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Class 11 - 12

Physics

Formula Book

Content

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Physics and Measurement

Important Formulae

1. Physical quantity-

- One physical quantity can be represented in terms of one or more units.

Amount of physical quantity(Q)=nu

n= numerical value or magnitude

u= unit

Types of the physical quantity

1) Scalar Quantity

- The quantities having magnitude only are known as scalar quantities.
- It does not specify the direction.
- Examples-Distance, time, work, energy, etc

2) Vector Quantity

- The quantities having both directions as well as magnitude are known as vector quantities.
- It has a specific direction.
- Examples - Displacement, force, velocity, acceleration, momentum, etc.

3) Tensor Quantity

Scalar and vector are special cases of a tensor.

- Tensors are represented in the multi-dimensional array, i.e., in different directions, tensors will have different magnitudes.
- If a tensor has only magnitude and no direction, it is called a scalar (a tensor of rank zero).
- If a tensor has magnitude and one direction, it is called a vector (a tensor of rank one).
- Examples are stress, The moment of inertia, coefficient of viscosity.

4) Ratio Quantity

When a physical quantity is the ratio of two similar quantities.

$$eg.\ Relative\ density = \frac{density\ of\ object}{density\ of\ water\ at\ 4^{\circ}c}$$

2. Fundamental and Derived Quantities -

1) Fundamental Quantities

- Those physical quantities are independent of all other quantities and cannot be expressed in terms of other basic quantities.
- Length, mass, time, electric current, temperature, amount of substance and luminous intensity.

2) Derived Quantities

- Derived Quantities are products and ratios of the fundamental quantities that exist in a system of units and these quantities can be expressed in terms of other basic quantities.
- e.g., Area, Density, Force, Pressure, etc.

Fundamental and derived units:-

Fundamental units:- The units of fundamental or basic quantities are called fundamental units or base units.

Derived units:- The units of those physical quantities which can be expressed as the combination of fundamental units are called derived units.

3. System of unit-

A complete set of units, for all kinds of physical quantities (both fundamental and derived), is known as a system of units.

Types of System of Unit

- **C.G.S. system-** In this system, fundamental units are centimetres (cm), grams (g) and second (s).

- **M.K.S. System-** In this system, fundamental units are meter(m), kilogram(kg) and second (s).
- **F.P.S. system-** In this system foot(ft), pound(lb) and second(s) are used for the measurement of length, mass and time respectively.
- **S.I. System-** It is known as the International System of Units. There are seven fundamental quantities in this system.

Fundamental Quantity	Fundamental Units	Symbol
Length	meter	m
Mass	Kilogram	Kg
Time	second	s
Electric current	Ampere	A
Temperature	Kelvin	K
Amount of substance	mole	mol
Luminous Intensity	candela	cd

1. **Practical units of length**

Name	Symbol	Conversion in m
1 fermi	1 fm	10^{-15} m
1 X-ray unit	1 XU	10^{-13}m
1 Angstrom	$1\mathring{A}$	10^{-10}m
1 micron	$1\mu m$	10^{-6}m
1 Astronomical unit	1 AU	$1.5 * 10^{11} m \approx 10^{8} km$
1 Light year	1 ly	$9.46 * 10^{15} m$
1 Parsec	1 Pc	3.26 light year

2. **Practical units of mass**

Name	Symbol	Conversion in kilogram (Kg)
1 Chandra Shekhar Unit	1 CSU	$2.8 \times 10^{30} kg$ = 1.4 times the mass of the sun
1 Metric tonne	1 Metric tonne	1000 kg
1 Quintal	1 Quintal	100 kg
1 Atomic mass unit	1 amu	$1.67 \times 10^{-27} kg$

3. **Practical Units of Time**

- 1 year =365.25 days =3.156×10^{7} Sec
- Lunar Month- 29.53 days (29 days 12 hours and 44 minutes)
- 1 Solar year- 366.25 sidereal days = 365.25 average solar day
- 1 Average Solar Day= $\left(\frac{1}{365.25}\right)^{th}$ part of the solar year
- 1 solar second = $\left(\frac{1}{86400}\right)^{th}$ part of the mean solar day

4. Dimension-

The dimension of physical quantity may be defined as the power to which fundamental quantities must be raised in order to express the given physical quantities.

For representing dimensions of different quantities, we use the following symbols:

1. Mass - M
2. Length - L
3. Time - T
4. Electric current - A

5. Temperature - K
6. Amount of substance- mol
7. Luminous intensity - cd

Dimension formula and SI units for some important physical quantities-

- **Frequency, angular frequency, angular velocity, velocity gradient**

All these quantities will have the same dimensional formula which is equal to $M^0L^0T^{-1}$

While the SI unit of Frequency and velocity gradient is sec^{-1},

And SI unit of angular frequency and angular velocity is radians per sec

Note:- Angle is a dimensionless quantity

- **Work, Potential Energy, Kinetic Energy, Torque**

All these quantities will have the same dimensional formula which is equal to ML^2T^{-2}

All these quantities will have the same unit in the SI system which is equal to N-m $or\ Joule$

- **Momentum, Impulse, Angular momentum, Angular impulse**

Momentum and Impulse both have the same dimensional formula which is equal to MLT^{-1}

And Both have the same SI unit which is equal to $kgms^{-1}$

Angular Momentum and Angular Impulse have the same dimensional formula which is equal to ML^2T^{-1}

and have the same SI unit which is equal to $kg(m)^2(sec)^{-1}$

- **Dimensionless Quantities**

The quantities which do not have dimensions are known as Dimensionless Quantities.

Because these quantities are the ratio of two similar quantities.

Example.

1. Strain
2. Refractive index
3. Relative density
4. Poisson's ratio

So all these quantities are dimensionless.

Or they have a dimensional formula which is equal to $M^0L^0T^0$.

And all these quantities are unitless.

- **Heat, Latent heat, Specific heat capacity and Temperature**

1. Temperature-

   Dimensional formula- $M^0L^0T^0K^1$(where K represents Kelvin)

   SI unit- Kelvin

2. Heat

   Dimensional formula- ML^2T^{-2}

   SI unit- Joule

3. Latent heat

   Its dimensional formula is equal to $M^0L^2T^{-2}$

   And its SI unit is equal to m^2s^{-2} or J/kg

4. Specific heat capacity

   Dimensional formula- $M^0L^2T^{-2}K^{-1}$

SI unit- $\frac{J}{kgK}$

- **Surface tension**

  Dimensional formula-$M^1L^0T^{-2}$

  SI unit- kgs^{-2}

- **Vander waals constant (a and b)**

$$\left(P+\frac{n^2a}{V^2}\right)(V-nb)=nRT$$

Where a and b are called Vander Waal's constant.

1) Vander Waal's constant (a)

Dimension- ML^5T^{-2}

Unit- $Newton-m^4$

2) Vander waal 's constant (b)

Dimension- $M^0L^3T^0$

Unit- m^3

- **Voltage, Resistance and resistivity**

1) Voltage (V)

Dimension- $ML^2T^{-3}A^{-1}$

Unit- Volt

2) Resistance (R)

Dimension- $ML^2T^{-3}A^{-2}$

Unit- Ohm

3) Resistivity (ρ)

Dimension- $ML^3T^{-3}A^{-2}$

Unit- Ohm - meter

- **Permittivity of free space and dielectric constant (k)**

1) The permittivity of free space(ϵ_o)

Dimension- $M^{-1}L^{-3}T^4A^2$

Unit- $C^{-2}N^1m^{-2}$ or farad/metre

2) dielectric constant (k)

Dimension- $M^0L^0T^0$

Unit- Unitless

- **Magnetic Field ,Permeability of free space, Magnetic flux and self inductance**

1) Magnetic Field (B)

Dimension- $M^1L^0T^{-2}A^{-1}$

Unit- $\frac{newton}{ampere-metre}$ or $\frac{volt-second}{metre^2}$

2)Permeability of free space

The dimension of permeability of free space (μ_o)-$M^1L^1T^{-2}A^{-2}$

SI unit- $\frac{newton}{ampere^2}$ or $\frac{henry}{metre}$

3) Magnetic flux (ϕ)

Dimension- $ML^2T^{-2}A^{-1}$

Unit- Weber or Volt-second

4) Coefficient of self-induction (L)

Dimension- $ML^2T^{-2}A^{-2}$

Unit- Henry

5. Application of Dimensional analysis-

- **To find the dimension of the physical constant**

We can find the dimension of a physical constant by substituting the dimensions of physical quantities in the given equation

1. Gravitational constant

$$F = G\frac{m_1 m_2}{r^2} \Rightarrow G = \frac{Fr^2}{m_1 m_2}$$

$$G = \frac{[MLT^{-2}][L^2]}{[M][M]} = [M^{-1}L^3T^{-2}]$$

$F \rightarrow force\ of\ Gravitation$

$G \rightarrow Universal\ Gravitational\ Constant$

$r \rightarrow distance\ between\ two\ masses$

$m_1, m_2 \rightarrow two\ masses$

2. Planck's Constant(h):-

$$E = hv \Rightarrow h = \frac{E}{v}$$

Dimensional formula- $M^1L^2T^{-1}$

SI unit- Joule-sec

3. Rydberg constant (R)

$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

Dimension- $M^0L^{-1}T^0$

Unit- m^{-1}

- **To convert a physical quantity from one system to another**

As we know, the measure of a physical quantity is constant, i.e., nu=constant.

$n_1[u_1] = n_2[u_2]$

If the dimension of a quantity in one system is $[M_1^a L_1^b T_1^c]$ and in another system, the dimension is $[M_2^a L_2^b T_2^c]$,

then $n_2 = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c$

- **Check the dimensional correctness**

It is based on the principle of homogeneity. According to this principle, both sides of an equation must be the same.

$L.H.S. = R.H.S.$

It also states that only those physical quantities with the same dimensions can be added or subtracted.

If the dimension of each term on both sides is the same, then the equation is dimensionally correct, otherwise not.

A dimensionally correct equation may or may not be physically correct.

- **To find the unit of physical quantity in a given system**

Let physical quantity be a force

So [F]=MLT^{-2}

If we replace M, L, and T in the dimensional formula of the physical quantity with fundamental units of the required system, we will get the unit of that physical quantity.

Now we want to find the unit of Force in the SI system

Which is $kgm\left(\sec^{-2}\right)$ or Newton

- **As a research tool to derive new relations**

For example, we can derive a relation for the Time period of a simple pendulum.

If $T = Km^a l^b g^c$

where

$T = time\, period$

$l = length$

$g = \ acceleration\, due\ to\, gravity$

So Equating exponents of similar quantities

a=0, b=1/2, c=-1/2

We get

$\therefore T = 2\pi\sqrt{\frac{l}{g}}$

6. Significant Figures:-

Significant figures are the figures of a number that express a magnitude to a specified degree of accuracy.

1) All non-zero digits are significant

For example-

42.3 -Three significant figure

238.4 -Four significant figure

33.123 -five significant figure

2) Zero becomes a significant figure if it exists between two non-zero digits

For example-

2.09 - Three significant figures

8.206 -Four significant figures

6.002 -Four significant figures

3) For leading zero(s), the zero(s) to the left of the first non-zero digits are not significant.

For example-

0.543 - three significant figures

0.069 - two significant figures

0.002 -one significant figure

4) The trailing zero(s) in a number without a decimal point are not significant. But if the decimal point is there then they will be counted in significant figures.

For example-

4.330- Four significant figures

433.00- five significant figures

343.000- six significant figures

5) Exponential digits in scientific notation are not significant.

For example- 1.32 X 10^{-2} -three significant figures

7. Rounding Off:-

Rounding off figures during calculation helps to make the calculation of big digits easier. While rounding off measurements, we use the following rules by convention:

(1) If the digit to be dropped is less than 5, then the preceding digit is left unchanged.

Example: x=7.82 is rounded off to 7.8, again x=3.94 is rounded off to 3.9.

(2) If the digit to be dropped is more than 5, then the preceding digit is raised by one.

Example: x = 6.87 is rounded off to 6.9, again x = 12.78 is rounded off to 12.8.

(3) If the digit to be dropped is 5 followed by digits other than zero, then the preceding digit is raised by one.

Example: x = 16.351 is rounded off to 16.4, again x = 6.758 is rounded off to 6.8.

(4) If the digit to be dropped is 5 or 5 followed by zeros, then the preceding digit is left unchanged if it is even.

Example: x = 3.250 becomes 3.2 on rounding off, again x = 12.650 becomes 12.6 on rounding off.

(5) If the digit to be dropped is 5 or 5 followed by zeros, then the preceding digit is raised by one if it is odd.

Example: x = 3.750 is rounded off to 3.8, and again x = 16.150 is rounded off to 16.2.

- **Significant Figures in Calculation:-**

1. Rules for addition and subtraction-

The result of an addition or subtraction in the number having different precisions should be reported to the same number of decimal places as are present in the number having the least number of decimal places.

For example:-

1) 33.3+3.11+0.313=36.723 but here the answer should be reported to one decimal place as the 33.3 has the least number of decimal place(i.e. only one decimal place), therefore the final answer=36.7

2) 3.1421+0.241+0.09=3.4731 but here the answer should be reported to two decimal places as the 0.09 has the least number of the decimal place(i.e. two decimal places), therefore the final answer=3.47

2 Rules for multiplication and division-

The answer to a multiplication or division is rounded off to the same number of significant figures as is possessed by the least precise term used in the calculation:-

For example:-

1) 142.06 x 0.23=32.6738 but here the least precise term is 0.23 which has only two significant figures, so the answer will be 33.

8. Errors of measurements

If $a_1, a_2, a_3 \ldots\ldots\ldots a_n$ are a measured value

then $a_m = \dfrac{a_1 + a_2 + \ldots\ldots a_n}{n}$

where a_m = true value

then

1)Absolute Error for n^{th} reading $= \Delta a_n = a_m - a_n$ = true value - measured value

So $\Delta a_1 = a_m - a_1$

$\Delta a_2 = a_m - a_2$

2) Mean absolute error

$$\Delta \bar{a} = \frac{|\Delta a_1| + |\Delta a_2| + \ldots. |\Delta a_n|}{n}$$

3) Relative error or Fractional error

The ratio of mean absolute error to the mean value of the quantity measured.

$$Relative\ \ error = \frac{\Delta \bar{a}}{a_m}$$

$\Delta \bar{a} = mean\ \ absolute\ \ error$

$a_m = mean\ \ value$

4) Percentage error

$$Percentage\ \ error = \frac{\Delta \bar{a}}{a_m} \times 100\%$$

8.1- Error in sum and Error in difference of two physical quantities

1) Error in sum (x=a+b)

- Absolute error in x is is given by

$\Delta x = \pm(\Delta a + \Delta b)$

where

Δa= absolute error in measurement of a

Δb= absolute error in measurement of b

Δx= absolute error in measurement of x

- The percentage error in the value of x is is given by

$$\frac{\Delta x}{x} \times 100 = \frac{(\Delta a + \Delta b)}{a+b} \times 100$$

2) Error in difference (x=a-b)

- Absolute error in x is is given by

$\Delta x = \pm(\Delta a + \Delta b)$

- Percentage error in the value of x is is given by

$$\frac{\Delta x}{x} \times 100 = \frac{(\Delta a + \Delta b)}{a-b} \times 100$$

8.2- Error in product and Error in division of two physical quantities

1) Error in product $x = ab$

- Maximum fractional error is given by

$$\frac{\Delta x}{x} = \pm\left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right)$$

where

Δa= absolute error in measurement of a

Δb= absolute error in measurement of b

Δx= absolute error in measurement of x

- The percentage error in the value of x is given by

$$\frac{\Delta x}{x} * 100 = \pm\left(\frac{\Delta a}{a} * 100 + \frac{\Delta b}{b} * 100\right)$$

2) Error in division $(x = \frac{a}{b})$

- The maximum fractional error in x is given by

$$\frac{\Delta x}{x} = \pm\left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right)$$

- The percentage error in the value of x is given by

$$\frac{\Delta x}{x} * 100 = \pm\left(\frac{\Delta a}{a} * 100 + \frac{\Delta b}{b} * 100\right)$$

8.3- Error in quantity raised to some power

For $(x = \frac{a^n}{b^m})$

- The maximum fractional error in x is given by

$$\frac{\Delta x}{x} = \pm\left(n\frac{\Delta a}{a} + m\frac{\Delta b}{b}\right)$$

- Percentage error in the value of x is given by

$$\frac{\Delta x}{x} * 100 = \pm\left(n\frac{\Delta a}{a} * 100 + m\frac{\Delta b}{b} * 100\right)$$

Kinematics

Important Formulae

Kinematics- In kinematics, we study ways to describe motion without going into the causes of motion.

1. Rest And Motion

1. Rest body is said to be at rest if it does not change its position with respect to its surroundings with the passage of time.

   e.g.: A book lying on the table.

2. Motion- Motion is known as a change in the position of an object with time.

   e.g. A moving bus.

- Note - Rest and motion are relative to each other.

  e.g. All passengers sitting inside the moving bus are at rest with respect to one another.

  But all appear to be in motion to a man standing outside the bus.

2.Types Of Motion

I. One Dimensional (1-D)-

- If only one coordinate is used to describe the motion of an object.
- Motion is a straight line is 1-D.
- E.g: Train running on singletrack, Apple falling from a tree

II. Two Dimensional (2-D)-

- When two coordinates are used to describe the motion of an object.
- Motion in-plane is 2-D.
- E.g Earth revolves around the sun.

III. Three Dimensional -

- When all three coordinates are used to describe the motion of an object.
- Motion in space is 3-D.
- e.g: object moving in space.

3.Mathematical tools (Differentiation and Integration)

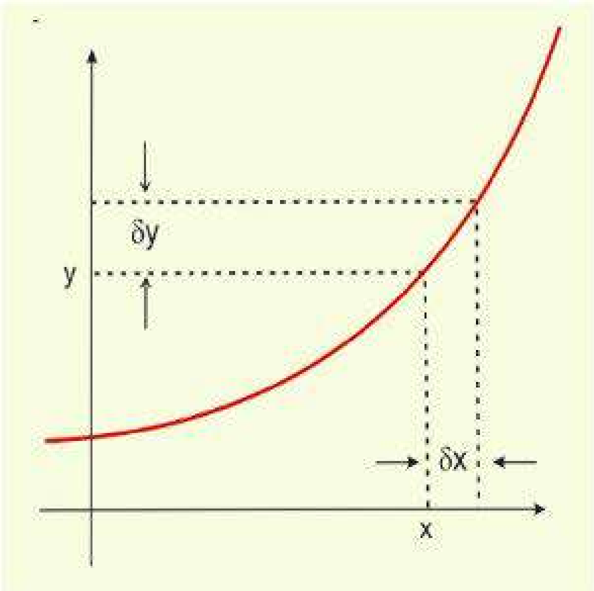
1. **Differentiation**

Differentiation is very useful when we have to find rates of change of one quantity compared to another.

- If y is one quantity and we have to find the rate of change of y with respect to x which is another quantity

Then the differentiation of y w.r.t x is given as $\frac{dy}{dx}$

- For a y V/s x graph



We can find the slope of graph using differentiation

I.e Slope of y V/s x graph = $\frac{dy}{dx}$

- **Some important Formulas of differentiation**

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}\sin x = \cos x$$

$$\frac{d}{dx}\cos x = -\sin x$$

$$\frac{d}{dx}\tan x = \sec^2 x$$

$$\frac{d}{dx}\cot x = -\csc^2 x$$

$$\frac{d}{dx}\sec x = \sec x \tan x$$

$$\frac{d}{dx}\csc x = -\csc x \cot x$$

$$\frac{d}{dx}e^x = e^x$$

$$\frac{d}{dx}a^x = a^x \ln a$$

$$\frac{d}{dx}\ln|x| = \frac{1}{x}$$

2. **Integration**

- Opposite process of differentiation is known as integration.
- Let x, y are two quantities

Using differentiation we can find the rate of change of y with respect to x i.e $\frac{dy}{dx}$

But using integration we can get direct relationship between quantities x and y

So let $\frac{dy}{dx} = K$ where K is constant

Or we can write $dy = Kdx$

Now integrating on both sides we get direct relationship between x and y

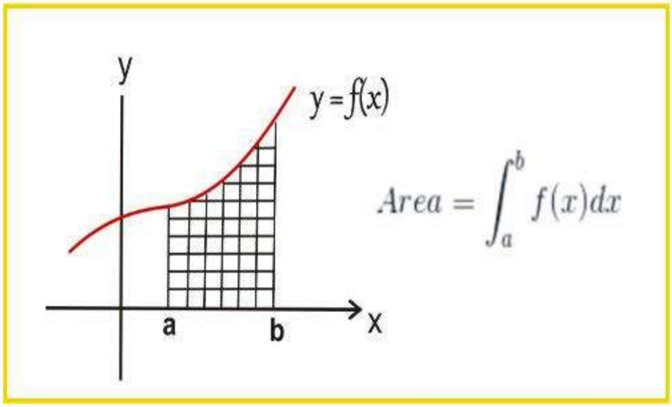
I.e $\int dy = \int K dx$

$y = Kx + C$

Where C is some constant

- For a y V/s x graph

We can find the area of graph using integration



- **Some important Formulas of integration**

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

$$\int \frac{dx}{x} = \ln|x| + C$$

$$\int e^x dx = e^x + C$$

$$\int a^x dx = \frac{1}{\ln a} a^x + C$$

$$\int \ln x dx = x \ln x - x + C$$

$$\int \sin x dx = -\cos x + C$$
$$\int \cos x dx = \sin x + C$$
$$\int \tan x dx = -\ln|\cos x| + C$$
$$\int \cot x dx = \ln|\sin x| + C$$
$$\int \sec x dx = \ln|\sec x + \tan x| + C$$
$$\int \csc x dx = -\ln|\csc x + \cot x| + C$$

$$\int \sec^2 x dx = \tan x + C$$
$$\int \csc^2 x dx = -\cot x + C$$
$$\int \sec x \tan x dx = \sec x + C$$
$$\int \csc x \cot x dx = -\csc x + C$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \frac{x}{a} + C$$

$$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$$

$$\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a} \sec^{-1} \frac{|x|}{a} + C$$

4.Scalars and vectors

1. **Scalars**

Physical quantities can be described completely by their magnitude only but no particular direction.

Examples- Distance, speed, work, charges, temperature, etc.

Tips for scalars-

- Scalar quantities can be positive, negative or zero.
- Represented by alphabet only A, B, C.
- These physical quantities follow normal algebraic rules of addition.

2. **Vectors**

Physical quantities can be described by their magnitude and direction.

Physical quantities like Displacement, force, velocity etc. are vectors.

Tips of vectors-

- Vectors can be positive, negative or zero.
- Represented by alphabet having an arrow on their head.

$\vec{A}, \vec{B}, \vec{C}$

- These physical quantities follow the laws of vector addition.
- Types of vectors

1. Equal vectors-

Two vectors are said to be equal if they have equal magnitude and the same directions.

The angle between these two vectors Θ is equal to zero.

2. Negative vectors-

Two vectors are said to be negative with respect to each other if they have equal magnitude but opposite directions.

The angle between negative vectors is equal to 180. i.e $\Theta = 180°$

3. Collinear vectors-

Two vectors are said to be collinear if they have a common line of action.

a. If two vectors are collinear and parallel then the angle between them is zero.

b. If two vectors are collinear and anti-parallel then the angle between them is 180^0.

4. Co-initial vectors-

Two vectors are said to be Co-initial vectors if they have the same initial point.

5. Vector addition and Vector Subtraction

1. **Vector addition-**

Vector quantities are not added according to simple algebraic rules, because their direction matters.

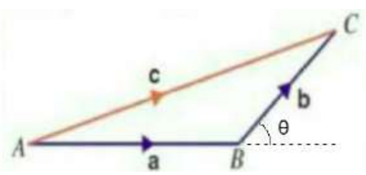
Case I- For the simple case in which both vectors have the same direction

When two vectors are in the same direction then upon addition the direction of the resultant vector is the same as any of the two vectors, while the magnitude of the resultant vector is simply the algebraic sum of two vectors.

Case II -For the case when both vectors do not have the same direction use the following laws

1. **Triangle law of vector addition**

If two vectors are represented by both magnitude and direction by two sides of a triangle taken in the same order then their resultant is represented by side of the triangle.



The figure represents the triangle law of vector addition

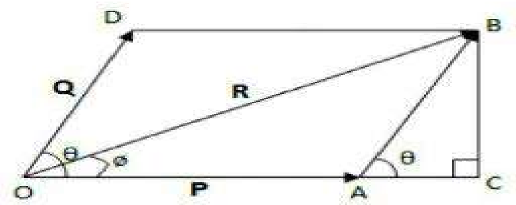
So resultant side C is given by

$c=\sqrt{a^2+b^2+2ab\cos\theta}$

Where θ= angle between two vectors.

2. **Parallelogram law of vector addition**

- If two vectors are represented by both magnitude and direction by two adjacent sides of a parallelogram taken from the same point then their resultant is also represented by both magnitude and direction taken from the same point but by the diagonal of the parallelogram.



The figure represents the law of parallelogram Vector Addition

Note- Commutative law-

The Sum of vectors remains the same in whatever order they may be added.

$\vec{P}+\vec{Q}=\vec{Q}+\vec{P}$

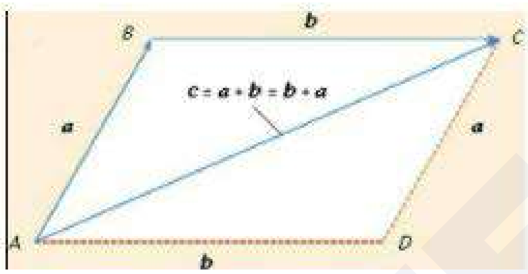


Fig shows the Commutative law of vector addition.

2) Vector Subtraction-

Vector subtraction of $\vec{B}$ from $\vec{A}$ is equal to Vector addition of $\vec{A}$ and negative vector of $\vec{B}$.

$\vec{A}-\vec{B}=\vec{A}+(-\vec{B})$

6. Unit vector-

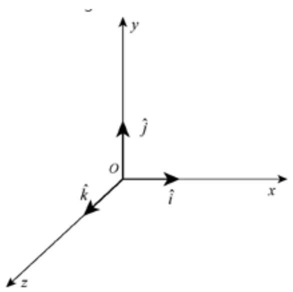
A vector having a magnitude of one unit is called a unit vector. It is represented by a cap/hat over the letter. Eg-$\hat{R}$ is called a unit vector of $\vec{R}$. Its direction is along the $\vec{R}$ and magnitude is unit.

Unit vector along $\vec{R}$ is

$$\hat{R}=\frac{\vec{R}}{|\vec{R}|}$$

- **Orthogonal unit vectors-**

It is defined as the unit vectors described under the three-dimensional coordinate system along x, y, and z-axis. The three unit vectors are denoted by i, j and k respectively.



Any vector (Let us say $\vec{R}$) can be written as

$$\vec{R} = x\hat{i} + y\hat{j} + z\hat{k}$$

Where x, y and z are components of $\vec{R}$ along x, y and z direction respectively.

Magnitude of $\vec{R}$-

$$\left|\vec{R}\right| = \sqrt{x^2 + y^2 + z^2}$$

Unit vector-

$$\hat{R} = \frac{x\hat{i} + y\hat{j} + z\hat{k}}{\sqrt{x^2 + y^2 + z^2}}$$

7. Multiplication of vectors

1. If a vector is multiplied by any scalar

$$\vec{Z} = n \cdot \vec{Y}$$

(n=1,2,3..)

Vector $\times$Scalar$=$ Vector

We get again a vector.

2. If a vector is multiplied by any real number (e.g. 2 or -2) then again, we get a vector quantity

3. Scalar or Dot or Inner Product

Scalar product of two vectors & $\vec{B}$ written as $\vec{A}\ .\ \vec{B}$

$\vec{A}\ .\ \vec{B}$ is a scalar quantity given by the product of the magnitude of $\vec{A}$ & $\vec{B}$ and the cosine of a smaller angle between them.

i.e $\vec{A} \cdot \vec{B} = A\,B \cdot \cos\Theta$

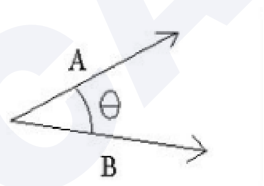


Figure showing the representation of scalar products of vectors.

Important results-

$$\hat{i}.\hat{j} = \hat{j}.\hat{k} = \hat{k}.\hat{i} = 0$$
$$\hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1$$

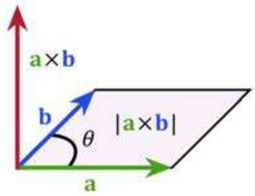
$$\vec{A}.\vec{B} = \vec{B}.\vec{A}$$

4. **Vector or cross-product**

Vector or cross product of two vectors $\vec{A}$ & $\vec{B}$ written as $\vec{A} \times \vec{B}$

$A \times B$ is a single vector whose magnitude is equal to the product of the magnitude of $\vec{A}$ & $\vec{B}$ and the sine of the smaller angle θ between them.

i.e $\vec{A} \times \vec{B} = A\,B \sin\theta$



The figure shows the representation of the cross-product of vectors.

Important results-

$\hat{i} \times \hat{j} = \hat{k},\ \hat{j} \times \hat{k} = \hat{i},\ \hat{k} \times \hat{i} = \hat{j}$
$\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = \vec{0}$

$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$

8. Distance and displacement

1. **Position vector**

- The position of a point, in space, is an important physical quantity which is also known as a position vector.
- Representation of position vector $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$
- Its magnitude is the distance between the initial point (tail) and the final point (head).

 (Generally, we take the initial point as the origin)

- Its direction is from the initial point and the final point.

 Magnitude of $r = \sqrt{x^2 + y^2 + z^2}$

- E.g. If $\vec{A} = 3\hat{i} - 4\hat{j} + 2\hat{k}$ then its magnitude

$= \sqrt{(3)^2 + (-4)^2 + (2)^2}$

$= \sqrt{9 + 16 + 4}$

$a = \sqrt{29}$

2. **Distance or Path length**

 - Length of the actual path between the initial and final positions of the body.
 - E.g. Distance travelled along a circle for one complete rotation is $2\pi r$
 - Tips of distance

 1. Distance is a scalar quantity.
 2. It is always positive.
 3. S.I. Unit: meter (m).
 4. Dimension $[L]$
 5. Distance depends on the path followed by the body.

3. **Displacement**

- The shortest path is between the initial and final positions.
- Tips for Displacement.

1. Displacement is a vector quantity.
2. It can be positive, zero or negative.
3. Displacement is independent of the path.
4. S.I. unit: Meter (m)

5. Dimension (L)

6. The magnitude of Displacement $\leqslant$ Distance.

- E.g. For a circle

$Distance = \pi r$ while $Displacement = 2r$

9. Speed and velocity

I. Speed

- Rate of change of distance with time.
- Formula-

$$Speed = \frac{Change\,in\,distance}{change\,in\,time}$$

$$v = \frac{distance}{time}$$

- Tips of Speed-

1. S.I. unit→ ms^{-1} or meters per second.

2. Dimensions=LT^{-1}

3. Speed is a scalar quantity.

- E.g: A body covers a distance of 18m in 1 sec

then speed is given by

$$v = \frac{distance}{time} = \frac{18m}{1s} \Rightarrow 18m/s$$

II . Average Speed and Instantaneous Speed

1. **Average Speed-**

- Amount of total distance covered in total time.
- Formula-

$$Average\ speed = \frac{total\ distance\ covered}{total\ time\ taken}$$

$$v_{av} = \frac{s}{t}$$

,

- E.g. A body covers a total distance of 50 m with variable speed in 5 sec.

$$v_{av} = \frac{s}{t}$$

$$\Rightarrow \frac{50m}{5s} = 10m/s$$

Average Speed = 10 m/s

- Tips for average speed-

If an object or body covers s_1 distance in t_1 time and s_2 distance in t_2 time then average speed is calculated by the

Formula- $V_{av} = \frac{s_1 + s_2}{t_1 + t_2}$

2. **Instantaneous Speed-**

- It is the speed at that particular instant or small interval of time.
- Formula-

$$v_{inst} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

III. Velocity

- Rate of change of displacement with time.
- Formula-

$$V = \frac{displacement}{time}$$

- Tips For Velocity-

1. S.I. unit $\rightarrow$ m/s
2. Dimensions-LT^{-1}
3. Velocity is a vector quantity.
4. Velocity is also called speed in a definite (particular) direction.

IV. Average Velocity and Instantaneous Velocity

1. **Average Velocity-**

- Amount of total displacement covered in total time.
- Formula-

$$Average \;\; Velocity = \frac{Total \;\; Displacement}{Total \;\; time \;\; taken},$$

$$\vec{V}_{avg} = \frac{\vec{S}_{net}}{t}$$

2. **Instantaneous Velocity-**

- It is Velocity at that particular instant or small interval of time.
- Formula-

$$\vec{V}_{inst} = \lim_{\Delta t \to 0} \frac{\Delta \vec{x}}{\Delta t}$$

10. Acceleration

Definition: Rate of change of velocity with time.

$$\vec{a} = \frac{change \; in \; velocity}{time \; taken} = \frac{\vec{v}_f - \vec{v}_i}{t}$$

Tips for Acceleration-

1. The body is said to have undergone acceleration if there is a change in velocity i.e.
   - Change in speed
   - Change in direction
   - Change in both
2. It is a vector quantity
3. Dimension = LT^{-2}
4. S.I unit = ms^{-2}

Types of acceleration-

- **Average acceleration**- Total change in velocity per unit time taken is the average acceleration

$$Avg. \; acceleration(\vec{a}_{avg}) = \frac{\Delta \vec{v}}{\Delta t}$$

- **Instantaneous acceleration** - Infinitesimal change in velocity per unit time taken is the average acceleration

$$Inst. \; acceleration(\vec{a}_{inst}) = \frac{d\vec{v}}{dt}$$

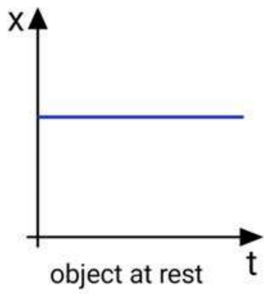
- **Uniform acceleration**- Change in velocity per unit time is constant.
- **Non-uniform acceleration**- Change in velocity per unit time is not constant.

11. Kinematics graphs-

I. Position time graph-

The slope of the position-time graph represents the velocity of the particle

1. Position time graph when the body is at rest

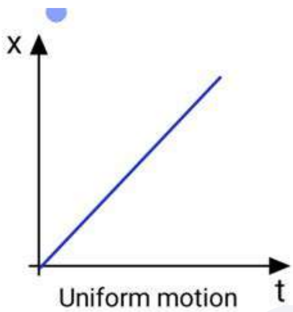


object at rest

Figure - 1

Figure 1 shows the position-time graph when the body is at rest

2. Position time graph for uniform motion



Uniform motion

Figure - 2

Figure 2 shows a position time graph for uniform motion.

Here the object is moving along a straight line and covers equal distances in equal intervals of time.

3. Position – time graph For an object in non-uniform motion

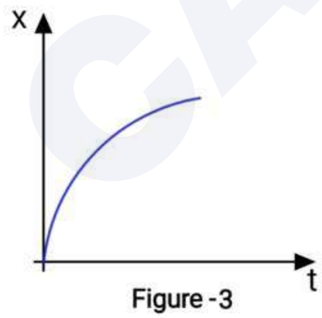


Figure -3

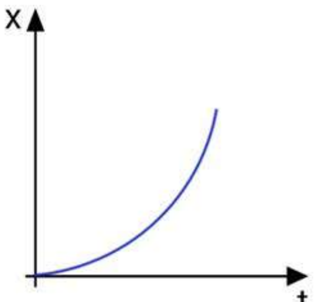


Figure - 4

The figure-3,4 shows a position-time graph for non-uniform motion.

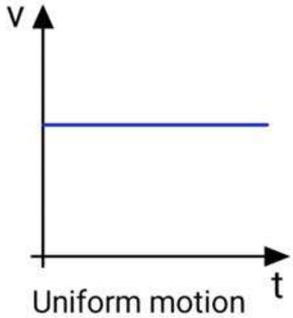
In figure-3, acceleration is positive and in Figure-4, acceleration is negative.

II. **Velocity Time Graph**

The graph is plotted by taking time t along the x-axis and the velocity of the particle on the y-axis.

- The area of the velocity v/s time graph for the particular time interval gives the displacement and distance travelled by the body for a given time interval.
- The slope of the velocity-time graph represents the acceleration of the particle.

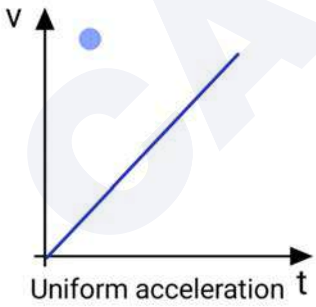
1. When a particle is moving with constant velocity.



Uniform motion
Figure - 5

Figure 5 shows constant velocity and zero acceleration.

2. For uniform acceleration of the particle



Uniform acceleration
Figure - 6

Figure 6 shows constant positive acceleration.

III. Acceleration-Time graph

The graph is plotted by taking time t along the x-axis and the acceleration of the particle on the y-axis.

- The area of the acceleration v/s time graph for the particular time interval gives the change in velocity of the body for a given time interval.
- The slope of the acceleration-time graph represents the jerk.

1. When a particle has Constant acceleration

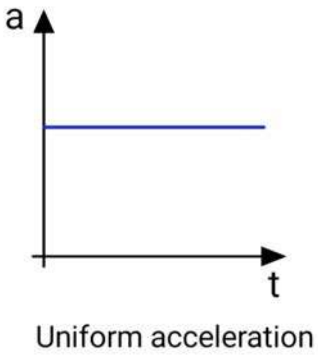


Figure - 7

Figure 7 represents uniform positive acceleration

2. A particle having uniformly increasing acceleration

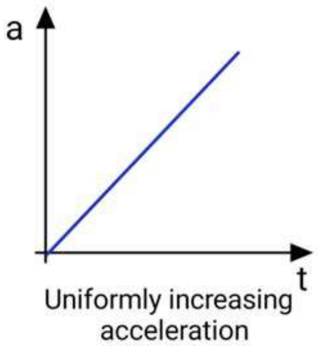


Figure - 8

Figure 8 represents positive and uniformly increasing acceleration.

12. Equation of motions

There are three equations of motion

1. **The first kinematical equation of motion (velocity-time equation)**

Formula-

$v = u + at$

v = Final velocity

u = Initial velocity

a= acceleration

t = time

2. **The second kinematical equation of motion (Position-time equation)**

Formula-

$s = ut + \frac{1}{2}at^2$

$s \rightarrow$ Displacement

$u \rightarrow$Initial velocity

$a \rightarrow$acceleration

$t \rightarrow$ time

3. **The third kinematical equation of motion (Velocity-displacement equation)**

Formula-

$v^2 - u^2 = 2as$

$v \rightarrow$ Final Velocity

$s \rightarrow$ Displacement

$u \rightarrow$Initial velocity

$a \rightarrow$acceleration

Displacement in the nth second

Formula: $S_n = u + \frac{a}{2}(2n - 1)$

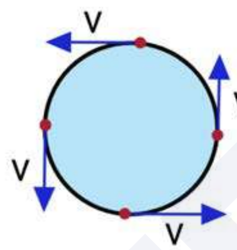
Where $u =$ Initial velocity

$a =$ uniform acceleration

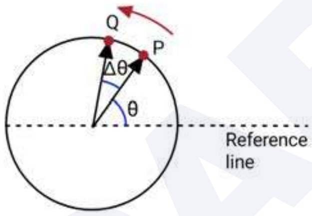
13. Uniform circular motion

13.1-Introduction -

Circular motion is one of the examples of motion in two dimensions. In the case of circular motion, the particle moves in a circular path on the circumference of a circle. The velocity of a particle moving on a circular path is along the tangent at that point.



13.2-Terms related to circular motion-



Radius vector

- The vector joining the centre of the circular path to the position on the circular path is called a radius vector

Angular position-

- The angle made by the radius vector with a reference line (arbitrarily chosen diameter) is called angular position.
- The direction of angular position can be clockwise or anticlockwise depending upon the choice of frame of reference.
- The angular position of the particle at position "P" is denoted by angle θ in the diagram above.

Angular displacement-

- The change in angular position is called angular displacement.
- It is the angle through which the radius vector rotates during the given circular motion.
- The angular displacement between positions 'P' and 'Q' is denoted by $\Delta\theta$ in the diagram above.
- S.I unit of angular position and angular displacement is Radian.

Angular velocity-

- Denoted by ω (omega)
- ω-Rate of change of angular displacement.

- Average angular velocity-
- $\omega_{avg} = \frac{\Delta\theta}{\Delta t}$
- Instantaneous angular velocity-
- $\omega = \frac{d\theta}{dt}$
- S.I. units- Radian per second (rad per sec)
- ω is a vector quantity
- The direction of ω is given by the Right-hand rule.
- According to the right-hand rule, if you hold the axis with your right hand and rotate the fingers in the direction of motion of the rotating body then the thumb will point in the direction of the angular velocity.
- **Relation between angular velocity and linear velocity-**
- $\vec{v} = \vec{\omega} \times \vec{r}$

Angular Acceleration-

- The rate of change of angular velocity with time is said to be Angular Acceleration.
- $\alpha = \frac{\Delta\omega}{\Delta t}$
- SI units- $rad.(sec)^{-2}$
- Angular Acceleration is a vector quantity.
- The direction of Angular Acceleration

a) If angular velocity is increasing then the direction of Angular Acceleration

is in the direction of angular velocity.

b) If angular velocity is decreasing then the direction of Angular Acceleration

is in the direction which is opposite to the direction of angular velocity.

Time period-

- Time is taken to complete one rotation
- Formula-

$$T = \frac{2\pi}{\omega}$$

Where $\omega = \text{angular velocity}$

If N= no. of revolutions and total time then

$T = \frac{t}{N}$ or $(\omega = \frac{2\pi N}{t})$

- S.I unit seconds (s)

5. Frequency-

- The total number of rotations in one second.
- Formula-

$$\nu = \frac{1}{T}$$

- S.I. unit = Hertz
- We can write the relation between angular frequency and frequency as

$$w = 2\pi\nu$$

6. Centripetal acceleration and Tangential acceleration -

a. Centripetal acceleration-

- Formula-

$$a_c = \frac{V^2}{r}$$

Where a_c=Centripetal acceleration,

V= linear velocity

r = radius

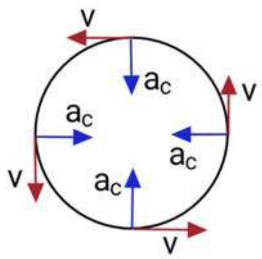


Figure Shows Centripetal acceleration

b. Tangential acceleration -

- During circular motion, if the speed is not constant, then along with centripetal acceleration there is also a tangential acceleration, Which is equal to the rate of change of magnitude of linear velocity.

$$a_t = \frac{\mathrm{d}v}{\mathrm{d}t}$$

- **Relation between angular acceleration and tangential acceleration-**

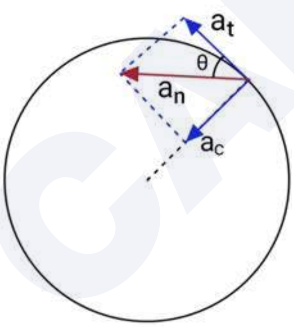
$$\vec{a_t} = \vec{\alpha} \times \vec{r}$$

Where $\vec{a_t}$ = tangential acceleration

$\vec{r} = radius\ \ vector$

$\alpha = angular\ acceleration$

c. Total acceleration-



- The vector sum of Centripetal acceleration and tangential acceleration is called Total acceleration.
- Formula-

$$a_n = \sqrt{a_c^2 + a_t^2}$$

d. Angle between Net acceleration and tangential acceleration (θ)

- From the above diagram-

$$tan\theta = \frac{a_c}{a_t}$$

13. Motion of Body Under Gravity (Free Fall)

Sign convention:

- Upward direction and right direction is taken as positive
- Downward direction and left direction is taken as negative

There are three cases basically in this -

1) If a body dropped from some height (initial velocity zero)

u = 0

a = g

$$v = gt$$
$$h = \frac{1}{2}gt^2$$
$$v^2 = 2gh$$
$$h_n = \frac{g}{2}(2n-1)$$

2) If a body is projected vertically downward with some initial velocity

Equation of motion: $v = u + gt$

$$h = ut + \frac{1}{2}gt^2$$
$$v^2 = u^2 + 2gh$$
$$h_n = u + \frac{g}{2}(2n-1)$$

3) If a body is projected vertically upward.

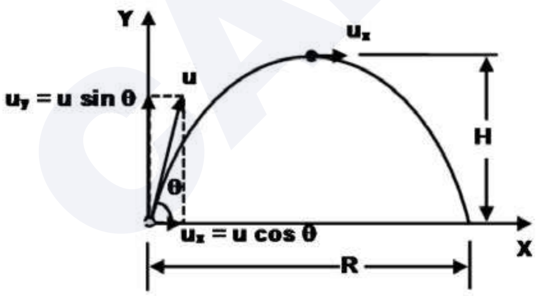
(i) Apply equation of motion :
Take initial position as origin and the direction of motion (vertically up) as
$a = -g$ [as acceleration due to gravity is downwards]
So, if the body is projected with velocity u,
and after time t it reaches up to height h then ,

$$v = u - gt; \quad h = ut - \frac{1}{2}gt^2; \; v^2 = u^2 - 2gh$$

(ii) For the case of maximum height $v = 0$
So from the above equation

$$u = gt$$
$$h = \frac{1}{2}gt^2$$
and $u^2 = 2gh$

14. Projectile Motion



1. Projectile Projected at an angle θ

- Initial Velocity- u

  Horizontal component = $u_x = u\cos\theta$

  Vertical component = $u_y = u\sin\theta$

- Final velocity = V

  Horizontal component = $V_x = u\cos\theta$

  Vertical component = $V_y = u\sin\theta - g.t$

So,

$$V = \sqrt{V_x^2 + V_y^2}$$

- Displacement = S

Horizontal component = $S_x = ucos\theta.t$

Vertical component = $S_y = usin\theta.t - \frac{1}{2}.g.t^2$

and, $S = \sqrt{S_x^2 + S_y^2}$

- Acceleration = a

Horizontal component = 0

Vertical component = -g

So, a = -g

Parameters in Projectile motion -

1. **Maximum Height -**

- Formula- $H = \frac{U^2 \sin^2\theta}{2g}$
- When the velocity of the projectile increases 'n' time then the Maximum height is increased by a factor of n^2
- Special Case-

If U is doubled, H becomes four times provided θ & g is constant.

2) Time of Flight

- Formula-

1. $T = \frac{2usin\theta}{g}$
2. Time of ascent =

$$t_a = \frac{T}{2}$$

3. Time of descent =

$$t_d = \frac{T}{2}$$

- When the velocity of the projectile increased 'n' time then the Time of ascent becomes n times
- When the velocity of the projectile increased 'n' time then Time of descent becomes n times
- When the velocity of the projectile increased 'n' time then the time of flight becomes n times.

3. **Horizontal Range-**

- Formula-

$$R = \frac{u^2 \sin 2\theta}{g}$$

- Special case of horizontal range

1. For maximum horizontal range.

$$\theta = 45^0$$

$$R_{max} = \frac{u^2 \sin 2(45)}{g} = \frac{u^2 \times 1}{g} = \frac{u^2}{g}$$

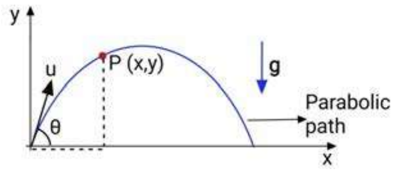
2. Range remains the same whether the projectile is thrown at an angle θ with the horizontal or at an angle θ with vertical (90-θ) with horizontal

3. When the velocity of the projectile increases 'n' time then the horizontal range is increased by a factor of n^2

4. When the horizontal range is n times the maximum height then

$$\tan\theta = \frac{4}{n}$$

4. Equation of trajectory-



$$y = x tan\theta - \frac{gx^2}{2u^2 cos^2\theta}$$

2. Projectile motion when projected horizontally

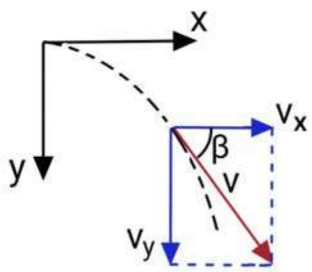
1. **Important equations**

- **Initial Velocity- u**

Horizontal component = $u_x = u$

Vertical component = $u_y = 0$

- **Velocity 'v' after time 't' sec-**



Horizontal component = $v_x = u$

Vertical component = $v_y = g.t$

and, $v = \sqrt{v_x^2 + v_y^2}$

i.e; $v = \sqrt{u^2 + (gt)^2}$

$$tan\beta = \frac{gt}{u}$$

Where, $\beta =$ angle that velocity makes with horizontal

- **Displacement=S**

Horizontal component = $S_x = u.t$

Vertical component = $S_y = \frac{1}{2}g.t^2$

and, $S = \sqrt{S_x^2 + S_y^2}$

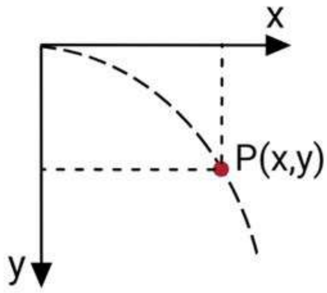
- Acceleration = a

Horizontal component= 0

Vertical component = g

So, a = g

- **Equation of path of a projectile**

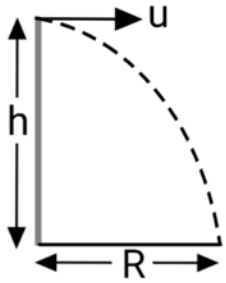


$y=\frac{g}{2u^2}.x^2$

$g \rightarrow$ Acceleration due to gravity

$u \rightarrow$ initial velocity

2. **Important Terms**



1. **Time of flight**

- Formula-

$$t=\sqrt{\frac{2h}{g}}$$

where $t=$ time of flight

$h=$ Height from which projectile is projected

2. **Range of projectile**

- Formula-

$$R=u.\sqrt{\frac{2h}{g}}$$

Where $R=$ Range of projectile

$u=$ horizontal velocity of projection from height h

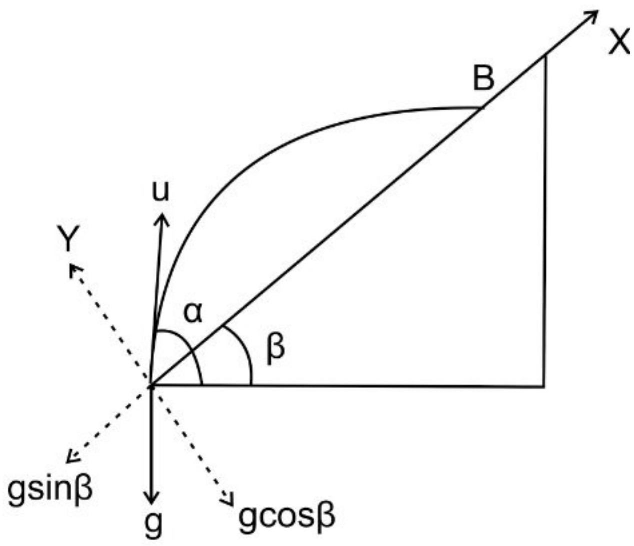
3. **Velocity at which projectile hit the ground**

$$v=\sqrt{u^2+2gh}$$

Where $v=$ velocity at which projectile hit the ground.

15. Projectile on an inclined plane

1. **Important equations**



U=Speed of projection

$\alpha =$ The angle of projection above-inclined plane (measured from the horizontal line)

$\theta =$ The angle of projection above-inclined plane (measured from the inclined plane)

$\beta =$ The angle of inclination.

a) Initial Velocity- U

Component along x or along inclined plane = $U_x = Ucos\theta$

Component along y or perpendicular to inclined plane = $U_y = USin\theta$

b) Final velocity =V

Component along x or along inclined plane = $V_x = Ucos\theta - (gsin\beta).t$

Component along y or perpendicular to inclined plane = $V_y = Usin\theta - (gcos\beta).t$

and, $V = \sqrt{V_x^2 + V_y^2}$

c) Displacement=S

Component along x or along inclined plane = $S_x = U_x t + \frac{1}{2}a_x.t^2$

Component along y or perpendicular to inclined plane = $S_y = U_y t + \frac{1}{2}a_y.t^2$

And $S = \sqrt{S_x^2 + S_y^2}$

d) Acceleration = a

Component along x or along inclined plane= $a_x = -gsin\beta$

Component along y or perpendicular to inclined plane = $a_y = -gcos\beta$

So a=-g

2. **Important Terms**

a) Time of flight

Formula-

$$T = \frac{2Usin\theta}{gcos\beta}$$

b) Range along incline plane

Formula-

$$R = \frac{2u^2 . \sin(\alpha - \beta) \cdot \cos\alpha}{g\cos^2\beta}$$

16. Relative Velocity

- The rate of change in the position of one object with respect to another object with time is defined as the relative velocity of one object with another.
- Formula-

Relative velocity of object A with respect to object B.

$$\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$$

- The relative velocity of A with respect to B is the velocity of A as observed by B.

Case of Relative Velocity-

1. **When A and B are moving along a straight line in the same direction.**

$\vec{V_A}$ = Velocity of object A.

$\vec{V_B}$ = Velocity of object B.

Then, the relative velocity of A w.r.t B is

$$\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$$

$\vec{V}_{AB}, \vec{V}_A, \vec{V}_B$ all are in the same direction. (If $\vec{V}_A > \vec{V}_B$)

And Relative velocity of B w.r.t A is

$$\vec{V}_{BA} = \vec{V}_B - \vec{V}_A$$

$$\& \ \vec{V}_{AB} = -\vec{V}_{BA}$$

2. **When A & B are moving along with straight line in the opposite direction.**

Relative velocity of A with respect to B is.

$$\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$$

$$V_{AB} = V_A + V_B$$

3. Relative Velocity when bodies moving at an angle θ to each other

- Relative velocity of a body, A with respected body B

$$V_{AB} = \sqrt{V_A^2 + V_B^2 + 2V_A V_B \cos(180 - \theta)}$$

$$= \sqrt{V_A^2 + V_B^2 - 2V_A V_B \cos(\theta)}$$

Where, $V_A = velocity\ of\ A$, $V_B = velocity\ of\ B$, $\theta = angle\ between\ A\ and\ B$

- If $\overrightarrow{V_{AB}}$ makes an angle β with the direction of $\vec{V}_A$, then

$$\tan\beta = \frac{V_B \sin(180 - \theta)}{V_A + V_B\cos(180 - \theta)}$$

$$\tan\beta = \frac{V_B \cdot \sin\theta}{V_A - V_B\cos\theta}$$

- If two bodies are moving at right angles to each other.

Relative Velocity of A with respect to B is

$$V_{AB} = \sqrt{{V_A}^2 + {V_B}^2}$$

17. Boat river Problem

1. **Important terms**

d = width of river

U = speed of river

V = Speed of Boat w.r.t. River

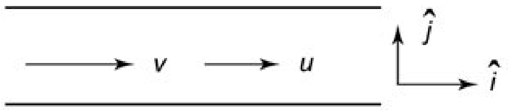
and V_b = Speed of boat w.r.t. Ground

So, the relation between u,v and V_b is

$$V_b = U + V$$

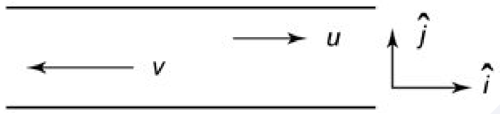
Let's try to find out V_b in some important cases

I) When the boat travels downstream (u and v have the same direction)



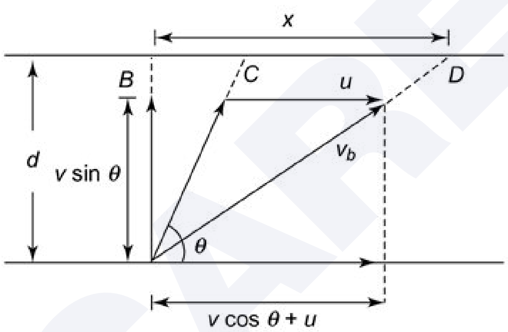
Then, $V_b = (U + V)\hat{i}$

II) When the boat travels upstream (u and v has opposite direction)



Then, $V_b = (U - V)\hat{i}$

III) If the boat travels at some angle θ with river flow (u)



Now resolve v in two-component

Component of v along $U = v_x = vcos\theta\hat{i}$

Component of v perpendicular to $U = v_y = vsin\theta\hat{j}$

So, $V_b = (vcos\theta + u)\hat{i} + vsin\theta\hat{j}$

and, $|V_b| = \sqrt{u^2 + v^2 + 2uvcos\theta}$

Now if the time taken to cross the river is t

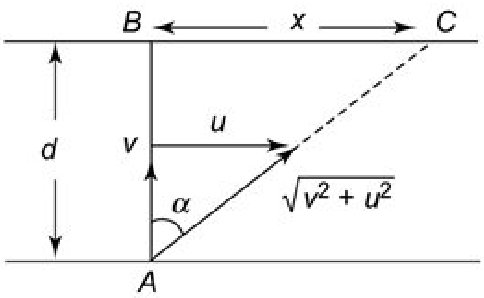
Then, $t = \frac{d}{vsin\theta}$

Here x = drift

And, $x=(u+v\cos\theta)t=\frac{(u+v\cos\theta)d}{v\sin\theta}$

2. Important cases-

I) To cross the river in the shortest time



This means v is perpendicular to u

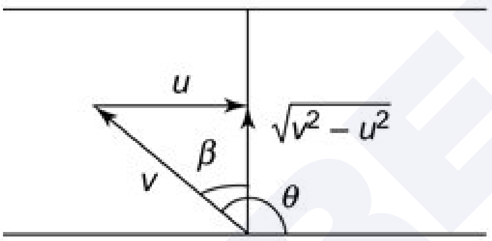
Or $Sin\theta=1\Rightarrow\theta=90^0$

So, $|V_b|=\sqrt{u^2+v^2}$

Time taken $t_{min}=\frac{d}{v}$

Drift along river flow, $x=d\left(\frac{u}{v}\right)$

II) To cross the river in the shortest path



Means drift = 0

$x=(u+v\cos\theta)t=0\Rightarrow\cos\theta=\frac{-u}{v}$

$|V_b|=\sqrt{v^2-u^2}$

The time taken to cross the river is $t=\frac{d}{v\sin\theta}$

$t=\frac{d}{\sqrt{v^2-u^2}}$

18. Rain - Man Problem

- **Terminology-**

  $\overrightarrow{V_m}=$ velocity of man in the horizontal direction

  $\overrightarrow{V_r}=$ velocity of rain w.r.t ground

  $\overrightarrow{V_{rm}}=$ velocity of rain w.r.t man

- **The velocity of rain w.r.t man is given by**

$$\overrightarrow{V_{rm}} = \overrightarrow{V_r} - \overrightarrow{V_m}$$

- For a Special case when the Velocity of rain falling vertically

Then, $tan\Theta = \frac{V_m}{V_r}$

Where Θ = angle which relative velocity of rain with

respect to man makes with the vertical

Laws of motion

Important Formulae

1.Inertia-

1. Definition- Inertia is the property of a body by which it continues to remain in its existing state of rest or uniform motion in a straight line unless an external force acts on it.

- It is a dimensionless and unitless quantity.

2. Types of inertia:-

    - The inertia of rest- Inability of a body to change its state of rest by itself.

      Ex- Person standing in the bus thrown backward when the bus starts suddenly.

- The inertia of motion- Inability of a body to change its state of motion by itself.

  Ex- Person standing in a moving bus thrown forward when the bus stops suddenly.

    - The inertia of direction- Inability of a body to change its direction of motion by itself.

  Ex- The raindrops falling vertically downwards cannot change their direction of motion.

2. Common forces in mechanics

Force-

1. Definition- Force is defined as an effect which causes a body to change its state.

- Force on 1 kg mass in the presence of gravity ($g = 9.8m/s^2$) is 1 kg-f=9.8 N

- Unit of force-

1. In SI unit- Newton(N)
2. In CGS- 1 dyne (1 newton = 100000 dyne)

- 1 Newton(N) is the force needed to accelerate an object with a mass of 1 kg at a rate of 1 m/s^2 (1 N = 1 kg · m/s^2)

Types of forces-

a) Contact forces-

- Contact forces are due to direct physical contact between objects.

Types of contact forces-

- Tension
- Normal reaction
- Spring force
- Friction

b) Non-contact forces-

- These forces act without the necessity of physical contact between objects.

- They depend on the presence of a "field" in the region of space surrounding the body under consideration.

Types of non-contact forces-

- Gravitational force
- Electrostatic force
- Magnetic force

c) Weak forces-

- Vanderwaal force

d) Nuclear forces

$F_{nuclear} > F_{electro} > F_{gravitation}$

$F_e/F_g = 10^{43}$

Therefore, $F_e >> F_g$

Common forces in mechanics-

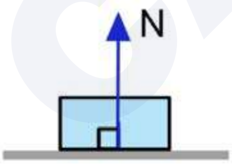
1. Gravitational force (Weight):-

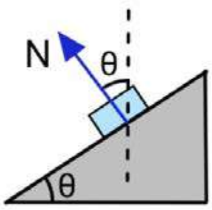
- Definition- Force with which earth attracts an object.
- It always acts towards the centre of the earth
- It is denoted by W=mg, where m=mass of the body and g= acceleration due to gravity.

2. Normal Reaction(N):-

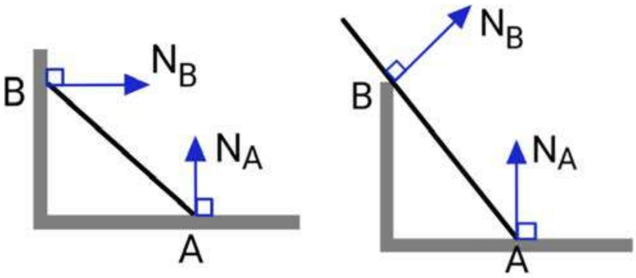
- Definition- A contact force between two bodies in physical contact which acts perpendicular to common surface in contact.
- Normal reaction is always a push force

Examples of Normal reaction acting on a block kept on horizontal surface and inclined plane is shown in figures below-





Examples of Normal reaction acting on a rod resting between the ground and wall is shown in the figures below-



3. Tension(T):-

- Force exerted by taut string, rope or chain against pulling force along the length.
- It acts away from the point of contact.
- It is always a pull force
- Tension remains the same as long as the string is the same only in case of a massless string.

4. Spring Force:-

Spring force is a type of restoring force which tries to come back to its natural length.

1. Spring force is given by:

$F_{sp} = -k\Delta x$

where,

F_{sp}=spring force

k= spring constant

Δx=net elongation or compression in the spring

2. Force at every point in a massless spring remains same, so we can solve questions of spring by considering it as string and spring force as tension.

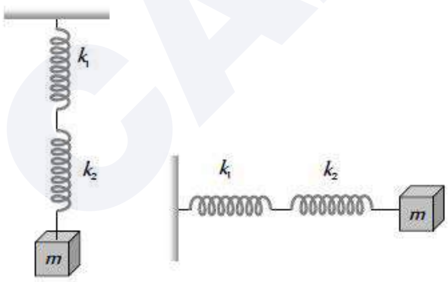
3. Spring constant:- $k\alpha\frac{1}{l}$

Where, k=spring constant

l=length of spring

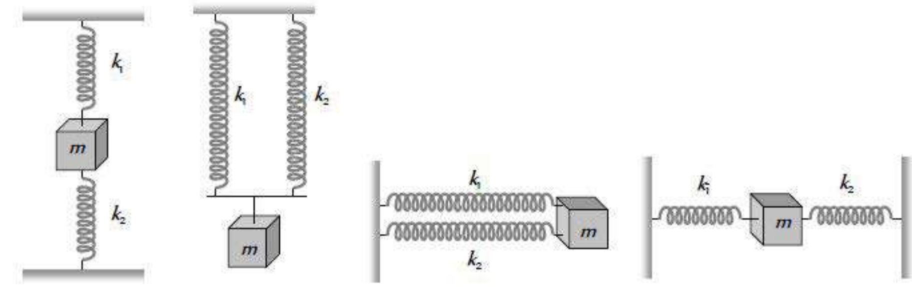
4. Combination of Spring:-

- Series combination:-



$\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2}$

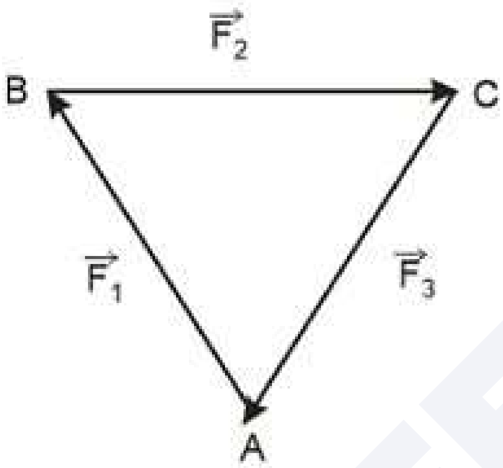
- Parallel combination:-



$k_{eq} = k_1 + k_2$

Equilibrium of concurrent forces-

- Concurrent Forces- If all the forces working on a body are acting on the same point then they are said to be concurrent.
- Three forces will be in equilibrium if they are represented by three sides of a triangle taken in order.

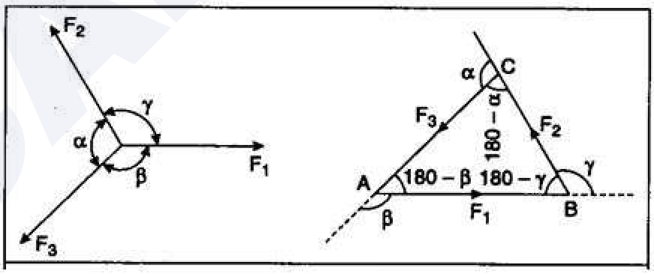


- For equilibrium,

$\sum \vec{F_{net}} = 0$,

$or \sum \vec{F_x} = 0, \sum \vec{F_y} = 0, \sum \vec{F_z} = 0$

- Lami's theorem: It states that for three concurrent forces in equilibrium,



$\frac{F_1}{sin\alpha} = \frac{F_2}{sin\beta} = \frac{F_3}{sin\gamma}$

3. Newton's First law of motion

Newton's 1st law of motion states that if the (vector) sum of all the forces acting on a particle is zero, then and only then does the particle remain unaccelerated, i.e., remains at rest or move with constant velocity.

- If $F_{net}=0 \Rightarrow a_{net}=0 \Rightarrow$ forces in all directions are zero,i.e, $\sum \vec{F_x} = 0, \sum \vec{F_y} = 0, \sum \vec{F_z} = 0$
- Newton's first law is also called the law of inertia.

- Newton's laws are valid in an inertial frame of reference but are not valid in a non-inertial frame of reference.

Frame of reference

- A frame of reference is a system of coordinate systems and clocks.
- Types of frame of reference:

1. Inertial frame of reference- A frame which is at rest or moving with uniform velocity.

Example- 1. Car moving with velocity v on a straight road.

2. Lift at rest.

2. A non-inertial frame of reference- a frame which is accelerated and does not have a constant velocity.

Example- 1. The frame travelling in a straight line but speeding up or slowing down.

2. The frame travelling along a curved path.

4. Linear Momentum

1. The linear momentum of a body is the quantity of motion contained in the body.
2. It is measured in terms of the force required to stop the body in a unit of time.
3. If a body of mass m is moving with velocity $\vec{v}$, then its linear momentum $\vec{P}$ is given by $\vec{p} = m\vec{v}$.
4. It is a vector quantity and its direction is the same as the direction of the velocity of the body.
5. S.I. Unit : kg-m/sec
6. Dimension- MLT^{-1}
7. If two objects of different masses have the same momentum, the lighter body possesses greater velocity.

As $p=m_1v_1=m_2v_2$=constant

$$\therefore \frac{V_1}{V_2} = \frac{m_2}{m_1} \Rightarrow V \ \alpha \ \frac{1}{m}$$

5. Newton's Second and Third Law of motion

Newton's second law of motion:-

- It states that the acceleration of the particle measured from an inertial frame is given by the (vector) sum of all the forces acting on the particle divided by its mass (only when mass is constant), i.e.,

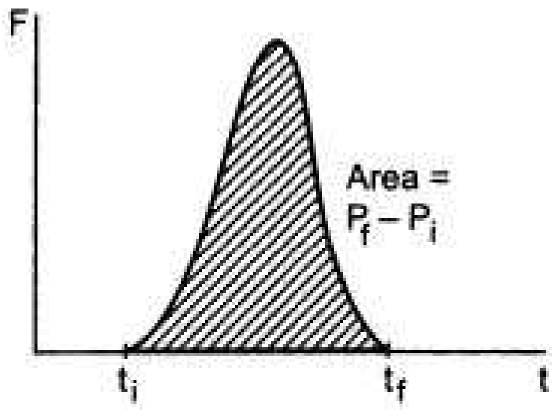
$$\vec{a} = \frac{\vec{F}}{m} \Rightarrow \vec{F} = m\vec{a}$$

Impulse:

1. The quantity $\vec{I} = \int_{t_1}^{t_2} \vec{F} \cdot dt$ is known as the impulse of the force F during the time interval t_1 to t_2 and

Is equal to the change in the momentum of the body on which it acts,

i.e. $P_f - P_1 = \int_{P_1}^{P_1} dP = \int \frac{dP}{dt} * dt = \int F dt$ $\Rightarrow$ Area under force and time graph is an impulse.



2. Dimension- MLT^{-1}

3. Unit- kg-m/sec

- Impulse Momentum Theorem- Newton's 2nd law can also be written as:

  Rate of change in momentum = Force Applied

  $\vec{F} = \frac{d\vec{p}}{dt}$

Inertial mass:

- This is defined by Newton's 2nd law- F = ma, which states that when a force F is applied to an object, it will accelerate proportionally, and that constar of proportion is the mass of that object.
- To determine the inertial mass, you apply a force of F Newtons to an object, measure the acceleration in m/s^2

  $Inertial\ mass(kg) = \frac{F}{a}$

Newton's third law of motion:-

- It states that "If a body A exerts a force F on another body B, then B exerts a force (-F) on A."
- Action and reaction never act on the same body.

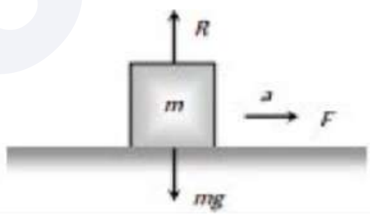
Law of Conservation of Linear Momentum:

- As we know, $\vec{F} = \frac{d\vec{p}}{dt}$, If the net external force acting on the system is zero then the change in momentum of system=0
- $\Rightarrow \Delta p = 0$

$\therefore p_{system} = p_1 + p_2 + p_3 + ... = constant$

6. Acceleration of Block on the horizontal smooth surface

1. When the pull is horizontal, and no friction



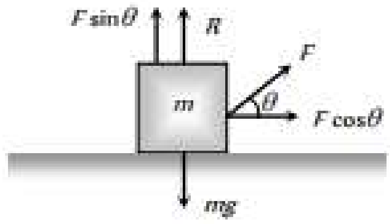
Balance forces-

the body is moving along the x-axis

$\because F_y = 0$
$R = mg$ & $F = ma$

$a = \frac{F}{m}$

2. Pull Acting at Angle (Upward)



Balancing forces in both X and Y directions,

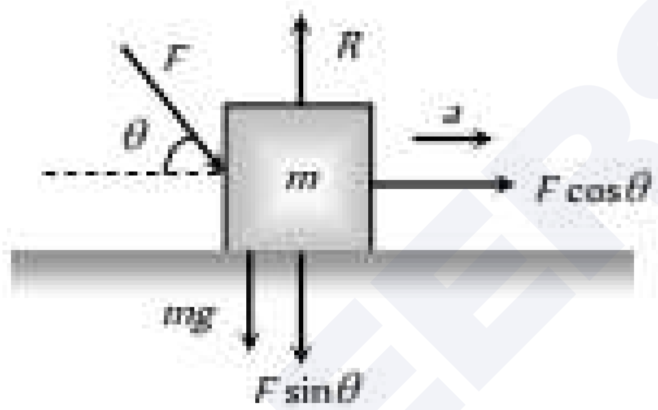
$R + F sin\theta = mg$

$R = mg - F sin\theta$ along Y-axis

$F cos\theta = ma$ along X-axis

$a = \frac{F cos\theta}{m}$

3. Push Acting at Angle (Downward)



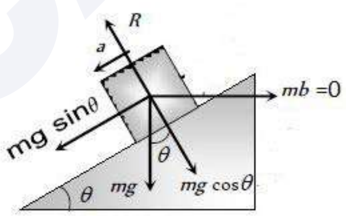
Balancing forces in both X and Y directions,

$R = mg + F sin\theta$ along Y-axis

$a = \frac{F cos\theta}{m}$ along X-axis

7.Acceleration of Block on Smooth Inclined Plane

1. **When an Inclined Plane is at rest**

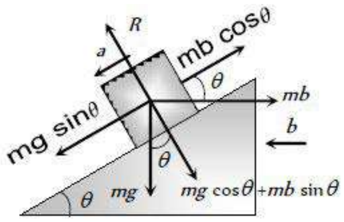


$R = mg\ cos\theta$ along with normal to the incline

$mg\ sin\theta = ma$ along the incline

$a = g\ sin\theta$

2. **When an Inclined Plane is given Acceleration 'b'**



$R = mg\, cos\theta + mb\, sin\theta$

$ma = mg\, sin\theta - mb\, cos\theta$

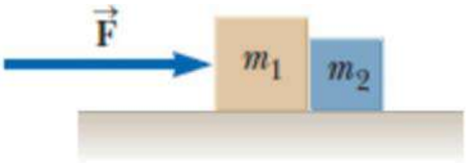
$a = g\, sin\theta - b\, cos\theta$

$*$ Condition for the body to be at rest relative to the inclined plane.

$a = g\, sin\theta - b\, cos\theta = 0$

$b = g\, tan\theta$

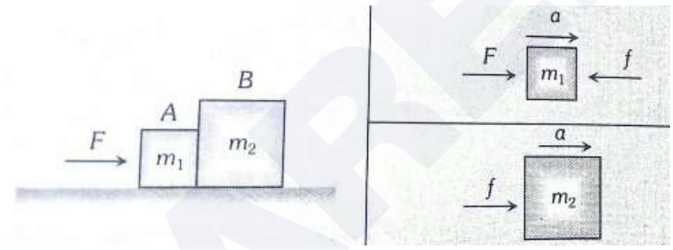
8. Motion of Block in Contact



$Use\ F_{net} = ma$

$$a = \frac{F_{net}}{m_1 + m_2}$$

1. When 2 Blocks are in Contact



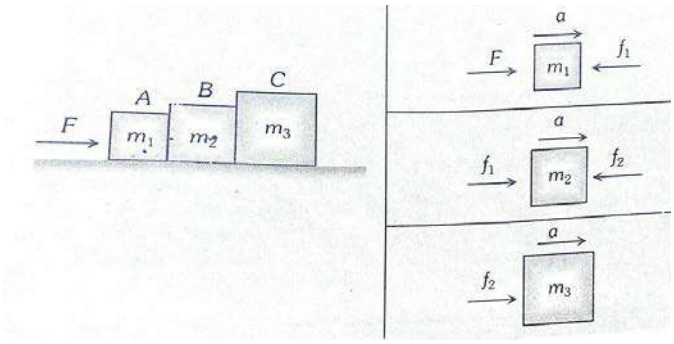
$F - f = m_1 a$

$f = m_2 a$

$$a = \frac{F}{m_1 + m_2}$$

$$f = \frac{m_2 F}{m_1 + m_2}$$

2. When 3 Blocks are in Contact



$$Use\ F_{net} = ma$$

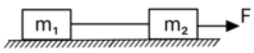
$$a = \frac{F}{m_1 + m_2 + m_3}$$

$$f_1 = \frac{(m_2 + m_3)F}{m_1 + m_2 + m_3}$$

$$f_2 = \frac{m_3 F}{m_1 + m_2 + m_3}$$

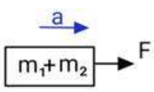
9. Motion of blocks when connected with string

1. Two blocks connected with a string on smooth horizontal surface



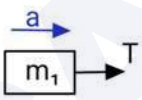
Let accelereration of the blocks be 'a', and Tension in the string be T.

F.B.D of both blocks combined-



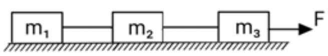
$$From\ F_{net} = M_{sys}a$$
$$F = (m_1 + m_2)a$$
$$\Rightarrow a = \frac{F}{m_1 + m_2}\ ...(1)$$

F.B.D of block of mass m_1-



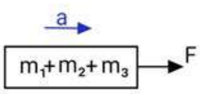
$$From\ F_{net} = M_{sys}a$$
$$T = m_1 a$$
$$From\ equation(1)$$
$$\Rightarrow T = \frac{m_1 F}{m_1 + m_2}\ ...(2)$$

2. Three blocks connected with a string on the smooth horizontal surface



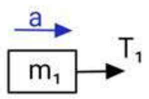
Let acceleration of the blocks be 'a', the tension in the string between m_1 and m_2 be T_1, and tension between m_2 and m_3 be T_2.

F.B.D of all the blocks combined-



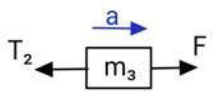
$$From\ F_{net} = M_{sys}a$$
$$F = (m_1 + m_2 + m_3)a$$
$$\Rightarrow a = \frac{F}{m_1 + m_2 + m_3} \ ...(1)$$

F.B.D of block m_1-



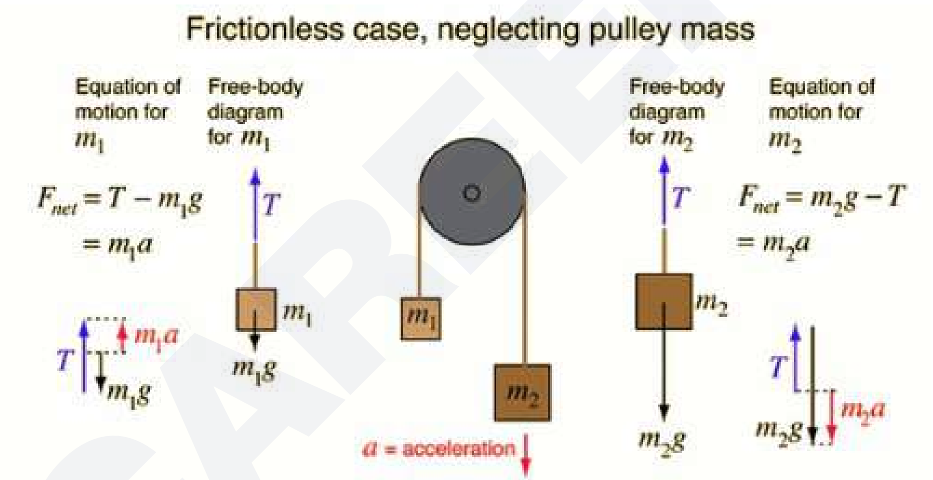
$$From\ F_{net} = M_{sys}a$$
$$T_1 = m_1 a$$
$$From\ equation\ (1) -$$
$$\Rightarrow T_1 = \frac{m_1 F}{m_1 + m_2 + m_3} \ ...(2)$$

F.B.D of m_3-



$$From\ F_{net} = M_{sys}a$$
$$F - T_2 = m_3 a$$
$$\Rightarrow T_2 = \frac{(m_1 + m_2)F}{m_1 + m_2 + m_3} \ ...(3)$$

10. Motion of connected blocks over a pulley



Equation of motion for m_1

$$F_{net} = T - m_1 g = m_1 a$$

Equation of Motion for m_2

$$F_{net} = m_2 g - T = m_2 a$$

$$a = \frac{[m_2 - m_1]g}{m_1 + m_2}$$

$$T = \frac{2m_1 m_2\, g}{m_1 + m_2}$$

11. Apparent weight of the body in a lift

If mass m is placed on a weighing machine which is placed in the lift.

Actual weight = mg

Apparent weight = Reaction force

R $\rightarrow$ Reaction force is given by reading of weighing machine.

1. Lift is at Rest

$V=0,\ a=0$

$use\ \ F_{net}=m\vec{a}$

$R-mg=0$

$R=mg$

Apparent weight $=$ Actual weight

2. Lift is moving upward or down with constant Velocity

$v=const,\ a=0$

$R=mg$

Apparent weight $=$ Actual weight

3. Lift is accelerating Upward with acceleration = a

$v=variable,$

$a<g$

$R-mg=ma$

$R=m(g+a)$

Apparent weight > Actual weight

4. Lift is moving up with a = g

$R-mg=mg$

$R=2mg$

Apparent weight = 2 (Actual weight)

5. **Lift accelerating down with 'a'** ($a<g$)

V= variable

$a<g$

$mg-R=ma$

$R=m(g-a)$

Apparent weight $<$ Actual weight

6. **Lift is moving down with a = g**

$a=g$

$mg-R=mg$

$R=0$

Apparent weight = 0 (weightlessness)

7. **Lift is moving down at the rate 'a > g'**

$V=Variable$

$a>g$

$mg-R=ma$

$R=-ve$

The body will rise from the floor of the lift & stick to the ceiling of the lift.

12. Recoiling of Gun

Let us consider a case of a gun held at rest. Let the mass of the gun be m_g, the mass of the bullet be m_b, the velocity of the bullet after firing the gun be V_b an the recoil velocity of the gun be V_g.

Assuming the net external force acting on the bullet and gun system is zero.

$\Rightarrow F_{ext} = 0$

Since the net external force on the gun and bullet system is zero the total momentum of the system will be constant.

Assuming the gun and bullet to be at rest initially, Initial momentum=0

Final momentum=$m_b V_b + m_g V_g$

So from momentum conservation, we get-

$$0 = m_b \vec{V_b} + m_g \vec{V_g}$$

$$\Rightarrow \vec{V_G} = -\frac{m_B}{m_G} \times \vec{V_B}$$

- -ve sign indicates that $\vec{V_G}$ is opposite to that of the velocity of the bullet
- Higher the mass of gun lesser will be recoil velocity i.e $\vec{V_G} \propto \frac{1}{m_G}$
- When the body of the shooter and the gun behave as one body/system

Then $\vec{V_G} \propto \frac{1}{m_G + m_{man}}$

$where, m_{man} \rightarrow mass\ of\ person\ holding\ gun$

- If n bullet each of mass m is fired per unit time from a gun

Then

$$F = V_{rel}\left(\frac{dm}{dt}\right) = V\,(mn)$$

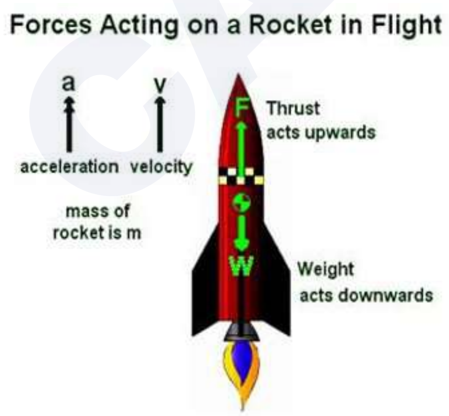
$$F = mnv$$

$$F = force\ required\ to\ hold\ the\ gun$$

$$n = no.\ of\ bullets$$

13. Rocket Propulsion

Let us assume a rocket of total initial mass (rocket + fuel) m_0, starts moving upward due to the thrust force of the fuel jet. Assuming the velocity of the fuel je with respect to the rocket to be u (assumed to be constant for this discussion) in a vertically downward direction and the mass of jet fuel emerging out of the rocket per unit time to be $\frac{dm}{dt}$. Let the velocity of the rocket after t time of motion be v and the acceleration of the rocket be a in vertically upward direction.



1. **Thrust on the rocket**

$$F = -\frac{udm}{dt}$$

Where F= Thrust

$$\frac{dm}{dt} = rate\ of\ ejection\ of\ the\ fuel$$

u=velocity of exhaust gas with respect to rocket

m=mass of the rocket at time t

Net force on rocket-

$$F_{net} = -\frac{udm}{dt} - mg$$

$$* F = -\frac{udm}{dt}\ [if\ gravity\ neglected]$$

2. **Acceleration of Rocket (a)**

$$a = -\frac{u}{m}\frac{dm}{dt} - g$$

- If g is neglected then

$$a = -\frac{u}{m}\frac{dm}{dt}$$

3. **Instantaneous Velocity of Rocket (v)**

If g is neglected then-

$$a = -\frac{u}{m}\frac{dm}{dt}$$

$$\frac{dv}{dt} = -\frac{u}{m}\frac{dm}{dt}$$

$$\int_0^v dv = -u\int_{m_0}^{m}\frac{1}{m}dm$$

$$\Rightarrow v = u\,log_e\left(\frac{m_\circ}{m}\right)$$

$$v = u * log_e(\frac{m_\circ}{m}) = 2.303u * log_{10}(\frac{m_\circ}{m})$$

4. **Burnt speed of Rocket**

- It is the speed attained by the rocket when complete fuel gets burnt.
- It is the maximum speed attained by the rocket
- Formula

$$V_b = V_{max} = u\,log_e\left(\frac{m_\circ}{m_r}\right)$$

$$V_b \rightarrow burnt\ speed$$

$$m_r \rightarrow residual\ mass\ of\ empty\ container$$

14. Friction

- The friction of the moving object is proportional to the normal force(numerically equal to the pressing force).

$$f \propto N$$

- The friction experienced by the object is dependent on the nature of the surface it is in contact with.
- Friction is independent of the area of contact as long as there is an area of contact(as for solid apparent area is not equal to actual area of contact).
- It acts tangentially along with the contact.
- The direction of friction is always opposite to the direction of relative motion.
- It can be also defined as the component of contact force which is parallel to the surfaces in contact.

Types of friction;-

- Static Friction
- Kinetic Friction

Kinetic Friction-

1. Kinetic friction occurs when there is relative motion between two bodies that are in contact.

2. When two bodies slip over each other, the force of friction is called kinetic friction.

3. It is a constant friction force, i.e, it does not depend upon the speed of relative motion.

4. The magnitude of the kinetic friction is proportional to the normal force acting between the two bodies.

5. It is denoted by:-

$f_K \ \alpha \ N$

$f_K = \mu_K N$

$f_K =$ the kinetic friction

$\mu_K =$ coefficient of kinetic friction is constant

N = reaction

Coefficient of friction (μ_k)**:**

- It is unitless and dimensionless.
- It depends on the nature and material of the two bodies in contact.
- It doesn't depend on the speed of the sliding bodies.

1. It occurs when there is a tendency of relative motion, i.e, the body is still at rest and is just about to move.

2. When two bodies do not slip over each other, then the force of friction is called static friction.

3. It is a variable force or self-adjusting force as it changes itself according to the applied force.

4. It is denoted by fs and the static friction is in between:- $0 < f_s < f_l$

where f_l is limiting friction.

5. Limiting friction is the maximum static friction that a body can exert on the other body in contact with

It and is given by:-

$f_l \ \alpha \ R$ or $f_l = \mu_s R$

$f_l =$ limiting friction

$\mu_s =$ coefficient of friction

R = reaction force

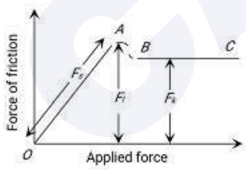
6. Generally,

$f_K < F_l$

$\therefore \ \mu_K < \mu_s$

The graph between Applied Force and the Force of Friction & Angle of Friction

1.

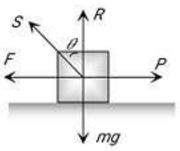


At A static friction is the maximum

OA = Represents static friction

Beyond A, the force of friction decreases slightly

2.



The angle between the normal reaction and resultant contact force is called the angle of friction

$tan\theta = \frac{F_l}{R}$

$tan\theta = \mu_s$

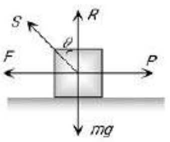
$\frac{F_l}{R} = \mu_s$

R = Reaction,

$F_l =$ Force of limiting friction

$\theta = tan^{-1}(\mu_s)$

3.



$S = \sqrt{F^2 + R^2}$

$S = \sqrt{(\mu mg)^2 + (mg)^2}$

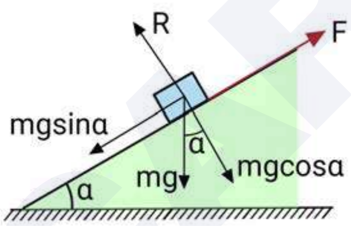
$S = mg\sqrt{\mu^2 + 1}$

S = Resultant force

$\mu =$ Coefficient of friction

If $\mu = 0$, S(will be minimum), I.e S = mg

Angle of Repose-



The angle of repose is defined as the angle of the inclined plane with horizontal such that the body is placed on it just begins to slide.

Here α is the angle of repose, F is the limiting friction, R is a normal reaction.

From the figure,

$R = mgcos\alpha$

$F = mgsin\alpha$ and

we know that

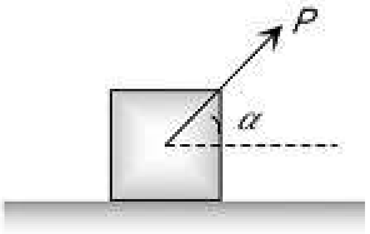
$\frac{F}{R} = tan\alpha$

$\frac{F}{R} = \mu_s = tan\theta = tan\alpha$

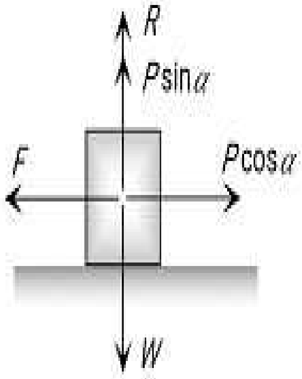
$\tan\alpha = \mu_s \Rightarrow \alpha = \tan^{-1}(\mu_s)$

15. Calculation of Required force in different situations

Case 1:- Minimum pulling force P at an angle α from the horizontal



By resolving P in the horizontal and vertical direction, we get:



where F is the friction force.

For the condition of equilibrium,

$F = P\ cos\ \alpha$

$R = W - P\ sin\ \alpha$

By substituting these values in F= μR, we get:

$Pcos\alpha = \mu(W - Psin\alpha)$

$Use = \mu = tan\theta$

$\Rightarrow Pcos\alpha = \frac{sin\theta}{cos\theta}(W - Psin\alpha)$

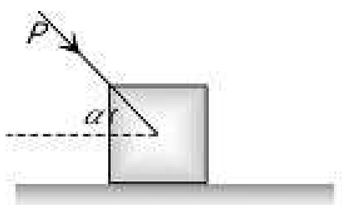
$P = \frac{Wsin\theta}{cos(\alpha - \theta)}$

where P is the pulling force,

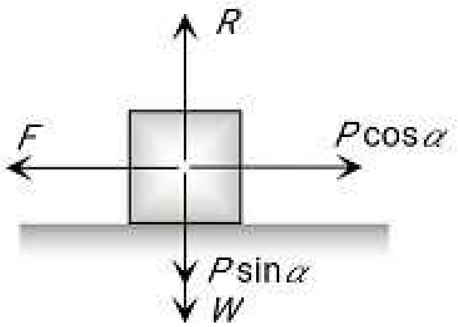
R is a normal reaction

W is the weight

Case 2:- Minimum pushing force P at an angle α from the horizontal



By resolving P in the horizontal and vertical direction, we get:



For the condition of equilibrium,

$F = Pcos\alpha$

$R = W + Psin\alpha$

By substituting these values in F=μR, we get:

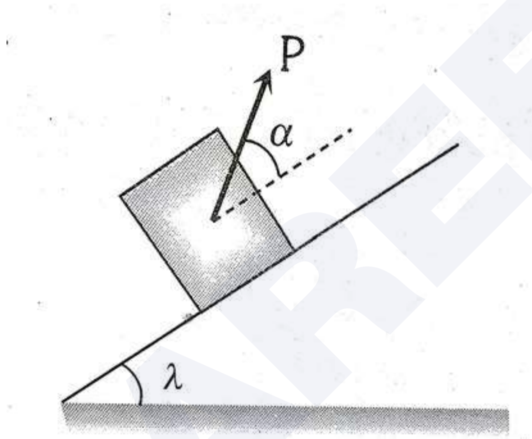
$Pcos\alpha = \mu(W + Psin\alpha)$

$Use = \mu = tan\theta$

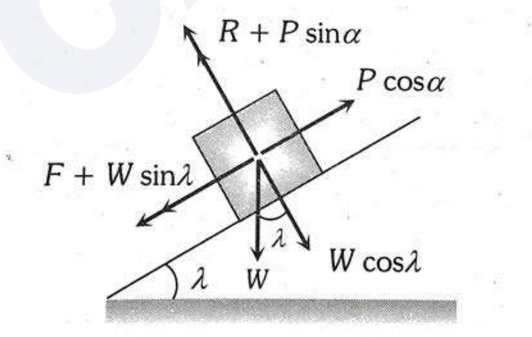
$\Rightarrow Pcos\alpha = \frac{sin\theta}{cos\theta}(W + Psin\alpha)$

$P = \frac{Wsin\theta}{cos(\alpha + \theta)}$

Case 3:- Minimum pulling force P to move the body upwards on an inclined plane



By resolving P in the direction of the plane and perpendicular to the plane, we get:



For the condition of equilibrium

$R + Psin\alpha = Wcos\lambda \Rightarrow R = Wcos\lambda - Psin\alpha$

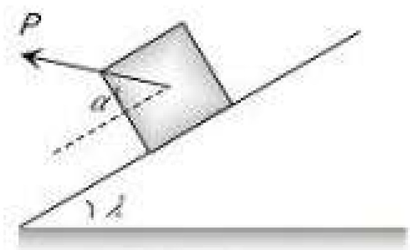
$F + Wsin\lambda = Pcos\alpha \Rightarrow F = Pcos\alpha - Wsin\lambda$

By substituting these values in F=μR, we get:

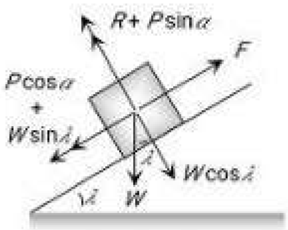
$$P = \frac{W sin(\theta + \lambda)}{cos(\alpha - \theta)}$$

Where θ is the angle of friction such that, $\mu = tan\theta$

Case 4:- Minimum force to move a body in a downward direction along the surface of the inclined plane



By resolving P in the direction of the plane and perpendicular to the plane, we get:



For the condition of equilibrium,

$$R + Psin\alpha = Wcos\lambda \Rightarrow R = Wcos\lambda - Psin\alpha$$

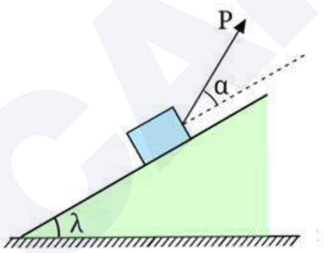
$$F = Pcos\alpha + Wsin\lambda$$

By substituting these values in F=μR, we get:

$$P = \frac{W sin(\theta - \lambda)}{cos(\alpha - \theta)}$$

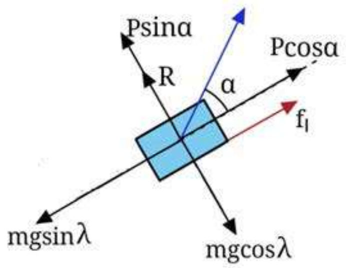
Where θ is the angle of friction such that, $\mu = tan\theta$

Case 5:- Minimum force to avoid sliding of a body down on an inclined plane



As the block has a tendency to slip downward, friction force will act up the incline. For the minimum value of P, the friction force is limiting and the block is in equilibrium.

Free Body Diagram of the block-



For the condition of equilibrium-

$$R + P\sin\alpha = mg\cos\lambda$$
$$\Rightarrow R = mg\cos\lambda - P\sin\alpha$$
Limiting friction−
$$f_l = \mu R$$

$$\Rightarrow f_l = \mu(mgcos\lambda - Psin\alpha)......(1)$$

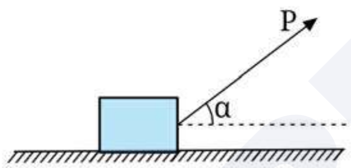
$$f_l = mgsin\lambda - Pcos\alpha \;...(2)$$

From equation (1) and (2)-

$$mgsin\lambda - Pcos\alpha = \mu(mgcos\lambda - Psin\alpha)$$
$\theta \rightarrow$ angle of friction
$$\mu = \tan\theta$$
$$P(\cos\alpha - \tan\theta\sin\alpha) = mg(\sin\lambda - \tan\theta\cos\lambda)$$

$$\Rightarrow P = \frac{mgsin(\theta - \lambda)}{cos(\theta + \alpha)}$$

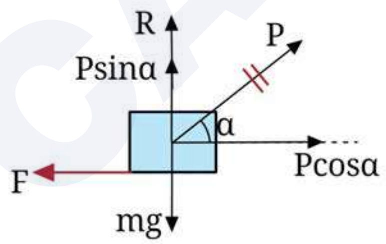
Case 6:- **Minimum Force of Motion along the horizontal surface and its direction**



Let the force P be applied at an angle α with the horizontal.

Let the friction force on the block be F.

F.B.D of the block-



For vertical equilibrium,

$$R + P\sin\alpha = mg$$
$$\Rightarrow \therefore R = mg - P\sin\alpha$$

and for horizontal motion,

$$P\cos\alpha \geq F \Rightarrow P\cos\alpha \geq \mu R$$

Substituting the value of R, we get:-

$$P\cos\alpha \geq \mu(mg - P\sin\alpha)$$

$$\Rightarrow P \geq \frac{\mu mg}{\cos\alpha + \mu\sin\alpha}$$

For the force P to be minimum $(\cos\alpha + \mu\sin\alpha)$ must be maximum i.e.,

$$\frac{d}{d\alpha}[\cos\alpha + \mu\sin\alpha] = 0$$

$$\Rightarrow -\sin\alpha + \mu\cos\alpha = 0$$

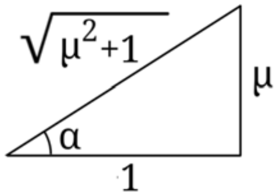
$$\therefore \tan\alpha = \mu$$

$\Rightarrow \alpha = \tan^{-1}(\mu)$= angle of friction.

i.e. For the minimum value of P, its angle from the horizontal should be

equal to the angle of friction.

As $\tan\alpha = \mu$, so from the figure,



$\sin\alpha = \frac{\mu}{\sqrt{1+\mu^2}}$ and $\cos\alpha = \frac{1}{\sqrt{1+\mu^2}}$

By substituting these values,

$$P \geq \frac{\mu mg}{\frac{1}{\sqrt{1+\mu^2}} + \frac{\mu^2}{\sqrt{1+\mu^2}}}$$

$$\Rightarrow P \geq \frac{\mu mg}{\sqrt{1+\mu^2}}$$

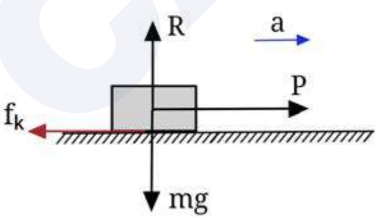
$$\therefore P_{min} = \frac{\mu mg}{\sqrt{1+\mu^2}}$$

16. Acceleration of block against friction

Case 1:- **Acceleration of a block on a horizontal surface**

- When the body is moving under the application of force P, then kinetic friction opposes its motion.

  Let a is the net acceleration of the body.



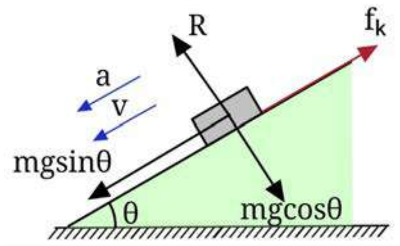
From the figure,

$$P - f_k = ma$$

$$a = \frac{P - f_k}{m}$$

Case 2:- **Acceleration of a block sliding down over a rough inclined plane**

- When the angle of the inclined plane is more than the angle of repose, the body placed on the inclined plane slides down with an acceleration a.



From the figure,

$ma = mg \sin\theta - f_k$

$ma = mg \sin\theta - \mu R$

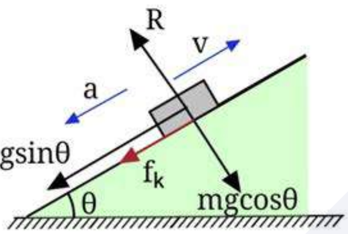
$ma = mg \sin\theta - \mu mg\cos\theta$

$a = g[\sin\theta - \mu\cos\theta]$

For $\mu = 0$ $\therefore$ $a = g \sin\theta$

Case 3:- **Retardation of a block sliding up over a rough inclined plane**

- When the angle of the inclined plane is less than the angle of repose, then for the upward motion (with some initial velocity)



$ma = mg \sin\theta + f_k$

$ma = mg \sin\theta + \mu mg \cos\theta$

$ma = g[\sin\theta + \mu\cos\theta]$

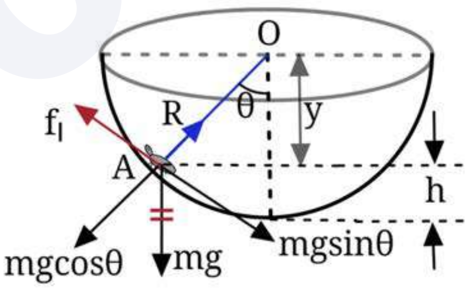
$a = g[\sin\theta + \mu\cos\theta]$

For $\mu = 0$

$a = g \sin\theta$

17. Motion of an Insect in the Rough Bowl-

As the insect crawls up, limiting friction friction force decreases and the component of weight along the suface (driving force) will decrease. Let's assume the insect can crawl upto height 'h' before it starts slipping. At that moment the frictional force will be limiting friction force as shown in the figure.



Let m=mass of the insect, r=radius of the bowl, μ= coefficient of friction for limiting condition at point A

$$\mathrm{R} = \mathrm{mg}\cos\theta \text{ ...(i)}$$

Limiting friction–

$$\mathrm{f_l} = \mu \mathrm{R} = \mu \mathrm{mgcos}\theta$$

$$\mathrm{f_l} = \mathrm{mg}\sin\theta \text{ ...(ii)}$$

From equation (1) and (2)-

$$\mathrm{mg}\sin\theta = \mu\mathrm{mgcos}\theta$$

$$\tan\theta = \mu$$

$$\tan^2\theta = \mu^2$$

$$\frac{(\mathrm{r}^2 - \mathrm{y}^2)}{\mathrm{y}^2} = \mu^2$$

$$\mathrm{y} = \frac{\mathrm{r}}{\sqrt{1+\mu^2}}$$

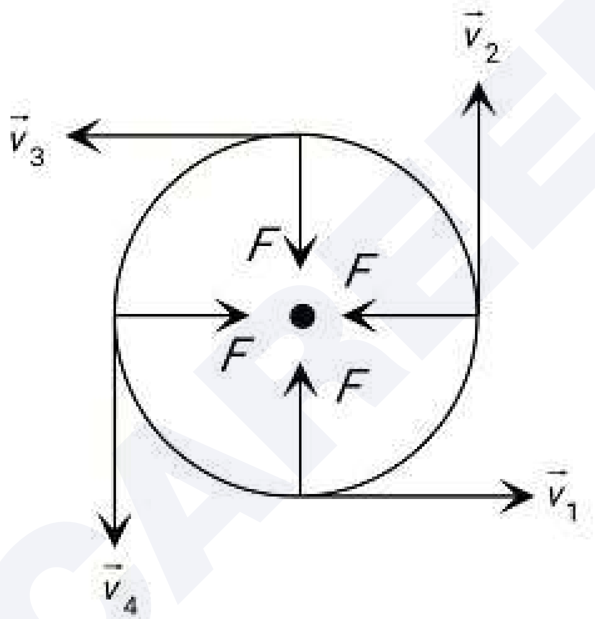
$$\mathrm{r} - \mathrm{h} = \frac{\mathrm{r}}{\sqrt{1+\mu^2}}$$

$$\mathrm{h} = \mathrm{r}\left[1 + \frac{1}{\sqrt{1+\mu^2}}\right]$$

18. Centripetal Force and Centrifugal Force

1. Centripetal Force-

- Force acts on the body along the radius and towards the center.



$$F = 4m\pi^2 n^2 r$$

$$F = \frac{4m\pi^2 n^2 r}{T^2}$$

Where

F = Centripetal force

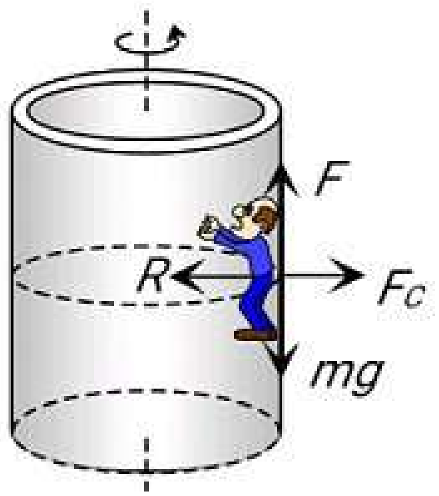
ω = Angular velocity

n = frequency

2. Centrifugal Force

- It is a fictitious force which has significance only in the rotating frame of reference.
- It is an Imaginary force due to the incorporated effects of inertia.

19. Sticking of Person with the wall of Rotor(Death well)



F = weight of person (mg)

$\mu R = mg$

$\mu F_c = mg$

$\mu m\omega_{min}^2 r = mg$

$\therefore \ \omega_{min} = \sqrt{\frac{g}{\mu r}}$

Where F = friction force

F_c = centrifugal force

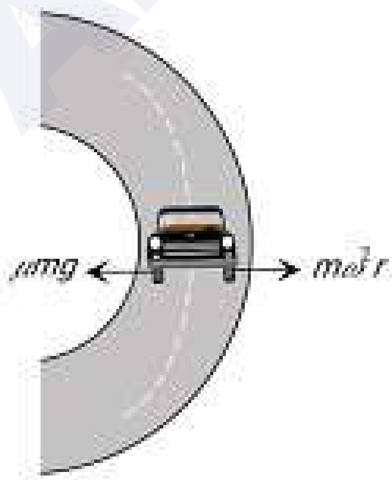
ω_{min} = minimum angular velocity

$\mu =$coefficient of friction

r = radius of Rotor

20. Skidding of Vehicle

Skidding of Vehicle on a Level Road-



Frictional force $\geq$ Req. centripetal Force

$\mu mg \geq \frac{mv^2}{r}$

$V_{safe} \leq \sqrt{\mu r g}$

$V_{safe} =$Safe vector move

r = radius of the curve

$\mu =$ coefficient of friction

- V_{safe} is the maximum velocity by which vehicle can turn on a circular path of radius r.

