

Correct Answer:
(1/10)th

Solution:
1000



Conservation of volume

$V_i = V_f$

$\left(\frac{4}{3}\pi R^3\right) \times 1000 = \frac{4}{3}\pi r^3$

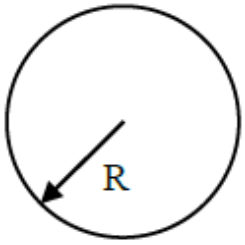
$r = \frac{R}{10}$

$(\text{ S.E. })_i = 1000\left(T \times 4\pi \times r^2\right)$

$(\text{ S.E. })_f = T\left(4\pi \times R^2\right)$

$= T\left(4\pi \times (10r)^2\right)$

$= \left(T \times 4\pi \times r^2\right) \times 100 = \frac{(\text{ S.E. })}{10} i$



Q. 8 A cricket player catches a ball of mass 120 g moving with 25 m/s speed. If the catching process is completed in 0.1s then the magnitude of force exerted by the ball on the hand of player will be (in SI unit):

Option 1:
30

Option 2:
24

Option 3:
12

Option 4:
25

Correct Answer:
30

Solution:

$F = ma$

$= \mathrm{m}\left(\frac{\mathrm{v}-\mathrm{u}}{\mathrm{t}}\right)$

$= \frac{120}{1000} \times \frac{25}{0.1}$

$F = 30N$

Hence, the answer is the option (1).

Q. 9 In a metre-bridge when a resistance in the left gap is 2Ω and unknown resistance in the right gap, the balance length is found to be $40\ \mathrm{cm}$. On shunting the unknown resistance with 2Ω. the balance length changes by:

Option 1:
62.5 cm

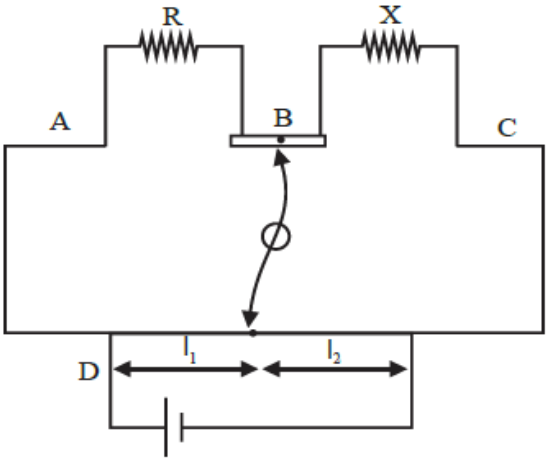
Option 2:
22.5 cm

Option 3:
20 cm

Option 4:
65 cm

Correct Answer:
22.5 cm

Solution:



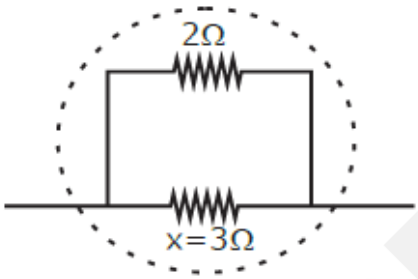
Given for $\ell_1 = 40\ \text{cm}$

$\frac{\text{R}}{\ell_1} = \frac{\text{X}}{\ell_2}$

$\Rightarrow \frac{2}{40} = \frac{\text{X}}{60}$

$\text{x} = 3\Omega$

After shutting of water uses



$\text{R}_{\text{eq}} = \frac{2\times3}{3+2} = \frac{6}{5}\Omega$

$\frac{\text{R}}{\ell} = \frac{\text{R}_{\text{eq}}}{100-\ell}$

$\Rightarrow \frac{2}{\ell} = \frac{6}{5(100-\ell)}$

$\Rightarrow 10(100-\ell) = 6\ell$

$500 - 5\ell = 3\ell$

$500 = 8\ell$

$\ell = \frac{500}{8} = \frac{125}{2} = 62.5\ \text{cm}$

Balance length change

$\text{by } = 62.5 - 40 = 22.5\ \text{cm}$

Q. 10 A diatomic gas $(\gamma = 1.4)$ does $200\ \mathrm{J}$ of work when it is expanded isobarically. The heat given to the gas in the process is:

Option 1:
800 J

Option 2:
600 J

Option 3:
700 J

Option 4:
850 J

Correct Answer:
700 J

Solution:

$$\mathrm{Q} = \mathrm{P}\Delta\mathrm{V} + \mathrm{nCv}\Delta\mathrm{T}$$

$$= \mathrm{P}\Delta\mathrm{V} + \mathrm{n}\frac{\mathrm{R}}{(\gamma-1)}\Delta\mathrm{T}$$

$$= \mathrm{P}\Delta\mathrm{V} + \frac{\mathrm{P}\Delta\mathrm{V}}{\gamma-1}$$

$$= \mathrm{P}\Delta\mathrm{V}\left(1 + \frac{1}{\gamma-1}\right)$$

$$= 200\left(1 + \frac{1}{0.4}\right)$$

$$= 200 \times 3.5$$

$$= 700\ \mathrm{J}$$

Hence, the answer is the option (3).

Q. 11 Train A is moving along two parallel rail tracks towards north with speed 72 km/h and train B is moving towards south with speed 108 km/h. Velocity of train B with respect to A and velocity of ground with respect to B are (in ms^{-1})

Option 1:
–50 and –30

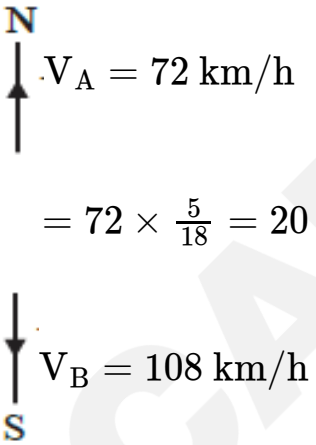
Option 2:
–50 and 30

Option 3:
–30 and 50

Option 4:
50 and –30

Correct Answer:
–50 and 30

Solution:



$= 108 \times \frac{5}{18} = 30\ \mathrm{m/s}$

$\mathrm{V_{B/A}} = \vec{\mathrm{V}}_\mathrm{B} - \vec{\mathrm{V}}_\mathrm{A}$

$= -30 - 20 = -50$

$\vec{\mathrm{V}}_{\mathrm{g/B}} = \vec{\mathrm{V}}_\mathrm{g} - \vec{\mathrm{V}}_\mathrm{B}$

$= 0 - (-30) = +30$

-50 and 30

Q. 12 A light planet is revolving around a massive star in a circular orbit of radius R with a period of revolution T. If the force of attraction between planet and star is proportional to $\mathrm{R}^{-3/2}$ then choose the correct option:

Option 1:
$T^2 \propto R^{7/2}$

Option 2:
$T^2 \propto R^3$

Option 3:
$T^2 \propto R^{5/2}$

Option 4:
$T^2 \propto R^{3/2}$

Correct Answer:
$T^2 \propto R^{5/2}$

Solution:
$\because F \propto \frac{1}{\mathrm{R}^{3/2}}$ or $F = \frac{K}{\mathrm{R}^{3/2}}$

Gravitational force provides centripetal force.

$\frac{\mathrm{K}}{\mathrm{R}^{3/2}} = \mathrm{m}\omega^2\mathrm{R}$

$\omega^2 = \frac{\mathrm{K}}{\mathrm{mR}^{5/2}}$

$\left(\frac{2\pi}{\mathrm{T}}\right)^2 = \frac{\mathrm{K}}{\mathrm{mR}^{5/2}} \Rightarrow \mathrm{T}^2 = \frac{4\pi^2\mathrm{mR}^{5/2}}{\mathrm{K}}$

$\mathrm{T}^2 \propto \mathrm{R}^{5/2}$

Hence, the answer is the option (3).

Q. 13 A microwave of wavelength 2.0 cm falls normally on a slit of width 4.0 cm. The angular spread of the central maxima of the diffraction pattern obtained on a screen 1.5m away from the slit, will be:

Option 1:
60°

Option 2:
45°

Option 3:
15°

Option 4:
30°

Correct Answer:
30°

Solution:
$\mathrm{d}\sin\theta = \lambda$ (Half angular speed)

$\sin\theta = \frac{\lambda}{\mathrm{d}} = \frac{2\times10^{-2}}{4\times10^{-2}} = \frac{1}{2}$

$\theta = 30^\circ$

Full angular speed $= 2\theta = 30^\circ$

Hence, the answer is the option (4).

Q. 14 If the root mean square velocity of hydrogen molecule at a given temperature and pressure is $2\ \mathrm{km/s}$, the root mean square velocity of oxygen at the same condition in $\mathrm{km/s}$ is:

Option 1:
1.0

Option 2:
1.5

Option 3:
2.0

Option 4:
0.5

Correct Answer:
0.5

Solution:

$$\frac{(V_{rms})_{H_2}}{(V_{rms})_{O_2}} = \frac{\sqrt{\frac{3RT}{M_{H_2}}}}{\sqrt{\frac{3RT}{M_{O_2}}}}$$

$$\Rightarrow \frac{2}{(V_{rms})_{O_2}} = \sqrt{\frac{M_{O_2}}{M_{H_2}}} = \sqrt{\frac{32}{2}}$$

$$\Rightarrow (V_{ms})_{O_2} = 0.5$$

Hence, the answer is the option (4).

Q. 15 If the frequency of an electromagnetic wave is 60 MHz and it travels in the air along z direction then the corresponding electric and magnetic field vectors will be mutually perpendicular to each other and the wavelength of the wave (in m) is:

Option 1:
2.5

Option 2:
5

Option 3:
10

Option 4:
2

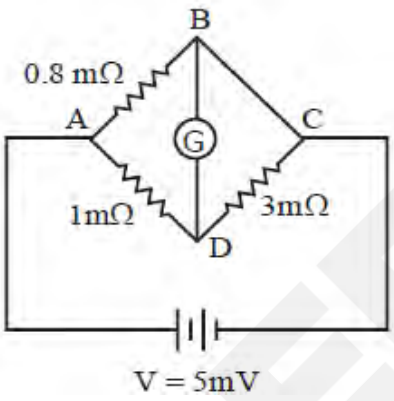
Correct Answer:
5

Solution:
Speed = Wavelength × Frequency

$$\Rightarrow \lambda = \frac{C}{f} = \frac{3\times10^8}{60\times10^6} = 5\ \text{m}$$

Hence, the answer is the option (2).

Q. 16 To measure the temperature coefficient of resistivity α of a semiconductor, an electrical arrangement shown in the figure is prepared. The arm BC is made up of the semiconductor. The experiment is being conducted at 25°C and resistance of the semiconductor arm is $3\ \text{m}\Omega$. Arm BC is cooled at a constant rate of 2°C/s. If the galvanometer G shows no deflection after $10\ \text{s}$, then α is:



Option 1:
$-1\times10^{-2}{}^\circ\text{C}^{-1}$

Option 2:
$-2\times10^{-2}{}^\circ\text{C}^{-1}$

Option 3:
$-2.5\times10^{-2}{}^\circ\text{C}^{-1}$

Option 4:
$-1.5\times10^{-2}{}^\circ\text{C}^{-1}$

Correct Answer:
$-2.5 \times 10^{-2}{}^{\circ}\mathrm{C}^{-1}$

Solution:
For no. deflection $= \frac{0.8}{1} = \frac{R}{3}$

$\mathrm{R} = 2.4\ \mathrm{m}\Omega$

Rate of fall of temp $= 2^{\circ}\mathrm{C/S}$

in $10\mathrm{sec}$ temp full by $= 2 \times 10 = 20^{\circ}\mathrm{C}$

$\Delta\mathrm{R} = \mathrm{R}\alpha\Delta\mathrm{T}$

$\Rightarrow \alpha = \frac{\Delta\mathrm{R}}{\mathrm{R}\Delta\mathrm{T}} = \frac{-0.6}{3\times 20}$

$= -10^{-2}{}^{\circ}\mathrm{C}^{-1}$

Q. 17 Conductivity of a photodiode starts changing only if the wavelength of incident light is less than $660\ \mathrm{nm}$. The band gap of photodiode is found to be $\left(\frac{X}{8}\right)\mathrm{eV}$. The value of X is: (Given, $\mathrm{h} = 6.6 \times 10^{-34}\mathrm{Js}, \mathrm{e} = 1.6 \times 10^{-19}\mathrm{C}$)

Option 1:
11

Option 2:
13

Option 3:
15

Option 4:
21

Correct Answer:
15

Solution:
Energy bond gap $\mathrm{E_g}$ is

$$E_g = \frac{hc}{\lambda} = \frac{1240}{\lambda(\text{ in nm })} = \frac{1240}{660} = 1.87\text{eV}$$

$$\text{or } E_g = \left(\frac{15}{8}\right)\text{eV}$$

$$\therefore x = 15$$

Hence, the answer is the option (3).

Q. 18 A transformer has an efficiency of 80% and works at 10V and 4 kW. If the secondary voltage is 240V, then the current in the secondary coil is:

Option 1:
1.33A

Option 2:
13.33A

Option 3:
1.59A

Option 4:
15.1A

Correct Answer:
13.33A

Solution:
Efficiency $\eta = 80\%, P_i = 4\text{ kW} = 4000\text{ W}$

$$V_P = 10\text{ V},\ V_S = 240\text{ V}$$

For primary coil : $I_p = \frac{P_i}{V_p} = \frac{4000}{10} = 400\text{ A}$

$$\eta = \frac{V_S I_S}{V_p I_p} \Rightarrow \frac{80}{100} = \frac{240 I_S}{10 \times 400}$$

$$\Rightarrow I_S = 13.33\text{ A}$$

Hence, the answer is the option (2).

Q. 19 In an ammeter, 5% of the main current passes through the galvanometer. If resistance of the galvanometer is G, the resistance of ammeter will be:

Option 1:
199 G

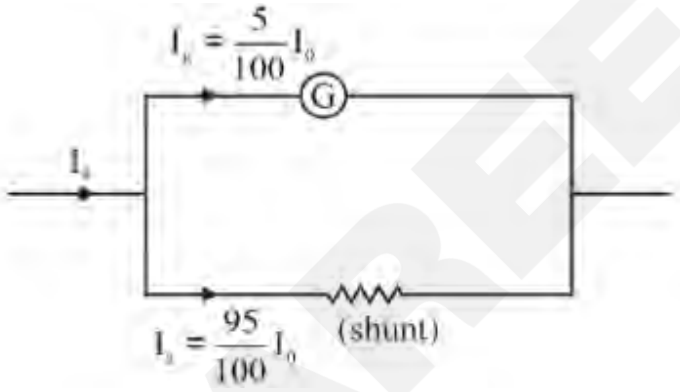
Option 2:
200 G

Option 3:
$\frac{G}{200}$

Option 4:
$\frac{G}{199}$

Correct Answer:
199 G

Solution:



By using KVL

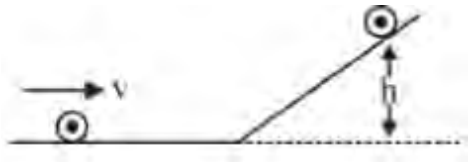
$$I_g \cdot G = I_S \cdot S$$

$$S = \frac{I_g G}{I_S} = \frac{\frac{5}{100} I_0}{\frac{95}{100} I} G = \frac{1}{19} G$$

$$R_A = \frac{SG}{S+G} = \frac{\frac{G}{19} \cdot G}{\frac{G}{19} + G} = \frac{G^2}{\frac{19 \times 20G}{19}} = \frac{G}{20}$$

CAREERS360

Q. 20 A disc of radius R and mass M is rolling horizontally without slipping with speed v. It then moves up an inclined smooth surface as shown in figure. The maximum height that the disc can go up the incline is:



Option 1:
$\frac{3v^2}{4g}$

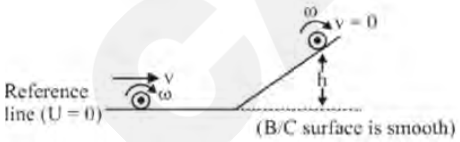
Option 2:
$\frac{v^2}{g}$

Option 3:
$\frac{2}{3}\frac{v^2}{g}$

Option 4:
$\frac{1}{2}\frac{v^2}{g}$

Correct Answer:
$\frac{1}{2}\frac{v^2}{g}$

Solution:



Rolling $V_0 = \omega R$

Conservation of enrgy

$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2}mv_0^2 + \frac{1}{2}\left[\frac{MR^2}{2}\right]\cdot\left(\frac{V_0}{R}\right)^2 + 0 = mgh + \frac{1}{2}\left[\frac{MR^2}{2}\right]\cdot\left(\frac{V_0^2}{R^2}\right)$$

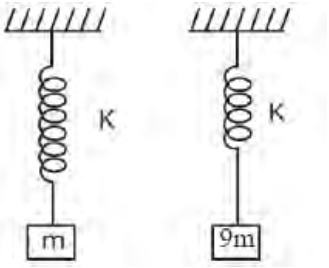
$$\frac{1}{2}mv_0^2 = mgh_{max}$$

$h_{max} = \frac{v_0^2}{2g}$

Q. 21 A mass m is suspended from a spring of negligible mass and the system oscillates with a frequency f_1. The frequency of oscillations if a mass $9\ m$ is suspended from the same spring is f_2. The value of $\frac{f_1}{f_2}$ is_____.

Correct Answer:
3

Solution:

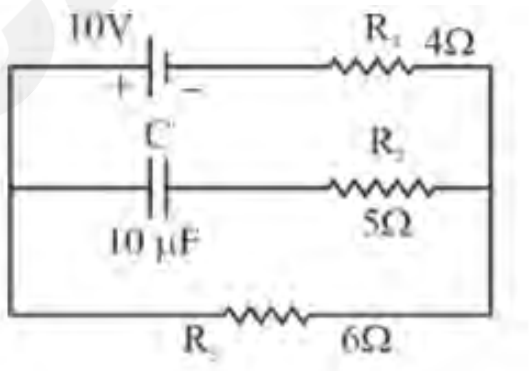


$\omega_1 = \sqrt{\frac{k}{m}} \quad \omega_2 = \sqrt{\frac{k}{9m}}$

$\omega_1 = 2\pi f_1 \quad \omega_2 = 2\pi f_2$

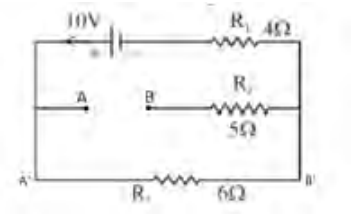
$\frac{f_1}{f_2} = \frac{\omega_1}{\omega_2} = \frac{\sqrt{\frac{k}{m}}}{\sqrt{\frac{k}{9m}}} = \frac{\sqrt{\frac{k}{m}}}{\frac{1}{3}\sqrt{\frac{k}{m}}} = 3$

Q. 22 In an electrical circuit drawn below the amount of charge stored in the capacitor is _____μC.



Correct Answer:
60

Solution:
In capacitor charge stored after full-charging L, means capacitor will act as open.



$$i_{net} = \frac{\Delta V}{R_{eq}}$$

$$\frac{\Delta V}{R_{eq}} = \frac{V}{R_1+R_3} = \frac{10}{4+6} = 1\ A$$

$$V_{A'} - V_{B'} = iR_3 = 1 \times 6 = 6\ V$$

$$q = C\Delta V = C\left(V_A - V_B\right) = 10\mu f \times 6 = 60\mu c$$

Q. 23 A particle initially at rest starts moving from reference point $x = 0$ along x-axis, with velocity v that varies as $v = 4\sqrt{x} m/s$. The acceleration of the particle is ms^{-2}.

Correct Answer:
8

Solution:
Given

$$v = 4\sqrt{x}$$



$$t = 0$$

$$t = 0$$

$$v = 0$$

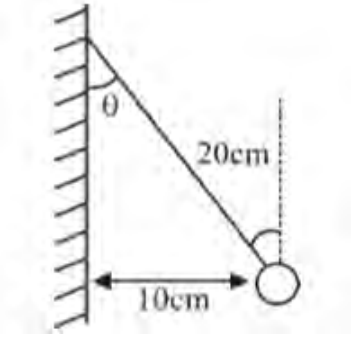
$$a = \frac{vdv}{dx} = (4\sqrt{x})\frac{d(4\sqrt{x})}{dx} = 16\sqrt{x} \cdot \frac{1}{2\sqrt{x}}$$

$16\sqrt{x} \cdot \frac{1}{2\sqrt{x}} = 8$

Q. 24 Suppose a uniformly charged wall provides a uniform electric field of $2 \times 10^4\ \mathrm{N/C}$ normally. A charged particle of mass $2\ \mathrm{g}$ being suspended through a silk thread of length $20\ \mathrm{cm}$ and remain stayed at a distance of $10\ \mathrm{cm}$ from the wall. Then the charge on the particle will be $\frac{1}{\sqrt{x}}\mu\mathrm{C}$ where $x =$_____. [use $g = 10\ \mathrm{m/s^2}$]

Correct Answer:
3

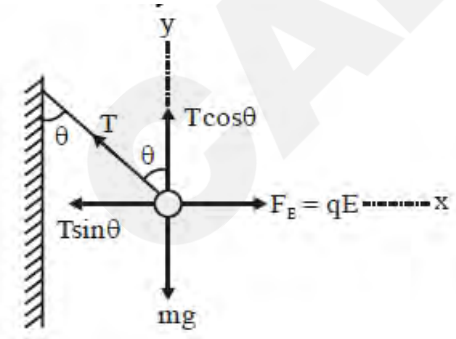
Solution:



$\sin\theta = \frac{10}{20} = \frac{1}{2}$

$\theta = 30^\circ$

Force analysis



A long x-axis

$\Sigma(f_{\text{net}})_x = 0$(for equilibrium)

$T\sin\theta = qE$.....(i)

A long y-axis

$\Sigma(f_{\text{Net}})_x = 0$ (for equilibrium)

$T\cos\theta = mg$.....(ii)

From (i) divide (ii)

$\Rightarrow \tan\theta = \frac{qE}{mg}$

$q = \frac{mg\tan\theta}{E}$

$q = \frac{2\times10^{-3}\times10}{2\times10^{4}} \times \frac{1}{\sqrt{3}} = \frac{10^{-6}}{\sqrt{3}}$

$q = \frac{1}{\sqrt{3}}\mu c \quad x = 3$

Q. 25 A moving coil galvanometer has 100 turns and each turn has an area of $2.0\ cm^2$. The magnetic field produced by the magnet is $0.01\ T$ and the deflection in the coil is 0.05 radian when a current of $10\ mA$ is passed through it. The torsinal constant of the suspension wire is $x \times 10^{-5}\ N-m/rad$. The value of x is_____.

Correct Answer:
4

Solution:
Given

$B = 0.01\ T, i = 10\ mA,\ N = 100$

$A = 2\times10^{-4}\ m^2 : K = ?$

$\tau = K\theta$.....(i)

$\uparrow$

torsional worst

$\tau = BiNA$ ....(ii)

From (i) \& (ii)

$K\theta = BiNA$

$$K = \frac{BiNA}{\theta}$$

$$K = \frac{(0.01)\times(10\times10^{-3}\ A)\times(100)\times(2\times10^{-4}\ m^2)}{(0.05)}$$

$$K = \frac{2\times10^{-4}}{5} = \frac{20}{5}\times10^{-5} = 4\times10^{-5}\ N-m/rad$$

$$\Rightarrow K = x\times10^{5}\ N-m/rad$$

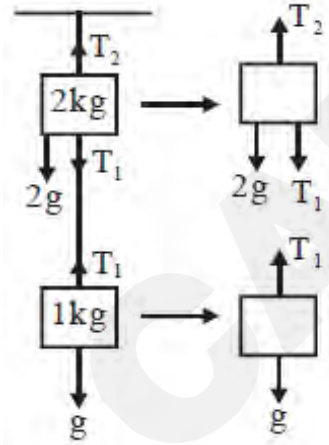
$$\text{so } X = 4$$

Hence, the answer is (4).

Q. 26 One end of a metal wire is fixed to a ceiling and a load of $2\ kg$ hangs from the other end. A similar wire is attached to the bottom of the load and another load of $1\ kg$ hangs from this lower wire. Then the ratio of longitudinal strain of upper wire to that of the lower wire will be [Area of cross section of wire $= 0.005\ cm^2$, $Y = 2\times10^{11}Nm^{-2}$ and $g = 10\ ms^{-2}$]

Correct Answer:
3

Solution:



$$\sum f_{net} = 0$$

$$T_2 = T_1 + 2\ g$$

$$= g + 2\ g = 3\ g$$

$$T_1 = g$$

$$(\text{ stress })_L = y(\text{ strain })_L$$

$$(\text{ strain })_{\mathrm{L}}=\frac{\text{stresS}}{\mathrm{Y}}=\frac{\mathrm{F}}{\mathrm{YA}}$$

$$(\text{ strain })_{\text{upper }}=\frac{\mathrm{T}_2}{\mathrm{YA}}$$

$$(\text{ strain })_{\text{lower }}=\frac{\mathrm{T}_1}{\mathrm{YA}}$$

$$\frac{(\text{ strain })_{\text{upper}}}{(\text{ strain })_{\text{lower}}}=\frac{\mathrm{T}_2}{\mathrm{T}_1}=\frac{3\,\mathrm{g}}{\mathrm{g}}=3$$

Q. 27 In Young's double slit experiment, monochromatic light of wavelength 5000 $\mathring{A}$ is used. The slits are $1.0\ \mathrm{mm}$ apart and screen is placed at $1.0\ \mathrm{m}$ away from slits. The distance from the centre of the screen where intensity becomes half of the maximum intensity for the first time is_____ $\times 10^{-6}\ \mathrm{m}$.

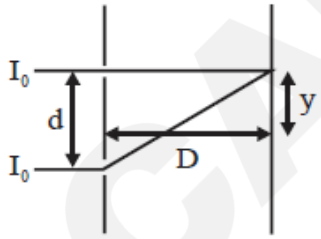
Correct Answer:
125

Solution:
$\lambda=5000\mathring{A}$

$\mathrm{d}=1\times10^{-3}\ \mathrm{m}$

$\mathrm{D}=1\ \mathrm{m}$

$\Delta\mathrm{x}=\mathrm{d}\sin\theta$.....(i)



$\Rightarrow$ For intensity

$\mathrm{I}_{\max}=4\mathrm{I}_0$

$\mathrm{I}_{\max}=\left(\sqrt{\mathrm{I}_0}+\sqrt{\mathrm{I}_0}\right)=\left(\sqrt[2]{\mathrm{I}_0}\right)^2=4\mathrm{I}_0$

$\mathrm{I}=\mathrm{I}_{\max}\cos^2\left(\frac{\Delta\phi}{2}\right)$

$\cos^2\left(\frac{\Delta\phi}{2}\right)=\frac{1}{\sqrt{2}}\ \frac{\Delta\phi}{2}=\frac{\pi}{4}\text{ or }-\frac{\pi}{4}$

$\Delta\phi = \frac{\pi}{2}$ or $\Delta\phi = -\frac{\pi}{2}$

$\frac{2\pi}{\lambda} \times \Delta x = \Delta\phi$

$\frac{2\pi}{\lambda} \times \Delta x = \frac{\pi}{2}\Delta x = \frac{\lambda}{4}$

From (i)

$\Delta x \approx d\tan\theta$

$\Delta x \approx \frac{dy}{D}$

$\frac{\lambda}{4} = \frac{dy}{D}$

$y = \frac{\lambda D}{4d} = \frac{(5000\times10^{-10})(1)}{4\times10^{-3}} = \frac{500}{4} \times 10^{-6} = 125 \times 10^{-6}$

Q. 28 A coil of 200 turns and area $0.20\ \mathrm{m}^2$ is rotated at half a revolution per second and is placed in uniform magnetic field of $0.01\ \mathrm{T}$ perpendicular to axis of rotation of the coil. The maximum voltage generated in the coil is $\frac{2\pi}{\beta}$ volt. The value of β is_____.