Skidding of object on a Rotating Platform-

Centrifugal force $\leq$ Force of friction

$m\omega^2 r \leq \mu m g$

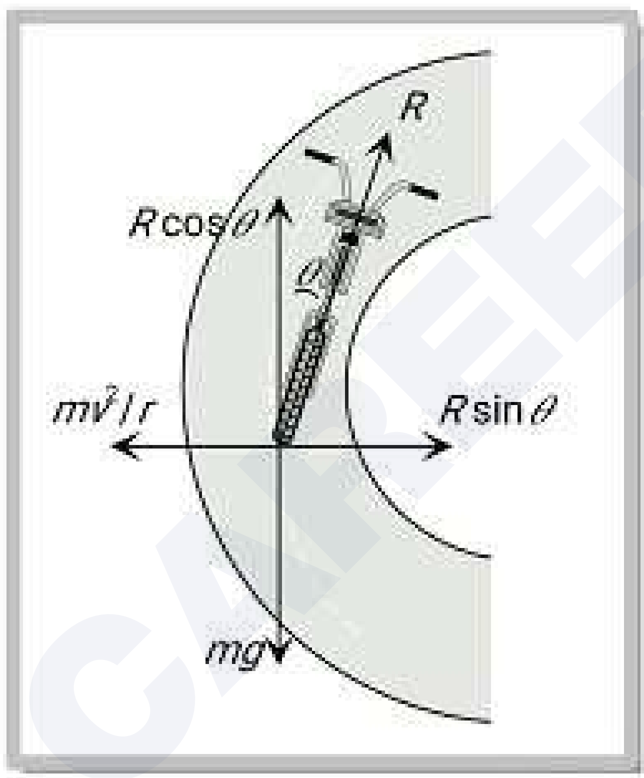
$\therefore\ \omega_{max} = \sqrt{\frac{\mu g}{r}}$ =It is the maximum angular velocity of rotation of the platform, so that object will not skid on it.

$\omega =$Angular velocity

r = radius

$\mu =$ coefficient of friction

21.Bending a Cyclist



From figure.

$R \sin\theta = \frac{mv^2}{r}$ (i)

$R \cos\theta = mg$ (ii)

(i) & (ii)

$tan\theta = \frac{v^2}{rg}$

$$\theta = tan^{-1}\left(\frac{v^2}{rg}\right)$$

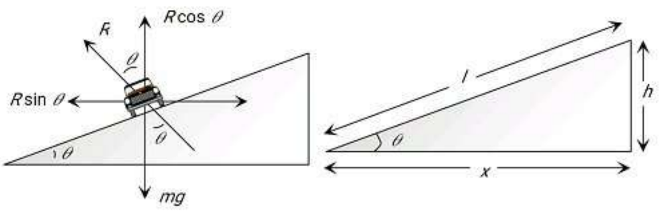
V = velocity

r = radius of track

θ = angle with which cycle leans

22. Banking of Road

1. **Without friction**



From figure

$R\cos\theta = mg$ (i)

$R\sin\theta = \frac{mv^2}{r}$ (ii)

$tan\theta = \frac{v^2}{rg}$

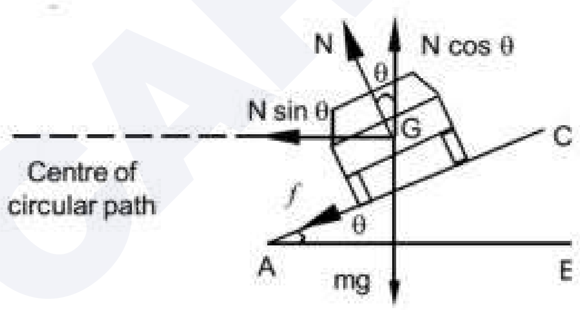
$tan\theta = \frac{\omega^2 r}{g} = \frac{V\omega}{g} = \frac{h}{l}$

h = height of outer edge from the ground level

l =width of the road

r = radius

2. If friction is also present



$$\frac{V^2}{rg} = \frac{\mu + tan\theta}{1 - \mu tan\theta}$$

Where θ = angle of banking

μ = coefficient of friction

V = velocity

- Maximum speed on a banked frictional road

$$V = \sqrt{\frac{rg(\mu + tan\theta)}{1 - \mu tan\theta}}$$

23. Force in non-uniform Circular Motion

$$F_c = ma_c = \frac{mv^2}{r} \qquad (\vec{F}_c \perp \vec{v})$$

$F_t = ma_t$

$$F_{net} = m\sqrt{a_c^2 + a_t^2}$$

m = mass

a_c= centripetal acceleration

a_t = tangential acceleration

F_c = centripetal force

Work Energy and Power

1. Work

Important Formulae

Work-

•

Work is said to be done when a force applied on the body displaces the body through a certain distance along the direction of the force.

- **Work done by a constant force-**

1. The scalar product of the force vector ($\overrightarrow{F}$) and the displacement vector ($\overrightarrow{S}$)

$$W = \vec{F}.\vec{S}$$

2. The product of the magnitude of force (F) magnitude of displacement (S) and cosine of the angle between them (Θ)

$$W = FS\cos\Theta$$

3. If the number of forces $\vec{F}_1, \vec{F}_2, \vec{F}_3 ......, \vec{F}_n$, are acting on a body and it shifts from position vector $\vec{r_1}$ to position vector $\vec{r_2}$

Then $W = \left(\vec{F}_1 + \vec{F}_2 + \vec{F}_3 ....+ \vec{F}_n\right) \cdot (\vec{r}_2 - \vec{r}_1) = \vec{F}_{net} \cdot \vec{r}_{net}$

4. Units-
   - SI Unit-Joule
   - CGS Unit- Erg
   - $1\ Joule = 10^7 Erg$
5. Dimension- ML^2T^{-2}
6. Dependence of work done by a constant force

Nature of Work Done-

1. **Positive Work-**
   - Positive work means that force (or its component) is parallel to displacement.
   - Means $0 \leq \Theta < \frac{\pi}{2}$

   Where Θ is the angle between force vectors and displacement vector
   - Maximum work = $W_{max} = FS,\ When\ \theta = 0^0$
   - E.g When you move a block by pulling it then work done by you on the block is positive
2. **Negative Work**
   - Negative work means that force (or its component) is opposite to displacement.

- Means $\frac{\pi}{2} < \Theta \leq \pi$

  Where Θ is the angle between force vectors and displacement vector

- Minimum work= $W_{min} = -FS,\ When\ \theta = 180^0$
- E.g When a body is made to slide over a rough surface, the work done by the frictional force is negative

3. **Zero work**

- Under three conditions, Work can be zero

  a. If the force is perpendicular to the displacement

  Means $\Theta = \frac{\pi}{2}$

  E.g-When a body moves in a circle the work done by the centripetal force is always zero.

  b. If there is no displacement (means s = 0)

  E.g- When a person tries to displace a wall by applying a force and can't able to move the wall

  So the work done by the person on the wall is zero.

  c. If there is no force acting on the body (means F=0)

  E.g-Motion of an isolated body in free space.

Work done by variable force-

- Force is a vector quantity. So it has a magnitude as well as direction. A variable force means when its magnitude or its direction or both varies with position.

  And work done by the variable force is given by -

$$W = \int \vec{F} \cdot \vec{ds}$$

  Where $\vec{F}$ is a variable force and $\vec{ds}$ is a small displacement

- **When Force is time-dependent**

  And we can write $\vec{ds} = \vec{v}dt$

  So, $W = \int \vec{F} \cdot \vec{v}\, dt$

  Where $\vec{F}$ and $\vec{v}$ are force and velocity vector at any instant.

- **Work Done Calculation by Force Displacement Graph**

  The area under the force-displacement curve with the proper algebraic sign represents work done by the force.

Work done by the frictional force-

1. **Work done by the frictional force is zero -**

   When the force applied on a body is insufficient to overcome the friction.

2. **Work done by the frictional force is negative**

   When the force is large enough to overcome the friction

3. **Work done by the frictional force is positive**

   When force is applied on a body, which is placed above another body ,the work done by the frictional force on the lower body maybe positive.

Work Done in Conservative and Non-Conservative Field-

1. **Conservative field-**

   - In the conservative field, work done by the force depends only upon the initial and final position.
   - In the conservative field, work done by the force does not depend on the path.
   - In the conservative field, work done by the force along a closed path is zero.

2. **Conservative force-**

- The forces of these type of fields are known as conservative forces.

  Example: Electrostatic forces, gravitational forces, the spring force

3. **Non-Conservative field-**

- In Non- conservative field, work done by the force depends on the path followed between any two positions/points.
- In Non-conservative field, work done by the force along a closed path is non-zero.

4. **Non-Conservative Force-**

- The forces corresponding to Non-Conservative field are known as non-conservative forces.
- Non-Conservative Force is dissipative.

  Example: Frictional force, Viscous force

2. Energy

Energy-

1. The energy of a body is defined as its capacity for doing work.
2. It is a scalar quantity
3. Dimension- ML^2T^{-2}
4. Unit-

   SI unit - Joule

   CGS - Erg

   and, $1 \text{ Joule} = 10^7 \text{ Erg}$

5. Mass energy equivalence-

   Einstein's special theory of relativity shows that material particle itself is a form of energy.

   The relation between the mass of a particle m and its equivalent energy is given as

   $$E = mc^2$$

   Where c = velocity of light in vacuum.

   E.g - If m =1kg then $E = 9 * 10^{16} J$

6. Various forms of energy
   - Mechanical energy (Kinetic and Potential)
   - Chemical energy
   - Electrical energy
   - Sound energy
   - Heat energy
   - Light energy

7. Transformation of energy-

   Conversion of energy from one form to another is possible through various devices and processes.

   Examples are -

   - Bulb- Electrical energy gets converted into light energy.
   - Speaker-Electrical energy gets converted into sound energy.
   - Heater- Electrical energy gets converted into heat energy

Mechanical Energy-

Mechanical energy is the sum of potential energy and kinetic energy. It is the energy associated with the motion and position of an object.

3. Kinetic energy

Kinetic energy-

The energy possessed by a body by virtue of its motion is called kinetic energy.

E.g-Moving vehicle possesses kinetic energy.

1. The expression for kinetic energy

$$K.E. = \frac{1}{2}mv^2$$

Where $m \rightarrow mass$

$v \rightarrow velocity$

2. Kinetic Energy is always positive.

3. Work-energy theorem-

Net work done by all the forces acting on a particle is equal to a change in its kinetic energy.

$$W = \frac{1}{2}mv^2 - \frac{1}{2}mv_0{}^2$$

$$W = k_f - k_i$$

Where $m = mass\ of\ the\ body$

$v_0 = initial\ velocity$

$v = final\ velocity$

This theorem is valid for a system in the presence of all types of forces (external or internal, conservative or non-conservative).

5. Relation of kinetic energy with linear momentum

$$k = \frac{mv^2}{2} \quad \text{..... (1)}$$

and, P = mv or v=p/m ..... (2)

put this value of v in equation (1)

Put we get $k = \frac{p^2}{2m}$

Where $p \rightarrow momentum$

$m \rightarrow mass$

4. Potential energy-

- **Definition-**

Potential energy is defined only for conservative forces.

In the space occupied by conservative forces, every point is associated with a certain energy which is called the energy of position or potential energy.

- **Change in potential energy -**

Change in potential energy between any two points is defined as the work done by the associated conservative force in displacing the particle between these two points without any change in kinetic energy.

$$U_i - U_f = \int_{r_i}^{r_f} \vec{f} \cdot d\vec{s} \quad \text{......(1)}$$

Where, $U_f - final\ potential\ energy$

$U_i - initial\ potential\ energy$

$f - force$

$ds - small\ displacement$

$r_i - initial\ position$

$r_f - final\ position$

We can define a unique value of potential energy only by assigning some arbitrary value to a fixed point called the reference point.

Whenever and wherever possible, we take the reference point at infinite and assume potential energy to be zero there.

i.e; if take $r_i = \infty$ and $r_f = r$ then from equation (1)

$$U_r = -\int_{\infty}^{r} \vec{f} \cdot d\vec{r} = -W$$

In the case of conservative force (field), potential energy is equal to the negative of work done in shifting the body from reference position to the given position.

- **Types of potential energy-**

Potential energy generally is of three types:

Elastic potential energy, Electric potential energy, and Gravitational potential energy

1. **Potential Energy stored when a particle displaced against gravity**

$$U = -\int f dx = -\int (mg)\, dx\, cos180^0$$

Where $m = mass\ of\ body$

$g = acceleration\ due\ to\ gravity$

$dx = small\ displacement$

2. **Potential Energy stored in the spring-**

- Restoring force = $f = -kx$ (or spring force)

  Where **k** is called the spring constant.

- Work done by restoring the force

$$W = -\frac{1}{2}kx^2$$

- Potential Energy

$$U = \frac{1}{2}kx^2$$

Where $K = spring\ constant$

$x = elongation\ or\ compression\ of\ spring\ from\ natural\ position.$

- **The relation between Conservative Force and Change in potential energy -**

For only conservative fields F equals the Negative of the rate of change of potential energy with respect to position.

$$F = \frac{-dU}{dr}$$

The three-dimensional formula for potential energy-

For only conservative fields F equals the negative gradient $(-\vec{\nabla})$ of the potential energy.

$$F = -\vec{\nabla}U$$

Where $\vec{\nabla}$ is del operator

And, $\vec{\nabla} = \frac{d}{dx}\vec{i} + \frac{d}{dy}\vec{j} + \frac{d}{dz}\vec{k}$

So, $F = -[\frac{dU}{dx}\vec{i} + \frac{dU}{dy}\vec{j} + \frac{dU}{dz}\vec{k}]$

Where $\frac{dU}{dx}$ =Partial derivative of U w.r.t. x (keeping y and z constant)

$\frac{dU}{dy}$ =Partial derivative of U w.r.t. y (keeping x and z constant)

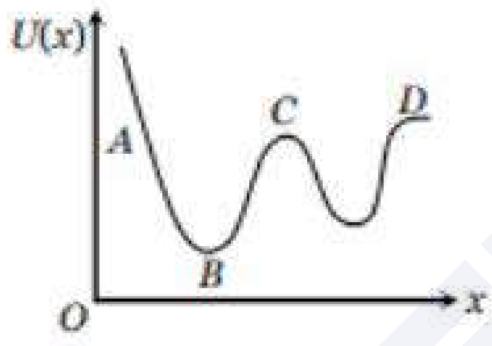
$\frac{dU}{dz}$ =Partial derivative of U w.r.t. Z (keeping x and y constant)

- **Potential energy curve**

A graph plotted between the potential energy of a particle and its displacement from the center of force is called a potential energy curve.

The figure shows a graph of the potential energy function U(x) for one-dimensional motion. As we know that negative gradient of the potential energy gives force.

$$-\frac{dU}{dx} = F$$



- **Nature of force-**

1. **Attractive force -**

- If $\frac{dU}{dx}$ is positive (means on increasing x, U is increasing)

  Then F is negative in direction i.e. force is attractive in nature.

- In the graph, this is represented in region BC.

2. **Repulsive force-**

- If $\frac{dU}{dx}$ is negative (means on increasing x, U is decreasing)

  Then F is positive in direction i.e. force is repulsive in nature.

- In the graph, this is represented in the region AB.

3. **Zero force**

- If $\frac{dU}{dx}$ is zero (means on increasing x, U is not changing) then F is zero
- Points B, C, and D represent the point of zero force.
- These points can be termed as a position of equilibrium.

- **Types of equilibrium**

If the net force acting on a particle is zero, it is said to be in equilibrium.

Means For equilibrium $\frac{dU}{dx} = 0$

Equilibrium of particles can be of three types-

1. **Stable equilibrium**

- When a particle is displaced slightly from a position, then a force acting on it brings it back to the initial position, it is said to be in the stable equilibrium position.
- $\frac{d^2U}{dx^2} > 0$ is positive.

i.e; the rate of change of $\frac{dU}{dx}$ is positive

- Potential energy is minimum.
- A marble is placed at the bottom of a hemispherical bowl.

2. **Unstable equilibrium**

- When a particle is displaced slightly from a position, then a force acting on it tries to displace the particle further away from the equilibrium position, it is said to be in unstable equilibrium.
- $\frac{d^2U}{dx^2}$ is negative

i.e; rate of change of $\frac{dU}{dx}$ is negative

- Potential energy is maximum.
- A marble balanced on top of a hemispherical bowl.

3. **Neutral equilibrium**

- When a particle is slightly displaced from a position then it does not experience any force acting on it and continues to be in equilibrium in the displaced position, it is said to be in neutral equilibrium.
- $\frac{d^2U}{dx^2} = 0$

i.e; the rate of change of $\frac{dU}{dx}$ is zero.

- Potential energy is constant.
- A marble is placed on a horizontal table.

5. Conservation of Energy-

1. Conservation of Mechanical Energy

Mechanical energy is the sum of potential energy and kinetic energy.

According to Conservation of Mechanical Energy, If only conservative forces act on a system,

The total mechanical energy remains constant.

By work-energy theorem, we have $W = k_f - k_i$ or $\Delta K = \int_{r_i}^{r_f} \vec{f}.d\vec{s}$ ....(1)

And the change in potential energy in a conservative field is $U_i - U_f = \int_{r_i}^{r_f} \vec{f} \cdot d\vec{s}$

Or, $-\triangle U = \int_{r_i}^{r_f} \vec{f} \cdot d\vec{s}$ ....(2)

From equation (1) and (2)

We get, $\Delta K = -\Delta U$

$\Delta K + \Delta U = 0$

Means, $K + U = E\,(constant)$

Or, E is constant in a conservative field i.e.; if the kinetic energy of the body increases its potential energy will decrease by an equal amount and vice versa.

2. **Law of conservation of total energy-**

If some non-conservative force like friction is also acting on the particle, the mechanical energy is no more constant.

It changes by the amount of work done by non-conservative forces.

i.e; $\Delta K + \Delta U = \triangle E = W_{fnc}$

The lost energy is transformed into heat or in other forms of energy. But the total energy remains constant.

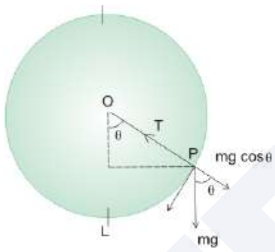
So, according to the Law of conservation of total energy "Energy may be transformed from one kind to another but it cannot be created or destroyed. The total energy in an isolated system is constant."

6. Vertical circular motion

- **This is an example of non-uniform circular motion.**

A particle of mass m is attached to a light and inextensible string. The other end of the string is fixed at O and the particle moves in a vertical circle of radius r is equal to the length of the string as shown in the figure.

- **Tension at any point on the vertical loop**



Consider the particle when it is at the point P and the string makes an angle θ with vertical.

Forces acting on the particle are:

T = tension in the string along its length,

And, mg = weight of the particle vertically downward.

Hence, the net radial force on the particle is

$$F_r = T - mgcos\theta$$

And, $$F_r = \frac{mv^2}{r}$$

Where r = length of the string

So, $$\frac{mv^2}{r} = T - mgcos\theta$$

Or, Tension at any point on the vertical loop

$$T = \frac{mv^2}{r} + mgcos\theta$$

Since the speed of the particle decreases with height,

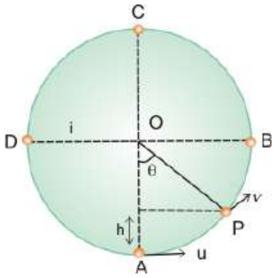
Hence, tension is maximum at the bottom, where cos θ = 1 (as θ = 0).

$$T_{max} = \frac{mv_{Bottom}^2}{r} + mg$$

Similarly,

$$T_{min} = \frac{mv_{Top}^2}{r} - mg$$

- **Velocity at any point on the vertical loop-**



If u is the initial velocity imparted to the body at the lowest point then, the velocity of the body at height h is given by

$$v = \sqrt{u^2 - 2gh} = \sqrt{u^2 - 2gr(1 - cos\theta)}$$

- **Velocity at the lowest point (A) for the various conditions in Vertical circular motion.**

1. Tension in the string will not be zero at any of the points and the body will continue the circular motion.

$$u_A > \sqrt{5gr}$$

2. Tension at highest point C will be zero and the body will just complete the circle.

$$u_A = \sqrt{5gr}$$

3. A particle will not follow the circular motion. Tension in the string becomes zero somewhere between points B and C whereas velocity remains positive. Particle leaves the circular path and follows a parabolic trajectory

$$\sqrt{2gr} < u_A < \sqrt{5gr}$$

4. Both velocity and tension in the string become zero between A and B and the particle will oscillate along a semi-circular path.

$$u_A = \sqrt{2gr}$$

5. The velocity of the particle becomes zero between A and B but the tension will not be zero and the particle will oscillate about the point A.

$$u_A < \sqrt{2gr}$$

- **Critical Velocity-**

It is the minimum velocity given to the particle at the lowest point to complete the circle.

$$u_A = \sqrt{5gr}$$

7. Power-

1. **Definition-**

Power is defined as the rate at which work is done or energy is transferred.

2. **Dimension** -

$$ML^2T^{-3}$$

2. **Units**-

- SI- Watt or Joule/sec
- CGS- Erg/sec

4. **Average power**-

$$P_{av} = \frac{\Delta w}{\Delta t} = \frac{\int_0^t p \cdot dt}{\int_0^t dt}$$

5. **Instantaneous power-**

$$P = \frac{dw}{dt} = P = \vec{F} \cdot \vec{v}$$

Where, $\vec{F} \rightarrow force$

$\vec{v} \rightarrow velocity$

i.e. power is equal to the scalar product of force with velocity

6. **Power expressed as the rate of change of kinetic Energy**

$$P = \frac{dk}{dt}$$

Where, $dk \rightarrow change\ in\ kinetic\ energy$

$dt \rightarrow interval\ of\ time$

8. Collision

The interaction between two or more objects is called a collision.And during this interaction strong force acts between two or more bodies for a short time as a result of which the energy and momentum of the interacting particle change.

- **Stages of collision-**

There are three distinct identifiable stages in collision

1. Before the collision.-

   The interaction forces are zero

2. During the collision-

   The interaction forces are very large and this forces act for a very short time. And because of this interaction forces the energy and momentum of the interacting particle change.

3. After the collision-

   The interaction forces are zero

- **Momentum and energy conservation in collision**

The magnitude of the interacting force is often unknown, therefore, Newton's second law cannot be used. But the law of conservation of momentum is useful in relating the initial and final velocities.

1. Momentum conservation-

   In a collision the effect of external forces such as gravity or friction is not taken into account as due to small duration of collision (t) average impulsive force responsible for collision is much larger than external force acting on the system and since this impulsive force is 'Internal' therefore the total momentum of the system always remains conserved.

2. Energy conservation-

   In a collision 'total energy' is also always conserved. Here total energy includes all forms of energy such as mechanical energy, internal energy, excitation energy, radiant energy, etc.

   But in a collision Kinetic energy may or may not be conserved.

- **Coefficient of restitution-**

  The ratio of the relative velocity of separation to the relative velocity of approach.

$$e = \frac{v_2 - v_1}{u_1 - u_2} = \frac{Relative\ velocity\ of\ separation}{Relative\ velocity\ of\ approach}$$

Types of collision-

1. **On the basis of conservation of kinetic energy**

   a. Perfectly elastic collision

   - In this collision, $(K.E)_{initial} = (K.E.)_{final}$
   - Coefficient of restitution e = 1
   - Ex- Bouncing of ball with same velocity after the collision with ground.

   b. Inelastic collision

- In this collision $(K.E)_{initial} \neq (K.E.)_{final}$
- Coefficient of restitution 0 < e < 1
- As $(K.E)_{initial} > (K.E.)_{final}$

  The loss in kinetic energy appears in other forms, such as heat , sound etc.

  Ex- Collision between two billiard balls. All majority of collision belong to this category.

c. Perfectly inelastic collision.

- If in a collision two bodies stick together or move with same velocity after the collision, the collision is said to be perfectly inelastic.
- Coefficient of restitution e = 0
- Ex-Collision between a bullet and a block of wood is an example of perfectly inelastic collision, if after collision the bullet remains embedded in the block, and block and bullet move together.

2. **On the basis of the direction of colliding bodies**

a. Head-on or one-dimensional collision

- In a head-on collision the motion of colliding particles before and after the collision is along the same line .

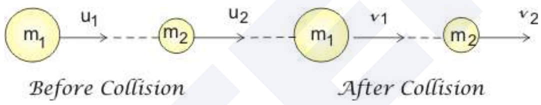
b. Oblique collision

- If directions of motion of colliding particles after collision is not along the initial line of motion of colliding particles , then the collision is called oblique.
- Example : Collision of billiard balls.

Perfectly Elastic Head on Collision-

- **In Perfectly Elastic Collision,**

Law of conservation of momentum and that of Kinetic Energy hold good.



$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 \quad ....(1)$$

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \quad ....(2)$$

$m_1, m_2 : masses$

$u_1, v_1 : initial\ and\ final\ velocity\ of\ the\ mass\ m_1$

$u_2, v_2 : initial\ and\ final\ velocity\ of\ the\ mass\ m_2$

From equation (1) and (2)

We get, $u_1 - u_2 = v_2 - v_1$ .....(3)

Or, we can say Relative velocity of approach = Relative velocity of separation

And $e = \frac{v_2 - v_1}{u_1 - u_2} = \frac{Relative\ velocity\ of\ separation}{Relative\ velocity\ of\ approach}$

So in Perfectly Elastic Collision

e = 1 ,

From equations (1),(2), (3)

We get

$$v_1 = (\frac{m_1 - m_2}{m_1 + m_2})u_1 + \frac{2m_2u_2}{m_1 + m_2} \quad ......(4)$$

Similarly, $v_2 = (\frac{m_2 - m_1}{m_1 + m_2})u_2 + \frac{2m_1u_1}{m_1 + m_2}$...... (5)

- **Special cases of head-on elastic collision**

1. Equal mass in case of perfectly elastic collision

Then, $v_1 = u_2 \ and \ v_2 = u_1$

Or, Velocity mutually interchange

2. If a massive projectile collides with a light target (i.e $m_1 >> m_2$)

Since $m_1 >> m_2$ so we use $m_2 = 0$

Putting $m_2 = 0$ in equation (4) and (5)

We get $v_1 = u_1 \ and \ v_2 = \ 2u_1 - u_2$

3. If the target particle is massive in case of elastic collision (i.e; $m_2 >> m_1$)

Since $m_2 >> m_1$

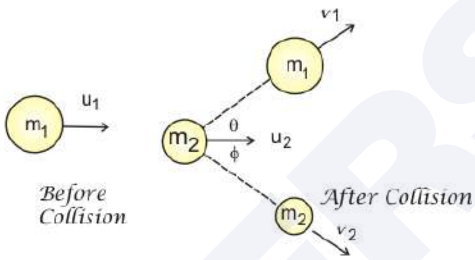
So, the lighter particle recoil with same speed and the massive target particle remain practically at rest.

i.e; $\bar{v}_2 = \bar{u}_2$

$\bar{v}_1 = -\bar{u}_1$

Perfectly elastic oblique collision-

- Let two bodies moving as shown in figure.



By law of conservation of momentum

Along x-axis-

$m_1u_1 + m_2u_2 = m_1v_1cos\theta + m_2v_2cos\phi$ ….. (1)

Along y-axis-

$0 = m_1v_1sin\theta - m_2v_2sin\phi$ …..(2)

By law of conservation of kinetic energy

$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$ ….(3)

And In Perfectly Elastic Oblique Collision

Value of e=1

So along line of impact (here along in the direction of v_2)

We apply e=1

And we get $e = 1 = \frac{v_2 - v_1cos(\theta + \phi)}{u_1cos\phi - u_2cos\phi}$ ….. (4)

So we solve these equations (1),(2),(3),(4) to get unknown.

- **Special condition**

$if \ m_1 = m_2 \ and \ u_2 = 0$

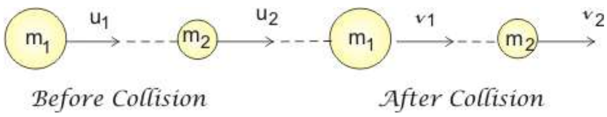
Then, from equation (1), (2) and (3)

We get, $\theta + \phi = \frac{\pi}{2}$

i.e; after perfectly elastic oblique collision of two bodies of equal masses (if the second body is at rest), the scattering angle $\theta + \phi$ would be 9(
.

Head on inelastic collision-

1. **In Inelastic Collision Law of conservation of momentum hold good but kinetic energy is not conserved .**



$$\frac{1}{2}m_1u_1^2+\frac{1}{2}m_2u_2^2\neq\frac{1}{2}m_1v_1^2+\frac{1}{2}m_2v_2^2$$

$m_1u_1+m_2u_2=m_1v_1+m_2v_2$ …… (1)

$m_1, m_2 : masses$

$u_1, v_1 : initial\ and\ final\ velocities\ of\ mass\ m_1$

$u_2, v_2 : initial\ and\ final\ velocities\ of\ mass\ m_2$

2. **In inelastic collision (0 < e < 1)**

$e=\frac{v_2-v_1}{u_1-u_2}$ ….. (2)

From equations (1),(2)

We get,

$$v_1=\left(\frac{m_1-em_2}{m_1+m_2}\right)u_1+\frac{(1+e)m_2}{m_1+m_2}u_2 \quad ……(3)$$

Similarly, $v_2=\left(\frac{m_2-em_1}{m_1+m_2}\right)u_2+\frac{(1+e)m_1}{m_1+m_2}u_1$ …… (4)

3. **Special case**

A sphere of mass m moving with velocity u hits inelastically with another stationary sphere of same mass.

As, $e=\frac{v_2-v_1}{u_1-u_2}$

So, $e=\frac{v_2-v_1}{u}$ or, $ue=v_2-v_1$ ....(5)

By conservation of momentum

As, $m_1u_1+m_2u_2=m_1v_1+m_2$

So $v_2+v_1=u$ ….(6)

From equation (5) and (6)

We get, $\frac{v_1}{v_2}=\frac{1-e}{1+e}$

4. **Loss in kinetic energy**

Loss in K.E = Total initial kinetic energy – Total final kinetic energy

$$\triangle K.E.=\left(\frac{1}{2}m_1u_1^2+\frac{1}{2}m_2u_2^2\right)-\left(\frac{1}{2}m_1v_1^2+\frac{1}{2}m_2v_2^2\right) \quad …. (7)$$

From equation (3) , (4) and (7)

We can write, Loss in kinetic energy in terms of **e** as

$$\triangle K.E.=\frac{1}{2}\left(\frac{m_1m_2}{m_1+m_2}\right)(1-e^2)(u_1-u_2)^2$$

Perfectly inelastic collision-

In Perfectly Inelastic Collision - Two bodies stick together after the collision ,so there will be a final common velocity (v)

1. **When the colliding bodies are moving in the same direction**

- By the law of conservation of momentum

$$m_1u_1 + m_2u_2 = (m_1 + m_2)v$$

$$v = \frac{m_1u_1 + m_2u_2}{(m_1 + m_2)}$$

- Loss in kinetic energy

$$\Delta K.E = \left(\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2\right) - \left(\frac{1}{2}(m_1 + m_2)V^2\right)$$

$$\Delta K.E = \frac{1}{2}\left(\frac{m_1m_2}{m_1 + m_2}\right)(u_1 - u_2)^2$$

2. **When the colliding bodies are moving in the opposite direction**

- By the law of conservation of momentum

$$m_1u_1 + m_2(-u_2) = (m_1 + m_2)v$$

$$v = \frac{m_1u_1 - m_2u_2}{m_1 + m_2}$$

If v is positive then the combined body will move along the direction of motion of mass m_1

If v is negative then the combined body will move in a direction opposite to the motion of mass m_1

- Loss in kinetic energy

$$\Delta K.E = \left(\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2\right) - \left(\frac{1}{2}(m_1 + m_2)V^2\right)$$

$$\Delta K.E = \frac{1}{2}\left(\frac{m_1m_2}{m_1 + m_2}\right)(u_1 + u_2)^2$$

Rotational Motion

Important Formulae

1. Rigid body rotation

- **Rigid body-**

  It is defined as a system of particles in which the distance between each pair of particles remains constant.

  This means the shape & size do not change during the motion.

- **Translation motion-**

  If a body is moving such that a line drawn between any two of its internal points remain parallel to itself.

  All the particles of the body move along parallel paths.

  All the particles of the body follows 1 D motion.

  Example- Motion of a body along a straight line.

- **Rotational motion-**

  A rigid body is said to be in pure rotation if every particle of the body moves in a circle and centre of all the circles lie on a straight line called the axis of rotation.

  The line joining any two internal points does not remain parallel.

  Example-motion of wheels, gears, motors.

- **Some important terms-**

  1. System-

     A system is a collection of any number of particles interacting with one another and are under observation for analysing the situation.

  2. Internal forces-

Internal forces are all the forces exerted by various particles of the system on one another. Internal forces between two particles are equal in magnitude and opposite in direction.

3. External forces-

External forces are the forces that we have to apply on the object/system from outside to move or stop the object/system.

2.Center of mass

1. **Definition-**

- Centre of mass of a body is defined as a single point at which the whole mass of the body or system is imagined to be concentrated and all external forces are applied there.
- It is the point where if a force is applied it moves in the direction of the force without rotating.

2. **x, y, and z coordinates of the centre of mass**

- For a system of N discrete particles

$$x_{cm} = \frac{m_1x_1 + m_2x_2.........}{m_1 + m_2.......}$$

$$y_{cm} = \frac{m_1y_1 + m_2y_2 + m_3y_3.........}{m_1 + m_2 + m_3.......}$$

$$z_{cm} = \frac{m_1z_1 + m_2z_2 + m_3z_3.........}{m_1 + m_2 + m_3.......}$$

Where m_1, m_2.......... are mass of each particle and x_1, x_2 ....y_1, y_2 .... z_1, z_2 are respectively x, y, & z coordinates of particles.

- It is the unique point where the weighted relative position of the distributed mass sums to zero.
- Centre of Mass of a continuous Distribution

$$x_{cm} = \frac{\int xdm}{\int dm}, \quad y_{cm} = \frac{\int ydm}{\int dm}, \quad z_{cm} = \frac{\int zdm}{\int dm}$$

Where dm is mass of small element. x, y, z are the coordinates of dm part.

3. **Important points about position of centre of mass**

- Its position is independent of the coordinate system chosen.
- Its position depends upon the shape of the body and distribution of mass.

  And depending on this it may lies inside of the body as well as outside the body.
- For symmetrical bodies having the homogenous distribution of mass ,the centre of mass coincides with the geometrical centre of the body.
- It changes its position only under the translatory motion whereas there is no effect on its position because of rotatory motion of the body.

4. **Centre of gravity-**

- Centre of gravity of a body is a point, through which the resultant of all the forces experienced by various particles of the body due to the attraction of earth, passes irrespective of the orientation of the body.
- If the body is located in a uniform gravitational field,then the centre of mass coincides with the centre of gravity of body, and if not then its centr of mass and centre of gravity will be at two different locations.

5. For a 2-dimensional body with uniform negligible thickness formulae for finding the position of the centre of mass can be rewritten a

$$r_{cm} = \frac{m_1\vec{r_1} + m_2\vec{r_2}....}{m_1 + m_2....} = \frac{\rho A_1 t\vec{r_1} + \rho A_2 t\vec{r_2}....}{\rho A_1 t + \rho A_2 t....} = \frac{A_1\vec{r_1} + A_2\vec{r_2}....}{A_1 + A_2....}$$

Where, $m = \rho.A.t$

6. Centre of mass when some mass is added in the body

$$\vec{r}_{cm} = \frac{m_1\vec{r_1} + m_2\vec{r_2}}{m_1 + m_2}$$

Where m_1 & $\vec{r_1}$ are mass and position of the centre of mass for the whole body. m_2 & $\vec{r_2}$ are mass and position of the centre of mass of added mass.

7. Position of centre of mass when some mass is removed

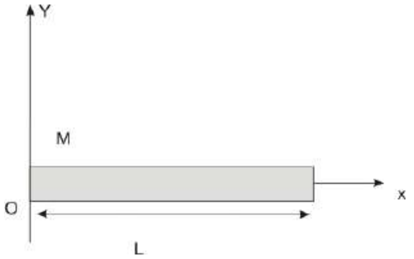
$$\vec{r}_{cm} = \frac{m_1\vec{r_1} - m_2\vec{r_2}}{m_1 - m_2}$$

Where $m_1$is value of whole mass and $\vec{r_1}$is position of centre of mass for whole mass.Similarly m_2 & $\vec{r_2}$ are values for mass which has been removed.

3. Center of mass of various bodies

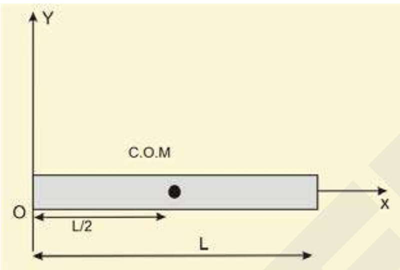
1.The uniform rod

Suppose a rod of mass M and length L is lying along the x-axis with its one end at x = 0 and the other at x = L

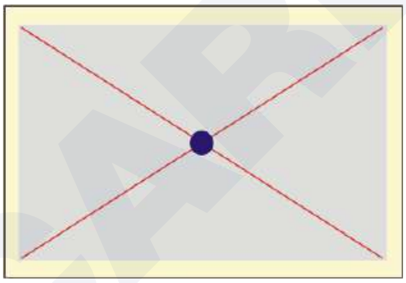


So the coordinates of COM of the rod are $(\frac{L}{2}, 0, 0)$

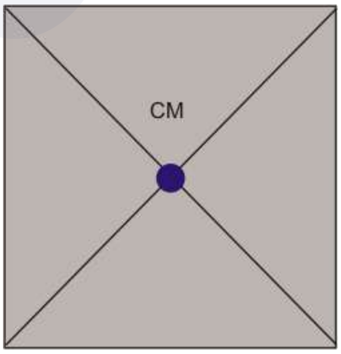
Means it lies at the centre of the rod.



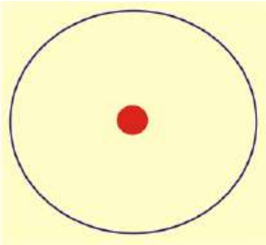
2. Rectangular plate



3. Square plate

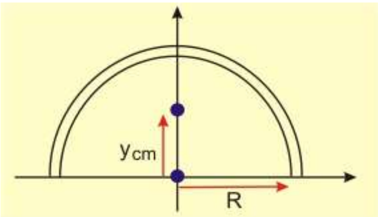


4. Circular plate



5. Semicircular ring-

Have a look at the figure of the semicircular ring.



Since it is symmetrical about the y-axis on both sides of the origin

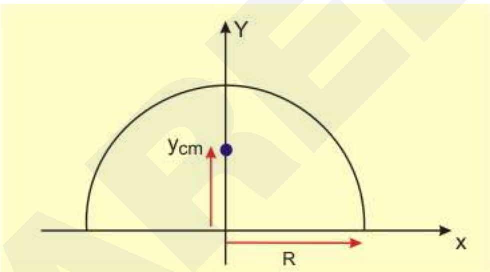
So we can say that its $x_{cm} = 0$

And its $z_{cm} = 0$ as z-coordinate is zero for all particles of semicircular ring.

and $y_{cm} = \frac{2R}{\pi}$

6. Semicircular disc

Have a look at the figure of the semicircular disc



Since it is symmetrical about y-axis on both sides of the origin

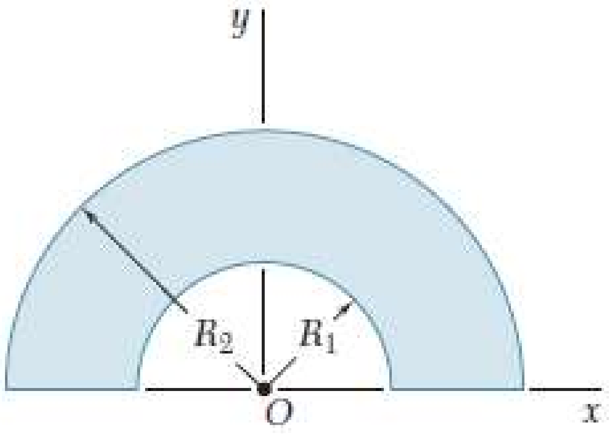
So, we can say that its $x_{cm} = 0$

And its $z_{cm} = 0$ as z-coordinate is zero for all particles of semicircular ring.

And $y_{cm} = \frac{4R}{3\pi}$

7.Semicircular annular ring

Have a look at the figure of the semicircular annular ring



It has inner radius as R_1 and outer radius as R_2 and centre as O

Since it is symmetrical about y-axis on both sides of the origin

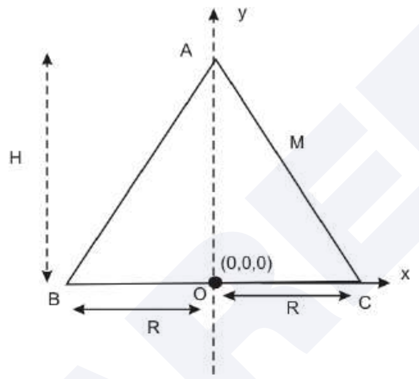
So we can say that its $x_{cm} = 0$

And its $z_{cm} = 0$ as z-coordinate is zero for all particles of semicircular ring.

And $y_{cm} = \frac{4}{3\pi} \times \frac{(R_2^3 - R_1^3)}{(R_2^2 - R_1^2)}$

8. Triangular plate

Have a look at the figure of A triangular plate as shown in figure.



Since it is symmetrical about y-axis on both sides of the origin

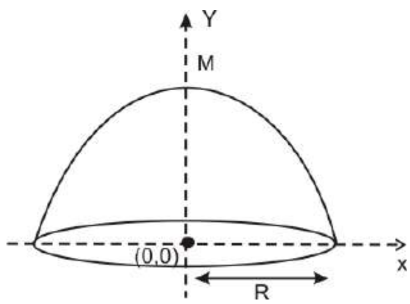
So we can say that its $x_{cm} = 0$

And its $z_{cm} = 0$ as z-coordinate is zero for all particles of semicircular ring.

and $\mathbf{y_{cm} = \frac{H}{3}}$ from base

9. Hollow Hemisphere

Have a look at the figure of Hollow Hemisphere



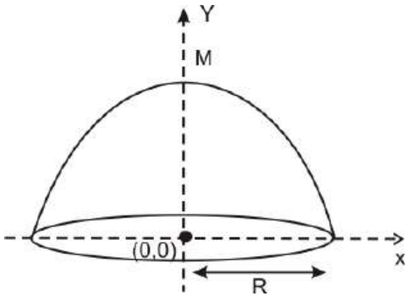
Since it is symmetrical about y-axis

So we can say that its $x_{cm} = 0$ and $z_{cm} = 0$

and $y_{cm} = \frac{R}{2}$ *from base*

10.Solid Hemisphere

Have a look at the figure of solid Hemisphere



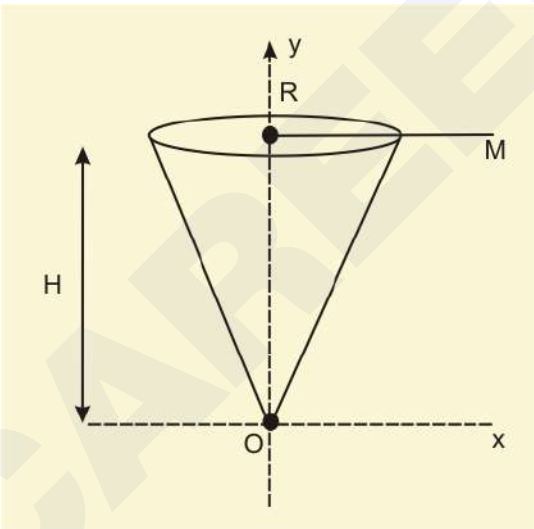
Since it is symmetrical about y-axis

So we can say that its $x_{cm} = 0$ and $z_{cm} = 0$

and $y_{cm} = \frac{3R}{8}$ from base

11. Hollow Cone

Have a look at the figure of Hollow Cone



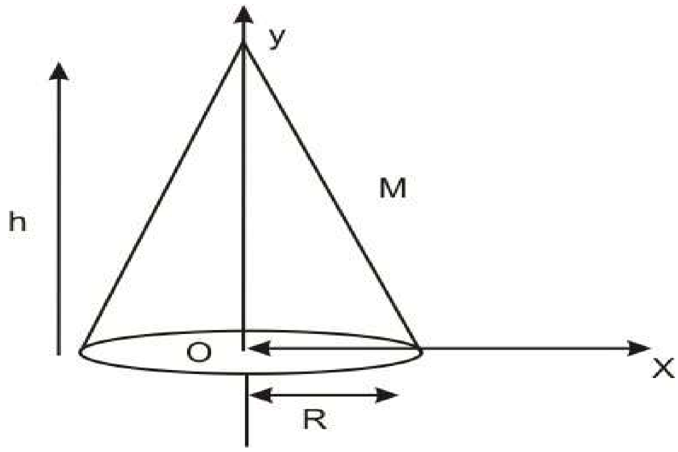
Since it is symmetrical about y-axis

So we can say that its $x_{cm} = 0$ and $z_{cm} = 0$

And $\mathbf{y_{cm}} = \frac{\mathbf{2H}}{\mathbf{3}}$ from O.

12. Solid cone

Have a look at the figure of a solid cone

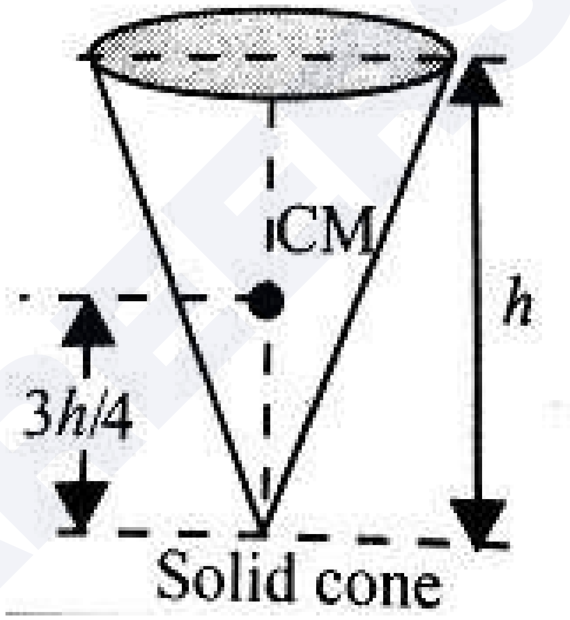


Since it is symmetrical about y-axis

So we can say that its $x_{cm} = 0$ and $z_{cm} = 0$

And, $\mathbf{y_{cm}} = \frac{\mathbf{H}}{\mathbf{4}}$ from bottom O

Or, Centre of Mass of a solid cone will lie at distance $\frac{3h}{4}$ from the tip of the cone.



4. Motion of the centre of mass

1. **Velocity of the centre of mass**

$$\vec{v}_{CM} = \frac{m_1\vec{v_1} + m_2\vec{v}_2........}{m_1 + m_2........}$$

m_1, m_2 ------- are mass of all the particles $\vec{v_1}, \vec{v}_2 \cdots\cdots$ are velocities of all the particles.

Similarly momentum of the system = $P_{sys} = M v_{cm}$

2. **Acceleration of centre of mass**

$$\vec{a}_{CM} = \frac{m_1\vec{a_1} + m_2\vec{a}_2........}{m_1 + m_2........}$$

m_1, m_2 are mass of all the particles $\vec{a_1}$, $\vec{a_2}\cdots$ are their respective acceleration.

Similarly Net force on the system = $F_{net} = Ma_{cm}$

And $F_{net} = \vec{F_{ext}} + \vec{F_{int}}$

And we know that both the action and reaction of an internal force must be within the system. In this way, vector summation will cancel all internal forces and hence net internal force on the system is zero.

So $\vec{F_{net}} = M\vec{a_{cm}}$

3. **If External Force = 0**

$$\vec{F}_{ext} = 0 \Rightarrow M\vec{a}_{cm} = 0 \Rightarrow \vec{a}_{cm} = 0$$

$$if\ \vec{a}_{cm} = 0 \Rightarrow v_{cm} = constant$$

If $v_{cm} = constant \Rightarrow P_{sys} = constant$

So it implies that the total momentum of the system must remain constant.

i.e. if no external force is acting on the system, the net momentum of the system remains constant. This is nothing but the principle of conservation of momentum in absence of external forces. Which says ìf resultant external force is zero on the system, then the net momentum of the system must remai constant.

- **Special case**

If External Force = 0 and Velocity of Centre of Mass = 0

Then centre of mass remains at rest. Individual components of a system may move and have non zero momentum due to mutual forces but the net momentum of the system remains zero.

5.Equations of Linear Motion and Rotational Motion.

	Linear Motion	Rotational Motion
I	If linear acceleration =a=0 Then u = constant and s = u t.	If angular acceleration=$\alpha = 0$ Then $\omega = constant$ and $\theta = \omega.t$
II	If linear acceleration= a = constant 1. $a = \frac{v-u}{t}$ 2. $v = u + at$ 3. $s = ut + \frac{1}{2}at^2$ 4. $s = \frac{v+u}{2} * t$ 5. $v^2 - u^2 = 2as$ 6. $S_n = u + \frac{a}{2}(2n-1)$	If angular acceleration=$\alpha = constant$ 1. $\alpha = \frac{\omega_f - \omega_i}{t}$ 2. $\omega_f = \omega_i + \alpha.t$ 3. $\theta = \omega_i.t + \frac{1}{2}.\alpha.t^2$ 4. $\theta = \frac{\omega_f + \omega_i}{2} * t$ 5. $\omega_f^2 - \omega_i^2 = 2\alpha\theta$ 6. $\theta_n = \omega_i + \frac{\alpha}{2}(2n-1)$
III	If linear acceleration= a $\neq$ constant 1. $v = \frac{dx}{dt}$ 2. $a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$ 3. $v.dv = a.ds$	If angular acceleration=$\alpha \neq constant$ 1. $\omega = \frac{d\theta}{dt}$ 2. $\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$ 3. $\omega.d\omega = \alpha.d\theta$

- Relation between linear and angular properties

1. $\vec{S} = \theta \times \vec{r}$

2. $\vec{v} = \omega \times \vec{r}$

3. $\vec{a} = \alpha \vec{\times} \vec{r}$

6. Torque

- Vector product of Force vector and position vector is known as **torque.**
- $\vec{\tau} = \vec{r} \times \vec{F}$
- Its direction is always perpendicular to the plane containing vector r and F and with the help of right hand screw rule we can find it.
- The magnitude of torque is calculated by using either
- $\tau = r_1 F \; or \; \tau = r \cdot F_1$

  r_1 = perpendicular distance from origin to the line of force.

  F_1 = component of force perpendicular to line joining force.
- $\tau = r.F.sin\phi$

  Where ϕ is the angle between vector r and F
- $\tau_{max} = r.F \; (when \; \phi = 90^0)$
- $\tau_{min} = 0 \; (when \; \phi = 0^0)$
- SI Unit- Newton-metre
- Dimension- ML^2T^{-2}
- If a body is acted upon by more than one force, then we get the resultant torque by doing vector sum of each torque.

  $\tau = \tau_1 + \tau_2 + \tau_3......$
- Just like force is the cause of translatory motion similarly Torque is the cause of rotatory motion.

7. Rotational Equilibrium

For Translational equilibrium $\sum \vec{F} = 0$

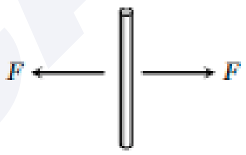
And For Rotational equilibrium $\sum \vec{\tau} = 0$

- For rotational equilibrium of system the resultant torque acting on it must be zero.

  i.e., $\sum \tau = 0$
- **Various cases of equilibrium**

  1. $\sum \vec{F} = 0$ and $\sum \vec{\tau} = 0$

  Forces are equal and act along the same line.

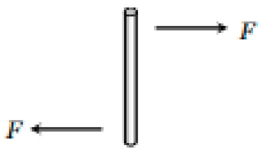


  Body will be in both Translational and Rotational equilibrium.

  i.e., It will remain stationary if initially it was at rest.

  2. $\sum \vec{F} = 0$ and $\sum \tau \neq 0$

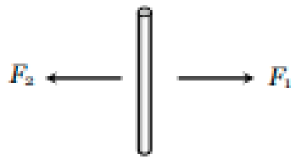
  Forces are equal and does not act along the same line.



Rotation of body will happen i.e. spinning of body.

3. $\sum F \neq 0$ and $\sum \vec{\tau} = 0$

Forces are unequal and act along the same line.

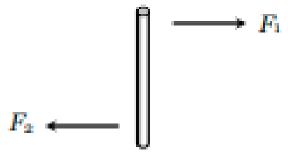


Body will be in Translational motion.

i.e., slipping of body

4. $\sum F \neq 0$ and $\sum \tau \neq 0$

Forces are unequal and does not act along the same line.



Body will be in both Rotation and translation motion.

i.e. rolling of a body.

8.Couple Force-

1. A couple is defined as combination of two equal and oppositely directed force but not acting along the same line.

i.e., $\sum \vec{F} = 0$ and $\sum \tau \neq 0$

2. Torque by a couple is given by

$\vec{\tau} = \vec{r} \times \vec{F}$

3. In case of couple both the forces are externally applied .

4. Work done by torque in twisting the wire is given by

$$W = \frac{1}{2} C.\theta^2$$

Where C is the coefficient of twisting

9. Moment of inertia

1. **Definition**

- Moment of inertia (I) of a body is a measure of its ability to resist change in its rotational state of motion.
- Moment of inertia play the same role in rotatory motion as is played by mass in translatory motion .

2. **Formula**

- Moment of inertia of a particle

$$I = mr^2$$

Where m is the mass of particle and r is the perpendicular distance of particle from rotational axis.

- Moment of inertia for system of particle

$$I = m_1r_1^2 + m_2r_2^2 + ...m_nr_n^2$$
$$= \sum_{i=1}^{n} m_ir_i^2$$

(This is Applied when masses are placed discreetly)

- Moment of inertia for continuous body

$$I = \int r^2 dm$$

Where r is the perpendicular distance of a particle of mass dm of rigid body from axis of rotation

3. **Dimension** = $[ML^2]$

4. **S.I. unit** = $kg - m^2$

5. It depends on mass, distribution of mass and on the position of the axis of rotation.

6. It does not depend on angular velocity, angular acceleration, torque, angular momentum and rotational kinetic energy.

7. **It is a tensor quantity.**

8. **Radius of gyration (K)-**

Radius of Gyration of a body about an axis is the effective distance from the axis where the whole mass can be assumed to be concentrated so that moment of inertia remains the same.

- Formula- $K = \sqrt{\frac{I}{M}}$

  Or, $I = MK^2$

- It does not depend on the mass of body
- It depends on the shape and size of the body, distribution of mass of the body w.r.t. the axis of rotation etc.
- Dimension- $M^oL^1T^o$
- S.I. unit: Meter.

8. Moment of Inertia of Two Point Masses About Their Centre of Mass

Let two masses m_1 and m_2 at a distance r and from their centre of mass they are at a distance r_1 and r_2 respectively.

Then,

1. $r_1 + r_2 = r$ .....(1)
2. $m_1r_1 = m_2r_2$ .....(2)
3. From equation (1) and (2)

$$r_1 = \frac{m_2}{m_1 + m_2} * r$$

And, $$r_2 = \frac{m_1}{m_1 + m_2} * r$$

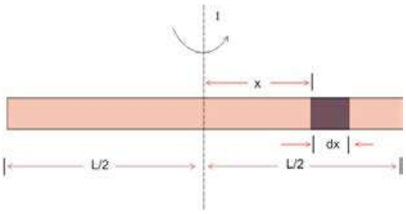
4. $I = I_1 + I_2 = m_1r_1^2 + m_2r_2^2$
5. $I = \frac{m_1m_2}{m_1 + m_2} * r^2$
6. $I_1 = m_2I, \ and \ \ I_2 = m_1I$

10. Moment of Inertia of various bodies

1.The uniform rod

I=Moment of inertia of an ROD about an axis through its centre and perpendicular to it

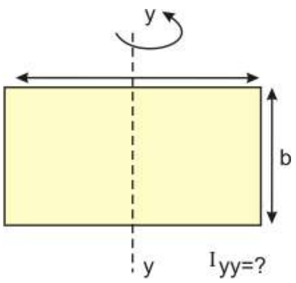
Consider a uniform straight rod of length L, mass M and having centre C



$$I=\int dI=\int x^2 dm=\int_{\frac{-L}{2}}^{\frac{L}{2}} \frac{M}{L} x^2 * dx=\frac{M}{L} \int_{\frac{-L}{2}}^{\frac{L}{2}} x^2 dx=\frac{ML^2}{12}$$

2. Uniform rectangular lamina

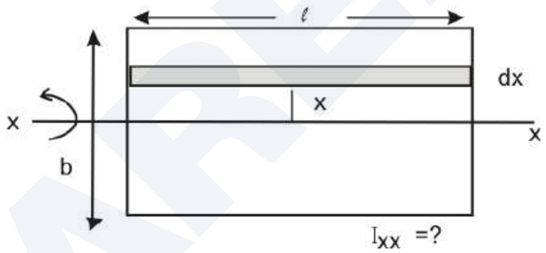
I_{yy}=Moment of inertia for uniform rectangular lamina about y-axis passing through its centre .



$$I_{yy}=\frac{Ml^2}{12}$$

Similarly

I_{xx} = Moment of inertia for uniform rectangular lamina about the x-axis passing through its centre.

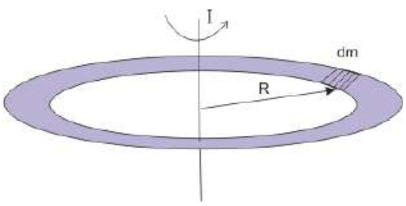


$$I_{xx}=\int dI=\int x^2 dm=\int_{\frac{-b}{2}}^{\frac{b}{2}} \frac{M}{lb} x^2 *(l) dx=\frac{M}{b} \int_{\frac{-b}{2}}^{\frac{b}{2}} x^2 dx=\frac{Mb^2}{12}$$

3. RING

I=Moment of inertia of a RING about an axis through its centre and perpendicular to its plane

Consider a ring of mass M, radius R and centre O.

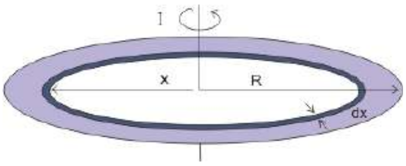


$$I=MR^2$$

4. DISC

I=Moment of inertia of a DISC about an axis through its centre and perpendicular to its plane

Consider a circular disc of mass M, radius R and centre O.

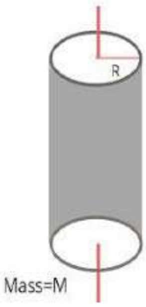


$$I = \frac{MR^2}{2}$$

5.Hollow cylinder

I= Moment of inertia of the hollow cylinder about its axis passing through its C.O.M

Consider a cylinder of mass M, radius R and length L as shown in figure

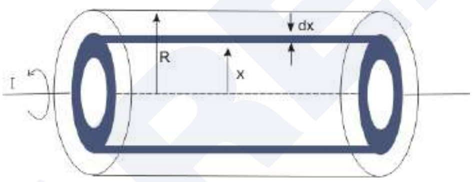


$$I = MR^2$$

6.SOLID CYLINDER

I= Moment of inertia of the CYLINDER about an axis through its centre

Consider a cylinder of mass M, radius R and length L.



$$I = \frac{MR^2}{2}$$

7. Hollow sphere

Let I = Moment of inertia of a hollow SPHERE about an axis through its centre

And I_x = Moment of inertia of a hollow SPHERE about x- axis through its centre

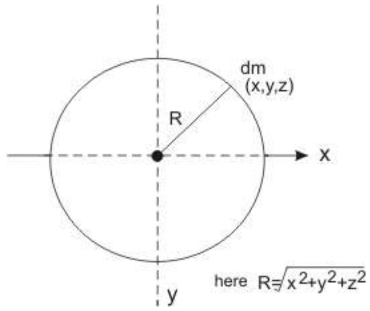
And I_y= Moment of inertia of a hollow SPHERE about y- axis through its centre

And I_z= Moment of inertia of a hollow SPHERE about z- axis through its centre

As hollow sphere is symmetric about any axis passing through its centre

So it will be symmetric about x, y, z axis passing through its centre

So we can say that $I_x = I_y = I_z = I$

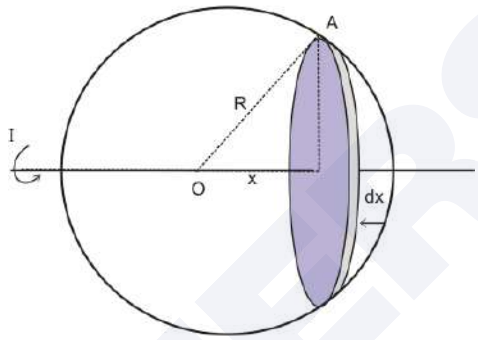


$$I = \frac{2}{3}MR^2$$

8. SOLID SPHERE

I=Moment of inertia of a SOLID SPHERE about an axis through its centre

Consider a sphere of mass M, radius R and centre O.



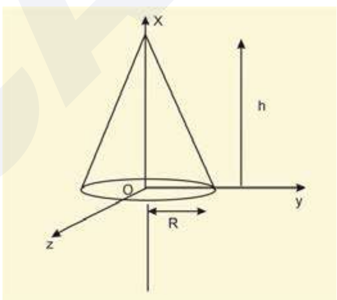
$$I = \frac{2}{5}MR^2$$

9. Solid cone

I=Moment of inertia of a solid cone about an axis through its C.O.M

Consider a solid cone of mass M, base radius R, and Height as h

As shown in Figure I is about the x-axis and through its C.O.M

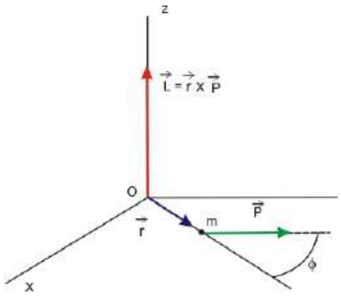


$$\mathbf{I} = \frac{\mathbf{3}}{\mathbf{10}} \times \mathbf{MR^2}$$

11. Angular Momentum

Angular Momentum-

- The moment of linear momentum of a body with respect to any axis of rotation is known as angular momentum. If P is the linear momentum of a particle and its position vector from the point of rotation is r then angular momentum is given by the vector product of linear momentum and position vector.



$$\vec{L} = \vec{r} \times \vec{P}$$

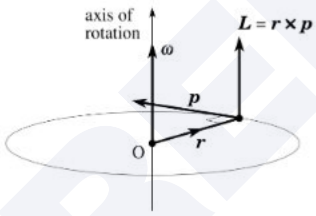
$$\vec{L} = \vec{r} \times \vec{P} = \vec{r} \times (m\vec{V}) = m(\vec{r} \times \vec{V})$$

$\left|\vec{L}\right| = rpsin\theta$, where θ is the angle between r and p.

$$|\vec{L}| = mvrsin\theta$$

- Its direction is always perpendicular to the plane containing vector r and P and with the help of right hand screw rule we can find it.

  Its direction will be perpendicular to the plane of rotation and along the axis of rotation
- $L_{max} = r * P \ (when \ \theta = 90^0)$
- $L_{min} = 0 \ (when \ \theta = 0^0)$
- SI Unit- Joule-sec or $kg - m^2/s$
- Dimension- ML^2T^{-1}
- In case of circular motion



As $\vec{r} \perp \vec{v}$ and $v = \omega r$ and $I = mr^2$

$L = mvr = mr^2\omega = I\omega$

So in vector form $\vec{L} = I\vec{\omega}$

- The net angular momentum of a system consisting of n particles is equal to the vector sum of angular momentum of each particle.

$\vec{L}_{net} = \vec{L}_1 + \vec{L}_2 .... + \vec{L}_n$

- **Analogy Between Translatory Motion and Rotational Motion for Common Terms**

	Translatory motion	Rotatory motion
1	Mass (m)	Moment of Inertia (I)
2	Linear momentum $P = mV$	Angular Momentum $L = I\omega$
3	Force F=ma	Torque $\tau = I\alpha$

Conservation Of angular momentum-

- From $\vec{L} = I\vec{\omega}$ we get $\frac{d\vec{L}}{dt} = I\frac{d\vec{\omega}}{dt} = I\vec{\alpha} = \vec{\tau}$

  i.e. the rate of change of angular momentum is equal to the net torque acting on the particle.

  This is **Rotational analogue of Newton's second law**

- Angular impulse = $\vec{J} = \int \vec{\tau} dt = \Delta\vec{L}$

  Or, $\vec{J} = I\left(\vec{w_f} - \vec{w_i}\right)$

  i.e., Angular impulse is equal to change in angular momentum

- As $\vec{\tau} = \frac{d\vec{L}}{dt}$

  So if the net external torque on a particle is zero then for that particle

  $$\frac{d\vec{L}}{dt} = 0 \Rightarrow \vec{L} = constant$$

  $$\Rightarrow L_i = L_f$$

  Similarly in case of system consists of n particles

  If the net external torque on a system is zero then for that system

  $$\frac{d\vec{L}}{dt} = 0 \Rightarrow \vec{L} = constant$$

  Or, $\vec{L}_{net} = \vec{L}_1 + \vec{L}_2 \ldots\ldots + \vec{L}_n = constant$

  I.e Angular momentum of a system remains constant if resultant torque acting on it zero.

  This is known as the **law of conservation of angular momentum.**

- For a system if $\vec{\tau}_{net} = 0$ then its

  $$\vec{L} = I\vec{\omega} = Constant$$

  Or, $I \; \alpha \; \frac{1}{\omega}$

  Example-In a circus during performance an acrobat try to bring the arms and legs closer to body to increase spin speed. On bringing the arms and legs closer to body, his moment of inertia I decreases. Hence ω increases.

12. Work, Energy and Power for Rotating Body

1. **Work-**

   For translation motion $W = \int F ds$

   So for rotational motion $W = \int \tau d\theta$

2. **Rotational kinetic energy-**

   The energy of a body has by virtue of its rotational motion is called its rotational kinetic energy.

	Rotational kinetic energy	Translatory kinetic energy
1	$K_R = \frac{1}{2}I\omega^2$	$K_T = \frac{1}{2}mV^2$
2	$K_R = \frac{1}{2}L\omega$	$K_T = \frac{1}{2}PV$

3 $K_R = \frac{L^2}{2I}$ $\qquad K_T = \frac{P^2}{2m}$

3. **Power =Rate of change of kinetic energy**

For translation motion $P = \vec{F}.\vec{V}$

So for rotational motion

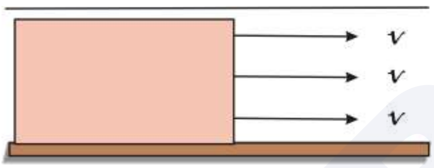
$$P = \frac{d(K_R)}{dt} = \frac{d(\frac{1}{2}I\omega^2)}{dt} = I\omega\frac{d\omega}{dt} = I\alpha\omega = \tau.\omega$$

Or $P = \vec{\tau}.\vec{\omega}$

13. Combined rotation and translation motion

1. **Pure Translational motion-**

If each particle of it has the same velocity/acceleration at a particular instant of time then A body is said to have pure translational motion.



- **Slipping-**

It is a motion in which the body slides on a surface without rotation.

Example- Motion of a wheel on a frictionless surface.

Here friction between the body and surface =f= 0

Wheel possess only translatory kinetic energy

i.e., - $K_T = \frac{1}{2}mv^2$

2. **Pure rotational motion-**

When a body rotates such that its axis of rotation does not move then that body is said to have pure rotational motion.

In pure rotational motion, each particle of the body has the same angular velocity/acceleration about its axis of rotation at a particular instant of time.

Example- Spinning of the wheel about a fixed axis

Here axis of rotation of a wheel is fixed.

Here body possesses only rotatory kinetic energy.

I.e $K_R = \frac{1}{2}I\omega^2$

Here Rotational angular momentum = $\vec{L} = I\vec{w}$

Where I = Moment of inertia about a fixed axis of rotation

ω = angular velocity of rotation

Another example is the motion of blades of a fan

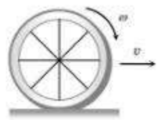
3. **Combined rotation and translation motion**

In this type of motion, the body is having both rotation and translation motion.

- **Rolling**

In the case of rolling motion, a body rotates about a fixed axis, and the axis of rotation also moves.

Example- Rolling of football on the ground



Here friction between the body and surface = $f \neq 0$

1. Kinetic energy-

The total kinetic energy of the body is the sum of both translational and rotational kinetic energy.

$$K_{net} = K_T + K_R = \frac{1}{2}mV^2 + \frac{1}{2}I\omega^2$$

Using $V = \omega R \ and \ I = mK^2$

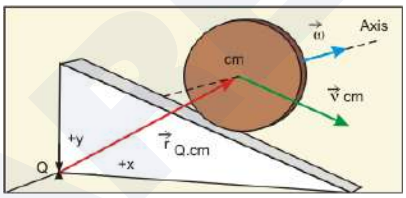
$$K_{net} = K_T + K_R = \frac{1}{2}mV^2(1 + \frac{K^2}{R^2})$$

2. Net Velocity at a point-

$$\vec{V}_{net} = \vec{V}_{translation} + \vec{V}_{rotation}$$

Where, $\vec{V}_{rot} = r\ w$

- **Angular momentum in case of Combined rotation and translation motion**



Angular momentum is always calculated at a particular point.

The net Angular momentum of a body is the sum of angular momentum due to both translational and rotational motion.

$$\vec{L} = \vec{L_{com}} + m\left(\vec{r} \times \vec{v_{com}}\right)$$

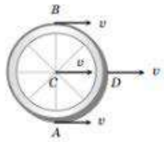
$$\Rightarrow \vec{L} = I_{com}\vec{\omega} + m\left(\vec{r} \times \vec{v_{com}}\right)$$

Where L_{com} represents the angular momentum of the body about the centre of mass and r is the position vector about which we have to calculate the angular momentum.

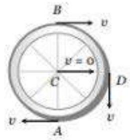
14. Rolling Without Slipping

- **The linear velocity of different points**

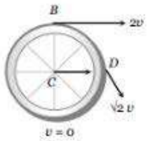
In pure Translation-



In pure Rotation-



And in Rolling all points of a rigid body have same angular speed (ω) but different linear speed.



I.e I.e

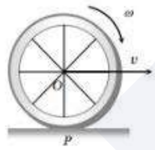
- During Rolling motion

If $V_{cm} > Rw \rightarrow slipping\ motion$

If $V_{cm} = Rw \rightarrow pure\ rolling$

If $V_{cm} < Rw \rightarrow skidding\ motion$

When the object rolls across a surface such that there is no relative motion of object and surface at the point of contact, the motion is called rolling without slipping.



Here the point of contact is P.

Friction force is available between object and surface but work done by it is zero because there is no relative motion between body and surface at the point of contact.

Or we can say No dissipation of energy is there due to friction.

I.e., Energy is conserved.

Which is $K_{net} = K_T + K_R = \frac{1}{2}mV^2 + \frac{1}{2}I\omega^2$

Now using $V = \omega.R$

And using $K_{net} = \frac{1}{2}mV^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}(I + mR^2)\omega^2$

Where I = moment of inertia of the rolling body about its centre 'O'

And using the Parallel axis theorem

We can write $I_p = I + mR^2$

So we can write $K_{net} = \frac{1}{2}I_p\omega^2$

Where I_p=moment of inertia of the rolling body about point of contact 'P'.

So this Rolling motion of a body is equivalent to a pure rotation about an axis passing through the point of contact (here through P) with the same angular velocity ω.

Here axis passing through the point of contact P is also known as Instantaneous axis of rotation.

(Instantaneous axis of rotation-Motion of an object may look as pure rotation about a point that has zero velocity.)

- **Net Kinetic Energy for different rolling bodies**

As $K_{net} = K_T + K_R = \frac{1}{2}mV^2(1+\frac{K^2}{R^2})$

So the quantity $\frac{K^2}{R^2}$ will have different values for different bodies.

Rolling body	$\frac{K^2}{R^2}$	K_{net}
Ring Or Cylindrical shell	1	mV^2
Disc Or solid cylinder	$\frac{1}{2}$	$\frac{3}{4}mV^2$
Solid sphere	$\frac{2}{5}$	$\frac{7}{10}mV^2$
Hollow sphere	$\frac{2}{3}$	$\frac{5}{6}mV^2$

- **The direction of friction-**

Kinetic friction will always oppose the rolling motion. While Static friction on the other hand only opposes the tendency of an object to move.

1. When an external force is in the upward diametric part

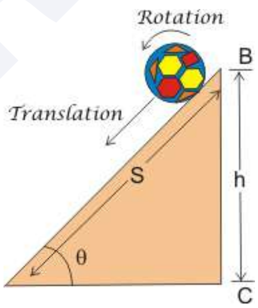
- If $K^2 = Rx$ then no friction will act
- If $K^2 > Rx$ then Friction will act in the backward direction
- If $K^2 < Rx$ then Friction will act in a forward direction

2. If an external force is in the lower diametric part,

Then friction always act backwards

15.Rolling without slipping on an Inclined Plane

When a body of mass m and radius R rolls down an inclined plane having an angle of inclination (θ) and at height 'h'



By conservation of mechanical energy

$$mgh = \frac{1}{2}mV^2(1+\frac{K^2}{R^2})$$

Where V=Velocity at the lowest point

And, $V = \sqrt{\frac{2gh}{1+\frac{K^2}{R^2}}}$

Similarly using $V^2 = u^2 + 2as$

$$\text{Acceleration} = a = \frac{g \sin \Theta}{1 + \frac{K^2}{R^2}}$$

And angular acceleration = $a = R\alpha$

And we know that $\tau = I\alpha$

And torque due to friction force = $\tau_f = fR = I\alpha = mK^2(Ra)$

So, $f = \dfrac{mg \sin \Theta}{1 + \frac{R^2}{K^2}}$

As $f = \mu N = \mu mgcos\theta$

So Condition for pure rolling on an inclined plane

$$\mu_s \geq \frac{\tan \Theta}{1 + \frac{R^2}{K^2}}$$

Where μ_s = limiting coefficient of friction

And let t= time taken by the body to reach the lowest point

So using $V = u + at$

We get, $\mathbf{t} = \dfrac{\mathbf{1}}{\mathbf{sin}\theta}\sqrt{\dfrac{\mathbf{2h}}{\mathbf{g}}\left(\mathbf{1} + \left[\dfrac{\mathbf{K^2}}{\mathbf{R^2}}\right]\right)}$

Gravitation

Important Formulae

1. Newton's law of Gravitation

- According to **Newton's law of gravitation,** the gravitational force is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centres.

  Due to gravitational force, Each particle in this universe attracts every other particle.

  The direction of this force is along the line joining the particles.

  Let two particles of masses m_1 and m_2 separated by a distance r exert a Force F on each other

  And Magnitude of F is given as

$$F \ \alpha \ \frac{m_1 m_2}{r^2}$$

Or, $F = \dfrac{G m_1 m_2}{r^2}$

Where

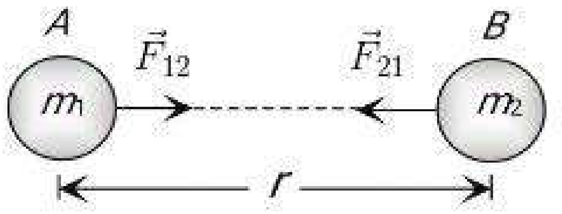
$F \rightarrow$ Force

$G \rightarrow$ Gravitational constant

$m_1, m_2 \rightarrow$ Masses

$r \rightarrow$ Distance between masses

- **The vector form of formula**



According to Newton's law of gravitation

$$\vec{F}_{12} = \frac{-Gm_1m_2}{r^2}(\hat{r}_{21})$$

Or $\vec{F}_{12} = \frac{-Gm_1m_2}{r^3}(\vec{r}_{21})$

Where $\hat{r}_{21} \rightarrow$ Position vector

Here negative sign indicates that the direction of $\vec{F}_{12}$ is opposite to that of $\vec{r}_{21}$

And $\hat{r}_{12} = -\hat{r}_{21}$

So, $\vec{F}_{12} = -\vec{F}_{21}$

Means Gravitational force between two bodies form an action and reaction pair.

i.e. the forces are equal in magnitude but opposite in direction.

This is in accordance with Newton's third law of motion

- **Universal Gravitational Constant (G)**

If $m_1 = m_2 = 1\ kg$ and r=1 m then F=G

I.e Universal gravitational constant is equal to the Gravitational force between two bodies each having unit mass and their centers are placed unit distance apart.

Value of G is $6.67 \times 10^{-11}\ N - m^2 kg^{-2}\quad (S.I.)$

Its Dimension Formula is $[M^{-1}L^3T^{-2}]$

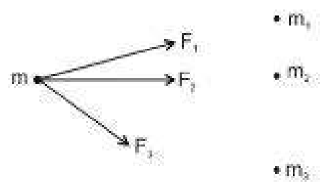
Value of G does not depend upon the nature & size of bodies

Also, it is also independent of the nature of medium between two bodies.

- Properties of Gravitational Force

1. Always attractive
2. It is the central force
3. Weakest force (Ratio of $F_g\ to\ F_e$ between two electrons is 10^{-43})
4. It is a conservative force
5. It is independent of the medium between the particles.
6. Gravitational force is long range-force.
7. Principle of superposition is valid for Gravitational Force

The gravitational force between two particles is independent of the presence or absence of other particles.



Force on a particle (m) due to the number of particles (m_1, m_2, m_3 , etc)

is the resultant of forces due to individual particles (F_1, F_2, F_3, etc)

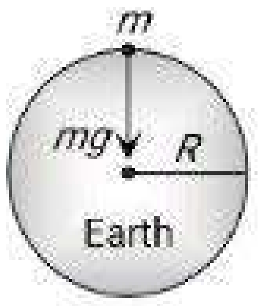
I.e- $\vec{F} = \vec{F_1} + \vec{F_2} + \vec{F_3}$.........

2. Acceleration due to gravity

The Gravitational Force exerted by the earth on a body is known as the gravitational pull of gravity. And this force will produce an acceleration in the motion of a body.

And this is known as the acceleration due to gravity.

This is denoted by g.



it is given by formula $g = \frac{GM}{R^2}$

And in term of $\rho \rightarrow$ density of earth

Using $M = \rho * (\frac{4}{3})\pi R^3$

we get $g = \frac{4}{3}\pi\rho GR$

- Its average value is $9.8\ m/s^2\ \ or\ \ 981 cm/sec^2\ or\ 32 feet/s^2$ on the surface of the earth.
- It is a vector quantity and its direction is always towards the centre of the earth/Planet.
- Dimension- LT^{-2}
- Its value depends upon the mass, radius, and density of the Earth/Planet.
- It is independent of mass, shape and density of the body situated on the surface of the Earth/planet.

i.e Value of g will be the same for a light as well as heavy body if both are situated on the surface of the Earth/planet.

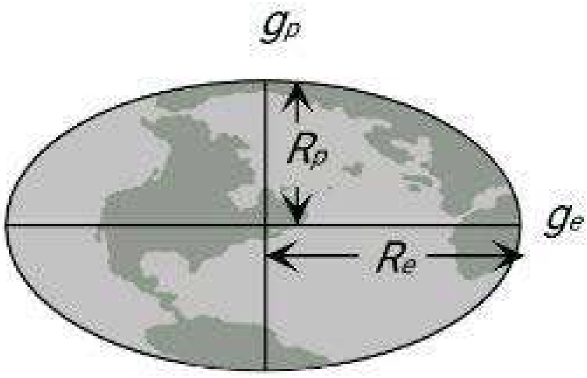
3. Factors affecting the value of acceleration due to gravity (g)

The value of acceleration due to gravity (g) changes its value due to the following factors

1. The shape of the earth
2. Height above the earth's surface
3. Depth below the earth's surface
4. Axial rotation of the earth.

Variation of 'g' due to the shape of the earth

Earth has an elliptical shape as shown in fig.



Where Equatorial radius is about 21 km longer than the polar radius.

Or $R_e > R_p$

Where $R_e \rightarrow$ Radius of the equator

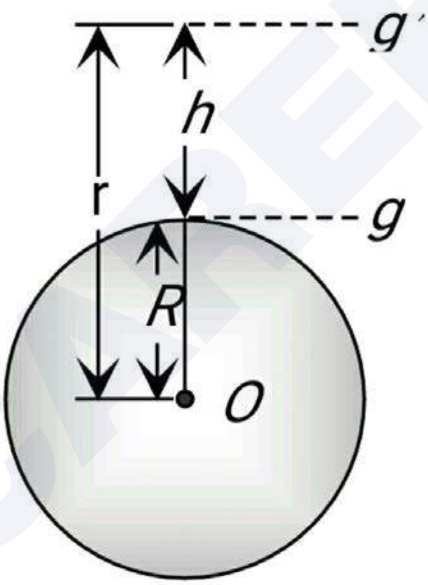
$R_p \rightarrow$ Radius of pole

So $g_p > g_e$

In fact $g_p = g_e + 0.018 m/s^2$

Or we can say Weight increases as the body is taken from equator to pole.

Variation in 'g' due to height-



Value of g at the surface of the earth (at distance r=R from earth center)

$$g = \frac{GM}{R^2}$$

Value of g at height h from the surface of the earth (at a general distance r=R+h from earth center)

$$g' \alpha \frac{1}{r^2}$$

Where $r = R + h$

As we go above the surface of the earth, the value of g decreases

So $g' = \frac{GM}{r^2}$

Where $g' \rightarrow$ gravity at a height h from the surface of the earth.

$R \rightarrow$ The radius of earth

$h \rightarrow$ height above the surface

- Value of 'g' at ∞

if $r = \infty \qquad g' = 0$

No effect of earth gravitational pull at infinite distance.

- Value of g when h < < R
- Formula

1. Value of g

$$g' = g(\frac{R}{R+h})^2 = g(1+\frac{h}{R})^{-2}$$

So $g' = g\left[1 - \frac{2h}{R}\right]$

2. The absolute decrease in the value of g with height

$$\Delta g = g - g' = \frac{2hg}{R}$$

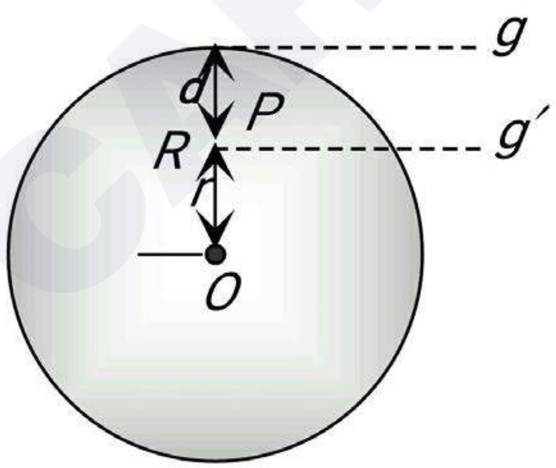
3. The fractional decrease in the value of g with height

$$\frac{\Delta g}{g} = \frac{g-g'}{g} = \frac{2h}{R}$$

4. Percentage decrease in the value of g with height

$$\frac{\Delta g}{g} \times 100\,\% = \frac{2h}{R} \times 100\,\%$$

Variation in 'g' due to depth-



Value of g at the surface of the earth (at d=0)

$$g = \frac{GM}{R^2} = \frac{4}{3}\pi\rho g R$$

Value of g at depth d from the surface of the earth (at a general distance r=(R-d) from earth centre)=g'

And $g' \propto (R-d)$

Means Value of g' decreases on going below the surface of the earth.

So $g' = g\left[1 - \frac{d}{R}\right]$

- Value of 'g' at the centre of the earth

  At the centre

  $\text{depth from surface}(d) = R$

  So $g' = 0$

  i.e., Acceleration due to gravity at the centre of the earth becomes zero.

- The absolute decrease in the value of g with depth

  $\Delta g = g - g' = \frac{dg}{R}$

- The fractional decrease in the value of g with depth

  $\frac{\Delta g}{g} = \frac{g - g'}{g} = \frac{d}{R}$

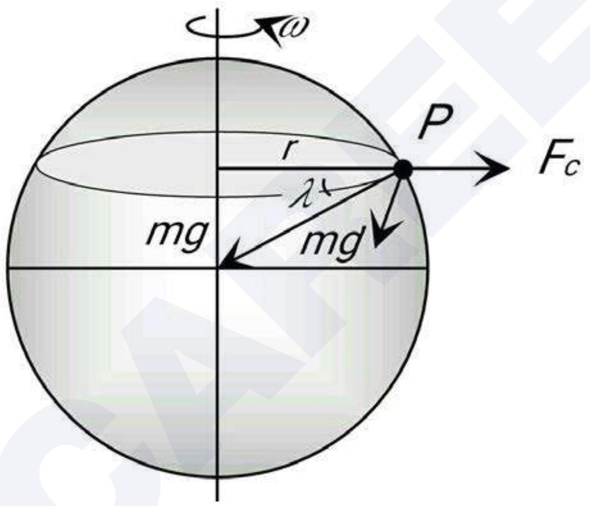
  The value of g decreases with depth.

- Percentage decrease in the value of g with depth

  $\frac{\Delta g}{g} \times 100\ \% = \frac{d}{R} \times 100\ \%$

  Note- The rate of decrease of gravity outside the earth (h<<R) is double that of inside the earth

  Variation in 'g' due to Rotation of earth-



As the earth rotates about its axis

Let its angular velocity is ω about an axis as shown in the figure..

As shown in the figure.

- λ=The angle between the equatorial plane at that point and line joining that point to the centre of the earth.
- For the poles λ=90 and for equator λ=0

  $r = RCos\lambda$

  $F_c = m\omega^2 r = m\omega^2 Rcos\lambda$

  From applying Newton's 2nd law along the line joining point P and centre.

  $mg - F_c Cos\lambda = mg'$

  Where g' is the value of acceleration due to gravity at point P.

So we get $g' = g - \omega^2 R\cos^2\lambda$

- The apparent weight of body decrease with an increase in angular velocity (ω)
- Apparent weight of the body varies from point to point because each point has different latitude and magnitude of centrifugal force varies with the latitude of the place.
- For Pole, λ=90

  So $g_{pole} = g$

  I.e value of g at the poles is independent of angular velocity of earth.
- For equator, λ=0

  $g_{equator} = g - \omega^2 R$

  I.e Decrease in the value of g is maximum at the equator
- Weightlessness due to rotation of the earth-

  Weightlessness means g'=0

  So $g' = g - \omega^2 R\cos^2\lambda$

  As $\lambda = 0$ (For equator)

  $O = g - \omega^2 R\cos^2 0$

  $g - \omega^2 R = 0$

  $\omega = \sqrt{\frac{g}{R}}$

Where $\omega \rightarrow$ Angular velocity for which a body at the equator will become weightless

- The time period of Rotation of earth for which body at the equator will become weightless

  $$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{R}{g}}$$

  Where $R \rightarrow$ Radius of earth

  And using $R = 6400 \times 10^3 m$

  $g = 10\, m/s^2$

  We get $\omega = \frac{1}{800}\frac{rad}{sec}$

  And $T = 1.40\ hr$
- Relation of gravity at the poles and equator

  After considering the effect of rotation, and the elliptical shape of the earth

  $g_p = g_e + 0.052\ m/s^2$

  Where $g_p \rightarrow$ gravity at the pole

  $g_e \rightarrow$ gravity at equator

4. Mass and Density of Earth

- Mass of Earth-

  Using Newton's law of gravitation we can estimate the mass of the earth

  $$M = \frac{gR^2}{G} \simeq\ 10^{25} kg$$

  $M \rightarrow$ mass of earth

  $G \rightarrow$ Gravitational constant
- Density of Earth

As $g=\frac{4}{3}\pi\rho GR$

Where $\rho \rightarrow$ density of earth

So $\rho=\frac{3g}{4\pi GR}$

$\rho=5478.4\ kg/m^3$

- Inertial mass

  Also known as the mass of material of body which measures its inertia.

  From Newton's second law of motion

  $F=m_i a$

  So $m_i=\frac{F}{a}$

  Where

  $m_i \rightarrow$ inertial mass

  $F \rightarrow$ external force

  $a \rightarrow\ acc^n$

1. Gravity has no effect on inertial mass.
2. Inertial mass is independent of size, shape, and state of the body.

- Gravitational Mass-

  It is mass which determines the gravitational pull acting upon it.

  Let F= gravitational pull on a body of mass

  applying Newton's law of gravitation

  We have $F=\frac{GMm_g}{R^2}$

  So we get $m_g=\frac{F}{GM/R^2}=\frac{F}{I}$

  Where $m_g=$ Gravitational mass

  $I \rightarrow$ Gravitational field intensity

  Tip-Spring balance measure gravitational mass.

- Mass (m)

1. It is the quantity of matter contained in the body.
2. Its SI unit- Kg
3. Its dimension is $[M]$
4. It is a scalar quantity.
5. It Can never be zero
6. Its value does not change with g.

- Weight (W)

1. It is an Attractive force exerted by the earth on anybody.
2. S.I. Unit: Newton or Kg - wt
3. Dimension-$[MLT^{-2}]$
4. It is a vector quantity

5. It changes its value according to the value of g

6. At ∞ and at the centre of earth g = 0, So W is equal to zero there.

5.Gravitational field Intensity

- Gravitational field-

  It is space or surrounding in which a material body feels the gravitational force of attraction.

- Gravitational field Intensity-

  It is the force experienced by a unit mass at a point in the field.

  It is denoted by I

  If the mass of a body is m then I is given by

$$\vec{I} = \frac{\vec{F}}{m}$$

$\vec{I} \rightarrow G.field\ Intensity$

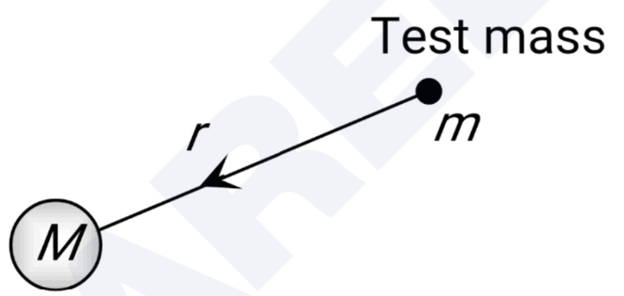
$m \rightarrow mass\ of\ object$

$\vec{f} \rightarrow Gravitational\ Force$

1. It is a vector quantity
2. If the field is produced by a body M the direction of its Gravitational field Intensity is always towards the center of gravity of M.
3. $Unit: \frac{Newton}{kg}\ or\ \frac{m}{s^2}$
4. $Dimension: [M^0LT^{-2}]$

Gravitational field due to Point mass-

If the point mass M is producing the field and test mass is at distance r as shown in fig



So Force is given as $F = \frac{GmM}{r^2}$

And the corresponding I is given by

$$I = \frac{F}{m} = \frac{GMm}{r^2m}$$

$$I = \frac{GM}{r^2}$$

Where $G \rightarrow Gravitational\ const$

$M \rightarrow mass\ of\ earth$

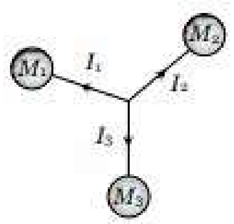
1. $I \propto \frac{1}{r^2}$

   Means As the distance (r) of test mass from point (M) Increases I decreases.

2. $I = 0\ at\ (r = \infty)$

Superposition of Gravitational field-

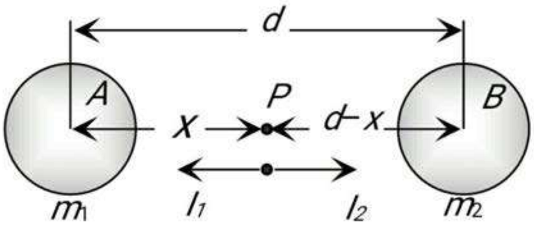
The net Intensity at a given point due to different point masses (M_1,M_2,M_3...) can be calculated by doing the vector sum of their individual intensities



$\vec{I}_{net} = \vec{I}_1 + \vec{I}_2 + \vec{I}_3 + ...$

- **Point of zero intensity-**

Let m_1 and m_2 are separated at a distance d from each other



And P is the point where net Intensity=$\vec{I}_{net} = \vec{I}_1 + \vec{I}_2$=0

Then P is the point of zero intensity

Let point P is at distance x from m1

Then For point P $\vec{I}_{net} = \vec{I}_1 + \vec{I}_2$=0

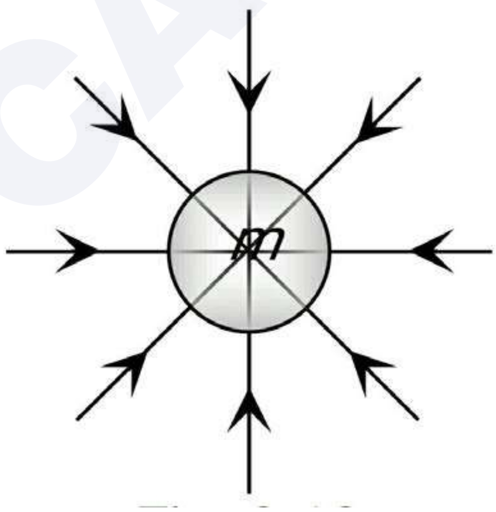
$$-\frac{Gm_1}{x^2} + \frac{Gm_2}{(d-x)^2} = 0$$

Then $x = \frac{\sqrt{m_1}\,d}{\sqrt{m_1} + \sqrt{m_2}}$

And $(d-x) = \frac{\sqrt{m_2}\,d}{\sqrt{m_1} + \sqrt{m_2}}$

- **Gravitational field line-**

Field line of Isolated mass-



Field lines are radially Inward

As $I = \frac{GM}{r^2}$

$$g = \frac{GM}{R^2} \Rightarrow I = g$$

So we can say that the intensity of the gravitational field at a point P in the field of Isolated mass is equal to the acceleration of test mass placed at that point P.

Properties of Gravitational field line-

1. The line includes arrows which represent the direction of the gravitational field.

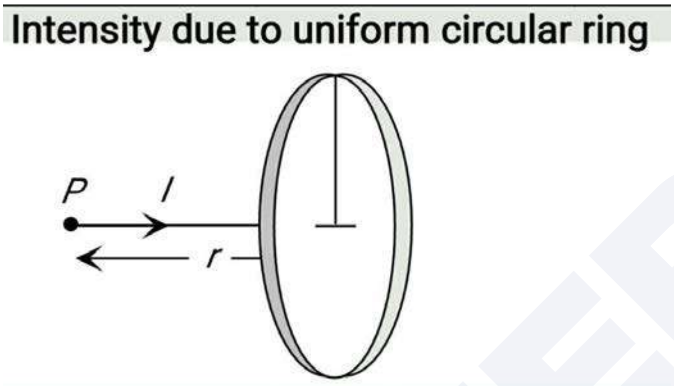
2. The magnitude of the gravitational field is proportional to the number of field lines crossing a unit area perpendicular to them.

The lines never cross

3. Lines do not form closed loops

6. Gravitational field due to various bodies

1. Gravitational field due to uniform circular ring



At the center of ring

$I = 0$

At a point on its Axis

$$I = \frac{GMr}{(a^2 + r^2)^{\frac{3}{2}}}$$

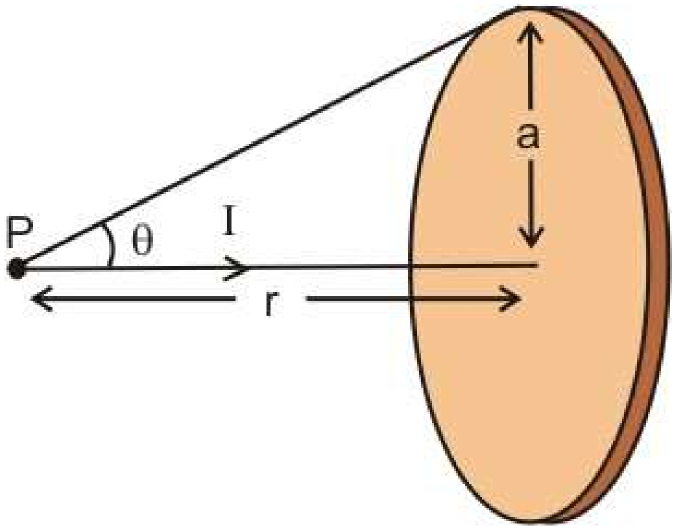
Where,

$r \rightarrow$ The distance of the point P along the Axis of the ring, from its center .

a= radius of the ring

2.Gravitational field Intensity due to uniform disc

For Uniform disc



$\Theta \rightarrow$ Angle with axis

$a \rightarrow$ Radius of disc

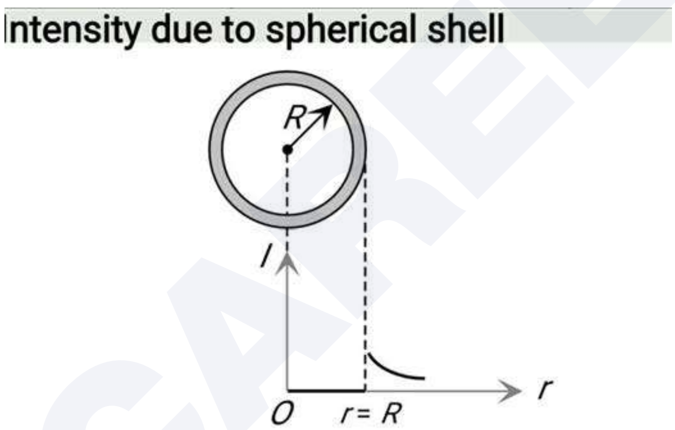
At the center of the disc

$I=0$

At a point on its axis

$I=\frac{2GM}{a^2}(1-\cos\Theta)$

3.Gravitational field Intensity due to spherical shell/hollow sphere



$R \rightarrow$ Radius of shell

$r \rightarrow$ Position of Point

$M \rightarrow$ Mass of spherical shell

- Inside the surface

  $r<R$

  $I=0$

- on the surface

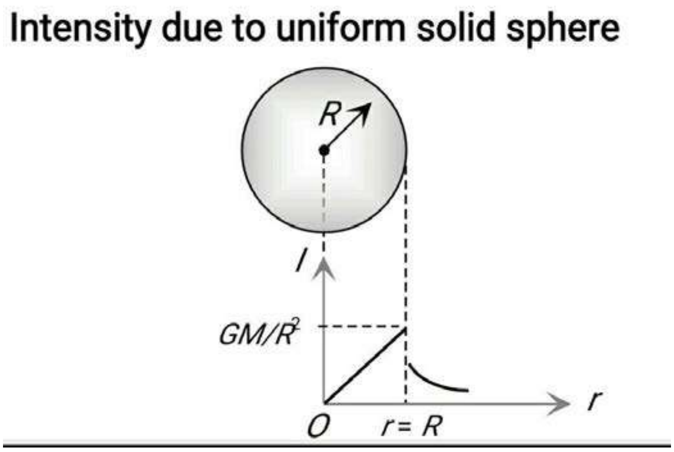
  $r=R$

  $I=\frac{GM}{R^2}$

- Outside the surface $r>R$

$$I = \frac{GM}{r^2}$$

4.Gravitational field Intensity due to uniform solid sphere



- Inside surface $r < R$

$$I = \frac{GMr}{R^3}$$

- on the surface

$$r = R$$

$$I = \frac{GM}{R^2}$$

- Outside surface $(r > R)$

$$I = \frac{GM}{r^2}$$

7.Gravitational Potential

In a gravitational field potential V at a point, P is defined as negative of work done per unit mass in changing the position of a test mass from some reference point to the given point.

Note-usually reference point is taken as infinity and potential at infinity is taken as Zero.

We know that $W = \int \vec{F} \cdot \vec{dr}$

So $V = -\frac{W}{m} = -\int \frac{\vec{F} \cdot \vec{dr}}{m}$

And $\vec{I} = \frac{\vec{F}}{m}$

$$V = -\int \vec{I} \cdot \vec{dr}$$

$V \rightarrow$ Gravitational potential

$I \rightarrow$ Field Intensity

$dr \rightarrow$ small distance

We can also write $I = -\frac{dV}{dr}$

Means a negative gradient of potential gives the intensity of the field .

The negative sign indicates that in the direction of intensity the potential decreases.

- It is a scalar quantity.
- $Unit \rightarrow \ Joule/kg \ or \ m^2/sec^2$
- $Dimension \ : \ [M^0L^2T^{-2}]$

Gravitational Potential at a distance 'r' -

If the field is produced by a point mass then

$$I = \frac{GM}{r^2}$$

So $V = -\int \vec{I} \cdot \vec{dr}$

$$V = -\frac{GM}{r}$$

at $r = \infty \qquad V = 0 = V_{max}$

Gravitational Potential difference -

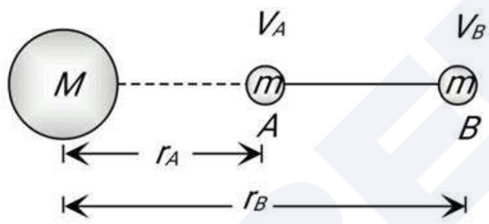
In the gravitational field, the work done to move a unit mass from one position to the other is known as Gravitational Potential difference.

If the point mass M is producing the field

Point A and B are shown in the figure.

V_A=Gravitational potential at point A

V_B=Gravitational potential at point B



$r_B \rightarrow$ the distance of mass at B

$r_A \rightarrow$ distance of mass at A

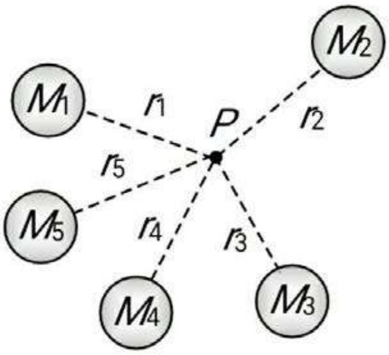
ΔV=The gravitational potential difference in bringing unit mass m from point A to point B in the gravitational field produced by M.

$$\Delta V = V_B - V_A = \frac{W_{A\rightarrow B}}{m}$$

$$\Delta V = -GM\left[\frac{1}{r_B} - \frac{1}{r_A}\right]$$

Superposition of Gravitational potential-

The net gravitational potential at a given point due to different point masses (M_1,M_2,M_3...) can be calculated by doing a scalar sum of their individual Gravitational potential.



$$V = V_1 + V_2 + V_3 \cdots$$

$$= -\frac{GM_1}{r_1} - \frac{GM_2}{r_2} - \frac{GM_3}{r_3} \cdots$$

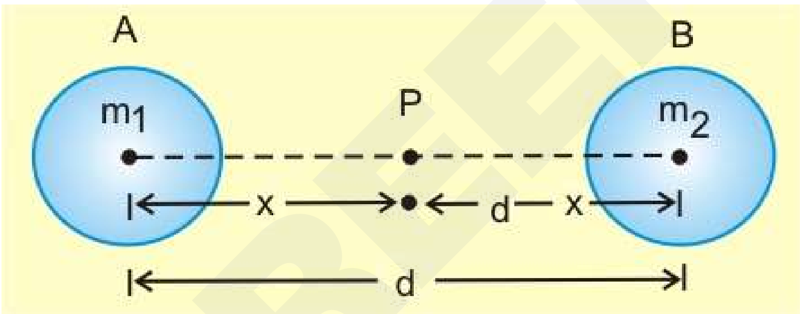
$$V = -G\sum_{i=1}^{i=n} \frac{M_i}{r_i}$$

$M_i \rightarrow$ mass

$r_i \rightarrow$ distances

Point of zero potential-

Let m_1 and m_2 are separated at a distance d from each other



And P is the point where net Gravitational potential $V = V_1 + V_2 = 0$

Then P is the point of zero Gravitational potential

Let point P is at distance x from m_1

Then For point P

$$V = V_1 + V_2 = 0$$

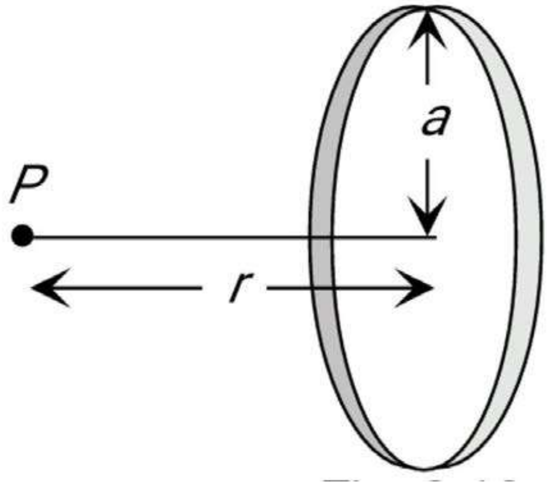
$$-\frac{Gm_1}{r_1} - \frac{Gm_2}{r_2} = 0$$

$$-\frac{Gm_1}{x} - \frac{Gm_2}{d-x} = 0$$

So $x = \dfrac{m_1 d}{m_1 - m_2}$

8. Gravitational potential due to various bodies

1. Gravitational potential due to Uniform circular ring-



$r =$ distance from ring

$a \rightarrow$ radius of Ring

$V \rightarrow$ Potential

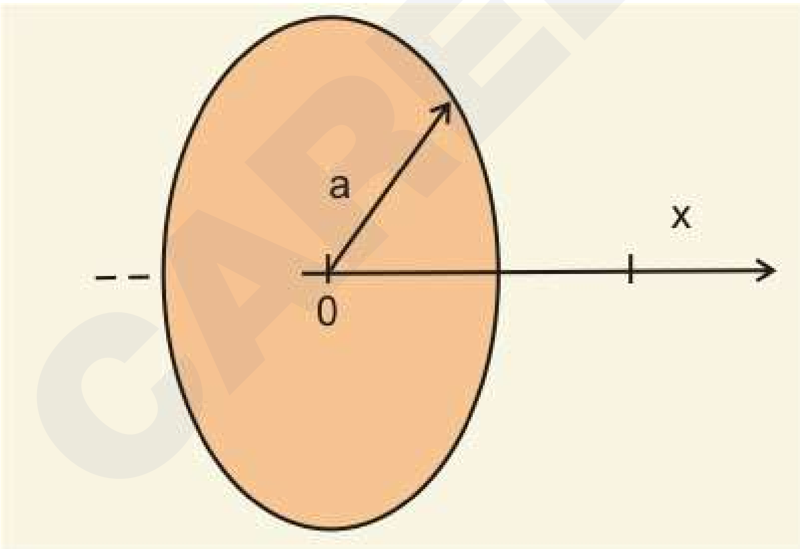
At a point on its Axis

$$V = -\frac{GM}{\sqrt{a^2 + r^2}}$$

At the centre

$$V = -\frac{GM}{a}$$

2. Gravitational Potential due to Uniform disc-



$a \rightarrow$ Radius of disc

M-mass of disc

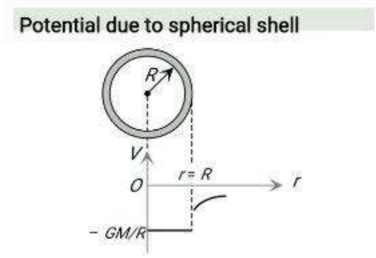
- At the center of the disc

$$V = -\frac{2GM}{a}$$

- At a point on its axis

$$V=-\frac{2GM}{a^2}(\sqrt{a^2+x^2}-x)$$

3. Gravitational Potential due to spherical shell -



$R \rightarrow$ Radius of shell

$r \rightarrow$ distance from the center of the shell

- **Inside the surface**

  $r<R$

  $$V=-\frac{GM}{R}$$

- **on the surface**

  $r=R$

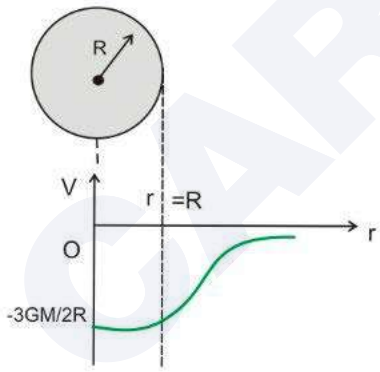
  $$V=-\frac{GM}{R}$$

- **Outside the surface**

  $r>R$

  $$V=-\frac{GM}{r}$$

4. Gravitational Potential due to Uniform solid sphere-



$R \rightarrow$ Radius of sphere

$M \rightarrow$ Mass of sphere

$r \rightarrow$ distance from the center of sphere

- **Inside the surface**

  $r<R$

  $$V=-\frac{GM}{2R}\left[3-\left(\frac{r}{R}\right)^2\right]$$

- **on the surface**

$$V_{surface} = -\frac{GM}{R}$$

- **Outside the surface**

$$V = -\frac{GM}{r}$$

- **Tip-** $V centre = \frac{3}{2} V surface$

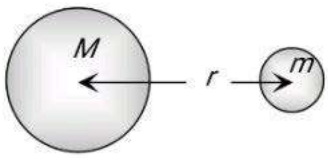
9. Gravitational Potential Energy

It is the amount of work done in bringing a body from ∞ to that point against gravitational force.