Correct Answer:
5

Solution:
$\phi = \mathrm{NBA}\cos\theta$

$\varepsilon = -\frac{\mathrm{d}\phi}{\mathrm{dt}}$

$\varepsilon = -\frac{\mathrm{d}}{\mathrm{dt}}(\mathrm{NBA}\cos\theta)$

$\varepsilon = \mathrm{NBA}\sin\theta \cdot \frac{\mathrm{d}\theta}{\mathrm{dt}}$

$\varepsilon = (\mathrm{NBA}\omega)\sin\theta$

$\varepsilon_{\max}$ at $\theta = 90^\circ$

$\varepsilon_{\max} = \mathrm{NBA}\omega$

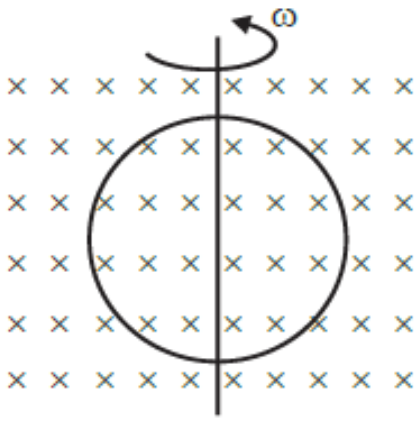
$\varepsilon_{\max} = 200 \times 0.01 \times 0.2 \times \pi$

$= 200 \times \frac{1}{100} \times \frac{2}{10} \times \pi$

$\varepsilon_{\max} = \frac{4\pi}{10} = \frac{2\pi}{5}$

$\varepsilon_{\max} = \frac{2\pi}{\beta}$

so $\beta = 5$



Q. 29 A uniform rod AB of mass $2\ \text{kg}$ and length $30\ \text{cm}$ at rest on smooth horizontal surface. An impulse of force 0.2 Ns is applied to end B. The time taken by the rod to turn through at right angles will be $\frac{\pi}{x}s$, where $x =$ _____.

Correct Answer:
4

Solution:
$\text{m} = 2\ \text{kg},\ \text{L} = 0.3,\ \text{J} = 0.2\ \text{N} - \text{s}$

Angular impulse about CM

Angular impulse $\vec{\text{I}} = \vec{\text{r}} \times \vec{\text{j}} = \vec{\text{L}}_f - \vec{\text{L}}_\text{i}$

$\frac{\text{L}}{2} \times 0.2 \times \sin 90^\circ = \text{I}_\text{CM}\omega - 0$

$\frac{\text{L}}{2} \times 0.2 \times 1 = \frac{\text{ML}^2}{12}\omega$

$\omega = \frac{0.2 \times 6}{ML}$

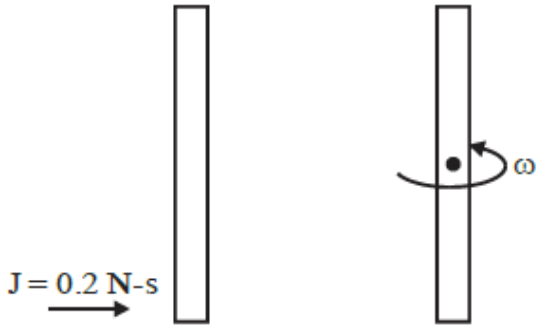
$= \frac{1.2}{2 \times 0.3} = \frac{4}{2} = 2$

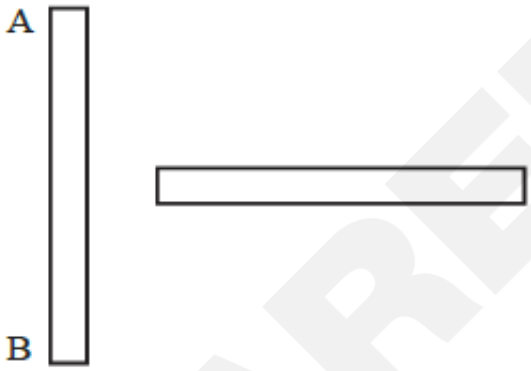
$\theta = \frac{\pi}{2}$

$\theta = \omega t$

$t = \frac{\theta}{\omega} = \frac{\pi}{2 \times 2} = \frac{\pi}{4}$

$t = \frac{\pi}{x}(\text{ given})\ x = 4$





Q. 30 A particular hydrogen-like ion emits the radiation of frequency 3×10^{15} Hz when it makes transition from $n = 2$ to $n = 1$. The frequency of radiation emitted in transition from $n = 3$ to $n = 1$ is $\frac{x}{9} \times 10^{15}$ Hz, when $x =$ _____.

Correct Answer:
32

Solution:

$E = 13.6\ \frac{1}{n_1^2} - \frac{1}{n_2^2}$

Energy released in transition

$E_1 = 13.6\ \frac{1}{1} - \frac{1}{4}\ eV$

$hf_1 = E_1 = 13.6 \times \frac{3}{4} eV$.....(i)

$E_2 = 13.6\ \frac{1}{1} - \frac{1}{9}\ eV$

$E_2 = 13.6 \times \frac{8}{9} eV$

$hf_2 = 13.6 \times \frac{8}{9} eV$....(ii)

$(ii)/(i)$

$\Rightarrow \frac{f_2}{f_1} = \frac{13.6 \times \frac{8}{9}}{13.6 \times \frac{3}{4}}$

$\frac{f_2}{f_1} = \frac{32}{27}$

$f_2 = \frac{32}{27} \times f_1$

$f_2 = \frac{32}{27} \times 3 \times 10^5 = \frac{32}{9} \times 10^{15}$

given

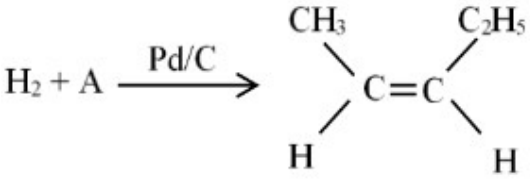
$f_2 = \frac{x}{9} \times 10^{15}\ Hz$

$x = 32$

Hence, the answer is (32).

Chemistry

Q. 1 In the given reactions identify A and B



$$CH_3 - C \equiv C - CH_3 + H_2 \xrightarrow{\text{Na/ Liquid } NH_3} "B"$$

Option 1:
A: n-Pentane & B: Cis $-2-$ butene

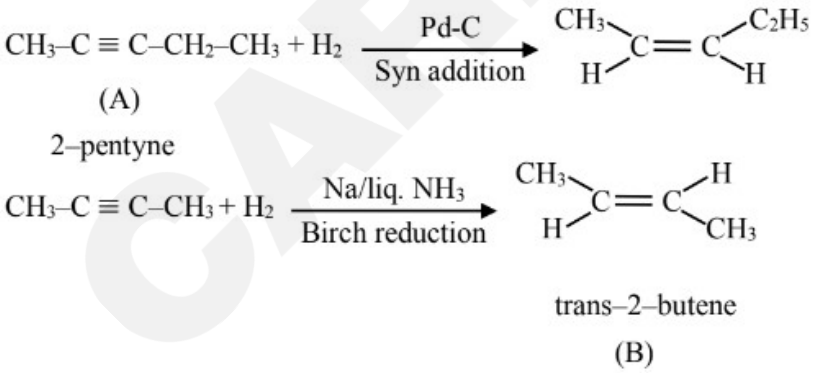
Option 2:
A: 2 - Pentyne & B: Cis - 2 - butene

Option 3:
A: n-Pentane & B: trans $-2-$ butene

Option 4:
A: $2-$ Pentyne & B: trans $-2-$ butene

Correct Answer:
A: $2-$ Pentyne & B: trans $-2-$ butene

Solution:



Q. 2 Solubility of calcium phosphate (molecular mass, M) in water is W_g per $100\ mL$ at $25^\circ C$. Its solubility product at $25^\circ C$ will be approximately.

Option 1:
$10^7\left(\frac{W}{M}\right)^3$

Option 2:
$10^3\left(\frac{W}{M}\right)^5$

Option 3:
$10^7\left(\frac{W}{M}\right)^5$

Option 4:
$10^5\left(\frac{W}{M}\right)^5$

Correct Answer:
$10^7\left(\frac{W}{M}\right)^5$

Solution:
At $25^\circ C$

$$Ca_3(PO_4)_2 \rightarrow 3Ca^{+2} + 2PO_4^{-3}$$
$$3\,S \quad 2\,S$$
$$K_{SP} = \left[Ca^{+2}\right]^3\left[PO_4^{-3}\right]^2$$
$$= [3\,S]^3[2\,S]^2$$

$\left[\frac{3\,W}{M}\right]^3\left[\frac{2\,W}{M}\right]^2$ as per $100\ mL$ but molarity is measured in litre

$$\left[\frac{3 \times W \times 1000}{M \times 100}\right]^3\left[\frac{2 \times W \times 1000}{M \times 100}\right]^2$$
$$\left[\frac{W}{M}\right]^5 \times 10^5 \times 108 \quad 108 \cong 10^2$$
$$= 10^7\left[\frac{W}{M}\right]^5$$

Hence, the answer is the option (3).

Q. 3 Given below are two statements:

Statement I: SiO_2 and GeO_2 are acidic while SnO and PbO are amphoteric in nature.

Statement II: Allotropic forms of carbon are due to property of catenation and $p\pi - d\pi$ bond formation. In the light of the above statements, choose the most appropriate answer from the options given below:

Option 1:
Statement I is false but statement II is true

Option 2:
Both statement I and statement II are false

Option 3:
Both Statement I and statement II are true

Option 4:
Statement I is true but statement II is false.

Correct Answer:
Statement I is true but statement II is false.

Solution:
Statement I : is correct $SiO_2 \& GeO_2$ amphoteric

Statement II : C - do not have de^-

So can't form $p\pi - d\pi$ bond incorrect.

Hence, the answer is the option (4).

Q. 4 The set of meta directing functional groups from the following sets is :

Option 1:
$-CN, -NH_2, -NHR, -OCH_3$

Option 2:
$-\mathrm{CN}, -\mathrm{CHO}, -\mathrm{NHCOCH_3}, -\mathrm{COOR}$

Option 3:
$-\mathrm{NO_2}, -\mathrm{NH_2}, -\mathrm{COOH}, -\mathrm{COOR}$

Option 4:
$-\mathrm{NO_2}, -\mathrm{CHO}, -\mathrm{SO_3H}, -\mathrm{COR}$

Correct Answer:
$-\mathrm{NO_2}, -\mathrm{CHO}, -\mathrm{SO_3H}, -\mathrm{COR}$

Solution:
All deactivating gps are meta directing gp except $-\mathrm{X} \& -\mathrm{NO}$.
Meta directing gps :- $-\mathrm{CN}, -\mathrm{CHO}, -\mathrm{COOR}, -\mathrm{COR}, -\mathrm{SO_3H}, -\mathrm{NO_2}, -\mathrm{COOH}$
O, P-directing gps :- $-\mathrm{NH_2}, -\mathrm{NHR}, -\mathrm{OCH_3}, -\mathrm{NHCOH_3}$

Hence, the answer is the option (4).

Q. 5 $[\mathrm{Co(NH_3)_6}]^{3+}$ and $[\mathrm{CoF_6}]^{3-}$ are respectively known as :

Option 1:
Inner orbital Complex, Spin paired Complex

Option 2:
Spin paired Complex, Spin free Complex

Option 3:
Spin free Complex, Spin paired Complex

Option 4:
Outer orbital Complex, Inner orbital Complex

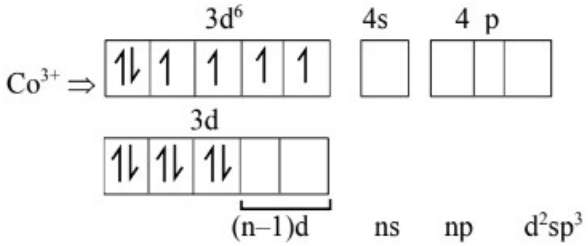
Correct Answer:
Spin paired Complex, Spin free Complex

Solution:
$\rightarrow [\mathrm{Co(NH_3)_6}]^{3+}$
$\mathrm{O.S\ of\ Co} = +3$

for $\mathrm{Co}^{+3}, \mathrm{NH_3}$ act as a strong ligand.

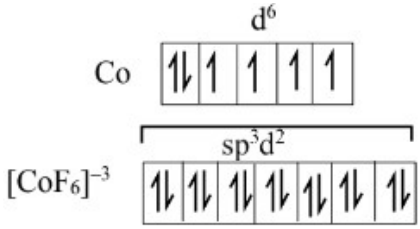
$Co = 4s^2 3d^7$

$Co^{+3} = 3d^6$



Low spin complex and spin paired that is inner orbital complex (IOC)

$[CoF_6]^{3-} \rightarrow sp^3\ d^2$ as F is weak field ligand



High spin and spin free, outer orbital complex (OOC)

Q. 6 The transition metal having highest 3rd ionization enthalpy is :

Option 1:
Mn

Option 2:
Fe

Option 3:
Cr

Option 4:
V

Correct Answer:
Mn

Solution:
Mn has Highest 3^{rd} IE

$IE_3\ kJ/mol$

$V \rightarrow 2833 \quad Mn \rightarrow 3260$
$Cr \rightarrow 2990 \quad Fe \rightarrow 2962$

Hence, the answer is the option (1).

Q. 7 Match list I with list II.

List I Compound		List II Use	
A.	Carbon tetrachloride	I.	Paint remover
B.	Methylene Chloride	II.	Refrigerators and air conditioners
C.	DDT	III.	Fire extinguisher
D.	Freons	IV.	Non- Biodegradable insecticide

Choose the correct answer from the options given below:

Option 1:
A-(II), B-(III), C-(I), D-(IV)

Option 2:
A-(III), B-(I), C-(IV), D-(II)

Option 3:
A-(I), B-(II), C-(III), D-(IV)

Option 4:
A-(IV), B-(III), C-(II), D-(I)

Correct Answer:
A-(III), B-(I), C-(IV), D-(II)

Solution:
Freons are used in refrigerators & air conditioners.

DDT is used as a Nonbiodegrable insecticides

CCl_4 is used on Fire Extinguisher.

Methylene chloride is used as a solvent for paint remover.

Hence, the answer is the option (2).

Q. 8 Match list I with list II.

	List I Reactants		**List II Product**
A.	Phenol, Zn/Δ	I.	Salicylaldehyde
B.	Phenol, $CHCl_3, NaOH, HCl$	II.	Salicylic Acid
C.	Phenol, $CO_2, NaOH, HCl$	III.	Benzene
D.	Phenol, Conc. HNO_3	IV.	Picric Acid

Choose the correct answer from the options given below:

Option 1:
A-(IV), B-(I), C-(II), D-(III)

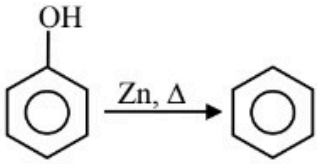
Option 2:
A-(III), B-(I), C-(II), D-(IV)

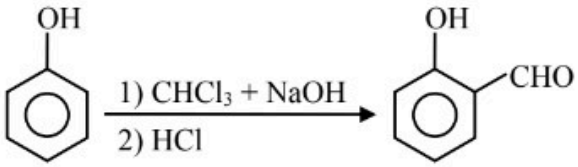
Option 3:
A-(IV), B-(II), C-(I), D-(III)

Option 4:
A-(III), B-(IV), C-(I), D-(II)

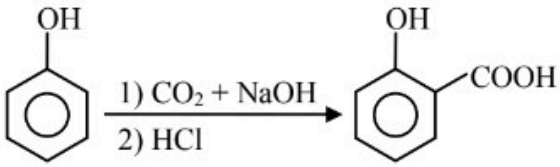
Correct Answer:
A-(III), B-(I), C-(II), D-(IV)

Solution:

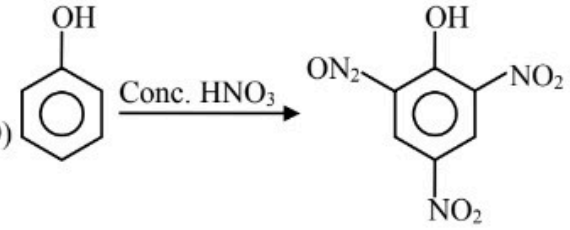
(A) 

(B) 

(Reimer-Tiemann Rxⁿ)

(C) 

(Kolbe's Rxⁿ)

(D) 

+ Oxidative products

Q. 9 Given below are two statements:

Statement I: Dimethyl glyoxime forms a six-membered covalent chelate when treated with $NiCl_2$ solution in presence of NH_4OH.

Statement II: Prussian blue precipitate contains iron both in (+2) and (+3) oxidation states.

In the light of the above statements, choose the correct answer from the options given below:

Option 1:
Statement I is false but statement II is true

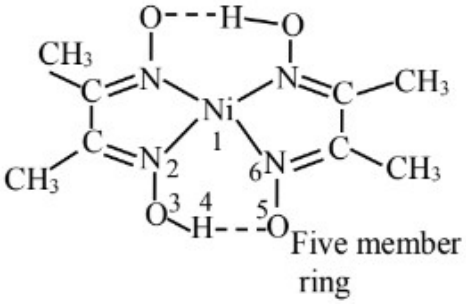
Option 2:
Both statement I and statement II are true

Option 3:
Statement I is true but statement II is false

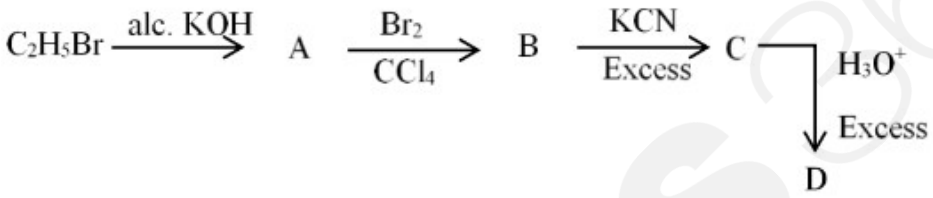
Option 4:
Both Statement I and statement II are false

Correct Answer:
Statement I is false but statement II is true

Solution:
$2(\mathrm{DMG}) + \mathrm{NiCl_2} \rightarrow \mathrm{Ni(DMG)_2}$



Q. 10



Acid D formed in above reaction is:

Option 1:
Malonic acid

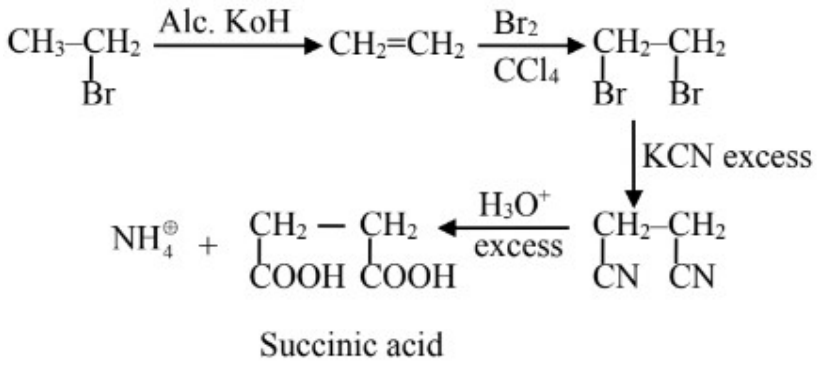
Option 2:
Oxalic acid

Option 3:
Succinic acid

Option 4:
Gluconic acid

Correct Answer:
Succinic acid

Solution:



Q. 11 Lassaigne's test is used for detection of :

Option 1:
Phosphorous and halogens only

Option 2:
Nitrogen, Sulphur and Phosphorous only

Option 3:
Nitrogen, Sulpher, Phosphorous and halogens

Option 4:
Nitrogen and Sulpur only

Correct Answer:
Nitrogen, Sulpher, Phosphorous and halogens

Solution:
Lassaigne's test is used for the detection of N, S, P and halogen.

Hence, the answer is the option (3).

Q. 12 The strongest reducing agent among the following is :

Option 1:
SbH_3

Option 2:
NH_3

Option 3:
BiH_3

Option 4:
PH_3

Correct Answer:
BiH_3

Solution:
BiH_3 is the best-reducing agent (when reducing nature increases down the group $NH_3 \rightarrow BiH_3$, as in case of other hydrides of Group IVA, VIA and VIIA).

Hence, the answer is the option (3).

Q. 13 Given below are two statements: one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A): In aqueous solutions Cr^{2+} is reducing while Mn^{3+} is oxidizing in nature. **Reason (R):** Extra stability to half filled electronic configuration is observed than incompletely filled electronic configuration.

In the light of the above statements, choose the most appropriate answer from the options given below:

Option 1:
(A) is true but (R) is false

Option 2:
Both (A) and (R) are true and (R) is the correct explanation of (A)

Option 3:
Both (A) and (R) are true but (R) is not the correct explanation of (A)

Option 4:
(A) is false but (R) is true.

Correct Answer:
Both (A) and (R) are true and (R) is the correct explanation of (A)

Solution:

$Cr^{2+} \rightarrow Cr^{3+}$ $Mn^{+3} \rightarrow Mn^{2+}$

$d^4 \rightarrow d^3$ $d^4 \rightarrow d^5$

Hence, the answer is the option (2).

Q. 14 The functional group that shows negative resonance effect is :

Option 1:
$-OH$

Option 2:
- OR

Option 3:
$-COOH$

Option 4:
$-NH_2$

Correct Answer:
$-COOH$

Solution:
$-COOH$ gp shows $-R$ or $-M$ effect
$-OR, -OH, -NH_2$ show $+R$ or $+M$ effects

Hence, the answer is the option (3).

Q. 15 The number of radial node/s for 3p orbital is:

Option 1:
3

Option 2:
2

Option 3:
1

Option 4:
4

Correct Answer:
1

Solution:
Radial Node : $\mathrm{n}-\ell-1$

$\Rightarrow \mathrm{n}=3, \ell=1, \text{ for } 3\mathrm{p}$
$=3-1-1$
$=1$

Hence, the answer is the option (3).

Q. 16 Which among the following has highest boiling point ?

Option 1:
$\mathrm{CH_3CH_2CH_2CH_2-OH}$

Option 2:
$\mathrm{CH_3CH_2CH_2CH_3}$

Option 3:
$\mathrm{CH_3CH_2CH_2CHO}$

Option 4:
$\mathrm{H_5C_2-O-C_2H_5}$

Hence, the answer is the option (1).

Correct Answer:
$\mathrm{CH_3CH_2CH_2CH_2-OH}$

Solution:
$\mathrm{CH_3CH_2CH_2CH_2-OH}$ has highest boiling point due to H-bonding.

Hence, the answer is the option (1).

Q. 17 Given below are two statements:

Statement I: A π bonding MO has lower electron density above and below the inter-nuclear axis.

Statement II: The π^* antibonding MO has a node between the nuclei.

In the light of the above statements, choose the correct answer from the options given below:

Option 1:
Both statement I and statement II are true

Option 2:
Both statement I and statement II are false

Option 3:
Statement I is true but statement II is false

Option 4:
Statement I is false but statement II is true

Correct Answer:
Statement I is false but statement II is true

Solution:
I : π BMO has larger e^- density above and below inter-nuclear axis due to overlapping $\&$ interference of atomic orbital.
hence statement I is false.
II : $\pi^*(\mathrm{ABMO})$ has nodal plane between nuclei. True

Hence, the answer is the option (4).

Q. 18 Given below are two statements:

Statement I: Both metals and non-metals exist in p and d-block elements.

Statement II: Non-metals have higher ionization enthalpy and higher electronegativity then the metals.

In the light of the above statements, choose the correct answer from the options given below:

Option 1:
Both statement I and statement II are false

Option 2:
Both statement I and statement II are true

Option 3:
Statement I is false but statement II is true

Option 4:
Statement I is true but statement II is false

Correct Answer:
Statement I is false but statement II is true

Solution:
Statement I : p-block has Metal as well non metals. while d-block has only metal. hence Ist is incorrect. Statement II : Non-Metal has high I.E. & E.N.

$\mathrm{F} \rightarrow$ highest E.N.
$\mathrm{He} \rightarrow$ Highest I.E.

Hence, the answer is the option (3).

Q. 19 Which of the following compounds show color due to d-d transition?

Option 1:
$K_2Cr_2O_7$

Option 2:
$CuSO_4.5H_2O$

Option 3:
$KMnO_4$

Option 4:
K_2CrO_4

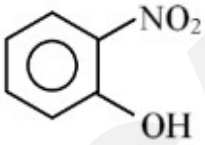
Correct Answer:
$CuSO_4.5H_2O$

Solution:
The water molecule in $CuSO_4.5H_2O$ causes d-orbital splitting in $CuSO_4$.

Hence d-d transition occurs therefore it shows a blue colour.

Hence, the answer is the option (2).

Q. 20 Select the compound from the following that will show intramolecular hydrogen bonding.

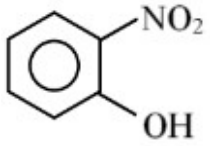
Option 1:


Option 2:
H_2O

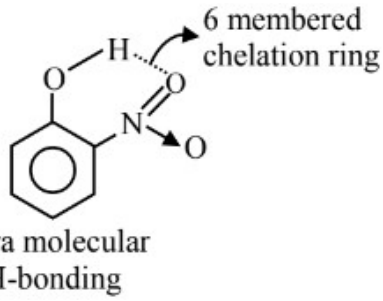
Option 3:
C_2H_5OH

Option 4:
NH_3

Correct Answer:



Solution:

→ 

→ $H_2O, EtOH \& NH_3$ form intermolecular H-bonding.

Q. 21 The number of tripeptides formed by three different amino acids using each amino acid once is-

---

Correct Answer:
6

Solution:
Suppose we have 3 amino acids glycine, Alanine & valine.
Possible Tripeptides are
Gly – Ala – Val

Gly – Val– Ala

Val – Gly – Ala

Val – Ala – Gly

Ala – Gly – Val

Ala – Val – Gly

Hence, the answer is (6).

Q. 22 Mass of ethylene glycol (antifreeze) to be added to $18.6\ \text{kg}$ of water to protect the freezing point at -24°C is_____ kg (Molar mass in gmol^{-1} for ethylene glycol 62, K_f of water $= 1.86\ \text{K kg mol}^{-1}$)

Correct Answer:
15

Solution:
$\Delta \text{T}_\text{f} = \text{K}_\text{f} \times \text{ molality}$

$$0 - -24 = 1.86 \times \frac{\text{w}}{62 \times 18.6}$$

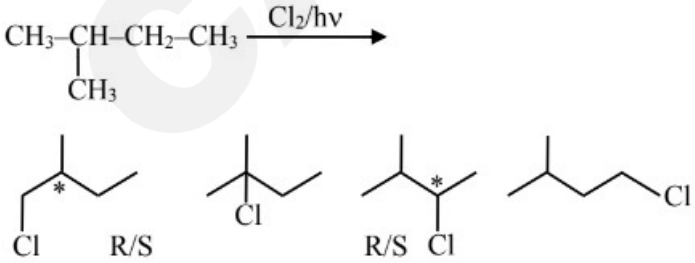
$$\Rightarrow \text{w} = \frac{62 \times 24 \times 18.6}{1.86} = 14880\text{gm}$$

$$= 14.88 \approx 15\text{Kg}$$

Hence, the answer is (15).

Q. 23 Total number of isomeric compounds (including stereoisomers) formed by monochlorination of 2-methylbutane is _____ .