- It is Scalar quantity
- SI Unit: Joule
- Dimension : $[ML^2T^{-2}]$

Gravitational Potential energy at a point-

If the point mass M is producing the field



Then gravitational force on test mass m at a distance r from M is given by $F = \frac{GMm}{r^2}$

And the amount of work done in bringing a body from ∞ to r

$$= W = \int_{\infty}^{r} \frac{GMm}{x^2} dx = -\frac{GMm}{r}$$

And this is equal to gravitational potential energy

So $U = -\frac{GMm}{r}$

$U \rightarrow$ gravitational potential energy

$M \rightarrow$ Mass of source-body

$m \rightarrow$ mass of test body

$r \rightarrow$ distance between two

Note- U is always negative in the gravitational field because Force is attractive in nature.

Means As the distance r increases U becomes less negative

I.e U will increase as r increases

And for $r = \infty$, U=o which is maximum

Gravitational Potential energy of discrete distribution of masses -

$$U = -G\left[\frac{m_1m_2}{r_{12}} + \frac{m_2m_3}{r_{23}} + \cdots\right]$$

$U \rightarrow$ Net Gravitational Potential Energy

$r_{12}, r_{23} \rightarrow$ The distance of masses from each other

Change of potential energy -

if a body of mass m is moved from r_1 to r_2

Then Change of potential energy is given as

$$\Delta U = GMm\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

$\Delta U \rightarrow$ change of energy

$r_1, r_2 \rightarrow$ distances

If $r_1 > r_2$ then the change in potential energy of the body will be negative.

I.e To decrease potential energy of a body we have to bring that body closer to the earth.

The relation between Potential and Potential energy -

As $U = \frac{-GMm}{r} = m\left[\frac{-GM}{r}\right]$

But $V = -\frac{GM}{r}$

So $U = mV$

Where $V \rightarrow$ Potential

$U \rightarrow$ Potential energy

$r \rightarrow$ distance

Gravitational Potential Energy at the center of the earth relative to infinity-

$$U_{centre} = mV_{centre}$$
$$V_{centre} \rightarrow Potential\ at\ centre$$

$$U = m\left(-\frac{3}{2}\frac{GM}{R}\right)$$

$m \rightarrow$ mass of body

$M \rightarrow$ Mass of earth

The gravitational potential energy at height 'h' from the earth's surface -

$$U_h = -\frac{GMm}{R+h}$$

Using $GM = gR^2$

$$U_h = -\frac{gR^2m}{R+h}$$

$$U_h = -\frac{mgR}{1+\frac{h}{R}}$$

$U_h \rightarrow$ The potential energy at the height h

$R \rightarrow$ Radius of earth

Relation between gravitational field and potential-

Gravitational field and potential are related as

$$\vec{E} = -\frac{dV}{dr}$$

Where E is Gravitational field

And V is Gravitational potential

And r is the position vector

And Negative sign indicates that in the direction of intensity the potential decreases.

$If\ \vec{r} = x\,\vec{i} + y\,\vec{j} + z\,\vec{k}$

Then $E_x = \frac{\delta V}{dx}, E_y = \frac{\delta V}{dy}, E_z = \frac{\delta V}{dz}$

Work Done Against Gravity-

The **gravitational potential energy** at height 'h' from the earth's surface

Is given by $U_h = -\frac{mgR}{1+\frac{h}{R}}$

So at the surface of earth put h=0

We get $U_s = -mgR$

So if the body of mass m is moved from the surface of earth to a point at height h from the earth's surface

Then there is a change in its potential energy.

And this change in its potential energy is known as work done against gravity to move the body from earth surface to height h.

$$W = \Delta U = GMm\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

Where W $\rightarrow$ work done

ΔU $\rightarrow$ change in Potential energy

r_1, r_2 $\rightarrow$ distances

Putting r_1=R , and r_2=R+h

So $W = \Delta U = GMm\left[\frac{1}{R} - \frac{1}{R+h}\right]$

1. when 'h' is not negligible

   $$W = \frac{mgh}{1+\frac{h}{R}}$$

2. when 'h' is very small

   As $W = \frac{mgh}{1+\frac{h}{R}}$

   But h is small as compared to earth's radius

   $\frac{h}{R} \rightarrow 0$

   So $W = mgh$

3. If h = nR then

   $$W = mgR * \frac{n}{n+1}$$

10. Kepler’s Laws of Planetary Motion

Kepler gives three empirical laws which govern the motion of the planets which are known as Kepler’s laws of planetary motion.

As we know that planets are large natural bodies rotating around a star in definite orbits.

So, Kepler laws are-

(a) **The law of Orbits**:

It is Kepler's First Law.

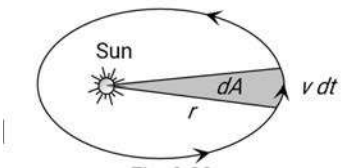
Every planet moves around the sun in an elliptical orbit. And the sun will be at one of the foci of the ellipse.

(b) **The law of Area**:

It is Kepler's 2nd law.

According to this, the line joining the sun to the planet sweeps out equal areas in equal intervals of time which clearly means that areal velocity is constant. So according to this law, a planet will move slowly when it is farthest from the sun and more rapidly when it is nearest to the sun. You can find it similar to the law of conservation of angular momentum.

For the below figure



Area of velocity = $\frac{dA}{dt}$

$\frac{dA}{dt} = \frac{1}{2}\frac{(r)(V\,dt)}{dt} = \frac{1}{2}rV$

Where

$\frac{dA}{dt} \rightarrow$ Areal velocity

$dA \rightarrow$ small area traced

Kepler's 2nd law is Similar to the Law of conservation of momentum

As $\frac{dA}{dt} = \frac{L}{2m}$

where

$L = mvr \rightarrow$ Angular momentum

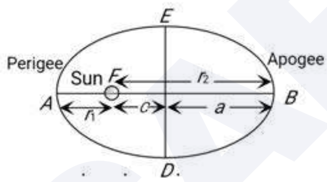
(c) **The law of periods**:

It is Kepler's 3rd law.

According to this, the square of the Time period of revolutions of any planet around the sun is directly proportional to

the cube of the semi-major axis of that particular orbit.

For the below figure



$AB = AF + FB$

$2a = r_1 + r_2$

$\therefore\ a = \frac{r_1 + r_1}{2}$

Where

$a =$ semi major Axis

$r_1 =$ The shortest distance of the planet from the sun (perigee)

$r_2 =$ Largest distance of the planet from the sun (apogee)

So if T=Time period of revolution

Then according to Kepler's 3rd law.

$T^2\ \alpha\ a^3$

or $T^2 \alpha \left(\frac{r_1+r_2}{2}\right)^3$

11. Escape Velocity-

Escape velocity is defined as the minimum velocity an object must have in order to escape from the planets gravitational pull.

- Escape velocity (in terms of the radius of the earth)

$$V_e=\sqrt{\frac{2GM}{R}}$$

Using $GM=gR^2$

We get $V_e=\sqrt{2gR}$

$V_e \rightarrow$ Escape velocity

$R \rightarrow$Radius of earth

And using $g=\frac{4}{3}\pi\rho GR$

$$V_e=R\sqrt{\frac{8}{3}\pi G\rho}$$

For the earth

$V_e=11.2Km/s$

- Escape velocity is independent of the mass of the body.
- Escape velocity is independent of the direction of projection of the body.
- Escape velocity depends on the mass and radius of the earth/planet.
- If the body projected with velocity less than escape velocity $(V<V_e)$

  In this case, the first body will reach a certain maximum height (H_{max})

  And after that, it may either move in an orbit around the earth/planet or may fall back down towards the earth/planet.

  $$H_{max}=R\left[\frac{V^2}{V_e^2-V^2}\right]$$

  $V_e \rightarrow$ escape velocity

  $V \rightarrow$ Projection velocity of the body

  $R \rightarrow$ Radius of planet

- If a body is projected with a velocity greater than escape velocity ($V>V_e$)

  Then By the law of conservation of energy

  Total energy at surface = Total energy at infinity

  $$\frac{-GMm}{R}+\frac{1}{2}mV^2=0+\frac{1}{2}m(V')^2$$

  And using $V_e=\sqrt{\frac{2GM}{R}}$

  We get

  $$V'=\sqrt{V^2-V_e^2}$$

  new velocity of the body at infinity= V'

  $V \rightarrow$ projection velocity

  $V_e \rightarrow$ Escape velocity

- **Escape energy-**

Energy to be given to an object on the surface of the earth so that it's total energy is 0

$\frac{GMm}{R} = Escape\ Energy$

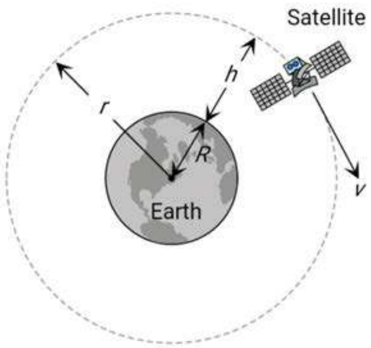
$M \rightarrow$ Mass of planet

$m \rightarrow$ mass of the body

$G \rightarrow$ Gravitational constant

12.Orbital Velocity of Satellite

Orbital velocity of a satellite is the velocity which is required to put the satellite into its orbit around the earth.



$v = \sqrt{\frac{GM}{r}}$

Where

$r \rightarrow$ Position of satellite from the centre of earth

$v \rightarrow$ Orbital velocity

- If r=(R+h) where R is the radius of the earth

  then:

  $v = \sqrt{\frac{gR^2}{R+h}} = R\sqrt{\frac{g}{R+h}} \quad [\text{ As } GM = gR^2 \text{ and } r = R+h]$

- **Dependence of Orbital Velocity**

  1. Orbital velocity is independent of the mass of satellite and is always along the tangent of the orbit.

  2.It depends upon the mass of the central body and radius of orbit

  means, Greater the value of radius of orbit, less be the orbital velocity

- **If satellite is close to the earth's surface,**

  As h<<<R or $h \approx 0$

  and using $GM = gR^2$

  So $V = \sqrt{\frac{GM}{R}} = \sqrt{gR}$

  $V = \sqrt{9.8 \times 6.4 \times 10^6}$

  $= 7.9\ km/s \simeq 8\ km/s$

  Where

  $V \rightarrow$ Orbital velocity

  $g \rightarrow 9.8\ m/s^2$

$R \rightarrow$ Radius of Earth

- **Angular momentum of satellite**

$L = mvr$

$L = \sqrt{m^2 GMr}$

$L =$ Angular momentum

$m \rightarrow$ mass of satellite

$v \rightarrow$ Orbital velocity of the satellite

- **Relation of escape velocity and orbital velocity**

we know that $V_e = \sqrt{\frac{2GM}{R}}$;

and $V = \sqrt{\frac{GM}{R}}$

$\Rightarrow V = \frac{V_e}{\sqrt{2}}$

Where

$V \rightarrow$ Orbital velocity

$V_e \rightarrow$ Escape velocity

or $V_{escape} = \sqrt{2} V_{orbital}$

Or we can say that

If the speed of satellite is made $\sqrt{2}$ times the original speed, then it will escape from the gravitational pull of the earth.

- **Shape of orbit of satellite**

If $0 < V < v_o$, then satellite does not remain in it's circular path rather it traces a spiral path and falls on earth

$V = v_o$ Satellite revolves in circular path

$V = v_e$ satellite move along the parabolic path and will escape from gravitational pull.

$V > v_e$ satellite will escape but now the of motion will be hyperbolic.

Here,

V= velocity of body

v_o - orbital velocity of a body

v_e - escape velocity of a body

13.Time period and energy of a satellite

The time period of satellite-

It is the time taken by satellite to go once around the earth.

And the time period (T) of the satellite is given by

$$T = 2\pi\sqrt{\frac{(R+h)^3}{gR^2}} = 2\pi\sqrt{\frac{R}{g}}\left(1+\frac{h}{R}\right)^{3/2} \quad [\text{As } r = R+h]$$

Where

$r =$ radius of orbit

$T \rightarrow$ Time period

$M \rightarrow$ Mass of planet

- If the satellite is very close to the earth's surface,

i.e., h<<<R,

then $T = 2\pi\sqrt{\frac{R}{g}} \cong 84.6\ minutes$

or $T \simeq 1.4hr$

- The time period of a satellite in terms of density

$$T = \sqrt{\frac{3\pi}{G\rho}}$$

$\rho \rightarrow$ Density of planet

$T \rightarrow$ Time period

$G \rightarrow$ Gravitational constant

$\rho = 5478.4 Kg/m^3$ for earth

Height of Satellite-

As we know, time period of satellite $T = 2\pi\sqrt{\frac{r^3}{GM}} = 2\pi\sqrt{\frac{(R+h)^3}{gR^2}}$

By squaring and rearranging both sides $\frac{gR^2T^2}{4\pi^2} = (R+h)^3$

$\Rightarrow \quad h = \left(\frac{T^2gR^2}{4\pi^2}\right)^{1/3} - R$

Putting the value of time period in the above formula we can calculate the height of the satellite from the surface of the earth.

The energy of Satellite-

When a satellite revolves around a planet in its orbit, it possesses both kinetic energy (due to orbital motion) and potential energy (due to its position against the gravitational pull of earth).

And these energies are given by

Potential energy: $U = mV = \frac{-GMm}{r} = \frac{-L^2}{mr^2}$

Kinetic energy : $K = \frac{1}{2}mv^2 = \frac{GMm}{2r} = \frac{L^2}{2mr^2}$

Total energy : $E = U + K = \frac{-GMm}{r} + \frac{GMm}{2r} = \frac{-GMm}{2r} = \frac{-L^2}{2mr^2}$

Where

$M \rightarrow mass\ of\ planet$

$m \rightarrow mass\ of\ satellite$

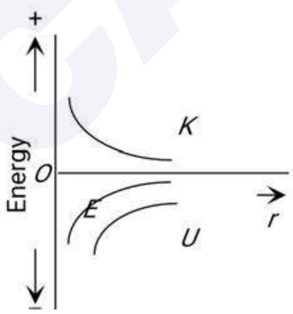
And

$K = -E$

$U = 2E$

$U = -2K$

- Energy Graph of satellite



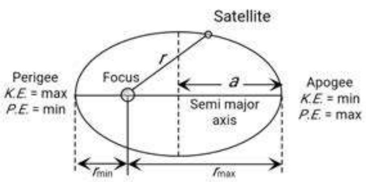
Where

$E \rightarrow Energy\ of\ satellite$

$K \rightarrow Kinetic\ energy$

$U \rightarrow Potential\ energy$

- Energy distribution in an elliptical orbit



In this **Total Energy**

$$E = -\frac{GMm}{2a} = const.$$

Where $a = semi - major\ axis$

- **Binding Energy (B.E.)-**

The minimum energy required to remove the satellite from its orbit to infinity is called Binding Energy.

And It is given by

$$B.E = \frac{GMm}{2r}$$

where

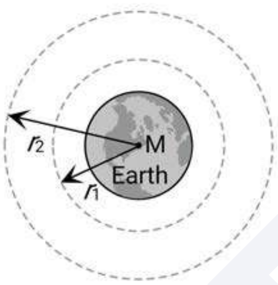
$B.E \rightarrow Binding\ energy$

$M \rightarrow mass\ of\ planet$

$m \rightarrow mass\ of\ satellite$

- Work done in changing the orbit-

When the satellite is transferred to a higher orbit i.e $(r_2 > r_1)$ as shown in the figure.



$$W = E_2 - E_1$$

$$W = \frac{GMm}{2}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

Where

$W \rightarrow work\ done$

$r_1 \rightarrow radius\ of\ 1st\ orbit$

$r_2 \rightarrow radius\ of\ 2nd\ orbit$

14.Weightlessness

There are basically three cases of weightlessness -

1. When objects fall freely under gravity -

   When a man is in a free-falling lift, then he will feel the weightlessness.

2. When a satellite revolves in its orbit around the earth -

   Because of this, the astronauts will feel weightlessness in the satellite.

3. When bodies are at null points in outer space -

As we go up from the earth's surface the gravitational pull of the earth goes on decreasing and the gravitational pull of the moon's increasing. There i a point when both the gravitational force will be equal and opposite, that null the weight of the body and we feel weightlessness.

Weightlessness in a Satellite.

The acceleration of the satellite is $\frac{GM}{r^2}$ towards the centre of the earth.

Let us suppose a body of mass m placed on a surface inside the satellite moving around the earth.

Then force on the body are -

$$\begin{aligned}
&\text{(i) The gravitational pull of earth } = \frac{GMm}{r^2} \\
&\text{(ii) The reaction by the surface } = R \\
&\text{By Newton's law } \frac{GmM}{r^2} - R = ma \\
&\quad \frac{GmM}{r^2} - R = m\left(\frac{GM}{r^2}\right) \quad \therefore \quad R = 0
\end{aligned}$$

As the reaction becomes 0.

And from the Laws of motion, we know that the reaction on a body will give its weight.

So, the body will feel weightlessness in the satellite.

Mechanical Properties of Solids

Important Formulae

1. Elasticity

The property of matter by virtue of which a body tends to regain its original shape and size after the removal of deforming force is called elasticity.

2. Stress and its types-

Stress-

When a force is applied on a body, it is deformed to a small or large extent depending upon the nature of the material of the body and the magnitude of the deforming force.

- The internal restoring force acting per unit area of the cross-section of the deformed body is called stress and is denoted by σ.
- The magnitude of stress, $\sigma = \frac{F}{A}$
- Unit of stress: N/m^2 or Pascal(Pa)
- Dimension of stress: $[ML^{-1}T^{-2}]$

Types of stress:

1. **Longitudinal stress/ Normal stress:** In Longitudinal stress, the force is applied normal to the surface.
    - It is of two types:
        a. **Tensile stress:** Longitudinal stress produced due to increase in length of a body under a deforming force is called tensile stress.
        b. **Compressive stress:** Longitudinal stress produced due to decrease in length of a body under a deforming force is called compressional stress.

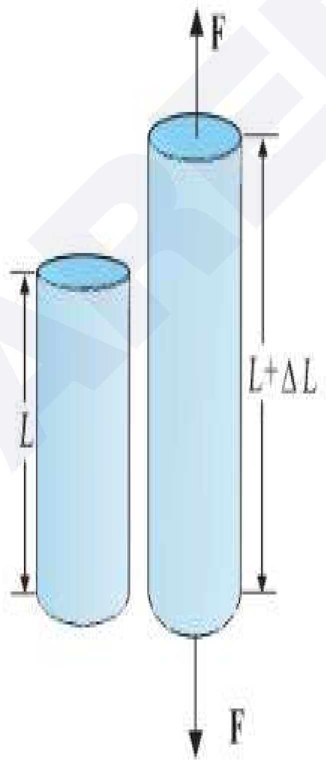


Fig: Tensile Stress

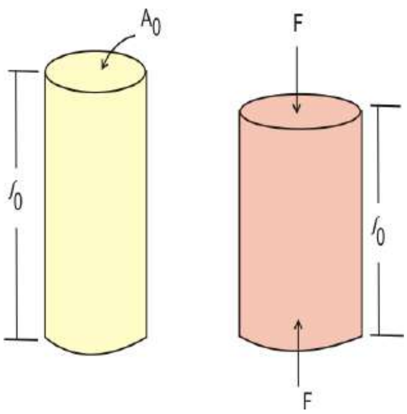
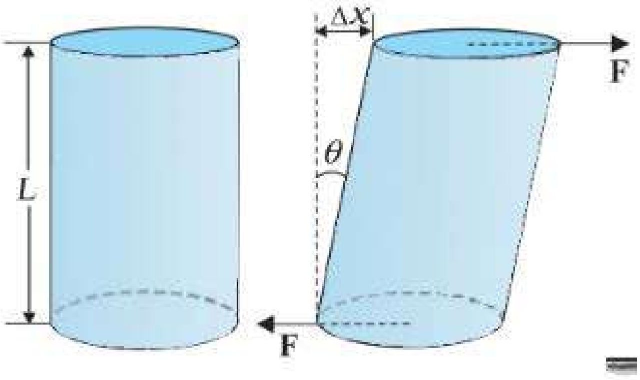


Fig: Compressive Stress

2. **Shearing stress/ tangential stress:** If two equal and opposite deforming forces are applied parallel to the cross-sectional area of the cylinder, there is a relative displacement between the opposite faces of the cylinder. The restoring force per unit area developed due to the applied tangential force is know as tangential or shearing stress.

- In this case, deforming force is applied tangential to one of the faces.
- Area for calculation is the area of the face on which force is applied.
- It produces change in shape, volume remaining the same.



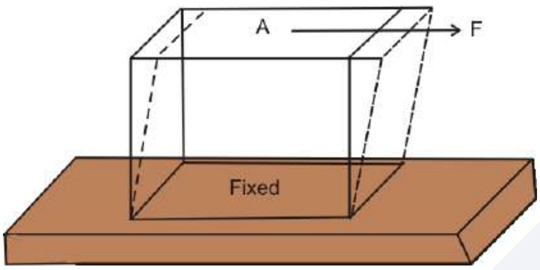
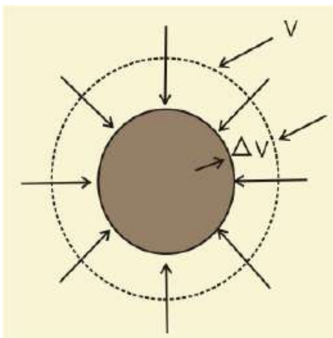


Fig:- Shearing stress

3. **Volume stress:**

- It produces change in volume and density, shape remaining the same.
- It occurs in solids, liquids or gases
- In case of fluids only bulk stress can be found.
- It is equal to change in pressure because change in pressure is responsible for change in volume.

$$Volume\ \ stress = \frac{F}{A} = P$$



3.Strain and it's types-

Strain -

- Strain is defined as the ratio of change in configuration to the original configuration.

- It has no dimensions and units as it is the ratio of two similar kind of physical quantities.

Types of strain:-

1. **Longitudinal strain:-** If the deforming force produces a change in length alone, the strain produced in the body is called longitudinal strain.

    - If the length increases from its natural length, the longitudinal strain is called **tensile strain.**
    - If the length decreases from its natural length, the longitudinal strain is called **compressive strain.**

$$Longitudinal\ strain = \frac{\Delta L}{L}$$

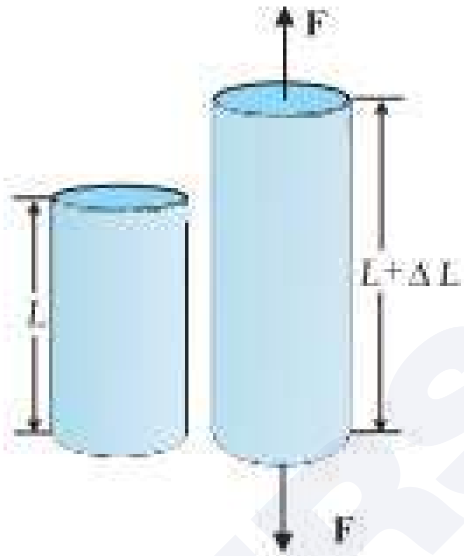


Fig: Tensile strain

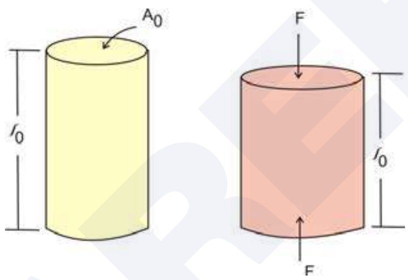


Fig: Compressive strain

2. **Shear strain:-** If the deforming force produces a change in the shape of the body without changing its volume, strain produced is called shearing strain

- It is defined as angle in radians through which a plane perpendicular to the fixed surface of the cubical body gets turned under the effect of tangential force.

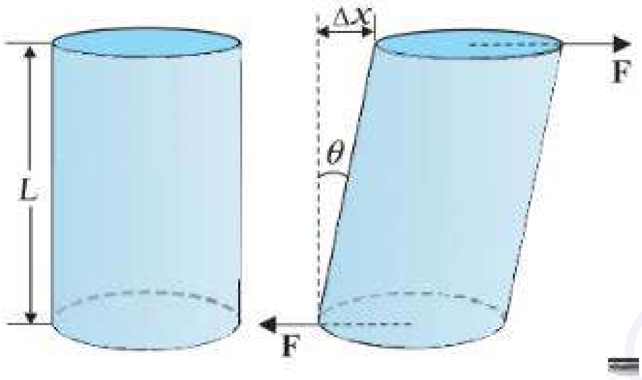


Fig:- Shearing strain

$$Shearing\ strain = \frac{\Delta x}{L}$$

- Example:- when a book is pressed with the hand and pushed horizontally.

Fig:- A book subjected to a shearing stress

3. **Volume Strain:-** If the deforming force produces a change in volume alone the strain produced in the body is called volumetric strain.

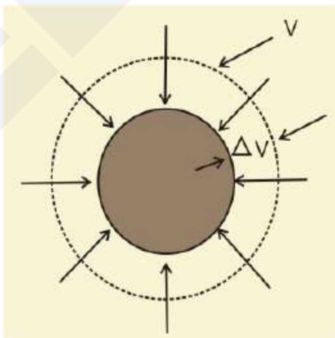


Fig:- Volumetric strain

$$Volume\ strain = \frac{\Delta V}{V}$$

4. Stress-strain Curve.

- The relation between the stress and the strain or a given material under tensile stress can be plotted on a graph called the strain stress curve.

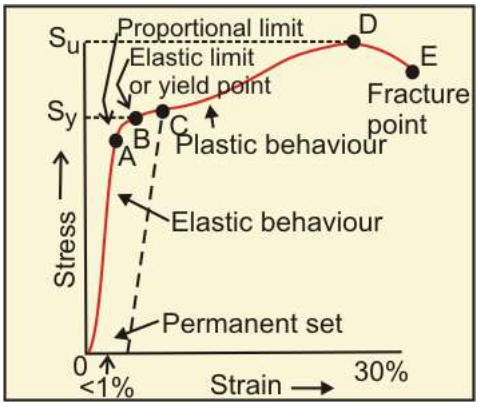


Fig:-A typical stress-strain curve for a metal.

The stress-strain curves vary from material to material. These curves help us to understand how a given material deforms with increasing loads.

1. When the strain is small (i.e., in region OA) stress is proportional to strain. This is the region where the Hooke's law is obeyed. The point A is called proportional limit and slope of line OA gives the Young's modulus (Y) of the material of the wire.
2. If the strain is increased a little bit, i.e., in the region AB, the stress is not proportional to strain. However, the wire still regains its original length after the removal of stretching force. This behaviour is shown up to point B known as elastic limit or yield-point. The region OAB represents the elastic behaviour of the material of wire.
3. If the wire is stretched beyond the elastic limit B, i.e., between BC, the strain increases much more rapidly and if the stretching force is removed the wire does not come back to its natural length. Some permanent increase in length takes place.
4. If the stress is increased further by a very small amount, a very large increase in strain is produced (region CD) and after reaching point D, the strain increases even if the wire is unloaded and ruptures at E. In the region DE, the wire literally flows. The maximum stress corresponding to D after which the wire begins to flow and breaks is called breaking or tensile strength. The region BCDE represents the plastic behaviour of the material of wire.

Types of materials:-

a. Ductile material:- If the large deformation in the material takes place between elastic limit and fracture point (or)

if the material is having large plastic region, then that material is called ductile material.

a. Brittle material:- If the material breaks down soon after the elastic limit is crossed, it is called as brittle material.

b. Elastomers:- These materials only have elastic region (i.e., no plastic region). For example:- rubber

5.Hooke's law

Hooke's law states that if the deformation is small, the stress in a body is proportional to the corresponding strain, i.e.,

$$Stress \ \alpha \ Strain$$

$$\Rightarrow Stress = E(Strain)$$

$$\Rightarrow E = \frac{Stress}{Strain}$$

Where E is called as Modulus of elasticity and it depends on the nature of the material and temperature of the body and is independent of the dimensions of tł body.

Unit of Modulus of elasticity= N/m^2

Modulus of elasticity is of three types:-

a. **Young's Modulus(Y):-** It is defined as the ratio of longitudinal stress to longitudinal strain.

$$Y = \frac{longitudinal \ stress}{longitudinal \ strain} = \frac{F/A}{\Delta L/L} = \frac{Fl}{A\Delta L}$$

b. **Shear Modulus or Modulus of rigidity(G):-** It is defined as the ratio of shearing stress to the shearing strain.

$$G = \frac{shearing \ stress}{shearing \ strain} = \frac{F/A}{x/L} = \frac{Fl}{Ax} = \frac{F}{A\phi}$$

c. **Bulk Modulus(B):-** It is defined as the ratio of volume stress to the volume strain.

$$Volume stress = \frac{F}{A} = Pressure$$

$$B = -\frac{P}{\Delta V/V}$$

where P=increase in pressure , V= original volume, ΔV=change in volume

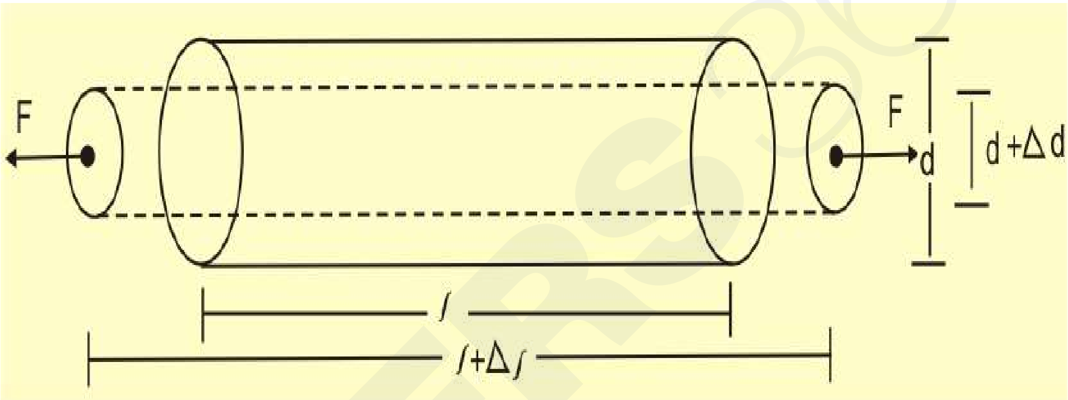
The negative sign indicates that with the increase in pressure, volume decreases by ΔV.

- **Compressibility(C):-** The reciprocal of bulk modulus is called as compressibility.

$$C = \frac{1}{B} = -\frac{\Delta V/V}{P}$$

- Unit of compressibility:- $N^{-1}m^2$
- **Poisson's ratio:-** The ratio of lateral strain to longitudinal strain is called Poisson's ratio (σ).

  a. Lateral strain : The ratio of change in radius or diameter to the original radius or diameter is called lateral strain.
  b. Longitudinal strain : The ratio of change in length to the original length is called longitudinal strain.

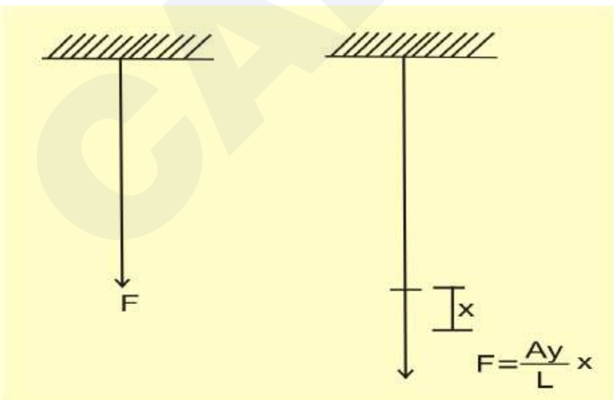


$$\sigma = -\frac{\Delta d/d}{\Delta l/l}$$

where negative sign is due to decrease in the transverse length

6.Work done in stretching a wire

When a body is in its natural shape, its potential energy corresponding to the molecular forces is minimum. When deformed, internal forces appear and work has to be done against these forces. Thus, the potential energy of the body is increased. This is called the elastic potential energy.



Suppose a wire having natural length L and cross-sectional area A is fixed at one end and is stretched by an external force applied at the other end.

The elastic potential energy of the stretched wire is:

$$U = \frac{1}{2}\frac{YA}{L}(\Delta L)^2$$

We can also write,

$$W=\frac{1}{2}\left[\frac{YA\Delta L}{L}\right]\Delta L=\frac{1}{2}(maximum\ stretching\ force)\times extension$$

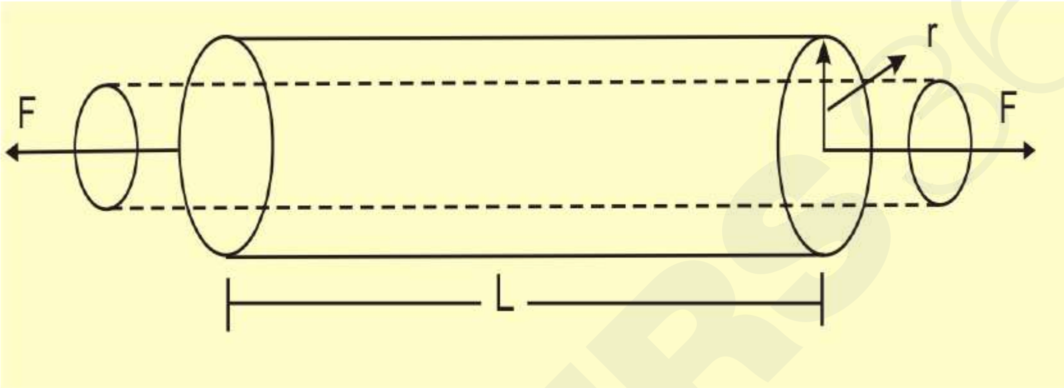
$$W=\frac{1}{2}Y(AL)\left[\frac{\Delta L}{L}\right]^2=\frac{1}{2}\times Y\times Volume\times(strain)^2$$

$$W=\frac{1}{2}\left[\frac{\Delta L}{L}\right]\left[Y\frac{\Delta L}{L}\right](AL)=\frac{1}{2}\times strain\times(Y strain)\times Volume$$

$$W=\frac{1}{2}\left[\frac{\Delta L}{L}\right]\left[Y\frac{\Delta L}{L}\right](AL)=\frac{1}{2}\times strain\times stress\times Volume$$

$$Also, Potential\ energy\ per\ unit\ volume=\frac{1}{2}\times strain\times stress$$

7. Relation Between Volumetric Strain, Lateral Strain and Poisson's Ratio



Let us long rod have a length L and radius 'r', then volume of this rod = $\pi r^2 L........(1)$

Now, Differentiating both the sides of (1), we get

$$dV=\pi r^2 dL+\pi 2rLdr$$

Now, dividing both the sides by volume of rod , i.e., $\pi r^2 L$, we get ,

$$\frac{dV}{V}=\frac{\pi r^2 dL}{\pi r^2 L}+\frac{\pi 2rLdr}{\pi r^2 L}=\frac{dL}{L}+2\frac{dr}{r}....(2)$$

So we can say that,

Volumetric strain = Longitudinal strain + 2(Lateral strain)

Also, $equation(2)$ can be written as,

$$\Rightarrow\frac{dV}{V}=\frac{dL}{L}-2\sigma\frac{dL}{L}=(1-2\sigma)\frac{dL}{L}$$

This is because , $\left[\sigma=\frac{-dr/r}{dL/L}\Rightarrow\frac{dr}{r}=-\sigma\frac{dL}{L}\right]$

Special case -

- When $\sigma=0.5$, then $dV=0$. It means that the substance is incompressible, so there is no change in volume.
- If a material having $\sigma=0$, it means lateral strain is zero. So, when a substance is stretched its length increases without any decrease in diameter.

For example - **cork has** $\sigma=0$. Also, in this case change in volume is maximum.

Mechanical Properties of Fluids

Important Formulae

1. Pressure In A Fluid

Fluids-

Fluids are the substances which began to flow when an external force is applied to it.

Fluids examples are liquids and gases.

Fluids don't have their own shape but they take shape of the containing vessel.

In Hydrostatic branch we study fluids which are at rest with respect to containing vessel.

While in Hydrodynamics we study fluids which are in motion with respect to containing vessel. For example study of flowing water from the tap comes under Hydrodynamics.

Ideal fluids- Assumption

1. It is Incompressible- Means the density and the specific volume of the fluid do not change during the flow.
2. It is Non-viscous- Layers of fluids does not exert any tangential force (especially friction force) on each other. And A true "non-viscous" fluid would flow along a solid wall without any slowing down because of friction.

Pressure-

1. Normal force exerted by liquid/fluid at rest per unit Area of the surface is called pressure of a liquid/fluid.

   If F is the normal force acting on a surface area A in a container with liquid.

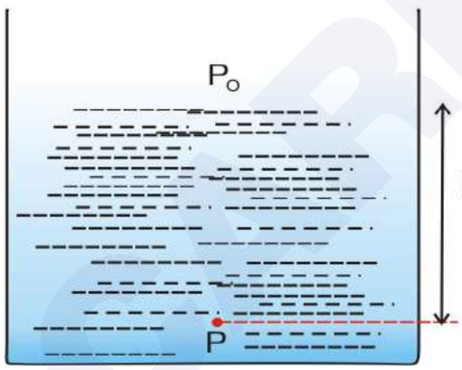
   then pressure exerted by the liquid on this surface is $P=\frac{F}{A}$
2. Units of pressure is N/m^2 or Pascal (S.I) and $dyne/cm^2$ (C.G.S)
3. The dimension of pressure is $ML^{-1}T^{-2}$
4. Atmospheric pressure- The pressure exerted by the atmosphere is called atmospheric pressure. 1 atm is the value of atmospheric pressure on the surface of the earth at sea level.

   And $1\ atm=1.01\times10^5 N/m^2=1.01\times10^5 Pascal=1.01\ bar=760\ torr$

   So Relation between bar and Pascal is $1 bar=10^5 Pa$

2.Variation Of Pressure

1. **Variation of pressure with depth**



Here P_0= Atmospheric pressure at the upper surface

And h= depth below the upper surface

ρ= density of liquid

P=Hydrostatic pressure for a point at depth h below the upper surface

Then P is given by $P=P_0+\rho gh$

Means Pressure increases with depth linearly.

- **Hydrostatic pressure=Absolute Pressure=**$P=P_0+\rho gh$

Absolute Pressure is always positive, It can never be zero.

From equation $P=P_0+\rho gh$

We can say that

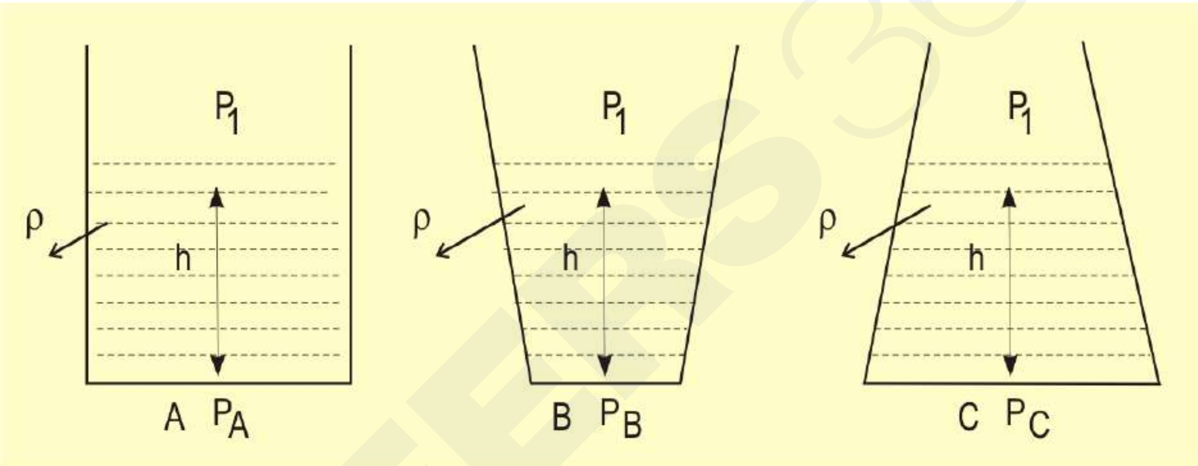
1. Hydrostatic pressure depends on

- h=depth of the point below the surface
- ρ=nature of liquid
- g=acceleration due to gravity

2. Hydrostatic pressure does not depend on

- amount of liquid
- The shape of the container

From this, we can say that for the below figure where the liquid is filled in vessels of different shapes to the same height,



The pressure at the base in each vessel will be the same, though

The volume or weight of the liquid in different vessels will be different.

I.e. In the above figure $P_A = P_B = P_C$

- **Gauge Pressure**- Gauge Pressure is known as the pressure difference between hydrostatic and atmospheric pressure.

So Gauge Pressure is given as $P - P_0 = gauge\ pressure$

In the equation $P = P_0 + \rho gh$

The term ρgh is known as pressure due to the liquid column of height h

We can rewrite the above equation as $\rho gh = P - P_0$

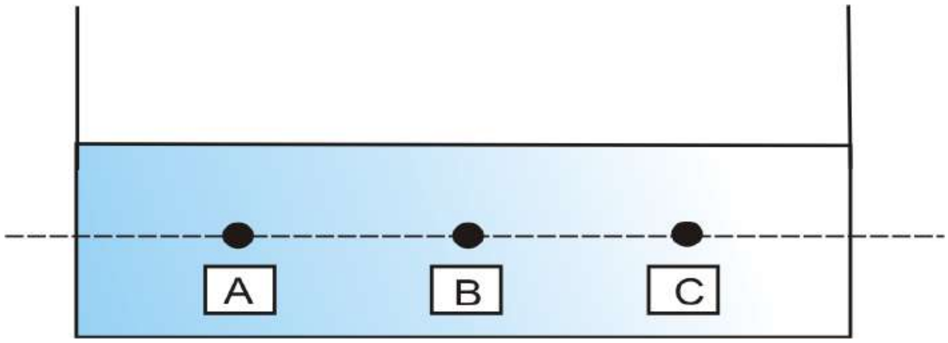
Or we can say that Gauge Pressure = $\rho gh = P - P_0$

It may be positive or negative or zero

2. **Variation of pressure along Horizontally**

The pressure is uniform on a horizontal line.

For the below figure



In horizontal line or in horizontal plane in stationary liquid

$P_A = P_B = P_C$

3. Pascal's Law

Pascal's law-

Pascal's law states that if the gravity effect is neglected then the pressure at a point in a fluid at rest is the same in all directions.

This law helps us to understand the isotropic nature of pressure.

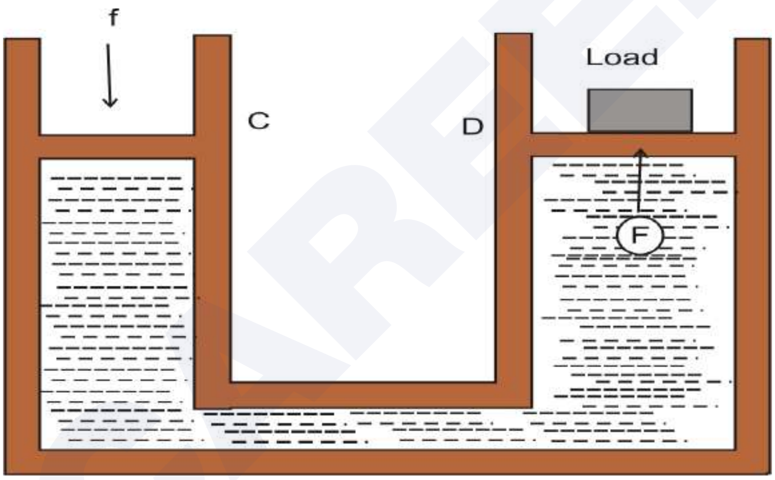
Pascal's law can be also stated as

The increase in pressure at one point of the enclosed liquid in the equilibrium of rest is transmitted equally to all other points of the liquid and also to the wall of the container, provided the effect of gravity is neglected.

The applications of this law can be seen in Hydraulic lift, hydraulic press, and hydraulic brakes, etc

Working of hydraulic lift-

A hydraulic lift is used to lift the heavy loads.



For the above figure

If a small force f is applied on the piston of C then the pressure exerted on the liquid

$P = \frac{f}{a}$

Where

a = Area of a cross-section of the piston in C

This pressure is transmitted equally to piston of cylinder D.

So $\frac{f}{a} = \frac{F}{A} \Rightarrow F = \frac{f}{a} A$

Where F=Upward force acting on the piston of cylinder D.

A=Area of a cross-section of the piston in D

Condition of Hydraulic Lift-

$A >> a \ therefore$

$F >> f$

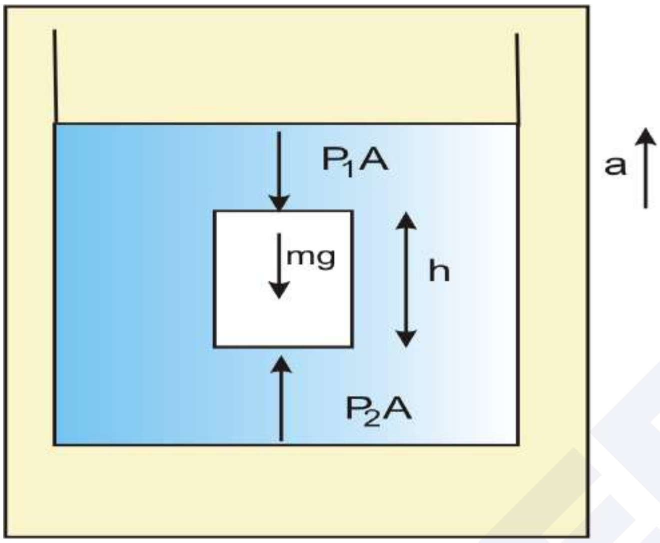
So heavy load placed on the larger Piston is easily lifted upward.

4. Variation Of Pressure In An Accelerated Fluid

Case I- When Acceleration in the vertical direction

1. **When the liquid container is moving with constant acceleration in an upward direction**

   Consider a cylindrical element of height h and Area A as shown in the below figure.



The force on the top face of the element $= P_1 A$

The force on the bottom face of the element $= P_2 A$

If a is the acceleration of the liquid then

and m is the mass of the element of the liquid and which is given by

$m = \rho h A$

Where ρ=density of liquid

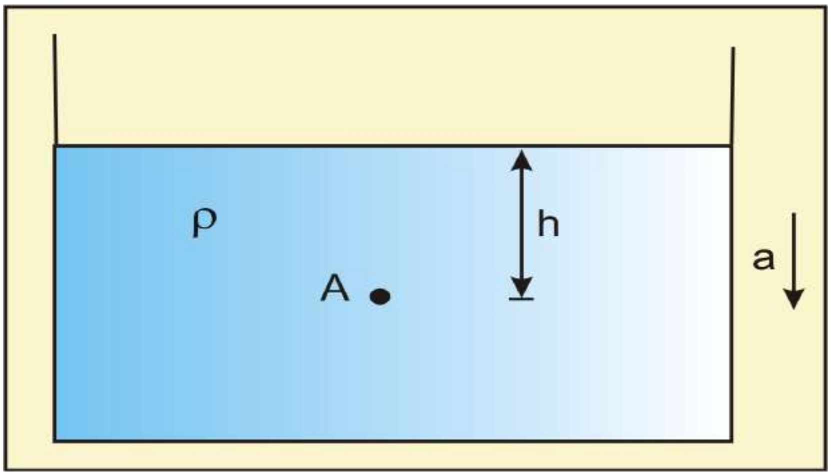
So using this we get

$P_2 - P_1 = \rho(g + a)h = \rho g_{eff} h$

2. **When the liquid container is moving with constant acceleration in a downward direction**

   I. constant downward acceleration (a< g)

   So g_{eff} for the below figure is given by $g_{eff} = (g - a)$



And Pressure at point A is given as

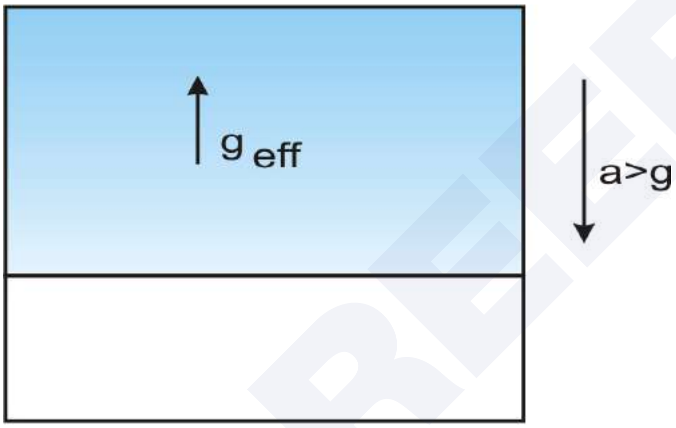
$$P = \rho(g-a)h = \rho g_{eff} h$$

II. constant downward acceleration (a=g)

The pressure became zero everywhere when a=g

III. Constant downward acceleration (a> g)

In this case, the fluid occupies the upper part of the container as shown in the figure.

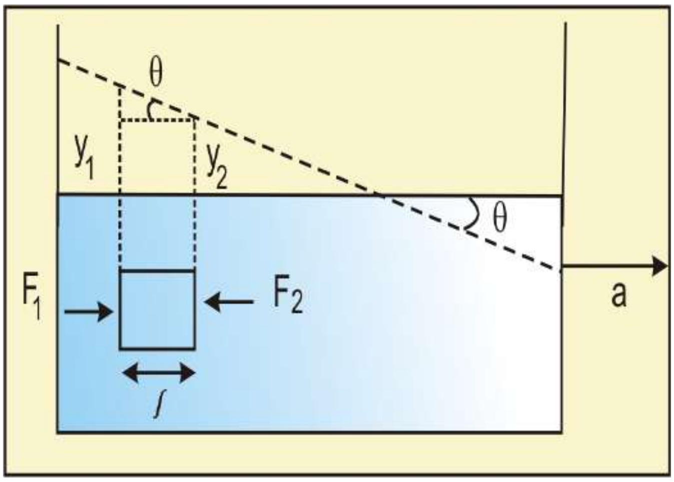


$g_{eff} = a - g$

Case II- When Acceleration in a Horizontal direction

If a liquid in the tank is moving on a horizontal surface with some constant acceleration a

Then the free surface of the liquid takes the shape as shown by the dotted line in the figure.



So using Newton's second law for the element

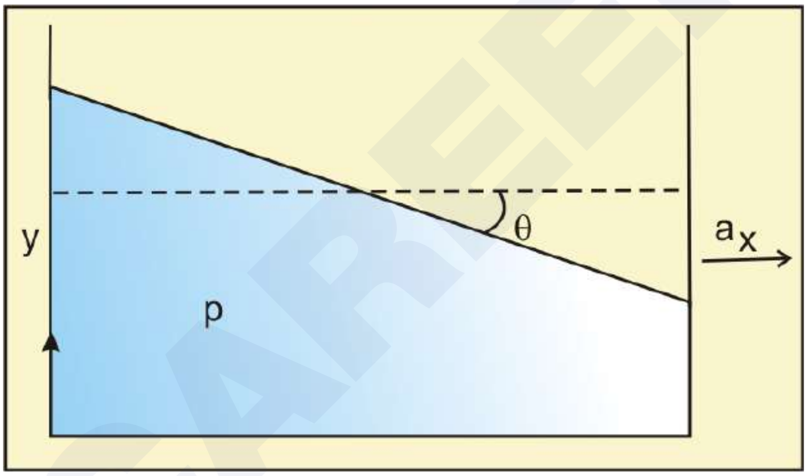
$$F_1 - F_2 = ma$$
$$or\ P_1A - P_2A = ma$$
$$or\ (\rho g y_1 - \rho g y_2)A = Al\rho a$$
$$or\ \frac{y_1 - y_2}{l} = \frac{a}{g} = tan\theta$$

So we can say that

The free surface of the liquid makes an angle θ with horizontal

Or the free surface of the liquid orient itself perpendicular to the direction of net effective gravity.

So for the below figure, we can say that



Pressure will vary in the horizontal direction.

And the Pressure gradient in the x-direction is given as

$$\frac{dp}{dx} = -\rho a_x$$

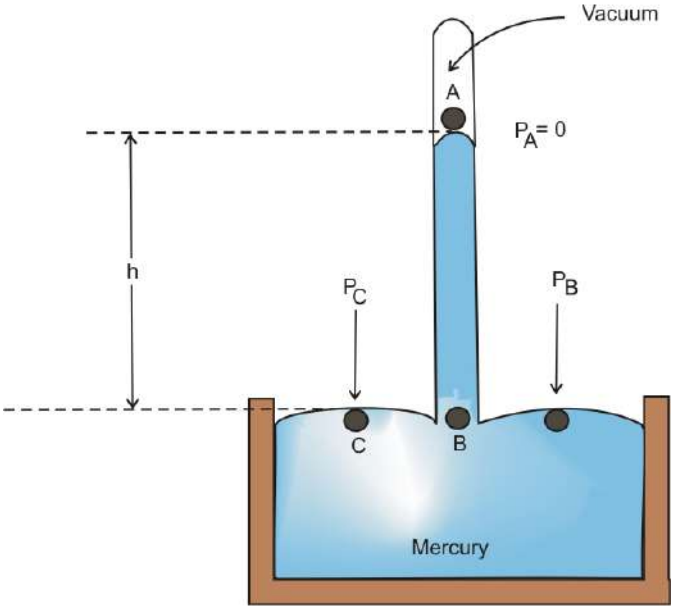
Where -ve sign indicates pressure increases in a direction opposite to the direction of acceleration.

5.Barometer And Manometer

Barometer-

Mercury Barometer was invented by Torricelli and it is a device used for measuring pressure.

Below is a figure showing the Mercury Barometer device



From the figure we can say that

At Point C only atmospheric pressure is there.

So $P_C = P_0$

Ans since B and C are at the same horizontal level

So $P_C = P_B = P_A + \rho g h = 0 + \rho g h = \rho g h = P_0 = Atmospheric\ pressure$

For mercury barometer h=76 cm and

And using $\rho = 13.6 * 10^3 kg/m^3$

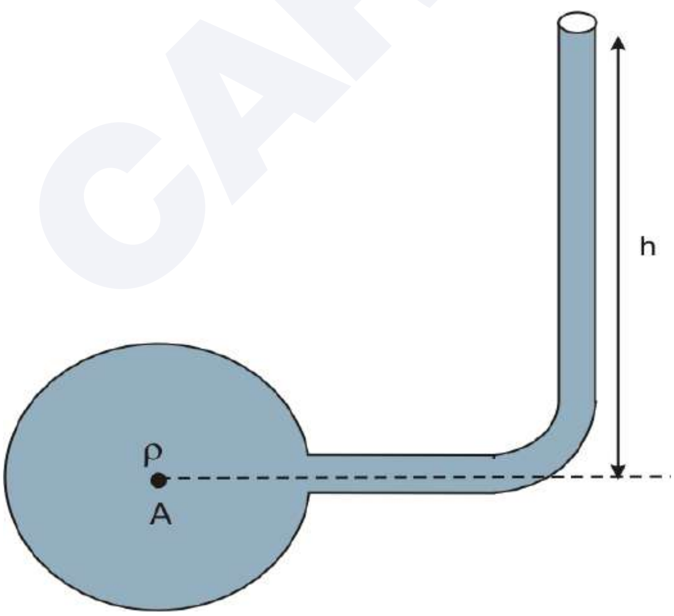
We get $P_0 = 1.013 * 10^5 N/m^2 = 1\ atm = 76\ cm\ of\ Hg$

Manometer-

Manometers are the devices used for measuring gauge pressure of fluids.

Simple Manometer-

The figure of Simple Manometer is given below



And from the figure we can say that

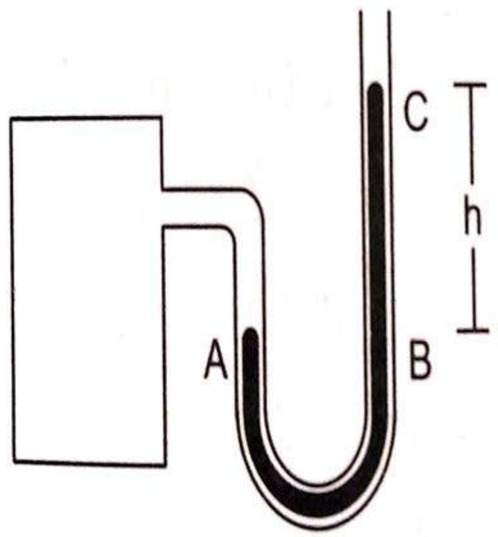
The gauge pressure at point A in the vessel is

$P_A = \rho g h$

Where ρ=density of the liquid

U-tube Manometer-

Figure of U-tube manometer is given below



And from the figure we can say that

$P_A = P_B$

And The gauge pressure at point A is given by

$P_A = \rho g h$

Where

ρ=density of the liquid

6.Archimedes Principle

Archimedes principle states that when a body is immersed partly or wholly in a fluid, then the liquid exerts an upward force/upthrust/buoyant force on the body which is equal to the weight of the fluid displaced by the body.

Buoyant force-

- The buoyant force is given as

$F_B = \rho V g$

Where F_B=Buoyant force

ρ= density of the fluid

V= Volume of the solid body immersed in the liquid or Volume of the fluid displaced

$As\ \rho = \frac{m}{V}$

So we can write $F_B = mg$=weight of the fluid displaced

Where m=mass of the fluid displaced

- The buoyant force acts vertically upwards (opposite to the weight of the body)
- The buoyant force is independent of mass, size, the density of the body inside the fluid.
- The buoyant force depends upon the nature/density of the displaced fluid.

Apparent weight-

- Apparent weight=(Actual weight)-(Buoyant force)
- The apparent weight of the body of density (ρ) when immersed in a liquid of density (σ) is given by

$$W_{app} = W - F_B = V\rho g - V\sigma g = V\rho g(1 - \frac{\sigma}{\rho})$$

$$W_{app} = W(1 - \frac{\sigma}{\rho})$$

Where W= Actual weight of the body

V= volume of the body immersed in a liquid

From this, we can say that

If a body of volume V is immersed in a liquid of density (σ) Then its weight reduces

And Loss in weight is given by

$$W_{loss} = W - W_{app} = V\sigma g$$

- **The relative density of a body**

$$R.D = \frac{density\ of\ body}{density\ of\ water}$$

- **Floatation-**

When a body of density ρ is immersed in a liquid of density σ,

Then the body will float if the buoyant force on the body which is equal to the weight of the fluid displaced by the body. Means body is in equilibrium.

1. If the density of the body is equal to that of liquid i.e $\rho = \sigma$

   Then the Weight of the body will be equal to upthrust.

   And the body will float but the body will fully be submerged in liquid.

2. If the density of the body is less than that of liquid $\rho < \sigma$

   Then the Weight of the body will be less than upthrust.

   And the body will float but the body will partially be immersed in liquid.

- If the density of the body is greater than that of liquid $\rho > \sigma$

Then the Weight of the body will be greater than upthrust So body will sink.

7. Type Of Flow

- **Steady flow-**

In this type of flow fluid characteristics like Velocity, Pressure and density etc at a Point do not change with time.

I.e $\frac{dv}{dt} = 0$, $\frac{dp}{dt} = 0$, $\frac{d\rho}{dt} = 0$

- **Unsteady flow-**

In this type of flow fluid Characteristics like Velocity, Pressure and density etc At a Point change with respect to time.

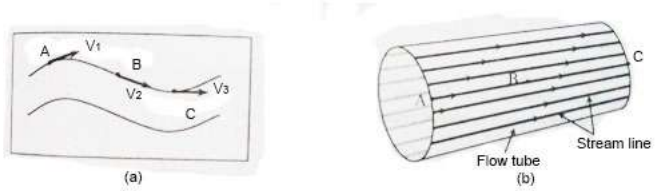
I.e $\frac{dv}{dt} \neq 0$, $\frac{dp}{dt} \neq 0$, $\frac{d\rho}{dt} \neq 0$

- **Streamline flow-**

Streamline the flow of a liquid is the type of fluid flow in which each particle of the fluid passing through a point travels along the same path and with the san velocity as the preceding element passes through that point.

Or

Streamline flow is defined as the path (straight or curved), the tangent to which at any point gives the direction of the flow of liquid.



(a) Stream-line in a liquid (b) Stream line flow of liquid.

For the above figure path ABC is streamlined.

And All the liquid particles passing through A, B, and C will have velocities as $V_1, V_2 \ \ and \ \ V_3$ respectively.

Property of streamline flow-

1. The direction of velocity at any point on the flow line is along the tangent.
2. Two streamlines cannot cross each other.

- **Laminar flow -**

If a liquid is flowing over a horizontal surface with a steady flow and moves in the form of infinitesimal parallel layers of different velocities which do not mi with each other, then the flow of liquid is called laminar flow.

This type of flow is also referred to as streamline flow.

In this flow, the velocity of liquid flow is always less than the critical

velocity of the liquid.

- **Turbulent Flow-**
When the velocity of liquid flow is greater than its critical velocity, then the motion of the particles of the liquid becomes disordered or irregular. Such a flow is called turbulent flow.

 In turbulent flow velocity of the fluid at a point is continuously undergoing changes in both magnitude and direction.

- **Critical velocity-**

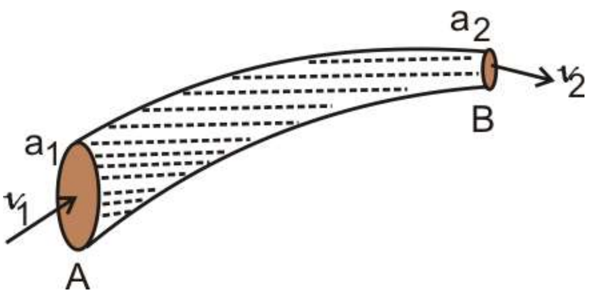
Critical velocity is defined as that velocity of the liquid, flow up to which it's streamlined/laminar and above which it's flow become turbulent.

8.Equation Of Continuity

It is applied when fluid is an ideal fluid. (means fluid is Incompressible and Non-viscous)

The equation of continuity is derived from the **principle of conservation of mass.**

Have a look at the flow of ideal fluid through the tube AB.



Equation of Continuity for the liquid flow in tube AB is given by

$$a_1 v_1 = a_2 v_2$$
$$or \quad av = constant$$

Or the Equation of Continuity states that for the liquid flow in the tube, the product of cross-section area and velocity remains the same at all points in the tub

From the Equation of Continuity, we can say that

- The velocity of flow will increase if cross-section decreases and vice-versa.

9. Bernoulli's Theorem

For a point in a fluid flow, **Bernoulli's Theorem** relates between its pressure, its velocity and its height from a reference point.

Bernoulli's Theorem states that the total energy (Pressure energy, Potential energy, and Kinetic energy) per unit volume or mass of an incompressible and non-viscous fluid in steady flow through a pipe remains constant throughout the flow. (Provided that there is no source or sink of the fluid along the length of the pipe).

Mathematically for a liquid flowing through a pipe.

We can write **Bernoulli's equation** as

$$P + \rho g h + \frac{1}{2}\rho v^2 = constant$$

$$P \rightarrow Pressure\ energy\ per\ unit\ volume$$

$$\rho g h \rightarrow Potential\ Energy\ per\ unit\ volume$$

$$\frac{1}{2}\rho v^2 \rightarrow Kinetic\ Energy\ per\ unit\ volume$$

Bernoulli's Theorem can be proved with the help of **work-energy theorem.**

Bernoulli's equation also represents the **conservation of mechanical energy** in the case of moving fluids.

- **Bernoulli's theorem for the unit mass of liquid flowing through the pipe is given by**

$$\frac{P}{\rho} + gh + \frac{1}{2}v^2 = constant$$

If we divide the above equation by g we get

$$\frac{P}{\rho g} + h + \frac{v^2}{2g} = constant$$

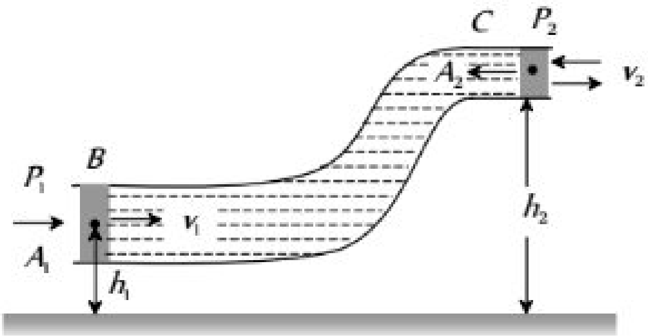
Where

h=gravitational head

$$\frac{P}{\rho g} \rightarrow Pressure\ head$$

$$\frac{v^2}{2g} \rightarrow velocity\ head$$

- For the below figure



With the help of Bernoulli's equation

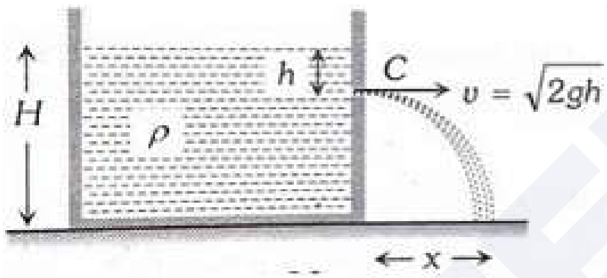
We can write

$$P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2 = constant$$

Applications of Bernoulli's Theorem-

1. The velocity of Efflux or Torricelli's Theorem-

If a liquid is filled in a vessel up to height H and a hole is made at a depth h below the free surface of the liquid as shown in Fig.



Now take the level of the hole as reference level (i.e., zero point of potential energy)

And by applying Bernoulli's equation we get

$$v = \sqrt{2gh}$$

This v is called the **Velocity of Efflux.**

This formula is only valid when (Area of Hole) <<< (Area of the vessel)

Thus Torricelli's Theorem relates the speed of fluid flowing out of an orifice.

(Note- The speed that an object would acquire in falling

from rest through a distance h is equal to $v = \sqrt{2gh}$

And this is same as that of Velocity of Efflux.)

- The velocity of efflux is independent of the nature of liquid (ρ), the quantity of liquid in the vessel and the area of the orifice/hole.
- The velocity of efflux depends on h (i.e depth below the free surface)

  I.e Means Greater is the distance of the hole from the free surface of the liquid, greater will be the velocity of efflux
- As the distance of hole from the ground is (H-h) and its initial vertical velocity at hole is zero.

  So Time taken by liquid to reach the Ground =t is given by

  $$T = \sqrt{\frac{2(H-h)}{g}}$$

  Where

  H - the height of the vessel

  And h= depth below the free surface

- **Range (x)-**

  During time t liquid is moving horizontally with constant velocity v,

  And it will hit the base level at a horizontal distance x as shown in the above figure.

  This horizontal distance x is also called a Horizontal range.

  Using x=vt

  We get Range as

  $x = R = 2\sqrt{h(H-h)}$

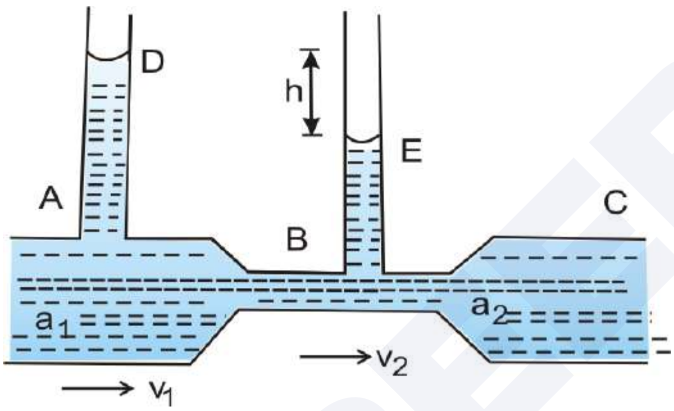
  This range will be maximum when $h = \frac{H}{2}$

  And Maximum value of the range is H

  Means $x_{max} = R_{max} = H$

2.Venturimeter-

- It is a device is used for measuring the rate of flow of liquid through pipes.
- This device based on application **Bernoulli's theorem.**
- The image of the Venturi Meter device is given below



For the above figure

a_1 and a_2 are an area of cross-section of tube A and B respectively

And v_1 and v_2 are the Velocities of the flow of liquid through A and B respectively

And $P_1 \;\; and \;\; P_2$ are the Liquid pressure at A and B respectively

and ρ= density of flowing liquid

And h=difference of fluid level between the vertical tube D and E

V=rate of the flow of liquid through pipe

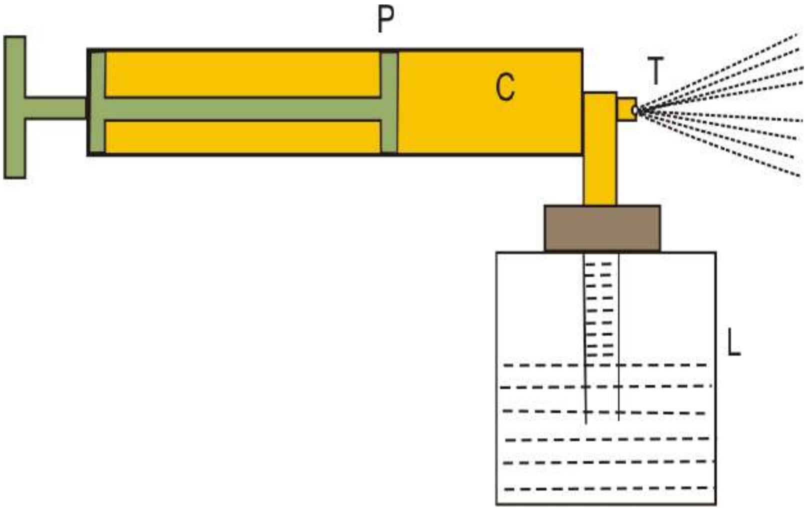
And V is given by

$$V = a_1 a_2 \sqrt{\frac{2gh}{a_1^2 - a_2^2}}$$

3. Aspirator pumps-

This works on the principle of Bernoulli's Theorem.

Example of Aspirator pumps is paint-gun, scent-spray or insect-sprayer, etc.



In such devices, high-speed air is passed over a tube T with the help of motion of a piston P in a cylinder C and this helps to spray the liquid L as shown in the above figure.

The high-speed air creates low pressure in the tube and because of the low-pressure liquid rise in it. And thus liquid gets sprayed with expelled air.

4. Change of plane of motion of spinning ball-

This can be with the help of the principle of Bernoulli's Theorem

Magnus effect- When a spinning ball is thrown it deviates from its usual path in flight. This effect is called the Magnus effect.

This effect plays a very important role in sports like cricket, tennis, and football, etc.

5.Working of an aeroplane-

This is also based on Bernoulli's principle.

6. During a tornado or hurricane, blowing off roofs by wind storms can be explained

with the help of the principle of Bernoulli's Theorem

10.Viscosity

- **Viscosity-**

Ideal fluids are non-viscous. But for real fluids, there is a viscous force between the adjacent layers of fluids which are in contact.

In case of a steady flow of a fluid when a layer of fluid slips or tends to slip on adjacent layers in contact, the tangential force/viscous force acting between two adjacent layers try to stop the relative motion between them.

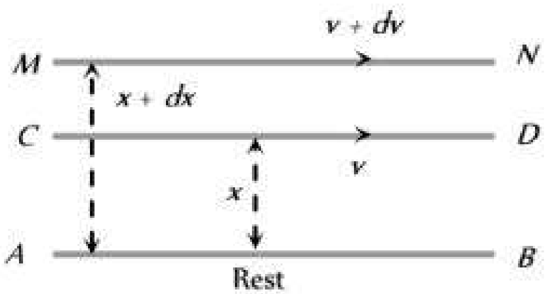
So The property of a fluid due to which it opposes the relative motion between its different layers is called viscosity.

Viscosity is also known as fluid friction or internal friction.

- **Velocity gradient -**

It is defined as the ratio of change in velocity to change in height.

I.e $Velocity\ gradient = \frac{chane\ in\ velocity}{change\ in\ height}$



For the above figure

Layer AB is at rest

While Layer CD is having velocity v and is at a distance x from layer AB

Similarly, Layer MN is having velocity (v+dv) and is at a distance (x+dx) from layer AB

Then Velocity gradient is given as $Velocity\ gradient = \frac{dv}{dx}$

Means Velocity gradient denotes the rate of change of velocity with distance x.

- **Viscous Force-**

In case of a steady flow of fluid, the force between the fluid layers opposing the relative motion is called viscous force.

1. Viscous force directly proportional to the area

I.e $F \alpha A$

Where A is the area

2. Viscous force directly proportional to the Velocity gradient

I.e $F \alpha \frac{dv}{dx}$

So we can write

$$F \alpha \frac{Adv}{dx} = F = -\eta A \frac{dv}{dx}$$

Where $A - Area$

$F - Viscous force$

$\eta = Co-efficient\ of\ viscosity$

$v - Velocity\ of\ liquid$

$x\ -\ Distance\ from\ refrence\ point$

And here Negative sign shows viscous force acts opposite to flow of liquid

- **Coefficient of viscosity-**

From the equation $F = -\eta A \frac{dv}{dx}$

$$If\ A = 1\ and\ \frac{dv}{dx} = 1$$
$$then\ \ \eta = F$$

So the coefficient of viscosity is defined as the viscous force acting per unit area between two layers moving with unit velocity gradient.

1. Coefficient of viscosity shows the nature of liquids.

2. Unit of viscosity is $dyne - s - cm^{-2}\ or\ Poise$ in CGS system

And $Newton\ - S - m^{-2}\ or\ Poiseuille\ or\ decapoise$ in the SI system

And 1 decapoise = 10 Poise

3. The dimension of viscosity is $ML^{-1}T^{-1}$

4. The cause of viscosity in liquids is cohesive forces among molecule whereas, in gases, it is due to the diffusion of molecules.

5. The viscosity of the liquid is much greater (about 100 times more) than that of gases.

6. With an increase in pressure, the viscosity of liquids (except water) increases while For gases viscosity is practically independent of pressure. And the viscosity of water decreases with increase in pressure.

7. The viscosity of gases increases with the increase of temperature, because on increasing temperature the rate of diffusion increases.

8. The viscosity of liquid decreases with the increase of temperature, because the cohesive force between the liquid molecules decreases with the increase of temperature

- **Poiseuille's Formula**

For the stream-line flow of liquid in capillary/narrow tube, If a pressure difference (P) is maintained across the two ends of a capillary tube of length 'l ' and radius r as shown in figure

Then according to Poiseuille's Formula

V= the volume of liquid coming out of the tube per second is

1. Directly proportional to the pressure difference (P).
2. Directly proportional to the fourth power of radius (r) of the capillary tube
3. Inversely proportional to the coefficient of viscosity (η) of the liquid.
4. Inversely proportional to the length (l) of the capillary tube.

And with the help of Dimension formula we get

$$V=\frac{KPr^4}{\eta l}$$

Where K is the constant of proportionality

And experimentally it is found that $K=\frac{\pi}{8}$

So Poiseuille's Formula is given as

$$V=\frac{\pi Pr^4}{8\eta l}$$

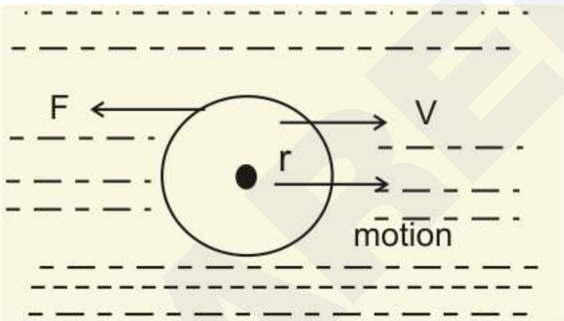
11.Stokes' Law And Terminal Velocity

- **Stokes' law-**

When a body moves through a fluid then The fluid exerts a viscous force on the body to oppose its motion.

And according to Stokes' law, the magnitude of the viscous force depends on the shape and size of the body, its speed and the viscosity of the fluid.

So for the below figure



If a sphere of radius r moves with velocity v through a fluid of viscosity η ,

Then using Stokes' law the viscous force (F) opposing the motion of the sphere is given by

$$F=6\pi\eta rv$$

Where

$\eta - coefficient\ viscosity$

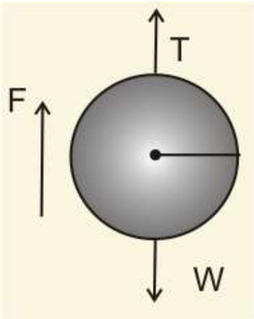
$r - radius$

$v - velocity$

- **Terminal Velocity-**

When the spherical body is dropped in a viscous fluid, it is first accelerated and then it's acceleration becomes zero and it attains a constant velocity and this constant velocity is known as terminal velocity.

For a spherical body of radius r is dropped in a viscous fluid, The forces acting on it are shown in the below figure.



So **Forces acting on the body are**

1. Weight of Body (W)

$$W = mg = \frac{4}{3}\pi r^3 \rho g$$

Where $\rho \rightarrow density\ of\ body$

2. Buoyant/ Thrust Force (T of F_B)

$$T = F_B = \frac{4}{3}\pi r^3 \sigma g$$

where $\sigma \rightarrow density\ of\ fluid$

3. Viscous force (F)

$$F = 6\pi\eta r v$$

So when the body attains terminal velocity the net force acting on the body is zero.

Apply force balance

$$F_B + F = W$$

$$\rightarrow 6\pi\eta r v + \frac{4}{3}\pi r^3 \sigma g = \frac{4}{3}\pi r^3 \rho g$$

$$\rightarrow 6\pi\eta r v = \frac{4}{3}\pi r^3 g (\rho - \sigma)$$

$$\rightarrow v_t = \frac{2}{9}\frac{r^2(\rho - \sigma)}{\eta} g$$

Where $v_T = terminal\ velocity$

From this formula, we can say that

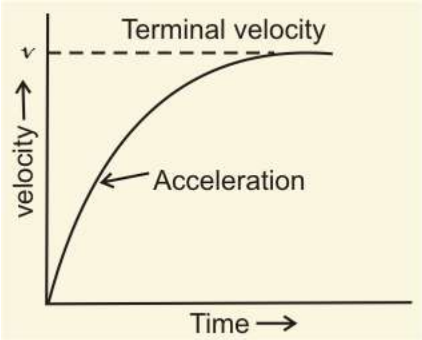
- Terminal velocity depends on the radius of the sphere/body.
- Greater the density of solid greater will be the terminal velocity
- Greater the density and viscosity of the fluid lesser will be the terminal velocity.
- If ϱ > σ then Terminal velocity will be positive.

I.e Spherical body attains constant velocity in a downward direction.

- If ϱ < σ then Terminal velocity will be negative.

I.e Spherical body attains constant velocity in an upward direction.

- Terminal velocity graph

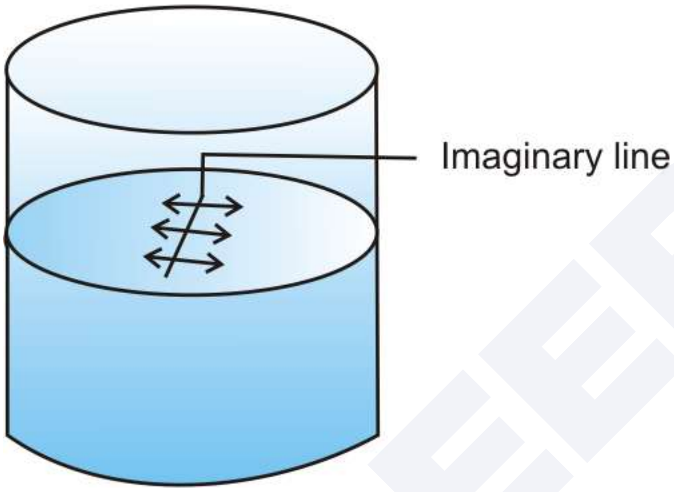


12. Surface Tension and Surface energy

Surface tension-

Surface tension is the elastic tendency of a fluid surface which makes it acquire the least surface area.

If we draw an imaginary line on the free surface of the liquid as shown in the below figure.



Then Surface tension of a liquid is measured by the force acting per unit

length on either side of an imaginary line.

So Surface tension of a liquid is given by

$$T = \frac{F}{l}$$

Where

$F \rightarrow force$

$l \rightarrow imaginary\ length$

The direction of this force is perpendicular to the line and tangential to the free surface of the liquid.

- It depends only on the nature of liquid and is independent of the area of surface or length of the imaginary line considered.
- It is a scalar quantity.
- Unit of Surface Tension-

  $\frac{N}{m} - in\ S.I.\ Unit$

  $\frac{dyne}{cm} - in\ c.g.s.\ Unit$

- Dimension- MT^{-2}
- Example-

  Raindrops are spherical in shape because each drop tends to acquire minimum surface area due to surface tension, and for a given volume, the surface area of the sphere is minimum.

- The surface tension of liquid decreases with the rise of temperature.

Surface energy-

The molecules on the liquid surface experience net downward force. And because of this force, these molecules tend to move downwards. So to fill the space we need to bring a molecule from the interior of the liquid to the free surface. And to do this some work is required to be done against the intermolecular forc of attraction. This work will be stored as the potential energy of the molecule on the surface.

And this stored potential energy of surface molecules per unit area of the surface is called **surface energy.**

Surface energy is also defined as the amount of work done in increasing the area of the surface film through unity.

I.e $surface\ \ energy = \frac{work\ done\ in\ increasing\ the\ surface\ area}{increase\ in\ surface\ area}$

or $surface\ \ energy = \frac{W}{\Delta A}.....(1)$

Where $W \rightarrow work\ done$

and $\Delta A \rightarrow increase\ in\ area$

And work done in increasing the surface area is given by

$W = T \times \Delta A....(2)$

where $T \rightarrow Surface\ tension$

and $\Delta A \rightarrow increase\ in\ area$

So we rewrite equation (2) as

$T = \frac{W}{\Delta A}....(3)$

So we can also define surface tension as the amount of work done in increasing the area of the liquid surface by unity against the force of surface tension.

Or we can say that *the surface tension of a liquid is numerically equal to its surface energy.*

As

$W = T\Delta A$

$If\ \Delta A = 1, then\ T = W$

$T \rightarrow Surface\ tension$

13. Excess Pressure

Excess Pressure-

Definition-Difference of pressure between the two sides of the liquid surface is known as Excess Pressure.

Cause of Excess Pressure-

Drop or bubble tends to contract and so compresses the matter enclosed, due to the property of surface tension. Thus to prevent further contraction, internal pressure inside Drop or bubble increases. This internal pressure increase until the equilibrium is achieved. So that is why in equilibrium the pressure inside a bubble or drop is greater than outside. And this difference of pressure between the two sides of the liquid surface is called excess pressure.

- Excess pressure in different cases

1. Excess pressure for plane surface

$\Delta P = 0$

means No difference in pressure.

2. Excess pressure for concave surface

$\Delta P = \frac{2T}{R}$

Where

T- Surface Tension

R- Radius

3. Excess pressure for a convex surface

$$\Delta P=\frac{2T}{R}$$

4. Pressure Difference in Water Droplet

$$\Delta P=\frac{2T}{R}$$

5. Change in Pressure of bubble in the air

$$\Delta P=\left(\frac{2T}{R}\right)\times 2=\frac{4T}{R}$$

- Excess pressure is inversely proportional to the radius of the bubble (or drop).

14. Contact Angle

The shape of the Liquid Meniscus-

- When a capillary tube is dipped in a liquid, the liquid surface becomes curved near the point of contact. The curved surface of the liquid is called the meniscus of the liquid.

  This curved surface is due to the resultant of two forces i.e. the force of cohesion and the force of adhesion.

- If liquid molecule A is in contact with solid (i.e. wall of capillary tube) then forces acting on molecule A are

1. Force of adhesion (F_a)

   This is force due to solid molecules on liquid molecules.

   Here it will act outwards at a right angle to the wall of the tube.

2. Force of cohesion (F_c)

   This is force due to liquid molecules on liquid molecules.

   Here it will act at an angle 45^0 to the vertical and towards liquid.

   So the resultant force (F_N) will be given by

   $$\vec{F_N}=\vec{F_a}+\vec{F_c}$$

   And If F_N makes an angle α with F_a

   Then $$tan\alpha=\frac{F_cSin(135)}{F_a+F_cCos(135)}=\frac{F_c}{\sqrt{2}F_a-F_c}$$

   As we know that the free surface of the liquid adjusts itself at a right angle to this resultant force.

   So by knowing the direction of resultant force we can find out the shape of the meniscus.

3. Shape of Liquid Meniscus in various cases.

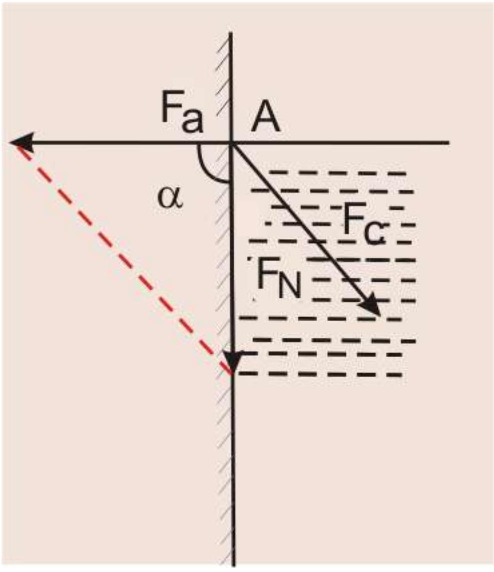
   Case I- When $F_c=\sqrt{2}F_a$

   As shown in the below figure

   The resultant force acts vertically downwards.

   Hence the liquid meniscus must be horizontal.

   Example-Pure water in a silver-coated capillary tube.

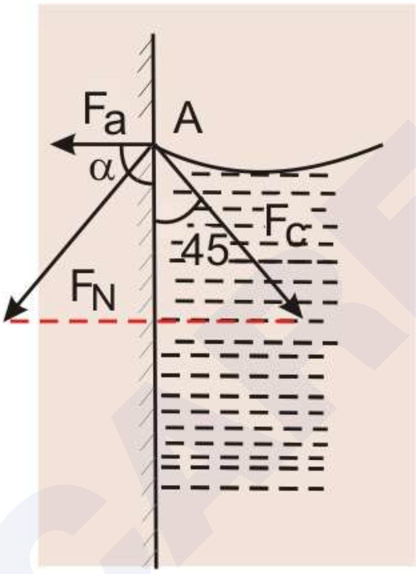


Case II- $F_c < \sqrt{2}F_a$

As shown in the below figure

The resultant force is directed outside the liquid. Hence the liquid meniscus must be concave upward.

Example-Example: Water in glass capillary tube.

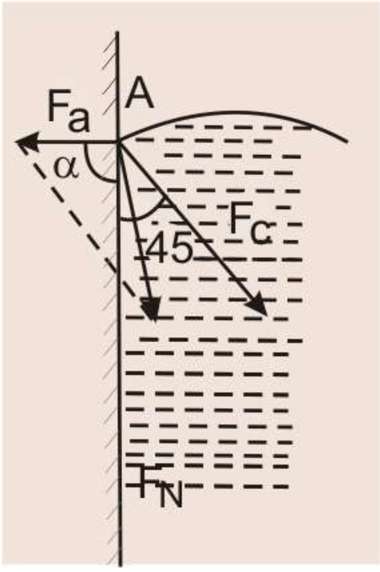


Case III- $F_c > \sqrt{2}F_a$

As shown in the below figure

The resultant force is directed inside the liquid. Hence the liquid meniscus must be convex upward.

Example: Mercury in glass capillary tube.



The angle of Contact (θ)-

The angle of contact between a liquid and a solid is defined as the angle enclosed between the tangents to the liquid surface and the solid surface inside the liquid.

While drawing tangent keeps following things in mind.

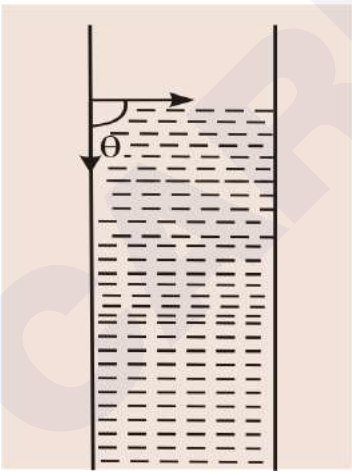
1.Both the tangents being drawn at the point of contact of the liquid with the solid.

2. Tangent to the liquid surface should be away from the solid surface.

3. Tangent to the solid surface should be towards the liquid surface.

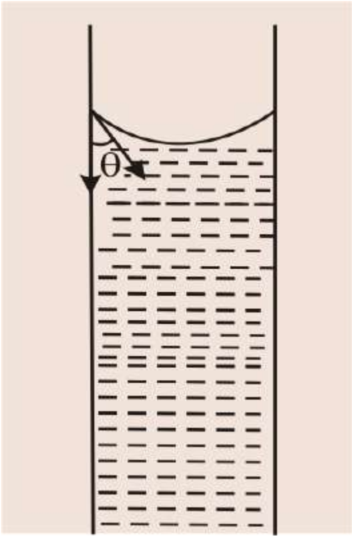
- Its value lies between 0^o and 180^o
- When $F_c=\sqrt{2}F_a$ then $\theta=90^0$

  i.e plane meniscus.
  In this case, Liquid does not wets the solid surface



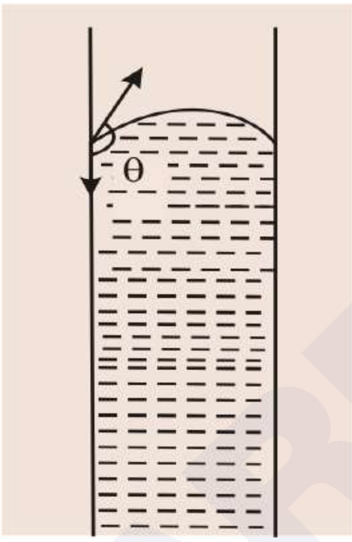
- When $F_c<\sqrt{2}F_a$ then $\theta<90^0$

  i.e concave meniscus.
  In this case, Liquid wets the solid surface



- When $F_c > \sqrt{2}F_a$ then $\theta > 90^0$

  i.e Convex meniscus.
  In this case, Liquid does not wet the solid surface.



- On increasing the temperature, angle of contact decreases.

15. Capillary Action

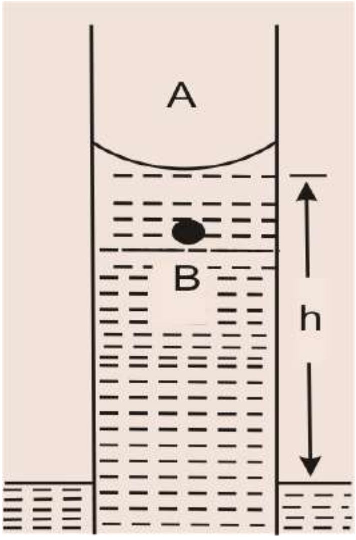
- **Capillarity -**

If a capillary tube is dipped in a liquid, it is found that the liquid in the capillary either ascends or descends relative to the surrounding liquid. This phenomenon is called capillarity.

Examples of capillarity- A towel soaks water.

- **Ascent Formula-**

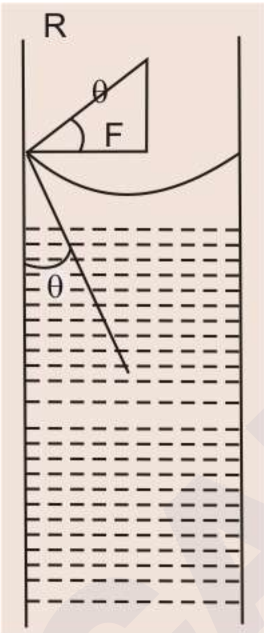
When one end of the capillary tube of radius r is immersed into a liquid of density ρ (For example- water and capillary tube of glass), And the shape of the liquid meniscus in the tube becomes concave upwards as shown in the figure.



Then the height h up to which the liquid level rises in the capillary tube is given by Ascent Formula

which says $h = \frac{2T\cos\Theta}{\rho g r}$

where



$T - surface\ Tension$

$r - radius\ of\ capillary\ tube$

$\rho - liquid\ density$

$\theta - Angle\ of\ contact$

1. The capillary rise depends on the nature of liquid and solid both. I.e on T, ρ, θ, r
2. For a given liquid and solid pair as T, ρ, θ are constat then $h\ \alpha\ \frac{1}{r}$. i.e., lesser the radius of capillary greater will be the rise and vice-versa.
3. Capillary action for various liquid-solid pair

I. For $\Theta < 90^{\circ}$ (I.e for water and capillary tube of glass)

So Meniscus will take Concave shape and liquid in the capillary will rise/ascend.

II. For $\Theta > 90^{\circ}$ (I.e for Mercury and glass capillary tube)

So Meniscus will take Convex shape and liquid in the capillary will fall/descend.

III. For $\Theta = 90^\circ$ (I.e for Pure water and silver-coated capillary tube.)

So Meniscus will take Plane shape and liquid in the capillary will show No rise, no fall.

Thermal Properties of Matter

Important Formulae

1. Temperature And Its Scales

Temperature-

- Temperature is the degree of hotness or coldness of a body. Heat always flow from high temperature to low temperature if there is no external work is applied.
- Temperature is one of the seven fundamental quantities and its dimension is $[\theta]$. S.I. unit of temperature is Kelvin.

Scales of Temperature-

To construct any scale of temperature, we have to take two fixed points . First fixed point is the freezing point (ice point) of water. The second fixed point is the boiling point (steam point) of water.

1. Celsius scale : In this scale ice point is taken 0° and steam point is taken 100°. The temperature measured on this scale all in degree Celsius(°C).

2. Farenheite scale : This scale of temperature has freezing point as 32°F and steam point as 212°F.

3. Kelvin scale : The Kelvin temperature scale is also known as **thermodynamic scale**. The temperature measured on this scale are in Kelvin (K).

Note - The triple point of water is also selected to be the zero of scale of temperature

Temperature on any scale can be converted into any other scale by using the following formula -

$$\frac{(Reading\ on\ any\ scale - Ice\ point)}{(Steam\ point - ice\ point)}$$

All the above mentioned temperature scale are related to each other by the following relationship -

$$\frac{C}{5} = \frac{F-32}{9} = \frac{K-273}{5}$$

The below table shows the range of various temperature scale -

Scale	Symbol for each degree	LFP	UFP	Number of divisions on the scale
Celsius	$^\circ C$	$0^\circ C$	$100^\circ C$	100
Fahrenheit	$^\circ F$	$32^\circ F$	$212^\circ F$	180
Reaumer	$^\circ R$	$0^\circ R$	$80^\circ R$	80
Rankine	$^\circ Ra$	460 Ra	672 Ra	212
Kelvin	K	273.15 K	373.15 K	100

2.Thermometer And Its Types

Thermometry:

A branch of science that deals with the measurement of the temperature of a substance is known as thermometry.

An instrument used to measure the temperature of a body is called a **Thermometer.** The principle on which it works is by absorbing heat from the body.

There are various kinds of thermometers which are briefly classified in three types -

- **Liquid thermometers -** In liquid thermometers, mercury is usually preferred over other liquids. The reason behind this is its expansion is large and uniform. And the main reason behind all these is that it has high thermal conductivity and low specific heat.

  Range of temperature :(freezing point of mercury) - (boiling point of mercury) which is $-50^0C\ to\ 350^0C$

  The formula for calculation of temperature at any length l : $$t = \frac{l - l_0}{l_{100} - l_0} \times 100^0 C$$

  Here - l= length of the mercury column at the given temperature t.

  l_0= length of the mercury column at the 0^oC temperature

l_{100} = length of the mercury column at the 100^oC temperature

- **Gas thermometers :** In this gases are used as thermometric material. Gas thermometers are more sensitive and accurate than liquid thermometers as th expansion of gases is more than that of liquids. In this gas are used as a thermoelectric substances are called **ideal gas** thermometers. These are basically of two types

(i) Constant pressure gas thermometers - If pressure is constant, then for ideal gas, volume is directly proportional to temperature. So,

$$V \alpha T$$

The formula for calculation of temperature at any volume V : $t = \frac{V - V_0}{V_{100} - V_0} \times 100^0C$

Here, V= volume of the gas column at the given temperature t.

V_0= volume of the gas column at the 0^oC temperature.

V_{100} = volume of the gas column at the 100^oC temperature.

(i) Constant volume gas thermometers - If the volume is constant, then for an ideal gas, pressure is directly proportional to temperature. So,

$$P \alpha T$$

The formula for calculation of temperature at any pressure P : $t = \frac{P - P_0}{P_{100} - P_0} \times 100^0C$

Here, P= pressure of the gas column at the given temperature t.

P_0= pressure of the gas column at the 0^oC temperature.

P_{100} = pressure of the gas column at the 100^oC temperature.

- **Resistance thermometers:** Usually **Platinum, Germanium** is used in resistance thermometers due to high melting point and large value of temperatu coefficient of resistance. This type of thermometer can be used for high temperatures.

The formula for calculation of temperature at any resistance R : $t = \frac{R - R_0}{R_{100} - R_0} \times 100^0C$

Here, R= Resistance of the material at the given temperature t.

R_0= Resistance of the material column at the 0^oC temperature.

R_{100} = Resistance of the material column at the 100^oC temperature.

3. Thermal Expansion

Thermal expansion is the tendency of material to change its shape, area, and volume in response to a change in temperature. So, if there is any change in temperature every material has tendency to change its dimension and the amount of change depends on the type of materials.

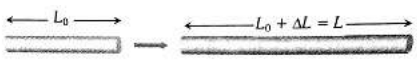
Thermal expansion is minimum in case of solids but maximum in case of gases because the intermolecular force is maximum in solids but minimum in gases

So, solids can expand in one dimension, two dimension and three dimension while liquids and gases usually suffers change in volume only.

Thermal expanision is basically of three types -

- **Linear expansion** : When a solid is heated and it's length increases, then the expansion is called linear expansion.

Let us take an specimen of length L_0. There is two scenario, first is before heating and the second image shows after heating. So,



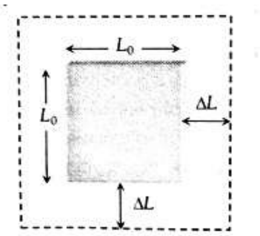
(i) Change in the length of the specimen is $\Delta L = L_o \alpha \Delta T$
(*Here*, $L =$ Original length, $\Delta T =$ Temperature change)

(ii) Final length of the specimen is $L = L_o(1 + \alpha \Delta T)$

(iii) Co-efficient of linear expansion $\alpha = \frac{\Delta L}{L_0 \Delta T}$

(iv) Unit of α is °C^{-1} or K^{-1} . It's dimension is $[\theta^{-1}]$

- **Superficial (areal) expansion** : When the temperature of a 2-Dimensiuonal specimen is changed, it's area changes, then the expansion is called superficial or areal expansion.



(i) Change in area is $\Delta A = A_o \beta \Delta T$

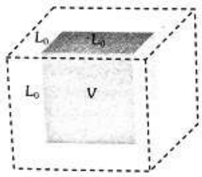
($A =$ Original area, $\Delta T =$ Temperature change)

(ii) Final area $A = A_o(1 + \beta \Delta T)$

(iii) Co-efficient of superficial expansion $\beta = \dfrac{\Delta A}{A_0 \Delta T}$

(iv) Unit of β is °C or K.

- **Volume or cubical expansion** : When a 3-Dimensional solid is heated and it's volume increases, then the expansion is called volume or cubical expansion.



Now there is one relation between the α, β *and* γ, which can be written as -

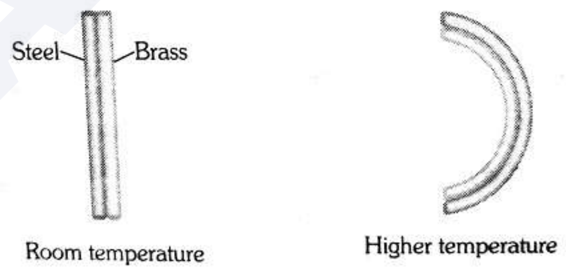
$$\alpha = \frac{\beta}{2} = \frac{\gamma}{3} \Rightarrow \alpha : \beta : \gamma = 1 : 2 : 3$$

Hence, for the same rise in temperature -

Percentage change in area = 2 times the percentage change in length.
Percentage change in volume = 3 times the percentage change in length.

Effects of thermal expansion on Solids-

(1) Bi-metallic strip : When two strips of equal lengths but of different materials (such that they have different value of coefficient of linear expansion) when join together, it is called **"bi-metallic strip"**, and it can be used in thermostat to break or make electrical contact. Bi-metallic strip has the characteristic property of bending on heating. This is due to unequal linear expansion of the two metal. The strip will bend with metal of greater α on outer side.



The above figure shows the condition before and after heating the bi-metallic strip.

(2) Effect of temperature on the time period of a simple pendulum : Let us suppose a pendulum clock keeps proper time at temperature θ. If the temperature is increased to $\theta' \ (\theta' > \theta)$ then due to linear expansion, length of pendulum and from the formula, we knows that the time period of simple pendulum is directly proportional to the square root of length of the pendulum hence its time period will increase.

Fractional change in time period $\dfrac{\Delta T}{T} = \dfrac{1}{2} \alpha \Delta \theta$

(i) In summer, the temperature will rise and due to this there will be increment in its time period. A pendulum clock becomes slow and will lose time.

Loss of time in a time period is given by - $\Delta T = \frac{1}{2}\alpha\Delta\theta T$

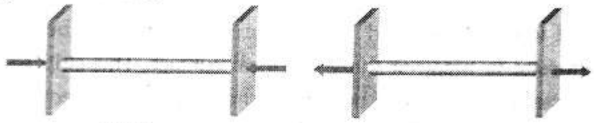
(ii) Time lost by the clock in a day - $\Delta T = \frac{1}{2}\alpha\Delta\theta t = \frac{1}{2}\alpha\Delta\theta(86400) = 43200\alpha\Delta\theta \sec$

(Time in one complete day on earth = 86400 seconds)

4. Thermal Stress And Thermal Strain

Thermal stress in a rod which is rigidly fixed : When a rod which rigidly fixed at ends such as to prevent expansion or contraction, when its temperature is increased or decreased. Due to preventing its thermal expansion or contraction, a compressive or tensile stress is developed in it. As the rod try to expand or contracts, then it apply a reaction force on the rigid support. If the change in temperature of a rod of length L is $\Delta\theta$ then -

$$\text{Thermal strain } = \frac{\Delta L}{L} = \alpha\Delta\theta \quad \left[\text{As } \alpha = \frac{\Delta L}{L} \times \frac{1}{\Delta\theta}\right]$$



As, If we know the strain then with the help of Hooke's law, we can find the stress also. If we know the stress , then we can find the force by multiplying cross sectional area with stress. Both stress and force can be written as -

$$\text{So, Thermal stress } = Y\alpha\Delta\theta$$
$$\text{or, Force on the supports } F = YA\alpha\Delta\theta$$

5. Thermal Expansion In Liquids And Gases

Thermal Expansion in Liquids-

- Like solids, liquids do not have linear and superficial expansion but liquid only undergoes volume expansion.
- We always need some solid vessel to keep the liquid, so liquids are always to be heated along with a vessel which contains them so initially on heating the system (System is liquid + vessel here). Initially, the level of liquid in the vessel falls (vessel expands more since it absorbs heat and liquid expands less) as the volume expansion co-efficient of solid is more than that of liquid but later on, it starts rising due to faster expansion of the liquid (because now solid transfer all the heat to liquid and that is the condition of steady-state)

$PQ \rightarrow$ represents expansion of vessel
$QR \rightarrow$ represents the real expansion of liquid
$PR \rightarrow$ represent the apparent expansion of liquid

So, from above we can conclude that the actual increase in the volume of the liquid = The apparent increase in the volume of liquid + the increase in the volume of the vessel.

Basically, liquids have two coefficients of volume expansion -

1. **Co-efficient of apparent expansion** γ_a: It is due to an apparent (Apparent means that appears but not real) increase in the volume of liquid. This happens when the expansion of the vessel containing the liquid is not taken into account.

$$\gamma_a = \frac{\text{Apparent expansion in volume}}{\text{Initial vdume } \times \Delta\theta} = \frac{(\Delta V)_a}{V \times \Delta\theta}$$

2. **Co-efficient of real expansion** γ_r : It is due to the actual increase in the volume of liquid due to heating. In this expansion of vessel containing the liquid is taken into account.

$$\gamma_r = \frac{\text{Real increase in volume}}{\text{Initial vdume } \times \Delta\theta} = \frac{(\Delta V)}{V \times \Delta\theta}$$

Also coefficient of expansion of flask $\gamma_{vessel} = \frac{\Delta V_{vessel}}{V \times \Delta\theta}$

So, $\gamma_{\text{Real}} = \gamma_{\text{Apporent}} + \gamma_{\text{Vessel}}$

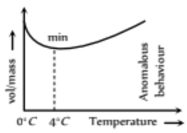
So the change (apparent change) in volume in liquid relative to the vessel is -

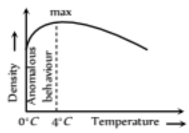
$$\Delta V_{app} = V\gamma_{app}\Delta\theta = V\left(\gamma_{\text{Real}} - \gamma_{\text{Vessel}}\right)\Delta\theta = V\left(\gamma_r - 3\alpha\right)\Delta\theta$$

Where, $\alpha =$ Coefficient of linear expansion of the vessel.