Correct Answer:
6

Solution:


Q. 24 The following data were obtained during the first order thermal decomposition of a gas A at constant volume:

$$A(g) \rightarrow 2\,B(g) + C(g)$$

S.NO.	Time/s	Total pressure(atm)
1.	0	0.1
2.	115	0.28

The rate constant of the reaction is_____ $\times 10^{-2}\ s^{-1}$ (nearest integer)

Correct Answer:
2

Solution:

$A(g) \rightarrow 2\,B(g) + C(g)$

$t = 0 \quad 0.1 \quad -$

$t = 115\,\sec\ 0.1 - x \quad 2x \quad x$

$\text{Total Pressure} = 0.1 + 2x = 0.28$

$x = 0.09$

$$K = \frac{2.303}{115}\log\frac{0.1}{0.1 - 0.09}$$

$K = 0.020$

$K = 2 \times 10^{-2}$

Hence, the answer is (2).

Q. 25 For a certain reaction at $300\ K,\ K = 10$, then ΔG° for the same reaction is_____ $\times 10^{-1}\ kJ\ mol^{-1}$. (Given $R = 8.314 JK^{-1}\ mol^{-1}$)

Correct Answer:
57

Solution:

$\Delta G^{\circ} = -2.303RT \log K_{eq}$

$= -2.303 \times 8.314 \times 300 \log 10$

$= -5744.17 \text{ J/mol}$

$= -5.744 \text{ kJ/mol}$

$= -57.44 \times 10^{-1} \text{ kJ/mol}$

$= 57$

Hence, the answer is (57).

Q. 26 The amount of electricity in Coulomb required for the oxidation of 1 mol of H_2O to O_2 is_____ $\times 10^5 C$.

Correct Answer:
2

Solution:
$H_2O \longrightarrow 2H^+ + \frac{1}{2}O_2 + 2e^-$

oxidation of 1 mole of water req. $96500 \times 2C$

$= 96500 \times 2$
$= 19300$
$= 1.93 \times 10^5 C \approx 2 \times 10^5 C$

Hence, the answer is (2).

Q. 27 Following Kjeldahl's method, 1 g of organic compound released ammonia, that neutralized 10 mL of $2M$ H_2SO_4. The percentage of nitrogen in the compound is_____ $\%$.

Correct Answer:
56

Solution:

$$\text{Normality of } H_2SO_4 = \text{Molarity} \times \text{n factor}$$
$$= 2 \times 2 = 4$$
$$\% \text{ of Nitrogen} = \frac{1.4 \times N_{H_2SO_4} \times V_{H_2SO_4}}{\text{wt.of organic compd.}}$$
$$= \frac{1.4 \times 4 \times 10}{1} = 56\%$$

Hence, the answer is (56).

Q. 28 Consider the following redox reaction:

$$MnO_4^- + H^+ + H_2C_2O_4 \rightleftharpoons Mn^{2+} + H_2O + CO_2$$

The standard reduction potentials are given as below: (E_{red}) :

$$E^\circ_{MnO_4^-/Mn^{2+}} = +1.51 \text{ V}$$
$$E^\circ_{CO_2/H_2C_2O_4} = -0.49 \text{ V}$$

If the equilibrium constant of the above reaction is given as $K_{eq} = 10^x$, then the value of $x =$_____ (nearest integer)

Correct Answer:
338

Solution:
$$MnO_4^- + H^+ + H_2C_2O_4 \rightleftharpoons Mn^{2+} + H_2O + CO_2$$
$$E^\circ_{Mn^{+7}/Mn^{+2}} = 1.51 \text{ V}$$
$$E^\circ_{C^{+4}/C^{+3}} = -0.49 \text{ V}$$
$$E^\circ_{cell} = E^\circ_{oxi.} + E^\circ_{red}$$
$$= 1.51 + (-0.49)$$
$$= 2 \text{ V}$$

Balance

$$2MnO_4^- + 5C_2O_4^{2-} + 16H^+ \rightarrow 2Mn^{2+} + 10CO_2 + 8H_2O$$

$$n = 10$$

$$E^0_{cell} = \frac{0.059}{n} \log K_{eq}$$

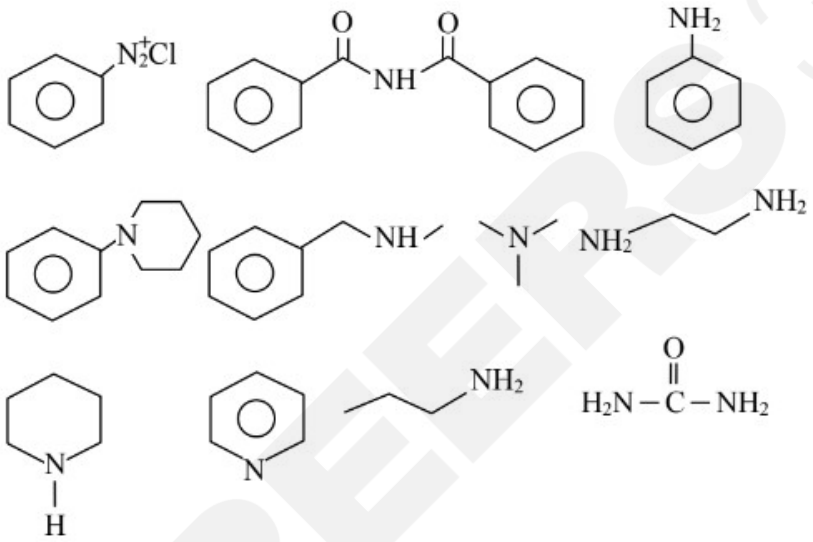
$$2 = \frac{0.059}{10} \log K_{eq}$$

$$\log K_{eq} = \frac{2 \times 10}{0.059} = 338 \Rightarrow K_{eq} = 10^{338}$$

$$= Keq = 10^{338}$$

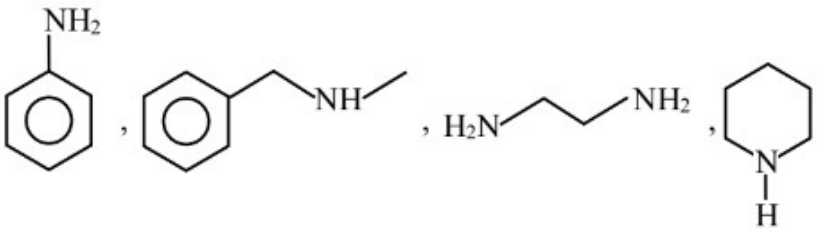
Hence, the answer is (338).

Q. 29 Number of compounds which give reaction with Hinsberg's reagent is _____ .



Correct Answer:
5

Solution:
1° Amines & 2° Amines give reaction with Hinsberg's reagent.



 give reaction with Hinsberg's reagent $(Ph - SO_2Cl)$

Q. 30 $10\ \mathrm{mL}$ of gaseous hydrocarbon on combustion gives $40\ \mathrm{mL}$ of $\mathrm{CO_2(g)}$ and $50\ \mathrm{mL}$ of water vapour. Total number of carbon and hydrogen atoms in the hydrocarbon is_____.

Correct Answer:
14

Solution:
general balancing of Hydrocarbon

$$\mathrm{C_xH_y} + \left(\mathrm{x} + \frac{\mathrm{y}}{4}\right)\mathrm{O_2} \longrightarrow \mathrm{xCO_2} + \frac{\mathrm{y}}{2}\mathrm{H_2O}$$
$$10\mathrm{ml}\ \left(\mathrm{x} + \frac{\mathrm{y}}{4}\right)\quad 40\mathrm{ml}\quad 50\mathrm{ml}$$

On applying POAC on carbon

$10 \times x = 40$
$x = 4$

On applying POAC on hydrogen

$10 \times y = 50 \times 2$
$y = 10$

Hence $\mathrm{C_4H_{10}}$

$10 + 4 = 14$

Hence, the answer is (14).

Maths

Q. 1 If the domain of the function $f(x) = \frac{\sqrt{x^2-25}}{(4-x^2)} + \log_{10}\left(x^2 + 2x - 15\right)$ is $(-\infty, \alpha) \cup [\beta, \infty)$, then $\alpha^2 + \beta^3$ is equal to:

Option 1:
140

Option 2:
175

Option 3:
125

Option 4:
150

Correct Answer:
150

Solution:

$f(x)=\frac{\sqrt{x^2-25}}{4-x^2}+\log_{10}\left(x^2+2x-15\right)$

to check domain :

$x^2-25\geq 0 \quad \ldots(i)$
$4-x^2\neq 0 \quad \ldots(ii)$
$x^2+2x-15>0 \quad \ldots(iii)$

Solving (i)

$(x-5)(x+5)\geq 0$
$x\in(-\infty,-5]\cup[5,\infty)\ldots\ldots\ldots(iv)$

Solvin (ii)

$x\neq 2$ or -2 . .(v)

Solving (iii)

$(x+5)(x-3)>0$
$x\in(-\infty,-5)\cup(3,\infty)\cdots\cdots\cdots(vi)$

domain $\Rightarrow$ intersection of (iv), (v), (vi)

$\therefore \mathrm{x}\in(-\infty,-5)\cup[5,\infty)$

on comparing $\alpha=-5 \quad \beta=5$
$\therefore \alpha^2+\beta^3=25+125=150$

Hence, the answer is the option (4).

Q. 2 If z is a complex number such that $|z| \geq 1$, then the minimum value of $z+\frac{1}{2}(3+4i)$ is :

Option 1:
2

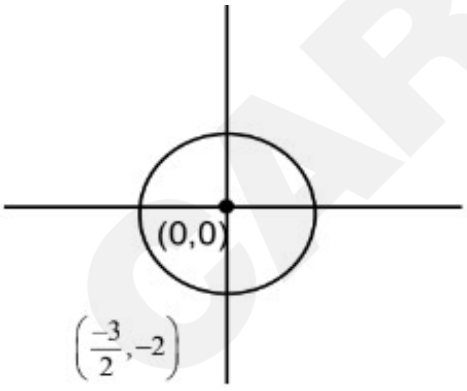
Option 2:
$\frac{5}{2}$

Option 3:
$\frac{3}{2}$

Option 4:
3

Correct Answer:
2

Solution:
Bonus or Ans. 0
$|Z| \geq 1$ is a region outside \& periphery of circle with centre $(0,0)\&$ radius $=$ one unit
minimum value of $\mathrm{Z}-\left(-\frac{3}{2}-2\mathrm{i}\right)$ is minimum distance of point in $|\mathrm{Z}| \geq 1$ from $\left(-\frac{3}{2}-2\right)$



Therefore minimum value of $\mathrm{Z}+\frac{1}{2}(3+4\mathrm{i}) = 0$

Q. 3 Consider a $\triangle \mathrm{ABC}$ where $\mathrm{A}(1,3,2), \mathrm{B}(-2,8,0)$ and $\mathrm{C}(3,6,7)$. If the angle bisector of $\angle \mathrm{BAC}$ meets the line BC at D, then the length of the projection of the vector $\overline{\mathrm{AD}}$ on the vector $\overline{\mathrm{AC}}$ is :

Option 1:
$\frac{37}{2\sqrt{38}}$

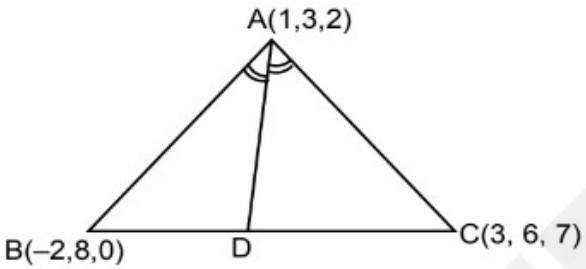
Option 2:
$\sqrt{19}$

Option 3:
$\frac{39}{2\sqrt{38}}$

Option 4:
$\frac{\sqrt{38}}{2}$

Correct Answer:
$\frac{37}{2\sqrt{38}}$

Solution:



D divides BC in the ratio of $|\overline{\mathrm{AB}}| : |\mathrm{AC}|$

$$\overrightarrow{\mathrm{AB}} = -3\hat{\mathrm{i}} + 5\hat{\mathrm{j}} - 2\hat{\mathrm{k}}$$
$$\overline{\mathrm{AC}} = 2\hat{\mathrm{i}} + 3\hat{\mathrm{j}} + 5\hat{\mathrm{k}}$$
$$|\overrightarrow{\mathrm{AB}} = \sqrt{38}$$
$$|\overrightarrow{\mathrm{AC}} = \sqrt{38}$$
$$\therefore \frac{\mathrm{BD}}{\mathrm{DC}} = \frac{\sqrt{38}}{\sqrt{38}} = \frac{1}{1}$$

$\therefore \mathrm{D}$ is mid point

coordinates of $\mathrm{D}\left(\frac{1}{2}, 7, \frac{7}{2}\right)$

$$\overline{\mathrm{AD}} = \frac{1}{2}\hat{\mathrm{i}} + 4\hat{\mathrm{j}} + \frac{3}{2}\hat{\mathrm{k}}$$

Projection of $\overline{AD}$ on $\overline{AC} = \frac{\overline{AD}\cdot\overline{AC}}{|\overline{AC}|} = \frac{-1+12+\frac{15}{2}}{\sqrt{38}}$

$= \frac{37}{2\sqrt{38}}$

Q. 4 Consider the relations R_1 and R_2 defined as $aR_1b \Leftrightarrow a^2+b^2=1$ for all $a,b \in R$ and $(a,b)R_2(c,d) \Leftrightarrow a+d=b+c$ for all $(a,b),(c,d) \in N \times N$. Then

Option 1:
R_1 and R_2 both are equivalence relations

Option 2:
Only R_1 is an equivalence relation

Option 3:
Only R_2 is an equivalence relation

Option 4:
Neither R_1 nor R_2 is an equivalence relation

Correct Answer:
Only R_2 is an equivalence relation

Solution:
$\mathrm{a,R,b} \Leftrightarrow \mathrm{a^2+b^2=1} \quad \forall \mathrm{a,b,} \in \mathrm{R}$

checking reflexive

$a=b$

$a^2+a^2=1$

$a=\pm\frac{1}{\sqrt{2}}$

$\therefore$ only $\left(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)\&\left(\frac{-1}{\sqrt{2}},\frac{-1}{\sqrt{2}}\right)$ will be present in R_1
So not reflexive, $\therefore$ not equivalence

$\mathrm{(a,b)R_2(c,d) \Leftrightarrow a+d=b+c \forall (a,b),(c,d) \in N \times N}$

Checking reflexive, put $\mathrm{a=b=c=d}$

$a+a=a+a$

Which is satisfied $\forall$ ordered pair $\in \mathrm{N} \times \mathrm{N}, \therefore \mathrm{R}_2$ is reflexive Checking symmetric
Let suppose ($a, b), (c, d))$ is in R_2
So, $\mathrm{a}+\mathrm{d}=\mathrm{b}+\mathrm{c}$
to check ((c, d), (a, b)) lies in R_2,
(i) can be expressed as,

c+b=d+a

$\therefore \mathrm{R}_2$ is symmetric

checking transitive,

Let suppose $((a, b), (c, d)) \& ((c, d)(e, f))$ are in R_2
So, $a+d=b+c$
$\mathrm{c}+\mathrm{f}=\mathrm{d}+\mathrm{e}$
adding (ii) \& (iii)
$a+f=b+e$
$\therefore (\mathrm{a}, \mathrm{b}), (\mathrm{e}, \mathrm{f})$ also in R_2
$\therefore \mathrm{R}_2$ is transitive
$\therefore \mathrm{R}_2$ is equivalence
$\therefore$ only R_2 is equivalence relation

Hence, the answer is the option (3).

Q. 5 Let the system of equations
$x+2y+3z=5, 2x+3y+z=9, 4x+3y+\lambda z=\mu$ have infinite number of solutions. Then $\lambda+2\mu$ is equal to :

Option 1:
22

Option 2:
17

Option 3:
15

Option 4:
28

Correct Answer:
17

Solution:
$x + 2y + 3z = 5$
$2x + 3y + z = 9$
$4x + 3y + \lambda z = \mu$

for infinite solution $D = D_1 = D_2 = D_3 = 0$

$$\mathrm{D} = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 4 & 3 & \lambda \end{vmatrix} = 0$$

$(3\lambda - 3) - 2(2\lambda - 4) + 3(6 - 12) = 0$

$\lambda = -13$

$$D_3 = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 9 \\ 4 & 3 & \mu \end{vmatrix} = 0$$

$3\mu - 27 - 2(2\mu - 36) + 5(6 - 12) = 0$

$\mu = 15$

$\lambda + 2\mu = -13 + 30 = 17$

Hence, the answer is the option (2).

Q. 6 If $\int_0^{\frac{\pi}{3}} \cos^4 x dx = a\pi + b\sqrt{3}$, where a and b are rational numbers, then $9a + 8b$ is equal to

Option 1:
2

Option 2:
1

Option 3:
3

Option 4:

$\frac{3}{2}$

Correct Answer:

$\frac{3}{2}$

Solution:

$$\int_0^{\frac{\pi}{3}} \cos^4 x dx = \int_0^{\frac{\pi}{3}} \cos^2 x \cdot \cos^2 x dx$$

$$= \int_0^{\frac{\pi}{3}} \cos^2 x \left(1 - \sin^2 x\right) dx$$

$$= \int_0^{\frac{\pi}{3}} \cos^2 x dx - \int_0^{\frac{\pi}{3}} 4\frac{\sin^2 x \cos^2 x}{4} dx$$

$$\int_0^{\frac{\pi}{3}} \left(\frac{1+\cos 2x}{2}\right) dx - \frac{1}{4}\int_0^{\frac{\pi}{3}} (\sin 2x)^2 dx$$

$$= \frac{1}{2}\left(\frac{1+\cos 2x}{2}\right)_0^{\frac{\pi}{3}} - \frac{1}{4}\int_0^{\frac{\pi}{3}} \left(\frac{1-\cos 4x}{2}\right) dx$$

$$= \frac{1}{2}\left[\frac{\pi}{3} + \frac{\sqrt{3}}{4}\right] - \frac{1}{8}\left[x - \frac{\sin 4x}{4}\right]_0^{\pi/3}$$

$$= \frac{\pi}{6} + \frac{\sqrt{3}}{8} - \frac{\pi}{24} - \frac{\sqrt{3}}{64}$$

$$= \frac{\pi}{8} + \frac{7\sqrt{3}}{64}$$

$$a = \frac{1}{8} \quad b = \frac{7}{64}$$

$$9a + 8b = \frac{9}{8} + \frac{7}{8} = 2$$

Hence, the answer is the option (4).

Q. 7 Let α and β be the roots of the equation $px^2 + qx - r = 0$, where $p \neq 0$. If p, q and r be the consecutive terms of a non constant G.P. and $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{3}{4}$, then the value of $(\alpha - \beta)^2$ is :

Option 1:
8

Option 2:
9

Option 3:
$\frac{20}{3}$

Option 4:
$\frac{80}{9}$

Correct Answer:
$\frac{80}{9}$

Solution:
$p, q, r,$ in G.P. $, \therefore q^2 = pr$

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{3}{4}$$

$$\frac{\alpha + \beta}{\alpha\beta} = \frac{3}{4}$$

$$-q$$

$$\frac{p}{-\frac{r}{p}} = \frac{3}{4}$$

$$|\alpha-\beta|=\frac{\sqrt{q^2+4pr}}{|P|}$$

$$(\alpha-\beta)^2=\frac{q^2+4pr}{p^2}$$

$$=\frac{5q^2}{p^2}=\frac{5q^2r^2}{q^4}=5\frac{r^2}{q^2}$$

$$=5\times\frac{16}{9}=\frac{80}{9}$$

Hence, the answer is the option (4).

Q. 8 Let Ajay will not appear in JEE exam with probability $\mathrm{p}=\frac{2}{7}$, while both Ajay and Vijay will appear in the exam with probability $\mathrm{q}=\frac{1}{5}$. Then the probability, that Ajay will appear in the exam and Vijay will not appear is :

Option 1:
$\frac{9}{35}$

Option 2:
$\frac{3}{35}$

Option 3:
$\frac{24}{35}$

Option 4:
$\frac{18}{35}$

Correct Answer:
$\frac{18}{35}$

Solution:

$$P(A) = \frac{5}{7}$$

$$P(A \cap V) = \frac{1}{5} = P(A) \cdot P(V) \quad \therefore P(V) = \frac{7}{25}$$

$$P(A \cap \overline{V}) = P(A) \cdot P(\overline{V})$$

$$= \frac{5}{7} \times \left(1 - \frac{7}{25}\right)$$

$$= \frac{18}{35}$$

Hence, the answer is the option (4).

Q. 9 Let P be a point on the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$. Let the line passing through P and parallel to y-axis meet the circle $x^2 + y^2 = 9$ at point Q such that P and Q are on the same side of the x-axis. Then, the eccentricity of the locus of the point R on PQ such that $PR : RQ = 4 : 3$ as P moves on the ellipse, is :

Option 1:
$\frac{13}{21}$

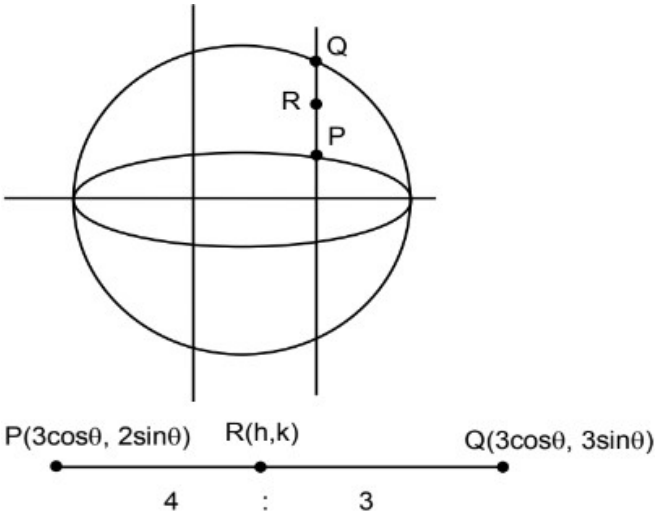
Option 2:
$\frac{\sqrt{139}}{23}$

Option 3:
$\frac{\sqrt{13}}{7}$

Option 4:
$\frac{11}{19}$

Correct Answer:
$\frac{\sqrt{13}}{7}$

Solution:



$h = \frac{12\cos\theta + 9\cos\theta}{7} \quad k = \frac{6\sin\theta + 12\sin\theta}{7}$

$h = 3\cos\theta \quad k = \frac{18\sin\theta}{7}$

$\frac{h^2}{3^2} + \frac{k^2}{\left(\frac{18}{7}\right)^2} = 1$

$$\left(\frac{18}{7}\right)^2 = 3^2\left(1 - e^2\right)$$

$$e^2 = 1 - \frac{36}{49}$$

$$e = \frac{\sqrt{13}}{7}$$

Q. 10 Consider 10 observations $x_1, x_2, \ldots, x_{10}$ such that $\sum_{i=1}^{10}(x_i - \alpha) = 2$ and $\sum_{i=1}^{10}(x_i - \beta)^2 = 40$, where α, β are positive integers. Let the mean and the variance of the observations be $\frac{6}{5}$ and $\frac{84}{25}$ respectively. Then $\frac{\beta}{\alpha}$ is equal to :

Option 1:
2

Option 2:
1

Option 3:
$\frac{5}{2}$

Option 4:

$\frac{3}{2}$

Correct Answer:

2

Solution:

$\bar{x}=\frac{\sum_{i=1}^{10} x_i}{10}=\frac{6}{5}, \quad \frac{\sum x_i^2}{10}-(\bar{x})^2=\frac{84}{25}$

$\sum_{i=1}^{10}(x_i-\alpha)=2 \quad \frac{\sum x_i^2}{10}-\frac{36}{25}=\frac{84}{25}$

$\sum_{i=1}^{10} x_i-10\alpha=2 \quad \frac{\sum x_i^2}{10}=\frac{120}{25}$

$10\alpha=10 \quad \sum x_i^2=48$

$\alpha=1$

$$\sum_{i=1}^{10}(x_i-\beta)^2=40$$

$$\sum_{i=1}^{10} x_i^2+10\beta^2-2\beta\sum_{i=1}^{10} x_i=40$$

$48+10\beta^2-24\beta=40$

$10\beta^2-24\beta+8=0$

$5\beta^2-12\beta+4=0$

$\beta=2$ or $\beta=\frac{2}{5}$ (reject)

$$\frac{\beta}{\alpha}=2$$

Hence, the answer is the option (1).

Q. 11 Let $f(x)=\ 2x^2+5\ x|-3|, x\in R$. If m and n denote the number of points where f is not continuous and not differentiable respectively, then $m+n$ is equal to:

Option 1:
5

Option 2:
3

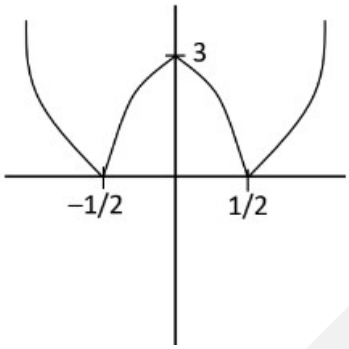
Option 3:
2

Option 4:
0

Correct Answer:
3

Solution:
$y = 2x^2 + 5\,x| - 3|$

By graphical transformation



Clearly f is continuous everywhere and not differentiable at three points.

$\therefore m = 0, n = 3$
$m + n = 3$

Q. 12 The number of solutions of the equation
$4\sin^2 x - 4\cos^3 x + 9 - 4\cos x = 0; x \in [-2\pi, 2\pi]$ is :

Option 1:
0

Option 2:
3

Option 3:
1

Option 4:
2

Correct Answer:
0

Solution:

$4\sin^2 x - 4\cos^3 x + 9 - 4\cos x = 0$

$4\left(1 - \cos^2 x\right) - 4\cos^3 x + 9 - 4\cos x = 0$

$4\cos^3 x + 4\cos^2 x + 4\cos x - 13 = 0$

Max value of $\cos x = 1$

So $4\cos^3 x + 4\cos^2 x + 4\cos x$ can never be greater than 12 therefore no solution.

Hence, the answer is the option (1).

Q. 13 Let the locus of the mid points of the chords of the circle $x^2 + (y-1)^2 = 1$ drawn from the origin intersect the line $x + y = 1$ at P and Q. Then, the length of PQ is :

Option 1:
$\frac{1}{2}$

Option 2:
1

Option 3:
$\frac{1}{\sqrt{2}}$

Option 4:

$\sqrt{2}$

Correct Answer:

$\frac{1}{\sqrt{2}}$

Solution:

Eq. of chord whose mid point is (h, k)

$$T = S_1$$

$$xh + yk - (y + k) = h^2 + k^2 - 2k$$

$$xh + yk - y = h^2 + k^2 - k$$

It passes through (0, 0)

$$\therefore \mathrm{h}^2 + \mathrm{k}^2 - \mathrm{k} = 0$$

$$\text{Locus: } \mathrm{x}^2 + \mathrm{y}^2 - \mathrm{y} = 0$$

Locus intersect x + y = 1 at P and Q

$$\therefore y^2 - y + (1 - y)^2 = 0$$

$$2y^2 - 3y + 1 = 0$$

$$(y - 1)(2y - 1) = 0$$

$$y = 1 \text{ or } y = \frac{1}{2}$$

$$x = 0 \quad x = \frac{1}{2}$$

$$\mathrm{P}(0, 1) \text{ and } \mathrm{Q}\left(\frac{1}{2}, \frac{1}{2}\right)$$

$$\mathrm{PQ} = \sqrt{\frac{1}{4} + \frac{1}{4}} = \frac{1}{\sqrt{2}}$$

Hence, the answer is the option (3).