Anomalous expansion of water:

Generally any material expands on heating and contracts on cooling. But in the case of water, it expands on heating if its temperature is greater than 4°C. In the range 0°C to 4°C, water contracts on heating and expands on cooling, i.e. γ is negative. So water has this special property, which is not found in any existing natural material. This behaviour of water in the range from 0°C to 4°C is called **anomalous expansion**. You can see it with the help of a graph. This is the anomalous behaviour of water which causes ice to form first at the surface of a lake in cold weather. So, as winter approaches, the water temperature increases initially at the surface. It results in the water sinking because of its increased density. Consequently, the surface reaches 0°C first and because of that the lake becomes covered with ice. This property of water makes the aquatic life survive the cold winter as the lake bottom remains unfrozen a temperature of about 4°C.
At 4°C, density of water is maximum while its specific volume is minimum.





Variation of Density with Temperature -

Most substances (solid and liquid) expand heat is supplied to them, i.e., the volume of a given mass of a substance increases on heating,

so the density should decrease $\left(\text{as } \rho \propto \frac{1}{V}\right)$. It means that the density is inversely proportional to the volume. From that, we can deduce the expression of density after heating or cooling as follows -

$$\frac{\rho'}{\rho} = \frac{V}{V'} = \frac{V}{V + \Delta V} = \frac{V}{V + \gamma V\Delta\theta} = \frac{1}{1 + \gamma\Delta\theta}$$

So, $$\Rightarrow \rho' = \frac{\rho}{1 + \gamma\Delta\theta} = \rho(1 + \gamma\Delta\theta)^{-1} = \rho(1 - \gamma\Delta\theta)$$

Here, ρ *and* ρ' *is the density before and after heating the material*

Expansion of Gases -

As we know that the gases have no definite shape. It takes the shape of the vessel in which it is kept. Therefore gases have only volume expansion. Since the expansion of the container (Because the container is solid) is negligible in comparison to the gases, therefore gases have only real expansion.

(1) Coefficient of volume expansion: At constant pressure, the unit volume of a given mass of a gas, increases with a 1°C rise of temperature, which is called the coefficient of volume expansion.

$$\alpha = \frac{\Delta V}{V_0} \times \frac{1}{\Delta\theta} \Rightarrow \text{ Final volume } V' = V(1 + \alpha\Delta\theta)$$

(2) Coefficient of pressure expansion :

$$\beta = \frac{\Delta P}{P} \times \frac{1}{\Delta\theta}$$

$$\therefore \text{ Final pressure } P' = P(1 + \beta\Delta\theta)$$

6.Heat

Heat -

- The form of energy which is exchanged among various bodies or system on account of temperature difference is defined as heat.So, we can say that the driving potential for the heat energy is the temperature differnece.
- Temperature of a body can be changed by giving heat (temperature rises) or by removing heat (temperature falls) from body.
- The amount of heat (Q) is given to a body depends upon it's mass (m), change in it's temperature and nature of material (C) i.e., $\mathbf{Q = mC\Delta\theta}$; where C = specific heat of material which depends on the material.
- There are various units of heat like Joule(J), erg, calorie(cal) etc.
- Heat is a scalar quantity.
- The calorie (cal) is defined as the amount of heat required to raise the temperature of 1 gram of water by 1^{o}C.
- 1 cal = 4.186 J

There are basically two types of specific heats -

1. **Gram specific heat :** It is defined as the amount of heat energy required to raise the temperature of unit mass of a body through 1°C (or K) is called gram specific heat of the material of the body. Actually it depends on the mass of the body which is in Gram.

If Q heat changes the temperature of mass m by $\Delta\theta$ then specific heat is given as - $c = \frac{Q}{m\Delta\theta}$

Based on this equation we can calculate the unit and dimension of this -

$$SI\ unit\ is = \frac{Joule}{kg - K}$$

Dimension is - $[L^2T^{-2}\boldsymbol{\theta}^{-1}]$

2. **Molar specific heat :** Molar specific heat of a substance is defined as the amount of heat required to raise the temperature of one gram mole o the substance through a unit degree. It is represented by C.

It can be written as - $C = M\frac{Q}{m\Delta\theta} = \frac{1}{\mu}\frac{Q}{\Delta\theta}$

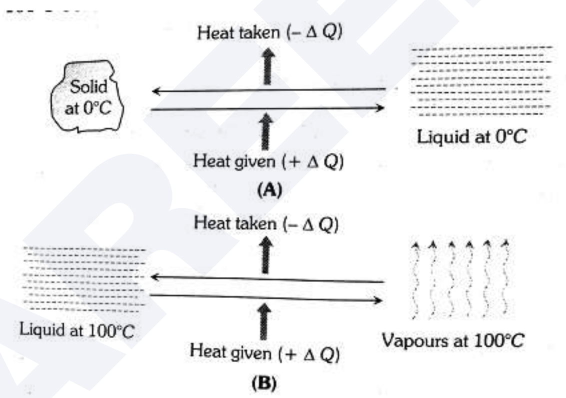
$$Here,$$
$$Q = Heat\ supplied,\ M = Molecular\ mass,\ m = Actual\ mass\ of\ the\ substance, \Delta\theta = Temperature\ difference$$

7.Change Of State

What is Phase?

We know that there are three states of matter. So, the term phase is used to describe a specific state of matter, such as solid, liquid or gas. A transition from on phase to another is called a phase change. So we need to supply or extract heat from any substance to change its phase or state. For any given pressure a phase change takes place at a definite temperature. So the temperature will not change during phase change.

Water is very common substance known to us. So at 0^0C temperature ice and liquid water can change its phase and at 100^0C the liquid water and steam can change its phase to each other at the atmospheric pressure.



Latent heat : Latent heat is also called hidden heat. In this there is no change in the temperature of the body and because of that it is said to be hidden or late as we are not feeling any change in temperature of the body. The amount of heat required to change the state of the mass m of the substance is written as : $Q = mL$, where L is the latent heat. It's unit is cal/gm or, J/kg and Dimension: $[L^2T^{-2}]$

Basically the latent heat is classified in two types -

(i) Latent heat of fusion : The latent heat of fusion is the heat energy required to change one kilogram of the material in its solid state at its melting point to one kilogram of the material in its liquid state. The latent heat of fusion for water (or latent heat of ice) is - $L_F = L_{\text{ice}} \approx 80\ \text{cal/gm} \approx 60\ \text{kJ/mol} \approx 336\ \text{kilo} - \text{joule/kg}$

(ii) Latent heat of vaporisation : The latent heat of vaporisation is the heat energy required to change one kilogram of the material in its liquid state at its boiling point to one kilogram of the material in its gaseous state. The latent heat of vaporisation of water (latent heat of steam) is $L_V = L_{\text{steam}} \approx 540\ \text{cal/gm} \approx 40.8\ \text{kJ/mol} \approx 2260\ \text{kilojoule/kg}$

Latent heat of vaporisation is more than the latent heat of fusion. This is because when a substance gets converted from liquid to vapour, so the increase i volume is large. Hence more amount of heat is required. But when a solid gets converted to a liquid, then there is negligible increase in volume. Hence very less amount of heat is required.

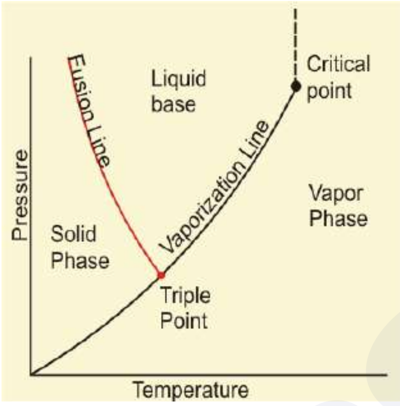
8.Triple Point Of Water

Some important terminologies -

- **Melting (or fusion) /freezing (solidification) :** The phase change of solid to liquid is called melting or fusion and the reverse phenomenon is calle freezing or solidification.
- **Vaporisation / liquefication (condensation) :** The phase change from liquid to vapour is called vaporisation. The reverse transition is calle liquefication or condensation.
- **Sublimation :** Sublimation is the conversion of a solid directly into vapours. So, in this the solid is directly converted to vapor without entering in liquid phase. Best example of this is the burning of Camphor.

TRIPLE POINT -

If we plot a graph between pressure and temperature for any material. Then there are three curves form on this graph, they are - fusion curve, vaporisatic curve and sublimation curve. Following graph shows variation of pressure with temperature of water -



Now, some description about the curves -

(i) Sublimation curve which connects points at which vapour (V) and solid (S) exist in equilibrium.
(ii) Vapourization curve which shows vapour and liquid (L) existing in equilibrium.
(iii) Fusion curve which shows liquid and solid existing in equilibrium.
The three curves meet at a single point which is called the **triple point.**

Triple point is that point for a substance where all the three phases co-exist in equilibrium.

For water - Triple point exist at - Pressure = 0.0062 bar or, 62 Pascal

Temperature = $0.01^{o}C$ or, 273.16 K

9.Joule's Law Of Heating

Joule's experiments conclusively established that heat is a form of energy. Spending a given amount of mechanical work, always the same amount of he will be produced. It does not depend on what type of arrangement is used for doing mechanical work.

Importance of Joule’s Experiments:

- This experiment shows that heat is a form of energy.
- Always the same amount of heat was produced by spending a given amount of mechanical work. It does not depend on what type of arrangement is use for doing mechanical work.
- It develops the relationship between Joule and Calories.

Whenever heat is converted into mechanical work or mechanical work is converted into heat, then the ratio of work done to the heat produced always remain constant. i.e. $J=\frac{W}{Q}$

This is Joule’s law and J is called the mechanical equivalent of heat.

10.Calorimetry Principle

Principle of Calorimetry -
Calorimetry means **'measurement of heat'.**
When two bodies (both being liquid or one being solid and other liquid) at different temperatures are mixed or come in contact, heat will be transferred from body at a higher temperature to a body at a lower temperature till both acquire the same temperature. The body at lower temperature absorbs heat while the body at higher temperature release it.

The principle of calorimetry represents the law of conservation of heat energy.

The temperature of the mixture always lies between the temperature of the liquid having the lowest temperature and the temperature of the liquid having the highest temperature.

Mixing of two substances when temperature changes only: It means no phase change. Suppose two substances having masses m_1 and m_2, gram specific heat C_1 and C_2 and temperature θ_1 and θ_2 then,

$$\text{Hence, Heat lost} = \text{Heat gained}$$

$$\Rightarrow m_1 c_1 (\theta_1 - \theta_{\text{mix}}) = m_2 c_2 (\theta_{\text{mix}} - \theta_2)$$

$$\theta_{\text{mix}} = \frac{m_1 c_1 \theta_1 + m_2 c_2 \theta_2}{m_1 c_1 + m_2 c_2}$$

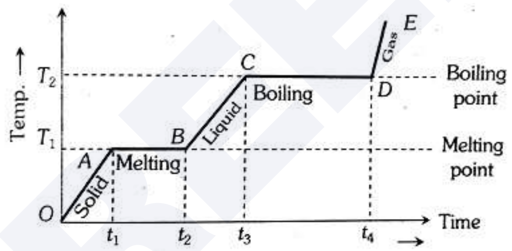
Similarly, we can derive formulas for different cases. The different cases and its result is mentioned in the table given below -

Condition	Temperature of mixture
If bodies are of same material ie $c_1 = c_2$	$\theta_{mix} = \frac{m_1\theta_1 + m_2\theta_2}{m_1 + m_2}$
If bodies are of same mass $m_1 = m_2$	$\theta_{mix} = \frac{c_1\theta_1 + c_2\theta_2}{m_1 + m_2}$
If $m_1 = m_2$ And $c_1 = c_2$	$\theta_{mix} = \frac{\theta_1 + \theta_2}{2}$

11.Heating Curve

Heating Curve

If a solid of mass (m) is heated at a constant rate such that it is undergoing change of phase from solid to liquid and liquid to gas on a graph of Temperature and time is called Heating curve.



Now we will discuss each phase one by one -

(1) In the region OA, temperature of solid is changing with time so, $Q = mc_S\Delta T \Rightarrow P\Delta t = mc_S\Delta T$ [as $Q = P\Delta t$]

Here, P is the power and Δt is the time interval.

So, from here we can deduce that - $c_s \propto \frac{1}{\text{Slope of line } OA}$

Because the $\frac{\Delta T}{\Delta t}$ is the slope of OA

(2) Now come to the region AB, here temperature is constant, so it represents change of phase, i.e., change of phase from solid state to liquid stat Now you can see that between A and B substance is partly solid and partly liquid.

If L_f is the latent heat of fusion $Q = mL_f$

$$L_F = \frac{P(t_2 - t_1)}{m} \quad [\text{ as } Q = P(t_2 - t_1)]$$

i.e., Latent heat of fusion is proportional to the length of line of zero slope. So, this line is parallel to the time axis.

(3) In the region BC temperature of liquid increases so, again $c_L \propto \frac{1}{\text{Slope of line } BC}$

Since it is sensible heat, so the temperature will change in this zone BC.

(4) In the region CD temperature is again constant, so it represents the change of phase, i.e., boiling with boiling point T_2 . In this the liquid is changing its phase from liquid to gas. The length of line CD is proportional to latent heat of vaporisation (L_v)

Here, $Q = mL_v$

So, $L_v = \frac{P(t_4 - t_3)}{m} \quad [\text{ as } Q = P(t_4 - t_3)]$

So, $L_v \propto Length\ of\ line\ CD$. It means that the line is parallel to the time axis.

(5) The line DE represents gaseous state of substance. Here, its temperature increases linearly with time. Just like solid and liquid, the reciprocal of slope of line will be proportional to specific heat or thermal capacity of substance in vapour state.

12. Heat Transfer

Heat transfer is the process of transfer of heat from high temperature reservoir to low temperature reservoir. In terms of the thermodynamic system, heat transfer is the movement of heat across the boundary of the system due to temperature difference between the system and the surroundings.

There are three modes of heat transfer which is described below -

1. **CONDUCTION -** The process of transmission of heat energy in which the heat is transferred from one particle to other particle without dislocation of the particle from their equilibrium position is called **conduction.**

2. CONVECTION - In this transfer of heat is by means of migration of material particles of medium is called convection. So, convection is the combination of conduction and advection. So, advection is the bulk flow of the particle. It is generally happens in fluids (liquid and gases)

3. RADIATION The process of the transfer of heat from one place to another place without need of medium is called radiation. So, radiation does not need any material medium to propogate. It is generally effective when a body is much higher temperature than the surroundings. In this the heat transfe is directly proportional to the fourth power of the absolute temperature.

13. Staedy Conduction Heat Transfer

Properties of conduction -
(1) Heat flows from high temperature to low temperature. In this, the particles of the medium simply vibrate about their mean position but do not leave the place.
(2) Conduction heat transfer is medium dependent.
(3) The temperature of the medium goes on increasing in the direction of heat transfer.
(4) Conduction mode of heat transfer is the process that is possible in all states of matter.
(5) In solids, the only mode of heat transfer is conduction.

(6) In metallic solids, free electrons carry is the heat energy carrier, therefore they are good conductors of heat.

Terminologies used in steady state or basics of conduction -

Steady-state: It means that the temperature in the system is not time-dependent. So, if we supply heat to a body then the temperature of the body increases but after some time, a state is reached when the temperature of every cross-section of the body becomes constant. In this state, no heat is absorbed by the body. This state of the body is called a **steady state.**

Isothermal surface : Any cross-section (within a conductor) having its all points at the same temperature, is called isothermal surface.

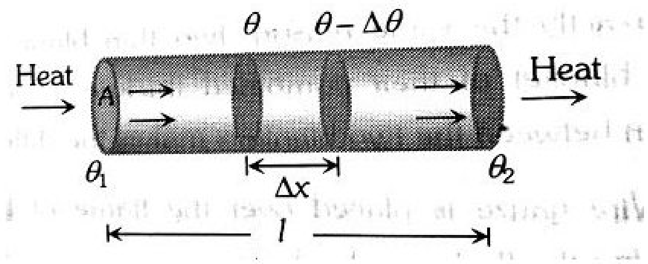
Temperature gradient : The rate of change of temperature with distance between two isothermal surfaces is called temperature gradient. It is denoted by -
So,

$\text{Temperature gradient } = \frac{-\Delta\theta}{\Delta x}$
The negative sign show that temperature θ decreases as the distance x increases in the direction of heat flow.

Thermal conductivity(K) - It is the measure of the ability of a substance to conduct heat through it. The magnitude of **K** depends only on nature of the material. Substances which do not permit easy flow of heat are called bad conductors. They possess low thermal conductivity due to very few free electrons.

14.Law Of Thermal Conductivity

Law of Thermal Conductivity -



Consider a rod of length 'l' , area of cross-section 'A' whose faces are maintained at temperature θ_1 and θ_2 respectively. In steady state the amount of heat flowing from one face to the other face in time t is given by -

$$Q = \frac{KA(\theta_1 - \theta_2)t}{l}$$

$$Q = Amount\ of\ heat\ transfer$$
$$t = Time\ of\ heat\ flow$$
$$K = Thermal\ conductivity\ of\ the\ material$$

So, from the above equation we can calculate the - Rate of flow of heat i.e. heat current which can be written as -

$$\frac{Q}{t} = H = \frac{KA(\theta_1 - \theta_2)}{l}$$

In the differential form, this heat current can also be written as -

$$\frac{dQ}{dt} = -KA\frac{d\theta}{dx}$$

In case of non-steady state or variable cross-section, this is the more general equation can be used to solve problems.

Relation of thermal conductivity of some material -

$$K_{Ag} > K_{Cu} > K_{Al}$$
$$K_{Solid} > K_{Liquid} > K_{Gas}$$
$$K_{Metals} > K_{Non-metals}$$

Thermal resistance (R_{th}) : The thermal resistance of a body is defined as the measure of its opposition to the flow of heat through it. It is defined as the ratio of temperature difference to the heat current $(\frac{Q}{t})$.

$$R_{th} = \frac{\theta_1 - \theta_2}{H} = \frac{\theta_1 - \theta_2}{KA(\theta_1 - \theta_2)/l} = \frac{l}{KA}$$

15. Electrical Analogy For Thermal Conduction-

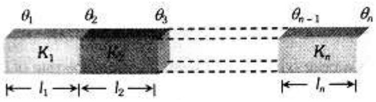
Electrical Conduction	Thermal conduction
1. Natural flow of electric charge is from higher potential to lower potential	1. Heat flows from higher temperature to lower temperature
2. The rate of flow of charge is defined as electric current. i.e., $I = \frac{dq}{dt}$	2. The rate of flow of heat may is called as heat current. i.e., $H = \frac{dQ}{dt}$
3. Ohm's law gives the relation between the electric current and the potential difference $I = \frac{V_1 - V_2}{R}$ where, R is the electrical resistance of the conductor	3. Similarly, the heat current may be related with the temperature difference as $H = \frac{\theta_1 - \theta_2}{R}$ where R, is the thermal resistance of the conductor
4. From the above point the electrical resistance is defined as $R = \frac{\rho l}{A} = \frac{l}{\sigma A}$	4.Similarly from the above point the thermal resistance may be defined as $R = \frac{l}{KA}$ where K = Thermal conductivity

where, $\rho = Resistivity$ $\sigma = Electrical\ conductivity$ $\frac{dq}{dt} = I = \frac{V_1 - V_2}{R} = \frac{\sigma A}{l}(V_1 - V_2)$	$\frac{dQ}{dt} = H = \frac{\theta_1 - \theta_2}{R} = \frac{KA}{l}(\theta_1 - \theta_2)$

16. Combination Of Metallic Rods

SERIES COMBINATION OF ROD/SLABS IN HEAT CONDUCTION -

Let n slabs each of cross-sectional area A, lengths $l_1, l_2, l_3 .... l_n$ and conductivities $K_1, K_2, K_3 .... K_n$ respectively be connected in series -



- **Heat current**: In the case of series combination, heat current is the same in all the conductors, So -

$$\frac{Q}{t} = H_1 = H_2 = H_3 .... = H_n$$

So, by law of thermal conductivity - $\frac{K_1 A(\theta_1 - \theta_2)}{l_1} = \frac{K_2 A(\theta_2 - \theta_3)}{l_2} = \frac{K_n A(\theta_{n-1} - \theta_n)}{l_n}$

- **Thermal resistance -** Net thermal resistance is equal to the sum of thermal resistance of all the slabs/rods. So, -

$$\text{Equivalent thermal resistance: } R = R_1 + R_2 + ...R_n$$

- **Thermal conductivity -** From the above equation of equivalent thermal resistance, equivalent thermal conductivity can be calculated as-

$$\text{From } R_S = R_1 + R_2 + R_3 + \ldots$$

$$\frac{l_1 + l_2 + \ldots l_n}{K_{eq} A_{eq}} = \frac{l_1}{K_1 A} + \frac{l_2}{K_2 A} + \ldots + \frac{l_n}{K_n A}$$

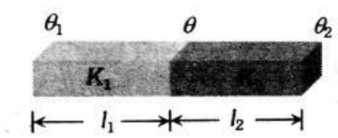
$$\Rightarrow K_{equivalent} = \frac{l_1 + l_2 + \ldots\ldots l_n}{\frac{l_1}{K_1} + \frac{l_2}{K_2} + \ldots\ldots + \frac{l_n}{K_n}}$$

$$A_{eq} L_{eq} = A_1 L_1 + A_2 L_2 + A_3 L_3 + \ldots + A_n L_n$$

For series combination: $L_{eq} = L_1 + L_2 + L_3 + \ldots + L_n$ (for each slab having constant area of cross section)

For parallel combination: $A_{eq} = A_1 + A_2 + A_3 + \ldots + A_n$ (for each slab having same length of slab)

- **The temperature of the interface of composite bar:** For the calculation of this, let the two bars be arranged in series as shown in the figure -

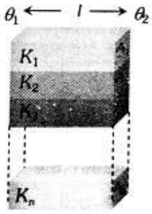


$$\text{ie., } \frac{Q}{t} = \frac{K_1 A(\theta_1 - \theta)}{l_1} = \frac{K_2 A(\theta - \theta_2)}{l_2}$$

$$\text{By solving, we get } \theta = \frac{\frac{K_1}{l_1}\theta_1 + \frac{K_2}{l_2}\theta_2}{\frac{K_1}{l_1} + \frac{K_2}{l_2}}$$

PARALLEL COMBINATION OF ROD/SLABS IN HEAT CONDUCTION -

Parallel combination : Let n slabs each of lengths l, cross-sectional area $A_1, A_2, A_3 \ldots\ldots A_n$ and conductivities $K_1, K_2, K_3 \ldots\ldots K_n$ respectively be connected in parallel -



- **Heat current** :If each slab will have different thermal conductivity, then Net heat current will be the sum of heat currents through individual slabs. i.e.,

$$H = H_1 + H_2 + H_3 + \ldots H_n$$

So, by law of thermal conductivity -

$$\frac{K(A_1 + A_2 + \ldots, +A_n)(\theta_1 - \theta_2)}{l}$$

$$= \frac{K_1 A_1 (\theta_1 - \theta_2)}{l} + \frac{K_2 A_2 (\theta_1 - \theta_2)}{l} + \ldots + \frac{K_1 A_n (\theta_1 - \theta_2)}{l}$$

- **Equivalent Thermal resistance -** Net thermal resistance in parallel combination -

$$\frac{1}{R_s} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots .. \frac{1}{R_n}$$

- **Thermal conductivity -** From the above equation of equivalent thermal resistance, equivalent thermal conductivity can be calculated as-

$$\frac{K(A_1 + A_2 + \ldots, +A_n)(\theta_1 - \theta_2)}{l}$$

$$= \frac{K_1 A_1 (\theta_1 - \theta_2)}{l} + \frac{K_2 A_2 (\theta_1 - \theta_2)}{l} + \ldots + \frac{K_1 A_n (\theta_1 - \theta_2)}{l}$$

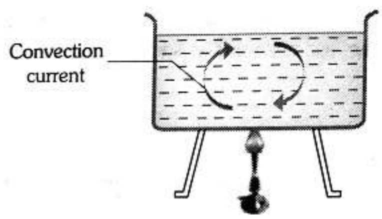
$$\Rightarrow K_{equivalent} = \frac{K_1 A_1 + K_2 A_2 + K_3 A_3 + \ldots .. K_n A_n}{A_1 + A_2 + A_3 + \ldots .. A_n}$$

- **Temperature of interface of composite bar :** Temperature gradient Same across each slab.

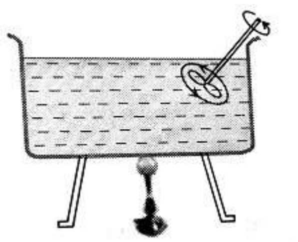
17. Heat Transfer By Convection

Convection is the heat transfer due to the bulk movement of molecules within fluids such as gases and liquids. Convection is of two types -

1. Natural convection : The main cause of this is the difference of densities at two places and is a consequence of gravity because on account of gravit the hot light particles rise up (density become less) and cold heavy particles (density become high) try setting down. It mostly occurs on heating a liquid/fluid.



2. Forced convection : If a fluid is **forced(by means of fan or draught or any external means)** to move to take up heat from a hot body then the convection process is called forced convection. In this case **Newton's law of cooling holds good**. According to which rate of loss of heat from a hot body due to moving fluid is directly proportional to the surface area of body and excess temperature of body over its surroundings.



18. Heat Transfer By Radiation

Radiation - The process of the transfer of heat from one place to another place without any requirement of the medium is called radiation. It means that the radiation does not need any material medium to propagate.

Characteristics of Radiation -

- The process of the transfer of heat from one place to another place without heating the medium is called radiation.
- The wavelength of thermal radiations ranges from $7.8 \times 10^{-7} m \;\; to \;\; 4 \times 10^{-7} m$. The radiation heat transfer belongs to the infra-red region of the electromagnetic spectrum. That is why thermal radiations are also called infra-red radiations.
- Every body whose temperature is above zero Kelvin emits thermal radiation. Practically it is not possible to reach 0 Kelvin in finite number of steps, so every material in this universe emit radiation.
- The intensity of thermal radiation is inversely proportional to the square of the distance of the point of observation from the source

$$(I \propto \frac{1}{d^2})$$

- As it is an electromagnetic wave, they follow laws of reflection, refraction, interference, diffraction, and polarisation.
- **Radiation pressure** - When these thermal radiations fall on a surface then exert pressure on that surface, which is called Radiation pressure.
- Radiation spectrum is obtained by **quartz or rock salt prism** because these materials do not have free electrons and interatomic vibrational frequency i greater than the radiation frequency, hence they do not absorb heat radiations.
- **Interaction of Radiation with Matter-**

When thermal radiations (Q) fall on a body, they are partly reflected, partly absorbed and partly transmitted

So we can write

$$Q = Q_a + Q_t + Q_r$$

or $$\frac{Q}{Q} = \frac{Q_a}{Q} + \frac{Q_t}{Q} + \frac{Q_r}{r}$$

or 1 = a + r + t

Where

$$\frac{Q_a}{Q} = a = Absorptance$$

$$\frac{Q_r}{Q} = r = Reflactance$$

$$\frac{Q_t}{Q} = t = Transmittance$$

So

1. If a = t = 0 and r = 1 then body is perfect reflector
2. If r = t = 0 and a = 1 then body is perfectly black body.
3. If, a = r = 0 and t = 1 then body is perfect transmitter
4. If $t = 0 \Rightarrow r + a = 1 \; or \; a = 1 - r$

   i.e. good reflectors are bad absorbers.

- **Prevost Theory of Heat Exchange-**

1. Every body emits heat radiations at all finite temperature (Except 0 K) as well as it absorbs radiations from the surroundings.
2. The amount of heat emitted/absorbed depends on the nature of the body, the temperature of the body and the cross-section of the body through which heat exchange is taking place.
3. The exchange of energy along various bodies takes place via radiation.
4. How the temperature of the body will vary will depend on the temperature of the surrounding

I. If surrounding temperature= body temperature

then $Q_{emission} = Q_{absorbed}$

i.e the body will emit and absorbed at the same rate

the temperature of the body remains constant (thermal equilibrium)

II. If surrounding temperature > body temperature

then $Q_{emission} < Q_{absorbed}$

i.e temperature of the body increases and it appears hotter.

III. If surrounding temperature < body temperature

then $Q_{emission} > Q_{absorbed}$

i.e temperature of the body decreases and consequently the body appears colder.

19. Black Body Radiation

The radiation radiated by the black body is known as **BLACK BODY RADIATION.**

Properties of Black Bodies -

- A perfectly black body is that which absorbs completely the radiations of all wavelengths incident on it.
- As a perfectly black body neither reflects nor transmits any radiation, therefore the absorptance of a perfectly black body is unity. I.e a=1
- The colour of an opaque body is the colour (wavelength) of radiation reflected by it. As a black body reflects no wavelength, so it appears black.
- When a perfectly black body is heated to a suitable high temperature, it emits radiation of all possible wavelengths. For example, the Sun is an example of a black body. As its temperature is very high and it emits all possible radiation.
- A perfectly black body is an ideal concept and it can't be realized in practice. But materials like Platinum black or Lampblack come close to being ideal black bodies. Such materials absorb 96% to 85% of the incident radiations.

20. Kirchhoff's Law

- **Emissive power-**

If the temperature of a body is more than it's surrounding then body emits thermal radiation.

1. **Spectral emissive power-** For a given surface **Spectral emissive power** is defined as the radiant energy emitted per sec per unit area of the surface within a unit wavelength around λ (i.e. lying between $(\lambda-\frac{1}{2})\ to\ (\lambda+\frac{1}{2})$. Spectral Emissive power for particular wavelength (λ) is denoted by e_λ

$$e_\lambda = \frac{Energy}{Area \times time \times wavelength}$$

So If the wavelength is changed then the value of Spectral Emissive power will also change.

2.Total Emissive Power (e) -Total Emissive power is It is defined as the total amount of thermal energy emitted per unit time, per unit area of the body for all possible wavelengths.

$$e = \int_0^\infty e_\lambda \, d\lambda$$

- **Absorptive Power**

1. **Spectral Absorptive power-** It is defined as the ratio of the amount of the energy absorbed in a certain time to the total heat energy incident upon it in the same time, both in the unit wavelength interval. It is denoted by a_λ

2. **Total Absorptive Power (a)-** It defined as the total amount of thermal energy absorbed per unit time, per unit area of the body for all possible wavelengths.

$$a = \int_0^\infty a_\lambda \, d\lambda$$

- **Emissivity** (ϵ) -

The emissivity of a body at a given temperature is defined as the ratio of the total emissive power of the body (e) to the total emissive power of a perfectly black body (E).

And it is given by $\varepsilon = \frac{e}{E}$

$\varepsilon = 1$ - for a perfectly black body

$\varepsilon = 0$ - for polished body

$(0 < \varepsilon < 1)$ - for practical bodies

- **Kirchhoff's law**

According to **Kirchhoff's law**, the ratio of emissive power to absorptive power is the same for all surfaces at the same temperature and is equal to the emissive power of a perfectly black body at that temperature.

I.e $\frac{e_1}{a_1} = \frac{e_2}{a_2} ...... = \frac{E}{A}$

And as for a perfectly black body A = 1

So $\frac{e_1}{a_1} = \frac{e_2}{a_2} ...... = \frac{E}{A}$

$$\frac{e_1}{a_1} = \frac{e_2}{a_2} ...... = E$$

or $\frac{e}{a} = E$

If emissive and absorptive powers are considered for a particular wavelength (λ)

then $\frac{e_\lambda}{a_\lambda} = E_\lambda$

This law also implies that a good absorber is a good emitter.

21. Wien's Displacement Law

Wien's displacement law states that the wavelength (i.e λ_{max}) for which the emissive power of a black body is maximum is inversely proportional to the absolute temperature (T) of the black body.

Or Mathematically we can write that

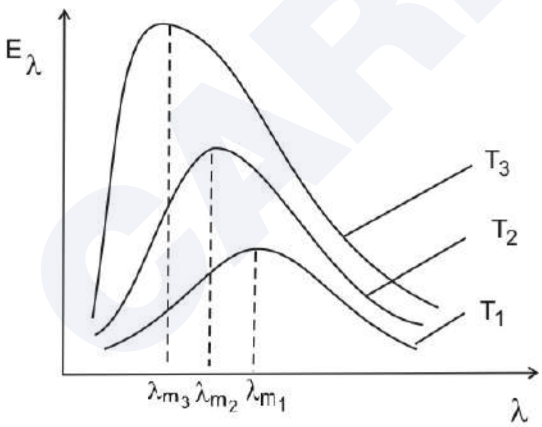
$$\lambda_{max} * T = b = constant$$

Where **b** is a constant of proportionality which is known as Wien's displacement constant.

Value of b is given as $b = 2.89 * 10^{-3}\ mK$

With the help of this law, we can say that

As the temperature of the body increases, the wavelength at which the spectral intensity (E_λ) is maximum shifts towards left, as shown in the below figure.



I.e

$$If\ \ T_1 < T_2 < T_3$$
$$Then\ \ \lambda_{m_1} > \lambda_{m_2} > \lambda_{m_3}$$

- Wien's displacement law is useful for determining the temperatures of hot radiant objects such as stars, and it is also useful for a determination of the temperature of any radiant object whose temperature is far above that of its surroundings.

22. Stefan Boltzmann Law

- According to **Stefan Boltzmann's law**, the radiant energy emitted by a perfectly black body per unit area per sec is directly proportional to the fourth power of its absolute temperature,

  or emissive power of the black body is directly proportional to the fourth power of its absolute temperature (θ).

  i.e $E \; \alpha \; \theta^4$

  $\Rightarrow \; E = \sigma\theta^4$

  where

  σ =Stefan's constant

  and its value is

  $\sigma = 5.67 \times 10^{-8} W/m^2K^4$

- **For ordinary body**

  1. **Emissive power** is given by $e = \epsilon E$

  So according to Stefan Boltzmann law

  $e = \epsilon E = \epsilon\sigma\theta^4$

  where ϵ = represent emissivity of the material

  2. **Radiant energy-**

  If Q is the total energy radiated by the ordinary body then

  $$e = \frac{Q}{A \times t} = \epsilon\sigma\theta^4 \Rightarrow Q = A\epsilon\sigma\theta^4 t$$

  3. **Radiant power (P):** It is defined as the energy radiated per unit area.

  i.e $P = \frac{Q}{t} = A\epsilon\sigma\theta^4$

  4. If an ordinary body at temperature θ is surrounded by a body at a temperature θ_0

  Then according to Stefan Boltzmann law

  $$e = \epsilon\sigma(\theta^4 - \theta_0^4)$$

23.Newton's Law Of Cooling

- **Rate of Loss of Heat**-

  If an ordinary body at temperature θ is placed in an environment of temperature $\theta_0 (\; and \;\; \theta_0 < \theta)$ then heat loss by radiation.

  And Rate of Loss of Heat is given by

  from **Stefan Boltzmann law**

  $$R_H = \frac{dQ}{dt} = A\epsilon\sigma(\theta^4 - \theta_0^4) \; .....(1)$$

- **Rate of Cooling-**

  If m is the body and c is the specific heat then $Q = mc\Delta\theta$

  And $\frac{dQ}{dt} = mc\frac{\Delta\theta}{dt}$ .....(2)

  Comparing equation 1 and 2

  we get Rate of Cooling as

  $$R_c = \frac{\Delta\theta}{dt} = \frac{A\epsilon\sigma}{mc}(\theta^4 - \theta_0^4)$$

  where

  c = specific heat capacity

  R_c= Rate of cooling.

As $mass = volume * density \Rightarrow \ m = \rho v$

So We can also write

$$R_c = \frac{A\epsilon\sigma}{v\rho c}(\theta^4 - \theta_0^4)$$

- If two bodies of the same material under identical environments

$$\frac{(R_c)_1}{(R_c)_2} = \frac{A_1 v_2}{A_2 v_1}$$

- Dependence of the rate of cooling (R_c)

As $R_c = \frac{A\epsilon\sigma}{mc}(\theta^4 - \theta_0^4) = \frac{A\epsilon\sigma}{v\rho c}(\theta^4 - \theta_0^4)$

So R_c will depend on

1. Nature of radiating surface i.e. greater the emissivity faster will be the cooling.
2. Area of the radiating surface, i.e. greater the area of the radiating surface, faster will be the cooling.
3. Specific heat of radiating body i.e. greater the specific heat of radiating body slower will be cooling.
4. Mass of radiating body i.e. greater the mass of radiating body slower will be the cooling.
5. The temperature of the radiating body i.e. greater the temperature of the body faster will be cooling.
6. The temperature of surrounding i.e. greater the temperature of surrounding slower will be cooling.

- **Newton's Law of Cooling**

According to Newton's Law of Cooling, if the temperature difference between the body and its surrounding is very small then the Rate of cooling is directly proportional to the temperature difference between the body and its surrounding.

I.e $\frac{d\theta}{dt}\alpha(\theta - \theta_0)$

1. Greater the temperature difference between the body and its surrounding greater will be the rate of cooling.
2. $if\ \theta = \theta_0 \Rightarrow \frac{d\theta}{dt} = 0$ i.e. a body can never be cooled to a temperature lesser than its surrounding by radiation.
3. When the body Cools by Radiation from $\theta_1^0 C$ to theta $\theta_2^0 C$ in time t

Then $\left[\frac{\theta_1 - \theta_2}{t}\right] = k\left[\frac{\theta_1 + \theta_2}{2} - \theta_0\right]$

Where $\theta_{av} = \frac{\theta_1 + \theta_2}{2}$

Variation of curves for Newton's Law of Cooling-

According to **Newton's Law of Cooling**

$\frac{d\theta}{dt}\alpha(\theta - \theta_0)$

or we can say that $\frac{d\theta}{dt} = k(\theta - \theta_0)$

where

k is the proportionality constant

$R = \frac{d\theta}{dt} = Rate\ of\ cooling$

$\theta = Temperature\ of\ the\ \ body$

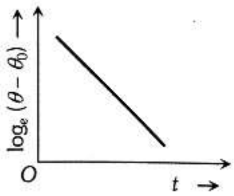
$\theta_0 = Temperature\ of\ the\ \ surrounding$

Using the above formula we can plot various curves

1.The curve between $log(\theta - \theta_0)\ Vs\ \ time(t)$

As $log_e(\theta - \theta_0) = -kt + c$

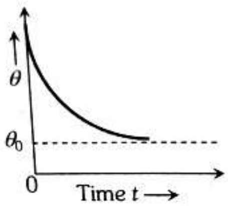
So the graph will be



2. **The curve between Temperature of body and time** i.e $\theta \ Vs \ t$

As $\theta - \theta_0 = Ae^{-k_t}$

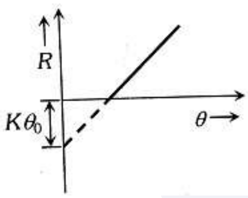
So the graph will be



3. **The curve between rate of Cooling and body temperature I.e** $R = \frac{d\theta}{dt} \ vs \ \theta$

As $R = \frac{d\theta}{dt} = K(\theta - \theta_0) = K\theta - K\theta_0$

So the graph will be

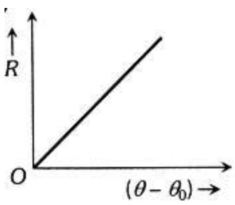


4. **The curve between the Rate of Cooling (R) and the Temperature difference between body and Surrounding**

I.e $R = \frac{d\theta}{dt} \quad Vs \quad (\theta - \theta_0)$

As $R \ \alpha \ (\theta - \theta_0)$

So the graph will be



Kinetic theory of Gases

Important Formulae

1.States Of Matter

The matter is defined as any substance that has mass and takes up space by having volume.

A **state of matter** is one of the distinct forms in which matter can exist. The states of matter are broadly classified in three states -

1. Solid
2. Liquid
3. Gas

However there is a fourth state (Plasma) also, but that is not in the scope of our syllabus.

- **Solid -** A solid is a state of matter in which particles are arranged such that their shape and volume are relatively stable. In this, the constituents of a solid tend to be packed together much closer than the particles in a gas or liquid.
- **Liquid -** A liquid is a state of matter which is a nearly incompressible fluid and it conforms to the shape of its container but retains a constant volume independent of pressure. It means that the volume is not changing with pressure.
- **Gas -** A gas is defined as a state of matter consisting of particles that have neither a defined volume nor a defined shape.

Comparison Chart of Solids, Liquids and Gaseous States.

Property	Solid	Liquid	Gas
Shape	Definite	Not definite	Not definite
Volume	Definite	Definite	Not definite
Density	Maximum	Less than solids but more than gases.	Minimum
Compressibility	Incompressible	Less than gases but more than solids.	Compressible
Crystallinity	Crystalline	Non-crystalline	
Interatomic or intermolecular distance	Constant	Non constant	Non constant
Relation between kinetic energy K and potential energy (U)	K<U	K>U	K>>U
Intermolecular force	Strongest	Less than solids but more than gases.	Weakest
Freedom of motion	Molecules vibrate about their mean position but cannot move freely.	Molecules have limited free motion.	Molecules are free to move.
Effect of temperature	Matter remains in solid form below a certain temperature.	Liquids are found at temperatures more than that of solid.	These are found at temperatures greater than that of solids and liquids.

2. Kinetic Theory Of Gases Assumptions

Ideal gas - It is a hypothetical gas (which is not real gas), whose molecules occupy negligible space and have no interactions (Force of interaction is very less), and which consequently obeys the gas laws exactly.

So, the ideal gas does not exist in real, but for study we take some assumption to make the gas ideal and we can apply some laws which are only valid for idea gases. These assumptions are -

1. The size of the molecules is negligible in comparison to intermolecular distance ($10^{-9} m$).
2. The molecules of a gas are identical, spherical, rigid and perfectly elastic point masses (It means that when they collide with each other, then there is no loss of energy while collision).
3. The molecules of a given gas are all identical but these molecules are different than those of another gas.
4. The volume of molecules is negligible in comparison to the volume of gas.
5. Molecules of a gas moves randomly in all possible direction with all possible velocities.
6. The speed of gas molecules varies from zero and infinity.
7. The gas molecules keep on colliding among themselves as well as with the walls of containing vessel. These collisions are perfectly elastic (no loss of energy).
8. The time spent in a collision between two molecules is negligible in comparison to time between two successive collisions (i.e., time required to travel mean free path).
9. The number of collisions per unit volume in a gas remains constant.
10. No attractive or repulsive force acts between gas molecules.
11. Gravitational attraction among the molecules is negligible due to extremely small masses and very high speed of molecules.
12. Molecules constantly collide with the walls of container due to which their momentum changes. The change in momentum is transferred to the walls of the container and due to this **Pressure** is exerted by gas molecules on the walls of the container.
13. The density of gas does not changes at any point of container.

3.The Gas Laws

BOYLE'S LAW-

It states that, for a given mass of an ideal gas at constant temperature, the volume of a gas is inversely proportional to its pressure.

$$V \propto \frac{1}{P}$$

$$\text{or,} \quad P.V = \text{constant}$$

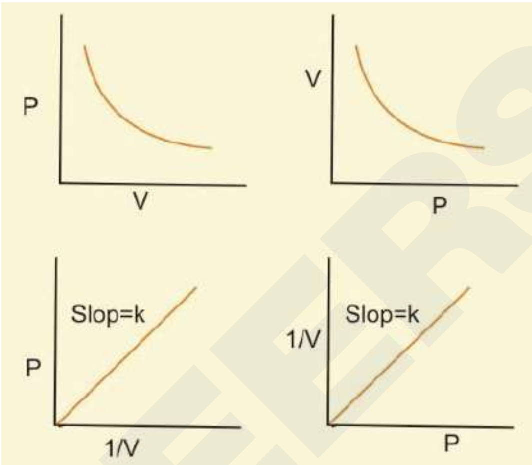
$$\Rightarrow P_1V_1 = P_2V_2$$

We can also write the above equation as,

$$PV = P\left(\frac{m}{\rho}\right) = \text{constant}$$

$$\text{So,} \Rightarrow \frac{P}{\rho} = \text{constant or } \frac{P_1}{\rho_1} = \frac{P_2}{\rho_2}$$

We can represent the Boyle's law through the various graph, which is shown as -



CHARLE'S LAW -

It states that, if the pressure remaining constant, the volume of the given mass of a gas is directly proportional to its absolute temperature.

From the above statement we can conclude the following equations -

$$\boldsymbol{V \propto T}$$

$$\frac{V}{T} = Constant$$

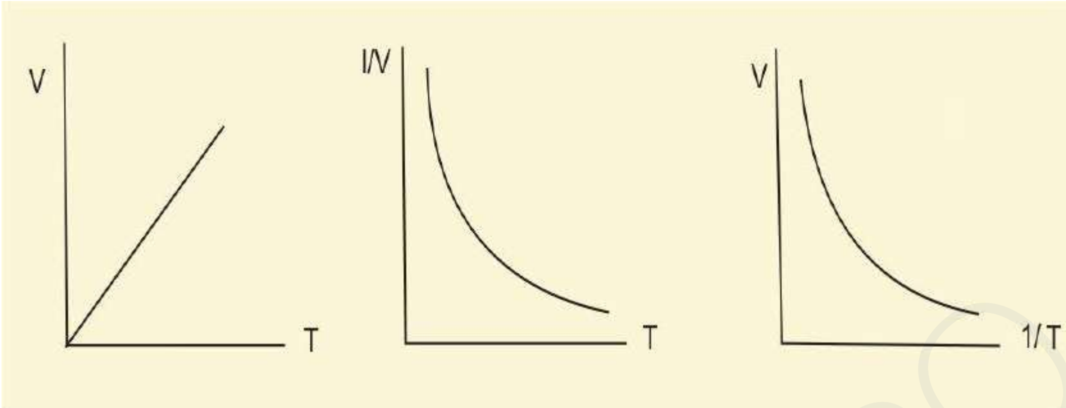
$$\text{So,} \frac{V_1}{T_1} = \frac{V_2}{T_2}$$

This equation can also be written in terms of density and temperature as -

$$\frac{V}{T} = \frac{m}{\rho T} = \text{constant} \left(\text{As volume } V = \frac{m}{\rho}\right)$$

$$\text{or,} \quad \rho T = \text{constant} \Rightarrow \rho_1 \mathbf{T_1} = \rho_2 \mathbf{T_2}$$

We can represent the Charle's law through the various graph, which is shown as -



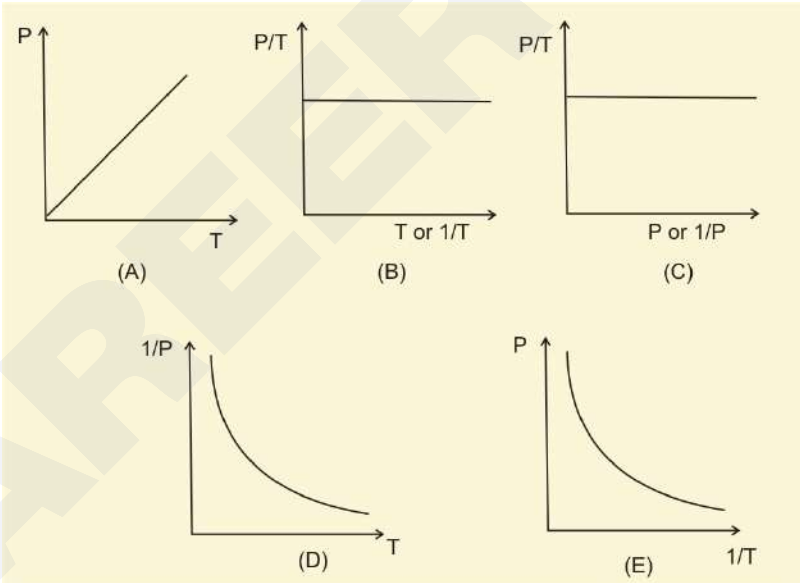
Gay-Lussac's law or pressure law -

If the volume remains constant, then the pressure of a given mass of a gas is directly proportional to its absolute temperature.

So, We can conclude the above statement in the following equation -

$$P \propto T \text{ or } \frac{P}{T} = \text{ constant } \Rightarrow \frac{P_1}{T_1} = \frac{P_2}{T_2}$$

The graphical representation of Gay-Lussac's law is -



AVAGADRO'S LAW -

Equal volume of all the gases under similar conditions of temperature and pressure contain equal number of molecules.

It implies that -

$$N_1 = N_2$$

N = Number of molecules in a particular gas.

GRAHAM'S LAW OF DIFFUSION

It states that when any two gases at the same pressure and temperature are allowed to diffuse into each other, then the rate of diffusion of each gas is inversely proportional to the square root of the density of the gas.

So we can say that,

$$r \propto \frac{1}{\sqrt{\rho}} \propto \frac{1}{\sqrt{M}} \alpha V_{rms}$$

Where, r = rate of diffusion of gas

ρ = Density of the gas

M = Molecular weight of the gas

V_{rms}= Root mean square velocity

Now, from the above equation, we can write,

$$\frac{r_1}{r_2} = \sqrt{\frac{\rho_2}{\rho_1}} = \sqrt{\frac{M_2}{M_1}}$$

DALTON'S LAW OF PARTIAL PRESSURE -

It states that the total pressure exerted by a mixture of non-reacting gases occupying a vessel is equal to the sum of the individual pressures which each gases exert if it alone occupied the same volume at a given temperature.

Now, let us have a mixture of 'n' gases, so from the above statement we can conclude that -

$$\text{For } n \text{ gases } P = P_1 + P_2 + P_3 + \ldots\ldots P_n$$

Here, P = Pressure exerted by the mixture of gases

$P_1, P_2, \ldots\ldots P_n$ = Partial pressure of the component gases.

4. Ideal Gas Equation

- **Ideal gas equation-**

  The equation which relates the pressure (P), volume (V) and temperature (T) of the given state of an ideal gas is known as an ideal gas equation or equation of state.

  From **Boyle's law,** we get $V \propto \frac{1}{P}$ .....(1)

  and From **Charle's Law,** we get $V \propto T$ ....(2)

  And from **Avogadro's Law,** we get $V \propto n$ .... (3)

  And from equation (1), (2), (3)

  we can write

$$V \propto \frac{nT}{P}$$
$$or \ \frac{PV}{nT} = constant$$
$$or \ \frac{PV}{nT} = R \ (where \ \ R \ \ is \ proportionality \ constant)$$
$$or \, PV = nRT$$

  So Ideal Gas Equation is given as

  $PV = nRT$

  where

  T= Temperature

  P= pressure of ideal gas

  V= volume

  n= numbers of mole

  R = universal gas constant

- Universal gas constant (R)-

  At S.T.P. the value of the universal gas constant is the same for all gases.

And its value is given as

$$R = 8.31\ \frac{Joule}{mole \times Kelvin} = 2\ \frac{cal}{mole \times Kelvin}$$

And its Dimension is : $[ML^2T^{-2}\theta^{-1}]$

- Boltzman's constant (k)-

It is represented by per mole gas constant.

i.e $k = \frac{R}{N} = \frac{8.31}{6.023 \times 10^{23}} = 1.38 \times 10^{-23} \mathrm{J/K}$

- Specific gas constant (r)-

It is represented by per gram gas constant.

i.e. $r = \frac{R}{M}$

It's unit is $\frac{\text{Joule}}{\text{gm} \times \text{kelvin}}$

5. Real Gas And Equation

- **Real gas-** The gases which do not obey gas Laws are called Real gas.

Two main factors because of which Real gas deviates from ideal gas are:

1) Presence of force of attraction between molecules.

2) The size of molecules are not negligible.

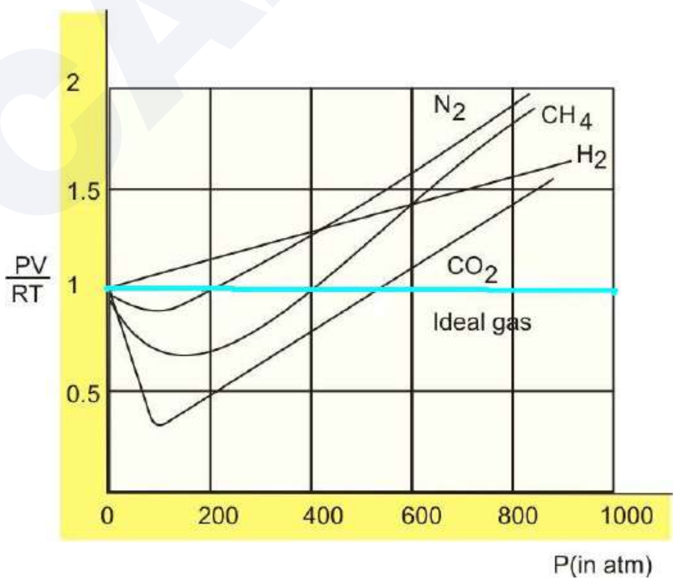
The gases actually found in nature are called real gases.

From the ideal gas equation, we get

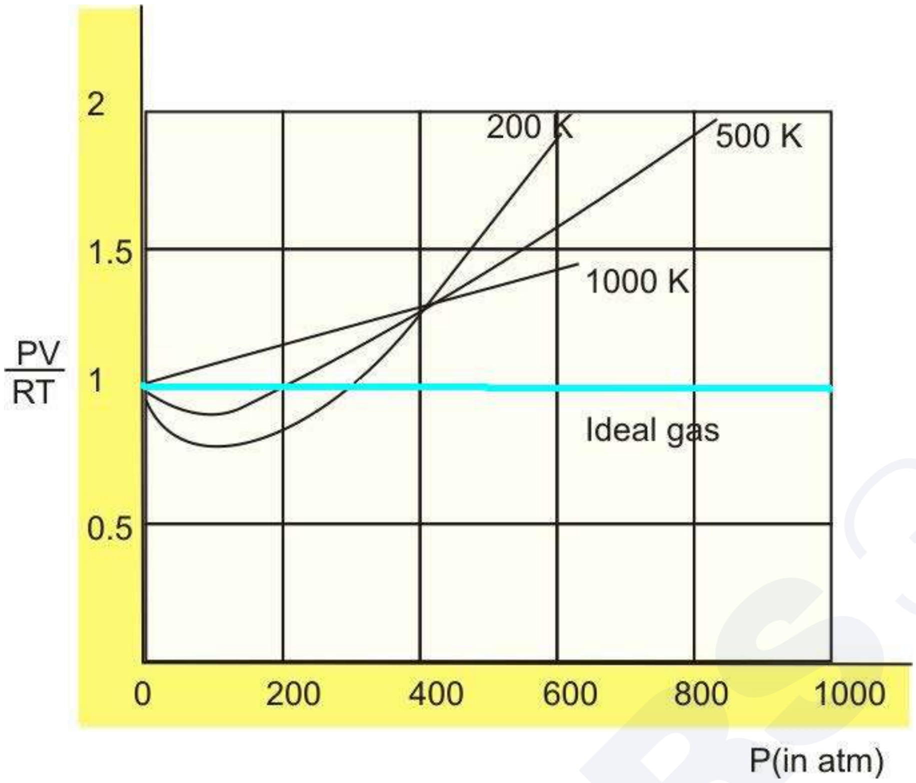
For exactly one mole of an ideal gas $\frac{PV}{RT} = 1$

The quantity $\frac{PV}{RT}$ is called the **compressibility factor** and should be a unit for an ideal gas.

Plotting the experimentally determined value of $\frac{PV}{RT}$ for exactly one mole of various real gases as a function of pressure P shows a deviation from identity as shown in the below graph.



Similarly, real gases show deviation from ideal behaviour as a function of temperature as shown in the below graph.



From the above graphs, we can say that A real gas behaves as an ideal gas most closely at low pressure and high temperature.

- **Real gas equation-** Real gas equation, For n moles of gas is given by

$$\left(P+\frac{n^2a}{V^2}\right)(V-nb)=nRT$$ .... (1)

Where a and b are called Vander wall's constant having dimensions and units as follows:

Dimension $: [a]=[ML^5T^{-2}]$ and $[b]=[L^3]$Dimension $: [a]=[ML^5T^{-2}]$ and $[b]=[L^3]$
Units $: a=N\times m$ and $b=m^3$ Units $: a=N\times m$ and $b=m^3$

As we know the ideal gas equation as PV=nRT...... (2)

From equations (1) and (2) we can say that

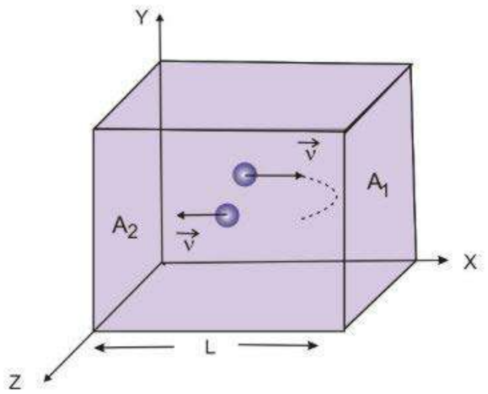
The real gas equation is nothing but the ideal gas equation with two corrections (i.e Volume correction and Pressure correction)

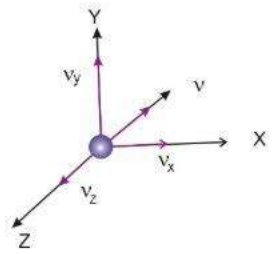
These corrections are given by Vander Waal's. So the real gas equation is also known as Vander Waal's gas equation.

1. **Volume correction-** Due to the finite size of the molecules the effective volume of gas becomes (V –nb).
2. **Pressure correction-** Due to the presence of intermolecular force in real gases, the effective pressure of gas becomes $P+\frac{n^2a}{V^2}$

6. Pressure Of An Ideal Gas

Consider an ideal gas (consisting of N molecules each of mass m) enclosed in a cubical box of side L as shown in the below figure.





1. **Instantaneous velocity-**

Any molecule of gas moves with velocity $\vec{v}$ in any direction

where $\vec{v}=v_x\hat{i}+v_y\hat{j}+v_z\hat{k}$

And Due to the random motion of the molecule

$v_x=v_y=v_z$

$$As\ v=\sqrt{v_x^2+v_y^2+v_z^2}$$
$$\Rightarrow v=3v_x^2=3v_y^2=3v_z^2$$

2. **The time during a collision-**Time between two successive collisions with the wall A_1

$$I.e\quad \Delta t=\frac{\text{Distance travelled by molecule between two successive collision}}{\text{Velocity of molecule}}$$

$$or\qquad \Delta t=\frac{2L}{v_x}$$

3. **Collision frequency (n):** It means the number of collisions per second.

$$I.e\ \ n=\frac{1}{\Delta t}=\frac{v_x}{2L}$$

4. **Change in momentum:** This molecule collides with A_1 wall (A1) with velocity v_x and rebounds with velocity ($-v_x$)
The change in momentum of the molecule is given by

$$\Delta p=(-mv_x)-(mv_x)=-2mv_x$$

As the momentum remains conserved in a collision,

$$\Delta p_{system}=0$$
$$\Delta p_{system}=\Delta p_{molecule}+\Delta p_{wall}=0$$
$$\Delta p_{wall}=-\Delta p_{molecule}$$

the change in momentum of wall A_1 is $\Delta p=2mv_x$

5. **Force on the wall:** Force exerted by a single molecule on the A_1 wall is equal to the rate at which the momentum is transferred to the wall by this molecule

$$\text{i.e. } F_{\text{Single molecule}}=\frac{\Delta p}{\Delta t}=\frac{2mv_x}{(2L/v_x)}=\frac{mv_x^2}{L}$$

The total force on the wall A_1 due to N molecules

$$F_x=\frac{m}{L}\sum v_x^2=\frac{m}{L}\left(v_{x1}^2+v_{x2}^2+v_{x3}^2+\ldots\right)=\frac{mN}{L}\overline{v_x^2}$$

where $\overline{v_x^2}=$ mean square of x component of the velocity.

6. **Pressure-**As pressure is defined as force per unit area, hence the pressure on A_1 wall

$$P_x=\frac{F_x}{A}=\frac{mN}{AL}\overline{v_x^2}=\frac{mN}{V}\overline{v_x^2}$$

$$As\ \overline{v_x^2} = \overline{v_y^2} = \overline{v_z^2}$$
$$So\ \ \overline{v^2} = \overline{v_x^2} + \overline{v_y^2} + \overline{v_z^2}$$
$$\Rightarrow \overline{v_x^2} = \overline{v_y^2} = \overline{v_z^2} = \frac{\overline{v^2}}{3}$$

So Total pressure inside the container is given by

$$P = \frac{1}{3}\frac{mN}{V}\overline{v^2} = \frac{1}{3}\frac{mN}{V}v_{rms}^2 \quad \left(\text{where } v_{rms} = \sqrt{\overline{v^2}}\right)$$

Using total mass= M=mN

Pressure due to an ideal gas is given as

$$P = \frac{1}{3}\rho\, v_{rms}^2 = \frac{1}{3}\left(\frac{M}{V}\right)\cdot v_{rms}^2$$

where

m = mass of one molecule

N = Number of the molecule

$$v_{rms}^2 = \frac{v_1^2 + v_2^2 + \dots\dots\dots}{n}$$

v_{rms} = RMS velocity

7. The Maxwell Distribution Laws

Various types of speeds of ideal gases-

- **Root mean square speed-** It is defined as the square root of the mean of squares of the speed of different molecules.

  $$\text{ie. } v_{ms} = \sqrt{\frac{v_1^2 + v_2^2 + v_3^2 + v_4^2 + \dots}{N}} = \sqrt{\bar{v}^2}$$

  1. As the Pressure due to an ideal gas is given as

  $$P = \frac{1}{3}\rho\, v_{rms}^2$$

  $$\Rightarrow v_{rms} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3PV}{\text{Mass of gas}}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3kT}{m}}$$

  Where

  R = Universal gas constant

  M = molar mass

  P = pressure due to gas

  ρ = density

  2. $v_{rms}\ \alpha\ \sqrt{T}$ I.e With the rise in temperature, rms speed of gas molecules increases.

  3. $v_{rms}\ \alpha\ \frac{1}{\sqrt{M}}$ I.e With the increase in molecular weight, rms speed of the gas molecule decreases.

  4. The rms speed of gas molecules does not depend on the pressure of the gas (if the temperature remains constant)

- **Most probable speed-**This is defined as the speed which is possessed by maximum the fraction of the total number of molecules of the gas.

  I.e $$v_{mps} = \sqrt{\frac{2P}{\rho}} = \sqrt{\frac{2RT}{M}} = \sqrt{\frac{2kT}{m}}$$

- **Average speed-**It is the arithmetic mean of the speeds of molecules in a gas at a given temperature.

  $$v_{avg} = \frac{v_1 + v_2 + v_3 + v_4 + \dots.}{N}$$

  and according to the kinetic theory of gases

$$v_{avg}=\sqrt{\frac{8P}{\pi\rho}}=\sqrt{\frac{8}{\pi}\frac{RT}{M}}=\sqrt{\frac{8}{\pi}\frac{kT}{m}}$$

- The relation between RMS speed, average speed, and most probable speed

$$V_{rms}>V_{avg}>V_{mps}$$

Maxwell's Law -

The v_{rms} (Root mean square velocity) gives us a general idea of molecular speeds in a gas at a given temperature. So, it doesn't mean that the speed of each molecule is v_{rms}.

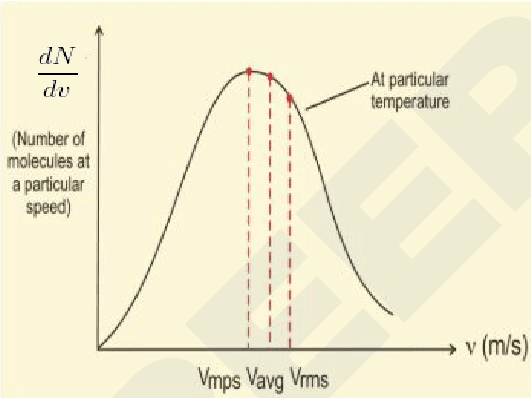
Many of the molecules have speed less than v_{rms} and many have speeds greater than v_{rms}. So, **Maxwell** derived an equation that describes the distribution of molecules in different speeds as -

$$\mathbf{dN}=4\pi\mathrm{N}\left(\frac{\mathbf{m}}{2\pi\mathbf{kT}}\right)^{3/2}\mathbf{v}^{2}\mathbf{e}^{-\frac{\mathbf{mv}^{2}}{2\mathbf{kT}}}\mathbf{dv}$$

where, $dN=$ Number of molecules with speeds between v and $v+dv$

So, from this formula, you have to remember a few key points -

1. $\frac{dN}{dv}\propto N$
2. $\frac{dN}{dv}\propto v^2$



Conclusions from this graph -

1. This graph is between number of molecules at a particular speed and speed of these molecules.

2. You can observe that the $\frac{dN}{dv}$ is maximum at most probable speed.

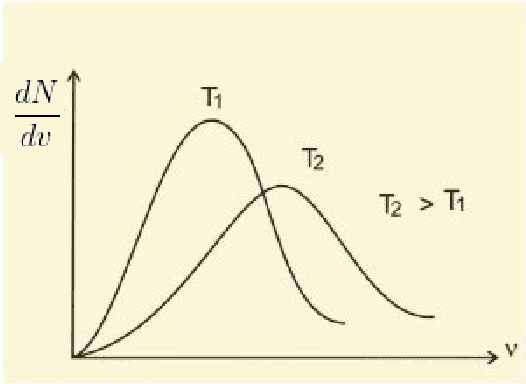
3. This graph also represent that $v_{rms}>v_{av}>v_{mp}$.

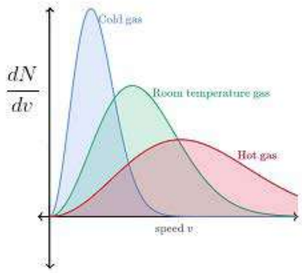
4. This curve is asymmetric curve.

5. From this curve we can calculate number of molecules corresponds to that velocity range by calculating area bonded by this curve with speed axis.

Effect of temperature on velocity distribution :

With rising of temperature, the curve starts shifting right side and become broader as shown as -

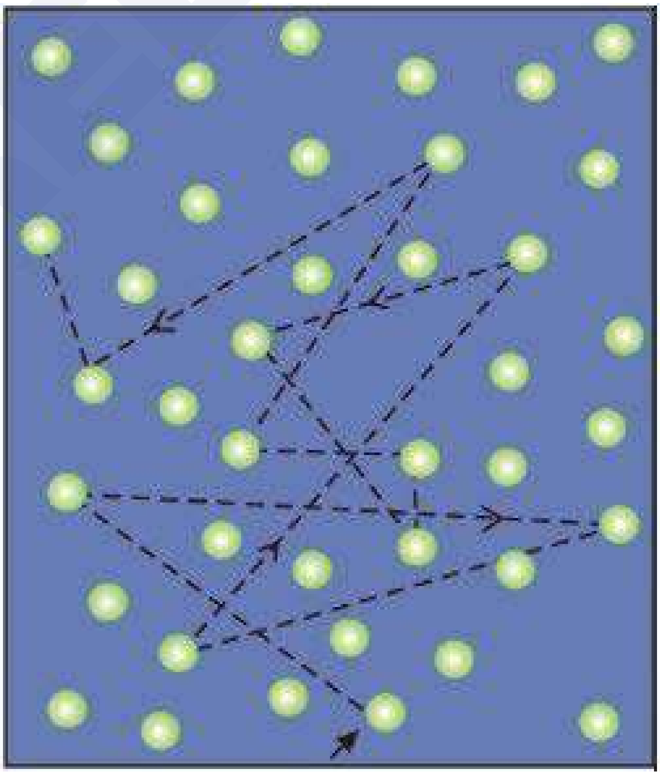




8. Mean Free Path

On the basis of kinetic theory of gases, it is assumed that the molecules of a gas are continuously colliding against each other. So, the distance travelled by a gas molecule between any two successive collisions is known as **free path**.

There are assumption for this theory that during two successive collisions, a molecule of a gas moves in a straight line with constant velocity. Now, let us discuss the formula of mean free path -



Let $\lambda_1, \lambda_2 \ldots\ldots \lambda_n$ be the distance travelled by a gas molecule during **n** collisions respectively, then the mean free path of a gas molecule is defined as -

$$\lambda = \frac{\text{Total distance travelled by a gas molecule between successive collisions}}{\text{Total number of collisions}}$$

Here, λ is the mean free path.

It can also be written as - $\lambda = \frac{\lambda_1 + \lambda_2 + \lambda_3 + \ldots + \lambda_n}{n}$

Now, let us take d = Diameter of the molecule,
N = Number of molecules per unit volume.

Also, we know that, PV = nRT

So, Number of moles per unit volume $= \frac{n}{V} = \frac{P}{RT}$

Also we know that number of molecules per unit mole $= N_A = 6.023 \times 10^{23}$

So, the number of molecules in 'n' moles = nN_A

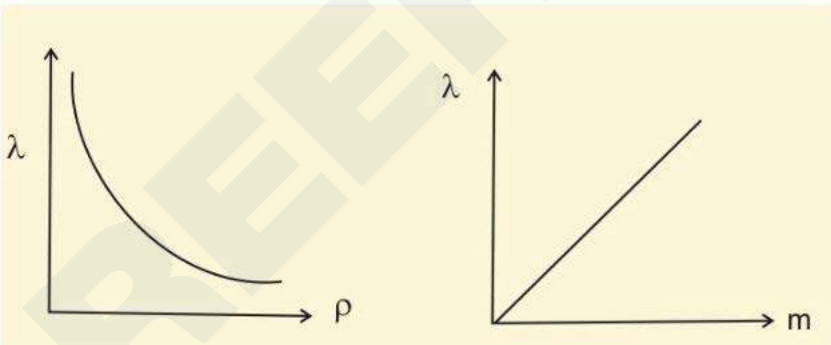
So the number of molecules per unit volume is $N = \frac{PN_A}{RT}$

So, $\lambda = \frac{RT}{\sqrt{2}\pi d^2 PN_A} = \frac{kT}{\sqrt{2}\pi d^2 P}$

If all the other molecules are not at rest then, $\lambda = \frac{1}{\sqrt{2}\pi N d^2} = \frac{RT}{\sqrt{2}\lambda d^2 PN_A} = \frac{kT}{\sqrt{2}\pi d^2 P}$

Now, if $\lambda = \frac{1}{\sqrt{2}\pi N d^2}$ and m = mass of each molecule then we can write - $\lambda = \frac{1}{\sqrt{2}\pi N d^2} = \frac{m}{\sqrt{2}\pi (mN) d^2} = \frac{m}{\sqrt{2}\pi d^2 \rho}$

So, $\lambda \propto \frac{1}{\rho}$ and $\lambda \propto m$



9. Degree of freedom

The degree of freedom of systems is defined as the possible independent motions, systems can have.

Or

The degree of freedom of systems is defined as the number of independent coordinates required to describe the system completely.

The independent motions can be **translational, rotational or vibrational** or any combination of these.

So the degree of freedom is of three types :
(i) Translational degree of freedom
(ii) Rotational degree of freedom
(iii) Vibrational degree of freedom

The degree of freedom is denoted by f.

And it is given by

$f = 3N - R$

Where

$N = no.\ of\ particle$

$R = no.\ of\ relation$

- **Value of degree of freedom for**

1.**Monoatomic gas-**

A monoatomic gas can only have a translational degree of freedom.

I.e $f = 3$

Example- He, Ne, Ar

2. **Diatomic gas**

A diatomic gas can have three translational degrees of freedom and two rotational degrees of freedom.

I.e $f = 5$

Example- H_2 , O_2 , N_2

3. Triatomic gas-

A triatomic gas can have three translational degrees of freedom and three rotational degrees of freedom.

I.e $f = 6$

Example- H_2O

- **Note-**

The above degrees of freedom are shown at room temperature. Further at high temperature, in the case of diatomic or polyatomic molecules, the atoms within the molecule may also vibrate with respect to each other. In such cases, the molecule will have 2 additional degrees of freedom, due to vibrational motion. I.e One for the potential energy and one for the kinetic energy of vibration.

So A diatomic molecule that is free to vibrate (in addition to translation and rotation) will have 7 degrees of freedom.

10. The kinetic energy of ideal gas-

In ideal gases, the molecules are considered as point particles. The point particles can have only translational motion and thus only translational energy. So for an ideal gas, the internal energy can only be translational kinetic energy.

Hence kinetic energy (or internal energy) of n mole ideal gas

$$E = \frac{1}{2} n M v_{ms}^2 = \frac{1}{2} n M \times \frac{3RT}{M} = \frac{3}{2} nRT$$

1. kinetic energy of 1 molecule

$$E = \frac{3}{2} kT$$

where k = Boltzmann's constant

and $k = 1.38 \times 10^{-23} J/K$

i.e Kinetic energy per molecule of gas does not depends upon the mass of the molecule but only depends upon the temperature of the gas.

2. kinetic energy of 1 mole ideal gas

$$E = \frac{3}{2} RT$$

i.e Kinetic energy per mole of gas depends only upon the temperature of the gas.

3. At T = 0, E = 0 i.e. at absolute zero the molecular motion stops.

- **The relation between pressure and kinetic energy**

As we know $P = \frac{1}{3} \frac{mN}{V} v_{ms}^2 = \frac{1}{3} \frac{M}{V} v_{ms}^2 \Rightarrow P = \frac{1}{3} \rho v_{ms}^2$ ......(1)

And K.E. per unit volume= $E = \frac{1}{2} \left(\frac{M}{V} \right) v_{ms}^2 = \frac{1}{2} \rho v_{ms}^2$ ...... (2)

So from equation (1) and (2), we can say that $P = \frac{2}{3} E$

i.e. the pressure exerted by an ideal gas is numerically equal to the two-third of the mean kinetic energy of translation per unit volume of the gas.

- **Law of Equipartition of Energy-**

According to this law, for any system in thermal equilibrium, the total energy is equally distributed among its various degrees of freedom.

I.e Each degree of freedom is associated with energy $E=\frac{1}{2}kT$

1. At a given temperature T, all ideal gas molecules will have the same average translational kinetic energy as $\frac{3}{2}kT$

2. Different energies of a system of the **degree of freedom f** are as follows

(i) Total energy associated with each molecule $=\frac{f}{2}kT$

(ii) Total energy associated with N molecules $=\frac{f}{2}NkT$

(iii) Total energy associated with 1 mole $=\frac{f}{2}RT$

(iv) Total energy associated with n mole $=\frac{nf}{2}RT$

11.Specific Heat Of A Gas

Specific heat - The specific heat is the amount of *heat* per unit mass required to raise the temperature by one Kelvin.

Now for gases, we have several types of specific heat, but here we will discuss basically two types of specific heat -

1. **Specific heat at constant volume(c_v) -**It is defined as the quantity of heat required to raise the temperature of unit mass of gas through 1°C or 1 Kelvin at constant volume.

   It is given as - $c_v=\frac{(\Delta Q)_V}{m\Delta T}$

   If 1 mole of gas is placed at the place of unit mass is considered, then this specific heat of gas is called **molar specific heat at constant volume** and is represented by **Cv** (Here C is capital)

   So, for molar specific heat -

$$C_V=Mc_V=\frac{M(\Delta Q)_V}{m\Delta T}=\frac{1}{\mu}\frac{(\Delta Q)_V}{\Delta T} \quad \left[\text{As } \mu=\frac{m}{M}\right]$$

2. **Specific heat at constant pressure (c_p) -**It is defined as the quantity of heat required to raise the temperature of unit mass of gas through 1°C or 1 Kelvin at constant pressure.

   It is given as - $c_p=\frac{(\Delta Q)_p}{m\Delta T}$

   If 1 mole of gas is placed at the place of unit mass is considered, then this specific heat of gas is called **molar specific heat at constant pressure** and is represented by $\mathbf{C_p}$ (Here C is capital)

   So, for molar specific heat at constant pressure -

$$C_p=Mc_p=\frac{M(\Delta Q)_p}{m\Delta T}=\frac{1}{\mu}\frac{(\Delta Q)_p}{\Delta T} \quad \left[\text{As } \mu=\frac{m}{M}\right]$$

12.Mayer's Formula

Molar Specific heat of the gas at constant volume=C_v

and Molar Specific heat capacity at constant pressure=C_p

Mayer's formula gives the relation between C_p and C_v as $C_p=C_v+R$

or we can say that molar Mayer's formula shows that specific heat at constant pressure is greater than that at constant volume.

- **Specific Heat in Terms of Degree of Freedom**

  1.Molar Specific heat of the gas at constant volume (C_v)

  For a gas at temperature T, the internal energy

$U=\frac{f}{2}nRT \Rightarrow$ Change in energy $\Delta U=\frac{f}{2}nR\Delta T\ldots$ (i)

Also, as we know for any gas heat supplied at constant volume

$(\Delta Q)_V=nC_V\Delta T=\Delta U.......(ii)$

From the equation (i) and (ii)

$C_v=\frac{fR}{2}$

where

f = degree of freedom

R= Universal gas constant

2. Molar Specific heat of the gas at constant pressure (C_P)

From Mayer's formula, we know that $C_P=C_v+R$

$$\Rightarrow C_P=C_V+R=\frac{f}{2}R+R=\left(\frac{f}{2}+1\right)R$$

3. Atomicity or adiabatic coefficient (γ)

It is the ratio of C_P to C_v

$$\gamma=\frac{C_P}{C_v}=1+\frac{2}{f}$$

Value of γ is always more than 1

for Monoatomic gas $\gamma=\frac{5}{3}$

for Diatomic gas $\gamma=\frac{7}{5}$

for Triatomic gas $\gamma=\frac{4}{3}$

- **Gaseous Mixture**

If two non-reactive gases A and B are enclosed in a vessel of volume V.

In the mixture n_1 mole of Gas A (having Specific capacities as C_{p1} and C_{v1} , Degree of freedom f_1 and Molar mass as M_1) is mixed with

n_2 mole of Gas B (having Specific capacities as C_{p2} and C_{v2} ,Degree of freedom f_2 and Molar mass as M_2)

Then Specific heat of the mixture at constant volume will be

$$C_{v_{mix}}=\frac{n_1C_{v_1}+n_2C_{v_2}}{n_1+n_2}$$

Similarly, Specific heat of the mixture at constant pressure will be

$$C_{p_{mix}}=\frac{n_1C_{p_1}+n_2C_{p_2}}{n_1+n_2}$$

And adiabatic coefficient (γ) of the mixture is given by

$$\gamma_{\text{mixure}}=\frac{C_{p_{mix}}}{C_{v_{mx}}}=\frac{\frac{(n_1C_{p_1}+n_2C_{p_2})}{n_1+n_2}}{\frac{(n_1C_{v_1}+n_2C_{v_2})}{n_1+n_2}}=\frac{(n_1C_{p_1}+n_2C_{p_2})}{(n_1C_{v_1}+n_2C_{v_2})}$$

Also

$$\frac{1}{\gamma_{mix}-1}=\frac{\frac{n_1}{\gamma_1-1}+\frac{n_2}{\gamma_2-1}}{n_1+n_2}$$

Similarly, the Degree of freedom of mixture is given as

$$f_{mix} = \frac{n_1 f_1 + n_2 f_2}{n_1 + n_2}$$

Similarly, the molar mass of the mixture

$$M_{mix} = \frac{n_1 M_1 + n_2 M_2}{n_1 + n_2}$$

Thermodynamics

Important Formulae

1.Introduction To Thermodynamics

Thermodynamics : It is a branch of science which deals with the exchange of energy in the form of heat or work between system and surroundings. It deals with the conversion of the heat energy into mechanical energy and vice-versa.

Thermodynamic system and surroundings : The collection of an extremely large number of atoms or molecules which are confined within certain boundaries (either fixed or moveable) such that it has a certain value of pressure, volume and temperature is called a **thermodynamic system**.

Anything outside the thermodynamic system to which energy or matter is exchanged is called its **surroundings.**

Example : Suppose there is Piston-cylinder arrangement which contains any gas within it, then the gas enclosed in a cylinder fitted with a piston forms the thermodynamic system but the atmospheric air which is outside the cylinder, movable piston are surroundings.

Thermodynamic system are classified in three major categories -

(i) **Open system** : It exchange both energy and matter with the surroundings.

(ii) **Closed system** : It exchange only energy (not matter) with the surroundings.

(iii) **Isolated system** : It exchange neither energy nor matter with the surroundings.

2.Thermodynamic State Variables And Equation Of State

Thermodynamic variables : Any thermodynamic system can be described by specifying some of the variables i.e; its pressure(P), volume(V), temperature(T), internal energy(U) and the number of moles(n). These parameters are called thermodynamic variables.

Extensive and Intensive properties/variables -

- Intensive properties do not depend on the amount of matter that is present. These are bulk properties. Examples of intensive properties are - Density, Temperature ets.
- Extensive properties are those properties which depend on the amount of matter that is present. Examples of extensive properties are - Volume, Weight etc.

Equation of state - The relation between the thermodynamic variables (P, V, T) of the system is called equation of state.

For n moles of an ideal gas, equation of state is PV = nRT

For n moles of a real gas equation of state is $(P + \frac{an^2}{V^2})(V - nb) = nRT$

Thermodynamic process : The process of change of state of a system involves change of thermodynamic variables such as pressure P, volume V and temperature T of the system. The process is known as thermodynamic process.

Some important processes are -

(i) Isothermal process (ii) Adiabatic process (iii) Isobaric process (iv) Isochoric process

(v) Cyclic and non-cyclic process (vi) Reversible and irreversible process

Later, we will study all these process one by one in detail.

State and path function -

State or Point function does not depend on the path followed by the thermodynamic process but it depends on the final and initial position of the process.

Path function depends on the path followed by a thermodynamic process and not on the initial and final states of the system. Example of point function is Internal energy and example of path function is Heat and work.

3.Thermodynamic Equilibrium

Thermodynamic equilibrium : When all the thermodynamic variables attain a steady value i.e. they do not change with respect to time, the system is said to be in the state of thermodynamic equilibrium. For the system to attain thermodynamic equilibrium, the following equilibrium must be attained -

(i) Mechanical equilibrium : There is no unbalanced force between the system and its surroundings. There is no pressure gradient.

(ii) **Thermal equilibrium :** There is a uniform temperature in all parts of the system and is same as that of surrounding. There is no temperature gradient.

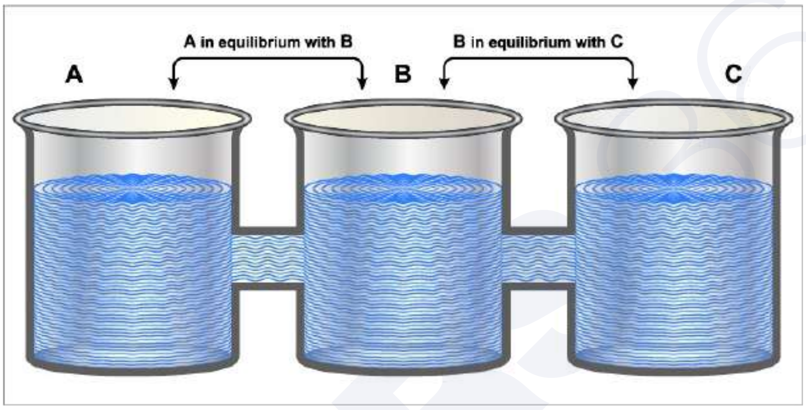
(iii) **Chemical equilibrium :** There is a uniform chemical composition throughout the system and the surrounding. There is no concentration gradient.

Quasi-static process - A **quasi-static process** is a thermodynamic process which happens slowly enough for the system such that each state will remain in internal equilibrium.

Example of quasi-static compression - when the volume of a system changes at enough slow rate to allow the pressure to remain constant throughout the system

Zeroth Law of Thermodynamics.

If systems A and B are each in thermal equilibrium and B and C are in thermal equilibrium with each other, then A and C are in thermal equilibrium with each other.



Zeroth law leads to the **concept of temperature**. All bodies in thermal equilibrium must have a common property. This common property is called temperature.

4.Heat, Internal Energy And Work - Thermodynamics

Quantities Involved in the First Law of Thermodynamics -

(a) **Heat (Q)** : It is the energy that is transferred between a system and its environment because of the temperature gradient.

(b) **Work (W)** : Work can be defined as the energy that is transferred from one body to the other owing to a force that acts between them.

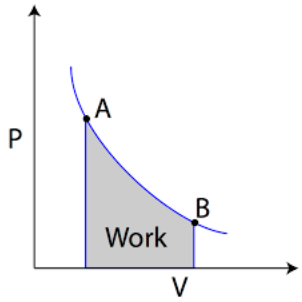
(C) **Internal energy (U)** : Internal energy of a system is the energy possessed by the system due to molecular motion and molecular configuration.

Types of internal energy -

- Due to molecular motion internal energy is kinetic internal energy (UK).
- Due to molecular configuration, it is called internal potential energy (UP).

Important points :

1. Heat and work are path-dependent quantities and Internal energy is point function.
2. $\Delta W = P\Delta V = P(V_f - V_i)$
   $\Delta W =$ positive if $V_f > V_i$ i.e. system expands against some external force.
   $\Delta W =$ negative if $V_f < V_i$ i.e. system contracts because of some external force exerted by the surrounding.
3. The area of P-v diagram on volume axis give the work done in a reversible process. Also for quasistatic process work is given by



$$W = \int_{V_1}^{V_2} P.dV$$

4. And for a cyclic process the clockwise area will show positive work and the anticlockwise area will show negative work done.
5. The internal energy of an ideal gas is totally kinetic and is given by

$$U = \frac{3}{2}.n.R.T$$

So, the Internal energy of an ideal gas is the function of temperature only.

6. For heat transfer -

$\Delta Q = mL$ (for change of state)

$\Delta Q = ms\Delta T$ (for change in temperature)

or,

$\Delta Q = nc\Delta T$

Where, c = molar specific heat capacity

Sign of dQ(Heat)

$dQ > 0$ if heat is given to the system

$dQ < 0$ if heat is extracted from the system

5.First Law Of Thermodynamics

First law of thermodynamics -

According to it heat given to a system (Q) is equal to the sum of increase in its internal energy (U) and the work done (W) by the system against the surroundings.

$$\Delta Q = \Delta U + \Delta W$$

or For cyclic process

$$\sum \Delta Q = \sum \Delta W$$

Drawback of First law of thermodynamics -

- First law of thermodynamics does not tell us reason about the direction of heat transfer.

Important points -

1. Q and W are the path functions but U is the point function.
2. First law of thermodynamics introduces the concept of internal energy.

6.Isobaric Process

Isobaric Process- A Thermodynamic process in which pressure remains constant is known as the isobaric process.

In this process, V and T change keeping P constant.I.e Charle's law is obeyed in this process

Key points in the Isobaric Process-

- Its Equation of state is given as $\frac{V}{T} = constant$

  I.e $\frac{V_1}{T_1} = \frac{V_2}{T_2} = \text{ constant}$

- P-V Indicator diagram for an isobaric process

  Its PV graph has slope=0 (i.e $\frac{dP}{dV} = 0$)

- Specific heat of gas during the isobaric process is given by

  $$C_P = \left(\frac{f}{2} + 1\right) R$$

- The bulk modulus of elasticity during the isobaric process is given by

  $$K = \frac{\Delta P}{-\Delta V/V} = 0$$

- Work done in the isobaric process-

  $$\Delta W = \int_{V_i}^{V_f} P dV = P \int_{V_i}^{V_f} dV = P\left[V_f - V_i\right]$$

  Or we can write $\Delta W = P\left(V_f - V_i\right) = nR\left[T_f - T_i\right] = nR\Delta T$

- Internal energy in an isobaric process

  $$\Delta U = nC_V\Delta T = n\frac{R}{(\gamma - 1)}\Delta T$$

- Heat in an isobaric process

  From FLTD $\Delta Q = \Delta U + \Delta W$

  $$\Delta Q = n\frac{R}{(\gamma - 1)}\Delta T + nR\Delta T = nR\Delta T\left[\frac{1}{\gamma - 1} + 1\right]$$

  So $\Rightarrow \Delta Q = nR\Delta T\frac{\gamma}{\gamma - 1} = n\left(\frac{\gamma}{\gamma - 1}\right)R\Delta T = nC_p\Delta T$

  So $\Delta Q = nC_p\Delta T$

- Examples of the isobaric process-

  1. Conversion of water into vapour phase (boiling process)

  From the first law of thermodynamics

  $$\Delta Q = \Delta U + \Delta W = \Delta U_K + \Delta U_P + \Delta W$$

  since $\Delta U_K = 0$ [as there is no change in temperature] and using, $\Delta Q = mL$

  $$\Delta Q = \Delta U_P + P\left[V_f - V_i\right]$$
  $$\Delta U_P = \Delta Q - P\left[V_f - V_i\right]$$
  $$\Delta U_P = mL - P\left[V_f - V_i\right]$$

Here, i- initial state and f-final state

  2. Conversion of ice into water

  From FLOT $\Delta Q = \Delta U + \Delta W$ and using, $\Delta Q = mL$

  we get $mL = \Delta U_P + \Delta U_K + \Delta W$

  $$mL = \Delta U_P + \Delta U_K + P\left(V_f - V_i\right)$$

  since $\Delta U_K = 0$ [as there is no change in temperature]

  and $\Delta W = 0$ [As $V_f - V_i$ is negligible, I.e, when ice convert into water then change in volume, is negligible]

Hence $\Delta U_P = mL$

7. Isochoric Process

Isochoric Process- A Thermodynamic process in which volume remains constant is known as the Isochoric Process.

In this process P and T changes keeping P constant. So Gay-Lussac's law is obeyed in this process.

Key points in the Isochoric Process

- Its Equation of state is given as $\frac{P}{T} = constant$

  or $\frac{P_1}{T_1} = \frac{P_2}{T_2} = \text{ constant}$

- P-V Indicator diagram for an isobaric process

Its PV graph has slope= infinity (i.e $\frac{dP}{dV} = \infty$)

- Specific heat of gas during the Isochoric process is given by

  $$C_V = \frac{f}{2}R$$

- The bulk modulus of elasticity during the Isochoric process is given by

  $$K = \frac{\Delta P}{-\Delta V/V} = \frac{\Delta P}{0} = \infty$$

- Work done in the Isochoric process-

  $$\Delta W = P\Delta V$$
  $$and\ as\ \Delta V = 0$$
  $$So\ \Delta W = 0$$

- Internal energy in the Isochoric process

  $$\Delta U = nC_V\Delta T = n\frac{R}{(\gamma - 1)}\Delta T$$

- Heat in the Isochoric process

  From FLTD $\Delta Q = \Delta U + \Delta W$

  But $\Delta W = 0$

  So $\Delta Q = \Delta U = nC_V\Delta T = n\frac{R}{(\gamma - 1)}\Delta T = \frac{P_fV_f - P_iV_i}{\gamma - 1}$

- Examples of Isochoric process-

1. Heating of water in a pressure cooker (Valve closed)

8.Isothermal Process

Isothermal process - When a thermodynamic system undergoes a thermodynamic process in such a way that its temperature remains constant, then that process is called an Isothermal process.

So, T = constant and $\Delta T = 0$.

Trick to recognize isothermal process -

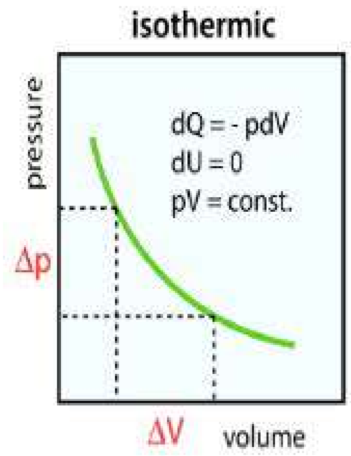
- The walls of the container must be perfectly conducting (no resistance) which allows the exchange of heat between the gas and surroundings.
- The process of compression or expansion should be infinitely slow so that the process gets proper time for the exchange of heat.

Equation of isothermal process -

As we know that the equation of state is given by - $PV = nRT$

If T = constant and for a particular amount of gas 'n' is also constant.

So we can write that - $P.V = Constant$



Points in graph of isothermal process -

i) Curves obtained on P-V graph are called isotherms and the graphs are hyperbolic in nature.

ii) Slope of isothermal curve :

By differentiating PV = C

$$PdV + VdP = 0 \Rightarrow PdV = -VdP \Rightarrow \frac{dP}{dV} = -\frac{P}{V}$$

$$tan\theta = \frac{dP}{dV} = \frac{-P}{V}$$

iii) The area between the isothermal curve and volume axis represents the work done in the isothermal process.

The formula of Work done in the isothermal process -

$$W = nRT \log_e\left(\frac{V_f}{V_i}\right) = 2.303nRT \log_{10}\left(\frac{V_f}{V_i}\right)$$
$$W = nRT \log_e\left(\frac{P_i}{P_f}\right) = 2.303nRT \log_{10}\left(\frac{P_i}{P_f}\right)$$

9.Adiabatic Process

Adiabatic process -

When a thermodynamic system undergoes a process, such that there is no exchange of heat takes place between the system and surroundings, this process is known as **adiabatic process.**

In this process P, V and T changes but $\Delta Q = 0$.

From first law of thermodynamics -

$$\Delta Q = \Delta U + \Delta W$$

Now in adiabatic -

$$0 = \Delta U + \Delta W$$

So, $\Delta U = -\Delta W$ **for adiabatic process**

Now, let us take two cases, first is for expansion in which the work done is positive and second one is compression in which the work done is negative -

If $\Delta W =$ positive then ΔU become negative so temperature decreases ie., adiabatic expansion produce cooling.
If $\Delta W =$ negative then ΔU become positive so temperature increases ie., adiabatic compression produce heating.

Equations of Adiabatic process -

1. $PV^\gamma =$ constant; where $\gamma = \frac{C_P}{C_V}$ - - - - Relating Pressure and volume

2. $TV^{\gamma-1} =$ constant $\Rightarrow T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$ or $T \propto V^{1-\gamma}$ - - - - -Relating Temperature and volume

3. $\frac{T^{\gamma}}{P^{\gamma-1}} =$ const. $\Rightarrow T_1^{\gamma} P_1^{1-\gamma} = T_2^{\gamma} P_2^{1-\gamma}$ or $T \propto P^{\frac{\gamma-1}{\gamma}}$ or $P \propto T^{\frac{\gamma}{\gamma-1}}$

For the slope of adiabatic curve on PV curve, we have to differentiate the adiabatic relation -

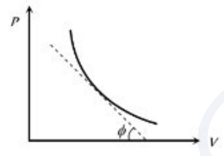
As, $PV^{\gamma} =$ constant

So,

$$dPV^{\gamma} + P\gamma V^{\gamma-1} dV = 0$$

$$\frac{dP}{dV} = -\gamma \frac{PV^{\gamma-1}}{V^{\gamma}} = -\gamma \left(\frac{P}{V}\right)$$

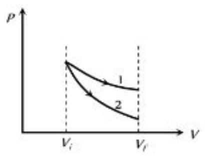
So we can say that in the given graph, the slope = $\tan(180^o - \phi) = -\gamma \left(\frac{P}{V}\right)$



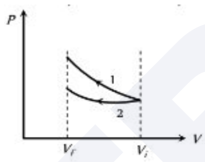
Also, we have studied that the slope of the isothermal curve on PV diagram is = $\frac{-P}{V}$

So, we can say that the - $(\text{Slope})_{adiabatic} = \gamma \times (\text{ Slope })_{isothermal}$, or $\frac{(\text{ Slope })_{adiabatic}}{(\text{ Slope })_{isothermal}} > 1$

With the help of graph we can see that the adiabatic curve is more steeper than the isothermal curve-



or,



Specific heat in the adiabatic process - Specific heat of gas during adiabatic change is zero. Mathematically -

$$C = \frac{Q}{m\Delta T} = \frac{0}{m\Delta T} = 0 \quad [\text{As } Q = 0]$$

Note- Even though heat is not supplied or taken out during the process but still, the temperature change is taking place. So we can say that Specific heat for an adiabatic process is zero.

Work done in the adiabatic process -

$$W = \int_{V_i}^{V_f} PdV = \int_{V_i}^{V_f} \frac{K}{V^{\gamma}} dV \Rightarrow W = \frac{[P_i V_i - P_f V_f]}{(\gamma - 1)} = \frac{\mu R(T_i - T_f)}{(\gamma - 1)}$$

So, if γ is increasing then the work done will be decreasing. As we know that -

$$\because \gamma_{mono} > \gamma_{diatomic} > \gamma_{triatomic} \Rightarrow W_{mono} < W_{diatomic} < W_{triatomic}$$

10.Polytropic Process

A process $PV^N = C$ is called polytropic process. So, any process in this world related to thermodynamics can be explained by polytropic process.

For example - 1. If N = 1 , then the process become isothermal.

2. If N=0, then the process become isobaric.

3. If N = γ, then the process become adiabatic

Work done by polytropic process -

$$W_{1-2} = \int PdV$$

For a polytropic process,

$$PV^N = P_1 V_1^N = P_2 V_2^N = C$$

$$P = \frac{C}{V^N}$$

Subsiting in Equation , we get,

$$\int PdV = \int \frac{CdV}{V^N} = C\int V^{-N}dv$$

$$= \left[V^{1-N}\right]_1^2 = \left(V_2^{1-N} - V_1^{1-N}\right)$$

$$W_{1-2} = \frac{P_2V_2 - P_1V_1}{1-N} \text{ or } \frac{P_1V_1 - P_2V_2}{N-1} ......(1)$$

$$P_1V_1 = nRT_1$$

$$P_2V_2 = nRT_2$$

So, equation (1) can be written as -

$$W_{1-2} = \frac{nR\left(T_2 - T_1\right)}{1-N}$$

$$\text{And for one mole, } W_{1-2} = \frac{R\left(T_2 - T_1\right)}{1-N}$$

Specific heat for polytropic process -

We can write equation of heat as - $Q = C\Delta T$

Here C = Molar specific heat -

From the first law of thermodynamics

$$Q = \Delta U + W$$

$$\text{or } C\Delta T = C_v\Delta T - \frac{R\Delta T}{(N-1)}$$

$$\therefore \quad C = C_v - \frac{R}{(N-1)} = \frac{R}{(\gamma-1)} - \frac{R}{(N-1)}$$

11. Cyclic And Non Cyclic Process

Cyclic Process - A cyclic process consists of a series of changes that return the system back to its initial state.

Non-cyclic Process - In the non-cyclic process, the series of changes involved do not return the system back to its initial state.

Now, as we know internal energy is the point function. So when the process returns to its initial point after completing the process then the final and initial internal energy will be the same. So, the change in internal energy is zero.

$$\text{In case of cyclic process as } U_{final} = U_{initial} \Rightarrow \Delta U = U_{final} - U_{initial} = 0$$

i.e., change in internal energy for cyclic process is zero

$$\text{So, we can say that } \Delta U \propto \Delta T \Rightarrow \Delta T = 0$$

By applying the first law of thermodynamics for cyclic process -

$$\Delta Q = \Delta U + \Delta W \Rightarrow \Delta Q = \Delta W \quad (As\ \Delta U = 0)$$

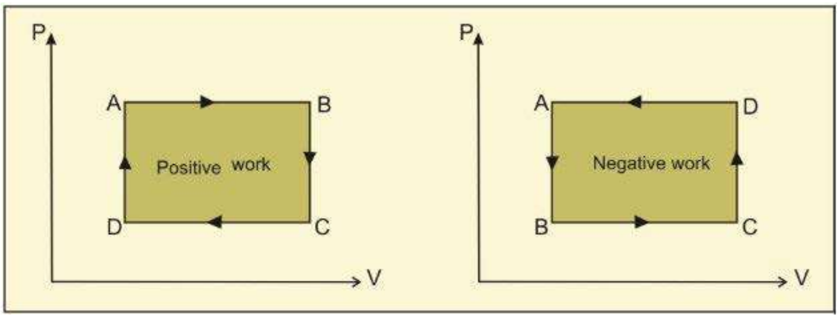
So, we can say that the heat given is equal to the work obtained in the cyclic process.

For the cyclic process, the initial point and final point is the same. So, the P-V graph is a closed curve and the area enclosed by the closed path gives the work done.

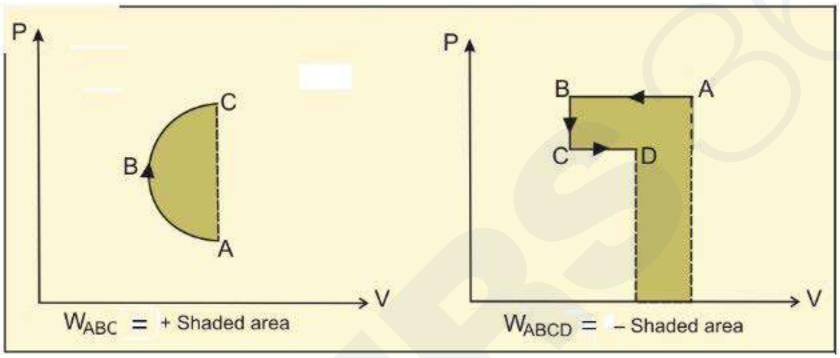
But, here is one assumption for the calculation of work done -

If the cycle is clockwise work done is positive and if the cycle is anticlockwise work done is negative.

From the given graph and its direction, you can see that the first graph is having positive work and the second graph had done negative work



Now, for the Non - cyclic process - The work done is equal to the area covered between the curve and the volume axis on the P-V diagram. It does not depend on the points or state but it depends on the process of the path. We can see this in the given graph.



12.Reversible And Irreversible Process

- **Reversible Process-**

A reversible process is one that can be reversed in such a way that all changes occurring in the direct process are exactly repeated in the opposite order and inverse sense.

And in the Reversible Process, no change is left in any of the bodies taking part in the process, or in the surroundings.

Or " A process is **reversible** only if it is quasi-static and there is no dissipative effect."

Condition of a reversible process

1) The complete absence of dissipative force.

2) The process should be infinitely slowing.

3) The temperature of the system must not differ appreciably from the surrounding.

Examples of the reversible process -

A reversible process is only an ideal concept. In the actual process, there is always a loss of heat due to friction, conduction, radiation, etc. I.e No process is reversible in true sense.

Some examples of reversible process are:

1. All isothermal and adiabatic changes are reversible if they are performed very slowly.
2. Very slow evaporation or condensation.
3. An extremely slow extension or contraction of spring without setting up oscillations.

- **Irreversible process-**

Any process which is not reversible exactly is an irreversible process. All-natural processes such as conduction, radiation, radioactive decay, etc. are irreversible.

Some examples of irreversible processes are :

1. Sudden expansion or contraction
2. Rapid evaporation or condensation
3. The sudden and fast stretching of a spring

13.Heat Engine

A **heat engine** is a device that converts heat into work continuously through a cyclic process.

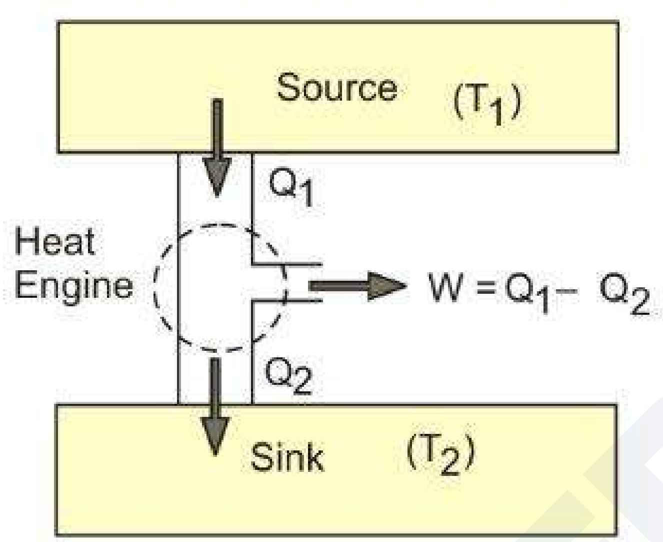
The essential parts of a heat engine are

1. **Source:** It is a reservoir of heat at **high temperatures** and infinite thermal capacity. Any amount of heat can be extracted from it.

2. **Working substance:** Steam, petrol, etc.

3. **Sink:** It is a reservoir of heat at **low temperatures** and infinite thermal capacity. Any amount of heat can be given to the sink.

- **Working of heat engine**



As shown in the above figure, The working substance absorbs heat Q_1 from the source, does an amount of work W returns the remaining amount of heat (i.e Q_2) to the sink and comes back to its original state and there occurs no change in its internal energy.

To obtain work continuously, the same cycle is repeated over and over again.

- **The efficiency of the heat engine** (η)**-** It is defined as the ratio of useful work obtained from the engine to the heat supplied to it. The performance of the heat engine is expressed by means of "efficiency".

$$I.e \quad \eta = \frac{\text{Work done}}{\text{Heat input}} = \frac{W}{Q_1}$$

For a cyclic process $\Delta U = 0$

so From the first law of thermodynamics, $\therefore \Delta Q = \Delta W \quad \text{so} \quad W = Q_1 - Q_2$

$$\eta = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$$

Practically efficiency of an engine is always less than 1.

14. Second Law Of Thermodynamics

- **Clausius's statement-**It is impossible for a self-acting machine to transfer heat from a colder body to a hotter one

  without the aid of an external agency.

- **Kelvin's statement-**It is impossible for a body or system to perform continuous work by cooling it to a a temperature lower than the temperature of the coldest one of its surroundings.
- **Kelvin-Planck's statement-**It is impossible to design an engine that extracts heat and fully utilizes it into work without producing any other effect.

These above statements are completely equivalent to the **Second Law of Thermodynamics.**

This explains that the efficiency of an engine is always less than unity because heat cannot be fully converted into work.

It also explains that heat cannot flow from a body at a low temperature to one at a higher temperature unless work is done by an external agency.

15.Entropy

Entropy- Entropy is a measure of the disorder of the molecular motion of a system. I.e Greater is the disorder, greater is the entropy.

The change in entropy is given as

$$dS = \frac{\text{Heat absorbed by system}}{\text{Absolute temperature}} \text{ or } dS = \frac{dQ}{T}$$

The relation $dS = \frac{dQ}{T}$ is called the mathematical form of the **Second Law of Thermodynamics.**

1. **Entropy for solid and liquid-**

i. When heat is given to a substance to change its state at a constant temperature.

Then change in entropy is given as

$$dS = \frac{dQ}{T} = \pm \frac{mL}{T}$$

where positive sign refers to heat absorption and negative sign to heat evolution.

And $L = Latent\ Heat$ and T is in kelvin.

ii. When heat is given to a substance to raises its temperature from $T_1\ to\ T_2$

Then change in entropy is given as

$$dS = \int \frac{dQ}{T} = \int_{T_1}^{T_2} mc\frac{dT}{T} = mc\log_e\left(\frac{T_2}{T_1}\right) = 2.303\ *\text{mc}\log_{10}\left(\frac{T_2}{T_1}\right)$$

where c = specific heat capacity

2. **Entropy for an ideal gas -**

For n mole of an ideal gas, the equation is given as PV = nRT

I.Entropy change for ideal gas in terms of T & V

From the first law of thermodynamics, we know that $dQ = dW + dU$

and $\Delta S = \int \frac{dQ}{T} = \int \frac{nC_V dT + PdV}{T}$

using PV = nRT

$$\Delta S = \int \frac{nC_V dT + \frac{nRT}{V}dV}{T} = nC_V \int_{T_1}^{T_2} \frac{dT}{T} + nR\int_{V_1}^{V_2} \frac{dV}{V}$$

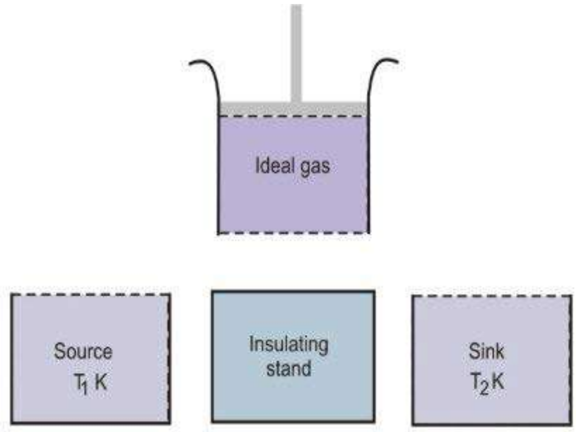
$$\Delta S = n\,C_V \ln\left(\frac{T_2}{T_1}\right) + nR\ln\left(\frac{V_2}{V_1}\right)$$

II.Entropy change for an ideal gas in terms of T & P

$$\Delta S = n\,C_P \ln\left(\frac{T_2}{T_1}\right) - nR\ln\left(\frac{P_2}{P_1}\right)$$

III.Entropy change for an ideal gas in terms of P & V

$$\Delta S = nC_V \ln\left(\frac{P_2}{P_1}\right) + n\,C_P\ln\left(\frac{V_2}{V_1}\right)$$



1. **A cylinder** with perfectly non-conducting walls and a perfectly conducting base containing an ideal gas as working substance and fitted with a non-conducting frictionless piston.

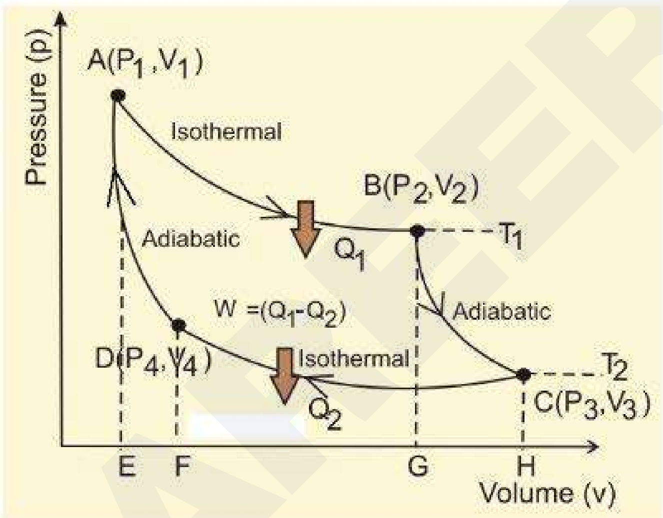
2. A source of infinite thermal capacity maintained at a constant higher temperature T_1

3. A sink of infinite thermal capacity maintained at a constant lower temperature T_2

4. A perfectly non-conducting stand for the cylinder.

- **Carnot cycle-**

  As shown in the below figure, It consists of the following 4 processes.



1. Isothermal expansion (curve AB)

The cylinder containing ideal gas as working substance allowed to expand slowly at this constant temperature T_1

So Work done = Heat absorbed by the system

$$W_1 = Q_1 = \int_{V_1}^{V_2} PdV = RT_1 \log_e \left(\frac{V_2}{V_1}\right) = \text{ Area of ABGE}$$

2. Adiabatic expansion (curve BC)

The cylinder is then placed on the non-conducting stand and the gas is allowed to expand adiabatically till the temperature falls from T_1 to T_2

$$W_2 = \int_{V_2}^{V_3} PdV = \frac{R}{(\gamma - 1)} [T_1 - T_2] = \text{ Area of } BCHG$$

3. Isothermal compression (curve CD)

The cylinder is placed on the sink and the gas is compressed at constant temperature T_2.
Work done = Heat released by the system

$$W_3 = Q_2 = -\int_{V_3}^{V_4} PdV = -RT_2 \log_e \frac{V_4}{V_3} = RT_2 \log_e \frac{V_3}{V_4} = \text{ Area of } CDFH$$

4. Adiabatic compression (curve DA)

Finally, the cylinder is again placed on a non-conducting stand and the compression is continued so that gas returns to its initial stage.

$$W_4 = -\int_{V_4}^{V_1} PdV = -\frac{R}{\gamma-1}(T_2 - T_1) = \frac{R}{\gamma-1}(T_1 - T_2) = \text{ Area of } ADFE$$

- **The efficiency of the Carnot cycle (η)**

The efficiency of the engine is defined as the ratio of work done to the heat supplied.

$$\eta = \frac{\text{work done}}{\text{Heat input}} = \frac{W}{Q_1}$$

Net work done during the complete cycle

$$W = W_1 + W_2 + (-W_3) + (-W_4)$$
$$As\ W_2 = W_4$$
$$\Rightarrow W = W_1 - W_3 = \text{ Area of } ABCD$$

$$\eta = \frac{W}{Q_1} = \frac{W_1 - W_3}{W_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{W_3}{W_1} = 1 - \frac{Q_2}{Q_1}$$

Putting the values we get

$$\eta = 1 - \frac{RT_2 \log_e (V_3/V_4)}{RT_1 \log_e (V_2/V_1)} \quad \text{..... (1)}$$

Since points, B and C lie on the same adiabatic curve

$$\therefore T_1 V_2^{\gamma-1} = T_2 V_3^{\gamma-1} \text{ or } \frac{T_1}{T_2} = \left(\frac{V_3}{V_2}\right)^{\gamma-1} \quad \text{......(2)}$$

Also, point D and A lie on the same adiabatic curve

$$\therefore T_1 V_1^{\gamma-1} = T_2 V_4^{\gamma-1} \text{ or } \frac{T_1}{T_2} = \left(\frac{V_4}{V_1}\right)^{\gamma-1} \quad \text{....(3)}$$

From the equation (2) and (3) we get

$$\frac{V_3}{V_2} = \frac{V_4}{V_1} \text{ or } \frac{V_3}{V_4} = \frac{V_2}{V_1} \Rightarrow \log_e\left(\frac{V_3}{V_4}\right) = \log_e\left(\frac{V_2}{V_1}\right) \quad \text{.....(4)}$$

Put equation (4) in equation (1) we get

The efficiency of the Carnot engine as

$$\eta = 1 - \frac{T_2}{T_1}$$

So $\eta = \frac{W}{Q_1} = 1 - \frac{T_2}{T_1}$

where

$T_1 =$ Source temperature , $T_2 =$ Sink Temperature and $(T_1 > T_2)$

and $T_1\ and\ T_2$ are in kelvin

From the above formula, we can conclude that

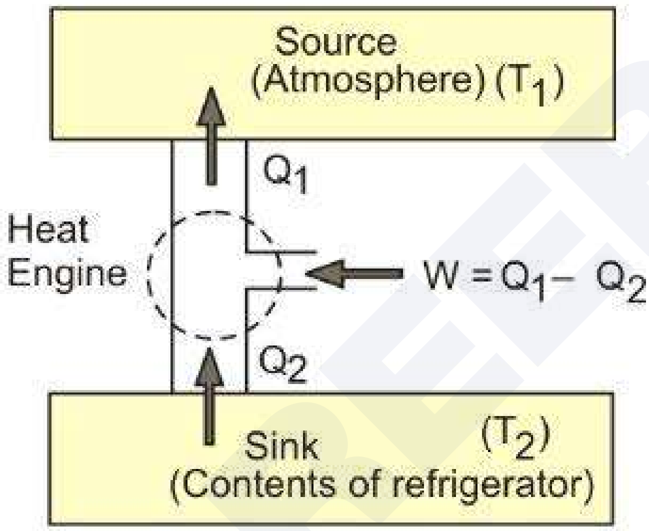
1. The efficiency of a heat engine depends only on temperatures of source and sink and is independent of all other factors.
2. As a Carnot engine is the ideal engine, So no heat engine can be more efficient than Carnot engine.
3. All reversible heat engines working between same temperatures are equally efficient
4. Since $T_1 > T_2$ So the efficiency of a Carnot engine is always lesser than unity.

17. Refrigerator Or Heat Pump

A **refrigerator or heat pump** is basically a heat engine run in the reverse direction.

It consists of three parts
1. **Source**: At higher temperature T_1
2. **Working substance**: It is called refrigerant. I.e liquid ammonia and freon works as a working substance.
3. **Sink**: At lower temperature T_2.

- **Working of refrigerator**



As shown in the above figure, The working substance takes heat Q_2 from a sink (contents of refrigerator) at lower temperature T_2, has a net amount of work done W on it by an external agent (usually compressor of refrigerator) and gives out a larger amount of heat Q_1 to a hot body at temperature T_1 (usually atmosphere).

- **Use of refrigerator**-

The cold body is cooled more and more with the help of a refrigerator. Because the refrigerator transfers heat from a cold to a hot body at the expense of mechanical energy supplied to it by an external agent.

- **Coefficient of performance** (β)**-**

The coefficient of performance is defined as the ratio of the heat extracted from the cold body to the work needed to transfer it to the hot body.

$$\beta=\frac{\text{Heat extracted}}{\text{work done}}=\frac{Q_2}{W}=\frac{Q_2}{Q_1-Q_2}$$

A perfect refrigerator is one which transfers heat from cold to a hot body without doing work.

i.e. $W=0$ so that $Q_1=Q_2$ and hence $\beta=\infty$

Oscillations

Important Formulae

1.Periodic And Oscillatory Motions

Periodic motion:- A motion, which repeats itself over and over again after a regular interval of time is called a periodic motion.

- The fixed interval of time after which the motion is repeated is called time period of the motion.
- If a particle moves along x -axis, its position depends upon time t. We express this fact mathematically by writing
  $x=f(t)$ or $x(t)$
  There are certain motions that are repeated at equal intervals of time. By this we mean that particle is found at the same position moving in the same direction with the same velocity and acceleration, after each period of time. Let T be the interval of time in which motion is repeated. Then
  $x(t)=x(t+T)$
  where T is the minimum change in time. And the function that repeats itself is known as a periodic function.
- Examples :
  1. Revolution of earth around the sun (period one year)
  2. Rotation of earth about its polar axis (period one day)
  3. Motion of hour's hand of a clock (period 12-hour)

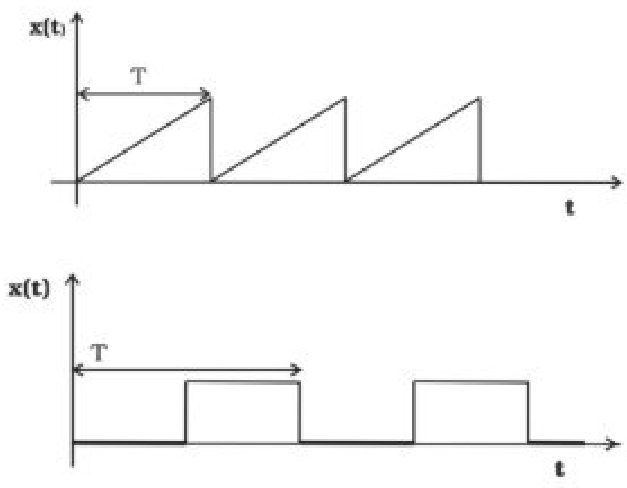
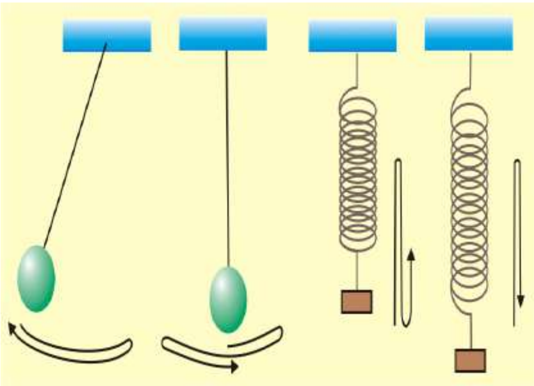
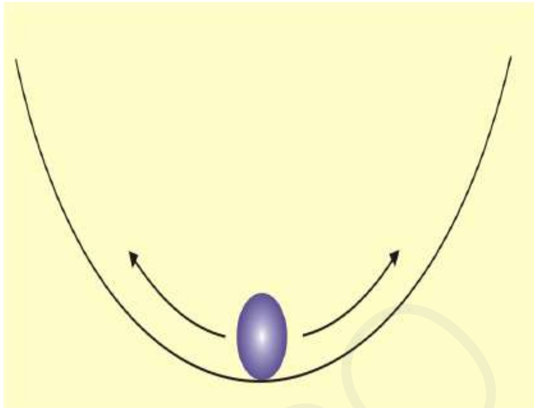
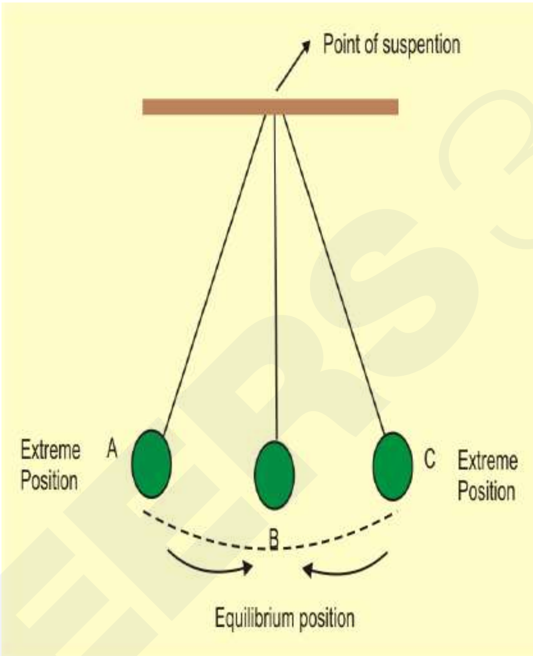


Fig:- Examples of Periodic motion

Oscillatory Motion:- Oscillatory motion is that motion in which a body moves to and fro or back and forth repeatedly about a fixed point in a definite interva of time.





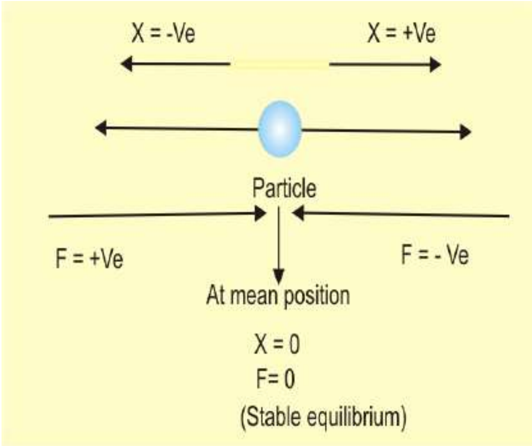


- Every oscillatory motion is periodic if energy is not lost anywhere, but every periodic motion need not be oscillatory. Circular motio is a periodic motion, but it is not oscillatory.
- **General equation of Oscillatory motion:-**

  When a body is given small displacement from the equilibrium position, a force starts acting towards the equilibrium position (or mean position) which tries to bring the body back to it's mean position. And that force is given by:-

  $F=-kx^n$, where x is measured from the mean position and n=1,3,5,7,9 etc

1. When x=positive, F=negative
2. When x=negative, F=positive
3. When x=0, F=0,i.e., at mean position

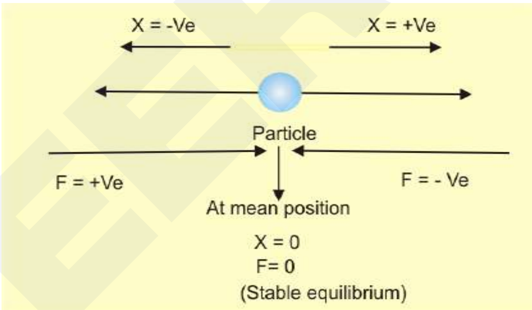


2. Simple Harmonic Motion (S.H.M.) And Its Equation

- Periodic motion is also called harmonic motion.
- Simple harmonic motion is the simplest form of oscillatory motion in which the particle oscillates on a straight line and the restoring force is always directed towards the mean position and it's magnitude at any instant is directly proportional to the displacement of the particle from the mean position a that instant i.e. Restoring force α Displacement of the particle from mean position.

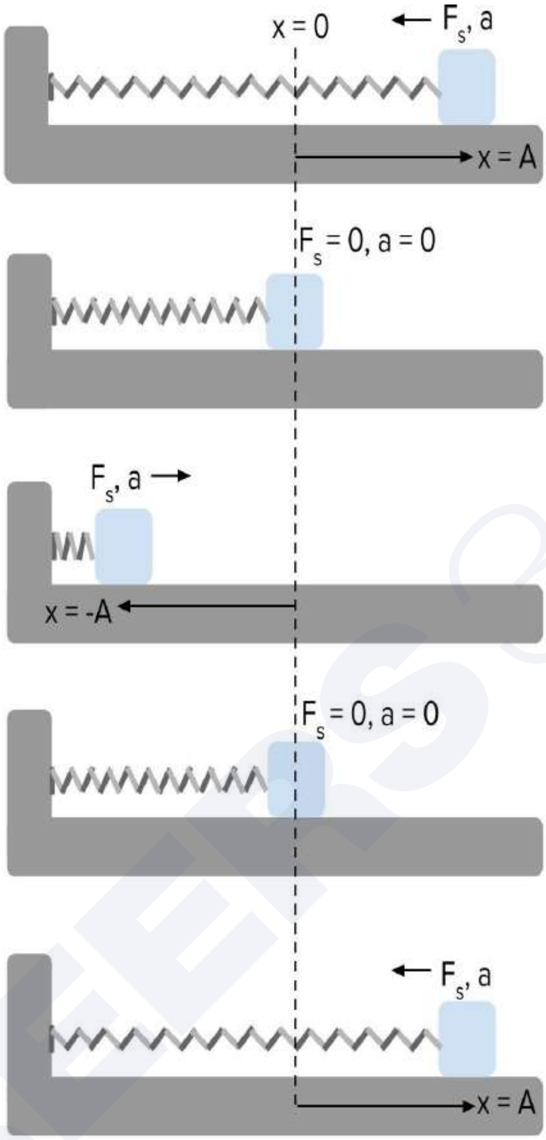
  $F=-kx$, where x is measured from mean position

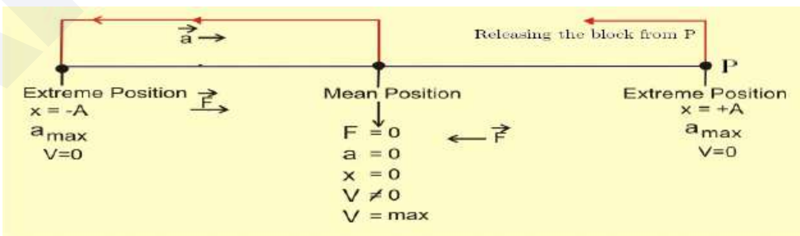
- All oscillations are not simple harmonic motions but all simple harmonic motions are oscillatory motions.



- Let's understand SHM with the help of the spring block system:

  Suppose we stretch the spring to the extreme position and then release it from there.





Here we can see that acceleration is always directed towards the mean position.

And $F = -kx$

$$Also,\ a = \frac{F}{m} \Rightarrow a = -\frac{k}{m}x \Rightarrow a = -\omega^2 x;\ where\ \omega^2\ is\ a\ positive\ constant\ and$$

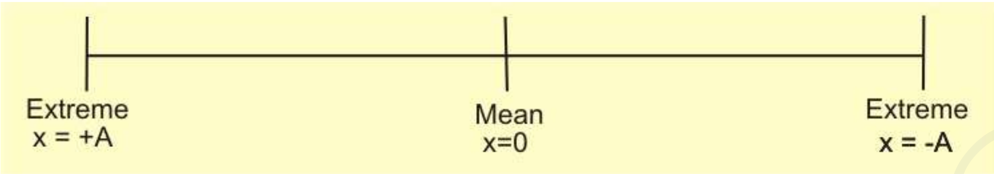
$$\omega = \sqrt{\frac{k}{m}} \Rightarrow k = m\omega^2,$$

where k is force or spring constant.

- v=0 at extreme position

- v=max at mean position
- a=0 at mean position
- $a = max\ at\ extreme\ position, i.e., at\ x = \pm A,\ \ a = \pm\omega^2 A$
- $Magnitude\ of\ maximum\ acceleration, |a_{max}| = \omega^2 A$

Equations of motions of SHM-



$As\ we\ know, a = -\omega^2 x$

$\Rightarrow \frac{dv}{dt} = -\omega^2 x \Rightarrow v\frac{dv}{dx} = -\omega^2 x \Rightarrow vdv = -\omega^2 xdx$

Let the particle is released from an extreme position, i.e., at x=+A, v=0 and it becomes v when the displacement becomes x.

On integrating both sides of the above equation, we get:

$\int_0^v vdv = \int_A^x -\omega^2 xdx$

$\Rightarrow \left[\frac{v^2}{2}\right]_0^v = -\omega^2 \left[\frac{x^2}{2}\right]_A^x$

$\Rightarrow v^2 - 0 = -\omega^2(x^2 - A^2)$

$\Rightarrow v^2 = \omega^2(A^2 - x^2)$

$\Rightarrow v = \pm\omega\sqrt{(A^2 - x^2)}$

$At\ x = 0, v_{max} = \pm\omega A$

Note-

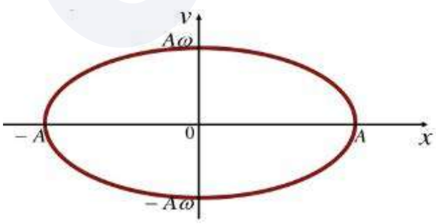
As the relation between velocity (v) and position (x) in SHM is given by

$v = \pm\omega\sqrt{(A^2 - x^2)}$

This can be rearranged as

$$\begin{aligned} & v^2 = \omega^2(A^2 - x^2) \\ \Rightarrow\ & v^2 = \omega^2 A^2 - \omega^2 x^2 \\ \Rightarrow\ & v^2 + \omega^2 x^2 = \omega^2 A^2 \\ \Rightarrow\ & \frac{v^2}{\omega^2 A^2} + \frac{x^2}{A^2} = 1 \end{aligned}$$

And this shows that the velocity-position graph is an ellipse (as shown in the below figure)



where

Major axis=2A

and Minor axis $=2\omega A$

- **General equation of SHM**

1. **For Displacement:-**

$x = ASin(wt+\phi)$; *where* ϕ *is initial phase or epoch and* $(\omega t+\phi)$ *is called as phase.*

Various displacement equations:-

(1) $x = ASin\omega t \Rightarrow$ *when particle starts from mean position towards right.*

(2) $x = -ASin\omega t \Rightarrow$ *when particle starts from mean position towards left.*

(3) $x = ACos\omega t \Rightarrow$ *when particle starts from right extreme position towards left*

(4) $x = -ACos\omega t \Rightarrow$ *when particle starts from left extreme position towards Right.*

2. **For Velocity (v):-**

$$x = ASin(\omega t + \phi)$$
$$\Rightarrow v = \frac{dx}{dt} = A\omega\ Cos(\omega t + \phi) = A\omega\ Sin(\omega t + \phi + \frac{\pi}{2})$$

3. **For Acceleration:-**

$$x = ASin(\omega t + \phi)$$
$$\Rightarrow v = \frac{dx}{dt} = A\omega\ Cos(\omega t + \phi) = A\omega\ Sin(\omega t + \phi + \frac{\pi}{2})$$
$$\Rightarrow a = \frac{dv}{dt} = -A\omega^2\ Sin(\omega t + \phi) = A\omega^2\ Sin(\omega t + \phi + \pi) = -\omega^2 x$$

So here we can see that the phase difference between x and v is $\frac{\pi}{2}$

similarly, the phase difference between v and a is $\frac{\pi}{2}$

similarly, the phase difference between a and x is π

- **Differential equation of SHM:-**

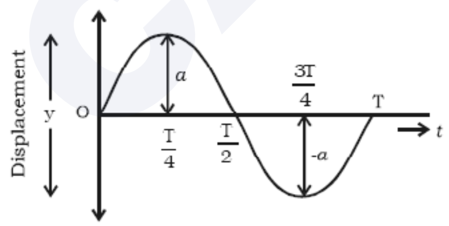
$$\frac{dv}{dt} = -\omega^2 x$$
$$\Rightarrow \frac{d}{dt}\left(\frac{dx}{dt}\right) = -\omega^2 x$$
$$\Rightarrow \frac{d^2x}{dt^2} + \omega^2 x = 0$$

If the motion of any particle satisfies this equation then that particle will do SHM.

- Different graphs in SHM

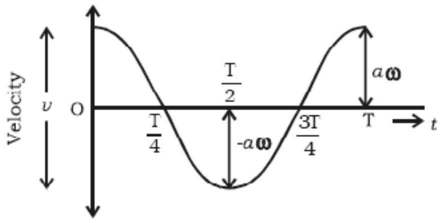
For $x = aSin(\omega t)$

Graph of displacement v/s time is given as



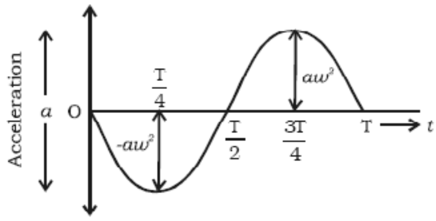
Graph of velocity V/s time

$v = a\omega\ Cos(\omega t)$



Graph of acceleration V/s time

$acceleration = -a\omega^2\ Sin(\omega t)$



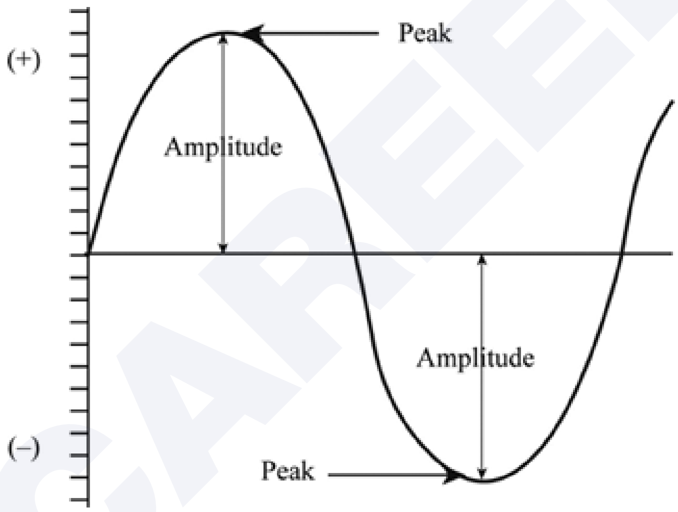
3.Important Terms In Simple Harmonic Motion

1. **Amplitude:-**

   We know that displacement of a particle in SHM is given by:

   $x = ASin(\omega t + \phi)$

   The quantity A is called the amplitude of the motion. It is a positive constant which represents the magnitude of the maximum displacement of the particle from mean position in either direction.



2. **Time period:-**

- In SHM, a particle repeats its motion after a fixed interval of time. And this time interval after which the particle repeats its motion is called time perioc It is denoted by T.
- Time period is also defined as the time taken to complete one oscillation. And after one time period, both displacement and velocity of the particle are repeated.
- We know that:-

$$x = ASin(\omega t + \phi)$$

If a motion is periodic with a period T, then the displacement x (t) must return to its initial value after one period of the motion; that is, x (t) must be equal to x (t + T) for all t and velocity v(t) must also return to its initial value, i.e., v(t) must be equal to v(t+T). So,

$$x(t) = x(t+T)$$

$$\Rightarrow ASin(\omega t + \phi) = ASin[\omega[t+T] + \phi]$$

$$\Rightarrow Sin(\omega t + \phi) = Sin[\omega[t+T] + \phi]$$

And

$$v(t) = v(t+T)$$
$$\Rightarrow A\omega Cos(\omega t+\phi) = A\omega Cos[\omega[t+T]+\phi]$$
$$\Rightarrow Cos(\omega t+\phi) = Cos[\omega[t+T]+\phi]$$

As we know both Sine and Cosine function repeats itself when their argument increases by 2π,i.e.,

$$\omega t+\phi+2\pi = \omega(t+T)+\phi$$
$$\Rightarrow 2\pi = T\omega$$
$$\Rightarrow T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}};$$
$$where\ k = force\ or\ spring\ constant\ and\ m = mass$$

- **Time period can also be written as:-**

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{\frac{Force}{displacement}}} = 2\pi\sqrt{\frac{m\times displacement}{m\times acceleration}}$$
$$\Rightarrow T = 2\pi\sqrt{\frac{displacement}{acceleration}}$$

3. **Frequency:-**

The reciprocal of T gives the number of repetitions that occur per unit time. This quantity is called the frequency of the periodic motion.

- It is denoted by f.

$$f = \frac{1}{T} = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$
$$\Rightarrow \omega = 2\pi f;\ where\ \omega\ is\ angular\ frequency$$

  - The unit of frequency is s^{-1} or Hertz(Hz).

4. **Phase:-**

  - $The\ quantity\ (\omega t+\Delta\phi)\ is\ called\ the\ phase.$
  - It determines the status of the particle in simple harmonic motion.
  - If the phase is zero at a certain instant, then:

$$x = A\ Sin(\omega t+\phi) = 0\ and\ v = A\omega\ Cos(\omega t+\phi) = A\omega$$

which means that the particle is crossing the mean position and is going towards the positive direction.

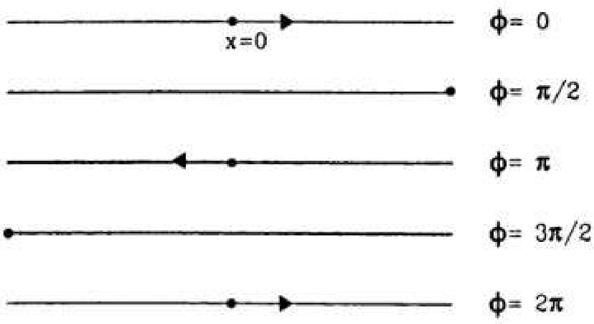


Fig:- Status of the particle at different phases

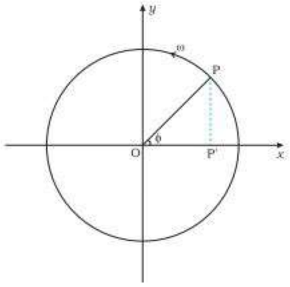
5. **Phase constant:-**

  - The constant ϕ is called the phase constant (or phase angle).
  - The value of ϕ depends on the displacement and velocity of the particle at t = 0 or we can say the phase constant signifies the initial conditions.
  - Any instant can be chosen as t = 0 and hence the phase constant can be chosen arbitrarily.

4. Simple Harmonic Motion And Uniform Circular Motion

Simple harmonic can be represented as a projection of circular motion.

If P moves uniformly on a circle as shown in the below figure, then its projection P′ on a diameter of the circle executes SHM.



As the particle P moves on the circle, The position of P′ on the x-axis is given by

x(t) = A cos (ωt + ϕ)

This is the equation of SHM on the x-axis with amplitude A and angular frequency as ω

Where A is the radius of the circle

and ϕ is the angle that the radius OP makes with the x-axis at t=0

Similarly, The position of P′ on the y-axis is given by

y(t)= A sin (ωt + ϕ)

This also an SHM of the same amplitude as that of the projection on the x-axis, but differing by a phase of π/2.

5. Composition Of Two SHM

If a particle is acted upon by two forces such that each force can produce SHM, then the resultant motion of the particle is a combination of SHM.

Composition of two SHM in the same direction

Let a force F_1 produces an SHM of amplitude A_1 whose equation is given by

$x_1 = A_1 sin\omega t$

Let another force F_2 produce an SHM of amplitude A_2 whose equation is given by

$x_2 = Asin(\omega t + \phi)$

Now if force F_1 and F_2 is acted on the particle in the same direction then the **resultant amplitude of the combination of SHM's is given by**

$A = \sqrt{A_1{}^2 + A_2{}^2 + 2A_1A_2.\cos\phi}$

$A_1 and A_2$ are the amplitude of two SHM's. ϕ is phase difference.

Note: Here the frequency of each SHM's are the same

And the resulting phase is given by

$\phi' = \tan^{-1}\left(\frac{A_2\sin\phi}{A_1 + A_2\cos\phi}\right)$

Composition of SHM in perpendicular direction:

Let a force F_1 on a particle produce an SHM given by

$x = Asin\omega t$

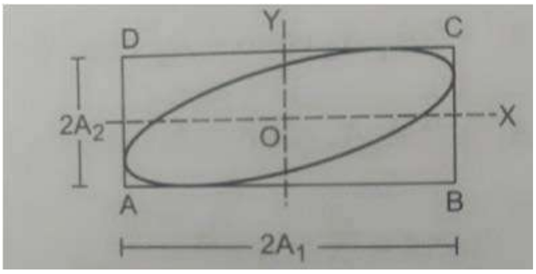
and a force F_2 alone produces an SHM given by

$x = Asin(\omega t + \phi)$

- Both the force F_1 and F_2 acting perpendicular on the particle will produce an SHM whose resultant is given by:

$$\frac{x^2}{A_1{}^2} + \frac{y_2{}^2}{A_2{}^2} - \frac{2xy\cos\phi}{A_1A_2} = \sin^2\phi$$

The above equation is the general equation of an ellipse. That is two forces acting perpendicular on a particle execute SHM along an elliptical path.

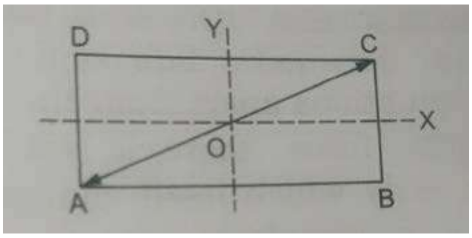


- When $\phi = 0$ resultant equation is given by

$$y = \frac{A_2}{A_1}.x$$

It is a straight line with slope

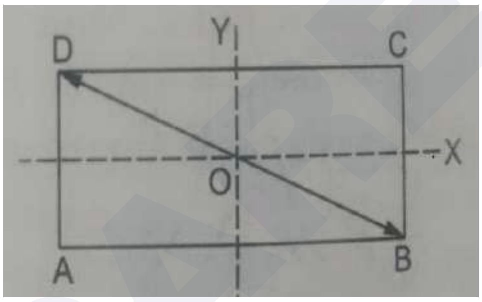
$\frac{A_2}{A_1}$ represented by the below figure



- When $\phi = \pi$ resultant equation

$$y = \frac{-A_2}{A_1}.x$$

which is represented by below straight line with slope $\frac{-A_2}{A_1}$



- When $\phi = \frac{\pi}{2}$ resultant equation

$$\frac{x^2}{{A_1}^2} + \frac{y^2}{{A_2}^2} = 1$$

It represents a normal ellipse

- if $A_1 = A_2 \; and \; \phi = \frac{\pi}{2}$ then it represents a circle

6.Energy In Simple Harmonic Motion

A particle executing S.H.M. possesses two types of energy: Potential energy and Kinetic energy

Potential energy-

- This is an account of the displacement of the particle from its mean position.
- Formula-

As restoring force is given as $F = -kx$

So $U=-\int dw=-\int_0^x F dx=\int_0^x kx dx=\frac{1}{2}kx^2$

using $\omega=\sqrt{\frac{k}{m}}$ or $k=m\omega^2$

we get $U=\frac{1}{2}m\omega^2x^2$

For $x=A\sin(wt)$

$U=\frac{1}{2}m\omega^2A^2\sin^2\omega t$

- Potential energy maximum and equal to total energy at extreme positions

i.e $U_{\max}=\frac{1}{2}kA^2=\frac{1}{2}m\omega^2A^2 \quad \text{when } x=\pm A; \omega t=\pi/2; \quad t=T/4$

- Potential energy is minimum at mean position

i.e $U_{min}=0 \quad \text{when } x=0; \omega t=0; t=0$

- The average value of potential energy with respect to t

$$Average\ of\ U=\frac{\int U\,dt}{\int dt}$$

$$\because U=\frac{1}{2}kx^2$$

So $$U_{avg}=\frac{\int \frac{1}{2}m\omega^2A^2\sin^2\omega t}{\int dt}=\frac{\int \frac{1}{4}m\omega^2A^2(1-\cos 2\omega t)dt}{dt}=\frac{1}{4}m\omega^2A^2$$

Kinetic energy-

- This is because of the velocity of the particle.
- Formula

$$K=\frac{1}{2}mv^2$$

or using $v=A\omega\cos\omega t$ we get $K=\frac{1}{2}mA^2\omega^2\cos^2\omega t$

And using $v=w\sqrt{A^2-x^2}$ and $k=m\omega^2$ we get $K.E.=\frac{1}{2}K\left(A^2-x^2\right)$

- Kinetic energy is maximum at the mean position and equal to total energy at the mean position.

i.e $K_{\max}=\frac{1}{2}m\omega^2A^2 \quad \text{when } x=0; t=0; \omega t=0$

- Kinetic energy is minimum at the extreme positions.

i.e $K_{\min}=0 \quad \text{when } y=A; t=T/4, \omega t=\pi/2$

- The average value of kinetic energy with respect to t

$$K_{avg}=\frac{\int K\,dt}{\int dt}$$

$$K_{avg}=\frac{\int \frac{1}{2}m\omega^2A^2\cos^2(\omega t)}{\int dt}=\frac{\int \frac{1}{4}m\omega^2A^2(1+\cos 2\omega t)dt}{dt}=\frac{1}{4}m\omega^2A^2$$

So $K_{avg}=U_{avg}$

Total energy-

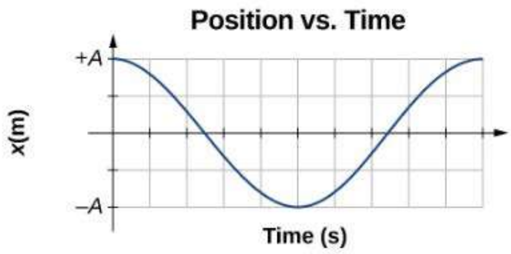
- Total mechanical energy = Kinetic energy + Potential energy or E=K+U

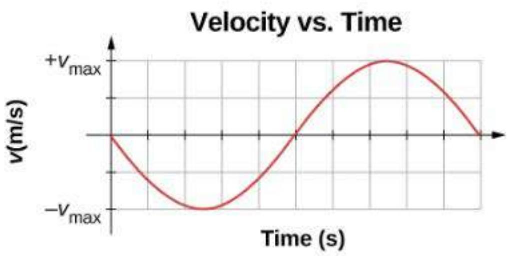
$$E=\frac{1}{2}m\omega^2\left(A^2-x^2\right)+\frac{1}{2}m\omega^2x^2=\frac{1}{2}m\omega^2A^2$$

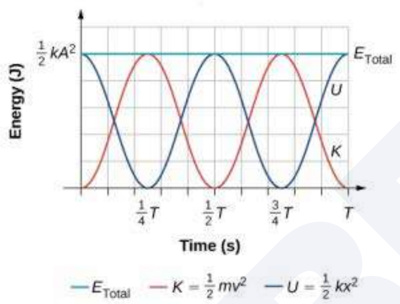
So Total energy does not depend on position(x) i.e. it always remains constant in SHM.

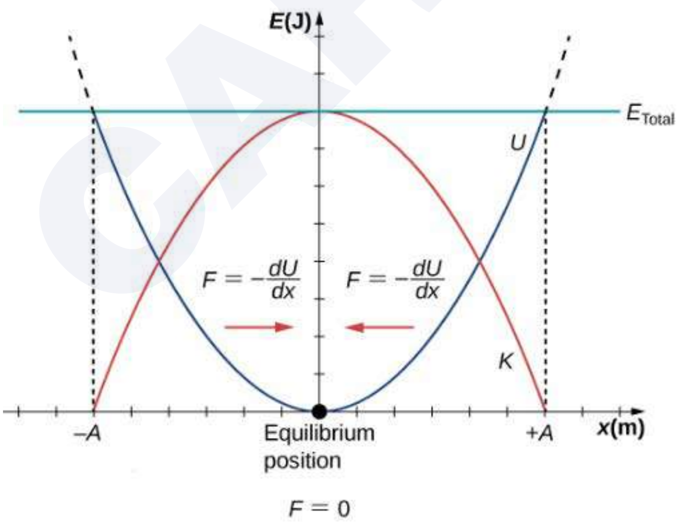
- Graph of Energy in S.H.M

At time t=0 sec, the position of the block is equal to the amplitude,









7.Oscillations Of A Spring-mass System

Spring Force:-

- Spring force is also called restoring force.

- $F = -kx$

  where k is the spring constant and its unit is N/m and x is net elongation or compression in the spring.

  Spring constant (k) is a measure of stiffness or softness of the spring

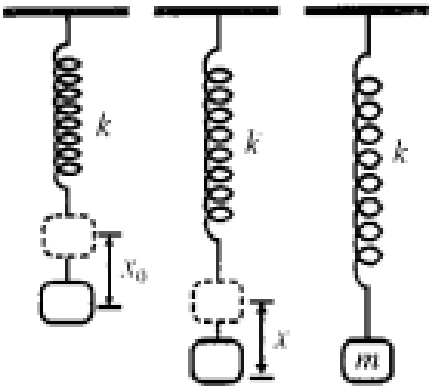
- Here -ve sign is because the force exerted by the spring is always in the opposite direction to the displacement.

The time period of the Spring mass system-

1. Oscillation of a spring in a verticle plane-

Let x_0 be the deformation in the spring in equilibrium. Then $kx_0 = mg.$

When the block is further displaced by x , the net restoring force is given by $F = -[k(x + x_0) - mg]$ as shown in the below figure.



using $F = -[k(x + x_0) - mg]$ and $kx_0 = mg.$

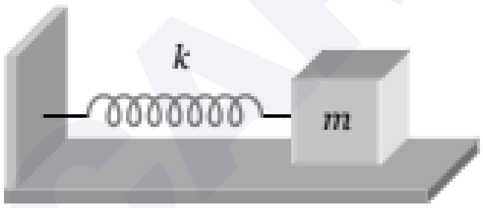
we get $F = -kx$

comparing it with the equation of SHM i.e $F = -m\omega^2 x$

we get $\omega^2 = \frac{k}{m} \Rightarrow T = 2\pi\sqrt{\frac{m}{k}}$

similarly $\text{Frequency} = n = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

2. **Oscillation of spring in the horizontal plane**



For the above figure, Time period of spring as $T = 2\pi\sqrt{\frac{m}{k}}$ and $\text{Frequency} = n = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

- Key points

1. The time period of a spring-mass system depends on the mass suspended

$$T \propto \sqrt{m} \quad \text{or} \quad n \propto \frac{1}{\sqrt{m}}$$

2. The time period of a spring-mass system depends on the force constant k of the spring

$$T \propto \frac{1}{\sqrt{k}} \quad \text{or} \quad n \propto \sqrt{k}$$

3. The time period of a spring-mass system is independent of acceleration due to gravity.

4. The spring constant k is inversely proportional to the spring length.

$$\text{As} \quad k \propto \frac{1}{\text{Extension}} \propto \frac{1}{\text{Length of spring }(l)}$$

i.e $kl = constant$

That means if the length of spring is halved then its force constant becomes double.

5. When a spring of length l is cut in two pieces of length l_1 and l_2 such that $l_1 = nl_2$

So using

$$l_1 + l_2 = l$$
$$nl_2 + l_2 = l$$
$$(n+1)l_2 = l \Rightarrow l_2 = \frac{l}{n+1}$$

$$\text{similarly } l_1 = nl_2 \Rightarrow l_1 = \frac{l*n}{(n+1)}$$

If the constant of a spring is k then

using $kl = constant$

i.e $k_1 l_1 = k_2 l_2 = kl$

we get

$$\text{Spring constant of first part } k_1 = \frac{k(n+1)}{n}$$
$$\text{Spring constant of second part } k_2 = (n+1)k$$

$$\text{and ratio of spring constant } \frac{k_1}{k_2} = \frac{1}{n}$$

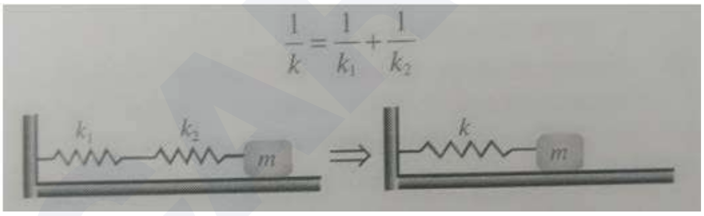
6. If the spring has a mass M and mass m is suspended from it, then its effective mass is given by $m_{eff} = m + \frac{M}{3}$

and $T = 2\pi\sqrt{\frac{m_{eff}}{k}}$

Oscillations in combination of springs-

1. Series combination of spring

If 2 springs of different force constant are connected in series as shown in the below figure



then k=equivalent force constant is given by

$$\frac{1}{K_{eq}} = \frac{1}{K} = \frac{1}{K_1} + \frac{1}{K_2}$$

Where $K_1 and\ K_2$ are spring constants of spring 1 & 2 respectively.

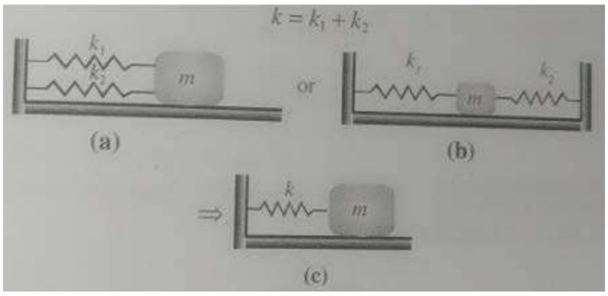
Similarly, If n springs of different force constant are connected in series having force constant $k_1, k_2, k_3 \ldots\ldots$ respectively

then $\frac{1}{k_{eff}} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} + \ldots\ldots\ldots$

If all the n spring have the same spring constant as K_1 then $\frac{1}{k_{eff}} = \frac{n}{k_1}$

2.The parallel combination of spring

If 2 springs of different force constant are connected in parallel as shown in the below figure



then k=equivalent force constant is given by

$$K_{eq} = K = K_1 + K_2$$

where $K_1 \, and \, K_2$ are spring constants of spring 1 & 2 respectively.

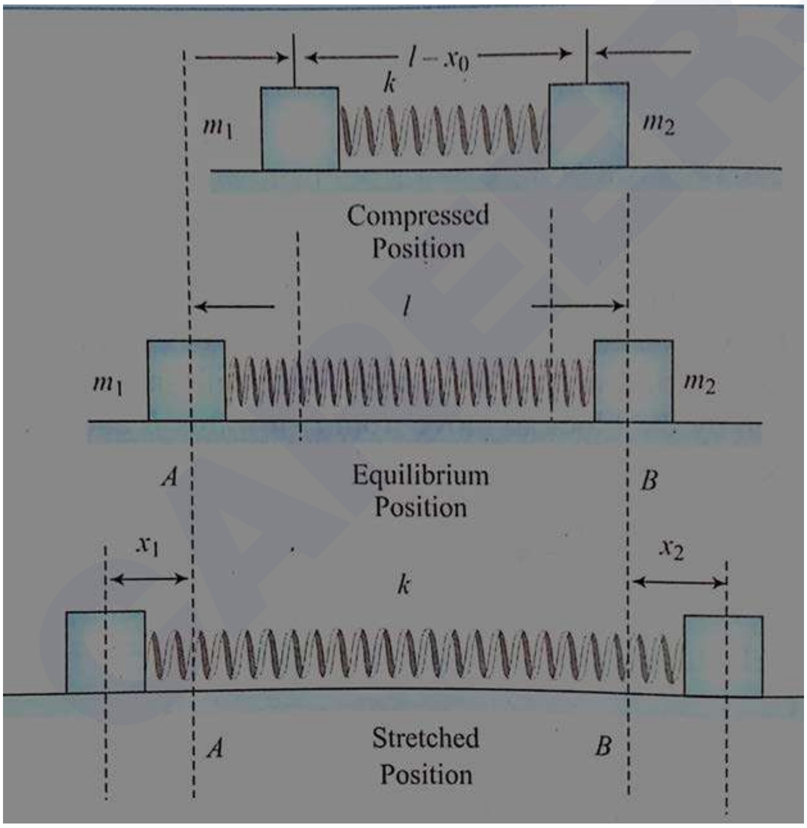
Similarly, If n springs of different force constant are connected in parallel having force constant $k_1, k_2, k_3 \ldots\ldots$ respectively

then $K_{eq} = K_1 + K_2 + K_3 \ldots.$

If all the n spring have the same spring constant as K_1 then $K_{eq} = nK_1$

8. Oscillation Of Two Particle System

Two blocks of masses m_1 and m_2 are connected with a spring of natural length l and spring constant k. The system is lying on a frictionless horizontal surface. Initially, the spring is compressed by a distance x_0 as shown in below Figure.



If we release these blocks from the compressed position, then they will oscillate and will perform SHM about their equilibrium position.

- The time period of the blocks-

In this case, the reduced mass m_r is given by $\frac{1}{m_r} = \frac{1}{m_1} + \frac{1}{m_2}$

and $T = 2\pi\sqrt{\frac{m_r}{k}}$

Or

- The amplitude of the blocks- Let the amplitude of the blocks as A_1 and A_2

$$then \quad m_1 A_1 = m_2 A_2$$

(As net external force is zero and initially the centre of mass was at rest

so $\Delta x_{cm} = 0$)

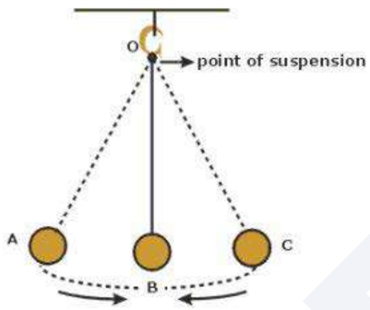
$$\begin{aligned}
&\text{By energy conservation,}\\
&\frac{1}{2}k\left(A_1 + A_2\right)^2 = \frac{1}{2}kx^2\\
&A_1 + A_2 = x_0 \quad or, \quad A_1 + \frac{m_1}{m_2}A_1 = x_0\\
&\quad A_1 = \frac{m_2 x_0}{m_1 + m_2}\\
&\text{Similarly, } A_2 = \frac{m_1 x_0}{m_1 + m_2}
\end{aligned}$$

9.Oscillation Of Pendulum

An **ideal simple pendulum** consists of a heavy point mass body suspended by a weightless, inextensible and perfectly flexible string from rigid support about which it is free to oscillate.

- **The time period of oscillation of simple pendulum (T)-**

When the bob is displaced to position B, through a small angle from the vertical as shown in the below figure.



Then Bob will perform SHM and its time period is given as

$$T = 2\pi\sqrt{\frac{l}{g}}$$

where

m=mass of the bob

l = length of pendulum

g = acceleration due to gravity.

- key points

1. The time period of a simple pendulum is independent of the mass of the bob.

I.e If the solid bob is replaced by a hollow sphere of the same radius but different mass, the time period remains unchanged.

2. $T \ \alpha \ \sqrt{l}$
where l is the distance between the point of suspension and center of mass of bob and is called effective length.

3. The period of a simple pendulum is independent of amplitude as long as its motion is simple harmonic.

Oscillation of Pendulum in different situations-

I.Pendulum in a lift-

1.The time period of a simple pendulum , If the lift is at rest or moving downward /upward with constant velocity.

$$T=2\pi\sqrt{\frac{l}{g}}$$

where

$l=$ the length of pendulum

$g=$ acceleration due to gravity.

2.The time period of a simple pendulum, If the lift is moving upward with constant acceleration a

$$T=2\pi\sqrt{\frac{l}{g+a}}$$

where

$l=$ the length of pendulum

$g=$ acceleration due to gravity.

$a=$ acceleration of pendulum.

3. The time period of simple pendulum If the lift is moving downward with constant acceleration a

$$T=2\pi\sqrt{\frac{l}{g-a}}$$

where

$l=$ the length of pendulum

$g=$ acceleration due to gravity.

$a=$ acceleration of pendulum.

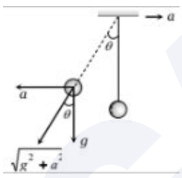
4. The time period of a simple pendulum , If the lift is moving downward with acceleration a =g

$$T=2\pi\sqrt{\frac{l}{g-g}}=\infty$$

It means there will be no oscillation in a pendulum as here $g_{eff}=0$

Similarly in the case of a satellite and at the center of the earth the $g_{eff}=0$ so in these cases, effective acceleration becomes zero and the pendulum will stop.

5. **The time period of a simple pendulum whose point of suspension moving horizontally with acceleration 'a'**



For the above figure $g_{eff}=(g^2+a^2)^{\frac{1}{2}}$

$$T=2\pi\sqrt{\frac{l}{(g^2+a^2)^{\frac{1}{2}}}}$$

Where

$l=$ the length of pendulum

$g=$ acceleration due to gravity.

$a=$ acceleration of pendulum.

6. The time period of simple pendulum accelerating down an incline

In this case $g_{eff}=gcos\theta$

$$T = 2\pi\sqrt{\frac{l}{g\cos\Theta}}$$

where

$l =$ the length of pendulum

$g =$ acceleration due to gravity.

$\Theta =$ angle of inclination

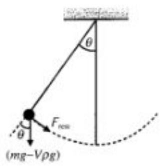
II. The time period of the pendulum in a liquid

If we immerse a simple pendulum in a liquid, the bob of the pendulum will experience a buoyant force in an upward direction in addition to the other forces such as gravity and tension.

If bob a simple pendulum of density σ is made to oscillate in some fluid of density ρ (where $\rho < \sigma$).

Then the buoyant force is given as $F_B = V\rho g$

As buoyant force will oppose its weight therefore $F_{net} = mg_{eff} = mg - F_B$



And for the above figure let bob is displaced for a small displacement x and is at an angle θ with the verticle.

For small displacement x of the bob, restoring force

$$F_{\text{rest}} = (mg - V\rho g)\sin\theta = -(mg - V\rho g)\frac{x}{l}$$

and acceleration $= -\left(g - \frac{V\rho g}{m}\right)\frac{x}{l}$

On comparing with standard equation of SHM, $a = -\omega^2 x$, we get

$$\omega = \sqrt{\frac{\left(g - \frac{V\rho g}{m}\right)}{l}} = \sqrt{\frac{g}{l}\left(1 - \frac{\rho}{\sigma}\right)}$$

and $T = 2\pi\sqrt{\frac{\ell}{g\left(1-\frac{\rho}{\sigma}\right)}}$

III. The time period of the Second's pendulum

Second, 's Pendulum: It is that simple pendulum whose time period of vibrations is two seconds.

Putting T=2 sec in $T = 2\pi\sqrt{\frac{l}{g}}$ we get the Length of a second's pendulum is nearly 1 meter on the earth's surface.

IV.Pendulum of large length but small amplitude

If the length of the pendulum is comparable to the radius of the earth

then $T = 2\pi\sqrt{\frac{1}{g\left(\frac{1}{l} + \frac{1}{R}\right)}}$

where

$l =$ length of pendulum

$g =$ acceleration due to gravity.

$R =$ Radius of earth

- **Various cases**

A. If $l << R$, then $\frac{1}{l} >> \frac{1}{R}$ so $T = 2\pi\sqrt{\frac{l}{g}}$

B. If $l >> R$ $(or\ \ l \to \infty)\ then\ \ \frac{1}{l} < \frac{1}{R}$

so $T = 2\pi\sqrt{\frac{R}{g}} = 2\pi\sqrt{\frac{6.4 \times 10^6}{10}} \cong 84.6$ minutes

and it is the maximum time period which an oscillating simple pendulum can have

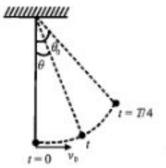
C. If $l = R$ so $T = 2\pi\sqrt{\frac{R}{2g}} \cong 1$ hour

10.Angular Simple Harmonic Motion

The general equation of linear SHM is given by $x = Asin(\omega t + \alpha)$

Similarly, The general equation of angular SHM is given by $\theta = \theta_0 sin(\omega t + \phi)$

where $\theta\ and\ \theta_0$ are angular displacement and angular amplitude of the bob respectively, as shown in the below figure



If l=length of the bob then we can write $\theta = \frac{x}{l}\ \ and\ \theta_0 = \frac{A}{l}$.

Similarly, The angular velocity if the bob which is in angular SHM is given by

$$\dot{\theta} = \frac{d\theta}{dt} = \theta_0 \omega Cos(\omega t + \phi)$$

$$or\ \ \dot{\theta} = \omega\sqrt{\theta_0{}^2 - \theta^2}$$

Similarly, The angular acceleration if the bob which is in angular SHM is given by

$$\alpha = \frac{d^2\theta}{dt} = -\theta_0 \omega^2 Sin(\omega t + \phi)$$

$$or\ \ \alpha = -\omega^2\theta$$

And Thus restoring torque on the body is given as

$$\tau_R = -I\alpha = -I\omega^2\theta$$

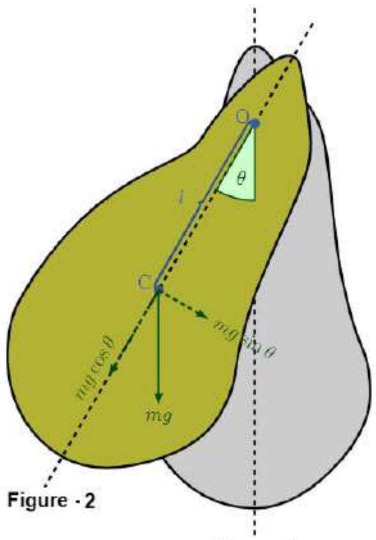
Thus we can state that in angular SHM, the angular acceleration of the body and the restoring torque on the body are directly proportional to the angular displacement of body from its mean position and are directed toward the mean position.

Similarly, a basic differential equation for angular SHM can be written as

$$\frac{d^2\theta}{dt^2} + \omega^2\theta = 0$$

11.Physical Pendulum

Physical pendulum- Any rigid body suspended from fixed support and can oscillate about that support then it is called a physical pendulum. e.g. A circular ring suspended on a nail in a wall etc.



The body is in equilibrium, as shown in the above fig-1 and it is pivoted about point O.

Now the body is displaced through a small angle θ as shown in the fig-2.

Let the distance between the point of suspension and centre of mass of the body=$OC = l$

Then torque on the body about O is given by $\tau = mglsin\theta$ ....(1)

Now if I=moment of inertia of the body about O, Then $\tau = I\alpha$... (2)

From the equation (1) and (2) we get

$$\tau = I\alpha = I\frac{d\theta^2}{dt} = -mglsin\theta$$

Since θ is very small so $I\frac{d\theta^2}{dt} = -mgl\theta$

Comparing with the equation $\frac{d\theta^2}{dt} = -\omega^2\theta$ we get

$$\omega = \sqrt{\frac{mgl}{I}} \Rightarrow T = 2\pi\sqrt{\frac{I}{mgl}}$$

Note-

$$\text{Time peried, } T = 2\pi\sqrt{\frac{I}{mgl}}; \quad I = I_{\text{CM}} + ml^2$$

Where I_{CM} is a moment of inertia relative to the axis which passes from the centre of mass and parallel to the axis of oscillation?

$$T = 2\pi\sqrt{\frac{I_{\text{CM}} + \text{ml}^2}{mgl}}, \text{ where } \text{I}_{\text{CM}} = \text{mk}^2$$

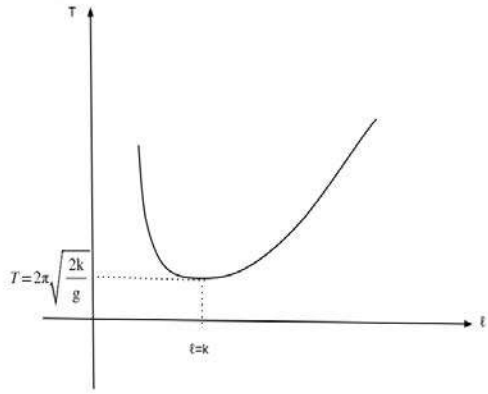
k is gyration radius (about an axis passing from centre of mass)

$$T = 2\pi\sqrt{\frac{mk^2 + ml^2}{mgl}}$$

$$= 2\pi\sqrt{\frac{k^2 + l^2}{gl}} = 2\pi\sqrt{\frac{l_{\text{eq}}}{g}}$$

$$L_{\text{eg}} = \frac{k^2}{l} + l = \text{ equivalent length of simple pendulum}$$

So the graph of the Time period (T) Vs length of a simple pendulum (l) is shown below

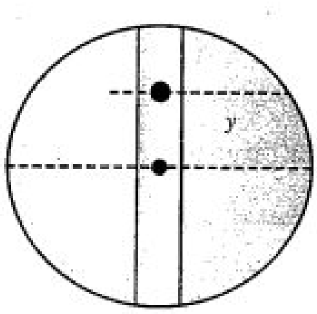


T is minimam when $l=k$

$\Rightarrow T_{\min}=2\pi\sqrt{\frac{2k}{g}}$

12.Motion Of A Ball In Tunnel Through The Earth

Case I: If the tunnel is along a diameter and the ball is released from the surface. If the ball at any time is at a distance y from the center of the earth as shown in the below figure,



So the restoring force will act on the ball due to gravitation between ball and earth.

Acceleration of the particle at the distance y from the center of the earth is given by

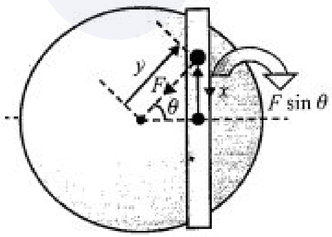
$a=\frac{-GMy}{R^3}$ and $g=\frac{GM}{R^2}$

$So\ \ a=\frac{-(gR^2)y}{R^3}\Rightarrow a=-\frac{g}{R}y$

Comparing with $a=-\omega^2 y$

$\omega^2=\frac{g}{R}\quad \omega=\sqrt{\frac{g}{R}}\Rightarrow T=2\pi\sqrt{\left(\frac{R}{g}\right)}=84.6\min$

Case II: If the tunnel is along a chord and ball is released from the surface. If the ball at any time is at a distance x from the centre of tunnel, as shown in the below figure



then the acceleration of the particle at the distance y from the center of the earth

$a=\frac{-GMy}{R^3}$

and using $g=\frac{GM}{R^2}$ we get $a=\frac{-\left(gR^2\right)y}{R^3} \Rightarrow a=-\frac{g}{R}y$

This acceleration will be towards the center of the earth.

So the component of acceleration towards the center of the tunnel.

$$a'=a\sin\theta=\left(-\frac{g}{R}y\right)\left(\frac{x}{y}\right)=-\frac{g}{R}x$$

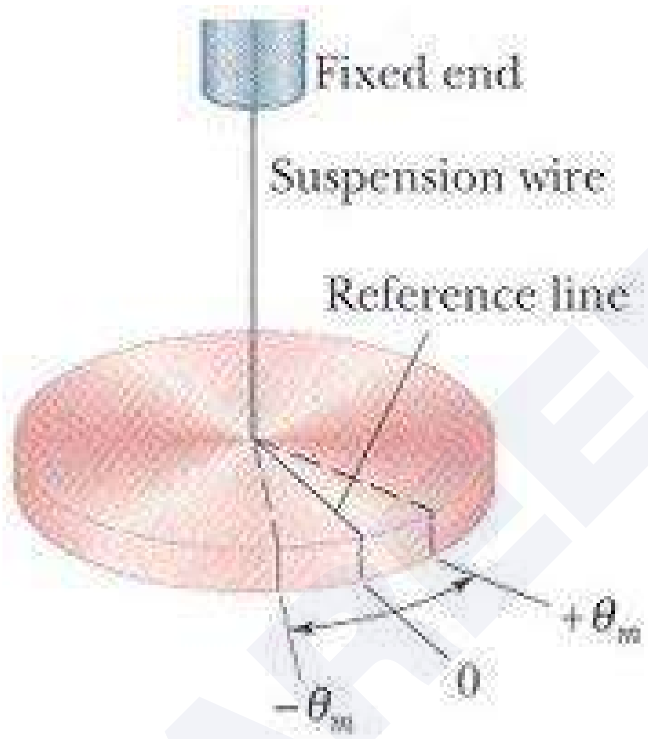
Comparing with $a'=-\omega^2 x$

$$\omega^2=\frac{g}{R} \Rightarrow \omega=\sqrt{\frac{g}{R}} \Rightarrow T=2\pi\sqrt{\frac{R}{g}}=84.6\text{min}$$

Note: The time period of oscillation is the same in both cases whether the tunnel is along a diameter or along the chord.

13.Time Period Of Torsional Pendulum

Below is the figure of the Torsional pendulum which consists of a rigid object suspended by a wire attached at the top to a fixed end.



When the object is twisted through some angle θ, the twisted wired exerts on the object a restoring torque and this restoring torque is proportional to the angular position.

That is $\tau=-k\theta$ where κ is called the torsion constant of the support wire.

Applying Newton's second law for rotational motion, we find that

$$\tau=-k\theta=I\frac{d^2\theta}{dt^2} \Rightarrow \frac{d^2\theta}{dt^2}=-\frac{k}{I}\theta$$

So the Time Period of Torsional pendulum is given as

$$T=2\pi\sqrt{\frac{I}{k}}$$

where

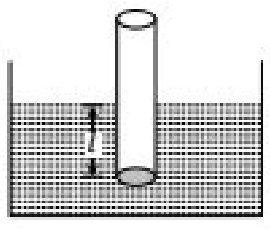
I = moment of inertia

k= torsional constant

14.The Oscillation Of Floating Bodies

A floating body is in a stable equilibrium. When it is displaced up and released, it accelerates down and when it is pushed down and released, it accelerates up. It means a floating body experiences a net force towards its stable equilibrium position. Hence, a floating body oscillates when displaced up or down from its mean position.

Consider a solid cylinder of density σ and height h, is floating in a liquid of density ρ as shown below figure, And $(\sigma < \rho)$.



If l is the length of cylinder dipping in liquid as shown in the above figure.

If it is depressed slightly and allowed to oscillate vertically.

Then the time period of the oscillation is given by

$$T = 2\pi\sqrt{\frac{l}{g}}$$

- The time period of the oscillation of the above SHM is also given in term of h, ρ, σ

$$\begin{aligned}&at\ \ mean\ position\\&F_{net} = 0 \Rightarrow Weight\ of\ \ solid = buoyant\ \ force \Rightarrow mg = V\rho g\\&As\ m = \sigma hA\\&\Rightarrow \sigma hAg = \rho lAg\\&\Rightarrow l = \frac{h\sigma}{\rho}\end{aligned}$$

So time period of the oscillation is given by

$$T = 2\pi\sqrt{\frac{h\sigma}{g\rho}}$$

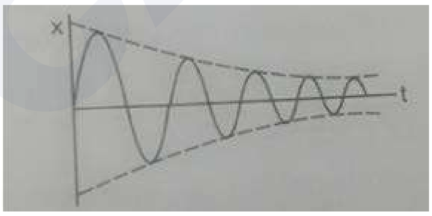
15.Free, Forced And Damped Oscillation

Free\undamped oscillation-

- The oscillation of a particle with fundamental frequency under the influence of restoring force is defined as free oscillations.
- The amplitude, frequency, and energy of oscillation remain constant.
- The frequency of free oscillation is called natural frequency because it depends upon the nature and structure of the body.

Damped oscillation-

- The oscillation of a body whose amplitude goes on decreasing with time is defined as damped oscillation.
- The amplitude of these oscillations decreases exponentially (as shown in the below figure) due to damping forces like frictional force, viscous force, etc



- These damping forces are proportional to the magnitude of the velocity and their direction always opposes the motion.
- Due to decrease in amplitude the energy of the oscillator also goes on decreasing exponentially
- The equation of motion of Damped oscillation is given by

$$m\frac{du}{dt} = -kx - bu$$

where

u=velocity

$-bu$ = damping force

b= damping constant

$-kx$ = restoring force

Or using $u = \frac{dx}{dt}$

where x=displacement of damped oscillation

we can write, The equation of motion of Damped oscillation as

$$m\frac{d^2x}{dt} = -kx - b\frac{dx}{dt}$$

The solution of the above differential equation will give us the formula of x as

$$x = A_0 e^{-\frac{bt}{2m}}.\sin(\omega' t + \delta)$$

where $\omega' = angular\ frequency\ of\ the\ damped\ oscillation$

and $\omega' = \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2} = \sqrt{{\omega_0}^2 - \left(\frac{b}{2m}\right)^2}$

- The amplitude in damped oscillation decreases continuously with time according to

  $$A = A_0.e^{-\frac{bt}{2m}}$$

- The energy in damped oscillation decreases continuously with time according to

  $E = E_0.e^{-\frac{bt}{m}}$ where $E_0 = \frac{1}{2}kA_0^2$

- Critical damping- The condition in which the damping of an oscillator causes it to return as quickly as possible to its equilibrium position without oscillating back and forth about this position.

  Critical damping happens at $\omega_0 = \frac{b}{2m}$

Forced Oscillation-

- The oscillation in which a body oscillates under the influence of an external periodic force is known as forced oscillation.
- The frequency of the forced oscillation is equal to the frequency of the external force.
- Let $f_t = f_0 \sin \omega_d t$ be an additional periodic force apart from the restoring force and the damping force,

  The differential equation for this motion is given by

  $$m\frac{du}{dt} = -kx - bu + f_0 \sin \omega_d t$$

  or $m\frac{d^2x}{dt} = -kx - b\frac{dx}{dt} + f_0 \sin \omega_d t$ .... (1)

  Where f_0 is a constant and ω_d is the angular frequency of the driving force.

The differential solution of the equation (1) will give us displacement (x_t) of the body as

$$x_t = A\sin(\omega_d t + \phi)$$

And $tan\phi$ is given as $tan\phi = \frac{-v_0}{\omega_d x_0}$

where $x_0\ and\ v_o$ are the displacement and velocity of the body respectively at t=0 (or at the moment when we apply external force)

- Amplitude in forced oscillation-

  $$A = \frac{F_0/m}{\sqrt{\left(\omega_d^2 - {\omega_0}^2\right)^2 + \left(\frac{b\omega_d}{m}\right)^2}}$$

  Where $\omega_0 = \sqrt{k/m}$ is called a natural angular frequency.

Case I-When difference between ω and ω_d is very large and damping is small (small b)

So in this case $\omega_d^2 - {\omega_0}^2 >> \frac{b\omega_d}{m}$

then Amplitude can be written as $A = \frac{F_0/m}{\omega_d^2 - {\omega_0}^2}$ ....(2)

Case II- When the difference between ω and ω_d is very small and small damping

In this case $\omega_d^2 - {\omega_0}^2 << \frac{b\omega_d}{m}$

then Amplitude can be written as $A = \frac{F_0}{\omega_d * b}$ and this is the maximum possible Amplitude. And this maximum possible Amplitude depends on ω_d and b

when $\omega_d = \omega_0$ and b is small, then $A = \infty$ (ideally maximum Amplitude) which in practical life is not possible.

- **Resonance-**

If we vary angular frequency ω of the applied force, the amplitude changes and becomes maximum when

$$w_d = \sqrt{{\omega_0}^2 - \left(\frac{b}{2m}\right)^2}$$

and this is the condition of resonance. For small damping (i.e., b is small),we can neglect b and the equation becomes $\omega = \omega_d$, so we can say that the state of resonance occurs when the frequency of the external will be equal to the natural frequency of the oscillator.

I.e when $\omega_d = \omega_0$

Waves

Important Formulae

1. Wave Motion

Wave motion:

Wave motion is defined as a form of disturbance transferred from one point to another involving transfer of energy but no transfer of matter.

Ex. A sound wave, a water wave, a wave on a string.

Particles in wave motion do not travel, they only oscillate about their position.

A wave is a form of energy or momentum that travels due to disturbances produced.

Types of wave-

Waves can classified on the basis of 3 different characterstics.

a) On the basis of medium

b) On the basis of vibration of the particles

c) On the basis of energy propagation

We are going to discuss each type in detail.

(a). On the basis of medium :

On the basis of medium waves can be classified into two categories.

1. Mechanical waves: The waves which require medium for their propagation are called mechanical waves.

Ex: Waves on string and spring, waves on water surface, sound waves, seismic waves.

2. Non-mechanical waves: The waves which do not require medium for their propagation are called non-mechanical or electromagnetic waves.

Ex: Light, heat (Infrared), radio waves, gamma rays, X-rays etc.

(b). On the basis of vibration of the particles :

On the basis of the vibration of the particles, waves can be classified into two categories.

1. Transverse waves: Particles of the medium execute simple harmonic motion about their mean position in a direction perpendicular to the direction of propagation of wave motion.

Ex. Movement of a string of a sitar or violin.

2. Longitudinal waves: If the particles of a medium vibrate in the direction of wave motion the wave is called longitudinal.

Ex. Sound waves travel through the air, Vibration of the air column in organ pipes.

(c). <u>On the basis of energy propagation</u> :

On the basis of energy, propagation waves can be classified into two categories.

1. Progressive waves: Progressive wave is formed due to continuous vibration of the particles of the medium. Progressive waves travel with a certain velocity. Progressive wave transport energy.

2. Stationary waves: A stationary wave is formed by the superposition of two identical progressive waves travelling in the opposite direction. Stationary wave doesn't travel in any direction. There is no flow of energy in stationary waves.

Ex. wave on a guitar string

General equation of travelling wave-

For wave travelling along positive x-axis,

$$y(x,t) = A\sin(k(x - vt)) = A\sin(\frac{2\pi x}{\lambda} - \frac{2\pi t}{T}) = A\sin(kx - \omega t)$$

where

k- propagation constant or angular wave number

A- Amplitude

$$k = \frac{2\pi}{\lambda}$$

$$v = f\lambda = \frac{\lambda}{T}$$

$$T = \frac{2\pi}{\omega}$$

For wave travelling along negative x-axis,

$$y(x,t) = A\sin(k(x + vt)) = A\sin(\frac{2\pi x}{\lambda} + \frac{2\pi t}{T}) = A\sin(kx + \omega t)$$

<u>GENERAL EQUATION OF TRAVELLING WAVE</u>

$$y(x,t) = A\sin(k(x \pm vt) + \phi) = A\sin(\frac{2\pi x}{\lambda} \pm \frac{2\pi t}{T} + \phi) = A\sin(kx \pm \omega t + \phi)$$

3.Travelling Sine Wave

The sine wave or sinusoid is a mathematical function that describes a smooth repetitive oscillation.

$$y(t) = A\sin(\omega t + \phi)$$

Here ω, is the angular frequency i.e,

$$\omega = \frac{2\pi}{T} = 2\pi f$$

and ϕ= phase angle

General form :

$y(x,t) = A\sin(kx - \omega t + \phi)$ when the wave is moving towards the right

$y(x,t) = A\sin(kx + \omega t + \phi)$ when the wave is moving towards the left.

The wavenumber is related to the angular frequency by**:**

$$k = \frac{\omega}{v} = \frac{2\pi f}{v} = \frac{2\pi}{\lambda}$$

Also,

Particle velocity = -(wave velocity) × (slope of y vs x graph)

$$\Longrightarrow V_p = -v\left(\frac{\partial y}{\partial x}\right)$$

$$\Longrightarrow \frac{\partial y}{\partial t} = -v\left(\frac{\partial y}{\partial x}\right)$$

Phase and phase difference-

Phase:

The quantity which expresses at any instant, the displacement of the particle and its direction of motion is called the phase of the particle.

If two particles of the medium, at any instant are in the same state of motion (parameters such as particle's displacement, velocity, and acceleration are same) then they are said to be in the same phase.

The phase of the wave is the quantity inside the brackets of the sin-function, and it is an angle measured either in degrees or radians.

$$\phi = \left(\frac{2\pi}{T}t - \frac{2\pi}{\lambda}x\right)$$

At a particular time t, The phase difference of the wave between point A (x_1) and point B (x_2) is given by

$$\phi_1 - \phi_2 = \left(\frac{2\pi}{\lambda}x_2 - \frac{2\pi}{\lambda}x_1\right)$$

$$\phi_1 - \phi_2 = \frac{2\pi}{\lambda}(x_2 - x_1)$$

The important result here is that the two waves can be:

(1). **In phase** if $x_2 - x_1 = n\lambda$, i.e the particles corresponding to positions x_1 and x_2 are in the same state of motion.

2) **Out of phase** if $x_2 - x_1 = \left(n + \frac{1}{2}\right)\lambda$, i.e one point in the string, x_1 say, is moving upwards while $x_2$is moving downwards but symmetrically.

4.Speed of transverse wave on a string

The distance between two successive crests is 1 wavelength i.e λ.

Thus in one time period, the wave will travel 1 wavelength in distance. Thus the speed of the wave, v is:

$$v = \frac{\lambda}{T} = \frac{\text{Distance travelled}}{\text{time taken}}$$

The speed of the traverse wave is determined by the restoring force set up in the medium when it is disturbed and the inertial properties (mass density) of the medium.

The inertial property will in this case be linear mass density μ .

$\mu = \frac{m}{L}$ where m is the mass of the string and L is length.

The dimension of μ is $[ML^{-1}]$ and T is like force whose dimension is $[MLT^{-2}]$. We need to combine these dimension to get the dimension of speed v which is $[LT^{-1}]$.

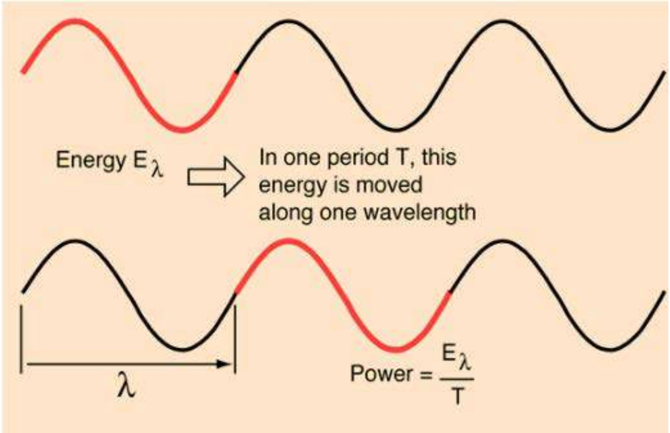
Therefore speed of wave in a string is given as :

$$v = \sqrt{\frac{T}{\mu}}$$

5.Power Transmitted Along The String

As a sinusoidal wave moves down a string, the energy associated with one wavelength on the string is transported down the string at the propagation velocity v.

From the basic wave relationship the distance traveled in one period is $vT = \lambda$, so the energy is transported one wavelength per period of the oscillation.



The energy associated with one wavelength of the wave is :

$$E_\lambda = \frac{1}{2}\mu\omega^2 A^2 \lambda$$

so the power transmitted would be :

$$P_\lambda = \frac{1}{2}\mu\omega^2 A^2 \frac{\lambda}{T}$$

since $v = \frac{\lambda}{T}$

Therefore $P_\lambda = \frac{1}{2}\mu\omega^2 A^2 v$

where $\omega =$ angular frequency, $\mu =$ mass per unit length of string, $A =$ wave amplitude

$v =$ wave propogation velocity

The intensity of the wave-

The flow of energy per unit area of cross-section of the string in the unit time is known as intensity of the wave.

As $P = \frac{1}{2}\mu\omega^2 A^2 v$

And using $I = \frac{P}{Area}$

we get $I = \frac{\frac{1}{2}\mu\omega^2 A^2 v}{Area}$

using $\mu = \frac{mass}{lenth} = \frac{m}{l}$ and $Volume = Area \times length$

We get $I = \frac{\frac{1}{2}m\omega^2 A^2 v}{lenth \times Area} = \frac{\frac{1}{2}m\omega^2 A^2 v}{Volume}$

And now using $\rho = \frac{mass}{volume}$

we get $I = \frac{1}{2}\rho\omega^2 A^2 v$

Where

$\rho =$ density

$\omega =$ angular frequency

$A =$ Amplitude

v= Wave speed

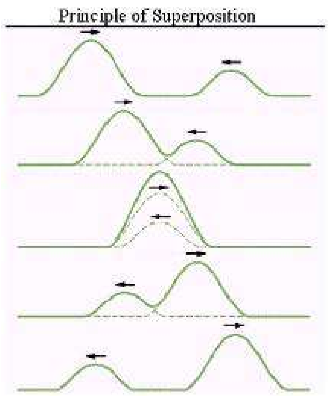
6. Interference and principle of superposition

When two waves of same frequency, the same wavelength, same velocity (nearly equal amplitude) moves in the same direction, Their superimposition results in the interference. Due to interference, the resultant intensity of sound at that point is different from the sum of intensities due to each wave separately. This modification of intensity due to the superposition of two or more waves is called interference.

The displacement at any time due to any number of waves meeting simultaneously at a point in a medium is the vector sum of the individual displacements due to each one of the waves at that point at the same time.

if $y_1, y_2, y_3 \ldots$ are the displacements at a particular time at a particular position, due to individual waves, then the resultant displacement would be :

$y = y_1 + y_2 + y_3 \ldots$



Interference of waves going in same direction-

Suppose two identical sources send sine waves of the same angular frequency ω in the positive x-direction. Also, the wave velocity and hence, the angular wave number (k) is same for the two waves. One source maybe started a little later than the other. The two waves arriving at a point then differ in phase . The two waves differ in phase by an angle ϕ . Their equations maybe written as :

$y_1 = A_1 \sin(kx - \omega t)$

$y_2 = A_2 \sin(kx - \omega t + \phi)$

According to the principle of superposition, the resultant wave is represented by

$y = y_1 + y_2 = A_1 \sin(kx - \omega t) + A_2 \sin(kx - \omega t + \phi)$

therefore the resultant we get,

$y = A \sin(kx - \omega t + \theta)$

where $A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos(\phi)}$ and $\tan(\theta) = \dfrac{A_2 \sin\phi}{A_1 + A_2\cos(\phi)}$

where

$A_1 =$ the amplitude of wave 1

$A_2 =$ the amplitude of wave 2

and $A_{\max} = A_1 + A_2$ *and* $A_{\min} = A_1 - A_2$

Resultant Intensity of two waves (I)-

Using $I \ \alpha \ A^2$

we get $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$

where

$I_1 =$ The intensity of wave 1

$I_2 =$ The intensity of wave 2

Constructive interferences :

- The phase difference between the waves at the point of observation is $\phi = 0^\circ$ or $2n\pi$
- The resultant amplitude at the point of observation will be maximum

$$i.e \quad A_{\max} = A_1 + A_2$$
$$\text{If } A_1 = A_2 = A_0 \Rightarrow A_{\max} = 2A_0$$

- Resultant intensity at the point of observation will be maximum

$$i.e \quad I_{\max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$$
$$I_{\max} = (\sqrt{I_1} + \sqrt{I_2})^2$$
$$\text{If} \quad I_1 = I_2 = I_0 \Rightarrow I_{\max} = 4I_0$$

Destructive interference :

- The phase difference between the waves at the point of observation is

$$\phi = 180^\circ \text{ or } (2n-1)\pi; n = 1, 2, \ldots$$
$$\text{or } (2n+1)\pi; n = 0, 1, 2 \ldots.$$

- The resultant amplitude at the point of observation will be minimum

$$i.e \quad A_{\min} = A_1 - A_2$$
$$\text{If } A_1 = A_2 \Rightarrow A_{\min} = 0$$

- Resultant intensity at the point of observation will be minimum

$$I_{\min} = I_1 + I_2 - 2\sqrt{I_1 I_2}$$
$$I_{\min} = (\sqrt{I_1} - \sqrt{I_2})^2$$
$$\text{If } I_1 = I_2 = I_0 \Rightarrow I_{\min} = 0$$

7.Reflection And Transmission Of Waves On A String

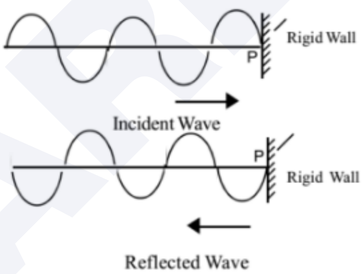
When waves are incident on a boundary between two media a part of incident waves returns back into the initial medium (reflection) while the remaining is partly absorbed and partly transmitted into the second medium (refraction).

<u>Boundary conditions :</u>

Assuming no transmission, no absorption and energy lost when a wave hits the boundary.

1. Rigid end: when the incident wave reaches a fixed end, it exerts an upward pull on the end, according to Newton's 3rd law at fixed end it exerts an equal and opposite downward force on the string. It results an **inverted pulse** or **phase change of π.**

Crest (C) reflects as trough(T) and vica-versa, Time changes by $\frac{T}{2}$ and path changes by $\frac{\lambda}{2}$.

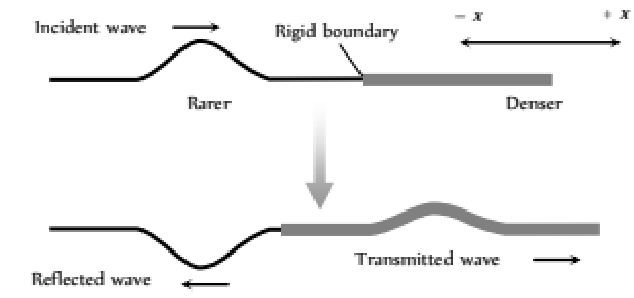


2) Free end: when a wave or pulse is reflected from a free end, then there is no change of phase (as there is no reaction force).

crest (C) reflects as the crest (C) and trough (T) reflects as a trough(T), Time changes by zero and path changes by zero.

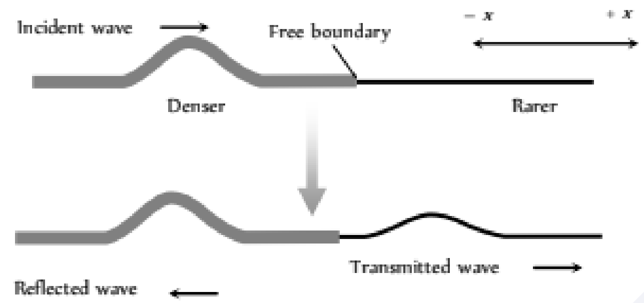
Wave in a combination of string-

1. Wave goes from rarer to a denser medium



Incident wave $\Rightarrow y_i = a_i \sin(\omega t - k_1 x)$
Reflected wave $\Rightarrow y_r = a_r \sin[\omega t - k_1(-x) + \pi] = -a_i \sin(\omega t + k_1 x)$
Transmitted wave $\Rightarrow y_t = a_t \sin(\omega t - k_2 x)$

2. Wave goes from denser to rarer medium



Incident wave $\Rightarrow y_i = a_i \sin(\omega t - k_1 x)$
Reflected wave $\Rightarrow y_r = a_r \sin[\omega t - k_1(-x) + 0] = a_i \sin(\omega t + k_1 x)$
Transmitted wave $\Rightarrow y_t = a_t \sin(\omega t - k_2 x)$

- **The amplitude of reflected wave-**

If $A_r =$ the amplitude of the reflected wave then

$$A_r = \frac{(V_2 - V_1) A_i}{(V_2 + V_1)}$$

Where

V_1 and V_2 are the velocity of the wave in the incident and transmitted wave.

$A_i =$ amplitude of incident wave.

Or we can write $\frac{A_r}{A_i} = \frac{k_1 - k_2}{k_1 + k_2} = \frac{v_2 - v_1}{v_2 + v_1}$

Where k_1 and k_2 are the angular wave number of the incident and transmitted wave respectively.

- Amplitude of transmitted wave

If $A_t =$ amplitude of the transmitted wave then

$$A_t = \left(\frac{2V_2}{V_1 + V_2}\right) A_i$$

Where

V_1 and V_2 are the velocity of the wave in the incident and transmitted wave.

$A_i =$ amplitude of incident wave.

Or we can write $\frac{A_t}{A_i} = \frac{2k_1}{k_1 + k_2} = \frac{2v_2}{v_2 + v_1}$

Where k_1 and k_2 are the angular wave number of the incident and transmitted wave respectively

8.Standing Wave On A String

Standing waves

When two sets of progressive wave of same type (both longitudinal or both transverse) having the same amplitude and same time period or frequency or wavelength travelling along the same straight line with same speed in opposite directions superimpose, a new set of waves are formed. These are called **stationary waves.**
Some of the characteristics of standing waves :

(1) In this the disturbance is confined to a particular region between the starting point and reflecting point of the wave.
(2) In this there is no forward motion of the disturbance from one particle to the adjoining particle and so on, beyond this particular region.
(3) The total energy in a stationary waves is twice the energy of each of incident and reflected wave. But there is no flow or transfer of energy along the stationary wave.

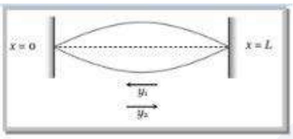
(4) Points in a standing wave, which are permanently at rest. These are called **nodes**. The distance between two consecutive nodes is $\frac{\lambda}{2}$

(5) The Points on the standing wave having maximum amplitude is known as **antinodes**. The distance between two consecutive antinodes is also $\frac{\lambda}{2}$
(6) All the particles execute simple harmonic motion about their mean position (except those are at nodes) with the same time period.

Note - In standing waves, if the amplitude of component waves are not equal. Resultant amplitude at nodes will not be zero. It will be minimum . Because of this, some energy will pass across nodes and waves will be partially standing.

Let us take an example to understand and derive equation of standing wave -

Let us take a string and when a string is under tension and set into vibration, transverse harmonic waves propagate along its length. If the length of string is fixed, reflected waves will also exist. These incident and reflected waves will superimpose to produce transverse stationary waves in a string



Incident wave $y_1 = a\sin\frac{2\pi}{\lambda}(vt + x)$

Reflected wave $y_2 = a\sin\frac{2\pi}{\lambda}[(vt - x) + \pi] = -a\sin\frac{2\pi}{\lambda}(vt - x)$

Now we can apply principle of superposition on this and get -

$$y = y_1 + y_2 = 2a\cos\frac{2\pi vt}{\lambda}\sin\frac{2\pi x}{\lambda}$$

So, $y = (2Asinkx)cos\omega t$

So, it can be written as - $y = Rcos\omega t$ .. . .. . (1)

where, $R = 2Asinkx$ . . . . . (2)

Equation (1) and (2) shows that after superposition of the two waves the medium particle execute SHM with same frequency and amplitude. Thus on superposition of two waves travelling in opposite direction, the resulting interference pattern will form Stationary waves.

Nodes and antinodes -

Points in a standing wave, which are permanently at rest. These are called **nodes**. The Points on the standing wave having maximum amplitude is known as **antinodes.**

For nodes -

From equation (2) we can say that - $kx = n\pi$

So, $x = \frac{n\pi}{k} = \frac{n\pi}{\frac{2\pi}{\lambda}} = \frac{n\lambda}{2}$

So, at point where $x = 0, \frac{\lambda}{2}, \lambda ... displacement\ is\ zero$

For antinodes -

From equation (2) we can say that - $kx = (2n+1)\frac{\pi}{2}$

So, $x = (2n+1)\frac{\lambda}{4}$

So, again using equation (2) $y = \pm 2A$

Thus at point for which $x=\frac{\lambda}{4},\frac{3\lambda}{4},\frac{5\lambda}{4}....., \ displacemnet\ is\ maximum\ (\pm 2A)$

Standing wave in a string fixed at both ends -

As we know that a string is said to vibrate if it vibrate according to the given equation -

$$y=(2Asinkx)cos\omega t$$

From this equation, for a point to be node,

$$x=\frac{n\lambda}{2},\ where\ n=0,1,2,3,....$$

In this the string is fixed at both the ends, so these ends are node. So, for $x=0\ and\ for\ x=L$ (which will be node). So, it can be written as -

$$L=\frac{n\lambda}{2},\ or,\ \lambda=\frac{2L}{n}\ where\ n=1,2,3,....$$

So, corresponding frequencies will be =

$$f=\frac{v}{\lambda}=n(\frac{v}{2L})\ ,\ where\ n=1,2,3....$$

here, v = speed of travelling waves on the string

By putting the values of 'n' , we are getting different frequencies. For example -

1. $for\ n\ =1,\ f=\frac{v}{2L}\ and\ it\ is\ called\ fundamental\ frequency\ or\ first\ harmonic$.

   The corresponding mode is called fundamental mode of vibration.

2. $If\ n\ =2,\ f_1=2(\frac{v}{2L})=\frac{v}{L}$

   This second harmonic or first overtone and $f_1=2f$

Similar to this, we can increase the value of 'n' and we get the respective harmonic and overtone.

Now, the **velocity of wave in string** is given by-

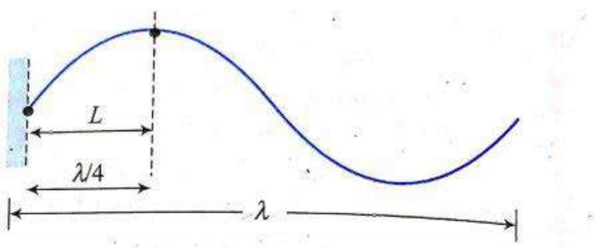
$$\nu=\sqrt{\frac{T}{\mu}}$$

So the natural frequency can be written as - $f_n=\frac{n}{2L}\sqrt{\frac{T}{\mu}}\ ;\ n=1,2,3,....$

<u>Standing wave in a string fixed at one end -</u>

In this case, one end is fixed and the other end is free. In the fundamental mode, the free end is an antinode, the length of string

$$L=\frac{\lambda}{4}$$

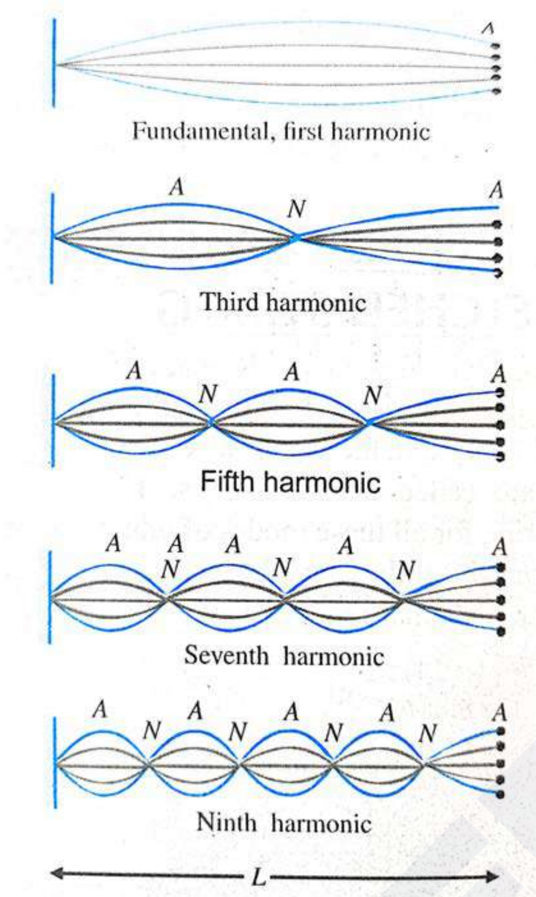


So, in the next mode-

$$L=\frac{3\lambda}{4}$$

So, in general we can write the equation =

$$L = \frac{n\lambda}{4}\ ,\ n = 1, 3, 5...$$



From this we can write the resonance frequency -

$$f_n = n\frac{v}{4L} = nf_1\ ;n = 1, 3, 5, ...$$

where, $f_1 = \frac{v}{4L}$ (Fundamental frequency)

9.Sound Wave

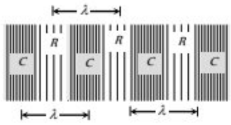
Sound is defined as the energy to which the human ears are sensitive is known as sound.

Sound waves always travel through any elastic material medium with a speed that depends on the properties of the medium. As sound waves travel through the air, the molecules of air vibrate to produce changes in density and pressure along the direction of motion of the wave. If the source o the sound waves vibrates as a Sine wave, the pressure variations are also like Sine waves. Because of this, the mathematical description of sinusoidal sound waves is very similar to that of sinusoidal waves on strings.

Transverse waves	Longitudinal waves
1.In this particles of the medium vibrates in a direction perpendicular to the direction of propagation of the wave.	1. In this particles of a medium vibrate in the direction of wave motion.
2. Transverse waves travels in the form of crests and troughs.	2. Longitudinal waves travels in the form of compression and rarefaction
3. It can be transmitted through solids, they can be set up on the surface of liquids. But they can not be transmitted into liquids and gases.	3. It can be transmitted through solids, liquids, and gases because, for these waves propagation, volume elasticity is necessary.

4. Transverse waves can be polarised.	4.Longitudinal waves can not be polarised.

Now, as the sound wave travels through the air, the element of air vibrate to produce a change in density and pressure along the direction of motion of the wave. So as we discussed in the table that the movement of the sound waves is like compression and rarefaction, this is shown in the given image. The detail of this with an example will be discussed in the latter concept.



Here, λ is the wavelength.

R = Rarefraction

C = Compression

Propagation of sound waves

Sound is a longitudinal wave that is created by a vibrating source such as a guitar string, the human vocal cords, or the diaphragm of a loudspeaker. As a soun wave is a mechanical wave, so, sound needs a medium having properties of inertia and elasticity.

.The equation of sound waves (in terms of pressure waves)

$$\Delta p=\Delta p_{\max} \sin [\omega(t-x / v)]$$

Relation between displacement wave and pressure wave -

$$dy = Ak\cos(kx-\omega t)dx$$

$$dV = Sdy = SAk\cos(kx-\omega t)dx$$

Where, S = Area of cross-section and V = Volume of section

$$\frac{dV}{V}=\frac{dy}{dx}=\frac{SAk\cos(kx-\omega t)dx}{Sdx}$$

$$\frac{dV}{V}=Ak\cos(kx-\omega t)$$

If B is the bulk modulus of the medium, then the excess pressure in section can be given as -

$$\Delta P=-B\left(\frac{dV}{V}\right)=-B\left(\frac{dy}{dx}\right)$$

$$\Delta P=-BAk\cos(kx-\omega t)$$

$$\Delta P=-\Delta P_{\max}\cos(kx-\omega t)$$

Here $\Delta P_{\max}$ is the pressure amplitude at a medium particle at position x from origin and ΔP is the excess pressure at that poi

So,

$$\Delta P_{\max}=BAk=\frac{2\pi}{\lambda}AB$$

In the compression zone, more particles stay in a unit volume of the medium. So, density and pressure of the region will be more. In refracted zone, lesser particles stay in any unit volume.

Let a sound wave is propagating in a medium of Bulk modulus B and density ρ.

So,

$$B=\left(-\frac{dp}{dV/V}\right)$$

Also, $\frac{dV}{V}=-\frac{d\rho}{\rho}$

From both Equation, we get, $d\rho = \frac{\rho}{B} dp$
The speed of sound is given by, $v = \sqrt{\frac{B}{\rho}} \Rightarrow \frac{\rho}{B} = \frac{1}{v^2}$
Hence, $d\rho = \frac{\rho}{B} \Delta p = \frac{1}{v^2} \Delta p$

So, this relation gives relation between pressure with density. So the variation of density is like variation of pressure -

$$\Delta \rho = (\Delta \rho)_m \sin(kx - \omega t)$$

$$\text{where, } (\Delta \rho)_m = \frac{\rho}{B} (\Delta p)_m = \frac{(\Delta p)_m}{v^2}$$

Note - Density equation is in phase with pressure equation and this is $\frac{\pi}{2}$ out of phase with the displacement equation

10. Velocity Of Sound In Different Media

Speed of sound wave in a material medium -

$$v = \sqrt{\frac{Y}{\rho}}$$

where Y=Youngs modulus of the medium

For longitudinal waves for liquid or gas -

$$v = \sqrt{\frac{B}{\rho}}$$

$where\ \rho = \text{Density of the medium}$
$and\ \ where\ \ B = \text{Bulk modulus of elasticity}$

Speed of sound wave in gas: Newton's formula

The main assumption before deriving the equation is when the sound propagates through a gas, temperature variation in compression and rarefaction is negligible. So, Newton assumed that the exchange of heat with the surrounding, the temperature of the layer will remain the same. Hence this process is isothermal. Thus by using the formula that we have studied in the last concept, we can write that -

$$v = \sqrt{\frac{B_{isothermal}}{\rho}} ......(i)$$

Where $B_{isothermal} = \text{Isothermal Bulk modulus}$

Now, in the isothermal process, PV = Constant

Differentiating both sides, we get -

$$Pdv = V(-dP)$$
$$B_{isothermal} = P = \frac{dP}{\frac{dV}{V}}$$

So from the definition of Bulk modulus, we can say that the **P = $B_{isothermal}$**

$$(As,\quad B_i = \frac{dP}{\frac{dV}{V}})$$

So from equation (i), We can write that -

$$\boldsymbol{v = \sqrt{\frac{P}{\rho}}}$$

This formula is given by Newton, So it is called Newton's formula.

Laplace correction-

Laplace Correction gives correction to the speed of sound in the gas. Newton's formula was formulated taking into consideration that sound travels in isothermal conditions, the result so obtained was not matching with the experimental value of the speed of sound.

Thus, Laplace came up with a correction to it that sound travelling through air is a sudden process, it is well known as a Laplace Correction to Newton's Formula.

$$v=\sqrt{\frac{B_{adiabatic}}{\rho}}$$

Where $B_{adiabatic}$ = adiabatic bulk modulus

Now, in the adiabatic process, PV^{γ} = Constant

Differentiating both sides, we get

$$P\gamma V^{\gamma-1}dv=V^{\gamma}(-dP)$$
$$B_{adiabatic}=-\frac{dP}{\frac{dV}{V}}=\gamma P$$

Factors affecting the speed of sound in the gas-

1. Effect of pressure -

We know that the speed of sound in gas = $\sqrt{\frac{\gamma P}{\rho}}$

Also, for gas, $Pv=nRT=\frac{m}{M}RT$

At constant temperature, we can write = $P\Delta V=\frac{\Delta m}{M}RT$

$$P=\frac{\Delta m}{\Delta V}\frac{RT}{M}$$

$$P=\rho\frac{RT}{M}$$

$$\frac{P}{\rho}=\text{ constant}$$

And as pressure changes, according to this the density changes. Thus we can say that the ratio will remain the same. So pressure does not create any effect on the speed of sound in the gas.

2. Effect of density -

For two gases of densities ρ_1 and ρ_2 at the same pressure with ratios of specific heat γ_1 and γ_2 -

$$\frac{v_1}{v_2}=\sqrt{\frac{\gamma_1\times\rho_2}{\gamma_2\times\rho_1}}$$

3. Effect of temperature -

As, $\frac{P}{\rho}=\frac{RT}{M}$

So $v=\sqrt{\frac{\gamma RT}{M}}\Rightarrow v\propto\sqrt{T}$

So, as the temperature increases the velocity will increase.

4. Effect of humidity -

Humidity is the percentage of water vapour present in the air. As the humidity increases, the percentage of water vapor in the air increases and this decreases the density of air resulting in the increased velocity of sound. So, with an increase in humidity, the density of air will decrease. And as we know that -

$$v\propto\frac{1}{\sqrt{\rho}}$$

So, the speed of sound will increase.

5. Effect of frequency -

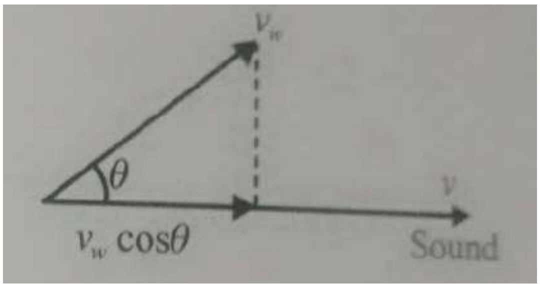
With the change of frequency, the wavelength also changes in the same proportion.

So, a product of both remains constant. From the equation - $f\lambda=v$,

So velocity remains constant.

6. Effect of wind -

As sound is carried by air, so as the velocity of wind changes then the velocity of the sound will change accordingly. Let the speed of the wind is v_w and it is blowing at an angle of θ with the direction of the sound. As shown in the figure -



The speed of sound gets extra effect from the speed of the wind as - $v_{sound} + v_w cos\theta$

$\theta\ may\ vary\ from\ 0\ to\ 180^o$

11. Intensity Of Sound Waves

The intensity of Periodic sound waves -

The intensity I of a wave is defined as the power per unit area, as the rate at which the energy transported by the wave transfers through a unit area A perpendicular to the direction of travel of the wave.

$$I = \frac{P}{A}$$

In this case, the intensity is therefore $I = \frac{1}{2}\rho v(\omega A)^2$

Also, for any sound waves -

$$\Delta P_m = ABk$$

$$A = \frac{\Delta P_m}{Bk}$$

Put this value in the equation of intensity

$$I = \frac{1}{2}\rho v\omega^2\left(\frac{\Delta P_m}{Bk}\right)^2 = \frac{1}{2}\rho v\omega^2\frac{\Delta P_m^2}{B^2k^2}$$

$$\text{As} \quad k = \omega/v \quad \text{and} \quad B = v^2\rho$$

$$\therefore I = \frac{1}{2}\rho v\omega^2\frac{\Delta P_m^2}{B^2\frac{\omega^2}{v^2}} = \frac{v\Delta P_m^2}{2B} = \frac{\Delta P_m^2}{2\rho v}$$

Now, the appearance of sound to the human ear is characterized by -

a. Pitch

b. Loudness

c. quality

Pitch - The pitch of a sound is an attribute of the sound that tells us about its frequency. A sound that is at a high pitch, has a high frequency. And a sound at low pitch has a lower frequency.

Loudness -

The loudness that a person sense is related to the intensity of sound though it is not directly proportional to it. Loudness can be defined and represented as -

$$\beta = 10log_{10}(\frac{I}{I_o})$$

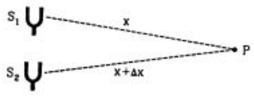
Where I = Intensity of the sound

I_o = Reference intensity (10^{-12} W-m^{-2})

For I = I_0 the sound level= $\beta = 0$

Interference of sound waves-

Let us take two tuning forks S_1 and S_2 placed side by side. which vibrate with equal frequency and equal magnitude. The point P is situated at a distance x from S_1 and $x+\Delta x$ from S_2 .