Q. 14 Let α be a non-zero real number. Suppose $f : R \to R$ is a differentiable function such that $f(0) = 2$ and $\lim_{x\to-\infty} f(x) = 1$. If $f'(x) = \alpha f(x) + 3$, for all $x \in R$, then $f(-\log_e 2)$ is equal to

Option 1:
7

Option 2:
9

Option 3:
3

Option 4:
5

Correct Answer:
9

Solution:
$y = f(x)$

$$\frac{dy}{dx} - \alpha y = 3$$

$$\text{I.F.} = e^{\int -\alpha dx} = e^{-\alpha x}$$

Sol. of differential equation

$$Y(I.F.) = \int (I.F)(3)dx + C$$

$$y \cdot e^{-\alpha x} = 3\int e^{-\alpha x}dx + C_1$$

$$e^{-\alpha x} \cdot y = \frac{-3e^{-\alpha x}}{\alpha} + C$$

$$\lim_{x\to-\infty} f(x) = 1$$

$$\lim_{x\to-\infty} f(x) = \frac{-3}{\alpha} + C \lim_{x\to-\infty} e^{\alpha x}$$

$$1 = \frac{-3}{\alpha}$$

$\alpha = -3$ (α should be positive for finite value of C)

$\mathrm{f(x) = 1 + Ce^{-3x}}$
$\mathrm{Put\ x = 0}$
$\mathrm{2 = 1 + C}$
$\mathrm{c = 1}$

$\therefore \mathrm{f(x) = 1 + e^{-3x}, f(-\ln 2) = 1 + e^{3\ln 2} = 9}$

Hence, the answer is the option (2).

Q. 15 Let P and Q be the points on the line $\frac{\mathrm{x+3}}{8} = \frac{\mathrm{y-4}}{2} = \frac{\mathrm{z+1}}{2}$ which are at distance of 6 units from the point $\mathrm{R}(1, 2, 3)$. If the centroid of the triangle PQR is (α, β, γ), then $\alpha^2 + \beta^2 + \gamma^2$ is :

Option 1:
18

Option 2:
24

Option 3:
26

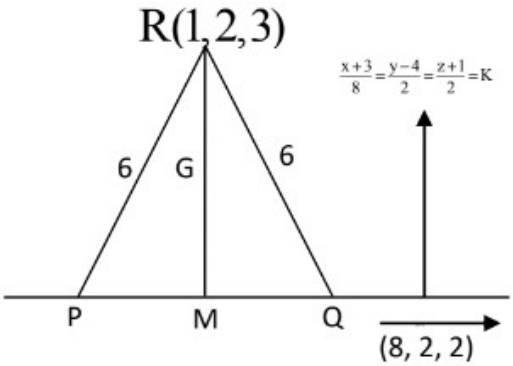
Option 4:
36

Correct Answer:
18

Solution:
M is mid point of PQ and $\perp^{\mathrm{r}}$ to line

G is centroid of $\triangle PQR$ such that $RG : GM \equiv 2 : 1$

Coordinates of $M(8k-3, 2k+4, 2k-1)$



$RM \cdot (8\hat{i} + 2\hat{j} + 2k) = 0$

$8(8K-4) + (2K+2)2 + (2K-4)2 = 0$

$72K - 36 = 0$

$K = \frac{1}{2}$

$M(1, 5, 0)$

$\therefore G\left(\frac{2\times 1+1\times 1}{3}, \frac{2\times 5+1\times 2}{3}, \frac{2\times 0+1\times 3}{3}\right)$

$G(1, 4, 1)$

$\alpha^2 + \beta^2 + \gamma^2 = 18$

Q. 16 The value of $\int_0^1 \left(2x^3 - 3x^2 - x + 1\right)^{\frac{1}{3}} dx$ is equal to :

Option 1:
-1

Option 2:
2

Option 3:
0

Option 4:
1

Correct Answer:
0

Solution:

$$I=\int_0^1 \left(2x^3-3x^2-x+1\right)^{1/3}\cdot dx$$

Applying king replace $x \to (1-x)$

$$I=\int_0^1 \left[2(1-x)^3-3(1-x)^2-(1-x)+1\right]^{1/3}\cdot dx$$

$$I=\int_0^1 \left[2-2x^3-6x+6x^2-3-3x^2+6x-1+x+1\right]^{1/3}\cdot dx$$

$$I=\int_0^1 \left[-2x^3+3x^2+x-1\right]^{1/3}\cdot dx$$

$$I=-\int_0^1 \left[2x^3-3x^2-x+1\right]^{1/3}\cdot dx$$

$$I=-I$$

$$2I=0\Rightarrow I=0$$

Hence, the answer is the option (3).

Q. 17 If the mirror image of the point $\mathrm{P}(3,4,9)$ in the line $\frac{\mathrm{x}-1}{3}=\frac{\mathrm{y}+1}{2}=\frac{\mathrm{z}-2}{1}$ is (α,β,γ), then $14(\alpha+\beta+\gamma)$ is :

Option 1:
102

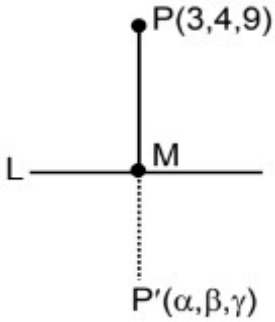
Option 2:
138

Option 3:
132

Option 4:
108

Correct Answer:
108

Solution:
$L: \frac{x-1}{3} = \frac{y+1}{2} = \frac{z-2}{1} = \lambda, p(3,4,9)$



$\mathrm{M}(3\lambda+1, 2\lambda-1, \lambda+2)$

$\mathrm{PM} = (3\lambda-2, 2\lambda-5, \lambda-7)$

DR's of $L: (3,2,1)$

$\overline{\mathrm{PM}} \perp$ line L,

$\text{So, } 3(3\lambda-2)+2(2\lambda-5)+1(\lambda-7)=0$
$9\lambda-6+4\lambda-10+\lambda-7=0$

$14\lambda = 23 \Rightarrow \lambda = \frac{23}{14}$
$\mathrm{M}\left(\frac{83}{14}, \frac{32}{14}, \frac{51}{14}\right)$

Now, As M is mid-point of pp'

$\therefore$ co-ordinates of p' are

$$\frac{\alpha+3}{2}=\frac{83}{14}\Rightarrow\alpha=\frac{62}{7}$$
$$\frac{\beta+4}{2}=\frac{32}{14}\Rightarrow\beta=\frac{4}{7}$$
$$\frac{\gamma+9}{2}=\frac{51}{14}\Rightarrow\gamma=\frac{-12}{7}$$

So, $14(\alpha+\beta+\gamma)=14\times\frac{54}{7}\Rightarrow 108$

Q. 18 Let S_n denote the sum of the first n terms of an arithmetic progression. If $S_{10}=390$ and the ratio of the tenth and the fifth terms is $15:7$, then $\mathrm{S}_{15}-\mathrm{S}_5$ is equal to :

Option 1:
800

Option 2:
890

Option 3:
790

Option 4:
690

Correct Answer:
790

Solution:
Given :

$\mathrm{S}_{10}=390,\qquad \frac{\mathrm{T}_{10}}{\mathrm{T}_7}=\frac{15}{7}$

$\mathrm{S}_{10}=\frac{10}{2}[2\mathrm{a}+9\,\mathrm{d}]=390$

$2\mathrm{a}+9\,\mathrm{d}=78 \qquad \ldots\ldots(1)$

$$\frac{T_{10}}{T_5}=\frac{a+9d}{a+4d}=\frac{15}{7}$$

$$Ta+63d=15a+60d$$

$$8a=3d\ldots\ldots(2)$$

From (1) & (2)

a = 3, d = 8

$$\text{Now, } S_{15} - S_5 = \frac{15}{2}[2a + 14d] - \frac{5}{2}[2a + 4d]$$

$$= \frac{5}{2}[6a + 42d - 2a - 4d]$$

$$= \frac{5}{2}[4\text{a} + 38\ \text{d}] = 5[2\text{a} + 19\ \text{d}]$$

$$= 5[6 + 19 \times 8]$$

$$= 790$$

Hence, the answer is the option (3).

Q. 19 Let m and n be the coefficients of seventh and thirteenth terms respectively in the expansion of $\left(\frac{1}{3}x^{\frac{1}{3}} + \frac{1}{2x^{\frac{2}{3}}}\right)^{18}$. Then $\left(\frac{n}{m}\right)^{\frac{1}{3}}$ is :

Option 1:
$\frac{1}{9}$

Option 2:
$\frac{1}{4}$

Option 3:
$\frac{4}{9}$

Option 4:
$\frac{9}{4}$

Correct Answer:
$\frac{4}{9}$

Solution:

$\left(\frac{1}{3}x^{1/3}+\frac{1}{2\cdot x^{2/3}}\right)^{18}$ coeff. of $T_7 \rightarrow m$

coeff. of $T_{13} \rightarrow n$

$$\frac{m}{n}=\frac{18c_6\left(\frac{1}{3}x^{1/3}\right)^{12}\cdot\left(\frac{1}{2x^{2/3}}\right)^{6}}{18c_{12}\left(\frac{1}{3}x^{1/3}\right)^{6}\cdot\left(\frac{1}{2x^{2/3}}\right)^{12}} \quad (\because {}^nc_r={}^nc_{n-r})$$

$$\left(\frac{m}{n}\right)^{1/3}=\left(\frac{1}{3^{12}}\times 3^6\times\frac{1}{2^6}\times 2^{12}\right)^{1/3}\Rightarrow\left(\frac{2^6}{3^6}\right)^{1/3}\Rightarrow\frac{4}{9}$$

Hence, the answer is the option (3).

Q. 20 Let $f(x)=\begin{cases}x-1, & x \text{ is even},\\ 2x, & x \text{ is odd},\end{cases}$ $x\in$. If for some $a\in N$, $f(f(f(a)))=21$, then $\lim_{x\to a^-}\left\{\frac{|x|^3}{a}-\left[\frac{x}{a}\right]\right\}$, where $[t]$ denotes the greatest integer less than or equal to t, is equal to :

Option 1:
169

Option 2:
121

Option 3:
225

Option 4:
144

Correct Answer:
144

Solution:

$$f(x)=\begin{cases}x-1, & x \text{ is even}\\ 2x, & x \text{ is odd}\end{cases}$$

$$f(\underbrace{f(f(a))}_{\text{even}})=21$$

$$\text{i.e. } f(f(a))=22$$

So, $a=12$

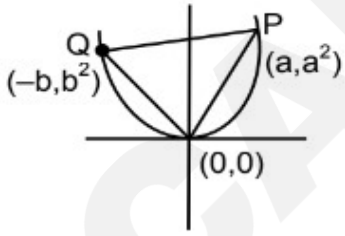
Now, $\lim_{x\to 12^-}\left\{\frac{12^3}{12}-\left[\frac{12}{12^-}\right]\right\}\Rightarrow\{(12)^2-0\}\Rightarrow 144$

Hence, the answer is the option (4).

Q. 21 Three points $O(0,0), P\left(a, a^2\right), Q\left(-b, b^2\right), a>0,\ b>0$, are on the parabola $y=x^2$. Let S_1 be the area of the region bounded by the line PQ and the parabola, and S_2 be the area of the triangle OPQ. If the minimum value of $\frac{S_1}{S_2}$ is $\frac{m}{n}, \gcd(m, n)=1$, then $m+n$ is equal to_____.

Correct Answer:
7

Solution:



$y=x^2 \quad PQ;\left(y-b^2\right)=\frac{a^2-b^2}{a+b}(x+b)$

$$PQ;(a-b)x-y+ab=0$$

Now, $S_1 = \int_{-b}^{a}[(a-b)x+ab] - [x^2] \cdot dx$

$$S_1 = \left[(a-b)\frac{x^2}{2} + abx - \frac{x^3}{3}\right]_{-b}^{a}$$

$$S_1 = (a-b)\frac{(a^2-b^2)}{2} + ab(a+b) - \frac{1}{3}[a^3+b^3]$$

$$S_1 = \frac{(a+b)}{6}[a^2+b^2+2ab] \Rightarrow \frac{(a+b)^3}{6}$$

$$\text{Area of } \triangle OPQ\,(S_2) = \frac{1}{2}\begin{matrix}0 & 0 & 1\\ a & a^2 & 1\\ -b & b^2 & 1\end{matrix} = \frac{1}{2}[ab(a+b)]$$

$$\frac{S_1}{S_2} = \frac{\frac{(a+b)^3}{6}}{\frac{ab}{2}(a+b)} \Rightarrow \frac{1}{3ab}(a+b)^2 \Rightarrow \frac{1}{3}\left[\frac{a}{b}+\frac{b}{a}+2\right]$$

as , a, b > 0

So, using A.M., G.M.

$$\frac{\frac{a}{b}+\frac{b}{a}}{2} \geq \left(\frac{a}{b}\cdot\frac{b}{a}\right)^{1/2}$$

$$\frac{a}{b}+\frac{b}{a} \geq 2$$

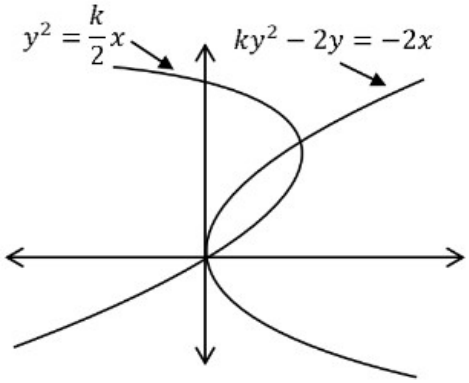
$$\min\left(\frac{S_1}{S_2}\right) = \frac{4}{3}$$

$\therefore m+n = 7$

Q. 22 The sum of squares of all possible values of k, for which area of the region bounded by the parabolas $2y^2 = kx$ and $ky^2 = 2(y-x)$ is maximum, is equal to_____.