The forks may be set into vibration with a phase difference δ_o. In case of tuning forks, the phase difference δ_o remains constant in time.

Suppose the two forks are vibrating in phase so that $\delta_o = 0$. Also, let p_{01} and p_{02} be the amplitudes of the waves from S_1 and S_2 respectively.

Let us examine the resultant change in pressure at a point P. The pressure-change at P due to the two waves are described by

$$p_1 = p_{01}\sin(kx-\omega t)$$

$$p_2 = p_{02}\sin[k(x+\Delta x)-\omega t]$$

$$= p_{02}\sin[(kx-\omega t)+\delta]$$

$$\text{where } \delta = k\Delta x = \frac{2\pi\Delta x}{\lambda}...(1)$$

Here, δ is the phase difference between the two waves reaching P. So, the resultant wave at P is given by -

$$p = p_0\sin[(kx-\omega t)+\varepsilon]$$

$$\text{where} \quad p_0^2 = p_{01}^2 + p_{02}^2 + 2p_{01}p_{02}\cos\delta$$

$$\text{and} \quad \tan\varepsilon = \frac{p_{02}\sin\delta}{p_{01}+p_0\cos\delta}$$

The resultant amplitude is maximum when = $2\pi n$ and is minimum when S = $(2n+1)\pi$, where n is an integer.

These are correspondingly the conditions for constructive and destructive interference.

A similar condition in terms of path difference can be written as -

$$\begin{array}{ll} \Delta x = n\lambda & \text{(constructive)} \\ \Delta x = (n+1/2)\lambda & \text{(destructive)} \end{array}$$

The above equation is obtained with the help of the (1) equation.

At constructive interference,

$$P_0 = P_{01} + P_{02}$$

At destructive interfernece -

$$P_0 = |P_{01} - P_{02}|$$

Constructive interference	Destructive interference
1. When the waves meet with the same phase, it forms constructive interference	1. When the waves meet with opposite phase, it forms destructive interference
2. Phase difference at the point of observation. $\delta = 0^o \ or \ 2n\pi$	2. Here, phase difference = $180^o \ or \ (2n-1)\pi \ where \ n = 1,2,3...$
3. Path difference = $n\lambda$	3. Path difference = $(2n-1)\frac{\lambda}{2}$
4. Resultant amplitude = $A_{max} = a_1 + a_2$	4. Resultant amplitude = $A_{min} = a_1 - a_2$
5. Resultant intensity will be maximum = $I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$	5. Resultant intensity will be minimum = $I_{max} = I_1 + I_2 - 2\sqrt{I_1 I_2}$

12. Standing Sound Waves

Standing Wave in a Closed Organ Pipe -

Organ pipes are musical instruments which are used for producing musical sound by blowing air into the pipe. In this longitudinal stationary waves are forme due to superimposition of incident and reflected longitudinal waves.

A closed organ pipe is a cylindrical tube having an air column with one end closed. Sound waves are enters from a source vibrating near the open end. An ingoing pressure wave gets reflected from the fixed end. This inverted wave is again reflected at the open end. After two reflections, it moves towards the fixe end and interferes with the new wave sent by the source in that direction. The twice reflected wave has travelled an extra distance of $2l$ causing a phase advance of $\frac{2\pi}{\lambda}.2l = \frac{4\pi l}{\lambda}$

Similarly at open ends, the twice reflected wave suffered a phase change of π at the open end.

So the phase difference is $\delta = \frac{4\pi l}{\lambda} + \pi$. Also the waves interfere constructively if phase difference is $2n\pi$

$$\frac{4\pi l}{\lambda} + \pi = 2n\pi$$

$$l = (2n-1)\frac{\lambda}{4}$$

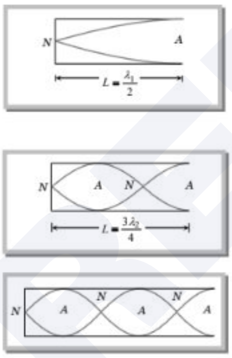
Here n = 1,2,3.... But if we take n = 0,1,2,.... then the above equation can also be written as - $l = (2n-1)\frac{\lambda}{4}$

So, the frequency can be written as - $\nu = \frac{v}{\lambda} = \frac{v.(2n-1)}{4l}$

Equation of standing wave is given by and explained earlier $= y = 2a\cos\frac{2\pi t}{\lambda}\sin\frac{2\pi x}{\lambda}$

As, general formula for wavelength defined earlier $= \lambda = \frac{4L}{(2n-1)}$

The minimum allowed frequency is obtained by putting n=1



(1) First normal mode of vibration : $n_1 = \frac{v}{4L}$

This is called fundamental frequency. The note so produced is called

fundamental note or first harmonic.

(2) Second normal mode of vibration : $n_2 = \frac{v}{\lambda_2} = \frac{3v}{4L} = 3n_1$

This is called third harmonic or frst overtone.

(3) Third normal mode of vibration : $n_3 = \frac{5v}{4L} = 5n_1$

This is called fifth harmonic or second overtone.

Standing Waves in Open Organ Pipes

General formula for wavelength - $\lambda = \frac{2L}{n}$ where $n = 1, 2, 3 \ldots\ldots\ldots$

Then the first normal mode of vibration is - $n_1 = \frac{v}{\lambda_1} = \frac{v}{2L}$

This is called fundamental frequency and the node so produced is called fundamental node or first harmonic.

(2) Second normal mode of vibration $n_2 = \frac{v}{\lambda_2} = \frac{v}{L} = 2\left(\frac{v}{2L}\right) = 2n_1 \Rightarrow n_2 = 2n_1$

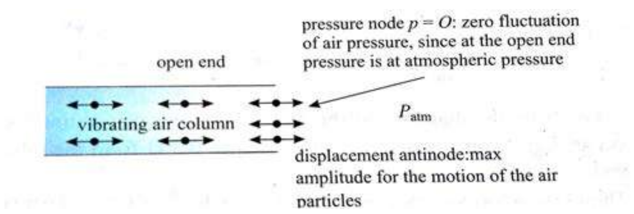
This is called second harmonic or first overtone.

(3) Third normal mode of vibration $n_3 = \frac{v}{\lambda_3} = \frac{3v}{2L}, n_3 = 3n_1$

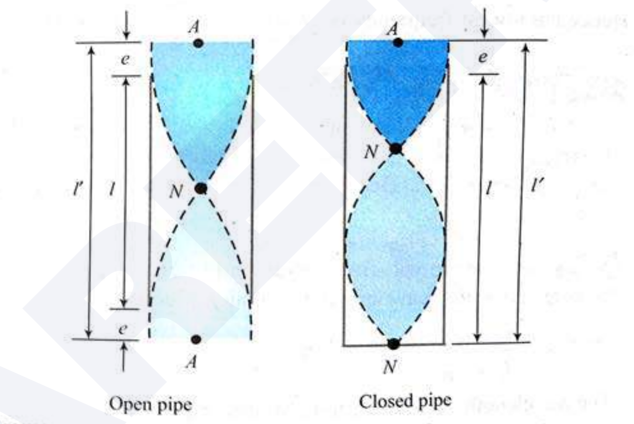
This is called third harmonic or second overtone.

End correction -

In the organ pipe, when the wave reaches the open end, due to collision particle sactters away from the pipe. Due to this the density reduces outside the pipe and form a rarer medium.



So, we can say that the wave is not exactly reflected back from the open end of the pipe. So, we can say that the antinodes will form always little away from the open ends. We can see this in the given figure. So the distance above the open end where an antinode is form is called **end correction.**



This end corrrection varies with radius of pipe and given as = $e = 0.6r$

Now taking the end correction into account, the frequency of a closed pipe of length l can be given as -

$$n_o = \frac{\nu}{4(l+e)} \quad \text{(One end open)}$$

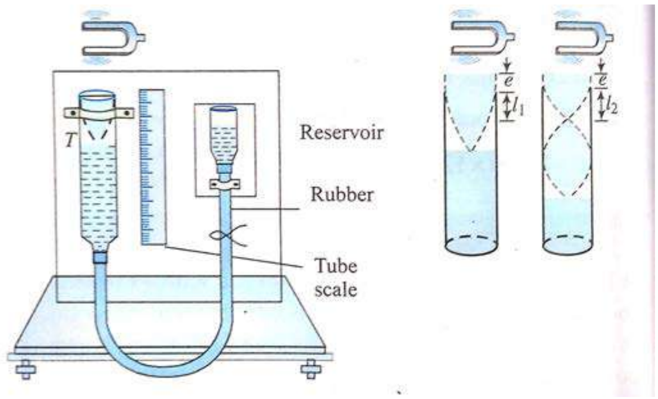
For open pipe -

$$n_o = \frac{\nu}{2(l+2e)} \quad \text{(Both ends open)}$$

Resonance column method

In this the equipment used is resonance tube. This apparatus is used to determine the velocity of sound in air and used to compare frequency of two turning fork.

It is closed organ pipe with variable length of air column. When we brought a turning fork near it, its air column vibrates with the frequency of fork. The length of air column varied until the frequency of fork and the air column become equal. When frequency becomes equal, column resonates and the note become loud.



Let at this position the length of air column is l_1. By further decreasing water level again after some distance maximum intensity of sound is obtained where the node is obtained. Let this level is l_2.

If l_1 *and* l_2 are the length of first and second resonance, then -

$$l_1 + e = \frac{\lambda}{4} \text{ and } l_2 + e = \frac{3\lambda}{4}$$

$$so, \lambda = 2(l_2 - l_1)$$

Speed of sound in air at room temperature $v = n\lambda = 2n(l_2 - l_1)$

Also,

$$\frac{l_2 + e}{l_1 + e} = 3$$

$$\Rightarrow l_2 = 3l_1 + 2e$$

So, the second resonance is obtained at length more than thrice the length of first resonance.

13.Beats

Beats -

When any two sound waves of slightly different frequencies, travelling along the same direction in a medium and superimpose on each other then the intensit of the resultant sound at a particular position rises and falls regularly with time. This phenomenon of regular variation in intensity of sound with time at a particular position is called beats.

If we struck two tuning forks of slightly different frequencies, one hears a sound of periodically varying amplitude. This phenomenon is called beating.

Beat frequency, equals the difference in frequency between the two sources which we will see below.

Let us consider two sound waves travelling through a medium having equal amplitude with slightly different frequencies f_1 and f_2. We use equations similar to equation $y = A\sin(kx - \omega t)$ to represent the wave functions for these two waves at a point such that $kx = \pi/2$:

$$y_1 = A\sin\left(\frac{\pi}{2} - \omega_2 t\right) = A\cos\left(2\pi f_1 t\right)$$

$$y_2 = A\sin\left(\frac{\pi}{2} - \omega_2 t\right) = A\cos\left(2\pi f_2 t\right)$$

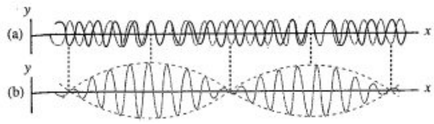
By using superposition principle -

$$y = y_1 + y_2 = A\left(\cos 2\pi f_1 + \cos 2\pi f_2 t\right)$$

We can also write the above equation by using trigonometric identity as -

$$y = \left[2A\cos 2\pi\left(\frac{f_1 - f_2}{2}\right)t\right]\cos 2\pi\left(\frac{f_1 + f_2}{2}\right)t$$

The graph is like this -



Graphs of the individual waves and the resultant wave are shown in the figure. We can see that the resultant wave has effective frequency equal to average frequency $\frac{f_1+f_2}{2}$. From the figure, we can see that this wave is multiplied by the envelope whose equation is given as -

$$y_{\text{envelope}} = 2A\cos 2\pi\left(\frac{f_1-f_2}{2}\right)t$$

A maximum in the amplitude of the resultant sound wave is detected whenever

$$\cos 2\pi\left(\frac{f_1-f_2}{2}\right)t = \pm 1$$

Hence, there are two maxima in each period of the envelope wave. Because the amplitude varies with frequency as $\frac{(f_1-f_2)}{2}$ the beat frequency is two times of this value and given by -

$$f_{\text{beat}} = |f_1 - f_2|$$

14. Doppler Effect

Doppler Effect -

Whenever there is a relative motion between a source of sound and the listener, the apparent frequency/wavelength of sound heard by the listener is different from the actual frequency/wavelength of sound emitted by the source.

When the distance between the source and listener is increasing the apparent frequency decreases. It means the apparent frequency is less than the actual frequency of sound. The reverse of this process is also true.

General expression for apparent frequency $f_{app} = \frac{[(v+v_m)-v_o]f}{[(v+v_m)-v_s]}$

Now, for different conditions the value of apparent frequency will change. Here f = Actual frequency; v_0= Velocity of observer; v_S = Velocity of source, v_M = Velocity of medium and v = Velocity of sound wave

There are some sign convention for the velocities - along the direction Source to Listener are taken as positive and all velocities along the direction Listener to Source are taken as negative.

If the velocity of the medium is zero then the formula become - $f_{app} = \left(\frac{v-v_o}{v-v_s}\right)f$

Now we will discuss some important cases and based on that the formulaes -

(1) Source is moving towards the listener and the listener at rest $f_{app} = \frac{v}{v-v_s}.f$

(2) Source is moving away from the listener and the listener is at rest $f_{app} = \frac{v}{v+v_s}.f$

(3) Source is at rest but listener is moving away from the source $f_{app} = \frac{v-v_L}{v}f$

(4) Source is at rest but the listener is moving towards the source $f_{app} = \frac{v+v_L}{v}f$

(5) When the Source and listener are approaching each other $f_{app} = \left(\frac{v+v_L}{v-v_S}\right)f$

(6) When the Source and listener moving away from each other $f_{app} = \left(\frac{v-v_L}{v+v_s}\right)f$

Note - Source and listener moves perpendicular to the direction of wave propagation i.e., f_{app} = f. It means there is no change in frequency of sound heard for the small displacement of source and listener at right angle to the direction of wave propagation but this is not true for large displacement. For a large displacement the frequency decreases because the distance between source of sound and listener increases.

Electric Charges and Fields

Important Formulae

1. Electric Charge

Electric charge:

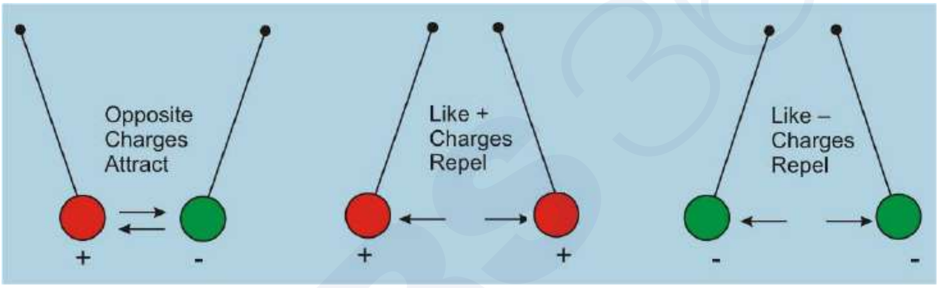
The charge is the property associated with the matter due to which it produces and experiences the electrical and magnetic effects.

The electric charge are of two types, namely positive charge and negative charge. A body is called to be positively charged when the body loses electrons and the negatively charged when the body gains electrons.

Electric charge is a scalar quantity and its SI unit is **Coulomb** represented as **C**. C= 1 ampere-second. The dimension of charge is **[AT]**.

Like charges repel each other (glass rods rubbed with wool or silk repel each other) and unlike charges attract each other (glass rod and wool attract each other).

Point charge: If the size of charged bodies is too small compared to the distance between them we treat them as a point charge.



Properties of charge:

1. **Additivity**: If a system contains n charge $q_1, q_2, q_3 \ldots\ldots q_n$, then the total charge of the system is $q_1 + q_2 + \ldots.. q_n$.
2. **The charge is conserved**: The charge can be neither created nor destroyed. When we rub a glass rod with silk there is a transfer of charge and not creation. The total charge of an isolated system is always conserved.
3. The charge is independent of the velocity of the particle. I.e The charge is non-relativistic.
4. **Quantization**: The charge on a body will be some integral multiple of e, where e is the charge of the electron.

   $e = 1.6 \times 10^{-19} C$

Conductors and insulators:

The materials which allow the passage of electricity are known as conductors and the materials which do not allow the passage of electricity are known as insulators.

Methods of charging

1. **By Friction:** When two bodies rub together both positive and negative charges in equal amounts appear simultaneously due to the transfer of electrons. When a glass rod is rubbed with a silk cloth, the electrons are transferred from the glass rod to the silk. The glass rod becomes positively charged and the silk rod becomes negatively charged.
2. **By induction:** When a charged body is brought near an uncharged body, one side of the neutral body becomes oppositely charged while the other side has the same charge. For example, when a positively charged glass rod is brought near a paper the paper gets attracted, This is because the rod attracts the electrons of paper towards it so that the edge of the paper near the rod becomes negatively charged and the other end becomes positively charged due to deficiency of electrons.
3. **By conduction:** When two conductors are brought in contact, the charges will spread over both the conductors. For example, when a negatively charged plastic rod is brought in contact with a neutral pith ball some of the electrons of the rod are transferred to the pith ball and the pith ball also becomes negatively charged.

2. Coulomb's Law

Coulomb's Law: The force of attraction or repulsion between two charges is directly proportional to the product of the two charges and inversely proportional to the square of the distance between them.

$$F \propto \frac{Q_1 Q_2}{r^2}$$

$$F = \frac{K Q_1 Q_2}{r^2}$$

K = Proportionality Constant

Q_1 and Q_2 are two-point charges

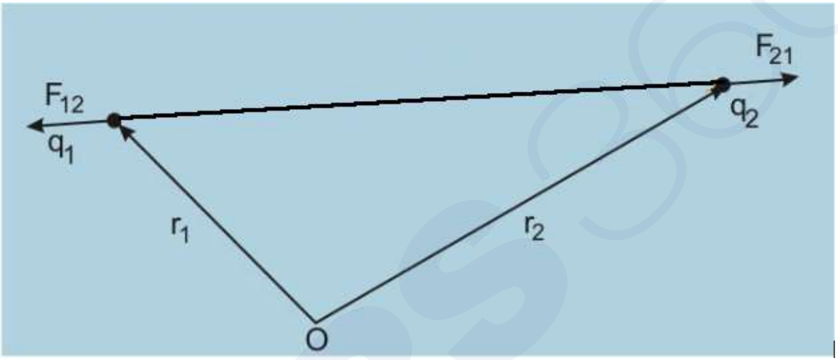
In SI unit value of K is

$$K = \frac{1}{4\pi\varepsilon_0}$$

Where,

$(\varepsilon_0) = 8.85 \times 10^{-12} \frac{C^2}{N-m^2}$ known as **absolute permittivity of air or free space**

The vector form of Coulomb's Law:



Consider two charges q_1 and q_2 separated by a distance r. Let the position vectors of q_1 be r_1 and that of q_2 be r_2. Then the force due to q_2 on q_1 as shown in figure F_{12} is directed along the unit vector r_{12} and

$$F_{12} = \frac{Kq_1q_2}{r^2}\hat{r}_{12}$$

$$here,\ \hat{r}_{12} = \frac{\vec{r}_1 - \vec{r}_2}{|r_1 - r_2|} = \frac{\vec{r}_{12}}{r}$$

$$F_{12} = \frac{Kq_1.q_2}{r^3}\vec{r}_{12}$$

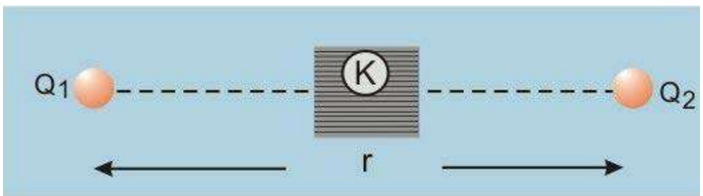
Force when the dielectric is inserted between the charges:

When a dielectric of dielectric constant k is completely filled between the charges then force

$$F_{med} = \frac{q_1q_2}{4\pi\varepsilon_0 kr^2} = \frac{q_1q_2}{4\pi\varepsilon_0\epsilon_r r^2}$$

ϵ_r is the relative permittivity / dielectric constant of the medium. The dielectric constant is the ratio of the permittivity of a substance to the permittivity of free space.

If the dielectric of thickness d is partially filled between the charges $Q_1 \quad and \quad Q_2$ then



$$F = \frac{Q_1Q_2}{4\pi\epsilon_0(r - d + \sqrt{k}d)^2}$$

Principle of Superposition:

It states that the total force acting on a given charge due to a number of charges is the Vector sum of the individual forces acting on that charge due to all the charges.

3. Electric Field

Electric field :

The space around a charge in which another charged particle experiences a force is said to have an electric field in it.

Electric Field Intensity:

The electric field intensity at any point is defined as the force experienced by a unit positive charge placed at that point.

i.e

$$E = \frac{F}{q_0}$$

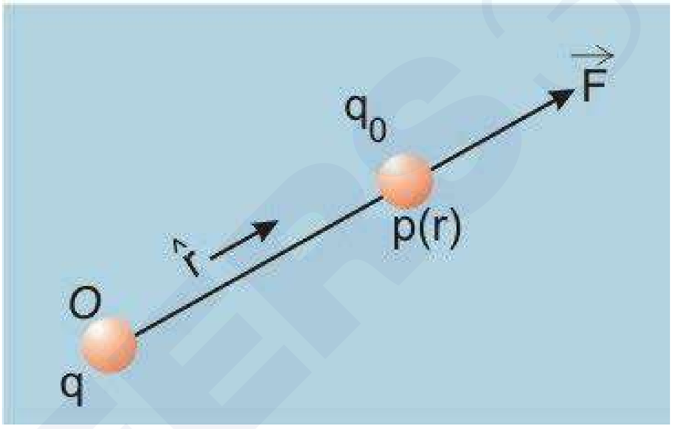
where F is the force experienced by q_0. The SI unit of E is,

$$\frac{Newton}{Columb} = \frac{Volt}{meter} = \frac{Joule}{Coulomb \times Meter}$$

The dimensional formula is $[MLT^{-3}A^{-1}]$

The electric field is a vector quantity and due to the positive charge is away from the charge and for the negative charge, it is towards the charge.

Electric field due to a point charge:



Consider a point charge placed at the origin O. Let a test charge q_0 is placed at P which is at a distance r from O. Force F on test charge q_0 is

$$F = \frac{1}{4\pi\epsilon_0}\frac{qq_0}{r^2}\hat{r}$$

The electric field at point P due to q is

$$E = \lim_{q_0 \to 0}\frac{F}{q_0} = \lim_{q_0 \to 0}\frac{1}{q_0}\frac{1}{4\pi\epsilon_0}\frac{qq_0}{r^2}\hat{r} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$$

(As q_o tends to zero the electric field produced by q is not affected by q_o.)

The magnitude of the electric field

$$E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$$

Electric field due to a system of charge:

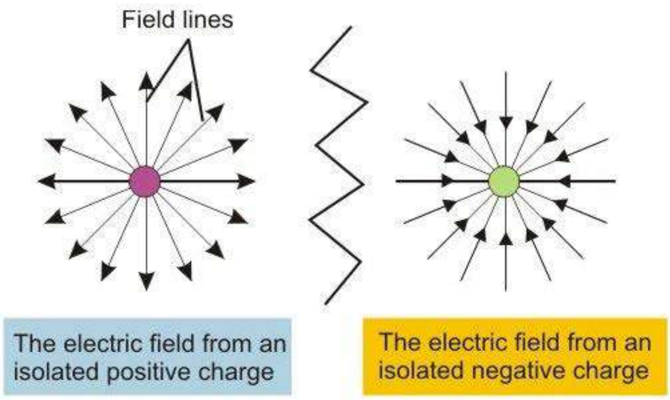
The electric field obeys the superposition principle. That is the electric field due to a system of charge at a point is equal to the vector sum of all the electric fields.

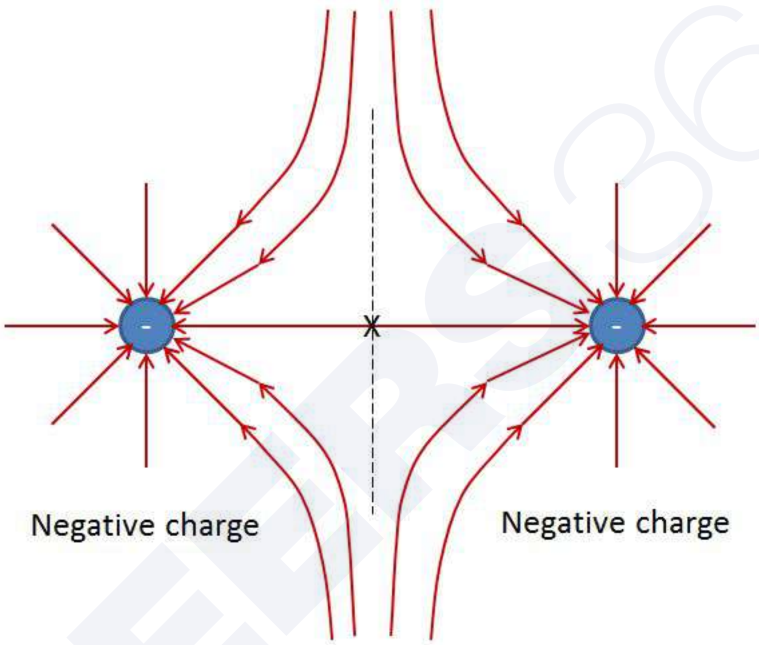
Electric lines of force:

An electric field line is an imaginary line or curve drawn through a region of space so that its tangent at any point is in the direction of the electric field at that point.

Properties of Electric lines of force (Electric field lines):

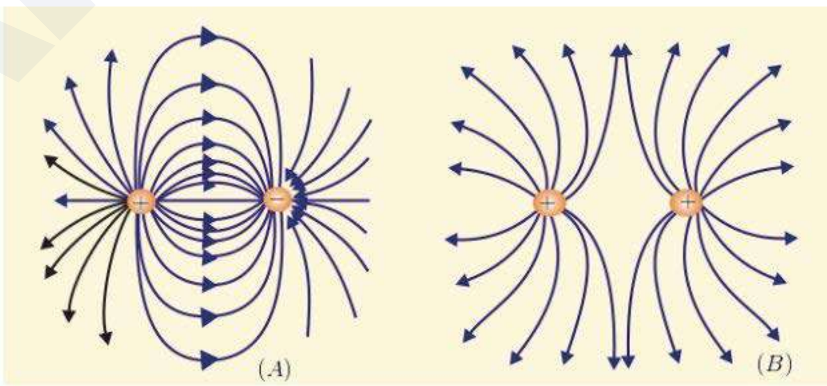
i) Electric lines of force diverge out from the positive charge and converge on the negative charge





ii) The number of field lines is proportional to the magnitude of the charge

iii) Lines of force never cross or intersect each other. If they intersect at any point then at that point electric field intensity will have two directions which is not possible.



iv) Electric field lines do not form a closed loop. Since electric field lines cannot start and end on the same charge.

v) The tangent at a point on electric field lines will give the direction of the force on a positively charged particle placed at that point.

vi) Electric field lines do not give the path of motion of the particle. It may show the path if the electric field lines are straight.

vii) Electric field lines are normal to the conductor if it is starting from a conductor or while ending on the conductor.

viii) The uniform electric field is represented by straight, parallel, and equidistant lines.

ix) The electric field does not exist inside a conductor.

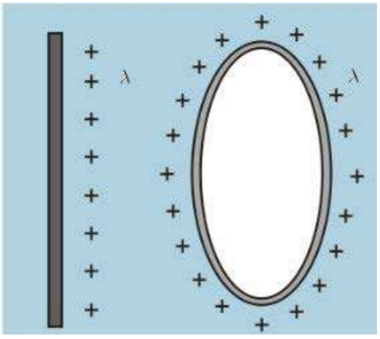
Discrete charge distribution: A system consisting of many individual charges.

Continuous charge distribution: An amount of charge distributed uniformly or non-uniformly on a body. It is of three types -

1. Linear charge distribution:

$(\lambda)-$ charge per unit length.

$\lambda=\frac{q}{L}=\frac{C}{m}=Cm^{-1}$

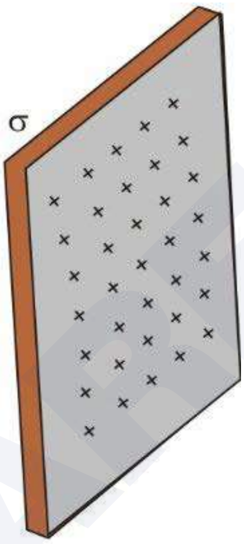


Example: wire, circulating ring

2. Surface charge distribution:

$(\sigma)-$ charge per unit Area

$\sigma=\frac{Q}{A}=\frac{C}{m^2}=Cm^{-2}$

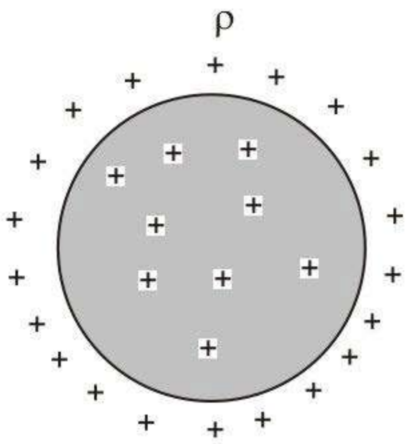


Example: plane sheet

3. Volume Charge distribution

$(\rho)-$ charge per unit volume.

$\rho=\frac{Q}{V}=\frac{C}{m^3}=Cm^{-3}$

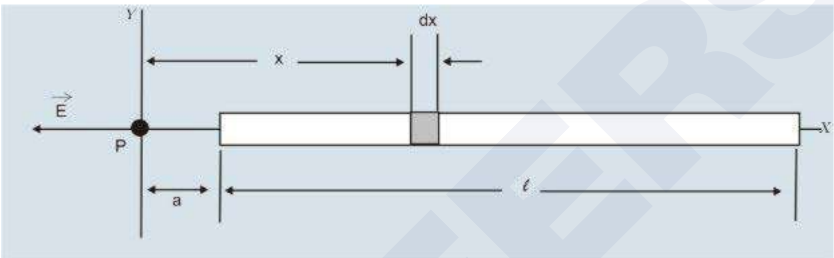


Example - charge on a dielectric sphere etc.

4. Electric Field Intensity due to various charged bodies

1. Electric field due to uniformly charged rod

So, let us consider a rod of length l which has uniformly positive charge per unit length lying on x-axis, dx is the length of one small section. This rod is having a total charge Q and dq is the charge on dx segment. The charge per unit length of the rod is λ. We have to calculate the electric field at a point P which is located along the axis of the rod at a distance of 'a' from the nearest end of Rod as shown in the figure -

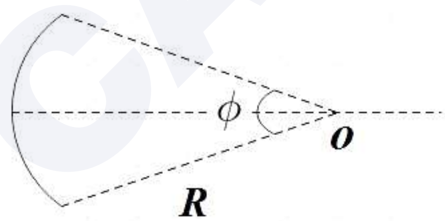


$$E = \frac{kQ}{a(l+a)}$$

Now if we slide the rod toward the origin and the $a \rightarrow 0$, then due to that end, the electric field is infinite.

2. Electric field strength due to a charged circular arc at its centre

E= The electric field at the centre of an arc of linear charge density λ, radius R subtending angle ϕ at the centre.

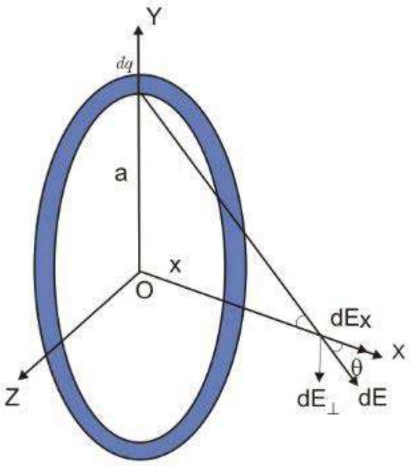


If Q is the total charge contained in the arc then,

$$\lambda = \frac{Q}{R\phi}$$

$$\therefore E = \frac{2\lambda}{4\pi\varepsilon_0 R}\sin\frac{\phi}{2}$$

3. Electric field on the axis of a charged ring-



In this one should notice that there is symmetry in this situation. Every element dq can be paired with a similar element on the opposite side of the ring.

Every component $d\vec{E}$ perpendicular to the x-axis is thus cancelled by a component $d\vec{E}$ in the opposite direction.
In the summation process, all the perpendicular components $d\vec{E}$ add to zero.

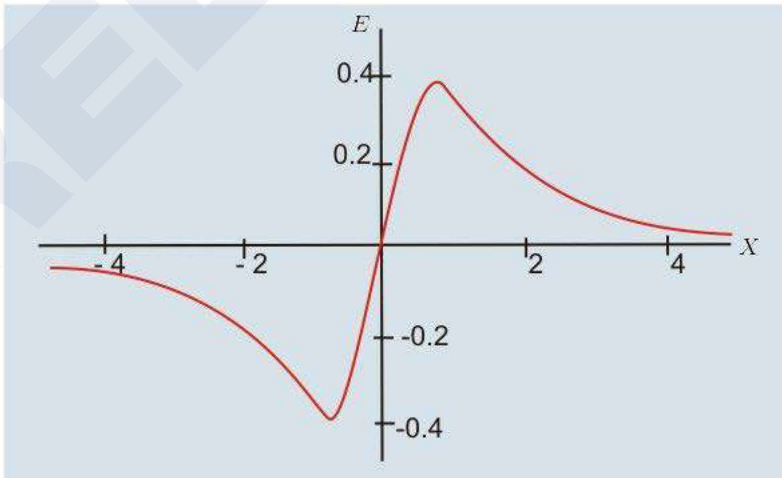
Thus we only add the dE_x components, which all lie along the +X direction

Also, $\int dq = Q$

The net electric field is -

$$E_{net} = \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{xQ}{(x^2+R^2)^{3/2}}$$

The graph between E and X -

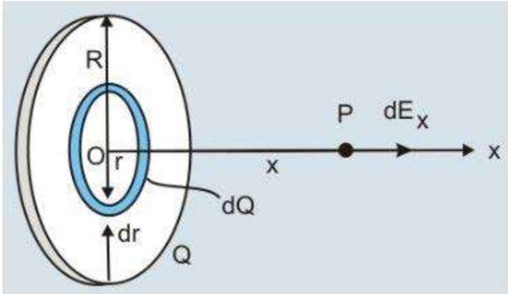


If, $x = \pm\frac{R}{\sqrt{2}}$

$$E_{max} = \frac{Q}{6\sqrt{3}\pi\varepsilon_0 R^2},$$

4. Electric field due to uniformly charged disk

Let us take a disk of radius R with a uniform positive surface charge density (charge per unit area) σ . E= The electric field at a point on the axis of the disk at a distance x from its centre.



$$E_x = \frac{\sigma x}{2\varepsilon_0}\left[-\frac{1}{\sqrt{x^2+R^2}}+\frac{1}{x}\right] = \frac{\sigma}{2\varepsilon_0}\left[1-\frac{x}{\sqrt{x^2+R^2}}\right]$$

As this disc is symmetric to x-axis, so the field in the rest of the component is zero i.e., $E_y = E_z = 0$

Special case -

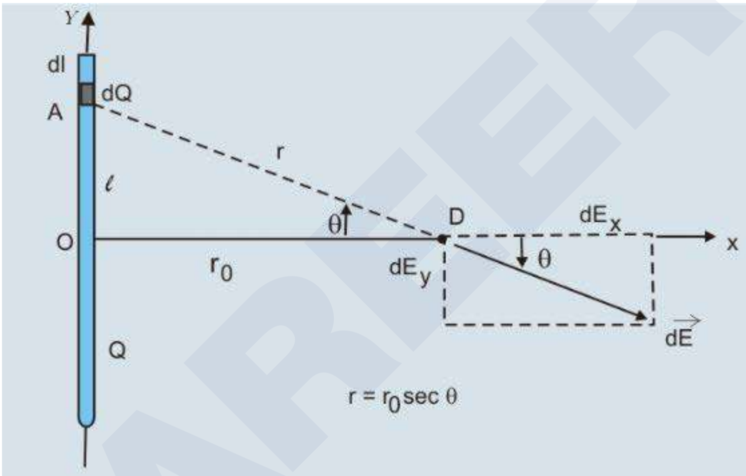
1) When $R >> x$**, then** $E_x = \frac{\sigma}{2\varepsilon_0}$ Note that this equation is independent of 'x'

2) When $x \to 0$ (i.e very near to disc), then $E_x = \frac{\sigma}{2\varepsilon_0}$

5.Electric field due to an infinite line charge -

Let us assume that positive electric charge Q is distributed uniformly along a line, lying along the y-axis.

E= The electric field at point D on the x-axis at a distance r_0 from the origin.



If the wire has finite length and the angles subtended by ends of wire at a point are θ_1 and θ_2
Then

$$E_x = \int_{-\theta_1}^{\theta_2} \frac{\lambda \cos\theta d\theta}{4\pi\varepsilon_0 r_0}$$

$$= \frac{\lambda}{4\pi\varepsilon_0 r_0}(\sin\theta_1 + \sin\theta_2)$$

$$E_y = \int_{-\theta_1}^{+\theta_2} \frac{\lambda \sin\theta d\theta}{4\pi\varepsilon_0 r_0}$$

$$= \frac{\lambda}{4\pi\varepsilon_0 r_0}(\cos\theta_1 - \cos\theta_2)$$

Special case-

1. If the line is infinite then the $\theta_1 = \theta_2 = 90^o$

Putting the value, we get -

$$E_x = \frac{\lambda}{2\pi\varepsilon_o r_o}$$

$$E_y = 0$$

2. If the line is semi-infinite then the $\theta_1 = 0, and\ \theta_2 = 90^o$

Putting the value, we get -

$$E_x = \frac{\lambda}{4\pi\varepsilon_o r_o}$$

$$E_y = \frac{\lambda}{4\pi\varepsilon_o r_o}$$

5. Motion of a charged particle in the uniform electric field

Whenever a charge is placed in an electric field, it will experience an electric force. There is an assumption that this whole system is placed in a gravity-free space.

For this condition, electrical force is the only force acting on the particle.

This net force will cause the particle to accelerate according to Newton's second law of motion.

So we can write -

$$\vec{F}_e = q\vec{E} = m\vec{a}$$

Acceleration will be constant if the Electric field is uniform and $\vec{a} = \frac{q\vec{E}}{m}$.

The direction of acceleration or motion of a charged particle depends on its nature.

If the charged particle is of a positive nature then it will move or accelerate in the direction of the electric field.

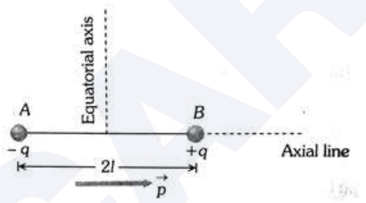
But in the case of a negatively charged particle, its motion or acceleration is in the opposite direction of the electric field.

Here we can use the kinematic equation of motion since the acceleration is constant.

6. Electric Dipole

An **electric dipole** is a system of two equal and opposite point charges separated by a very small and finite distance.

Below is the figure showing an electric dipole consisting of two equal and opposite point charges $-q\ \ and\ \ +q$ separated by a small distance $2l$.



Dipole moment- The strength of an electric dipole is measured by a vector quantity known as the electric dipole moment.

Its magnitude is equal to the product of the magnitude of either charge and the distance between the two charges,

i.e. for the dipole, as shown in the above figure dipole moment is given as

$$\left(\vec{P}\right) = q\left(2\vec{l}\right)$$

And its direction is along the line from –q to +q.

Its S.I unit is C-m

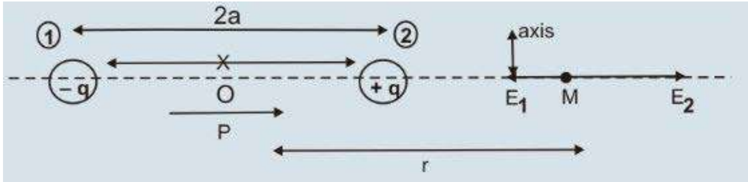
and its cgs unit is Debye ($1\ Debye = 3.3 \times 10^{-30} C - m$)

The axial line of dipole- A line passing through the negative and positive charges of the electric dipole is called the axial line of the electric dipole.

Centre of dipole- The midpoint of the line joining the two charges is called the centre of the dipole.

Equatorial line- An **equatorial line** of a **dipole** is the **line** perpendicular to the **axis of the dipole** and passing through the **Centre of the dipole.**

Electric Field Intensity due to an Electric Dipole at a Point on the Axial Line-



E=Electric Field Intensity due to an Electric Dipole at a Point M which is on axial line and at a distance r from the centre of a dipole.

Where $E_1 \quad and \quad E_2$ is the Electric Field Intensity at M due to $-q \quad and \quad +q$ charges respectively.

The intensities $E_1 \quad and \quad E_2$ are along the same line but in opposite directions.

$E_1 = \frac{kq}{(r+a)^2}$

$E_2 = \frac{kq}{(r-a)^2}$

$E_{net} = E_2 - E_1$

$$E_{net} = \frac{Kq}{(r-a)^2} - \frac{Kq}{(r+a)^2} \quad = \left[\frac{4Kqar}{(r^2-a^2)^2}\right]$$

Using $P = q(2a)$

So $$E_{net} = \left[\frac{2KPr}{(r^2-a^2)^2}\right]$$

- For short/Ideal dipole (i.e. r>>a)

then $$\vec{E}_{net} = \frac{2K\vec{P}}{r^3} = \frac{2\vec{P}}{4\pi\epsilon_0 r^3}$$ (This is the value of E_{net} when the dipole is placed in the vacuum.)

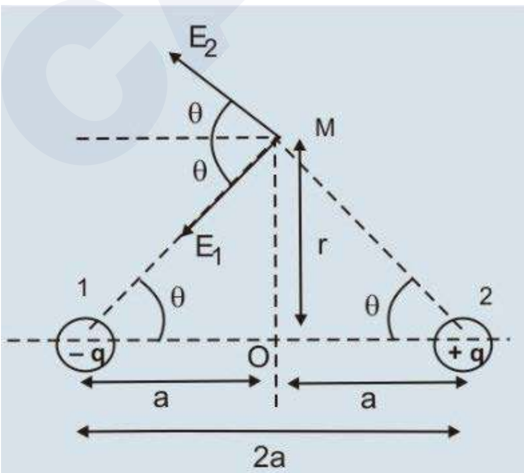
If the dipole is placed in the medium having the permittivity as ϵ_m

Then $$\vec{E}_{net} = \frac{2\vec{P}}{4\pi\epsilon_m r^3} = \frac{2\vec{P}}{4\pi\epsilon_0\epsilon_r r^3}$$

Note: The direction of the electric field E is in the direction of $\vec{P}$.

i.e Angle between E_{axi} and $\vec{P}$ is 0^0.

Electric Field Intensity due to an Electric Dipole at a Point on the Equatorial line-



E=Electric Field Intensity due to an Electric Dipole at a Point M which is on the Equatorial line and at a distance r from the center of a dipole.

Where $E_1 \quad and \quad E_2$ is the Electric Field Intensity at M due to $-q \quad and \quad +q$ charges respectively.

$$|\overrightarrow{E_1}| = \frac{1}{4\pi\epsilon_0} \times \frac{q}{r^2 + a^2}$$

$$|\overrightarrow{E_2}| = \frac{1}{4\pi\epsilon_0} \times \frac{q}{r^2 + a^2}$$

So $|\overrightarrow{E_1}| = |\overrightarrow{E_2}| = |\overrightarrow{E}|$

$$\begin{aligned}|\vec{E}| &= 2|E_1|\cos\theta \\ &= \frac{2}{4\pi\epsilon_0} \cdot \frac{q}{(r^2 + a^2)}\cos\theta \\ &= \frac{2}{4\pi\epsilon_0} \cdot \frac{q}{(r^2 + a^2)} \frac{a}{\sqrt{r^2 + a^2}} \\ &= \frac{q \times 2a}{4\pi\epsilon_0 \left(r^2 + a^2\right)^{3/2}}\end{aligned}$$

Using $P = q(2a)$

$$\therefore \vec{E} = \frac{-\vec{P}}{4\pi\epsilon_0 \left(r^2 + a^2\right)^{3/2}}$$

And

- For short/Ideal dipole (i.e. r>>a)

  then $\vec{E}_{net} = \frac{-K\vec{P}}{r^3} = \frac{-\vec{P}}{4\pi\epsilon_0 r^3}$ (This is the value of E_{net} when the dipole is placed in the vacuum.)

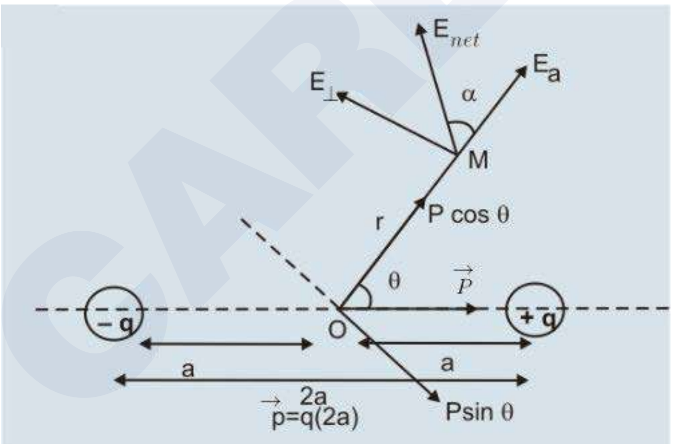
  If the dipole is placed in the medium having the permittivity as ϵ_m

  Then $\vec{E}_{net} = \frac{-\vec{P}}{4\pi\epsilon_m r^3} = \frac{-\vec{P}}{4\pi\epsilon_0\epsilon_r r^3}$

Note: Here the direction of the electric field E is opposite to the direction of $\vec{P}$.

i.e Angle between E_{equi} and $\vec{P}$ is 180^0.

Electric field due to a dipole at any general point-



E=Electric Field Intensity due to an Electric Dipole at a Point M which at a distance r from the center of a dipole and making an angle θ with the axial line.

From the figure, M is at the axial line of dipole having dipole moment as $Pcos\theta$ and M is at the Equatorial line of dipole having dipole moment as $Psin\theta$

if r>>a

Then $E_a = \frac{1}{4\pi\varepsilon_0} \times \frac{2P\cos\theta}{r^3}$ and $E_\perp = \frac{1}{4\pi\varepsilon_0} \times \frac{P\sin\theta}{r^3}$

$$\left|\vec{E}_{net}\right| = \sqrt{E_a^2 + E_\perp^2}$$

$$|\vec{E}_{net}| = \sqrt{\left\{\frac{2KP\cos\theta}{r^3}\right\}^2 + \left\{\frac{KP\sin\theta}{r^3}\right\}^2}$$

$$= \sqrt{\left(\frac{KP}{(r^3}\right)^2\{4\cos^2\theta + \sin^2\theta\}}$$

$$= \frac{KP}{r^3}\sqrt{1+3\cos^2\theta}$$

- Let make an angle α with E_a

then $\tan\alpha = \frac{E_\perp}{E_a} = \frac{sin\theta}{2\cos\theta} = \frac{1}{2}\tan\theta$

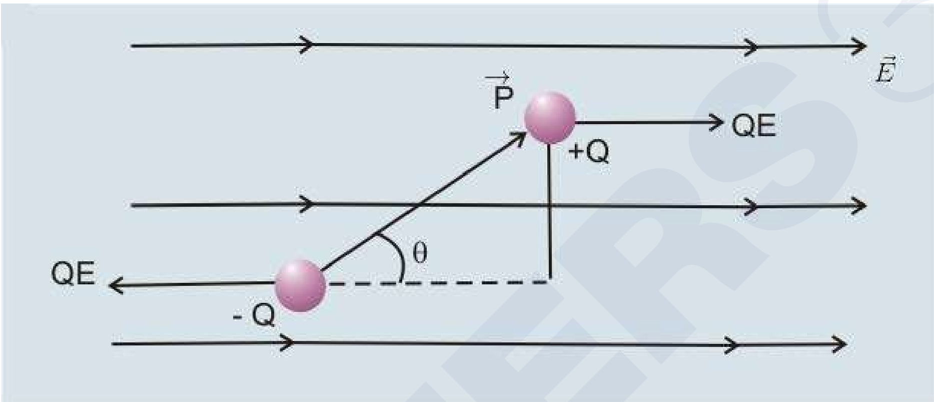
Note- The above results are valid only for short/Ideal dipole.

7. Dipole in Uniform electric field

Net Force-

When a dipole is kept in a uniform electric field. The net force experienced by the dipole is zero as shown in the below figure.

I.e $F_{net} = 0$



Hence dipole will not make any linear motion.

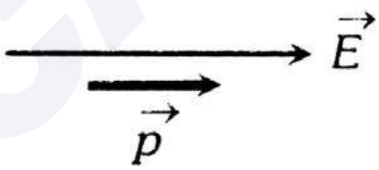
Torque on dipole-

Net torque about the center of the dipole is given as $\tau = QEd\sin\theta$

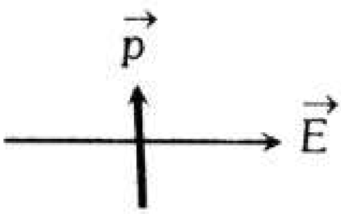
Using $P = Qd$ we get $\tau = PE\sin\theta$

So $\vec{\tau} = \vec{P} \times \vec{E}$

- The direction of the torque is normal to the plane containing dipole moment P and electric field E and is governed by right-hand screw rule.
- If Dipole is parallel to E the torque is Zero. I.e $\Theta = 0°$ $\tau = 0$ (This is the position of **stable equilibrium** of dipole)



- Torque is maximum when Dipole is perpendicular to E. I.e $\Theta = \frac{\pi}{2}$ $\tau = PE = maximum \quad torque$



Oscillation of dipole -If a dipole experiencing a torque in an electric field is allowed to rotate, then it will rotate to align itself to the Electric field. But when it reaches along the direction of E the torque becomes zero. But due to inertia, it overshoots this equilibrium condition and then starts oscillating about this mean position.

The time period of this oscillation is given as

$$T = 2\pi\sqrt{\frac{I}{PE}}$$

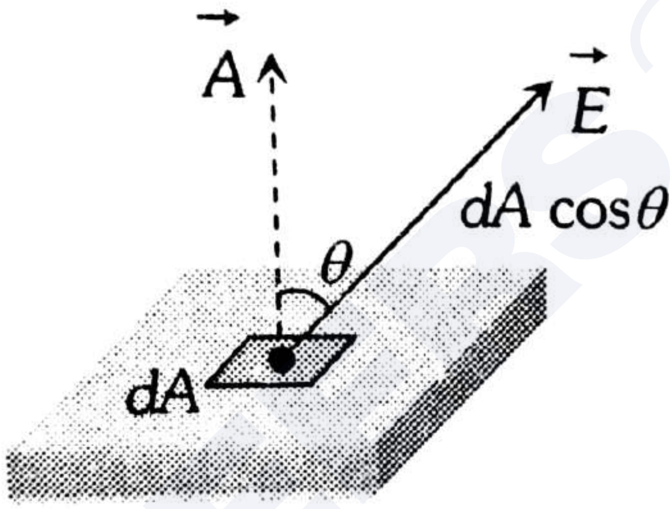
where I= moment of inertia of **dipole** about the axis passing through its centre and perpendicular to its length.

Dipole in Non-Uniform Electric Field- In case the Electric field is non-uniform, the magnitude of the force on +q and –q will be different. So $F_(net) \neq 0$ and At the same time due to a couple of forces acting, a torque will also be acting on it.

8. Electric Flux

Electric flux (ϕ):

The electric flux through an area is the number of electric field lines passing normally through the area.



Flux through an area dA is given by

$$d\phi = \vec{E}.\vec{dA} = EdAcos\theta$$

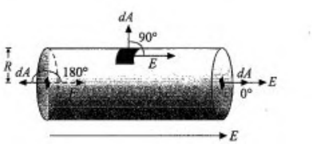
here, θ is the angle between the area vector and the electric field.

- Total flux through area A is $\phi = \int \vec{E}.\vec{d}A$
- Flux is a scalar quantity so they can be added algebraically.
- The SI unit of flux is volt-meter

<u>**Electric flux through a closed surface -**</u>

Consider a cylindrical surface of radius R, length l, in a uniform electric field E. The axis of the cylinder is parallel to the field direction. We can divide the entire surface into three parts, right and left plane faces and curved portion of its surface. Hence, the surface integral consists of the sum of the three terms:

$$\phi_E = \oint E \cdot dA = \oint_{\text{left end}} E \cdot dA + \oint_{\text{right end}} E \cdot dA + \oint_{\text{curved}} E \cdot dA$$



Here, left end and electric field is making 180° and the right end and the electric field is making 0° . Also one can notice that the curved surface is making 90° with the direction of electric field. So,

$$(\phi_E)_{\text{left end}} = \oint_{\text{left end}} \vec{E} \cdot d\vec{A} = \oint_{\text{left end}} \vec{E} d\vec{A} \cos 180^\circ = -E\pi R^2$$

$$(\phi_E)_{\text{right end}} = \oint_{\text{right end}} \vec{E} \cdot d\vec{A} = \oint_{\text{right end}} \vec{E} d\vec{A} \cos 0^\circ = E\pi R^2$$

$$(\phi_E)_{\text{curved}} = \oint_{\text{curved surfice}} \vec{E} \cdot d\vec{A} = \oint_{\text{curved surface}} E dA (\cos 90^\circ) = 0$$

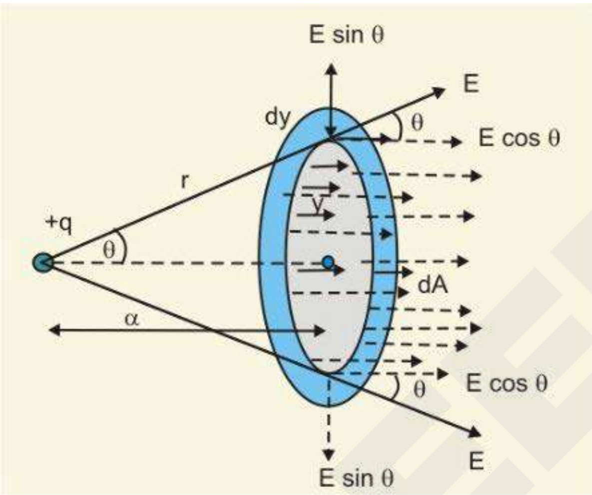
$$\text{Total flux} = (\phi_E)_{\text{right end}} + (\phi_L)_{\text{left end}} + (\phi_E)_{\text{curved surface}}$$

$$= (+E\pi R^2) + (-E\pi R^2) + 0 = 0$$

Similarly, we can find the electric flux through any closed surface by an electric field.

Electric flux through cone or disc-

Consider a point charge q at a distance 'a' from a disc of radius R as shown in the given figure.



The total flux can be given as -

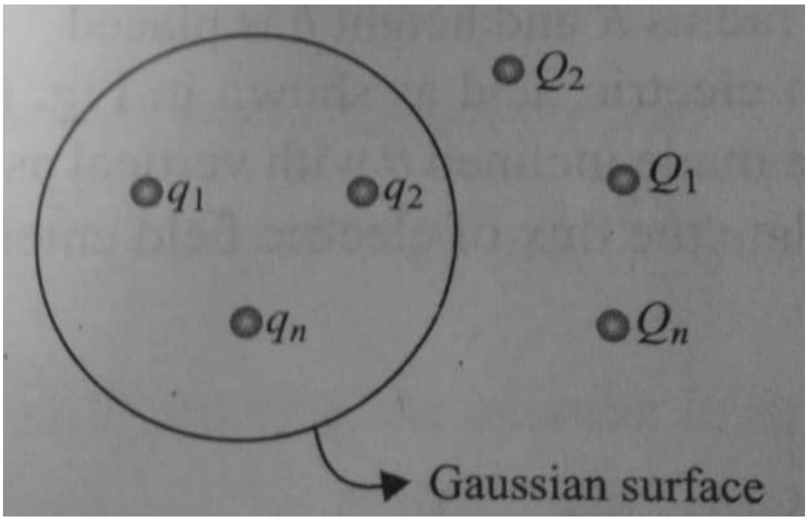
$$\phi = \frac{q}{2\varepsilon_0}\left(1 - \frac{a}{\sqrt{a^2+R^2}}\right)$$

9. Gauss Law and its Application

Gauss's law:

Gauss's law states that the net flux of an electric field in a closed surface is directly proportional to the enclosed electric charge.

The surface on which Gauss's law is applied is called Gaussian Surface (as shown in the below figure).



Note- Remember that the closed surface in Gauss's law is imaginary. There need not be any material object at the position of the surface.

Gauss's law for a closed surface states that :

$\phi_{\text{closed}} = \frac{q_{net}}{\epsilon_0}$, q_{net} is the total charge inside the closed surface. The closed surface on which we apply Gauss law is called the Gaussian surface.

Also, the flux can be written in the integral form as:

$\phi = \int \bar{E} \dot{d} \bar{S}$ S is the area enclosed and E is the electric field intensity passing through it.

<u>The usefulness of Gauss's law :</u>

1. Gauss's law is useful when the Gaussian surface has symmetry about the charge.

2. We can take any Gaussian surface but the Gaussian surface should not pass through the charge. iI can pass through the charge but it can pass through continuous charge distribution.

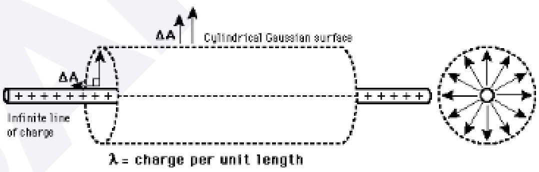
The limitation of Gauss law-

Gauss law is applicable to certain symmetrical shapes it cannot be used for disks and rings and an electric dipole etc.

Applications of Gauss Law (to find the electric field due to various charged bodies):

Electric field due to infinite linear charge :

Consider a Gaussian surface in the form of a cylinder at radius r, the electric field has the same magnitude at every point of the cylinder and is directed outward.



Here the Gaussian surface will be the cylinder around the linear charge of length l .

Gauss's law :

$$\phi = \oint E \cdot dA = \frac{Q_{inside}}{\epsilon_0}$$

Here cylinder has 3 surfaces: 1. Upper 2. lower 3. Curved

$$\phi_{cylinder} = \phi_1 + \phi_2 + \phi_3$$

$$= 0 + 0 + \int EdA \cos 0$$

$$\implies \phi = \int EdA = \frac{Q_{inside}}{\epsilon_0}$$

$$\implies \phi = E \int dA = \frac{Q_{inside}}{\epsilon_0}$$

$$\implies \phi = E(2\pi rl) = \frac{Q_{inside}}{\epsilon_0}$$

$$\implies E = (2\pi rl) = \frac{\lambda l}{2\pi a \times l \times \epsilon_0} \text{ since, } \lambda = \frac{Q_{inside}}{l}$$

$$\implies E = \frac{\lambda}{2\pi \epsilon r}$$

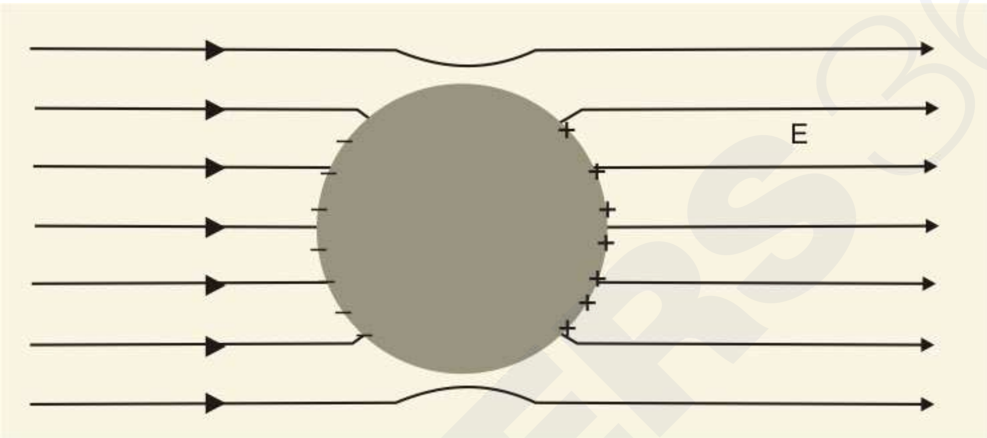
Therefore E is inversely proportional to r.

The electric field in a conductor:

Conductors have a large number of free electrons that are free to move inside the conductors but metal ions are fixed. Now if we place this conductor in an electric field, due to the electric field, electrons will experience a force.

The total electric field at any point in the conductor is the vector sum of the original electric field and the electric field due to the redistributed charged particles. Since they are oppositely directed, the two contributions to the electric field inside the conductor tend to cancel each other.

The electric field is zero at all points inside the conductor, and, while the total charge is still zero, the charge has been redistributed as in the following diagram:



These charges which are appearing on the surface are called induced charges. Due to these induced charges, the electric field will be produced and that is the induced electric field.

The direction of this electric field will be from positive to negative.

$E_{net} = E_{in} + E_{ext}$

and $E_{net} = 0$ inside the conductor.

now if we make a gaussian surface inside the conductor

We know that $E_{inside} = 0$

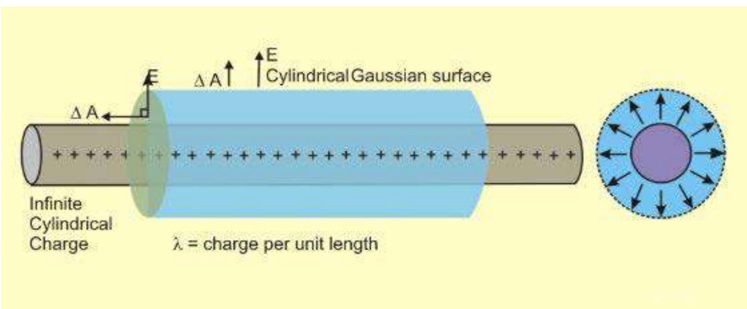
Therefore, $\phi = \oint E \cdot ds = \frac{q_{inside}}{\epsilon_0} = 0$

Hence, $q_{inside} = 0$ charge inside the conductor zero.

Electric field due to cylinders :

1. Solid conducting/ hollow conducting

Consider a Gaussian surface in the form of a cylinder at radius **r > R**, the electric field has the same magnitude at every point of the cylinder and is directed outward.



when the charge is on the surface we need to take account of the surface charge density.

$\sigma = \frac{\text{charge}}{\text{area}}$

For a uniformly charged cylinder of radius, R. We have to consider these three areas where

a) Inside the cylinder (r<R)

b) At the surface (r=R)

c) outside the cylinder (r>R).

(a). Inside (r<R)

E =0

(b). Outside (r>R)

$E = \frac{\sigma R}{\epsilon_0 r}$ or $E = \frac{\lambda}{2\pi\varepsilon_0 r}$

(c). At the surface (r=R)

$E = \frac{\sigma}{\epsilon_0}$ or $E = \frac{\lambda}{2\pi\varepsilon_0 R}$

2. Solid non-conducting

In the case of a solid non-conducting cylinder, the charge is not only on the surface but also distributed through the whole volume. Therefore, volume charge density, $\rho = \frac{\text{charge}}{\text{volume}}$

(a). Inside (r<R)

$E = \frac{\rho r}{2\epsilon_0}$ or $E = \frac{\lambda r}{2\pi\varepsilon_0 R^2}$

(b). Outside (r>R)

$E = \frac{\rho R^2}{2\epsilon_0 r}$

(c). At the surface (r=R)

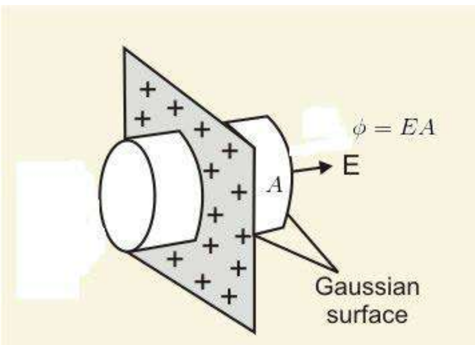
$E = \frac{\rho R}{2\epsilon_0}(\text{max.})$

FIeld of an infinite plane sheet -

Consider a thin, flat, infinite sheet which consists of uniform positive charge per unit area σ.

We can see that there is symmetry in this lamina. So, to take advantage of these symmetry properties,

we use a cylinder as our Gaussian surface whose axis is perpendicular to the sheet of charge, with ends of area A .



We can also observe that the charged sheet passes through the middle of the cylinder's length and because of this flux through each end is EA. This is because $\vec{E}$ is perpendicular to the charged sheet and parallel to the area vector of the flat face. The $\vec{E}$ is along the curved surface i.e perpendicular to the area vector so, the flux will be zero through this. Then the total flux will be 2EA. Now the net charge within the Gaussian surface can be calculated as -

$$Q_{enclosed} = \sigma A$$

So we can write that by Gauss's law -

$$2EA = \frac{\sigma A}{\varepsilon_o}$$

$$E = \frac{\sigma}{2\varepsilon_o}$$

If the charge is negative, $\vec{E}$ will be toward the sheet.

Electric field due to uniform charged sphere-

The sphere may be hollow or solid and both hollow and solid, spheres may be conducting or non-conducting. So let us know the electric field due to all these cases -

In the case of a conducting sphere, the whole charge will come on the surface of the sphere but when the sphere is non-conducting then the whole charge is distributed all over the sphere.

Electric field due to hollow conducting/ Non-conducting and solid conducting sphere -

1. For a point outside the sphere -

We first consider the field outside the conductor, so we choose $r > R$. The entire conductor is within the Gaussian surface enclosed charge is q . The area of the Gaussian surface is $4\pi r^2$;
$\vec{E}$ is uniform over the surface and perpendicular to it at each point. The flux integral $\oint E_{\perp} dA$ in Gauss's law is therefore which gives

$$E\left(4\pi r^2\right) = \frac{q}{\varepsilon_0}$$

or, $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$ (outside a charged conducting sphere)

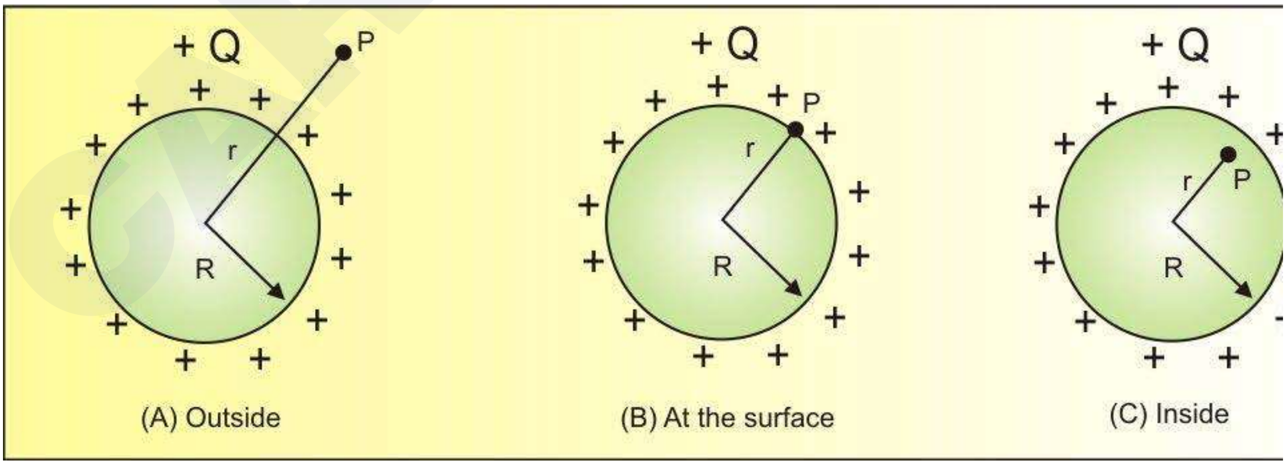
For all points outside the shell, the field due to the uniformly charged shell is such that the entire charge is concentrated at the centre of shell.

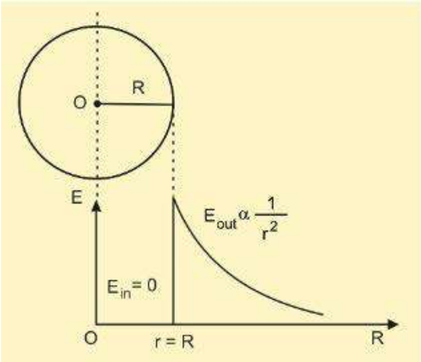
2. For a point inside the sphere (r<R) -

Since the charge will be zero in this case for the hollow sphere and conducting solid sphere. So, **E = 0** inside the sphere.

3. At surface (r = R)

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}$$





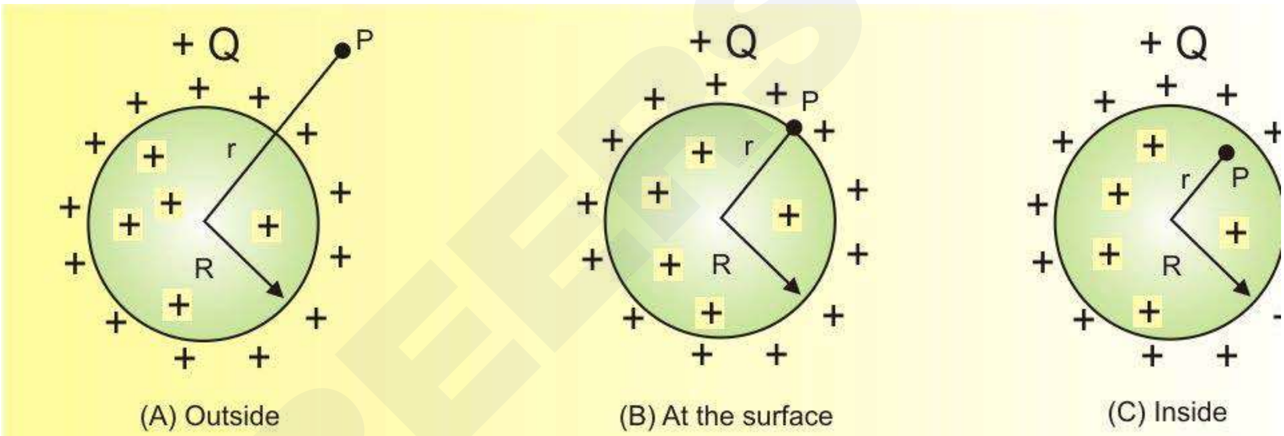
Electric field due to solid non-conducting -

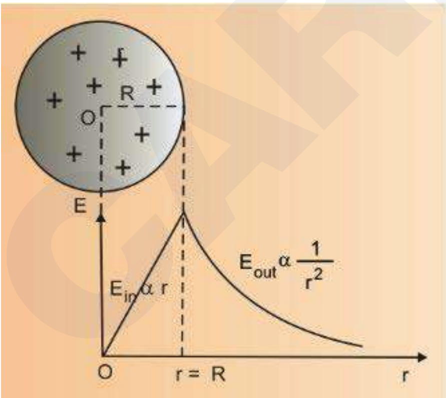
1. Outside (r>R)

$$E = \frac{\rho R^3}{3\varepsilon_o r^2}$$

2. Inisde (r<R) -

$$E = \frac{\rho r}{3\varepsilon_o}$$





Electrostatic Potential and Capacitance

Important Formulae

1.Electric Potential

Electric Potential-

In an Electric field Electric potential V at a point, P is defined as work done per unit charge in changing the position of test charge from some reference point to the given point.

Note-usually reference point is taken as infinity and potential at infinity is taken as Zero.

We know that $W_{ext} = \int \overrightarrow{F_{ext}} \cdot \overrightarrow{dr}$

Since

$$W_{ext} = \Delta U \quad and \quad \Delta KE = 0$$
$$\Rightarrow \text{For all time} \quad F_{net} = 0$$
$$\Rightarrow \vec{F}_{ext} + \vec{F}_{system} = 0$$

i.e $\vec{F}_{ext} = -\vec{F}_{system}$

So $V = \frac{W_{ext}}{q_0} = \int \frac{\vec{F}_{ext} \cdot \vec{dr}}{q_0} = -\int \frac{\vec{F}_{system} \cdot \vec{dr}}{q_0}$

where

$V \rightarrow$ Electric potential

- It is a scalar quantity.
- $SI \;\; Unit \rightarrow \frac{J}{C} = volt$ while CGS unit is stat volt

  $1 \; volt = \frac{1}{300} \; stat \; volt.$
- Dimension - $[V] = \left[\frac{W}{q_0}\right] = \left[\frac{ML^2T^{-2}}{AT}\right] = [ML^2T^{-3}A^{-1}]$.

Electric Potential at a distance 'r' -

If the Electric field is produced by a point charge q then

$$F = \frac{Kqq_0}{r^2}$$

Using $V = \frac{W_{ext}}{q_0} = \int \frac{\vec{F}_{ext} \cdot \vec{dr}}{q_0} = -\int \frac{\vec{F}_{system} \cdot \vec{dr}}{q_0}$

$$V = \frac{Kq}{r}$$

at $r = \infty \qquad V = 0 = V_{max}$

Electric Potential difference -

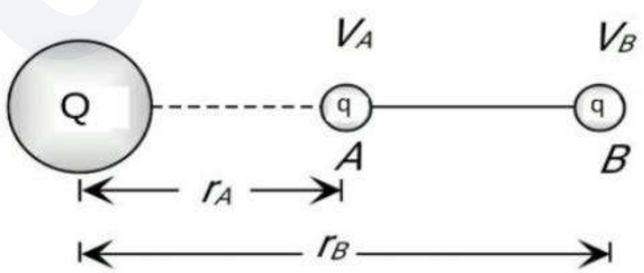
In the Electric field, the work done to move a unit charge from one position to the other is known as Electric Potential difference.

If the point charge Q is producing the field

Point A and B are shown in the figure.

V_A=Electric potential at point A

V_B=Electric potential at point B



$r_B \rightarrow$ the distance of charge at B

$r_A \rightarrow$ distance of charge at A

ΔV=The Electric potential difference in bringing charge q from point A to point B in the Electric field produced by Q.

$$\Delta V = V_B - V_A = \frac{W_{A\to B}}{q}$$

$$\Delta V = KQ\left[\frac{1}{r_B} - \frac{1}{r_A}\right]$$

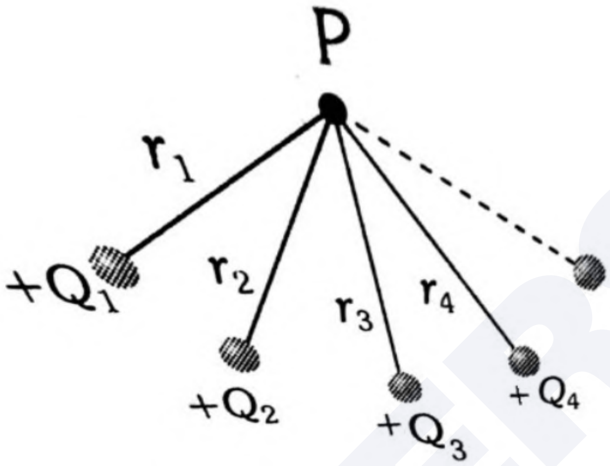
Superposition of Electric potential-

Statement- Total electric potential at a given point in space due to all the charges placed around it is the scalar or algebraic addition of electric potential due to individual charges at that point.

i.e

The net Electric potential at a given point due to different point masses (Q_1,Q_2,Q_3…) can be calculated by doing a scalar sum of their individuals Electric potential.

$$V = V_1 + V_2 + V_3 + \cdots = \frac{kQ_1}{r_1} + k\frac{Q_2}{r_2} + \frac{k\,(Q_3)}{r_3} + \cdots = \frac{1}{4\pi\varepsilon_0}\sum\frac{Q_i}{r_i}$$



- Electric Potential due to Continuous charge distribution

$$V = \int dV = \int \frac{dq}{4\pi\varepsilon_0 r}$$

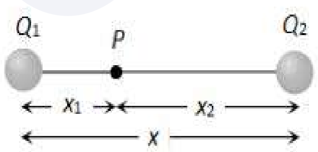
Zero potential due to a system of a two-point charge -

1.For internal point

$(It\ is\ assumed\ that\ |Q_1| < |Q_2|)$

Let at P, V is zero

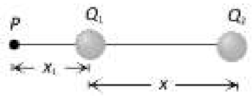
$$V_P = 0 \Rightarrow \frac{Q_1}{x_1} = \frac{Q_2}{(x - x_1)}$$
$$\Rightarrow x_1 = \frac{x}{(Q_2/Q_1 + 1)}$$



If both charges are like then the resultant potential is not zero at any finite point.

2.For external point

Let at P, V is zero



$$V_P \Rightarrow \frac{Q_1}{x_1} = \frac{Q_2}{(x + x_1)}$$
$$\Rightarrow x_1 = \frac{x}{(Q_2/Q_1 - 1)}$$

Relation between electric field and potential -

Electric field and potential are related as

$$\vec{E} = -\frac{dV}{dr}$$

Where E is Electric field

And V is Electric potential

And r is the position vector

And Negative sign indicates that in the direction of intensity the potential decreases.

$$If\ \ \vec{r} = x\,\vec{i} + y\,\vec{j} + z\,\vec{k}$$

Then $E_x = \frac{\delta V}{dx}, E_y = \frac{\delta V}{dy}, E_z = \frac{\delta V}{dz}$

where

$E_x = -\frac{\partial V}{dx}$ (partial derivative of V w.r.t. x)

$E_y = -\frac{\partial V}{dy}$ (partial derivative of V w.r.t. y)

$E_z = -\frac{\partial V}{dz}$ (partial derivative of V w.r.t. z)

When an electric field is a uniform (constant)-

As Electric field and potential are related as $dV = \int_{r_0}^{r} -\vec{E}.d\vec{r}$

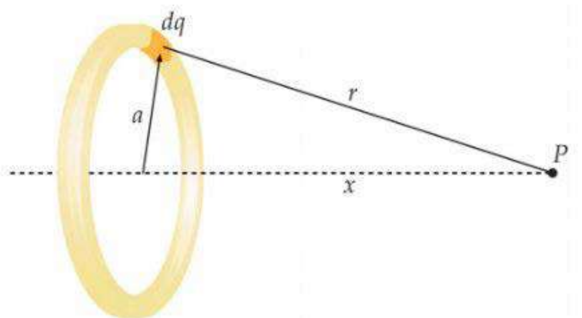
and E=constant then $dV = -\vec{E}\int_{r_0}^{r} d\vec{r} = -\vec{E}dr$

- If at any region E = 0 then V = constant
- If V = 0 then E may or may not be zero.

2. Electric potential due to various charged bodies

1.Electric Potential due to uniformly charged ring-

We want to find the electric potential at point P on the axis of the ring as of radius a, shown in the below figure



Total charge on ring: Q
Charge per unit length: $\lambda = Q/2\pi a$

Take a small elemental arc of charge dq

Charge on an arc: dq

So $$dV = K\frac{dq}{r} = \frac{K\,dq}{\sqrt{x^2+a^2}}$$

$$V(x) = K\int\frac{dq}{\sqrt{x^2+a^2}} = \frac{K}{\sqrt{x^2+a^2}}\int dq = \frac{KQ}{\sqrt{x^2+a^2}}$$

Special cases-

- **The potential at the center of the ring**

  $V_c = \frac{KQ}{a}$ **(since x=0)**

- **If x>>a**

  $$V = \frac{KQ}{x}$$

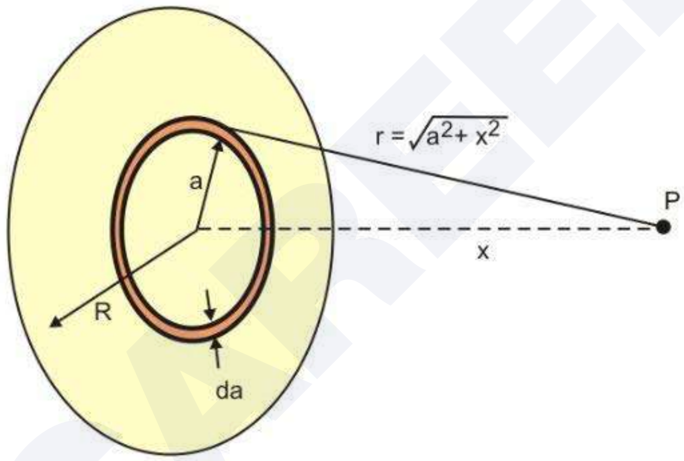
  As x increases, V will decrease.

  $As \quad x \to \infty, \quad V = 0.$

  So the maximum potential is at the centre of the ring.

2.Electric Potential due to uniformly charged Disc-

V(x)=Electric potential at point P on the axis of the disk of radius R, as shown in the below figure



Total charge on ring: Q
Charge per unit Area: $\lambda = Q/\pi R^2$

$$V(x) = 2\pi\sigma K[\sqrt{x^2+R^2} - |x|]$$

We can also write

$$V(x) = \frac{\sigma}{2\epsilon_0}\left[\sqrt{x^2+R^2} - |x|\right]$$

Special cases-

- **The potential at the centre of the disc**

  $V_c = \frac{2KQ}{R}$ **(since x=0)**

- **If x>>R**

$$V(x) = 2\pi\sigma K|x|[\sqrt{1+\frac{R^2}{x^2}} - 1] \simeq 2\pi\sigma K|x|\left[1+\frac{R^2}{2x^2} - 1\right] = \frac{K\sigma\pi R^2}{|x|}$$

$$\Rightarrow V(x) = \frac{KQ}{|x|}$$

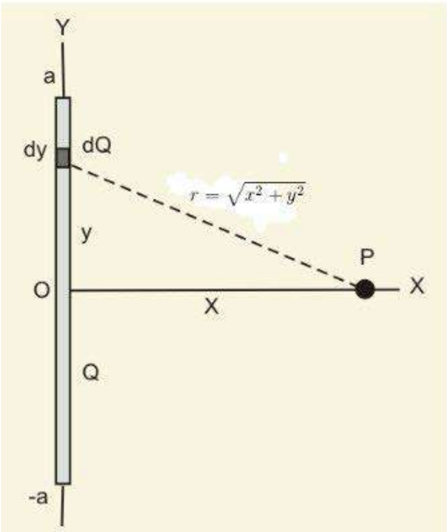
As |x| increases, V will decrease.

$$As \quad |x| \to \infty, \quad V = 0$$

So the maximum potential is at the centre of the disc.

3.Electric Potential due to a finite uniform line of charge-

V_P=The potential due to a finite uniform line of positive charge at point P which is at a distance x from the rod on its perpendicular bisector, as shown in the below figure.



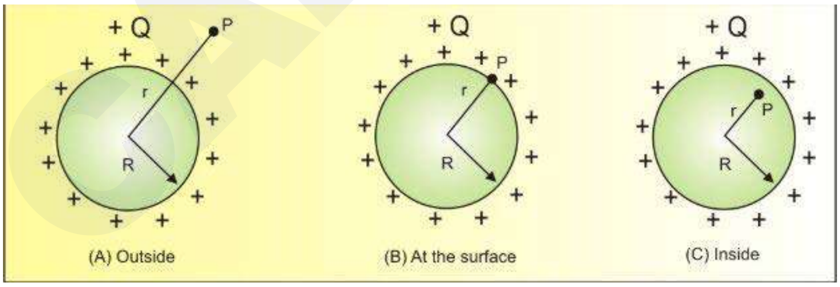
$\lambda = Q/2a$ Uniform linear charge density

We get $$v_P = \frac{kQ}{2a} \ln\left[\frac{(x^2+a^2)^{1/2}+a}{(x^2+a^2)^{1/2}-a}\right]$$

4. Electric Potential due to Hollow conducting, Hollow non conducting, Solid conducting Sphere-

In the case of Hollow conducting, Hollow non conducting, Solid conducting Spheres charges always resides on the surface of the sphere.

If the charge on a conducting sphere of radius R is Q. And we want to find V at point P at distance r from the center of the sphere.



- Outside the sphere (P lies outside the sphere. I.e r>R)

$$E_{out} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2} = \frac{\sigma R^2}{\epsilon_0 r^2}$$

$$V(r) = -\int_{r=\infty}^{r=r} \vec{E}.d\vec{r} = \frac{1}{4\pi\varepsilon_0}\frac{\vec{Q}}{r}$$

- Inside the sphere (P lies inside the sphere. I.e r<R)

$$E_{in} = 0$$

$V_{in} = constant$ and it is given as

$$V(r) = -\int_{r=\infty}^{r=r} \vec{E}.d\vec{r} = -\int_{\infty}^{R} E_r(dr) - \int_{R}^{r} E_r(dr) = \frac{1}{4\pi\varepsilon_0} \times \frac{q}{R} + 0$$

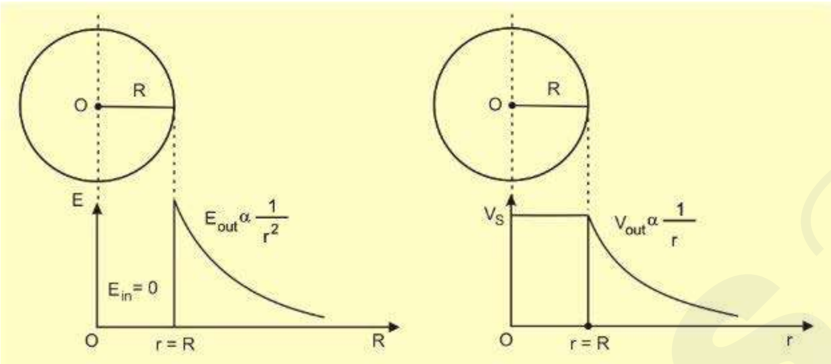
$$\Rightarrow V(r) = \frac{1}{4\pi\varepsilon_0} \times \frac{q}{R}$$

- At the surface of Sphere (I.e at r=R)

$$E_s = \frac{1}{4\pi\epsilon_0}\frac{Q}{R^2} = \frac{\sigma}{\epsilon_0}$$

$$V_s = \frac{1}{4\pi\epsilon_0}\frac{Q}{R} = \frac{\sigma R}{\epsilon_0}$$

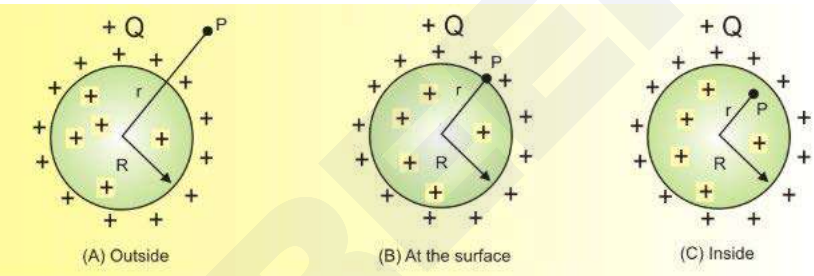
- The graph between (E vs r) and (V vs r) is given below



5. Electric Potential due to Uniformly charged Non conducting sphere-

Suppose charge Q is uniformly distributed in the volume of a non-conducting sphere of radius R.

And we want to find V at point P at distance r from the center of the sphere.



- Outside the sphere (P lies outside the sphere. I.e r>R)

$$E_{out} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2} \qquad V_{out} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r}$$

$$E_{out} = \frac{\rho R^3}{3\epsilon_0 r^2} \qquad V_{out} = \frac{\rho R^3}{3\epsilon_0 r}$$

- Inside the sphere (P lies inside the sphere. I.e r<R)

$$E_{in} = \frac{1}{4\pi\epsilon_0}\frac{Qr}{R^3} \quad V_{in} = \frac{Q}{4\pi\epsilon_0} * \frac{3R^2 - r^2}{2R^3}$$

$$E_{in} = \frac{\rho r}{3\epsilon_0} \qquad V_{in} = \frac{\rho(3R^2 - r^2)}{6\epsilon_0}$$

- At the surface of Sphere (I.e at r=R)

$$E_s = \frac{1}{4\pi\epsilon_0}\frac{Q}{R^2} \qquad V_s = \frac{1}{4\pi\epsilon_0}\frac{Q}{R}$$

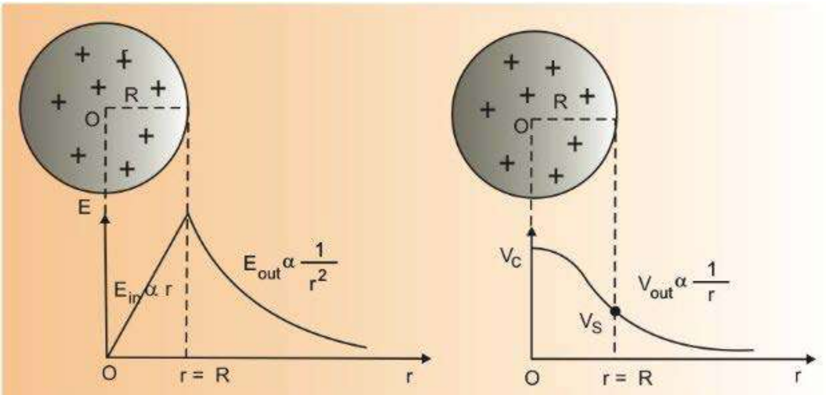
$$E_s = \frac{\rho R}{3\epsilon_0} \qquad V_s = \frac{\rho R^2}{3\epsilon_0}$$

Note - If P lies at the centre of the uniformly charged non-conducting sphere (I.e at r=0)

$$V_{centre} = \frac{3}{2} \times \frac{1}{4\pi\epsilon_0}\frac{Q}{R} = \frac{3}{2}V_s$$

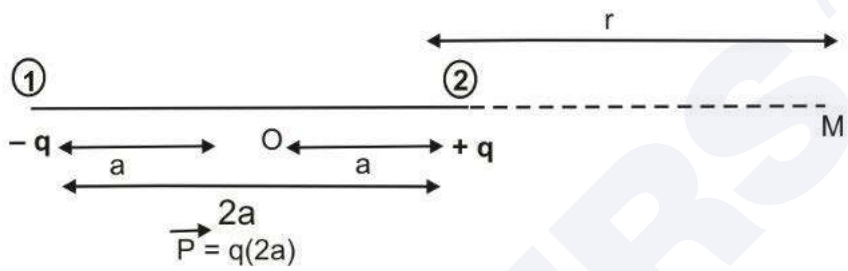
i.e $V_c > V_s$

- The graph between (E vs r) and (V vs r) is given below



3.Electric potential due to an electric dipole

Electric Potential due to an Electric Dipole at a Point on the Axial Line-



V_{net}=Electric Potential due to an Electric Dipole at a Point M which is on axial line and at a distance r from the center of a dipole.

Where $V_1 \;\; and \;\; V_2$ is the Electric Potential at M due to $-q \;\; and \;\; +q$ charges respectively.

$V_1 = \frac{kq}{(r+a)}$

$V_2 = \frac{kq}{(r-a)}$

$V_{net} = V_2 - V_1$

$$V_{net} = V_1 + V_2$$
$$= \frac{-kq}{(r+a)} + \frac{k_q}{(r-a)}$$
$$= k_q\left\{\frac{1}{r-a} - \frac{1}{r+a}\right\}$$
$$= kq\left\{\frac{(r+a)-(r-a)}{(r-a)(r+a)}\right\}$$

So $V_{net} = \frac{2kqa}{r^2 - a^2}$

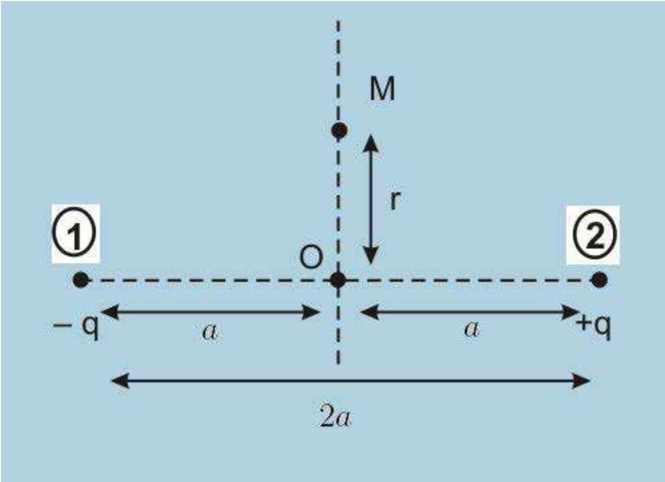
Using $P = q(2a)$

So $V_{net} = \frac{kP}{r^2 - a^2}$

- if r>>a

then $V_{net} = \frac{KP}{r^2} = \frac{P}{4\pi\epsilon_0 r^2}$

Electric potential due to an Electric Dipole at a Point on the Equitorial line-



V_{net}=Electric potential due to an Electric Dipole at a Point M which is on the Equitorial line and at a distance r from the center of a dipole.

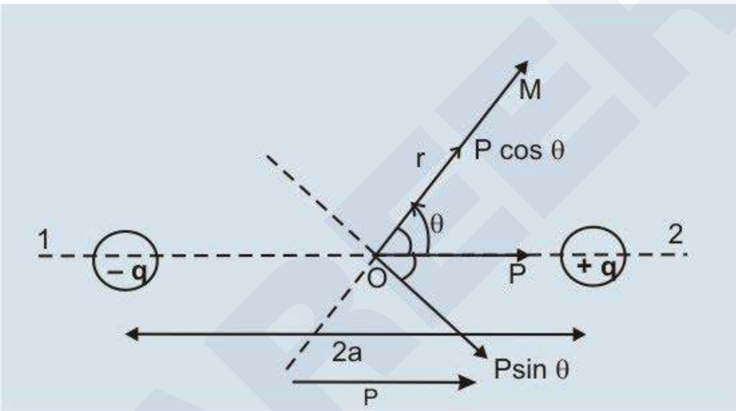
Where $V_1 \quad and \quad V_2$ is the Electric Field Intensity at M due to $-q \quad and \quad +q$ charges respectively.

$$V_1 = -\frac{1}{4\pi\epsilon_0} * \frac{q}{\sqrt{r^2+a^2}}$$

$$V_2 = \frac{1}{4\pi\epsilon_0} * \frac{q}{\sqrt{r^2+a^2}}$$

$$V_{net} = V_2 - V_1 = 0$$

Electric potential due to a dipole at any general point-



V_{net}=Electric potential due to an Electric Dipole at a Point M which at a distance r from the center of a dipole and making an angle θ with the axial line.

From the figure, M is at the axial line of dipole having dipole moment as $Pcos\theta$ and M is at the Equitorial line of dipole having dipole moment as $Psin\theta$.

So $Psin\theta$ has no contribution in electric potential at point M.

if r>>a

then $V_a = \frac{1}{4\pi\varepsilon_0} \times \frac{2P\cos\theta}{r^2}$ and $V_\perp = 0$

So $V_{net} = V_a = \frac{KPcos\theta}{r^2}$

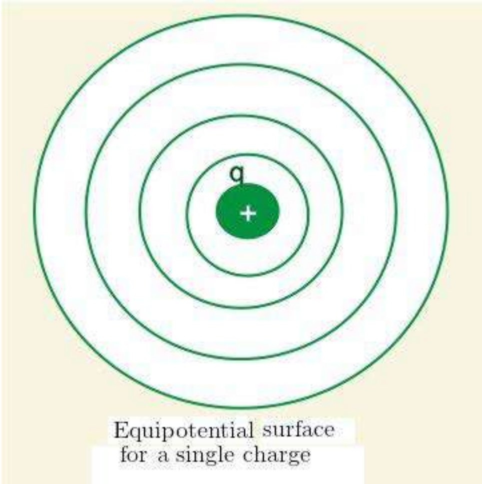
4. Equipotential Surface

A real or imaginary surface in an electric field that has the same potential at every point is called an equipotential surface.

Equipotential surfaces can be of any shape.

For example for a point charge of having charge q the potential at a distance, r is given as $V = \frac{kq}{r}$

So For V=constant, we get r=constant means for a point charge of having charge q, the equipotential surfaces are the concentric spherical surfaces having a charge q at their center as shown in the below figure.

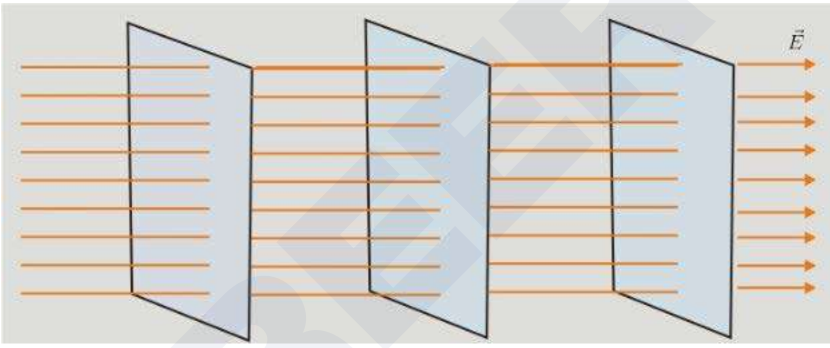


All points on the spherical surface of radius r centered on q have the same V.

Properties of the equipotential surface-

- The potential difference between any two points on the Equipotential surfaces is zero.
- No work is done by the electric force to move the charge from one point to another point on an equipotential surface.
- Equipotential surfaces can never cross each other, otherwise potential at a point will have two values which is not possible.
- An equipotential surface is always perpendicular to electric lines of force.

  For example, An equipotential surface for a uniform electric field is shown below.



From the figure, it is clear that the Direction of the electric field is perpendicular to the equipotential surface.

5.Electrostatic Potential energy

Electrostatic Potential energy -

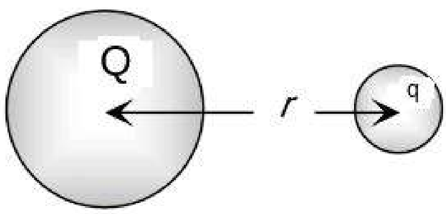
It is the amount of work done by external forces in bringing a body from ∞ to a given point against electric force.

or It is defined as negative work done by the electric force in bringing a body from ∞ to that point.

- It is Scalar quantity
- SI Unit: Joule
- Dimension : $[ML^2T^{-2}]$

Electric Potential energy at a point

If the point charge Q is producing the electric field



Then electric force on test charge q at a distance r from Q is given by $F = \frac{KQq}{r^2}$

And the amount of work done by the electric force in bringing a test charge from ∞ to r is given by

$$W = \int_{\infty}^{r} \frac{KQq}{x^2} dx = -\frac{KQq}{r}$$

And negative of this work done is equal to electric potential energy

So $U = \frac{KQq}{r}$

$U \rightarrow$ electric potential energy

$r \rightarrow$ distance between two

Change of potential energy-

if a charge q is moved from r_1 to r_2 in a electric field produced by charge Q

Then Change of potential energy is given as

$$\Delta U = KQq\left[\frac{1}{r_2} - \frac{1}{r_1}\right]$$

$\Delta U \rightarrow$ change of energy

$r_1, r_2 \rightarrow$ distances

Potential Energy Of System Of two Charge-

$U = \frac{KQ_1Q_2}{r}$ $(S.I)$ where $K = \frac{1}{4\pi\epsilon_0}$

Potential Energy For a system of 3 charges-

$$U = K\left(\frac{Q_1Q_2}{r_{12}} + \frac{Q_2Q_3}{r_{23}} + \frac{Q_1Q_3}{r_{13}}\right)$$

Work energy relation-

$W = U_f - U_i$

Where W=work done by an external force

$U_f - final\, P.E$

$U_i - initial\, P.E$.

The relation between Potential and Potential energy-

As $U = \frac{KQq}{r} = q\left[\frac{KQ}{r}\right]$

But $V = \frac{KQ}{r}$

So $U = qV$

Or potential is defined as Potential energy Per unit charge.

i.e $V = \frac{W}{Q} = \frac{U}{Q}$

Where $V \rightarrow$ Potential

$U \rightarrow$ Potential energy

Electron Volt-

$1\, ev = 1.6 \times 10^{-19} J$=$1.6 \times 10^{-12} erg$.

It is the smallest practical unit of energy which is used in atomic and nuclear physics.

Electric potential Energy of Uniformly charged sphere-

$$U = \frac{3Q^2}{20\pi\epsilon_0 R}$$

Where R - Radius and Q - total charge.

Energy density- It is defined as the energy stored for unit volume.

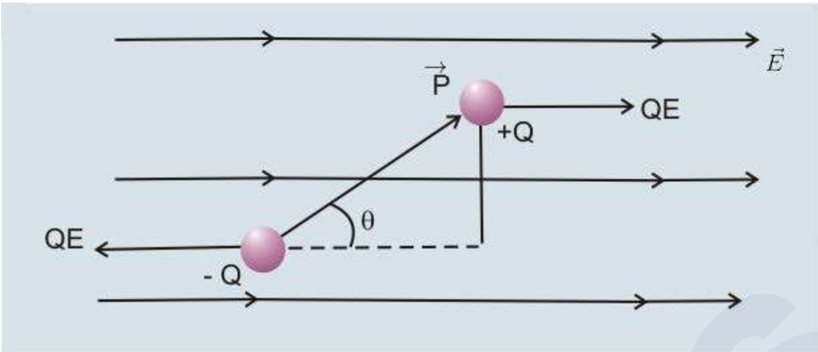
$U_v = \frac{U}{V}$

Where $U - Potential\,Energy$ and $V - Volume$.

6.Electric potential energy of an electric dipole

When a dipole is kept in a uniform electric field. The net force experienced by the dipole is zero as shown in the below figure.

I.e $F_{net} = 0$



But it will experience torque. And Net torque about the center of dipole is given as

$\tau = QEd\sin\theta$ or $\tau = PE\sin\theta$ or $\vec{\tau} = \vec{P} \times \vec{E}$

Work done in rotation-

Then work done by electric force for rotating a dipole through an angle θ_2 from the equilibrium position of an angle θ_1

$$W_{ele} = \int \tau d\theta = \int_{\theta_1}^{\theta_2} \tau d\theta cos(180^0) = -\int_{\theta_1}^{\theta_2} \tau d\theta$$
$$\Rightarrow W_{ele} = -\int_{\theta_1}^{\theta_2} (P \times E) d\theta = -\int_{\theta_1}^{\theta_2} (PESin\theta) d\theta = PE(\cos\theta_2 - \cos\theta_1)$$

And So work done by an external force is $W = PE(\cos\theta_1 - \cos\theta_2)$

For example

if $\theta_1 = 0^\circ \; and \; \theta_2 = \theta$

$W = PE(1 - \cos\theta)$

$if \; \theta_1 = 90^\circ \; and \; \theta_2 = \theta$

$W = -PE\cos\theta$

Potential Energy of a dipole kept in Electric field-

As $\Delta U = -W_{ele} = W$

So change in Potential Energy of a dipole when it is rotated through an angle θ_2 from the equilibrium position of an angle θ_1 is given as
$\Delta U = PE(\cos\Theta_1 - \cos\Theta_2)$

$if \; \Theta_1 = 90^\circ \; and \; \Theta_2 = \Theta$

$\Delta U = U_{\theta_2} - U_{\theta_1} = U_\theta - U_{90} = -PE\cos\Theta$

Assuming $\Theta_1 = 90^\circ$ and $U_{90^0} = 0$

we can write $U = U_\theta = -\vec{P} \cdot \vec{E}$

Equilibrium of Dipole-

1. Stable Equilibrium-

$\Theta = 0^\circ$

$\tau = 0$

$U_{min} = -PE$

2. Unstable Equilibrium-

$\Theta = 180^\circ$

$\tau = 0$

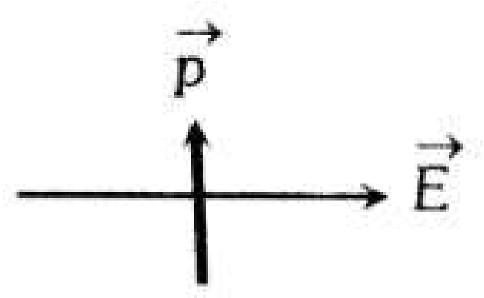
$U_{max} = PE$

Note-

When $\Theta = 90^\circ$

then $\tau_{max} = PE$ and $U = 0$

and it is important to note here that dipole is not in equilibrium since $\tau_{max} \neq 0$



7. Capacitor

A capacitor is a passive two-terminal electrical component used to store energy electrostatically in an electric field.

Capacitance: Capacitance is the ability of a capacitor to hold an electrical charge.

For an isolated capacitor, capacitance is defined as the ratio of the charge to the potential of a capacitor.

$$C = \frac{Q}{V}$$

As Q increases V also increases because potential depends on the charge.

$$Q \propto V$$
$$Q = CV$$

Therefore Capacitance is a constant quantity.

Unit of Capacitance-

$S.I unit - \frac{C}{V} = farad(f)$

Smaller S.I unit is mf , μf , nf and Pf

$C.G.S - Stat\ farad\ \ and\ \ 1F = 9 \times 10^{11} statfarad$

$Dimension = M^{-1}L^{-2}T^{4}A^{2}$

- If V=1V, C=Q. Hence, we define the capacitance of an conductor as the charge required to rise the potential of the conductor by 1V. In the SI system, the unit of capacitance is farad.

CAPACITANCE OF A SPHERICAL CONDUCTOR OR CAPACITOR:

A single conductor can also act as a capacitor. For this, let a charge q be given to a spherical conductor of radius R, then potential on it is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$$

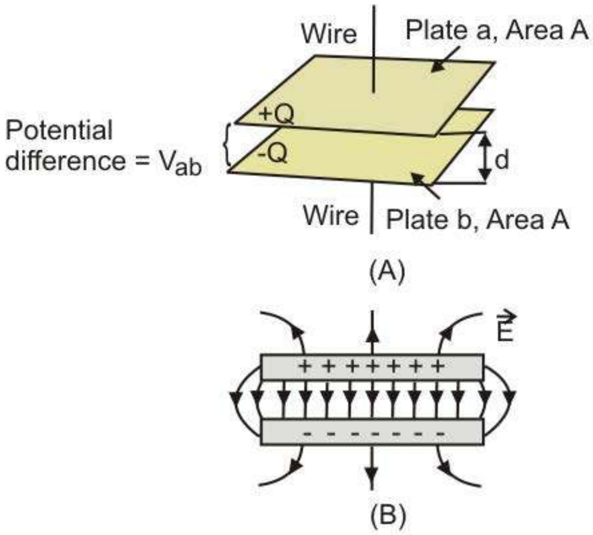
The other conductor is supposed to be at infinity, whose potential will be taken as zero. So the potential difference between the sphere and the conductor at infinity becomes V-0=V.

$$C = \frac{q}{V} = 4\pi\varepsilon_0 R$$

Thus, the capacitance of a spherical conductor is $C = 4\pi\varepsilon_0 R$, so we can C depends on the medium and dimension of conductor.

Parallel plate capacitor -

It consists of two large plates placed parallel to each other with a separation d which is very small in comparison to the two dimensions (length and breadth) of the plates . In an ideal capacitor, electric field resides in the region within the plates i.e., no electric field is outside the plates . Electric field is directed from positive plate to negative plate in such a way that the lines emerge perpendicularly from positive plate and terminate perpendicularly to the negative plate.



(A) A charged parallel plate capacitor.
(B) When the separation is small as compared to their size, there is a slight fringing of the electric field $\vec{E}$ at the edges

Electric field between the plates -

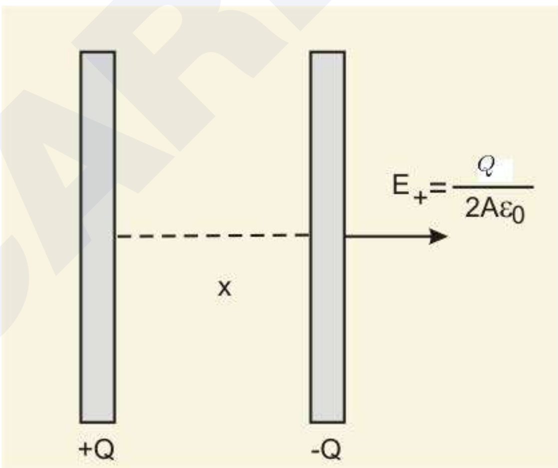
$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}$$

Therefore, potential difference between the plates is

$$V = Ed = \frac{Qd}{A\varepsilon_0}$$

and therefore capacitance $C = \frac{Q}{V} = \frac{\varepsilon_0 A}{d}$

Force Between the Plates of a Parallel-Plate Capacitor-



Let us consider a parallel-plate capacitor with plate area A.

Suppose a positive charge +Q is given to one plate and a negative charge -Q to the other plate.

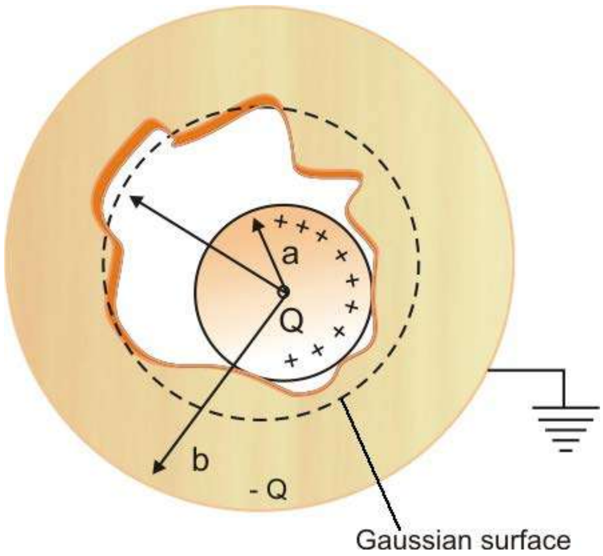
The electric field due to only the positive plate is $E = \sigma/2\varepsilon_0 = Q/2A\varepsilon_0$ at all points if the plate is large.

The negative charge -Q on the other plate finds itself in the field of this positive charge.

Therefore, force on this plate is $F = EQ = Q^2/2A\varepsilon$. This force will be attractive.

Spherical capacitors:

Spherical capacitors has two concentric spherical conducing shells of radii a and b, say b>a. The shell on the outer side is earthed. We place a charge +Q on the inner shell. It will reside on the outer surface of the shell. A charge -Q will be induced on inner surface of outer shell. A charge +Q will flow from outer shell to the earth.



Consider a Gaussian spherical surface of radius r such that a<r<b.
From Gauss's law, electric field at distance r>a is

$$E=\frac{Q}{4\pi\varepsilon_0 r^2}$$

The potential difference is :

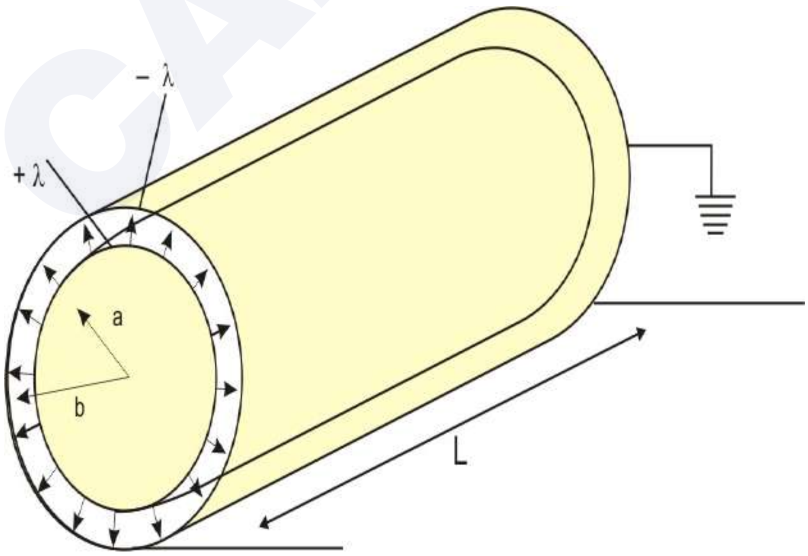
$$V_b-V_a=-\int_0^b \bar{E}\cdot d\bar{r}=-\int_a^b \frac{Q}{4\pi\varepsilon_0 r^2}dr$$

Since $V_b=0, we\ have$
$$V_b=\frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right)=\frac{Q(b-a)}{4\pi\varepsilon_0 ab}$$

$$\text{Therefore, capacitance, } C=\frac{Q}{V_b-V_a}=\frac{Q}{V_a}=\frac{4\pi\varepsilon_0 ab}{b-a}$$

Cylinderical capacitor

It consists of two coaxial cylinders of radii a and b. Assume that b>a. The cylinders are long enough so that we can neglect fringing of electric field at the ends.The outer one is earthed. Electric field at a point between the cylinders will be radial and its magnitude will depend on the distance from the central axis. Consider a Gaussian surface of length y and radius r such that a<r<b . Flux through the plane surface is zero because electric field and area vector are perpendicular to each other.



For the curved part,

$$\phi = \int \vec{E} \cdot d\vec{s} = \int E ds$$
$$= E \int ds = E \cdot 2\pi r y$$

Charge inside the gaussian surface, $q = \frac{Qy}{L}$

From Gauss's law $\phi = E 2\pi r y = \frac{Qy}{L\varepsilon_0} \Rightarrow E = \frac{Q}{2\pi\varepsilon_0 L r}$.

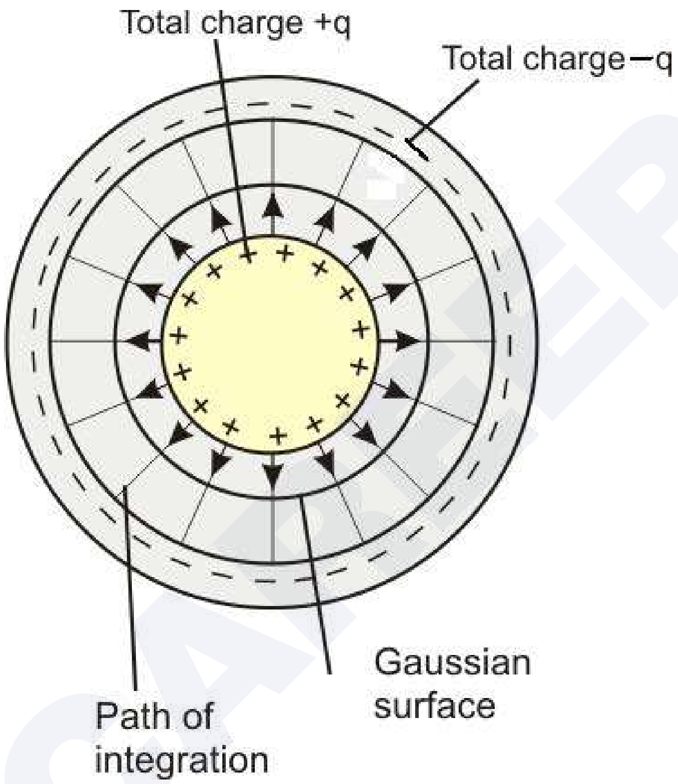
Potential difference:

$$V_b - V_a = -\int_a^b \bar{E} \cdot d\bar{r} = -\int_a^b \frac{Q}{2\pi\varepsilon_0 L r} dr = -\frac{Q}{2\pi\varepsilon_0 L_a} \int_a^b \frac{1}{r} dr$$

$$V_a = \frac{Q}{2\pi\varepsilon_0 L} \ln \frac{b}{a} \quad (\text{ since } V_b = 0)$$

Therefore, Capacitance of cylindrical capacitor is,

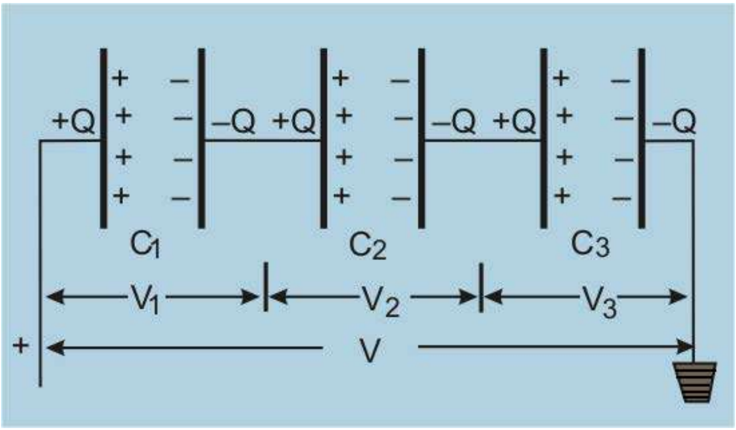
$$C = \frac{Q}{V_\alpha - V_a} = \frac{Q}{V_a} = \frac{2\pi\varepsilon_0 L}{\ln\left(\frac{b}{a}\right)}$$



8.Combination Of Capacitors - Parallel And Series

Series combination:

If capacitors are connected in such a way that we can proceed from one point to another by only one path passing through all capacitors then all these capacitors are said to be in series.



Here three capacitors are connected in series and are connected across a battery of potential difference 'V'.

Charge: 'q' given by battery deposits at first plate of first capacitor. Due to induction it attract '–q' on the opposite plate.The pairing +ve q charges are repelled to first plate of Second capacitor which in turn induce -q on the opposite plate. Same action is repeated to all the capacitors and in this way all capacitors get q charge. As a result ; the charge given by battery q, every capacitor gets charge q.

Potential difference: V is the sum of potentials across all capacitors. Therefore

$$V = v_1 + v_2 + v_3$$

$$v_1 = \frac{q_1}{c_1}, v_2 = \frac{q_2}{c_2}, v_3 = \frac{q_3}{c_3}$$

Equivalence equation: The equivalent capacitance for the combination of capacitance in series can be calculated as $C_e = \frac{q}{V}$.

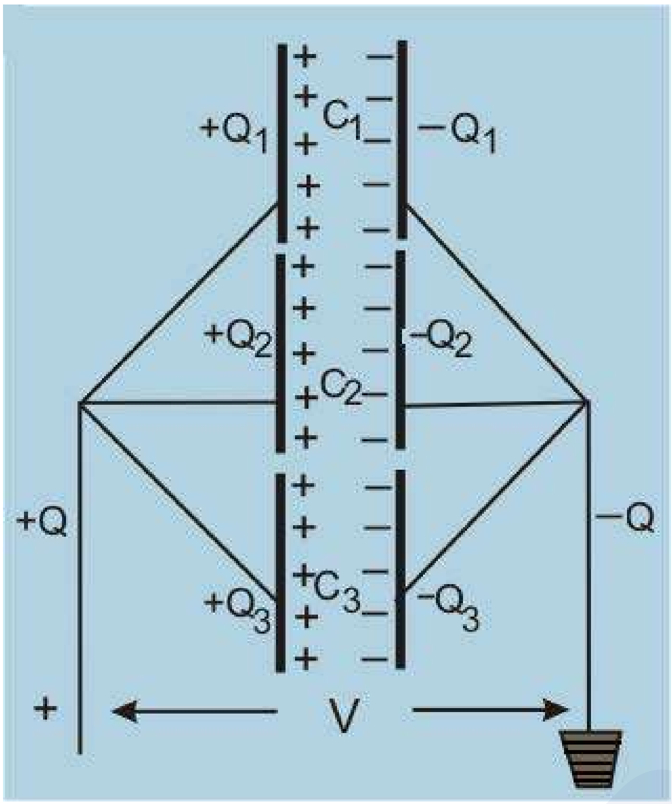
Or,

$$\begin{aligned} 1/\mathrm{C_e} &= \mathrm{V/q} \\ &= (\mathrm{v_1 + v_2 + v_3})/\mathrm{q} \\ &= \mathrm{v_1/q + v_2/q + v_3/q} \\ 1/\mathrm{C_e} &= 1/\mathrm{C_1} + 1/\mathrm{C_2} + 1/\mathrm{C_3} \end{aligned}$$

For 2 capacitor system $C = \frac{c_1 c_2}{c_1 + c_2}$, and $v_1 = \frac{c_2}{c_1 + c_2} \cdot V$

If n capacitor of capacitance 'c' are joint in series then equivalent capacitance $C_e = \frac{c}{n}$.

Parallel combination: If capacitors are connected in such a way that there are many paths to go from one point to other. All these paths are parallel and capacitance of each path is said to be connected in parallel.



Here three capacitors are connected in parallel and are connected across a battery of potential difference 'V'.

The potential difference across each capacitor is equal and it is same as P.D. across Battery. The charge given by source is divided and each capacitor gets some charge. The total charge $q = q_1 + q_2 + q_3$.

Therefore, each capacitor has charge

$q_1 = C_1V_1, q_2 = C_2V_2, q_3 = C_3V_3$

Equivalent Capacitance : We know that, $q = q_1 + q_2 + q_3$ when divided by v both sides , $\frac{q}{v} = \frac{q_1}{v} + \frac{q_2}{v} + \frac{q_3}{v}$.

Therefore the equivalence capacitance will be:

$\mathbf{C} = c_1 + c_2 + c_3$

The equivalent capacitance in parallel increases, and it is more than largest in parallel. In parallel combination V is same therefore $v = \frac{q_1}{c_1} = \frac{q_2}{c_2} = \frac{q_3}{c_3}$. In parallel combination $q \propto c$. Larger capacitance larger is charge.

Charge distribution : $q_1 = c_1v, q_2 = c_2v, q_3 = c_3v$.

In 2 capacitor system charge on one capacitor $q = \frac{c_1}{c_1 + c_2 + \ldots} \cdot q$.

Capacitors in parallel give C = nc.

9.Energy Stored In Capacitor

Energy stored in capacitor: When the charge is added to a capacitor then charge already present on the plate repel any new incoming charge. Hence a new charge has to be sent by applying force and doing work on it. All this work done on charges become energy stored in the capacitor. If Q is the amount of charge stored when the whole battery voltage appears across the capacitor, then the stored energy is obtained from the integral:

$$U = \int_0^Q \frac{q}{C} dq = \frac{1}{2} \frac{Q^2}{C}$$

This energy expression can be put in three equivalent forms by just permutations based on the definition of capacitance $C = \frac{Q}{V}$.

$$U = \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} QV = \frac{1}{2} CV^2$$

This energy is stored in the form of an Electric field between the plates.

The energy density per unit volume (u)

$$u = \frac{1}{2}cv^2/V = \frac{1}{2}\frac{\varepsilon_0 AE^2 d^2}{dAd}$$

$$u = \frac{1}{2}\varepsilon_0 E^2.$$

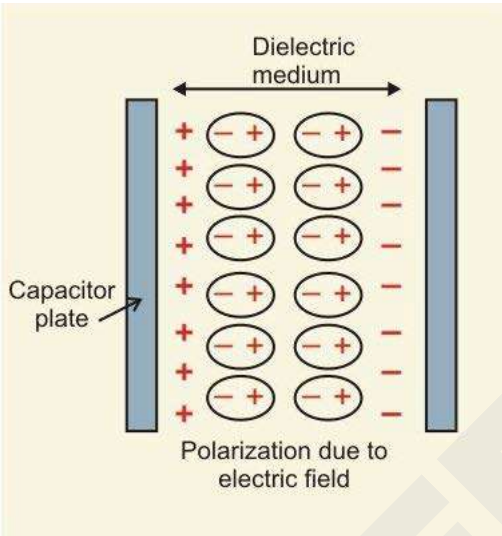
Note- The energy density per unit volume (u) is numerically equal to Electrostatic pressure.

10.Dielectrics

Dielectric: A dielectric is an insulating material in which all the electrons are tightly bounded to the nuclei of the atoms and no free electrons are available for the conduction of current. They are non-conducting materials. They do not have free charged particles like conductors have. They are of two types.

1. Polar : The centre of +ve and –ve charges do not coincide. Example HCl, H_2O, They have their own dipole moment

2. Non-Polar : The centers of +ve and –ve charges coincide. Example $CO_2, C_6H_6.$ They do not have their own dipole moment.



When a dielectric slab is exposed to an electric field, the two charges experience force in opposite directions. The molecules get elongated and develops a surface charge density σ_p . This leads to development of an induced electric field Ep , which is in opposition direction of external electric field Eo . Then net electric field E is given by $E = E_o - E_i$.

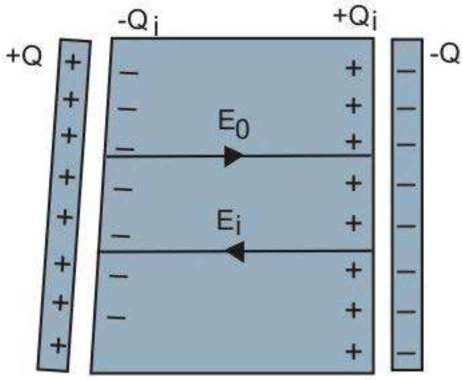
This indicates that net electric field is decreased when dielectric is introduced.

The ratio $\frac{E_0}{E} = K$ is called dielectric constant of the dielectric. Hence, Electric field inside a dielectric is $E_i = \frac{E_0}{K}$.

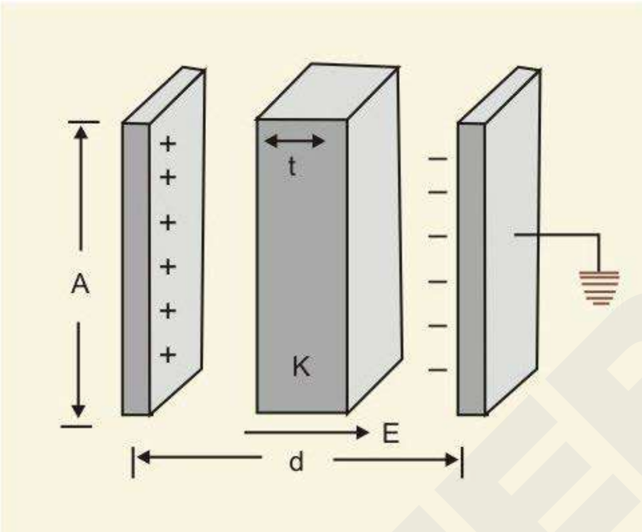
$$E = E_0 - E_i \text{ and } E = \frac{E_0}{k}$$

$$So,\ E_0 - E_i = \frac{E_0}{K}$$

$$\text{or } E_0 K - E_i K = E_0$$
$$\text{or } E_0 K - E_0 = E_i K$$
$$\text{or } E_i = \frac{K-1}{K} E_0$$
$$\text{or } \frac{\sigma_i}{\varepsilon_0} = \frac{K-1}{K}\frac{\sigma}{\varepsilon_0}$$
$$or\ \sigma_i = \frac{K-1}{K}\sigma$$
$$\text{or } \frac{Q}{A} = \frac{K-1}{K}\frac{Q}{A}$$
$$\text{or } Q_i = Q\left(1 - \frac{1}{K}\right)$$



This is irrespective of the thickness of the dielectric slab,i.e., whether it fills up the entire space between the charged plates or any part of it.



$$C' = \frac{\epsilon_0 A}{d - t + \frac{t}{k}}$$

Current Electricity

Important Formulae

1. Current

- **Definition**: Electric current is defined as the rate of flow of electric charge through any cross-section.

  i.e If a charge of ΔQ flows through an area for time Δt then

1. Average electric current through the area is defined as

$$\bar{i} = \frac{\Delta Q}{\Delta t}$$

2. The instantaneous current at any time t is given by

$$i = \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt}$$

And If the flow is uniform then Current

$$I = \frac{Q}{t}$$

If a current I flows through an area for time t then, total charge flowing is

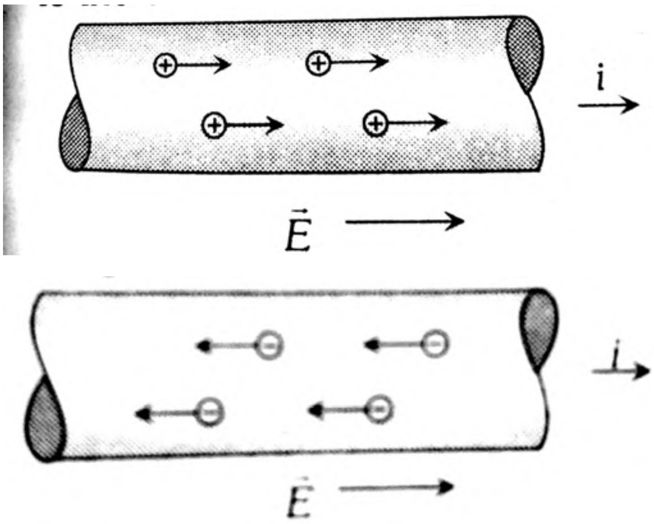
$$Q = \int I dt$$

- **The direction of current**:-

As Current is the rate of flow of charge, Charge can be +ve or -Ve.

The direction of flow of positive charge is considered as the conventional direction of current.

Or the direction opposite to the direction of flow of negative charge is considered as the conventional direction of the current .



- **Tips about current:-**

1. Current flows through a circuit only if the circuit is closed
2. Current can be added algebraically.
3. Unit of Electric Current: The unit of electric current is Ampere or A.

The current is said to be 1 ampere when 1 coulomb of charge flows through any cross-section in every second.

- **Calculating current in different situation-**

1. When charge is performing translatory motion-

If n particles each having charge q pass a given area in time t

$$i=\frac{nq}{t}$$

2. When charge is performing rotatory motion-

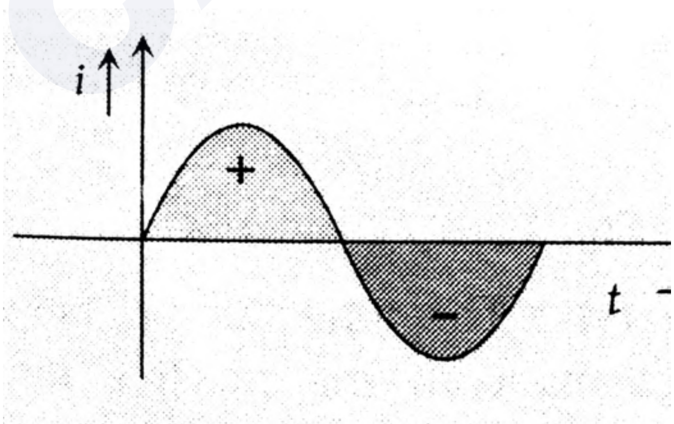
If a point charge q is moving in a circle of radius r

With Velocity V (frequency ν , angular speed \omega and time period T)

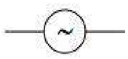
Then $i=\frac{q}{t} \Rightarrow \frac{q}{T}=q\nu=\frac{qV}{2\pi r}=\frac{qw}{2\pi}$

- **Types of current**

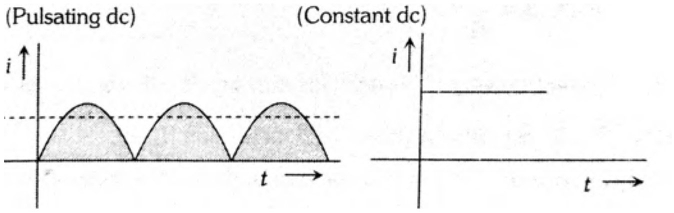
1. Alternating current

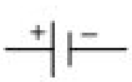


- Magnitude and direction both varies with time.
- Shows heating effect only.

- Its symbol 

2. Direct current

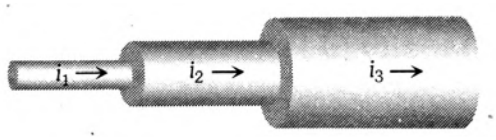


- No change direction with time.
- Shows heating effect , chemical effect and magnetic effect of current.
- It's a symbol 

- **Current independent of Area-**

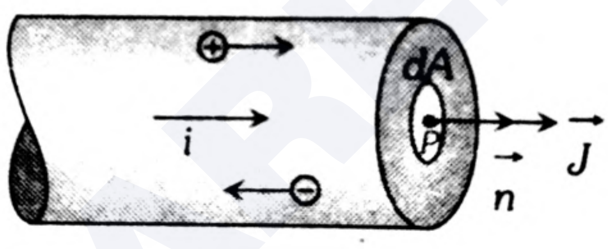
Current does not change with change in cross-sectional area

Here $i_1 = i_2 = i_3$



2. Current Density

- Current density: The amount of electric current flowing per unit cross-sectional area of a material.
- It is a vector quantity.

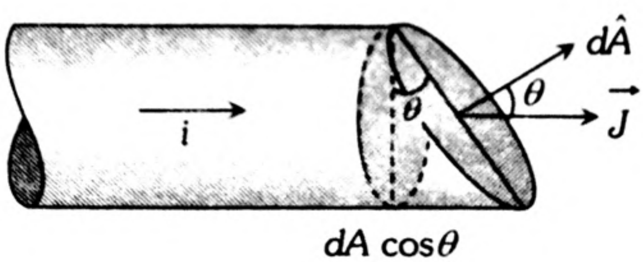


If a current of Δi flows through an area ΔA the average current density $\bar{j} = \frac{\Delta i}{\Delta A}$ in the direction of the current.

At point P :

$j = \lim_{\Delta A \to 0} \frac{\Delta i}{\Delta A}$ in the direction of the current

- If the current is not Perpendicular to Area



$J_{av} = \frac{di}{dA \cos\theta}$

$di = JdA\cos\theta = \vec{J}\cdot d\vec{A}$

θ is the angle between the normal to the area and the direction of the current.

- If the current density $\vec{J}$ is uniform for a normal cross-section $\vec{A}$ then

$$i = \int \vec{J}.d\vec{A}$$

- Unit and dimension -

The unit of current density is Amp/m^2

The dimension of current density is $[L^{-2}A]$

3. Drift Velocity:-

Drift velocity is the average velocity that a particle such as an electron attains in a material due to an electric field.

$$v_d = \frac{-e\vec{E}}{m}\tau$$

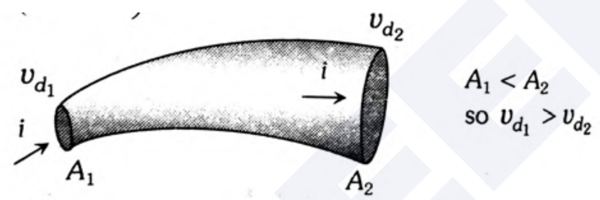
Where v_d is the drift velocity, E is the electric field applied, e and m are the charge and mass of electrons respectively and τ is the relaxation time.

Drift velocity and current-

$$J = \frac{I}{A} = \frac{neAv_d}{A}$$

$$J = nev_d$$

- v_d is inversely proportional to area: $v_d \ \alpha \ \frac{1}{A}$:



4. Ohms Law

- **Ohm's law**

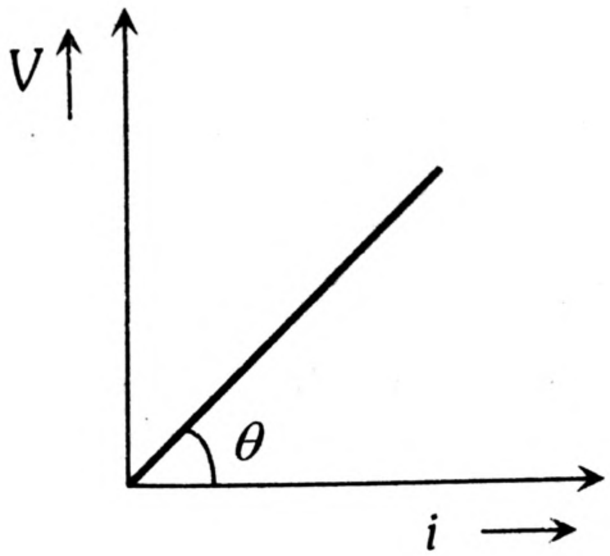
In a conductor, if all external physical conditions like temperature and pressure are kept constant the Current flowing through a conductor is directly proportional to the Potential difference across two ends.

$$V \propto I$$

$$V = IR$$

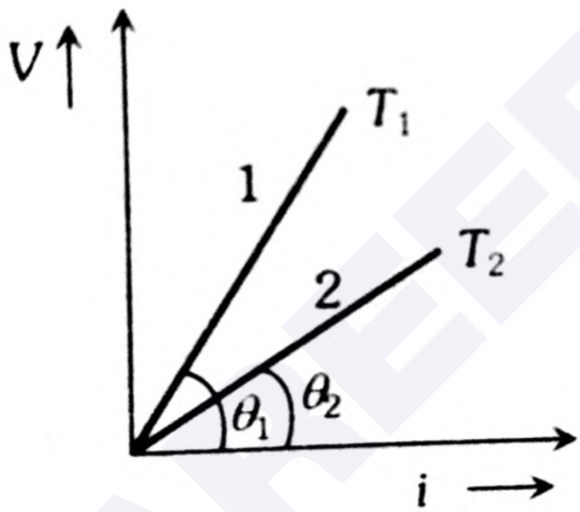
$R-$ Electric Resistance

- **The graph between V and I**



The slope gives the resistance

- **The graph between V and I at different temperatures**



Here $T_1 > T_2$. The resistance of a conductor increases with an increase in temperature

- **Miniature form of Ohm's law**

Drift velocity and current are related as

$I = neAV_d$

So $J = \frac{I}{A} = neV_d$

And $v_d = \frac{-e\vec{E}}{m}\tau$

So $\vec{J} = \frac{ne^2\tau}{m}\vec{E} = \sigma\vec{E}$

As $\vec{J} = \sigma\vec{E}$

$\vec{J} = \frac{\vec{E}}{\rho}$

Where $\sigma \rightarrow$ Conductivity

$\rho \rightarrow$ Resistivity

Both J and E will have the same direction.

- **Mobility-**

Drift velocity per unit Electric field is called the mobility of electrons.

$\mu = \frac{v_d}{E}$

$\mu -$ Mobility

$v_d -$ Drift velocity

Relation between conductivity and Mobility:

$\sigma = \mu ne$

Where n is the number of electrons per unit volume, e is the charge of the electron and σ is the conductivity.

In case of a conductor conductivity decrease with the increase of temperature and in the case of semiconductor conductivity increase with increase in temperature.

5. Resistance and Resistivity

Resistance-

The resistance is known as the property of a substance by virtue of which it opposes the flow of current through it.

Formula-

For a conductor of resistivity ρ having a length of a conductor= l

and Area of a crosssection of conductor= A

Then the resistance of a conductor is given as

$$R = \rho \frac{l}{A}$$

Where $\rho \rightarrow$ Resistivity

- Its S.I unit is $Volt/Amp$ or ohm (Ω)
- Its Dimensions is $ML^2T^{-3}A^{-2}$
- Reciprocal of resistance is known as conductance.

The resistance of a conductor depends on the following factors

1. Length -

As $R = \rho \frac{l}{A}$

So Resistance of a conductor is directly proportional to its length

i.e. $R \ \alpha \ l$

2. Area of cross-section-

As $R = \rho \frac{l}{A}$

The resistance of a conductor is inversely proportional to its area of cross-section

i.e. $R \ \alpha \ \frac{1}{A}$

3. Material of the conductor-

As $R = \rho \frac{l}{A}$

And For a conductor, if n = No. of free electrons per unit volume in the conductor, τ= relaxation time then the resistance of the conductor

Then $\rho = \frac{m}{ne^2\tau}$

for different conductors n is different

And ρ depends on n

So R is also different.

4. Temperature-

As $R=\rho\frac{l}{A}$

And $\rho=\frac{m}{ne^2\tau}$

So $R\ \alpha\ \frac{1}{\tau}$

And as temperature increases τ decrease

So as the temperature increases resistance increases

Temperature-dependent resistance is given by

$R_T=R_{T_0}\left[1+\alpha\left[T-T_0\right]\right]$

R_T- Resistance at temperature T

R_0- Resistance at temperature T_o

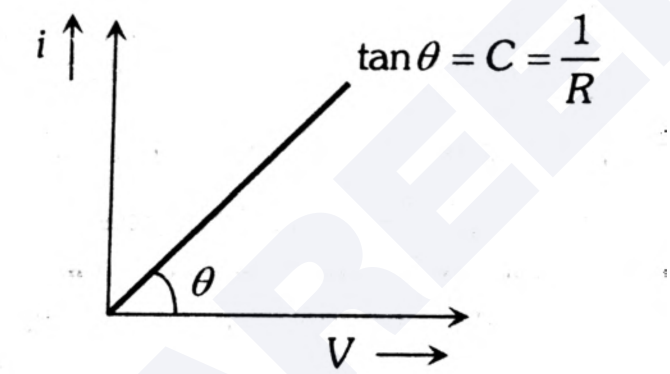
$\alpha-$ temperature coefficient of resistance

$\alpha=\frac{R_T-R_o}{R_o\left(T-T_o\right)}$

Where the value of α is different at different temperatures

Type of substances according to Ohm's law

1. Ohmic Substance: The substance which obeys Ohm's law are known as Ohmic substance. I-V graph is linear and the slope gives conductance which is reciprocal of resistance



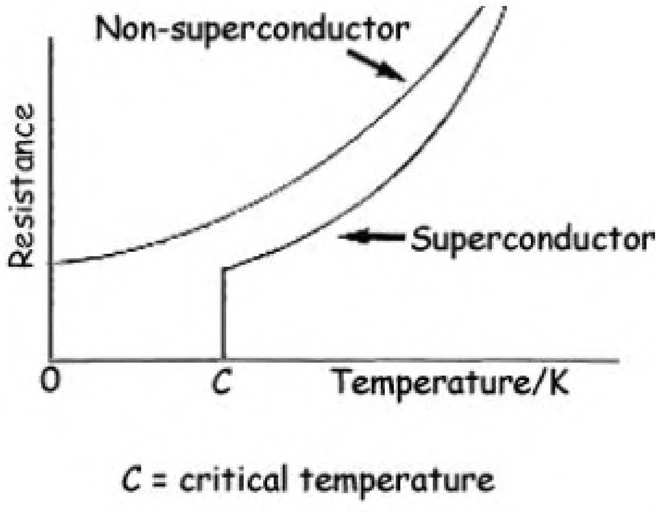
2. Non-ohmic substances

Those substances which don't obey Ohm's law are known as Non-ohmic or non-linear conductors.

For example gases, crystal rectifiers, etc.

3. Superconductor: For certain materials, resistivity suddenly becomes zero below a certain temperature (critical temperature). The material in this state is called a superconductor.

In Superconductor, resistivity is zero



Resistivity or Specific Resistance (ρ) -

As $R = \rho \frac{l}{A}$

If $l = 1m$ and A= 1 m^2

Then R=ρ

Resistivity is numerically equal to the resistance of a substance having a unit area of cross-section and unit length.

- Where m is the mass, n is the number of electrons per unit volume, e is the charge of electron and τ is the relaxation time

  Then $\rho = \frac{m}{ne^2\tau}$

- S.I Unit - Ohm.m

- Dimensions-$ML^3T^{-3}A^{-2}$

And as reciprocal of Resistivity is known as conductivity.

So the dimension of conductivity is $M^{-1}L^{-3}T^3A^2$

- Resistivity is independent of the shape and size of the body as it is an intrinsic property of the substance.

The resistivity of a conductor depends on the following factors

1. Nature of the body-

As $\rho = \frac{m}{ne^2\tau}$

for different conductors n is different

And ρ depends on n

Soρ is also different.

2. Temperature-dependent Resistivity :

$\rho = \rho_o\left(1 + \alpha\left(T - T_o\right)\right)$

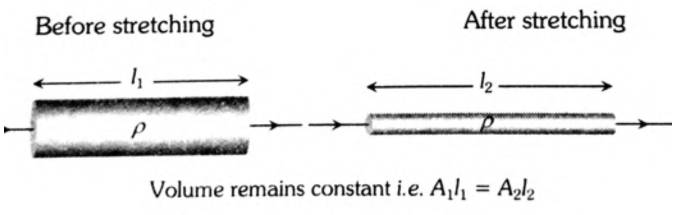
ρ : Resistivity at temperature T

ρ_0: Resistivity at the temperature T_0

3. Resistivity increases with impurity and mechanical stress.

6. Stretching of wire

If a conducting wire stretches its length increases area of cross-section decreases but the volume remains constant



Suppose for a conducting wire before stretching

it's length = l_1, area of cross-section =A_1, radius =r_1, diameter = d_1,

and resistance $R_1 = \rho\frac{l_1}{A_1}$

After stretching length =l_2, area of cross-section =A_2, radius = r_2, diameter = d_2

and resistance $R_2 = \rho\frac{l_2}{A_2}$

So $\frac{R_1}{R_2} = \frac{l_1}{l_2} * \frac{A_2}{A_1}$

But Volume is constant So

$$A_1l_1 = A_2l_2$$

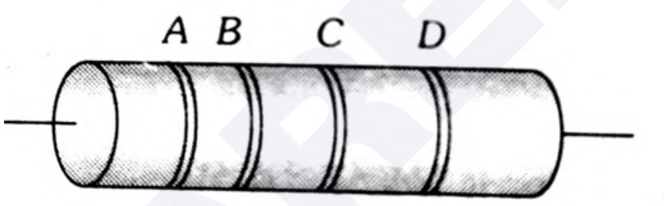
$$\Rightarrow \frac{l_1}{l_2} = \frac{A_1}{A_2}$$

Now $\frac{R_1}{R_2} = \frac{l_1}{l_2} * \frac{A_2}{A_1} = (\frac{l_1}{l_2})^2 = (\frac{A_2}{A_1})^2 = (\frac{r_2}{r_1})^4 = (\frac{d_2}{d_1})^4$

- If a wire of resistance R and length l is stretched to length nl, then new resistance of wire is n^2R

7. Colour coding of Resistance

The carbon resistance has normally four coloured rings bands A, B, C and D.



Colour bands A and B Indicates the decimal figure.

Colour band C Indicates the decimal multiplier.

Colour band D Indicates the tolerance in per cent about the indicated value. Tolerance represents the percentage accuracy of the indicated value.

- **To remember the sequence of colour code**

B B ROY Great Britain Very Good Wife

Tolerance of Gold is ± 5 %

Tolerance of Silver is ± 10%

Tolerance if no colour ± 20%

Letters as an aid to memory	Colour	Figure (A, B)	Multiplier (C)
B	Black	0	10^0
B	Brown	1	10^1
R	Red	2	10^2
O	Orange	3	10^3
Y	Yellow	4	10^4
G	Green	5	10^5
B	Blue	6	10^6
V	Violet	7	10^7
G	Grey	8	10^8
W	White	9	10^9

8.Heat and Power developed in a resistor

Heat developed in a resistor: When a steady current flows through a resistance R for time t, the loss in electric potential energy appears as increased thermal energy(Heat H) of resistor and $H = i^2Rt$

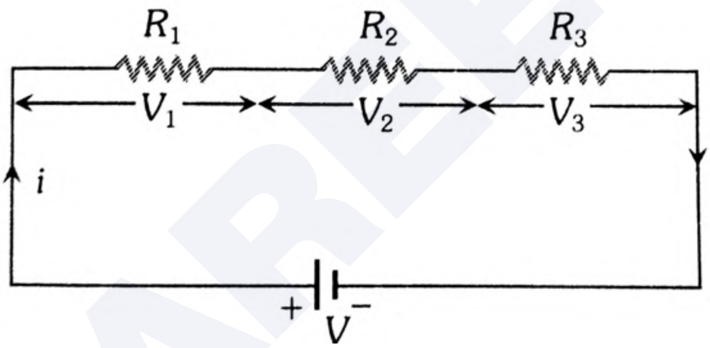
The power developed = $\frac{energy}{time} = i^2R = iR = \frac{V^2}{R}$

Unit of heat is the joule (J)

Unit of power is watt (W)

9. Series grouping of Resistance

In this case , Potential drop is different across each resistor and Current is the same



$R_{eq} = R_1 + R_2 + R_3 + \cdots + R_n$

$R_{eq} =$ Equivalent Resistance

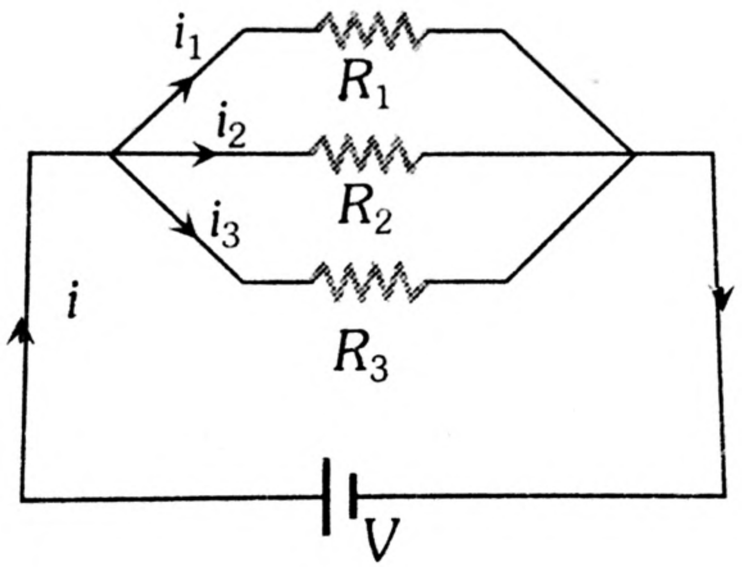
For n identical resistance: $R_{eq} = nR$

$V' = \frac{V}{n}$

10.Parallel Grouping of Resistance

In this case,

Potential is Same across each resistors and current is different

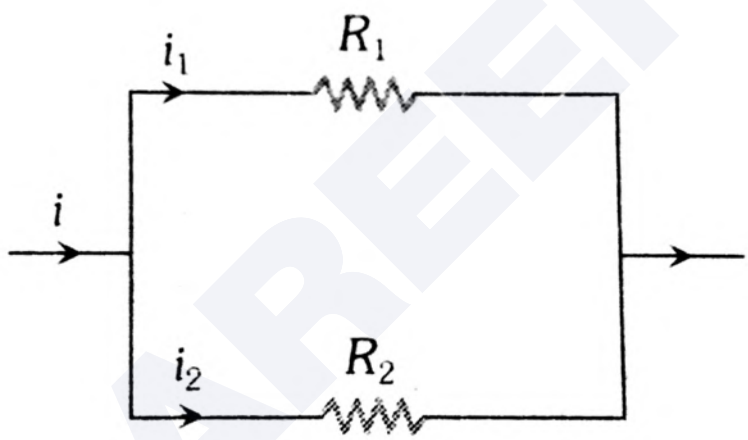


$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}$$

If two resistances are in Parallel:

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

Current through any resistance:



$$i' = i\left(\frac{Resistance\ of\ opposite\ Branch}{total\ Resistance}\right)$$

The required current of the first branch $i_1 = i\left(\frac{R_2}{R_2 + R_2}\right)$

The required current of the second branch $i_2 = i\left(\frac{R_1}{R_1 + R_2}\right)$

11. Cell

- Cell

The device which converts Chemical energy into electrical energy is known as an electric cell.

It maintains the flow of charge inside the circuit by supplying energy.

- **The direction of flow of current:**

1. inside the cell is from negative to positive electrode
2. while outside the cell is from positive to negative electrode.

- **Electromotive force (Emf) of a cell:**

It is the work done / energy carried by unit charge passing through one complete cycle of the circuit.

E=W/q

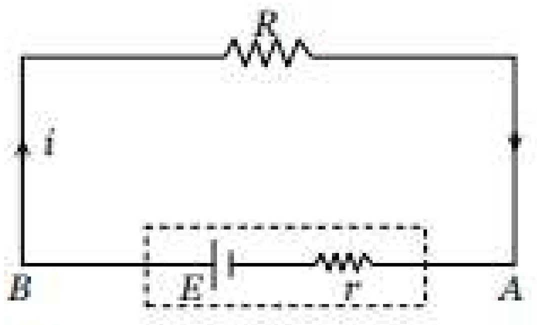
Unit of emf is volt.

The emf of cell is also known as potential difference across the terminals of a cell when it is not giving any current.

Note-Cell is a source of constant emf but not constant current.

- **Potential difference (V)-**

It is the work done / energy carried by unit charge passing through external part (excluding the cell) of the circuit.



Potential difference is equal to the product of current and resistance of that given part.

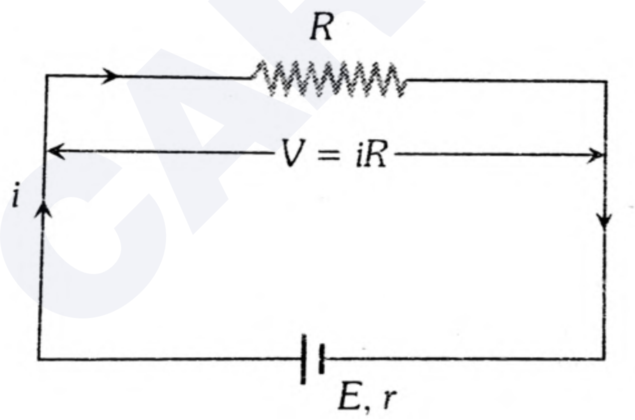
i.e. $V_{AB} = iR.$

- **Internal resistance-**

In case of a cell the opposition of electrolyte to the flow of current through it is the internal resistance of the cell. It is shown by r.

1. The internal resistance of a cell depends on the distance between electrodes i.e $r \propto d$
2. The internal resistance of a cell depends on the area of the electrodes i.e $r \propto A$
3. Internal resistance of a cell depends on the concentration of electrolyte i.e $r \propto c$
4. Internal resistance of a cell depends on the temperature of the electrolyte i.e $r \propto \frac{1}{temp}$

- **The current supplied by the cell-**



Cell supplies a constant current in the circuit.

i.e $i = \frac{E}{R+r}$,

$R-$ External resistance

r- internal resistance

Potential drop inside the cell=ir

The internal resistance of the cell

$r = (\frac{E}{V} - 1)R$

Power dissipated in external resistance

$P = \left(\frac{E}{R+r}\right)^2 R$

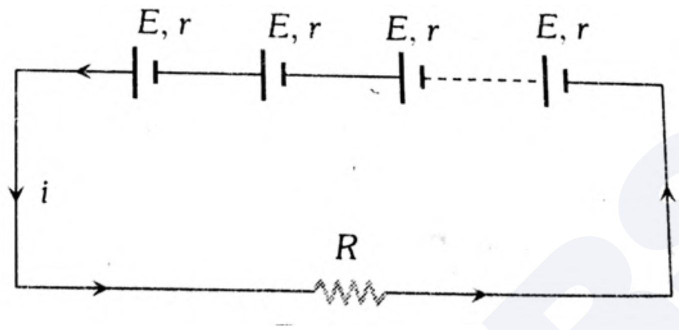
Maximum power is obtained when the resistance value of the load is equal in value to that of the voltage source's internal resistance.

Maximum power $P_{max} = \frac{E^2}{4r}$

12. Grouping of cell

Series grouping of cells:

In series grouping anode of one cell is connected to the cathode of other cells

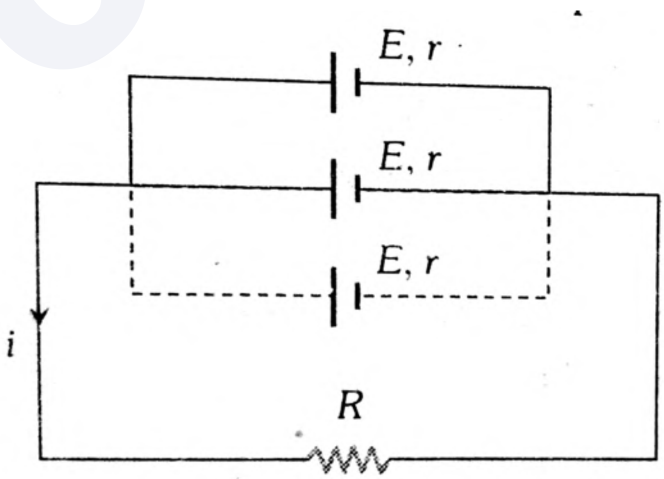


$n =$ identical cells which are connected in series, then

- Equivalent e.m.f of combination is $E_{eq} = nE$
- Equivalent internal resistance $r_{eq} = nr$
- Main current / current from each cell $i = \frac{nE}{R + nr}$
- Power dissipated in the external circuit is $\left(\frac{nE}{R+nr}\right)^2 \cdot R$
- Conditions for Maximum Power is $R = nr$
- $P_{max} = n(\frac{E^2}{4r})$ when $nr << R$

Parallel grouping of cell-

In parallel grouping, all anodes are connected to one point and all cathode together at other points



For n cells connected in parallel

Equivalent e.m.f $E_{eq} = E$

Equivalent internal resistance $R_{eq} = \frac{r}{n}$

The main current is

$$i = \frac{E}{R + \frac{r}{n}}$$

The potential difference across the external resistance

$V = iR$

Current from each cell

$$i' = \frac{i}{n}$$

The power dissipated in the circuit

$$P = \left(\frac{E}{R + \frac{r}{n}}\right)^2 \cdot R$$

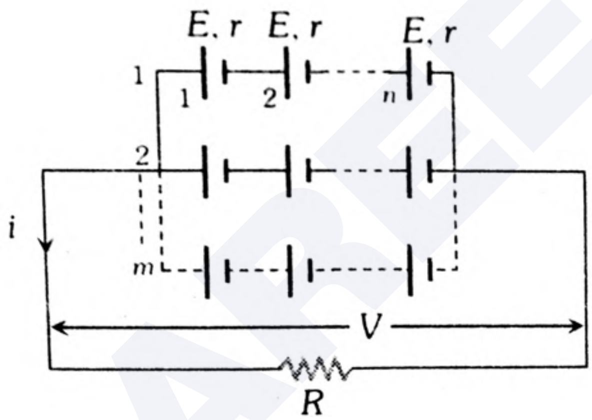
Condition for Maximum Power

$$R = \frac{r}{n}$$

$P_{max} = n\left(\frac{E^2}{4r}\right)$ when $r >> nR$

Mixed grouping of cells-

if n identical cells are connected in a row and such m rows are connected in parallel



Equivalent e.m.f is $E_{eq} = nE$

Equivalent internal resistance $r_{eq} = \frac{nr}{m}$

The main current flowing through the load $i = \frac{nE}{R + \frac{nr}{m}} = \frac{mnE}{mR + nr}$

The potential difference across load $V = iR$

The potential difference across each cell $V' = \frac{V}{n}$

Current from each cell $i' = \frac{i}{m}$

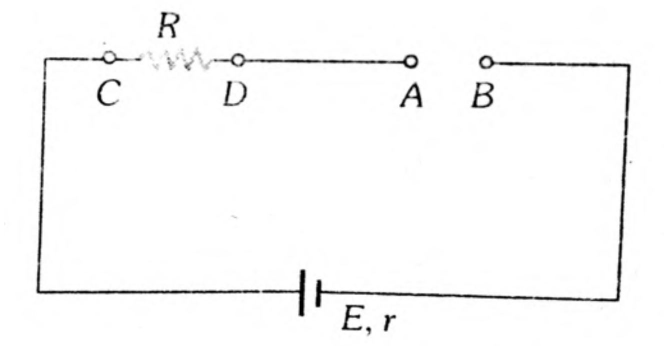
Condition for Maximum Power $R = \frac{nr}{m}$

$$P_{max} = (mn)\frac{E^2}{4r}$$

mn - the total number of cells.

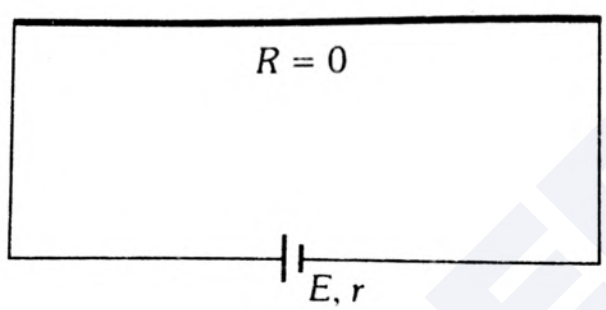
Open circuit and Short circuit-

Open circuit:-



- no current is taken from the cell. $i = 0$
- The potential difference between A and B i.e $V_{AB} = E$
- The potential difference between C and D i.e $V_{CD} = 0$

Short circuit-



- Two terminals of a cell are joined together by a thick conducting wire
- Maximum current $i = \frac{E}{r}$
- $V = 0$

Emf of a cell when the cell is charging and discharging-

1. When supplying the current: $E = V + iR$
2. When the cell is being charged: $E = V - iR$

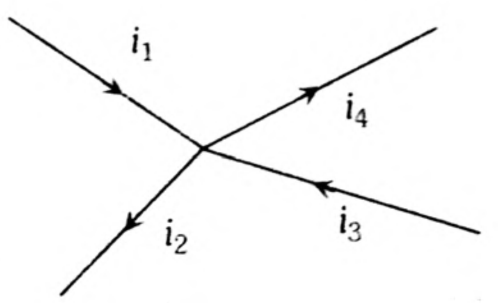
13.Kirchhoff laws-

1. **Kirchoff's first law-**

In a circuit, at any junction, the sum of the currents entering the junction must equal the sum of the currents leaving the junction.

This law is also known as the Junction rule or current law (KCL)

$\sum i = 0$



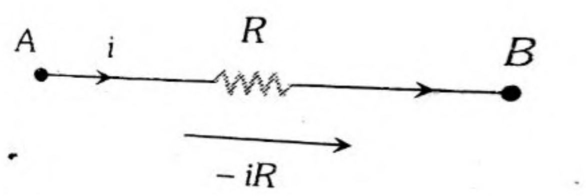
i.e $i_1 + i_3 = i_2 + i_4$

2. Kirchoff's second law-

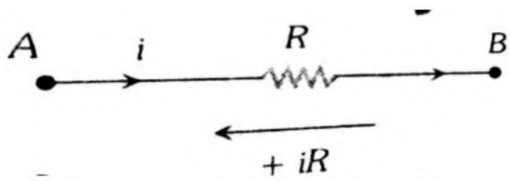
The algebraic sum of all the potential across a closed loop is zero.

This law is also known as Kirchhoff's Voltage law (KVL)

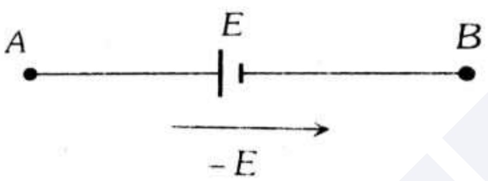
- Change in Potential in traversing a resistance is -iR



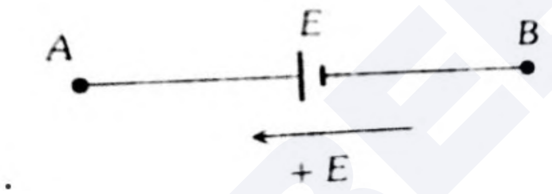
- Change in Potential in the opposite direction is iR



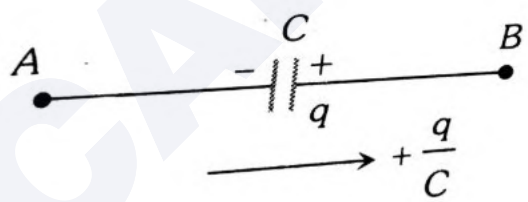
- Traversing an e.m.f source from negative to positive terminal is -E



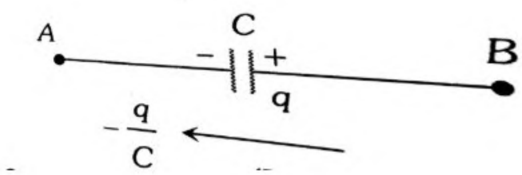
- While in the opposite direction is E



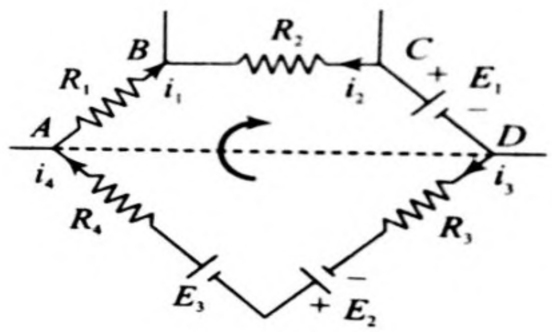
- Change in Potential in traversing a capacitor from negative to positive $\frac{q}{C}$



- While in the opposite direction is $-\frac{q}{C}$



- In closed-loop



$-i_1R_1 + i_2R_2 - E_1 - i_3R_3 + E_2 + E_3 - i_4R_4 = 0$

Conservation of charge and Energy-

KCL is simply based on the conservation of charge. That is charge cannot accumulate at a junction.

KVL is based on the conservation of energy. That is energy supplied by the source will be equal to the energy consumed by the circuit elements

$\sum V = 0$

14. Galvanometer,Ammeter and Voltmeter

Galvanometer-

It is an instrument used to detect small current passing through it by showing deflection.

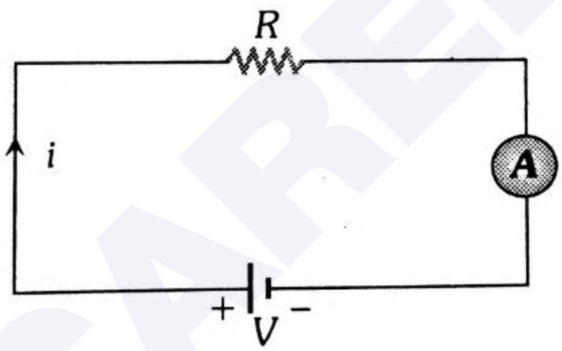
Example for types of galvanometers are

Moving coil galvanometer

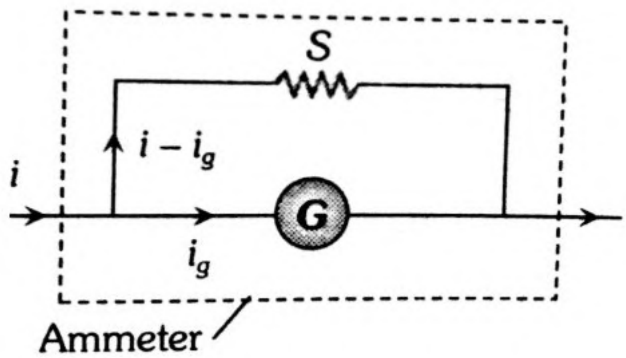
Moving magnet galvanometer

Ammeter-

It is a device used to measure current and always connected in series with the circuit



- In the above circuit, A represents the ammeter.
- Conversion of galvanometer into ammeter: Connect a low resistance (shunt) in parallel to the galvanometer.

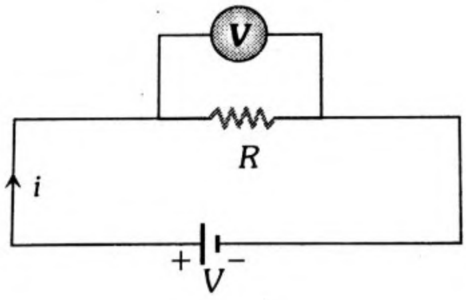


- The equivalent resistance of the combination is $\frac{Gs}{G+s}$ where G is the galvanometer resistance and s is the shunt resistance.
- Required shunt $s=\frac{i_g G}{(i-i_g)}$. Where i is the total current, i_g is the galvanometer current
- To pass n^{th} part of main current through galvanometer that is If $i_g=i/n$

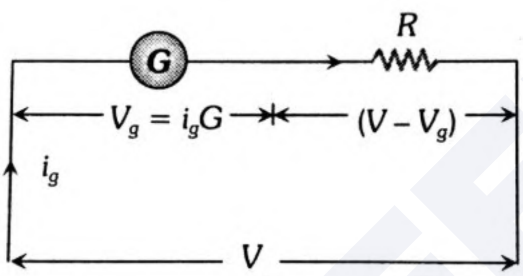
Then the required shunt resistance $s=\frac{G}{(n-1)}$

Voltmeter-

It is a device used to measure Potential difference and is always put in parallel with the circuit element whose voltage is to be measured



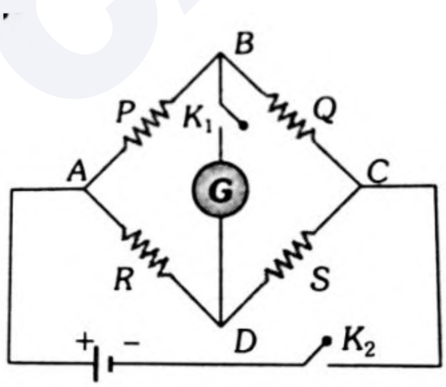
Conversion of galvanometer into voltmeter: by Connecting a large Resistance R in series.



- The equivalent resistance of the combination is $G+R$
- The required value of high resistance to be connected in series with the galvanometer is $R=\frac{V}{I_g}-G=\left[\frac{V}{V_g}-1\right]G$. V is the total voltage applied across the circuit and V_g is the total voltage drop across the galvanometer.
- if n^{th} part of the applied voltage across galvanometer, that is $V_g=V/n$, then $R=(n-1)G$.

15.Wheatstone's bridge and Meter Bridge

Wheatstone's Bridge-



It is an arrangement of four resistances which can be used to measure one of them in terms of rest. Let R is an adjustable resistance, Pand Q are known resistance and S is an unknown resistance. To obtain valur of S we will close the keys K_1 and K_2 and will adjust R to obtain null deflection in the galvanometer. Then the balancing condition of bridge is

$$\frac{P}{Q}=\frac{R}{S}$$

Case I-

$V_B = V_D$

(Balanced condition)

No current will flow through the galvanometer

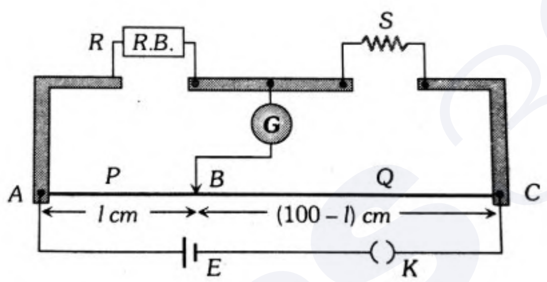
Case II-

unbalanced condition: $V_B > V_D$

$(V_A - V_B) < (V_A - V_D)$

Current will flow from A to B

Meter bridge -



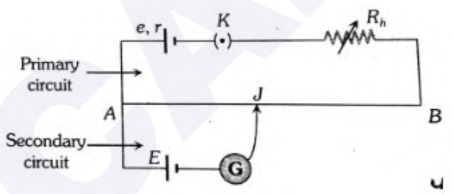
- It is used to find the resistance of a given wire using a meter bridge and hence determine the specific resistance of its materials.
- It works on the principle of Wheatstone's bridge
- The meter bridge arrangment is shown in the above figure. The wire connected between A and C is of 1 merer length and have uniform crossection. A constant current is passed through the wire AC so that potential of the wire is proportional to the length of the wire. R is the known resistance which is selected from the resistance box. S is the unknown resistance whose value can be measured. The arrangment also has a galvanometer with a jokey
- We will slide the galvanometer through wire AC so as to obtain null deflection in the galvanometer. Let B is the point on AC where null deflection is obtained and length AB=l. Pand Q resistance of the portion AB and BC respectively then then by principle of Wheatstone's bridge

$$\frac{P}{Q}=\frac{R}{S} \Rightarrow S=\frac{(100-l)}{l}R$$

16. Potentiometer and its applications

Potentiometer-

- The potentiometer is a device which does not draw current from the given circuit and still measures the potential difference.



Potentiometer consists of wire of length 5 to 10 meters arranged on a wooden block as parallel strips of wires with 1-meter length each and end of wires are joined by thick coppers. The wire has a uniform cross-section and is made up of the same material. A driver circuit that contains a rheostat, key, and a voltage source with internal resistance r. The driver circuit sends a **constant current (I)** through the wire.

Potential across the wire AB having length L is given as V=IR, Where R is the resistance of the wire AB

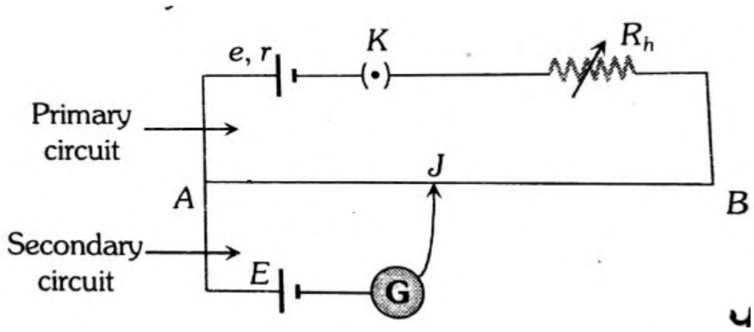
Since the driver circuit sends a constant current (I) through the wire So $V \quad \alpha \quad R$

Using $R=\frac{\rho L}{A}$ we can say that $R \quad \alpha \quad L$ since area and resistivity are constant.

Therefore we get V is proportional to length. I.e $V \propto L$

The secondary circuit contains cell/resistors whose potential is to be measured. Whose one end is connected to a galvanometer and another end of the galvanometer is connected to a jockey which is moved along the wire to obtain a point where there is no current through the galvanometer. So that potential of the secondary circuit is proportional to the length at which there is no current through the galvanometer. This is how the potential of a circuit is measured using the potentiometer.

Calibration of potentiometer-



In the potentiometer, a battery of known emf E is connected in the secondary circuit. A constant current I is flowing through AB from driver circuit (that is circuit above AB). The jockey is slide on potentiometer wire AB to obtain null deflection in the galvanometer. Let l be the length at which galvanometer shows null deflection.

Since the potential of wire AB (V) is proportional to the length AB(L).

Similarly $E \quad \alpha \quad l$

So we get

$$\frac{V}{E} = \frac{L}{l}$$
$$V = E\frac{L}{l}$$

Thus we obtained the potential of wire AB when a constant current is passing through it. This is known as calibration.

Potential gradient-

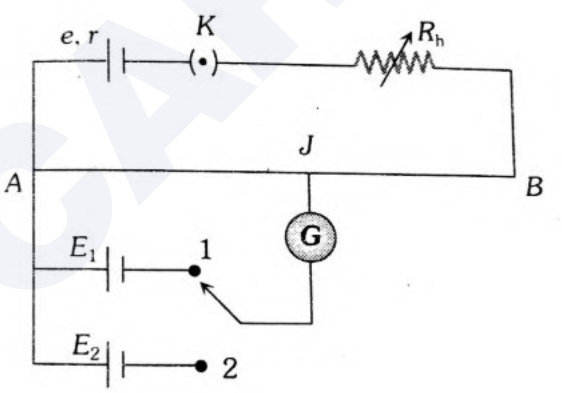
The potential difference per unit length of wire i.e $x = \frac{V}{L}$

or Using $V = iR = \left(\frac{e}{R + R_h + r}\right)R$

$$x = \frac{V}{L} = \frac{e}{(R + R_h + r)}\frac{R}{L}$$

Applications of potentiometer-

1. Comparison of emf of cell-



For the above figure

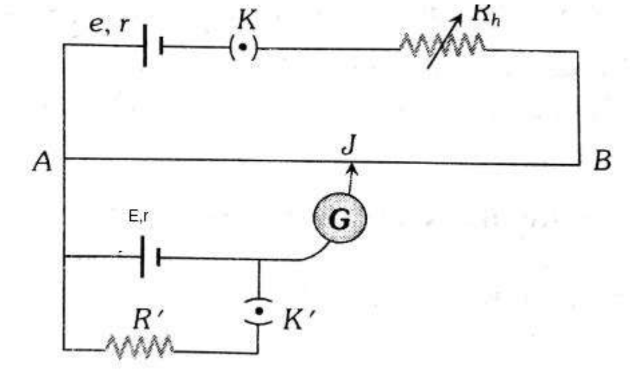
l_1 is the balancing length obtained when cell with emf E_1 is included in the secondary circuit. That is key is at position 1. l_2 is the balancing length obtained when cell with emf E_2 is included in the secondary circuit. That is key is at position 2.

So since $E \quad \alpha \; l$ we get

$$\frac{E_1}{E_2} = \frac{l_1}{l_2}$$

With the help of the above ratio, we can compare the emf of these cells.

2.Determine the internal resistance of a cell-



Note-The cell in the secondary circuit has emf E and internal resistance r

Here l_1 is the balancing length obtained when key K' is open that is we include only the cell in the secondary circuit. So corresponding potentials of wire of balancing length l_1 is E. And we know that $E \quad \alpha \quad l_1.....(1)$

Similarly l_2 is the balancing length obtained when key K' is closed that is both cell and R' is connected in the secondary circuit.So corresponding potentials of wire of balancing length l_2 is V.

And we know that $V \quad \alpha \quad l_2....(2)$

or we can say that

$$IR' \quad \alpha \quad l_2$$
$$\Rightarrow \frac{E}{r+R'} * R' \quad \alpha \quad l_2....(3)$$

So taking ratio of equation (1) to equation (2)

we get

$$\frac{E}{V} = \frac{l_1}{l_2}$$

$$\frac{E}{\frac{ER'}{r+R'}} = \frac{l_1}{l_2}$$

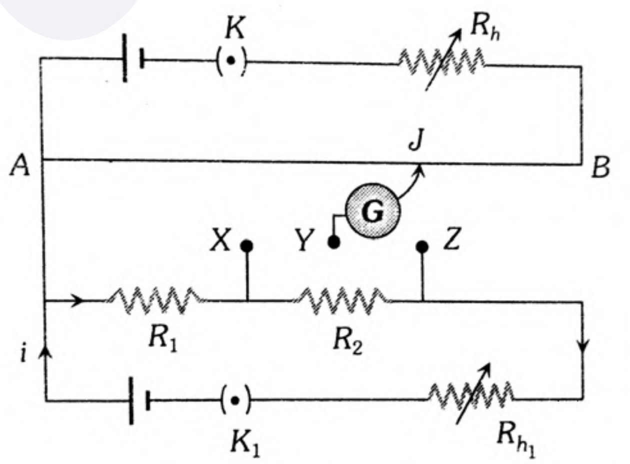
$$\Rightarrow \frac{r+R'}{R'} = \frac{l_1}{l_2}$$

Then the internal resistance is given by

$$r = (\frac{l_1 - l_2}{l_2})R'$$

or $r = (\frac{E}{V} - 1)R'$

3.Comparison of resistances-



The balance point is at a length l_1 cm from A when jockey J is plugged in between Y and X, while the balance point is at a length l_2 cm from A when jockey J is plugged in between Y and Z.

Then we get a ratio of resistances as

$$\frac{R_2}{R_1} = \frac{l_2 - l_1}{l_1}$$

With the help of this ratio, we can compare these resistances.

17. Faraday's laws of electrolysis

Faraday's first law-

According to the Faraday's first law, "The amount of substance or quantity of chemical reaction at electrode is directly proportional to the quantity of electricity passed into the cell".

$$\mathrm{W \text{ or } m \propto q}$$

$$\mathrm{W \propto It}$$

$$\mathrm{W = ZIt}$$

$$\mathrm{Z = \frac{M}{nf}}$$

$$\mathrm{Z = \frac{M}{nf}}$$

$$\mathrm{Z = Electrochemical\ equivalence}$$

$$\mathrm{M\ =\ molarmass}$$

$$\mathrm{F = 96500}$$

$$\mathrm{n\ = Number\ of\ electrons\ transfer}$$

$$\mathrm{q\ =\ amount\ of\ charge\ utilized}$$

Electrochemical equivalent is the amount of the substance deposited or liberated by one-ampere current passing for one second (that is, one coulomb, I x t = Q or one coulomb of charge.
One gram equivalent of any substance is liberated by one faraday.

$$\mathrm{Eq.\ Wt. = Z \times 96500}$$

$$\mathrm{\frac{W}{E} = \frac{q}{96500}}$$

$$\mathrm{w = \frac{E.q}{96500}}$$

$$\mathrm{W = \frac{Eit}{96500}}$$

As w = a x 1 x d that is, area x length x density
Here a = area of the object to be electroplated
d = density of metal to be deposited
l = thickness of layer deposited
Hence from here, we can predict charge, current strength time, thickness of deposited layer etc.

NOTE: One faraday is the quantity of charge carried by one mole of electrons.

$$\mathrm{E\ \alpha\ Z}$$
$$\mathrm{E = FZ}$$
$$\mathrm{1F = 1.6023 \times 10^{-19} \times 6.023 \times 10^{23}}$$
$$\mathrm{= 96500\ Coulombs}$$

Faraday's second law-

According to Faraday's second law, "When the same quantity of electricity is passed through different electrolytes, the amounts of the products obtained at the electrodes are directly proportional to their chemical equivalents or equivalent weights".

$$\text{As } \frac{W}{E} = \frac{q}{96500} = \text{No of equivalents constant}$$

So

$$\frac{E_1}{E_2} = \frac{M_1}{M_2} \text{or} \frac{W_1}{W_2} = \frac{Z_1}{Z_2 It} = \frac{Z_1}{Z_2}$$

E_1 = equivalent weight mass

E_2 = equivalent weight mass

W or M = mass deposited

From this law, it is clear that 96500 coulomb of electricity gives one equivalent of any substance.

Application of Faraday's Laws

- It is used in electroplating of metals.
- It is used in the extraction of several metals in pure form.
- It is used in the separation of metals from non-metals.
- It is used in the preparation of compounds

NOTE:
Current Efficiency: It is the ratio of the mass of the products actually liberated at the electrode to the theoretical mass that could be obtained

$$\text{C.E.} = \frac{\text{desired extent}}{\text{Theoretical extent of reaction}} \times 100\%$$

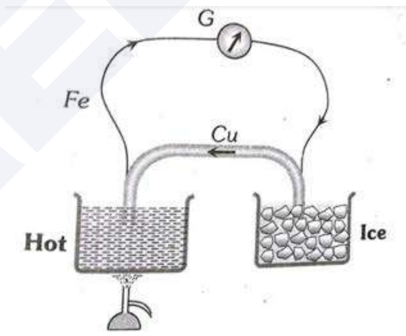
18.Thermo Couple

Thermocouple

Thermocouple: Two wires of different metals connected at two points to form two junctions. This thermoelectric device used to measure the temperature is called a thermocouple. If one junction of the thermocouple is at lower temperature and the other is at a higher temperature then a current starts flowing through the thermocouple.

Seeback Effect -

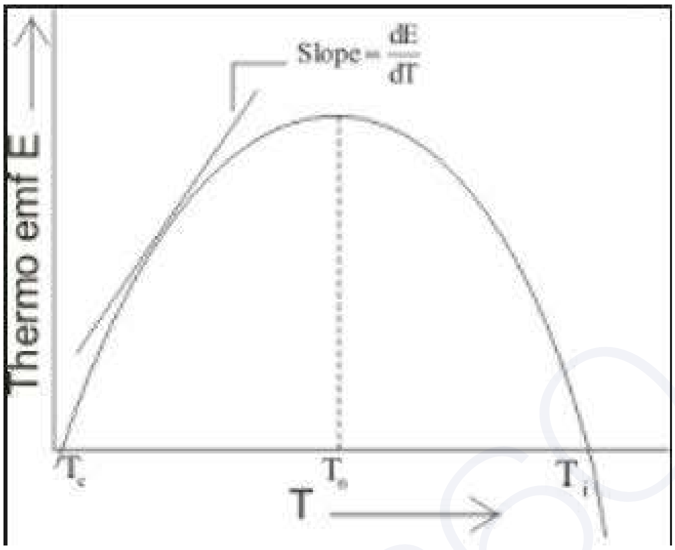
According to this when the two junctions of a thermocouple are kept and maintained at different temperatures, then a current starts flowing through the loop made by conductors known as thermo-electric current. Because of this potential difference will develop between the junctions which is called thermo electric emf which is of the order of a few micro-volts per degree temperature difference.



Seebeck arranged different metals in the decreasing order of their electron density. Few metals forming the series are as below.

Sb, Fe, Cd, Zn, Ag, Au, Cr, Sn, Pb, Hg, Mn, Cu, Pt, Co, Ni, Bi

Neutral temperature:



Keeping the temperature of cold junction constant and increasing the temperature of the hot junction, the emf increases and become maximum at a particular temperature. This temperature of the hot junction is called neutral temperature(θ_n). If the temperature is further increased the thermal emf start decreasing and at a particular temperature, thermal emf becomes zero. If the temperature is further increased the thermal emf start reversing. The temperature of the hot junction at which the thermal emf start reversing is known as inversion temperature(θ_i)

$$\Theta_n = \frac{\Theta_i + \Theta_c}{2}$$

$\Theta_n =$ Neutral Temperature

$\Theta_i =$ Inversion Temperature

$\Theta_c =$ Cold Temperature

Thermo electric emf is given by the equation -

$$E = \alpha t + \frac{1}{2}\beta t^2$$

where α and β are thermo electric constant (t = temperature of hot junction).

For E to be maximum at $t=t_n$, we will differentiate Electric field with respect to temperature of the hot junction and we get -

$$\frac{dE}{dt} = 0 \text{ i.e. } \alpha + \beta t_n = 0 \Rightarrow t_n = -\frac{\alpha}{\beta}$$

If the temperature of hot junction increases beyond neutral temperature, then there is decrease in the thermo emf and at a particular temperature it becomes zero, if heat is supplied further, the direction of emf is reversed. This temperature of hot junction is called temperature of inversion (t_i).

$$t_n = \frac{t_i + t_c}{2}$$

Here, t_c is the temperature of cold junction.

19.Charging of capacitor and inductor-

- **Charging of a Capacitor:**

  When a capacitor with zero charges is connected to a battery of emf V through connecting wires, total resistance including internal resistance of the battery and of the connecting wires be R then after a time t let the charge on capacitor be q, current be i and $V_c = \frac{q}{C}$,

  charge deposited on the positive plate in time dt is $dq = idt$
  so that $i = \frac{dq}{dt}$

  Using Kirchhoffs loop law, $\frac{q}{C} + Ri - V = 0$

  or, $Ri = E - \frac{q}{C}$

or, $R\frac{dq}{dt} = \frac{VC-q}{C}$

or, $\int_0^q \frac{dq}{VC-q} = \int_0^t \frac{1}{CR}dt$

or, $-\ln\frac{VC-q}{VC} = \frac{t}{CR}$

or, $1 - \frac{q}{VC} = e^{-t/CR}$

or, $q = VC\left(1 - e^{-t/CR}\right)$

Where RC is the time constant (τ) of the circuit.

At $t = \tau = RC$

$$q = CV\left(1 - \frac{1}{e}\right) = 0.63CV$$

- **Discharging of capacitors:**

If initially a capacitor has a charge Q and is discharged through an external load. Let after a time t the remaining charge in the capacitor be q then

Using Kirchhoff's loop law,

$$\frac{q}{C} - Ri = 0$$

Here $i = -\frac{dq}{dt}$ because the charge q decreases as time passes.

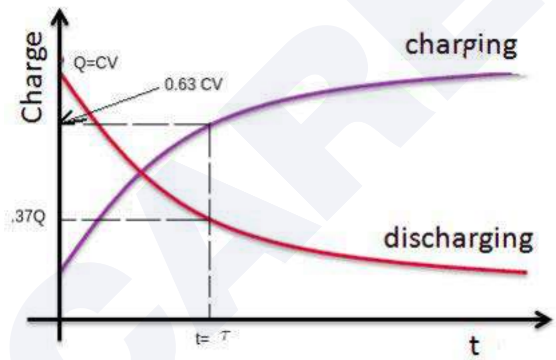
Thus, $R\frac{dq}{dt} = -\frac{q}{C}$

or, $\frac{dq}{q} = -\frac{1}{CR}dt$

or, $\int_Q^q \frac{dq}{q} = \int_0^t -\frac{1}{CR}dt$

or, $\ln\frac{q}{Q} = -\frac{t}{CR}$

or, $q = Qe^{-t/CR}$



- Note: At a steady-state capacitor connected to the DC battery acts as an open circuit. The capacitor does not allow a sudden change in voltage.

Charging and discharging of an inductor:

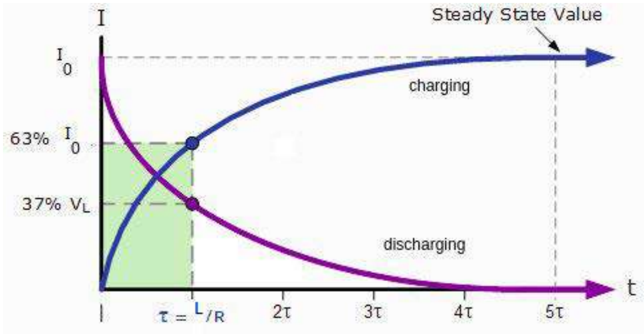
When an inductor is connected to a DC source of emf V through a resistance R the inductor charges to maximum current $(i_0 = \frac{V}{R})$ at steady state. If the inductor current is increased from zero at time=0 to i at time=t then-current i is given by

$$i = i_0(1 - e^{\frac{-t}{\tau}})\ where\ \tau = \frac{L}{R}$$

τ is the time constant of the circuit. Here the current is exponentially increasing.

- While discharging of inductor current decreases exponentially and is given by

$$i = i_0(e^{\frac{-t}{\tau}})$$



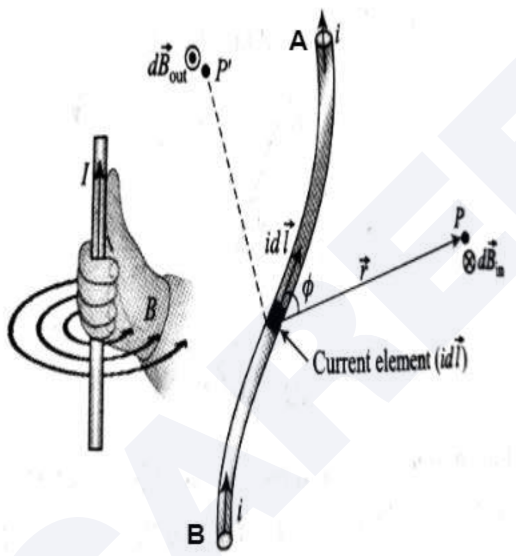
- Note: At steady state, an inductor connected to the DC battery acts as a short circuit. The inductor does not allow a sudden change in current.

Moving Charges and Magnetism

Important Formulae

1. Biot-Savart Law:-

- If a point charge q is kept at rest near a current-carrying wire, It is found that no force acts on the charge. It means a current-carrying wire does not produce an electric field.
- However, if the charge q is projected in the direction of the current with velocity v, then it is deflected towards the wire (q is assumed positive). There must be a field at P that exerts a force on the charge when it is projected, but not when it is kept at rest. This field is different from the electric field which always exerts a force on a charged particle whether it is at rest or in motion. This new field is called the magnetic field and is denoted by the symbol B. The force exerted by a magnetic field is called magnetic force.



According to Biot Savart's Law, the magnetic induction dB at point P due to the elemental wire segment AB as shown in the figure depends upon four factors which are given as

(i) dB is directly proportional to the current in the element.

$$dB \propto I$$

(ii) dB is directly proportional to the length of the element

$$dB \propto dl$$

(iii) dB is inversely proportional to the square of the distance r of the point P from the element

$$dB \propto \frac{1}{r^2}$$

Combining the above factors, we have

$$dB \propto \frac{I dl \sin \theta}{r^2}$$

$$dB = K \frac{I dl \sin \theta}{r^2}$$

Where K is a proportionality constant and its value depends upon the nature of the medium surrounding the current carrying wire. Its SI Units its value is given as

$$K = \frac{\mu_0}{4\pi} = 10^{-7}\ \mathrm{T-m/A}$$

Here, i is the current, $d\vec{l}$ is the length-vector of the current element and $\vec{r}$ is the vector joining the current element to the point P and θ is the angle between $d\vec{l}$ and $\vec{r}$.

μ_0 is called the permeability of vacuum or free space. Its value is $4\pi \times 10^{-7}$T-m/A.

The magnetic field at a point P , due to a current element in vacuum, is given by:

Vector form: $$d\vec{B} = \frac{\mu_0}{4\pi}\frac{(id\vec{l} \times \vec{r})}{r^3}$$

Scalar form: $$dB = \frac{\mu_0}{4\pi}\frac{idl\sin\theta}{r^2}$$

For medium other than vacuum, μ_0 will be replaced by μ

$\mu = \mu_0 \times \mu_r$

where, μ_r is the relative permeability of the medium (also known as the diamagnetic constant of the medium)

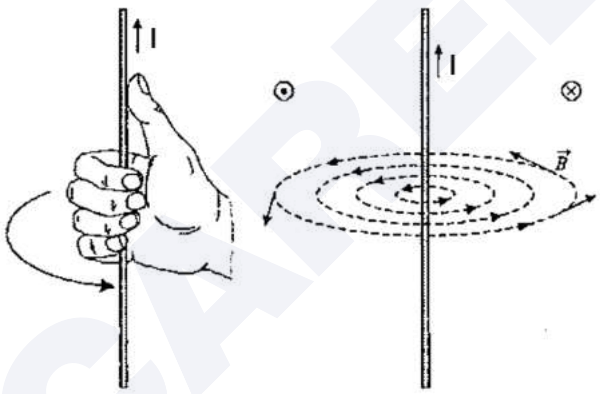
Direction of magnetic field:

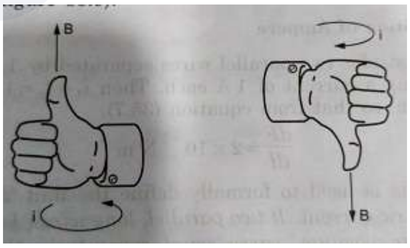
1. The rule of cross product

The direction of the field is perpendicular to the plane containing the current element and the point P according to the rules of cross-product. If we place the stretched right-hand palm along $d\vec{l}$ in such a way that the fingers curl towards $\vec{r}$, the cross product $d\vec{l} \times \vec{r}$ is along the thumb. Usually, the plane of the diagram contains both $d\vec{l}$ and $\vec{r}$. The magnetic field $d\vec{B}$ is then perpendicular to the plane of the diagram, either going into the plane or coming out of the plane. We denote the direction going into the plane by an encircled cross and the direction coming out of the plane by an encircled dot.

2. Right hand thumb rule

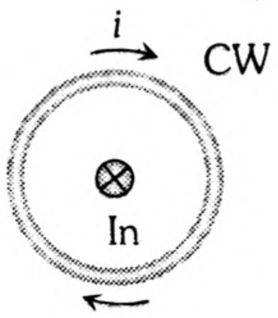
The direction of this magnetic induction is given by right hand thumb rule stated as "Hold the current carrying conductor in the palm of the right hand so that the thumb points in the direction of the flow of current, then the direction in which the fingers curl, gives the direction of magnetic field lines"



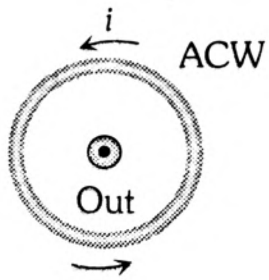


Various Cases:

Case 1. If the current is in a clockwise direction then the direction of the magnetic field is away from the observer or perpendicular inwards.



Case 2. If the current is in an anti-clockwise direction then the direction of the magnetic field is towards the observer or perpendicular outwards

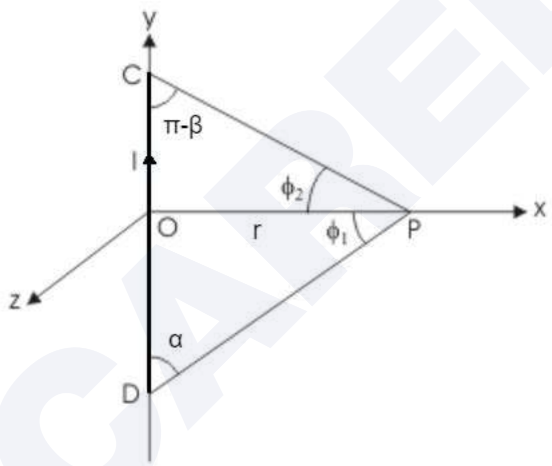


2. Magnetic Field due to current in a straight wire:

Magnetic field lines around a current-carrying straight wire are concentric circles whose centre lies on the wire.

Magnetic field due to a current-carrying wire at a point P which lies at a perpendicular distance r from the wire, as shown, is given as:

$$B=\frac{\mu_0}{4\pi}\frac{I}{r}(\sin\phi_1+\sin\phi_2)$$

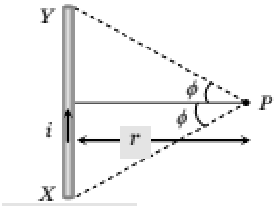


From figure, $\alpha=(90^\circ-\phi_1)$ and $\beta=(90^\circ+\phi_2)$

Hence, it can be also written as $B=\frac{\mu_o}{4\pi}\frac{I}{r}(\cos\alpha-\cos\beta)$

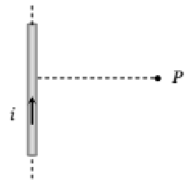
Different cases:

Case 1: When the linear conductor XY is of finite length and the point P lies on it's perpendicular bisector as shown



$$B = \frac{\mu_0}{4\pi} \cdot \frac{i}{r}(2\sin\phi)$$

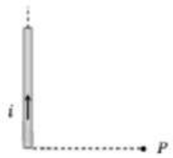
Case 2: When the linear conductor XY is of infinite length and the point P lies near the centre of the conductor



$\phi_1 = \phi_2 = 90^o$.

$$B = \frac{\mu_0}{4\pi}\frac{i}{r}[\sin 90^\circ + \sin 90^\circ] = \frac{\mu_0}{4\pi}\frac{2i}{r}$$

Case 3: When the linear conductor is of semi-infinite length and the point P lies near the end Y or X



$\phi_1 = 90^o$ and $\phi_2 = 0^o$

$$B = \frac{\mu_0}{4\pi}\frac{i}{r}[\sin 90^\circ + \sin 0^\circ] = \frac{\mu_0}{4\pi}\frac{i}{r}$$

Case 4: When point P lies on the axial position of the current-carrying conductor then magnetic field at P,

$\alpha = \beta = 0^o$

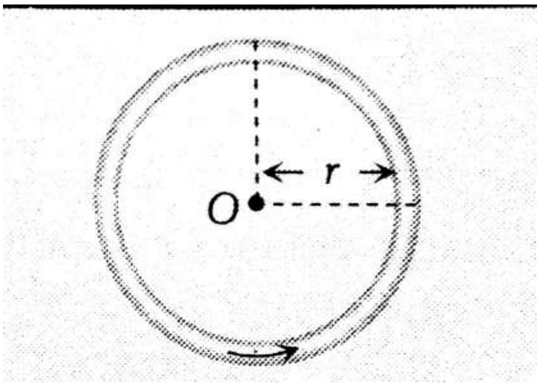
$$B = \frac{\mu_o}{4\pi}\frac{i}{r}(\cos\alpha - \cos\beta) = \frac{\mu_o}{4\pi}\frac{i}{r}(\cos 0 - \cos 0) = 0$$

Note:

- The value of magnetic field induction at a point, on the centre of separation of two linear parallel conductors carrying equal currents in the same direction, is zero.
- If the direction of current in the straight wire the known then the direction of the magnetic field produced by a straight wire carrying current is obtained by maxwell's right-hand thumb rule.

3. Magnetic Field due to circular current loop at its centre:

Magnetic Field due to circular coil at Centre-



Consider a circular coil of radius a and carrying current I in the direction shown in Figure. Suppose the loop lies in the plane of the paper.

B = The magnetic field at the centre O of the coil.

and $\vec{r}$ is the position vector of point O from the current element.

$$\therefore B=\frac{\mu_0 I}{4\pi r^2}2\pi r$$
$$\therefore B=\frac{\mu_0 I}{2r}$$

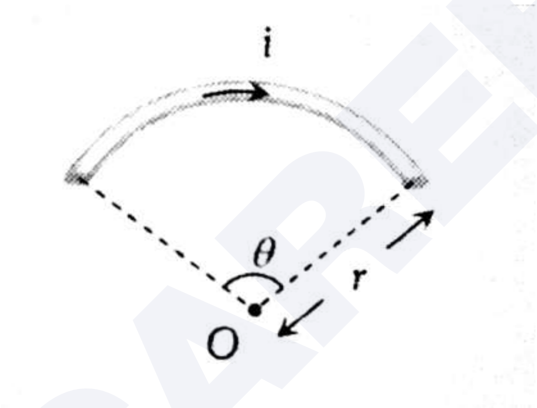
For N turns,

$$B_0=B_{Centre}=\frac{\mu_0}{4\pi}\frac{2\pi Ni}{r}=\frac{\mu_0 Ni}{2r}$$

where N=number of turns, i= current and r=radius of a circular coil.

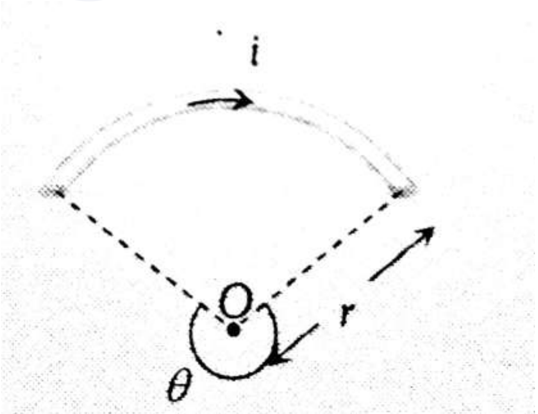
Magnetic field due to a current-carrying circular arc

Case 1: Arc subtends angle theta at the centre as shown below then $B_0=\frac{\mu_0}{4\pi}\frac{i\theta}{r}$

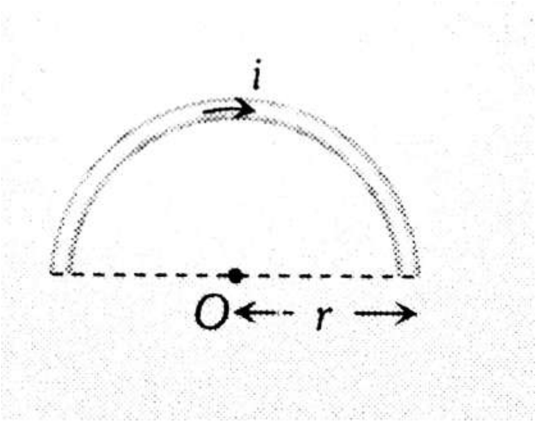


where the angle θ is in radians.

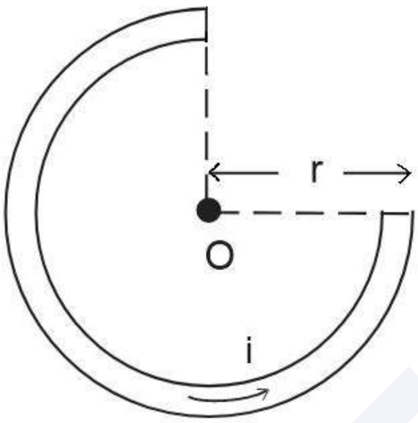
Case 2: Arc subtends angle $(2\pi-\theta)$ at the centre then $B_0=\frac{\mu_0}{4\pi}.\frac{(2\pi-\theta)i}{r}$



Case 3: The magnetic field of the Semicircular arc at the centre is $B_0=\frac{\mu_o}{4\pi}\frac{\pi i}{r}=\frac{\mu_o i}{4r}$

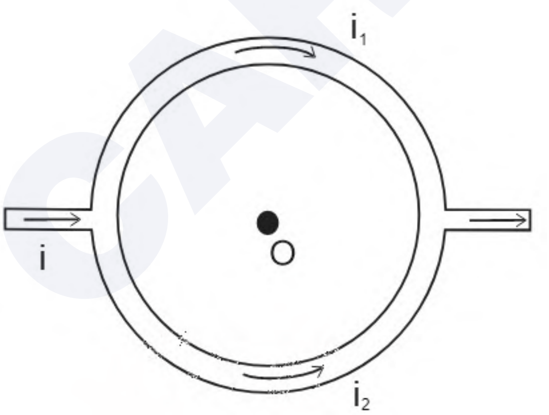


Case 4: Magnetic field due to three-quarter Semicircular Current-Carrying arc at the centre $B_0=\frac{\mu_o}{4\pi}\frac{(2\pi-\frac{\pi}{2})i}{r}$

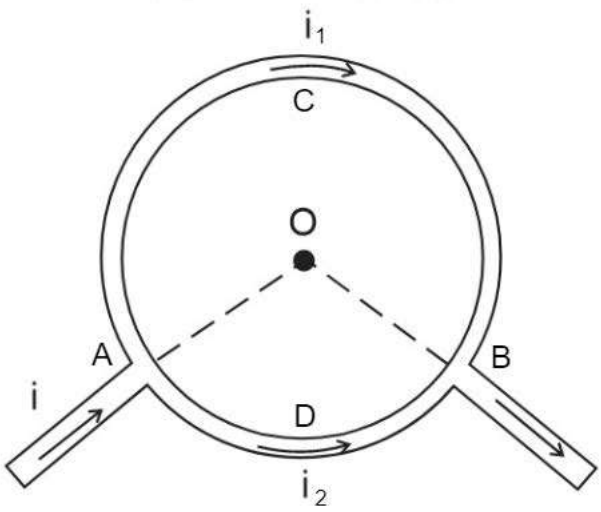


Special cases

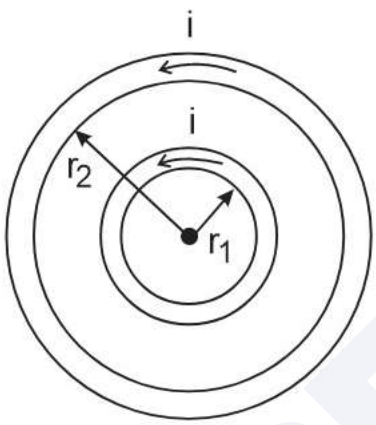
1. If the Distribution of current across the diameter then $B_0=0$



2. If Current between any two points on the circumference then $B_0=0$



3. Concentric co-planar circular loops carrying the same current in the Same Direction-

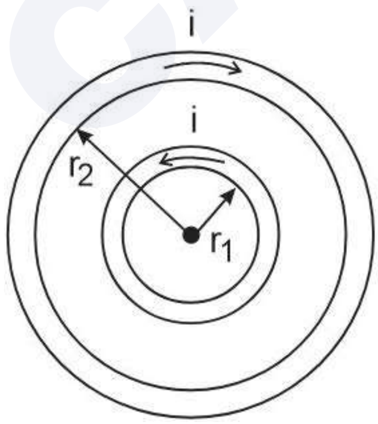


$$B_{centre} = \frac{\mu_o}{4\pi}\,(2\pi i)\left[\frac{1}{r_1} + \frac{1}{r_2}\right]$$

If the direction of currents are the same in concentric circles but have a different number of turns then

$$B_{centre} = \frac{\mu_o}{4\pi}\,(2\pi i)\left[\frac{n_1}{r_1} + \frac{n_2}{r_2}\right]$$

4. Concentric co-planar circular loops carrying the same current in the opposite Direction

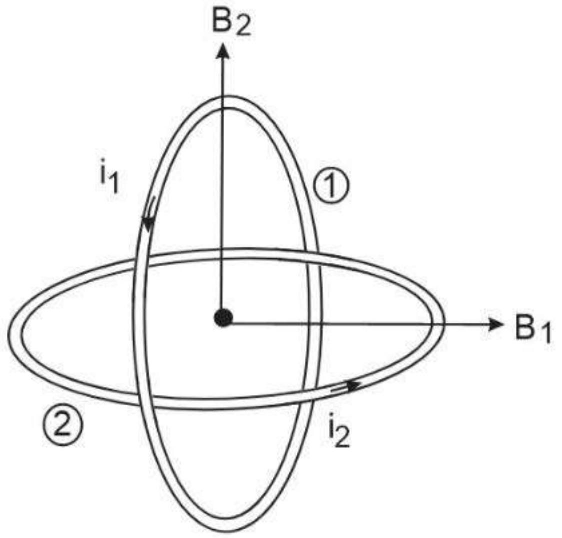


$$B_{centre} = \frac{\mu_o}{4\pi}\,(2\pi i)\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

If the number of turns is not the same i.e $n_1 \neq n_2$

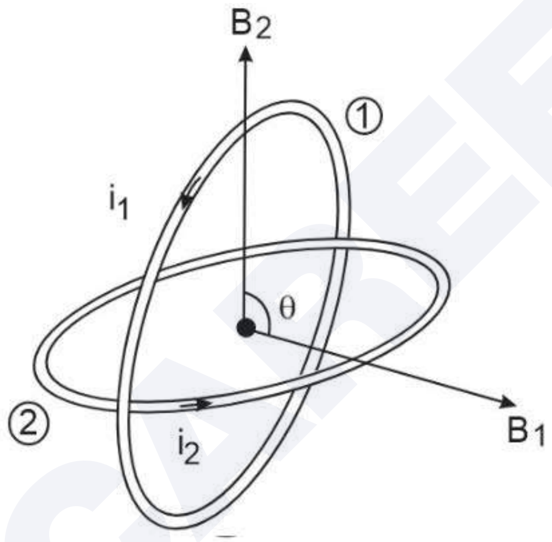
$$B_{centre} = \frac{\mu_o}{4\pi}(2\pi i)\left[\frac{n_1}{r_1} - \frac{n_2}{r_2}\right]$$

5. Concentric loops but their planes are perpendicular to each other



Then $B_{net} = \sqrt{B_1{}^2 + B_2{}^2}$

6. Concentric loops but their planes are at an angle θ with each other



$$B_{net} = \sqrt{B_1{}^2 + B_2{}^2 + 2B_1B_2\cos\theta}$$

4. Magnetic Field On The Axis Of Circular Current Loop-

In the figure, it is shown that a circular loop of radius R carrying a current I.

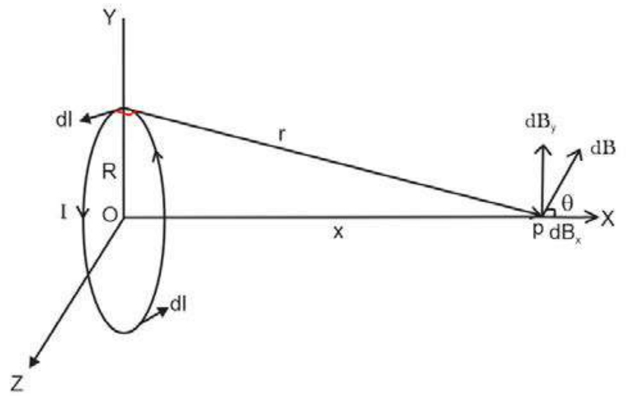
Application of Biot-Savart law to a current element of length dl at angular position θ with the axis of the coil.

the current in the segment $d\ell$ causes the field $d\bar{B}$ which lies in the x-y plane as shown below.

Another symetric $d\bar{\ell}'$ element that is diametrically opposite to previously $d\ell$ element cause $d\vec{B}'$.

Due to symmetry the components of $d\vec{B}$ and $d\vec{B}'$ perpendicular to the x-axis cancel each other. i.e., these components add to zero.

The x-components of the $d\vec{B}$ combine to give the total field $\vec{B}$ at point P.



We can use the law of Biot-Savart to find the magnetic field at a point P on the axis of the loop, which is at a distance x from the center.

$d\vec{\ell}$ and $\hat{r}$ are perpendicular and the direction of field $d\vec{B}$ caused by this particular element $d\vec{\ell}$ lies in the x-y plane.

we get

$$\Rightarrow B_{axis} = \frac{\mu_0 I R^2}{2\left(x^2+R^2\right)^{3/2}} \text{ (on the axis of a circular loop)}$$

- If x>>R, then $B = \frac{\mu_0 I R^2}{2x^3}$.
- At centre , $x = 0 \Rightarrow B_{\text{centre}} = \frac{\mu_0}{4\pi} \cdot \frac{2\pi \mathrm{N}i}{R} = \frac{\mu_0 N i}{2R} = B_{\max}$

5. Ampere's Circuital Law And Its Applications

Ampere's circuital law-

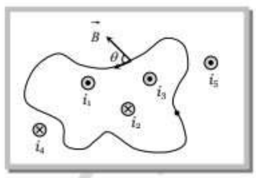
Amperes law is also a method to calculate the magnetic field due to a given current distribution like Biot-Savart's law.

Statement: The line integral of the magnetic field $\vec{B}$ around any closed curve is equal to μ_o times the total current i passing through the area enclosed by the curve.

$$\text{Mathematical statement: } \oint \vec{B} d\vec{l} = \mu_0 \sum i = \mu_0 \left(i_1 + i_3 - i_2\right)$$

Also using $\vec{B} = \mu_0 \vec{H}$ $\quad \ldots . (where \vec{H} = magnetising\ field)$

$$\oint \mu_0 \vec{H} \cdot \overrightarrow{dl} = \mu_0 \Sigma i \Rightarrow \oint \vec{H} \cdot \overrightarrow{dl} = \Sigma i$$



Fingers are curled in the loop direction, the current in the direction of the thumb is taken as positive whereas in the direction opposite to that of the thumb is taken as negative.

Now, we can see that the total current crossing the above area is $\left(i_1+i_3-i_2\right)$, so any current outside the given area will not be considered. So we have to assume $(\text{ Outward } \odot \rightarrow +ve, \text{ Inward } \otimes \rightarrow -ve)$

General guidelines for the selection of Ampere's path for its application in different situations

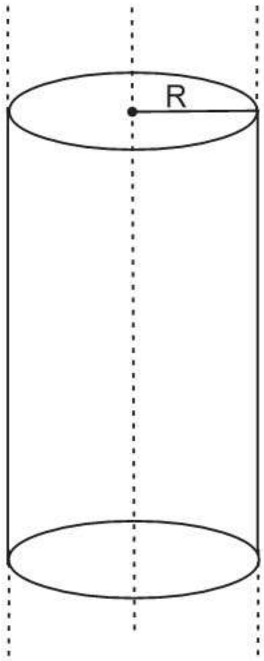
(i) Path should be chosen in such a way that at every point of the path magnetic induction should be either tangential to the path elements or normal to it so that the 'dot' product can be easily handled.

(ii) Path should be chosen in such a way that at every point of the path magnetic induction should either be uniform or zero so that calculations become easy.