Correct Answer:
8

Solution:



$$\text{Area } = A = \int_0^{\frac{2k}{k^2+4}} \left[\left(\frac{ky^2-2y}{-2}\right) - \left(\frac{2y^2}{k}\right)\right] dy$$

$$= \left[\left(\frac{-k}{2} - \frac{2}{k}\right)\frac{y^3}{3} + \frac{y^2}{2}\right]_0^{\frac{2k}{k^2+4}}$$

$$= \frac{-\left(k^2+4\right)}{2k} \times \left(\frac{2k}{k^2+4}\right)^3 \times \frac{1}{3} + \left(\frac{2k}{k^2+4}\right)^2 \times \frac{1}{2}$$

$$= -\left(\frac{2k}{k^2+4}\right)^2 \times \frac{1}{3} + \left(\frac{2k}{k^2+4}\right)^2 \times \frac{1}{2}$$

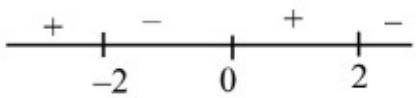
$$= \frac{1}{6} \times \frac{4k^2}{\left(k^2+4\right)^2}$$

$$A(k) = \frac{2}{3}\frac{k^2}{\left(k^2+4\right)^2}$$

$$A'(K) = \frac{2}{3}\frac{\left[2K\left(K^2+4\right)^2 - 2K^2\left(K^2+4\right)(2K)\right]}{\left(K^2+4\right)^4} = 0$$

$$\Rightarrow \frac{2}{3}(2K)\left(K^2+4\right)\left[K^2+4-2K^2\right] = 0$$

$K = 0$ or $K = 2$ or $K = -2$



$\therefore$ $K = 0$ is local minima

$K = -2 \,\&\, 2$ are local maxima

$\therefore$ $(-2)^2 + 2^2 = 8$

Q. 23 If $y = \frac{(\sqrt{x}+1)(x^2-\sqrt{x})}{x\sqrt{x}+x+\sqrt{x}} + \frac{1}{15}\left(3\cos^2 x - 5\right)\cos^3 x$, then $96y'\left(\frac{\pi}{6}\right)$ is equal to_____.

Correct Answer:
105

Solution:

$$y = \frac{(\sqrt{x}+1)\left(x^2-\sqrt{x}\right)}{x\sqrt{x}+x+\sqrt{x}} + \frac{1}{15}\left(3\cos^2 x - 5\right)\cos^3 x$$

$$y = \frac{(\sqrt{x}+1)\cdot\sqrt{x}(x\sqrt{x}-1)}{x\sqrt{x}+x+\sqrt{x}} + \frac{1}{15}\left(3\cos^2\cdot x - 5\right)\cos^3 x$$

$$y = \frac{(\sqrt{x}+1)\cdot\sqrt{x}(\sqrt{x})^3-1^3)}{x\sqrt{x}+x+\sqrt{x}} + \frac{1}{15}\left[3\cos^5 x - 5\cos^3 x\right]$$

$$y = \frac{(\sqrt{x}+1)\cdot\sqrt{x}(\sqrt{x}-1)(x+\sqrt{x}+1)}{\sqrt{x}(x+\sqrt{x}+1)} + \frac{1}{15}\left[3\cos^5 x - 5\cos^3 x\right]$$

$$y = (x-1) + \frac{1}{15}\left[3\cos^5 x - 5\cos^3 x\right]$$

Now, diff w.r.t x

$$y' = 1 - 0 + \frac{1}{15}\left[15\cos^4 x \cdot (-\sin x) - 15\cos^2 x(-\sin x)\right]$$

$$y' = 1 - \sin x \cdot \left[\cos^4 x - \cos^2 x\right]$$

$$y'(\pi/6) = 1 - \sin\frac{\pi}{6}\left[\cos^4\frac{\pi}{6} - \cos^2\frac{\pi}{6}\right]$$

$$= 1 - \frac{1}{2}\left[\frac{9}{16} - \frac{3}{4}\right]$$

$$= 1 - \frac{1}{2}\left[\frac{9-12}{16}\right] \Rightarrow 1 + \frac{3}{32} \Rightarrow \frac{35}{32}$$

$$96 \cdot y'(\pi/6) = 96 \times \frac{35}{32} \Rightarrow 105$$

Hence, the answer is the (105).

Q. 24 If $\frac{dx}{dy} = \frac{1 + x - y^2}{y}, x(1) = 1$, then $5x(2)$ is equal to_____.

Correct Answer:
5

Solution:

$$\frac{dx}{dy} = \frac{1+x-y^2}{y}$$

$$ydx = dy + x \cdot dy - y^2 dy$$

$$ydx - xdy = dy - y^2 \cdot dy$$

divide by y^2 both sides

$$\frac{ydx - xdy}{y^2} = \frac{1}{y^2} \cdot dy - dy$$

$$\int d(x/y) = \int \frac{1}{y^2} \cdot dy - \int dy$$

$$\frac{x}{y} = \frac{-1}{y} - y + c$$

$$1 = -1 - 1 + C$$

$$1 = -1 - 1 + C \Rightarrow [C = 3]$$

$$\frac{x}{y} = \frac{-1}{y} - y + 3$$

$$x = 3y - y^2 - 1$$

$$5x(2) \Rightarrow 5\left(3 \times 2 - (2)^2 - 1\right) \Rightarrow 5(1) = 5$$

Hence, the answer is (5).

Q. 25 Let $f : (0, \infty) \to R$ and $F(x) = \int_0^x tf(t)dt$. If $F\left(x^2\right) = x^4 + x^5$, then $\sum_{r=1}^{12} f\left(r^2\right)$ is equal to_____.

Correct Answer:
219

Solution:

$$F(x) = \int_0^x t \cdot f(t)dt$$

$$F'(x) = xf(x)$$

Given $F\left(x^2\right) = x^4 + x^5$ Let $^2 = t$

$$\mathrm{F(t)} = \mathrm{t}^2 + \mathrm{t}^{5/2}$$

$$\mathrm{F'(t)} = 2\mathrm{t} + 5/2\mathrm{t}^{3/2}$$

$$\mathrm{t \cdot f(t)} = 2\mathrm{t} + 5/2\mathrm{t}^{3/2}$$

$$\mathrm{f(t)} = 2 + 5/2\mathrm{r}^{1/2}$$

$$\sum_{r=1}^{12} f\left(r^2\right) = \sum_{r=1}^{12} 2 + \frac{5}{2}r$$

$$= 24 + 5/2\left[\frac{12(13)}{2}\right]$$

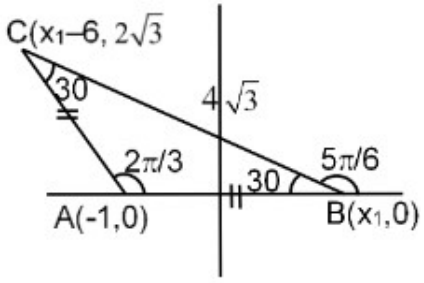
$$= 219$$

Hence, the answer is the (219).

Q. 26 Let ABC be an isosceles triangle in which A is at $(-1, 0), \angle\mathrm{A} = \frac{2\pi}{3}$, $\mathrm{AB} = \mathrm{AC}$ and B is on the positve x-axis. If $\mathrm{BC} = 4\sqrt{3}$ and the line BC intersects the line $\mathrm{y} = \mathrm{x} + 3$ at (α, β), then $\frac{\beta^4}{\alpha^2}$ is_____.

Correct Answer:
36

Solution:



co-ordinates of 'C' are

$$\frac{x-x_1}{\cos\frac{5\pi}{6}}=\frac{y-0}{\sin\frac{5\pi}{6}}=4\sqrt{3}$$

$$x=4\sqrt{3}\cos\frac{5\pi}{6}+x_1\Rightarrow 4\sqrt{3}\times\left(-\frac{\sqrt{3}}{2}\right)+x_1\Rightarrow(x_1-6)$$

$$y=4\sqrt{3}\sin\frac{5\pi}{6}\Rightarrow 4\sqrt{3}\times\frac{1}{2}\Rightarrow 2\sqrt{3}$$

$$C\left(x_1-6,2\sqrt{3}\right)$$

$$m_{AC}=-\sqrt{3}$$

$$\frac{0-2\sqrt{3}}{-1-x_1+6}=-\sqrt{3}\Rightarrow x_1=3$$

$$BC: x+\sqrt{3}y=3, y=x+3$$

$$y-3+\sqrt{3}y=3$$

$$y=\frac{6}{\sqrt{3}+1}, x=\frac{6}{\sqrt{3}+1}-3\Rightarrow\frac{6-3\sqrt{3}-3}{\sqrt{3}+1}$$

$$x=\frac{3-3\sqrt{3}}{\sqrt{3}+1}$$

$$\frac{-6}{(1+\sqrt{3})^2}$$

$$\text{So, }\frac{\beta^4}{\alpha^2}=36$$

Q. 27 Let $A=I_2-2MM^T$, where M is a real matrix of order 2×1 such that the relation $M^TM=I_1$ holds. If λ is a real number such that the relation $AX=\lambda X$ holds for some non-zero real matrix X of order 2×1, then the sum of squares of all possible values of λ is equal to_____.

Correct Answer:
2

Solution:

$$A = I_2 - 2MM^T$$

$$A^2 = (I_2 - 2MM^T)(I_2 - 2MM^T)$$

$$= I_2 - 2MM^T - 2MM^T + 4MM^TMM^T$$

$$= I_2 - 4MM^T + 4MM^T$$

$$= I_2$$

$$AX = \lambda X$$

$$A^2X = \lambda AX$$

$$X = \lambda(\lambda X)$$

$$X = \lambda^2 X$$

$$X(\lambda^2 - 1) = 0$$

$$\lambda^2 = 1$$

$$\lambda = \pm 1$$

The sum of the square of all possible values = 2

Hence, the answer is (2).

Q. 28 Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = -\hat{i} - 8\hat{j} + 2\hat{k}$ and $\vec{c} = 4\hat{i} + c_2\hat{j} + c_3\hat{k}$ be three vectors such that $\vec{b} \times \vec{a} = \vec{c} \times \vec{a}$. If the angle between the vector c and the vector $3\hat{i} + 4\hat{j} + \hat{k}$ is θ, then the greatest integer less than or equal $\tan^2\theta$ is_____.

Correct Answer:
38

Solution:

$a = \hat{i} + \hat{j} + \hat{k}, b = -\hat{i} - 8\hat{j} + 2\hat{k}, c = 4\hat{i} + c_2\hat{j} + c_3\hat{k}$

Given, $\bar{b} \times \mathbf{a} = \mathbf{c} \times \vec{a}$

$$-10\hat{\mathrm{i}} + 3\hat{\mathrm{j}} + 7\hat{\mathrm{k}} = (\mathrm{c}_2 - \mathrm{c}_3)\hat{\mathrm{i}} + (\mathrm{c}_3 - 4)\hat{\mathrm{j}} + (4 - \mathrm{c}_2)\hat{\mathrm{k}}$$

$\mathrm{c}_2 - \mathrm{c}_3 = -10; \mathrm{c}_3 - 4 = 3; 4 - \mathrm{c}_2 = 7$

$\mathrm{c}_3 = 7 \quad \mathrm{c}_2 = -3$

So, $\mathrm{c} = 4\hat{\mathrm{i}} - 3\hat{\mathrm{j}} + 7\hat{\mathrm{k}}$

angle $\mathrm{b/w}$ c \& $(3\hat{\mathrm{i}} + 4\hat{\mathrm{j}} + \hat{\mathrm{k}})$ is θ

$$\therefore \cos\theta = \frac{12 - 12 + 7}{\sqrt{74} \cdot \sqrt{26}} \Rightarrow \frac{7}{\sqrt{74} \cdot \sqrt{26}}$$

$$\tan^2\theta = \sec^2\theta - 1 \Rightarrow \frac{74 \times 26}{49} - 1$$

$$\tan^2\theta = \frac{1924 - 49}{49} = \frac{1875}{49} = 38.26$$

$$\left[\tan^2\theta\right] = 38$$

Hence, the answer is the (38).

Q. 29 If three successive terms of a G.P. with common ratio $\mathrm{r}(\mathrm{r} > 1)$ are the lengths of the sides of a triangle and $[\mathrm{r}]$ denotes the greatest integer less than or equal to r, then $3[r] + [-r]$ is equal to_____.

Correct Answer:

1

Solution:

Let side of triangle be a, ar, ar^2

$\text{Now,} \quad a + ar > ar^2$

$1 + r > r^2$

$\text{or} \quad r^2 - r - 1 < 0$

$\left(r - \frac{1+\sqrt{5}}{2}\right)\left(r - \frac{-1+\sqrt{5}}{2}\right) < 0$

$r \in \left(\frac{-1+\sqrt{5}}{2}, \frac{1+\sqrt{5}}{2}\right)$

$\text{but, r} > 1 \text{ (given)}$

$\therefore r \in \left(1, \frac{1+\sqrt{5}}{2}\right)$

$[r] = 1$

$[-r] = -2$

$\text{So, } 3[r] + [-r] = 3 - 2 \Rightarrow 1$

Hence, the answer is the (1).

Q. 30 The lines $\mathrm{L_1, L_2, \ldots, L_{20}}$ are distinct. For $\mathrm{n = 1, 2, 3, \ldots, 10}$ all the lines $\mathrm{L_{2n-1}}$ are parallel to each other and all the lines L_{2n} pass through a given point P. The maximum number of points of intersection of pairs of lines from the set $\{\mathrm{L_1, L_2, \ldots, L_{20}}\}$ is equal to_____.

Correct Answer:
101

Solution:
Given: $\mathrm{L_1, L_3, L_5 \ldots . L_{19}}$ are parallel $\mathrm{L_2, L_4, L_6 \ldots . . L_{20}}$ (concurrent)

Total At. of intersection

$$= 20c_2 - \underset{\substack{\downarrow \\ (\text{due to parallellines})}}{10c_2} - \underbrace{10c_2 + 1}_{\substack{\downarrow \\ (\text{dueto concurent lines})}}$$

$$= 190 - 45 - 45 + 1$$

$$= 101$$

Hence, the answer is the (101).

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