Application of Ampere's law (To find magnetic field due to various bodies)-

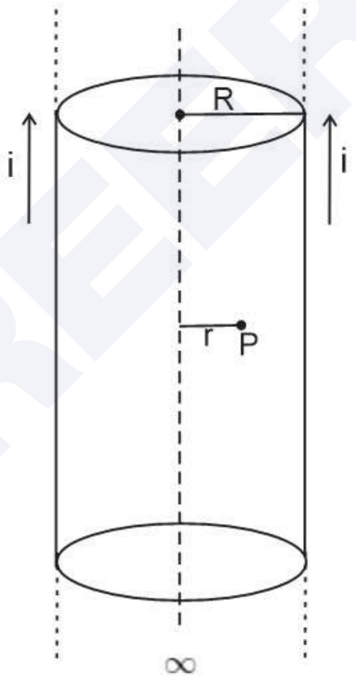
1. Magnetic field due to infinitely long cylindrical wire due to current 'i' flowing through its surface only -

Let us consider an infinitely long cylindrical wire of radius R and the current is distributed on the surface of the wire, then this wire will behave as a hollow cylindrical wire.

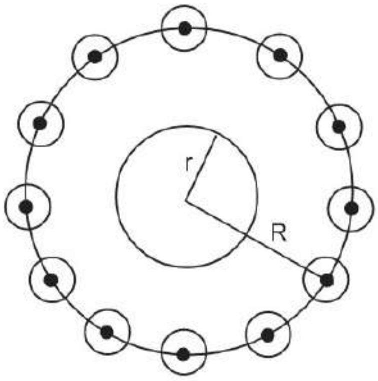


Now let us take different situations -

a) For a point inside the wire - (r<R)



From the top view, the Ampere's loop will look like this -

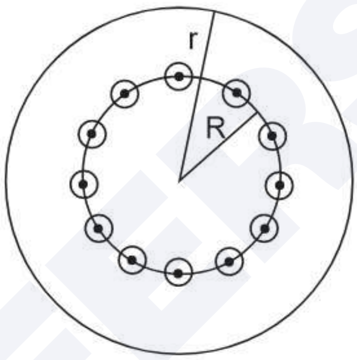


Since there is no current inside the Ampere's loop, so there will be no magnetic field in this loop because -

$$\oint \vec{B}.\vec{dl} = \mu_0 i$$

b) For a point outside the wire (r>R) -

Then from the top, it can be seen as -



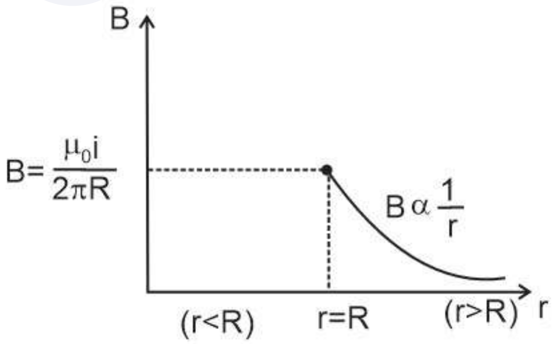
It is just like the concept of a current carrying wire which we have studied in the last concept with the help of Ampere's circuital law as well as by Biot-savart law. So again by applying same Ampere's circuital law we can deduce that -

$$B = \frac{\mu_o i}{2\pi r}$$

c) On the surface (r=R)

$$B_s = \frac{\mu_o i}{2\pi r}$$

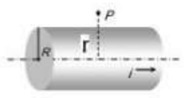
From the above equations, we can plot a graph between B and different positions 'r'.



2. Magnetic field due to infinitely long cylindrical wire due to current 'i' distributed uniformly across its cross-section -

Magnetic field due to a cylindrical wire is obtained by the application of Ampere's law. Here also we consider few cases one by one -

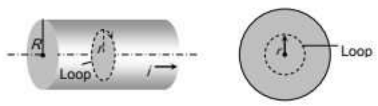
a) Outside the cylinder -



It is just like the concept of a current carrying wire which we have studied in the last concept with the help of Ampere's circuital law as well as by Biot-savart law. So again by applying same Ampere's circuital law we can deduce that -

$$B=\frac{\mu_o i}{2\pi r}$$

b) Inside the solid cylinder : Current enclosed by loop (i') is lesser then the total current (i) -



Since the current density will remain same. So,

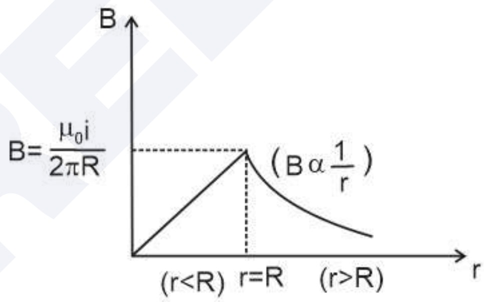
$$J=J' \Rightarrow \quad i=i\times\frac{A'}{A}=i\left(\frac{r^2}{R^2}\right)$$

$$\text{Hence at inside point } \oint \overrightarrow{B_{\text{in}}}\cdot d\vec{l}=\mu_0 i \Rightarrow B=\frac{\mu_0}{2\pi}\cdot\frac{ir}{R^2}$$

c) At surface (r=R) -

$$B_s=\frac{\mu_o i}{2\pi r}$$

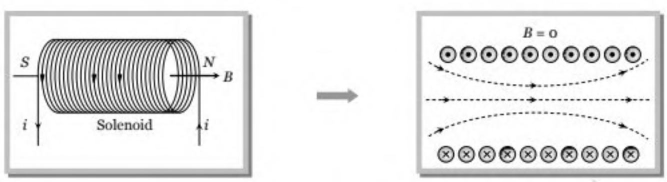
The variation of B with r can be drawn as -



6. Solenoid

Solenoid -

A solenoid is defined as a cylindrical coil of many tightly wound turns of insulated wire with a general diameter of the coil smaller than its length. The solenoid have two ends and one end behaves like the north pole while the opposite end behaves like the south pole. As the length of the solenoid increases, the interior field becomes more uniform and the external field becomes weaker which can be seen from the diagram.



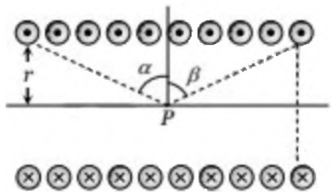
As the current flows a magnetic field is produced around and within the solenoid. The magnetic field within the solenoid is uniform and parallel to the axis of solenoid. Here we will discuss two cases, one with solenoid having finite length and other when the solenoid is of infinite length.

(i) Finite-length solenoid :

Let n = number of turns per unit length $\frac{N}{L}$

whrere, N = total number of turns,

l = length of the solenoid



The magnetic field inside the solenoid at point P is given by -

$$B=\frac{\mu_0}{4\pi}(2\pi ni)[\sin\alpha+\sin\beta]$$

(ii) Infinite length solenoid -

If the solenoid is of infinite length and the point is well inside the solenoid. So in this case the angle $\alpha \ and \ \beta$ will be $\frac{\pi}{2}$. So if we put this value in the equation of finite length you will get -

$$B_{in}=\mu_0 ni$$

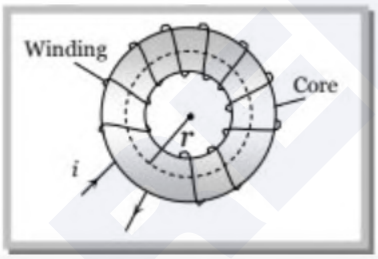
Here again, n = number of turns per unit length.

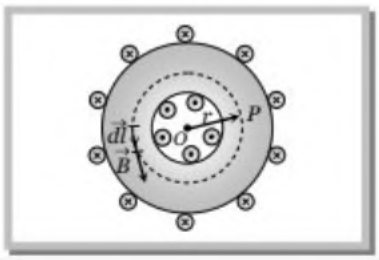
Note - **The magnetic** field outside the solenoid is zero.

7. Toroid

Toroid -

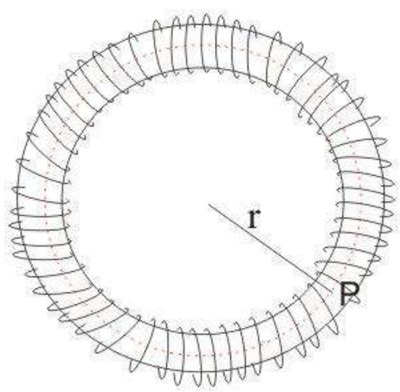
If we try to bend a solenoid in the form of a ring then the obtained shape is a Toroid. So, a toroid can be considered a ring-shaped closed solenoid. Hence it is like an endless cylindrical solenoid. From the given figure we can understand Toroid much better.





Now to obtain the magnetic field by a toroid, let us consider a toroid having N turns.

Here, we will now apply Ampere circuital law to calculate the magnetic field of a toroid. Suppose we have to find the magnetic field B at a point P inside the toroid as shown below in figure -



Let us take an amperian loop which is a circle through point P and concentric inside the toroid. By symmetry, the field will have equal magnitude at all points of this circle and this field is tangential to every point in the circle
Thus,

$$\oint B.dl = \mu_0 \text{NI}$$
or,

$$2\pi r \text{B} = \mu_0 \text{NI}$$
or,

$$\text{B} = \frac{\mu_0 \text{NI}}{2\pi r}$$

From the above result, B varies with r i.e. field B is not uniform over the cross-section of the core because as we increase 'r' the B varies.

8. Force On A Moving Charge In Magnetic Field

The magnetic force on a free moving charge is perpendicular to both the velocity of the charge and the magnetic field with direction given by the right hand rule. The force is given by the charge times the vector product of velocity and magnetic field.

The force is always perpendicular to both the magnetic field and velocity.

$$F = qvB \sin\theta$$
$$F = qvB \text{ if } \theta = 90$$

If the velocity is perpendicular to the magnetic field then the force is given by the simple product :

Force = charge x velocity x B-field

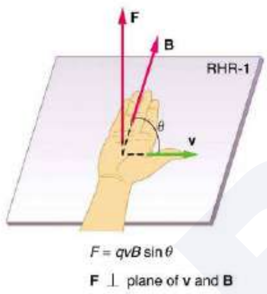
Right-hand rule:

The magnitude of the *magnetic force F* on a charge *q* moving at a speed *v* in a magnetic field of strength *B* is given by

$F = qvB \sin\theta$,

where θ is the angle between the directions of v and B. This force is often called the Lorentz force. In fact, this is how we define the magnetic field strength *B*—in terms of the force on a charged particle moving in a magnetic field. The SI unit for magnetic field strength *B* is called the tesla (T). Therefore magnetic field strength is given as :

$$B = \frac{F}{qv \sin\theta}$$



The unit of Tesla is :

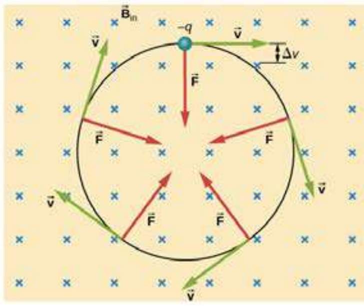
$$1\text{T} = \frac{1\text{N}}{\text{C} \cdot \text{m/s}} = \frac{1\text{N}}{\text{A} \cdot \text{m}}$$

- The *direction* of the force on a moving charge is given by the right-hand rule . Point the thumb of the right hand in the direction of *v*, the fingers in the direction of *B*, and perpendicular to the palm points in the direction of *F*.
- The force is perpendicular to the plane formed by ***v*** and ***B***. Since the force is zero if ***v*** is parallel to ***B***, charged particles often follow magnetic field lines rather than cross them.

9. Motion Of A Charged Particle In Uniform Magnetic Field

1.If the velocity is perpendicular to the magnetic field

In the figure a negatively charged particle moves in the plane of the paper in a region where the magnetic field is perpendicular to the paper. The magnetic force is perpendicular to the velocity, so velocity changes in direction but not magnitude. The result is uniform circular motion. Note that because the charge is negative, the force is opposite in direction to the prediction of the right-hand rule.



In this situation, the magnetic force supplies the centripetal force $F_c = \frac{mv^2}{r}$. Noting that the velocity is perpendicular to the magnetic field, the magnitude of the magnetic force is reduced to $F = qvB$. Because the magnetic force *F* supplies the centripetal force F_c we have,

$$qvB = \frac{mv^2}{r}$$

Solving for *r* gives

$$r = \frac{mv}{qB}$$

Here, *r* is the radius of curvature of the path of a charged particle with mass *m* and charge *q*, moving at a speed *v* that is perpendicular to a magnetic field of strength *B*. The time for the charged particle to go around the circular path is defined as the period, which is the same as the distance traveled (the circumference) divided by the speed. Based on this and the Equation, we can derive the period of motion as:

$$T = \frac{2\pi r}{v} = \frac{2\pi}{v}\frac{mv}{qB} = \frac{2\pi m}{qB}$$

Therefore frequency of revolution is

$$\frac{1}{T} = \frac{qB}{2\pi m}$$

This frequency is called the cyclotron frequency.

2. If the velocity is not perpendicular to the magnetic field

$$F_c = \frac{mv^2}{r}$$

The radius of curvature as

$$r = \frac{mv}{qB}$$

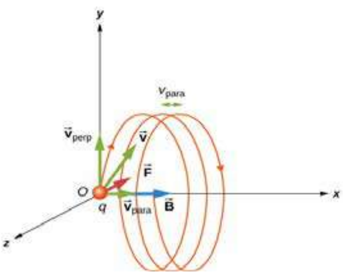
If the velocity is not perpendicular to the magnetic field, then we can compare each component of the velocity separately with the magnetic field. The component of the velocity perpendicular to the magnetic field produces a magnetic force perpendicular to both this velocity and the field:

$$v_{perp} = v\sin\theta$$
$$v_{para} = v\cos\theta$$

where θ is the angle between *v* and *B*. The component parallel to the magnetic field creates constant motion along the same direction as the magnetic field, also shown in Equation. The parallel motion determines the pitch p of the helix, which is the distance between adjacent turns. This distance equals the parallel component of the velocity times the period:

$$p = v_{para}T$$

This results in a helical motion, as shown in the following figure:



While the charged particle travels in a helical path, it may enter a region where the magnetic field is not uniform. In particular, suppose a particle travels from a region of strong magnetic field to a region of weaker field, then back to a region of stronger field. The particle may reflect back before entering the stronger magnetic field region. This is similar to a wave on a string traveling from a very light, thin string to a hard wall and reflecting backward. If the reflection happens at both ends, the particle is trapped in a so-called magnetic bottle.

The radius of helical path

$$r = \frac{m(v \sin\theta)}{qB}$$

Time period of helical path

$$T = \frac{2\pi m}{qB}$$

Frequency of helical path

$$F = \frac{1}{T} = \frac{qB}{2\pi m}$$

Pitch: The pitch is the horizontal distance between two consecutive circles.

$$P = (V\cos\theta)T = \frac{2\pi m}{qB}(V\cos\theta)$$

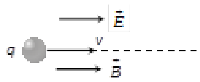
10. Lorentz Force

Lorentz force-

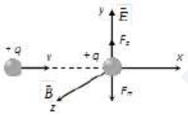
When the moving charged particle is subjected simultaneously to both electric field $\vec{E}$ and magnetic field $\vec{B}$, the moving charged particle will experience electric $\vec{F}_e = q\vec{E}$ and magnetic force $\vec{F}_m = q(\vec{v} \times \vec{B})$; so the net force on it will be $\vec{F} = q[\vec{E} + (\vec{v} \times \vec{B})]$ Which is our lorentz-force equation. Depending on the directions of $\vec{v}$, E and $\vec{B}$ following situations are possible.

(i) When $\vec{v}$, $\vec{E}$ and $\vec{B}$ all the three are collinear : In this situation the magnetic force on it will be zero and only electric force will act and so $\vec{a} = \frac{\vec{F}}{m} = \frac{q\vec{E}}{m}$.

(ii) The particle will pass through the field following a straight-line path (parallel field) with change in its speed. So in this situation speed, velocity, momentum and kinetic energy all will change without change in direction of motion as shown



(iii) $\vec{v}$, $\vec{E}$ and $\vec{B}$ are mutually perpendicular : in this situation if $\vec{E}$ and $\vec{B}$ are such that $\vec{F} = \vec{F}_e + \vec{F}_m = 0$ ie. $\vec{a} = (\vec{F}/m) = 0$



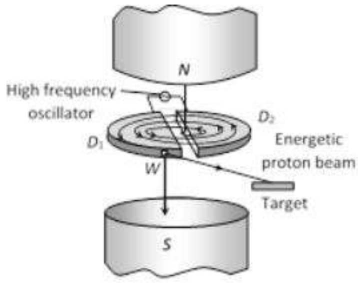
as shown in the figure, the particle will pass through the field with the same velocity, without any deviation in the path.

And in this situation, as $F_e = F_m$ i.e. $qE = qvB \Rightarrow v = \frac{E}{B}$.

11.Cyclotron

A cyclotron is a device used to accelerate positively charged particles (like α-particles, deutrons etc.) to acquire enough energy to carry out nuclear disintegration etc. It is based on the fact that the electric field accelerates a charged particle and the magnetic field keeps it revolving in circular orbits of constant frequency.

It consists of two hollow D-shaped metallic chambers D_1 and D_2 called dees. The two dees are placed horizontally with a small gap separating them. The dees are connected to the source of high frequency electric field. The dees are enclosed in a metal box containing a gas at a low pressure of the order of 10^{-3} mm mercury. The whole apparatus is placed between the two poles of a strong electromagnet NS as shown in fig. The magnetic field acts perpendicular to the plane of the dees.



Radius of the path travelled by the particle can be given as :

$$r = \frac{mV}{qB}$$

where V is the velocity, q is the charge and B is magnitude of magnetic field applied.

Cyclotron frequency : Time taken by an ion to describe a semicircular path is given by $t = \frac{\pi r}{v} = \frac{\pi m}{qB}$

If T= time period of oscillating electric field then, $T = 2t = \frac{2\pi m}{qB}$

therefore the cyclotron frequency $\nu = \frac{1}{T} = \frac{Bq}{2\pi m}$

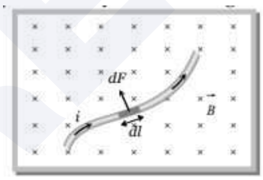
Maximum energy of particle : Maximum energy gained by the charged particle $E_{max} = \left(\frac{q^2B^2}{2m}\right) r_0^2$

where r_0=maximum radius of the circular path followed by the positive ion.

12. Force On A Conductor Carrying Current In A Magnetic Field

Magnetic force on a current carrying conductor -

In case of current carrying conductor in a magnetic field force experienced by its small length element is $d\vec{F} = i(d\vec{l} \times \vec{B})$



For total force, we will integrate the above equation. So the total magnetic force -

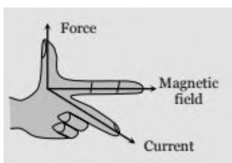
$$\vec{F} = \int d\vec{F} = \int i(d\vec{l} \times \vec{B})$$

If magnetic field is uniform i.e., $\vec{B}$= constant and
$\int d\vec{l} = \vec{L} =$ vector sum of all the length elements from initial to final point. Which is in accordance with the law of vector addition is equal to length vector $\vec{L}'$ joining initial to final point.

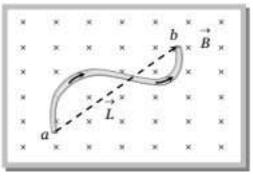
Then, $\vec{F} = i\left[\int d\vec{l}\right] \times \vec{B} = i\left(\vec{L} \times \vec{B}\right)$

Direction of force -

According to **Fleming's left-hand rule** - Stretch the fore-finger, central finger and thumb left hand mutually perpendicular. Then if the fore-finger points in the direction of field $\vec{B}$ and the central in the direction of current i, the thumb will point in the direction of force. For better understanding, look at the image given below,



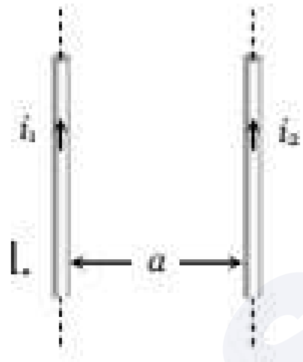
Note - If curved wire is given in the question then the length will be taken as shown in the figure -



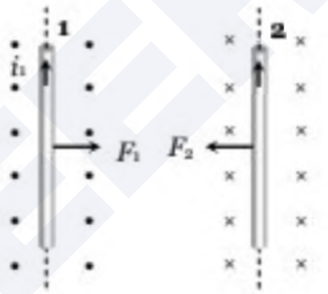
And the direction of the length vector should be in the direction of current.

Force between two parallel current carrying infinite wires-

Let us take two long straight conductors carrying currents i_1 and i_2 placed parallel to each other at a distance 'a' from each other as shown in the figure -



The conductor 2 experiences the same magnetic field at every point along its length due to the conductor 1. Because of this there will be some force acting on conductor 2 and the direction of magnetic force is indicated in the figure and it can be visualised by using the right-hand thumb rule.



Now, if we apply Ampere's circuital law on the first conductor then the magnitude of the magnetic field can be obtained as -

$$B_1 = \frac{\mu_0 I_1}{2\pi a}$$

Then the force on a segment of length L of the conductor 2 due to the conductor 1 can be given as,

$$F_{21} = I_2 L B_1 = \frac{\mu_0 I_1 I_2}{2\pi a} L$$

Similarly, we can calculate the force exerted by the conductor 2 on the conductor 1. We see that, the conductor 1 experiences the same force due to the conductor 2 but the direction of force is opposite. Thus we can say that,

$$F_{21} = F_{12}$$ (But the direction will be opposite.)

But if the direction of current flowing through the conductor is opposite in both the conductors then both the wire will repel each other.

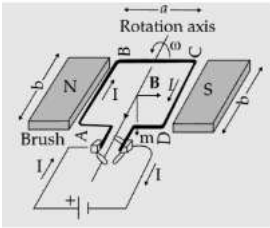
Also, the magnitude of the force acting per unit length can be given as -

$$f_{12} = f_{21} = \frac{\mu_0 I_1 I_2}{2\pi a}$$

13. Torque on a rectangular current loop in a uniform magnetic field

Let us consider a case when the rectangular loop is placed such that the uniform magnetic field B is in the plane of the loop. This is illustrated in the given figure.The field exerts no force on the two arms AD and BC of the loop. It is perpendicular to the arm AB of the loop and exerts a force F_1 on it which is directed into the plane of the loop. Its magnitude is,

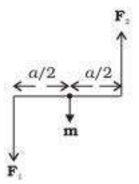
$$F_1 = IbB$$



Similarly it exerts a force F_2 on the arm CD and F_2 is directed out of the plane of the paper.

$$F_2 = IbB = F_1$$

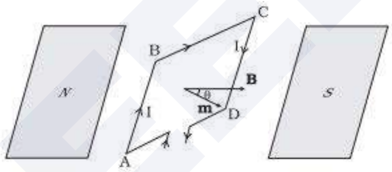
Thus, the net force on the loop is zero. But these two forces are actingat a distance 'a' between them. This torque on the loop due to the pair of forces F_1 and F_2 . From the figure given below shows that the torque on the loop tends to rotate it anti-clockwise. This torque is (in magnitude),



$$\begin{aligned} \tau &= F_1 \frac{a}{2} + F_2 \frac{a}{2} \\ &= IbB \frac{a}{2} + IbB \frac{a}{2} = I(ab)B \\ &= IAB \end{aligned}$$

where A = ab is the area of the rectangle.

Now we will discuss the case when the plane of the loop is making an angle θ with magnetic field.



Here again you can see that the forces on arms AB and CD are F_1 and F_2 -

$$F_1 = F_2 = IbB$$

Then the torque will be the

$$\begin{aligned} \tau &= F_1 \frac{a}{2} \sin\theta + F_2 \frac{a}{2} \sin\theta \\ &= \text{Iab}\, B \sin\theta \\ &= IAB \sin\theta \end{aligned}$$

From the above equations we can see that the torques can be expressed as vector product of the **magnetic moment** of the coil and the magnetic field. We define the magnetic moment of the current loop as,

$$m = IA$$

If the coil has N turns then the magnetic moment formula becomes -

$$m = NIA$$

And its direction is defined by the direction of Area vector.

So, Torque equation can be written as,

$$\tau = \mathbf{m} \times \mathbf{B}$$

14. <u>Circular current loop as magnetic dipole-</u>

The magnetic field due to a current (I) carrying circular wire (Radius = R) on its axis at a distance 'x' is -

$$B=\frac{\mu_0 I R^2}{2\left(x^2+R^2\right)^{3/2}}$$

If $x >>> R$, then R will become negligible and the equation becomes -

$$B=\frac{\mu_0 I R^2}{2 x^3}$$

Now, as the area of this loop is πR^2, so the equation becomes -

$$B=\frac{\mu_0 I A}{2 \pi x^3}$$

As, m = I A So,

$$\mathbf{B}=\frac{\mu_0 \mathbf{m}}{2 \pi x^3}$$

$$=\frac{\mu_0}{4 \pi} \frac{2 \mathbf{m}}{x^3}$$

The expression shown above is very similar to an expression obtained earlier for the electric field of a dipole.

The similarity may be seen if we substitute,

$$\mu_0 \rightarrow 1 / \varepsilon_0$$

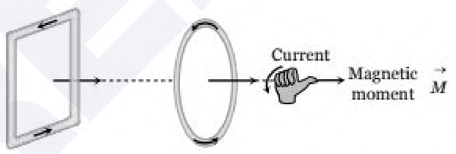
$$\mathbf{m} \rightarrow \mathbf{p}_e \text{ (electrostatic dipole)}$$

$$\mathbf{B} \rightarrow \mathbf{E} \text{ (electrostatic field)}$$

So the equation becomes,

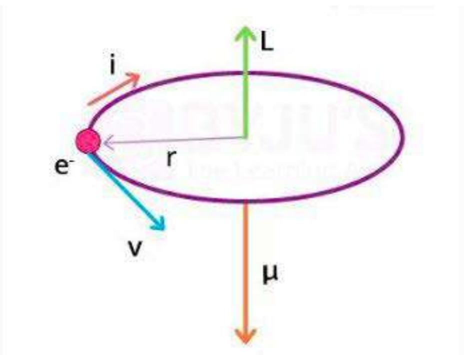
$$\mathbf{E}=\frac{2 \mathbf{p}_e}{4 \pi \varepsilon_0 x^3}$$

So, We can say from the above analogy that the circular current loop can act as a magnetic dipole. The direction of the magnetic moment can be obtained as -



But there is a fundamental difference: an electric dipole is built up of two elementary units — the charges (or electric monopoles). In magnetism, a magnetic dipole (or a current loop) is the most elementary element. The equivalent of electric charges, i.e., magnetic monopoles, are not known to exist.

15.Magnetic dipole moment of a revolving electron-



Let us consider an electron that is revolving around in a circle of radius r with a velocity v.

The charge of the electron is e and its mass is m, both of which are constant. The time period T of the electrons' orbit is -

$$T=\frac{\text{Circumference}}{\text{Velocity}}=\frac{2 \pi r}{v}$$

So the current due to motion of electron is -

$$i=\frac{q}{T}=\frac{-e}{\frac{2\pi r}{v}}=\frac{-ev}{2\pi r}$$

Now, as we know that the direction of current is opposite to the direction of motion of electron. Now the magnetic moment is defined as -

$$\mu=iA$$

So the Magnetic moment of an electron:

$$\mu=\frac{-ev}{2\pi r}A=\frac{-ev}{2\pi r}\pi r^2$$

$$\mu=\frac{-erv}{2}$$

If we divide and multiply by the mass of the electron,

$$\mu=\frac{-e}{2m_e}m_e v r$$

As we have studied that that the angular momentum L is given by:

L = mvr

So the above equation can be written as -

$$\mu=\frac{-e}{2m_e}L$$

The negative sign shows that the velocity and current are on opposite directions as shown in the figure given above. Also in the vector form it is written as -

$$\vec{\mu}=\frac{-e}{2m_e}\vec{L} \qquad \text{..............................(1)}$$

Now, by Niels Bohr , Angular momentum of the electron is given as -

$$L=n\frac{h}{2\pi}, n=0,\pm1,\pm2\ldots$$

Where n is the orbit quantum number and h is the Planck's constant,

Now by using the equation (1)

$$\mu=n\frac{-e}{2m_e}\frac{h}{2\pi}$$

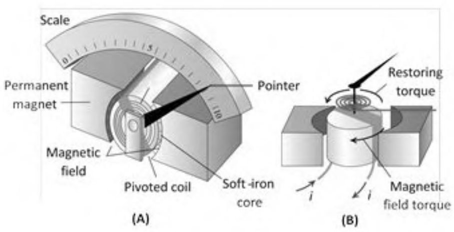
$$\mu=-n\frac{eh}{4\pi m_e}$$

If we put n =1, then the equation become -

$$\mu_B=-\frac{eh}{4\pi m_e}=9.27\times10^{-24}J/T$$ (This is called Bohr Magneton μ_B)

16.Moving coil galvanometer-

A moving coil galvanometer is an electromagnetic device which is used to measure small values of current. It consists of a permanent horse-shoe magnet, coil, soft iron core, pivoted spring, non-metallic frame, scale, and pointer as shown in the figure



As we have studied the torque acts on a current-carrying coil suspended in a uniform magnetic field. Due to this, the coil rotates. Hence, the deflection in the coil of a moving coil galvanometer is directly proportional to the current flowing in the coil.

In this, the coil is suspended between the pole pieces of a strong horse-shoe magnet. The magnetic field is made radial and for this, the pole pieces are made cylindrical and a soft iron cylindrical core is placed within the coil without touching it. The benefit of this type of field is that the plane of the coil always remains parallel to the field. Therefore θ=90° and the deflecting torque always has the maximum value.

$$\tau_{\text{deflection}} = NBiA$$

Now if the coil deflects, a restoring torque is set up in the pivoted spring. If α is the angle of twist, the restoring torque is

$$\tau_{\text{restoring}} = C\alpha$$

where C is the torsional constant of the fibre.

When the coil is in equilibrium, then -

$$NBiA = C\alpha,$$

So, $$i = \frac{C}{NBA}.\alpha \quad \Rightarrow i = K\alpha$$

$$\text{where } K = \frac{C}{NBA} \text{ is the galvanometer constant.}$$

This linear relationship between i and α makes the moving coil galvanometer useful for current measurement and detection.

Here we will discuss two important terminologies -

1. **Current sensitivity (S_i) :** The current sensitivity of a galvanometer is defined as the deflection produced in the galvanometer per unit current flowing through it. So it can be written as -

$$S_i = \frac{\alpha}{i} = \frac{NBA}{C}$$

2. **Voltage sensitivity (S_V) :** Voltage sensitivity of a galvanometer is defined as the deflection produced in the galvanometer per unit voltage applied to it. So it can be written as -

$$S_V = \frac{\alpha}{V} = \frac{\alpha}{iR} = \frac{S_i}{R} = \frac{NBA}{RC} \qquad (By\ using\ Ohm's\ law)$$

Magnetism and Matter

Important Formulae

1. Magnetic field lines

Important points regarding Magnetism-

- The earth behaves as a magnet with the magnetic field lines pointing from the geographic south to the north.
- When a bar magnet is freely suspended, it points in the north-south direction. The tip which points to the geographic north is called the north pole and the tip which points to the geographic south is called the south pole of the magnet.
- There is a repulsive force when like poles of two magnets are brought close together and there is an attractive force when unlike poles of two magnets are brought close together.
- The north and south pole cannot be separated by splitting the magnet into two parts.

i.e If a magnet is broken into a number of pieces each piece becomes a magnet.

- It is possible to make magnets out of iron and its alloys.

Magnetic field-

Space around a magnetic Pole or magnet or current-carrying wire within which its effect can be experienced.

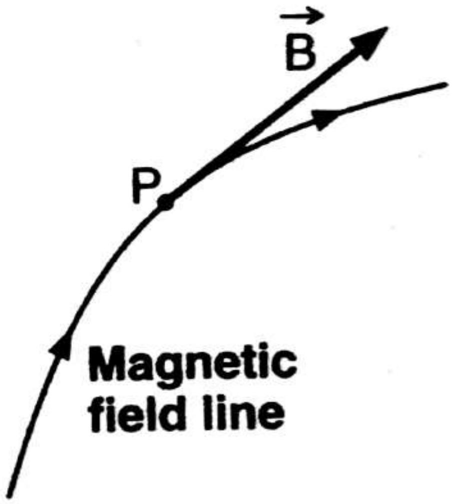
Magnetic field lines-

The magnetic field line is not real. The magnetic field lines are a visual and intuitive realization of the magnetic field.

Properties of magnetic field lines-

- The magnetic field lines of a magnet form continuous closed loops. Outside the magnet, magnetic field lines start from the north pole and end at the south pole, whereas inside the magnet its direction is from south pole to north pole.
- The tangent to the magnetic field line at a given point represents the direction of the net magnetic field ($\vec{B}$) at that point.

For the below figure The tangent to the magnetic field line at point P represents the direction of the net magnetic field ($\vec{B}$) at point P.



- The larger the number of magnetic field lines crossing per unit area, the stronger is the magnitude of the magnetic field ($\vec{B}$) at that region.
- The two magnetic field lines do not intersect at any point.

2. Bar Magnet As An Equivalent Solenoid

Bar Magnet-

A bar magnet consists of two equal and opposite magnetic poles separated by a small distance.

Pole strength (m)-

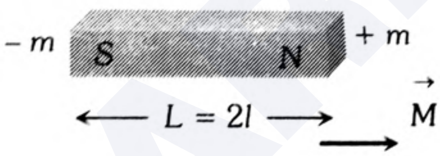
The strength of a magnetic pole to attract magnetic materials towards itself is known as pole strength.

It is a scalar quantity and it is represented by +m and -m.

It depends on the nature of the material of the magnet and the area of the cross-section i.e, independent from the length.

Magnetic dipole moment ($\vec{M}$)- It represents the strength of the magnet. Mathematically it is defined as the product of the strength of either pole and effective length.

i.e for the below figure $\vec{M} = mL = m(2l)$

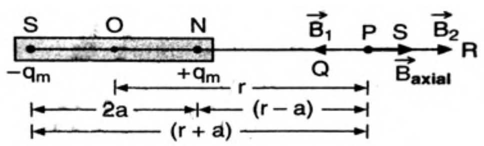


It is a vector quantity directed from south to north.

This is analogous to the electrical dipole moment which was given by $\vec{P} = qL$

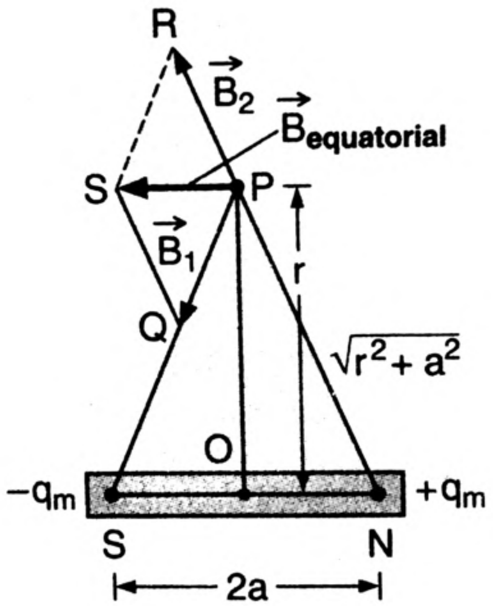
And using this analogy we can calculate

- The magnetic field on the Axial Position of a bar magnet-



$$For \quad r >> a \quad \Rightarrow B_{axial} = \frac{\mu_o\, 2M}{4\pi\, r^3}$$

- Magnetic Field at the equatorial position of a magnet-



$$B_e = \frac{\mu_o}{4\pi}\frac{M}{(r^2 + a^2)^{\frac{3}{2}}}$$

And for $For \;\; r >> a \quad \Rightarrow B_e = \frac{\mu_o}{4\pi}\frac{M}{r^3}$

Magnetic Field at any general point due to bar magnet-

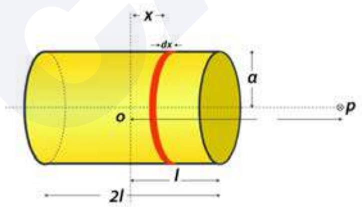
$$B_g = \frac{\mu_o}{4\mu}\frac{M}{r^3}\sqrt{3cos^2\Theta + 1}$$

Solenoid-

The solenoid is defined as a cylindrical coil of many tightly wound turns of insulated wire with generally diameter of the coil smaller than its length.

Bar magnet as an equivalent solenoid-

By calculating the axial field of a finite solenoid carrying current and equating it with the magnetic field of bar magnet we can demonstrate a Bar magnet as an equivalent solenoid.



For the above figure

Let n = number of turns per unit length $\frac{N}{L}$

whrere, N = total number of turns,

$L = 2l$ = length of the solenoid

for $r >> R \quad and \quad r >> x$

Using $N = n(2l)$

we get $\vec{B} = \frac{\mu_0 N I \pi R^2}{2\pi r^3}$

Now if we consider the above solenoid as a Bar magnet then its dipole moment is given by $\vec{M} = NIA$

Now using $A = \pi R^2$ we can write $\vec{B} = \frac{\mu_0 NIA}{2\pi r^3} = \frac{\mu_0 \vec{M}}{2\pi r^3} = \frac{2\mu_0 \vec{M}}{4\pi r^3}$

$\vec{B} = \frac{2\mu_0 \vec{M}}{4\pi r^3}$ This is equivalent to the magnetic field on the Axial Position of a bar magnet.

3. The Dipole In A Uniform Magnetic Field

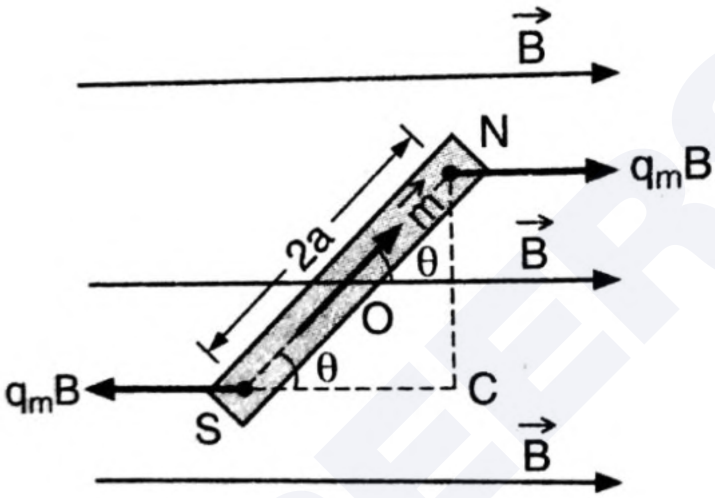
Net Force-

As magnetic dipole is analogous to an electric dipole.

So we can use $m = q_m$

when a magnetic dipole is kept in a uniform magnetic field. The net force experienced by the dipole is zero as shown in the below figure.

I.e $F_{net} = 0$



Hence magnetic dipole will not make any linear motion.

Torque on dipole-

Net torque about the center of dipole is given as $\tau = q_m B(2a)\sin\theta$

Using $\vec{M} = q_m 2a$ we get $\tau = MB\sin\theta$

So $\vec{\tau} = \vec{M} \times \vec{B}$

- The direction of the torque is normal to the plane containing dipole moment M and magnetic field B and is governed by right-hand screw rule.
- If Dipole is parallel to B the torque is Zero. I.e $\Theta = 0^\circ$ $\tau = 0$ (This is the position of **stable equilibrium** of dipole)
- Torque is maximum when Dipole is perpendicular to B. I.e $\Theta = \frac{\pi}{2}$ $\tau = MB = maximum \quad torque$

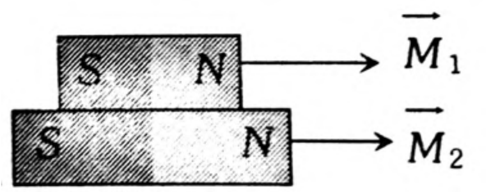
Oscillation of dipole -If a dipole experiencing a torque in a magnetic field is allowed to rotate, then it will rotate to align itself to the magnetic field. But when it reaches along the direction of B the torque becomes zero. But due to inertia, it overshoots this equilibrium condition and then starts oscillating about this mean position.

The time period of this oscillation is given as

$$T = 2\pi\sqrt{\frac{I}{MB}}$$

where I= moment of inertia of **dipole** about the axis passing through its centre and perpendicular to its length.

- For two magnets having Magnetic Moments in the same direction (i.e sum position of the magnetic moment)



$M_s = M_1 + M_2$

$I_s = I_1 + I_2$

M_s - Net Magnetic Moment

I_s - Net Moment of Inertia

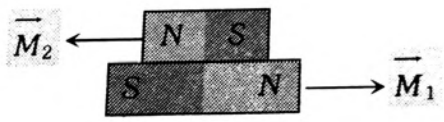
So Time period is

$$T = 2\pi\sqrt{\frac{I_s}{M_s B}}$$

Similarly, Frequency is given as

$$\nu = \frac{I}{T_s} = \frac{1}{2\pi}\sqrt{\frac{(M_1 + M_2)B}{I_s}}$$

- For two magnets having Magnetic Moments in the opposite direction (i.e difference position of the magnetic moment)



$M_d = M_1 - M_2$

$I_d = I_1 + I_2$

So Time period is

$$T = 2\pi\sqrt{\frac{I_d}{M_d B}} \text{ or } T_d = 2\pi\sqrt{\frac{I_1 + I_2}{(M_1 - M_2)B}}$$

Similarly, Frequency is given as

$$\nu_d = \frac{1}{T_d} = \frac{1}{2\pi}\sqrt{\frac{(M_1 - M_2)B}{I_1 + I_2}}$$

- The ratio of difference and sum position of the magnetic moment

$$\frac{T_s}{T_d} = \sqrt{\frac{M_1 - M_2}{M_1 + M_2}}$$

$$\frac{M_1}{M_2} = \frac{T_d^2 + T_s^2}{T_d^2 - T_s^2} = \frac{\nu_s^2 + \nu_d^2}{\nu_s^2 - \nu_d^2}$$

Dipole in Non-Uniform Magnetic Field- In case the magnetic field is non-uniform, the magnitude of the force on $+{+q_m} \quad and \quad -q_m$ will be different. So $F_(net) \neq 0$ and At the same time due to a couple of forces acting, a torque will also be acting on it.

Work done in rotation-

The work done by magnetic force for rotating a magnetic dipole through an angle θ_2 from the equilibrium position of an angle θ_1 is given as

$$W_{mag} = \int \tau d\theta = \int_{\theta_1}^{\theta_2} \tau d\theta cos(180^0) = -\int_{\theta_1}^{\theta_2} \tau d\theta$$

$$\Rightarrow W_{mag} = -\int_{\theta_1}^{\theta_2} (M \times B)d\theta = -\int_{\theta_1}^{\theta_2} (MBSin\theta)d\theta = MB(\cos\Theta_2 - \cos\Theta_1)$$

And So work done by an external force is $W = -W_{mag} = MB(\cos\Theta_1 - \cos\Theta_2)$

For example

if $\Theta_1 = 0^\circ \ and \ \Theta_2 = \Theta$

$W = MB(1 - \cos\Theta)$

$if \ \Theta_1 = 90^\circ \ and \ \Theta_2 = \Theta$

$W = -MB\cos\Theta$

Potential Energy of a dipole kept in a magnetic field-

As $\Delta U = -W_{mag} = W$

So change in the Potential Energy of a dipole when it is rotated through an angle θ_2 from the equilibrium position of an angle θ_1 is given as $\Delta U = MB(\cos\Theta_1 - \cos\Theta_2)$

$if \ \Theta_1 = 90^\circ \ and \ \Theta_2 = \Theta$

$\Delta U = U_{\theta_2} - U_{\theta_1} = U_\theta - U_{90} = -MB\cos\Theta$

Assuming $\Theta_1 = 90^\circ$ and $U_{90^0} = 0$

we can write $U = U_\theta = -\vec{M}\cdot\vec{B}$

Equilibrium of Dipole-

1. Stable Equilibrium-

$\Theta = 0^\circ$

$\tau = 0$

$U_{min} = -MB$

2. Unstable Equilibrium-

$\Theta = 180^\circ$

$\tau = 0$

$U_{max} = MB$

3. **Not in equilibrium-**

$\Theta = 90^\circ$

$\tau_{max} = MB$

$U = 0$

4. Gauss Law Of Magnetism

Magnetic flux -

It is defined as the magnetic lines of force passing normally through a surface called magnetic flux.

As we learn in electrostatic, the Gauss law for a closed surface states that :

$\phi_{\text{closed}} = \frac{q_{net}}{\epsilon_0}$,

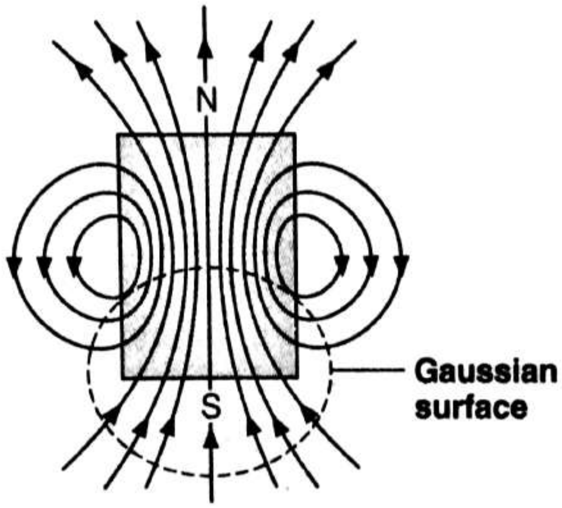
where $\phi = \int \bar{E}d\bar{S}$

S is the area enclosed and E is the electric field intensity passing through it.

and q_{net} is the total charge inside the closed surface.

But **Gauss's Law of magnetism** states that the flux of the magnetic field through any closed surface is zero (as shown in the below figure).

It is because inside the closed surface simplest magnetic element is a magnetic dipole with both the poles (since magnet with monopole does not exist). So a number of magnetic field lines entering the surface are equal to the number of magnetic field lines leaving the surface. So the net magnetic flux through any closed surface is zero.



I.e Gauss law for closed surface-

$$\oint \vec{B} \cdot \vec{ds} = 0$$

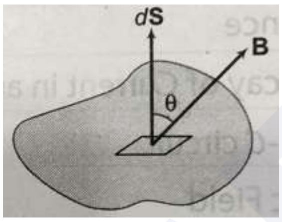
Gauss law if the surface is open

$$\int \vec{B} \cdot \vec{ds} = \phi_B$$

i.e Consider an element of the area dS on an arbitrarily shaped surface is shown in the figure. If the magnetic field at this element is $\vec{B}$, the magnetic flux through the element is $d\phi_B = \vec{B}.d\vec{S} = BdS\cos\theta$

So, the total flux through the surface is

$$\phi_B = \int \vec{B}.d\vec{S} = \int BdS\cos\theta$$

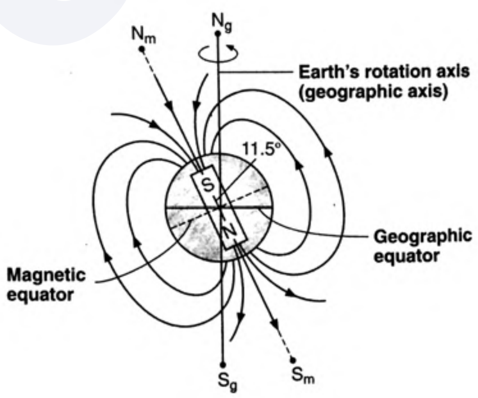


5. Earth's Magnetic Field

The reason why, A bar magnet, when suspended freely, points in a north-south direction is due to the earth’s giant magnetic field.

The branch of Physics which deals with the study of Earth's magnetic field is called Terrestrial magnetism. It is also known as geomagnetism.

Various Terminologies about Earth’s Magnetism-



Geographic axis- The Axis of rotation of Earth is called the Geographic axis.

Geographic Meridian- A vertical plane passing through the geographical axis is called a Geographic meridian.

Geographic Poles-The points where the Geographic axis cuts the surface of Earth are called Geographic poles (i.e. Ng, Sg)

Magnetic axis- The axis of the huge magnet assumed to be lying inside the earth is called the magnetic axis.

The magnetic Equator- The circle on the earth's surface perpendicular to the magnetic axis is called the magnetic equator.

The angle between Magnetic and Geographical Axis- They make an angle of 11.5° with each other.

or we can say that the Earth's magnetic field is similar to that of a bar magnet tilted 11 degrees from the spin axis of the Earth.

THE MAGNETIC ELEMENTS-

These define the Earth's magnetic field $\vec{B}$ at any point.

Following are the three magnetic elements of the earth:

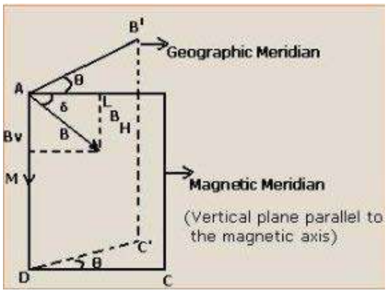
1. Magnetic declination (θ)

2. The angle of Dip or Magnetic Inclination **(**δ**)**

3. The horizontal component of Earth's magnetic field (B_H)

Magnetic declination (θ**)-**

Magnetic Declination is defined as the angle between geographic and magnetic meridian planes.



The angle of Dip or Magnetic Inclination (δ**) -**

Magnetic dip or magnetic inclination at a place is defined as the angle which the direction of the total strength of Earth's magnetic field makes with a horizontal line in the magnetic meridian.

At poles, the angle of dip = 90^0 and at the equator, the angle of dip = 0^0

Horizontal component (H) of Earth's magnetic field (B_H)-

The intensity of the earth's magnetic field can be resolved into two components

- Horizontal Component(B_H
- Vertical Component(B_V)

So we can write $\tan \delta = \frac{B_V}{B_H}, \quad \sin \delta = \frac{B_V}{B}, \cos \delta = \frac{B_H}{B}$

Resultant Magnetic Field due to earth

$$B_H = B \cos \delta$$

$$B_V = B \sin \delta$$

$$B = \sqrt{B_H^2 + B_V^2}$$

Earth Magnetic field is horizontal only at the magnetic equator i.e when $\delta = 0^0$ then $B_H = B$ and $B_V = 0$

Earth Magnetic Field at the Pole- Since $\delta = 90^0$ So $B_H = 0$ and $B_V = B$

Important points-

- Isoclinic lines- The lines that pass through different places having the same angle of dip.
- Aclinic line-A line which passes through places having an angle of dip as 0^0
- Isodynamic line-The lines drawn through places having the same of B_H

Tangent law-

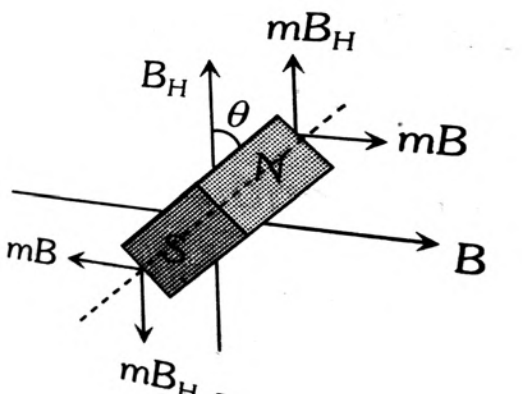
When a small magnet is suspended in two uniform magnetic field B and B_H which are at right angles to each other.

The magnet comes to rest at an angle Θ.

i.e for the below figure when

Magnet in Equilibrium

$$Then\ MB_H Sin\Theta = MBSin(90-\Theta)B \Rightarrow B_H tan\Theta\ \ (tangent\ \ law)$$
$$or\ \ tan\theta = \frac{B}{B_H}$$



6. Magnetisation And Magnetic Intensity

The magnetic intensity (H)-

The magnetic intensity of the magnetizing field is given by $H = \frac{B_0}{\mu_0}$

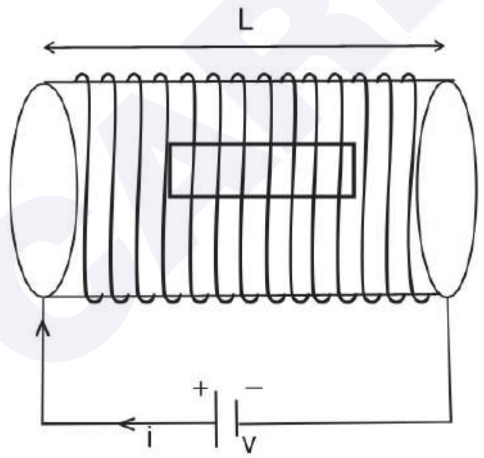
And its S.I. unit is A/m while its C.G.S. Unit is oersted.

Magnetization (M) -

Magnetization is a process in which a normal material is converted into a magnetic material by exposing it to an external magnetic field. The magnetic intensity is the reason due to which a normal material changes into magnetic material.

We define magnetization M of a sample to be equal to its net magnetic moment per unit volume i.e $M = \frac{m_{net}}{V}$

Consider a long solenoid of n turns per unit length and carrying a current i



The magnetic field in the interior of the solenoid is given by $\mathbf{B}_0 = \mu_0 \mathbf{nI}$

If $n = \frac{N}{L}$ then $\mathbf{B}_0 = \frac{\mu_0 \mathbf{NI}}{L}$ where N=number of turns and L=length of solenoid

Using $H = \frac{B_0}{\mu_0}$ So we can write $H = \frac{B_0}{\mu_0} = \frac{NI}{L}$

If the interior of the solenoid is filled with a material with non-zero magnetization then the material will magnetize.

And the field inside the solenoid will be greater than B_0.

The net B field in the interior of the solenoid may be expressed as

$B = B_0 + B_m$

B - total magnetic field

B_0 - the magnetic field in a vacuum

B_m - magnetic field due to magnetization of the material

And B_m is proportional to the magnetization M of the material and is expressed as $B_m = \mu_0 M$

And using $H = \frac{B_0}{\mu_0}$ we can write $B_o = \mu_0 H$

So we get $B = B_0 + B_m = \mu_0 H + \mu_0 M = \mu_0(H + M)$

So we get $H = \frac{B}{\mu_0} - M$

The Magnetization (M) of material is influenced by The magnetic intensity (H)

So the relation between M and H is given as $\mathrm{M} = \chi \mathrm{H}$

where χ is called magnetic susceptibility. And it is a measure of how a magnetic material responds to an external field.

Using $\mathrm{M} = \chi \mathrm{H}$ in $B = \mu_0(H + M)$

we get $\mathbf{B} = \mu_0(\mathbf{1} + \chi)\mathbf{H} = \mu_0 \mu_\mathbf{r} \mathbf{H} = \mu \mathbf{H}$

where $\mu_r = (1 + \chi)$ is called relative magnetic permeability of the substance.

and $\mu = \mu_0 \mu_r = \mu_0(1 + \chi)$ is the magnetic permeability of the substance

7. Magnetic Properties Of Materials

Depending on the magnetic properties, the magnetic materials are classified as

- Diamagnetic substance
- Paramagnetic substance
- Ferromagnetic substance

Diamagnetic substance-

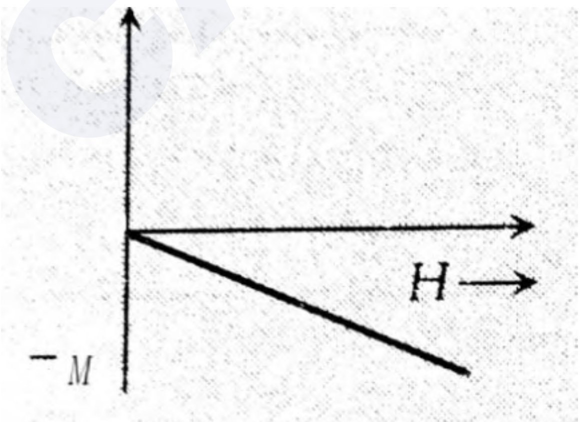
The substance which is feebly magnetized in a direction opposite to that of the magnetizing field in which those are placed.

bismuth, copper, lead, silicon, etc are diamagnetic substances.

And for diamagnetic substances

$$\chi < 0$$
$$0 \leq \mu_r < 1$$
$$\mu < \mu_0$$

Magnetization(M) Vs The magnetic intensity (H) curve-



Magnetic moment (m) for diamagnetic substances is Very low or nearly 0.

The cause of magnetism for Diamagnetic substances is the Orbital motion of electrons.

Diamagnetic substances are those which have a tendency to move from the stronger to the weaker part of the external magnetic field.

The magnetic field lines are expelled by these substances.

Behavior in a non-uniform magnetic field In diamagnetic substance-These are repelled in an external magnetic field.

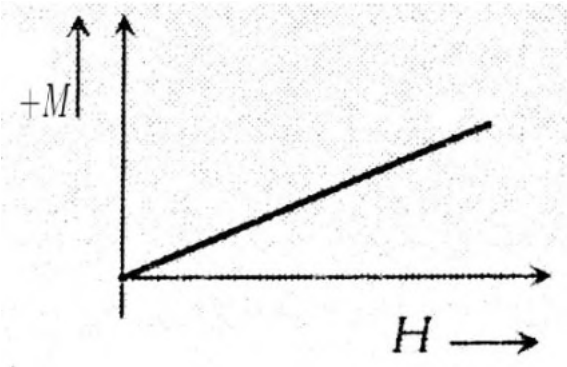
Paramagnetic substances-

Paramagnetic substances are those which get weakly magnetized when placed in an external magnetic field.

And for Paramagnetic substances

$$0 < \chi < \varepsilon$$
$$1 < \mu_r < 1 + \varepsilon$$
$$\mu > \mu_0$$

Magnetization(M) Vs The magnetic intensity (H) curve-



They have a tendency to move from a region of a weak magnetic field to a strong magnetic field.

i.e., they get weakly attracted to a magnet.

Cause of magnetism-Spin motion of electrons

The magnetic moment (m) for Paramagnetic substances is Very low.

Behavior in a non-uniform magnetic field In a Paramagnetic substance- These are feebly attracted in an external magnetic field.

Ferromagnetic substances-

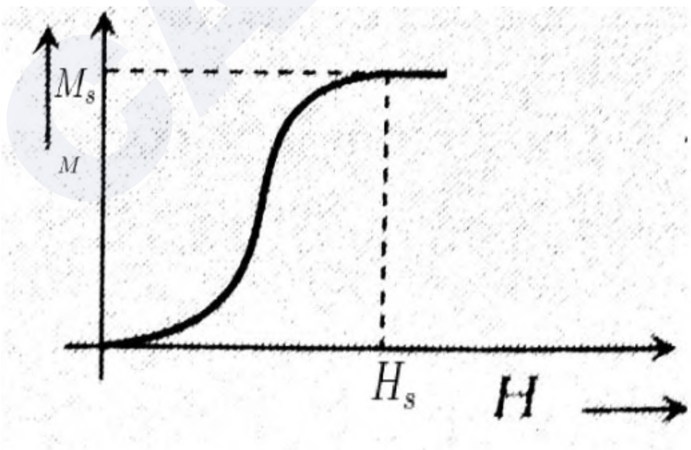
The substance which is strongly magnetized in the direction of the magnetizing field in which they are placed.

Iron, cobalt, nickel, gadolinium, and the number of alloys are ferromagnetic in nature.

And for ferromagnetic substances

$$\chi >> 1$$
$$\mu_r >> 1$$
$$\mu >> \mu_0$$

Magnetization(M) Vs The magnetic intensity (H) curve-



The cause of magnetism is the Formation of domains.

Magnetic moment (m) for ferromagnetic substances is Very high.

They have strong tendency to move from a region of weak magnetic field to strong magnetic field.

Magnetic field lines tend to crowd into a ferromagnetic substance.

Behavior in a non-uniform magnetic field In ferromagnetic substance- These are strongly attracted in an external magnetic field

- **Curie Temperature or Curie Point-**

It is the temperature above which increasing the temperature the susceptibility of ferromagnetic materials decreases.

i.e At a temperature above the Curie Point, a ferromagnetic becomes an ordinary Paramagnetic

It is denoted by T_c.

Curie-Weiss curve-

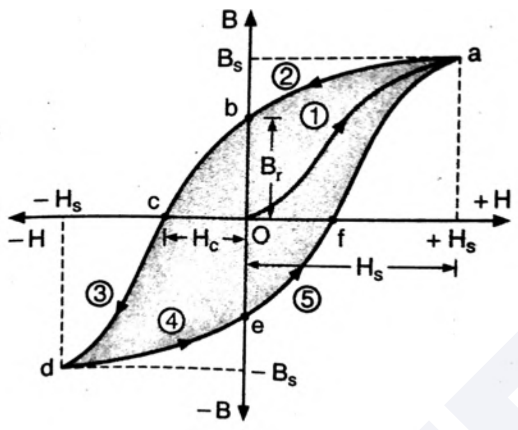
(For $T>T_c$) $\chi_m \propto \frac{1}{T-T_c}$ or $\chi_m = \frac{C}{T-T_c}$ where C is some constant.

8. Hysteresis Curve

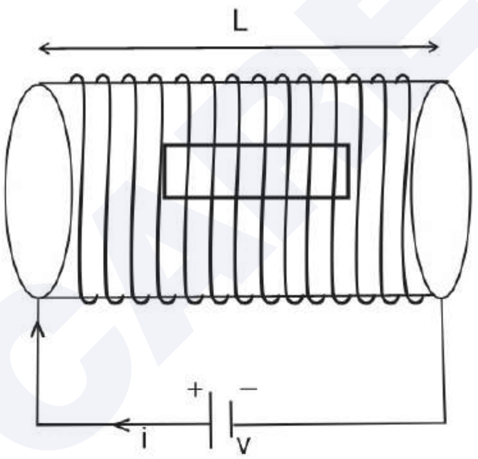
Hysteresis-

It is the property of the Lagging of magnetic induction (B) behind magnetic intensity (H) in the case of ferromagnetic substances.

Hysteresis Curve- This is nothing but the graph of (B Vs H)or (I Vs H) as shown below.



When a non-magnetized material is placed in the long solenoid which is carrying current i as shown in the below figure.



Initially $When\ \ i=0\ \ then\ \ B=0, H=0, I=0$ I.e at Point O.

Now if we increase i, it will result in an increase in B and H and I, till saturation point (a) I.e path 1 or Path Oa

Now we decrease H and reduce it to zero by decreasing i $\Rightarrow$ I.e path 2 or Path ab.

So at point b, H=0 but $B \neq 0 \Rightarrow B = B_r$ where B_r is called **retentivity** or **remanence** or **residual magnetism.**

This is happening because, For ferromagnetic materials, by removing the external magnetic field, i.e. H = 0, the magnetic moment of some domains remains aligned in the applied direction of the previous magnetising field, resulting in a residual magnetism.

Now we have to remove this residual magnetism of the material or demagnetize the material completely. For this, we will reverse the direction of the current in the solenoid.

So, the process of demagnetizing a material completely (i.e path bc) by applying the magnetizing field in a negative direction is defined as **Coercivity**.

So At point c we have B = 0 and H = H_c where H_c is called coercivity.

Coercivity signifies magnetic hardness or softness of substance:
I.,e Magnetic hard substance (steel) ——> High coercivity
Magnetic soft substance (soft iron) ——> Low coercivity.

If, after the magnetization has been reduced to zero, the value of H is further increased in the 'negative' i.e. reversed direction, the material again reaches a state of magnetic saturation, represented by point d.

Next, the current is reduced (curve de) and reversed (curve ea) then The cycle repeats itself till point a.

Electromagnetic Induction

Important Formulae

1. Magnetic flux-

The total number of magnetic lines of force passing normally through an area placed in a magnetic field is equal to the magnetic flux linked with that area.

Net magnetic flux through the surface is given by

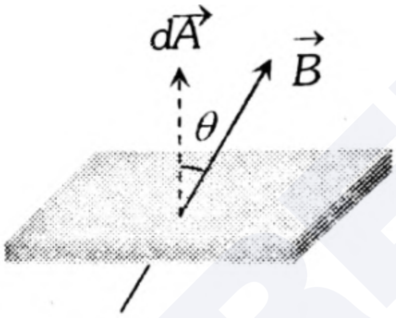
$$\phi_B = \oint \vec{B} \cdot d\vec{A} = BA\cos\Theta$$

where

$\phi_B =$ Magnetic Flux

$B =$ Magnetic field

$\Theta =$ The angle between the area vector and magnetic field vector



- Magnetic flux is a scalar quantity.
- Unit of magnetic flux -

  It's S.I. unit is Weber (wb) or $Tesla \times m^2$ and its C.G.S. unit is maxwell(Mx).

  and $1\ wb = 1\ Tm^2$ and $1\ Mx = 10^{-8}\ wb$

- The dimension of magnetic flux is $ML^2T^{-2}A^{-1}$
- if θ = 0 then $\phi = BA$ and Flux will be positive.
- If $\theta = \frac{\pi}{2}$ then Flux will be zero (i.e $\phi = 0$)

2. Faraday's Law Of Induction

Faraday's First Law-

Whenever the number of magnetic lines of force (Magnetic Flux) passing through a circuit changes an emf called induced emf is produced in the circuit. The induced emf persists only as long as there is a change of flux.

Faraday's Second Law-

The induced emf is given by the rate of change of magnetic flux linked with the circuit.

i.e Rate of change of magnetic Flux= $\varepsilon = \frac{-d\phi}{dt}$

where $d\phi \rightarrow \phi_2 - \phi_1$= change in flux

And For N turns it is given as $\varepsilon = \frac{-N\,d\phi}{dt}$ where N= Number of turns in the Coil .

The negative sign indicates that induced emf (e) opposes the change of flux.
And this Flux may change with time in several ways

I.e As $\phi = BA\cos\Theta$ So $\varepsilon = N\frac{-d}{dt}(BA\cos\Theta)$

1.If Area (A) change then $\varepsilon = -NB\cos\Theta\left(\frac{dA}{dt}\right)$

2.If Magnetic field (B) change then $\varepsilon = -NA\cos\Theta\left(\frac{dB}{dt}\right)$

3. If Angle (θ) change then $\varepsilon = -NAB\frac{d(\cos\Theta)}{d\Theta} \times \frac{d\Theta}{dt}$ or $\varepsilon = +NBA\,\omega\sin\Theta$

- **Induced Current-**

$$I = \frac{\varepsilon}{R} = \frac{-N}{R}\frac{d\phi}{dt}$$

where

$R \rightarrow$ Resistance

$\frac{d\phi}{dt} \rightarrow$ Rate of change of flux

- Induced Charge-

$$dq = i.dt = \frac{-N}{R}\frac{d\phi}{dt}.dt$$

$$dq = \frac{-N}{R}d\phi$$

I.e Induced Charge time-independent.

- Induced Power-

$$P = \frac{\varepsilon^2}{R} = \frac{N^2}{R}\left(\frac{d\phi}{dt}\right)^2$$

i.e - Induced Power depends on both time and resistance

3.Lenz's Law

Lenz's law-

This law gives the direction of induced emf /induced current.

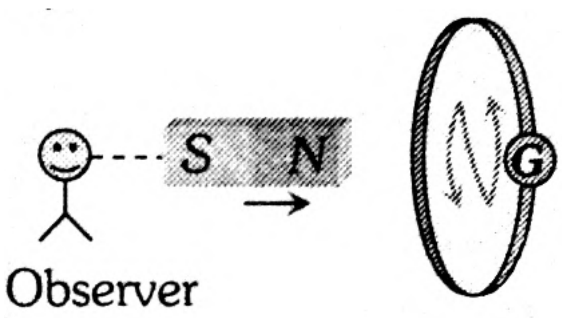
According to **Lenz's law,** the direction of induced emf or current in a circuit is such as to oppose the cause that produces it.

And this law is based upon the law of conservation of energy.

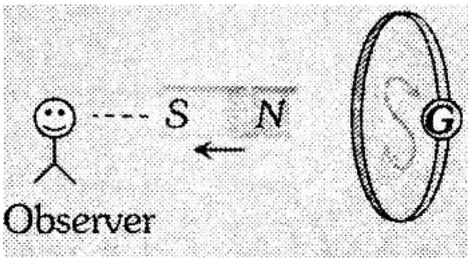
- The induced current in a closed-loop circuit

1. When N pole of a bar magnet moves towards the coil the flux associated with the loop increases and an emf is induced in it.

To repel the approaching north pole, the induced current is set up in the loop (if the loop is closed) in such a direction so that the front face of the loop behaves as the north pole. Therefore induced current as seen by observer O is in an anticlockwise direction (as shown in the figure).

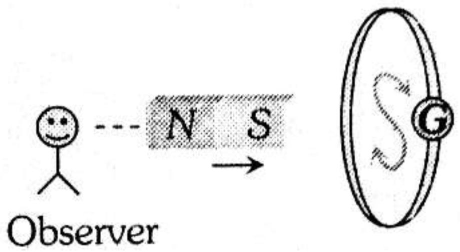


2. Similarly When N pole of a bar magnet moves away from the loop as shown in the figure.



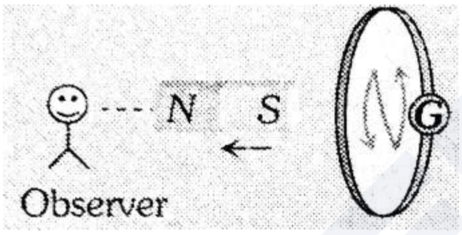
To attract the north pole, the induced current is set up in the loop (if the loop is closed) in such a direction so that the front face of the loop behaves as the south pole. Therefore induced current as seen by observer O is in a clockwise direction.

3. Similarly When S pole of a bar magnet moves towards the loop as shown in the figure.



To repel the approaching south pole, the induced current is set up in the loop (if the loop is closed) in such a direction so that the front face of the loop behaves as the south pole. Therefore induced current as seen by observer O is in a clockwise direction (as shown in the figure).

4. Similarly When S pole of a bar magnet moves away from the loop as shown in the figure.

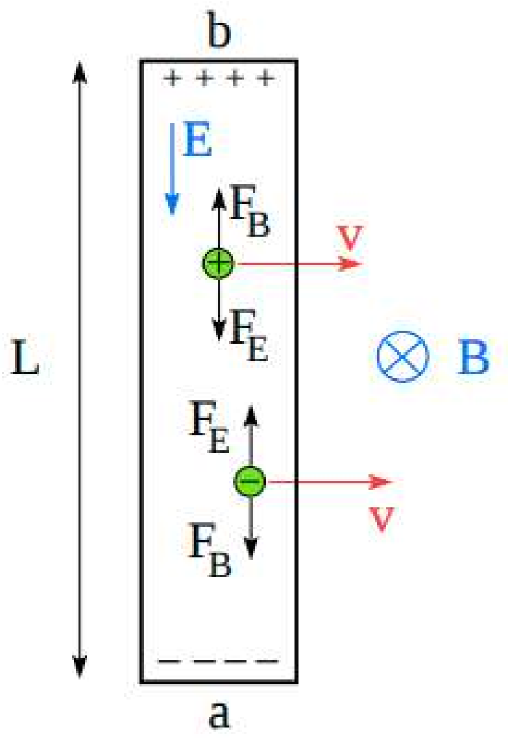


To attract the south pole, the induced current is set up in the loop (if the loop is closed) in such a direction so that the front face of the loop behaves as the north pole. Therefore induced current as seen by observer O is in an anticlockwise direction.

- If the loop is free to move the cause of induced emf in the coil can also be termed as relative motion. Therefore to oppose the relative motion between the approaching magnet and the loop, the loop will itself start moving in the direction of motion of the magnet.

4. Motional Electromotive Force

If a conducting rod of length L is moving with a uniform velocity $\vec{V}$ perpendicular to the region of the uniform magnetic field ($\vec{B}$) which directed into the plane of the paper as shown in the below figure.



Then the magnetic force on +ve charges is given by $\vec{F}_B = q(\vec{v} \times \vec{B}) = e(\vec{v} \times \vec{B})$ toward side b.

And similarly the magnetic force on -ve charges is given by $\vec{F}_B = q(\vec{v} \times \vec{B}) = e(\vec{v} \times \vec{B})$ toward side a.

So positive and negative charges will accommodate at side b and side a respectively. This will create an electric field having direction from b to a. And electric force due to this field on charges will be given as $\vec{F}_E = q\vec{E}$

Applying Equilibrium condition between electric and magnetic force

$F_E = F_B \quad \Rightarrow qE = qvB \quad \Rightarrow \quad E = vB$

So Potential difference induced between endpoints of the rod is given by

$V_{ab} \equiv V_b - V_a = EL \quad \Rightarrow V_{ab} = vBL$

this Potential difference (V_{ab}) is known as motional emf.

So Motional EMF is given by

$\varepsilon = BLv$
where

$B \rightarrow$ magnetic field

$L \rightarrow$ length of conducting

$v \rightarrow$ the velocity of the rod perpendicular to a uniform magnetic field.

If this conducting rod is part of a closed circuit and r is the resistance of the rod

(And assuming resistance of other parts of the circuit is negligible)

Then **Induced Current** in the conducting rod is given as $I = \frac{\varepsilon}{r} = \frac{BLv}{r}$

Magnetic force on the conducting rod is given as

$$F = ILB = B\left(\frac{BLv}{r}\right)L$$

$$F = \frac{B^2vL^2}{r}$$

The power dissipated in moving the conducting rod -

$$P_{mech} = P_{ext} = F \cdot v = \left(\frac{B^2vL^2}{r}\right) \cdot v$$

$$P_{mech} = P_{ext} = \frac{B^2L^2v^2}{r}$$

Electric Power or the rate of heat dissipation across the resistance is given as

$$P_E = I^2r = \left(\frac{BLv}{r}\right)^2 \cdot r = \frac{B^2L^2v^2}{r}$$

Since $P_{mech} = P_E$ So we can say that the principle of conservation of energy is applicable for the motional emf.

General Case-

Motional emf when $\vec{B}\ and\ \vec{V}\ and\ \vec{l}$ are at some angle with each other as shown in the below figure.

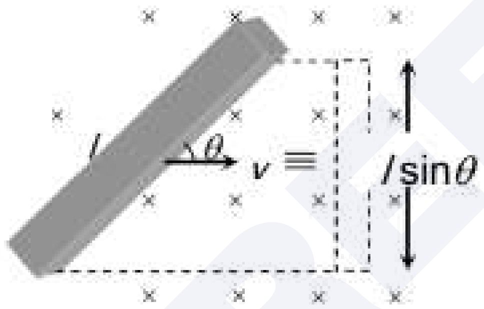
$$\text{Then At steady state, } |F_e| = |F_m|$$
$$\Rightarrow F_e = -F_m$$
$$\Rightarrow e\vec{E} = -\ell(\vec{V} \times \vec{B})$$
$$\Rightarrow \vec{E} = -(\vec{V} \times \vec{B})$$

$$And\ Poential\ difference = dv = -\vec{E} \cdot \vec{dl}$$
$$\Rightarrow dv = \int (\vec{V} \times \vec{B}) \cdot \vec{dl}$$
$$\Rightarrow \Delta v = (\vec{V} \times \vec{B}) \cdot \vec{l}$$

$$\Rightarrow \varepsilon = (\vec{V} \times \vec{B}) \cdot \vec{l}$$

For example-

- If the rod is moving by making an angle θ with the direction of the magnetic field or length as shown in the below figure.

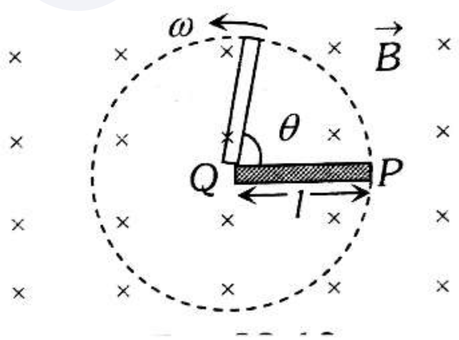


then Induced emf $\Rightarrow \varepsilon = BlV\,sin\theta$

$\Delta v = potential\ \ difference$
$B = Magnetic\ field$
$V = velocity\ of\ the\ rod$

Motional E.m.f due to rotational motion-

If a conducting rod PQ is rotating with angular velocity ω about its one end (Q) in a uniform magnetic field as shown in the below figure.



then $\varepsilon = \frac{1}{2}Bl^2\omega = Bl^2\pi\nu$

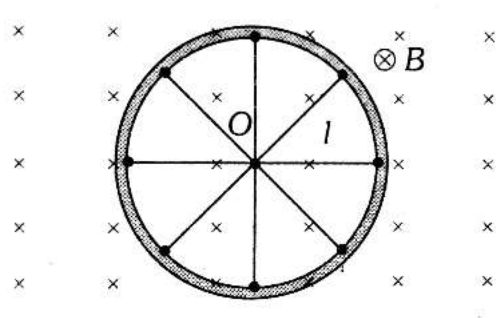
where

$$\nu = \frac{\omega}{2\pi} = \frac{1}{T} \to frequency$$

$$T \to Time\ period$$

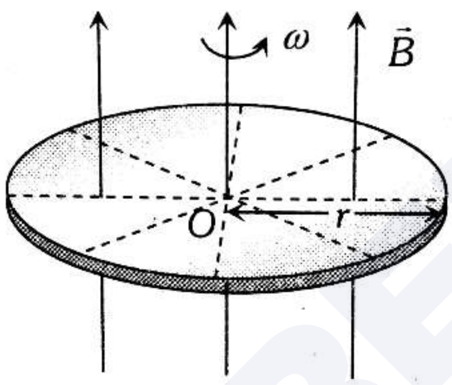
Similarly

- For Cycle wheel rotating with angular velocity ω about O.



$$\varepsilon = \frac{1}{2}B\omega l^2$$

- For Metal Disc



$$\varepsilon = \frac{1}{2}B\omega r^2$$

5. Induced Electric Field

Whenever a magnetic field is varying with time, an induced electric field E_{in} is produced in any closed path, whether in the matter or in empty space. This Induced electric field is directly proportional to induced emf as $\varepsilon = \oint \vec{E_{in}} \cdot \vec{dl}$.

And this E_{in} is given as

$$\varepsilon = \oint \vec{E_{in}} \cdot \vec{dl} = \frac{-d\phi}{dt}$$

This is known as an integral form of Faraday's laws of EMI.

Properties of Induced electric field-

- The induced electric field is different from the electrostatic field. As it is non-conservative and non-electrostatic in nature.
- Its field lines are concentric circular closed curves.
- This field is not created by source charges.
- Its direction is along the tangent to its field lines.

6. Time Varying Magnetic field

Induced electric field is given by

$$\varepsilon = \oint \vec{E_{in}} \cdot \vec{dl} = \frac{-d\phi}{dt}$$

But using $\phi = B.A$ so we can also write

$$\varepsilon = \oint \vec{E_{in}} \cdot \vec{dl} = \frac{-d\phi}{dt} = -A\frac{dB}{dt}$$

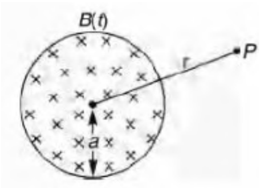
Where

A $\rightarrow$ constant Area

B $\rightarrow$ Varying Magnetic field

For example-

A uniform but time-varying magnetic field B(t) exists in a circular region of radius 'a' and is directed into the plane of the paper as shown in the below figure, the magnitude of the induced electric field (E_{in}) at point P lies at a distance r from the centre of the circular region is calculated as follows.



As due to the time-varying magnetic field induced electric field will be produced whose electric field lines are concentric circular closed curves of radius r.

$$if \quad r \leq a$$

$$then \ E_{in} \times (2\pi r) = \pi r^2 \left|\frac{dB}{dt}\right|$$

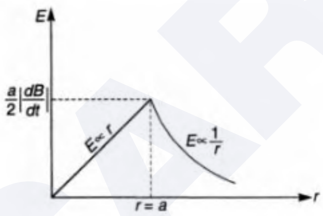
$$\Rightarrow E_{in} = \frac{r}{2}\left|\frac{dB}{dt}\right|$$

For $r > R$,

$$E_{in} \times 2\pi r = \pi a^2 \left|\frac{dB}{dt}\right|$$

$$\Rightarrow E_{in} = \frac{a^2}{2r}\left|\frac{dB}{dt}\right|$$

- The graph of E vs r

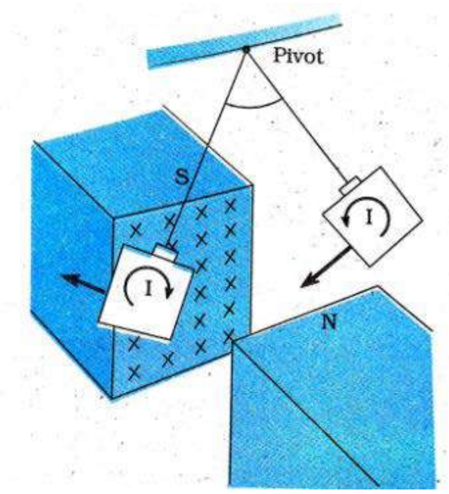


where E=induced electric field

7. Eddy currents

An eddy current is a current set up in a conductor in response to a changing magnetic field. They flow in closed loops in a plane perpendicular to the magnetic field. By Lenz law, the current swirls in such a way as to create a magnetic field opposing the change. They are known as eddy currents as they are in the pattern of eddies in the water. Because of the tendency of eddy currents to oppose, eddy currents cause a loss of energy. Eddy currents are undesirable since they heat up the core and dissipate electrical energy in the form of heat.

Direction of eddy current:



For the above figure, the magnetic flux associated with the plate keeps on changing as the plate moves in and out of the region between magnetic poles. The flux change induces eddy currents in the plate. Directions of eddy currents are opposite when the plate swings into the region between the poles and when it swings out of the region as shown in the above figure.

Eddy currents are used to advantage in certain applications like-

1. Magnetic braking in trains-Strong electromagnets is situated above the rails in some electrically powered trains. When the electromagnets are activated, the eddy currents induced in the rails oppose the motion of the train. As there are no mechanical linkages, the braking effect is smooth.
2. Induction furnace-Induction furnace can be used to produce high temperatures and can be utilized to prepare alloys, by melting the constituent metals. A high-frequency alternating current is passed through a coil that surrounds the metals to be melted. The eddy currents generated in the metals produce high temperatures sufficient to melt it.
3. Electromagnetic damping
4. Electric power meters

8. Self Inductance

Inductance-

It is the property of electrical circuits that oppose any change in the current in the circuits.

Inductance is analogous to inertia in mechanics.

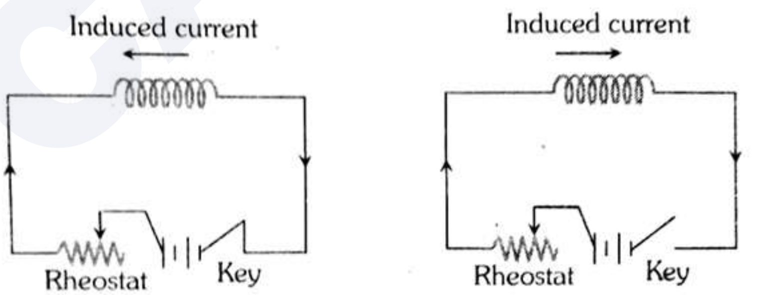
Self Inductance-

Whenever the electric current passing through a coil or circuit changes, the magnetic flux linked with it will also change. And to oppose this flux change according to Faraday's laws of electromagnetic induction, an emf is induced in the coil or the circuit. This phenomenon is called 'self-induction'.

or Self-inductance is defined as the induction of a voltage in a current-carrying wire when the current in the wire itself is changing.

And the emf induced is called back emf, current so produced in the coil is called induced current.

And the direction of induced current for case A and case B is shown below.



(A) Main current increasing

(B) Main current decreasing

Coefficient of self induction (L)-

If ϕ is the flux linkages associated with 1 turn of the coil. And if N is the number of turns in the coil.

Then total flux linkage associated with the coil is $N\phi$

And this total flux linkage is directly proportional to the current in the coil. I.e $N\phi \ \alpha \ i$

or we can write $\phi_{total} = \phi_T = N\phi = Li$

where L=coefficient of self-induction.

So the coefficient of self-induction is given as $L = \frac{N\phi}{I}$

- If $i = 1$ amp, $N = 1$ then, $L = \phi$

i.e The coefficient of self-induction of a coil is equal to the flux linked with the coil when the current in it is 1 amp.

Faraday Second Law of Induction emf-

Using $\phi_{total} = N\phi = Li$ and $\varepsilon = \frac{-d\phi_T}{dt}$

we get $\varepsilon == -N\frac{d\phi}{dt} = -L\frac{di}{dt}$

- If $\frac{di}{dt} = 1\frac{amp}{sec}$ $and\ N = 1$ then $|\varepsilon| = L$

i.e The coefficient of self-induction is equal to the emf induced in the coil when the rate of change of current in the coil is unity.

Units and dimensional formula of 'L'-

S.I. Unit - Henry (H)

And $1H = \frac{1V.sec}{Amp}$

And its dimensional formula is $ML^2T^{-2}A^{-2}$

Dependence of self-inductance (L)-

It depends upon the number of turns (N), Area (A) and permeability of medium (μ).

'L' does not depend upon current flowing or change in current flowing.

Coefficient of Self inductance for long Solenoid-

Let us consider a long solenoid of N turns with length l and area of cross-section $A = \pi r^2$. It carries current i .

Let n = number of turns per unit length $\frac{N}{L}$

whrere, N = total number of turns,

l = length of the solenoid

If B is the magnetic field at any point inside the solenoid, then $B = \mu_0 ni$

The magnetic flux per turn = B × area of each turn

i.e $\text{Magnetic flux per turn} = \phi_0 = \frac{\mu_o NiA}{l}$

So total flux is given as $\phi_T = N\phi_0 = N * \frac{\mu_o NiA}{l} = \frac{\mu_o N^2 iA}{l}....(1)$

If L is the coefficient of self induction of the solenoid, then

$$\phi_T = \text{Li}....(2)$$

From equations (1) and (2)

$$\text{Li} = \frac{\mu_o \text{N}^2\text{IA}}{l}$$

$$\therefore \quad \text{L} = \frac{\mu_o \text{N}^2\text{A}}{l}$$

If the core is filled with a magnetic material of permeability μ,
then, $\text{L} = \frac{\mu \text{N}^2\text{A}}{l}$

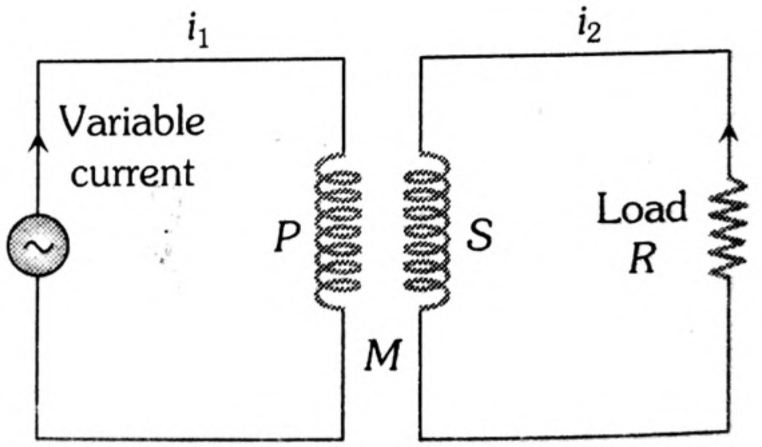
9. Mutual Inductance

Whenever the current passing through a coil or circuit changes, the magnetic flux linked with a neighboring coil or circuit will also change. Hence an emf will be induced in the neighboring coil or circuit. This phenomenon is called 'mutual induction'.

or The phenomenon of producing an induced emf in a coil due to the change in current in the other coil is known as mutual induction.

Coefficient of mutual induction (M)-

If two coils (P-primary coil or coil 1, S-Secondary Coil or coil 2) are arranged as shown in the below figure.



If we change the current through the coil P (i.e i_1) then flux passing through Coil S (i.e ϕ_2) will change.

I.e $N_2\phi_2 \,\alpha\, i_1 \Rightarrow N_2\phi_2 = M_{21}i_1 = Mi_1$

where

M_{21} = mutual induction of Coil 2 w.r. t Coil 1

$N_1 =$ Number of turns in the primary coil

$N_2 =$ Number of turns in the secondary coil

$i_1 =$ current through the primary coil or coil 1

Similarly, if we exchange the position of Coil 1 and Coil 2

then

If we change the current through the coil S (i.e i_2) then flux passing through Coil P (i.e ϕ_1) will change.

I.e $N_1\phi_1 \,\alpha\, i_2 \Rightarrow N_1\phi_1 = M_{12}i_2 = Mi_2$

where

M_{12} = mutual induction of Coil 1 w.r. t Coil 2

$N_1 =$ Number of turns in the primary coil

$N_2 =$ Number of turns in the secondary coil

$i_2 =$ current through the coil 2 or Coil S

- As $N_2\phi_2 = Mi_1$

  If $i_1 = 1$ amp, $N_2 = 1$ then, $M = \phi_2$

I.e coefficient of mutual induction of two coils is numerically equal to the magnetic flux linked with one coil when unit current flows through the neighboring coil.

- Using Faraday Second Law of Induction emf we get

$$\varepsilon_2 = -N_2\frac{d\phi_2}{dt} = -M\frac{di_1}{dt}$$

If $\frac{di_1}{dt} = 1\frac{amp}{sec}$ $and\ N_2 = 1$ then $|\varepsilon_2| = M$

I.e The coefficient of mutual induction of two coils is numerically equal to the emf induced in one coil when the rate of change of current through the other coil is unity.

Units and dimensional formula of 'M'-

S.I. Unit - Henry (H)

And $1H = \frac{1V.sec}{Amp}$

And its dimensional formula is $ML^2T^{-2}A^{-2}$

Dependence of mutual inductance

- Number of turns (N_1, N_2) of both coils
- Coefficient of self inductances (L_1, L_2) of both the coils

and the relation between M, L_1, L_2 is given as

$$M = K\sqrt{L_1 L_2}$$

where K =coeffecient of coupling.

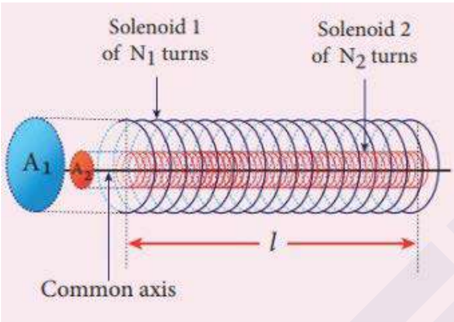
If L=0 then M = 0

If K = 0 i.e case of No coupling then M = 0.

- Distance(d) between two coils (i.e As d increases then M decreases)
- The magnetic permeability of medium between the coils (μ_r)

Mutual Inductance for two coaxial long solenoids-

Consider two long co-axial solenoids of the same length l..Let A_1 and A_2 be the area of cross-section of the solenoids with A_1 being greater than A_2 as shown in the below figure.



The turn density of these solenoids are n_1 and n_2 respectively are given as $n_1 = \frac{N_1}{l}$ and $n_2 = \frac{N_2}{l}$

Let i_1 be the current flowing through solenoid 1, then the magnetic field produced inside it is given as

$$B_1 = \mu_o n_1 i_1$$

As the field lines of $\vec{B_1}$ are passing through the area A_2

So the magnetic flux linked with each turn of solenoid 2 due to solenoid 1 and is given by

$$\Phi_{21} = \int_{A_2} \bar{B}_1 \cdot d\vec{A} = B_1 A_2 = (\mu_0 n_1 i_1)\, A_2$$

The total flux linkage of solenoid 2 with total turns N_2 is

$$(\phi_{21})_{total} = N_2 \Phi_{21} = (n_2 l)\,(\mu_0 n_1 i_1)\, A_2$$
$$\Rightarrow (\phi_{21})_{total} = N_2 \Phi_{21} = (\mu_0 n_1 n_2 A_2 l)\, i_1$$

And Using $(\phi_{21})_{total} = N_2 \Phi_{21} = M_{21} i_1$ we get

$$M_{21} = \mu_0 n_1 n_2 A_2 l$$

Where M_{21} is the mutual inductance of the solenoid 2 with respect to solenoid 1.

Similarly M_{12} =mutual inductance of solenoid 1 with respect to solenoid 2 is given as

$$M_{12} = \mu_0 n_1 n_2 A_2 l$$

Hence $M_{21} = M_{12} = M$

So, In general, the mutual inductance between two long co-axial solenoids is given by

$$M = \mu_0 n_1 n_2 A_2 l$$

If a dielectric medium of permeability μ is present inside the solenoids, then

$$\mathrm{M} = \mu \mathrm{n_1 n_2 A_2 l} \quad \text{or} \quad \mathrm{M} = \mu_0 \mu_r \mathrm{n_1 n_2 A_2 l}$$

Mutual Inductance for a pair of concentric coils-

Consider two circular coils one of radius 'r_1' and the other of radius' r_2'placed coaxially with their centers coinciding as shown in the below figure.

And $r_1 >>> r_2$,so we can assume coil 2 is at the center of coil 1.

Suppose a current i_1 flows through the outer circular coil.
Then Magnetic field at the center of the coil 1 is given as

$$B_1 = \frac{\mu_0 N_1 i_1}{2r_1}$$

So the total flux passing through coil 2 will be given as

$$(\phi_2)_{total} = N_2 B_1 A_2 = \frac{\mu_0 N_1 N_2 i_1 A_2}{2r_1}$$

And using $(\phi_2)_{total} = M i_1$

we get $M = \frac{\mu_0 N_1 N_2 A_2}{2r_1} = \frac{\mu_0 N_1 N_2 (\pi r_2^2)}{2r_1}$

Where M=mutual inductance between two concentric coils

10. Energy Stored In An Inductor

Energy stored in an inductor (U)-

In building a steady current in the circuit, the source emf has to do work against of self-inductance of the coil and whatever energy consumed for this work stored in the magnetic field of coil this energy called as magnetic potential energy (U) of the coil.

When an electric current i is flowing in an inductor, there is energy stored in the magnetic field. Considering a pure inductor L, the instantaneous power which must be supplied to initiate the current in the inductor is

$$P = iv = Li\frac{di}{dt}$$

The work done by the voltage source during a time interval dt is

$$dW = Pdt = iL\frac{di}{dt}dt = Lidi$$

total work W done in establishing the final current I in the inductor

$$\mathrm{W} = \int_0^t Pdt = \int_0^I Lidi = \frac{1}{2}LI^2$$

So Energy stored in the magnetic field of the inductor is given as

$$U = \frac{1}{2}LI^2$$

The energy density (u)/Energy per unit volume-

using $U = \frac{1}{2}LI^2$

for the solenoid field, we can write

$$U = \frac{1}{2}(Li)i = \frac{N\phi i}{2}$$

$$u = \frac{U}{V} = \frac{B^2}{2\mu_0}$$

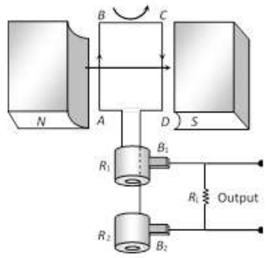
11. AC Generator

AC generator:

An electrical machine used to convert mechanical energy into electrical energy is known as ac generator.

Principle: It works on the principle of electromagnetic induction i.e., when a coil is rotated in a uniform magnetic field, an induced emf is produced in it.

Construction: The main components of ac generator are -



- **Armature:** Armature coil (ABCD) consists of a large number of turns of insulated copper wire wound over a soft iron core.
- **Strong field magnet:** A strong permanent magnet or an electromagnet whose poles (N and S) are cylindrical in shape in a field magnet. The armature coil rotates between the pole pieces of the field magnet. The uniform magnetic field provided by the field magnet is perpendicular to the axis of rotation of the coil.
- **Slip rings:** The two ends of the armature coil are connected to two brass slip rings R_1 and R_2 . These rings rotate along with the armature coil.
- **Brushes:** Two carbon brushes (B_1 andB_2), are pressed against the slip rings. The brushes are fixed while slip rings rotate along with the armature. These brushes are connected to the load through which the output is obtained.

Working: When the armature coil ABCD rotates in the magnetic field provided by the strong field magnet, it cuts the magnetic lines of force. Thus the magnetic flux linked with the coil changes and hence induced emf is set up in the coil. The direction of the induced emf or the current in the coil is determined by Fleming's right-hand rule.

The current flows out through the brush B_1 in one direction of half of the revolution and through the brush B_2 in the next half revolution in the reverse direction. This process is repeated. Therefore, emf produced is of alternating nature.

$$e = -\frac{N d\varphi}{dt} = NBA\omega \sin \omega t = e_0 \sin \omega t$$

where $e_0 = NBA\omega$AC generator

$$i = \frac{e}{R} = \frac{e_0}{R} \sin \omega t = i_0 \sin \omega t R$$ where R is the resistance of the circuit.

Alternating Current

Important Formulae

1. Average And Rms Value Of Alternating Current And Voltage

Average or Mean value:

The average voltage (or current) of a periodic waveform whether it is a sine wave, square wave or triangular waveform is defined as the quotient of the area under the waveform with respect to time. In other words, the averaging of all the instantaneous values along time axis with time being one full period (T).

The average value of alternating quantity for one complete cycle is zero.

The average value of ac over half cycle (t=0 to T/2)

$$i_{av} = \frac{\int_{\frac{T}{2}}^{0} i\,dt}{\int_{\frac{T}{2}}^{0} dt} = \frac{2i_0}{\pi} = 0.637 i_0 = 63.7\% \text{ of } i_0$$

Similarly $V_{av} = \frac{2V_0}{\pi} = 0.637 V_0 = 63.7\% \text{ of } V_0$

Peak to peak value :

The peak-to-peak value of an AC voltage is defined as the difference between its positive peak and its negative peak.

Peak to peak value = $V_0 + V_0 = 2V_0$

Form factor and peak factor :

The ratio of r.m.s value of ac to its average during half cycle is defined as form factor. The ratio of peak value and r.m.s value is called peak factor.

Root mean square (RMS) value:

The root mean square value of a quantity is the square root of the mean value of the squared values of the quantity taken over an interval.

Root of mean of square of voltage or current in an ac circuit for one complete cycle is called r.m.s value .It is denoted by V_{rms} or i_{rms}.

$$V_{rms} = \sqrt{\frac{V_1^2 + V_2^2 + V_3^2 + V_4^2 \ldots + V_n^2}{n}} = \sqrt{\bar{V}^2}$$

$$V_{rms} = \sqrt{\frac{\int_{\frac{T}{2}}^{0} V\,dt}{\int_{\frac{T}{2}}^{0} dt}} = \frac{V_0}{\sqrt{2}} = 0.707V_0 = 70.7\% \text{ of } V_0$$

Similarly , $i_{rms} = \frac{i_0}{\sqrt{2}} = 0.707 i_0 = 70.7\% \text{ of } i_0$

(i) The r.m.s. value of alternating current is also called virtual value or effective value.

(ii) In general when values of voltage or current for alternating circuits are given, these are r.m.s. value.

(iii) ac ammeter and voltmeter are always measured r.m.s. value. Values printed on ac circuits are r.m.s. values.

(iv) In our houses ac is supplied at 220 V, which is the r.m.s. value of voltage. It's peak value is $\sqrt{2} \times 200 = 311V.$

(v) r.m.s. value of ac is equal to that value of dc, which when passed through a resistance for a given time will produce the same amount of heat as produced by the alternating current when passed through the same resistance for same time.

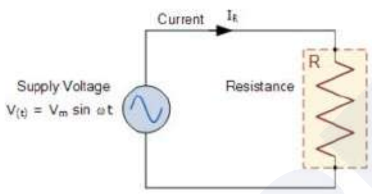
Mean square value $(\bar{V}^2 \text{ or } \bar{i}^2)$:

The average of square of instantaneous values in one cycle is called mean square value. It is always positive for one complete cycle. for example:

$$\bar{V}^2 = \frac{1}{T}\int_0^T V^2 dt = \frac{V_0^2}{2} \text{ or } \bar{i}^2 = \frac{i_0^2}{2}$$

2. AC voltage applied to a resistor:

When a constant voltage source or battery is applied across a resistor current is developed in resister. This current has a unique direction and flows from the negative terminal of a battery to positive terminal. The magnitude of the current remains constant as well. If Direction of current through resistor changes periodically then current is called alternating current.



Voltage V(t) is applied across resistance R. V(t) is sinusoidal voltage with peak V_m and time period T.

$$T = \frac{1}{f} = \frac{2\pi}{\omega}$$

Where f is frequency and ω is angular frequency . This kind of circuit is a purely resistive circuit. According to Kirchhoff's law –

$$v(t) = Ri(t)$$
$$i(t) = \frac{v(t)}{R}$$
$$i(t) = \frac{V_m \sin(\omega t)}{R}$$
$$i_m = \frac{V_m}{R}$$
$$i(t) = i_m \sin(\omega t)$$

Here voltage and current has same frequency and both are in same phase. Therefore phase difference between current and voltage is 0.

The maximum value of voltage is achievd at t=T/4.

Peak current , $i_0 = \frac{V_0}{R}.$

Power factor:

Ratio of resistance and impedance. The power factor also denoted by $cos\phi$.

power factor = $\cos(\phi) = 1$

Power:

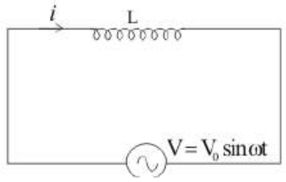
$$P = V_{rms} i_{rms} = \frac{V_0 i_0}{2}$$

Time difference

$$T.D. = 0$$

3.AC voltage applied to an inductor

Voltage applied in the circuit is $V=V_0\sin\omega t$ is applied to pure inductor coil of inductance L. As the current through the inductor varies and opposing induced emf is generated in it and is given by $-L\frac{di}{dt}$.



From Kirchhoff's loop rule:

$$V_0 \sin\omega t - L\frac{di}{dt} = 0$$
$$or$$
$$di = \frac{V_0}{L}\sin\omega t dt$$

Integrating both sides we get,

$$i = -\frac{V_0}{\omega L}\cos\omega t + C$$

Where C is the constant of integration .This integration constant has dimensions of current and is independent of time. Since source has an emf which oscillates symmetrically about zero, the current it sustains also oscillates symmetrically about zero, so there is no time independent component of current that exists. Thus constant C=0.

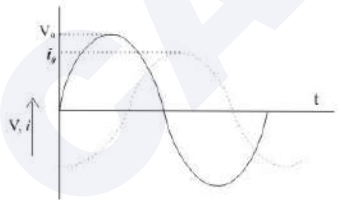
So we have,

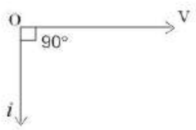
$$i = \frac{-V_0}{\omega L}\cos\omega t$$
$$= \frac{V_0}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right)$$
$$i = i_0 \sin\left(\omega t - \frac{\pi}{2}\right)$$

Where $i_0 = \frac{V_0}{\omega L}$ is called peak value of the current.

From instantaneous values of current and voltage we see that in pure inductive circuit the current lags behind emf by a phase angle of π/2.

This phase relationship is graphically shown below in the figure-





Since peak value of current in the coil is $i_0 = \frac{V_0}{\omega L}$.
Comparing it with the ohm's law we find product ωL has dimension of resistance and it can be represented by
$X_L = \omega L$
where X_L is known as reactance of the coil which represents the effective opposition of the coil to the flow of alternating current.

Phase difference (between voltage and current):

$$\phi = +\frac{\pi}{2}$$

Power factor :

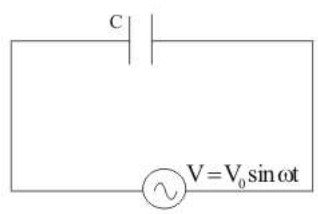
$$\cos(\phi) = 0$$

Time difference:

T.D = $\frac{T}{4}$

4. AC voltage applied to a capacitor:

The circuit containing alternating voltage source $V=V_0 \sin\omega t$ connected to a capacitor of capacitance C.



Suppose at any time t, q be the charge on the capacitor and i be the current in the circuit. Since there is no resistance in the circuit, so the instantaneous potential drop q/C across the capacitor must be equal to applied alternating voltage so,

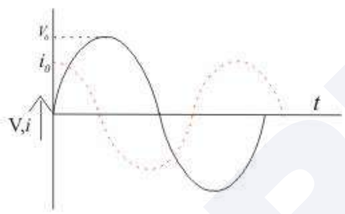
$$\frac{q}{C} = V_0 sin\omega t$$

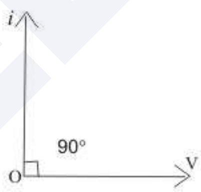
Since $i = dq/dt$ is the instantaneous current in the circuit so,

$$\begin{aligned} i = \frac{dq}{dt} &= \frac{d}{dt}(CV_0 \sin\omega t) \\ &= CV_0\omega \cos\omega t \\ &= \frac{V_0}{(1/\omega C)} \cos\omega t \\ &= i_0 \cos\omega t = i_0 \sin\left(\omega t + \frac{\pi}{2}\right) \end{aligned}$$

Where, $i_0 = \frac{V_0}{(1/\omega C)}$ is the peak value of current.

Comparing equation of current with $V=V_0 \sin\omega t$,we see that in a perfect capacitor current leads the emf by a phase angle of π/2.





Again comparing peak value of current with ohm's law ,we find that quantity 1/ωC has the dimension of the resistance.

Thus the quantity $X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$ is known as capacitive reactance.

Phase difference (between voltage and current):

$$\phi = -\frac{\pi}{2}$$

Power :

P= 0

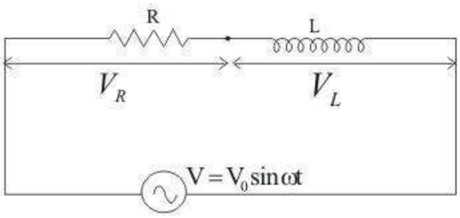
Power factor :

$$\cos(\phi) = 0$$

Time difference:

T.D = $\frac{T}{4}$

5.Series LR circuit-



The above figure shows that pure inductor of inductance L connected in series with a resistor of resistance R through sinusoidal voltage, which is given by - $V = V_0 \sin(\omega t + \phi)$.

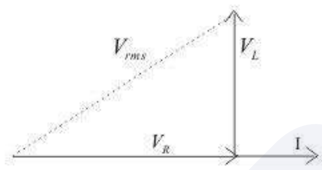
The alternating current I, which is flowing in the circuit gives rise to voltage drop V_R across the resistor and voltage drop V_L across the coil. As we have studied in previous concept that the voltage drop V_R across R would be in phase with current but voltage drop across the inductor will lead the current by a phase factor π/2.

So, the voltage drop across R is - $V_R = IR$

voltage drop across the inductor L is - $V_L = I(\omega L)$

Where, I is the value of current in the circuit at a given instant of time

So, the voltage phasor diagram is -



In the above figure, we have taken current as a reference quantity because same amount of current flows through both the components. Thus from phasor diagram -

$$V = \sqrt{V_R^2 + V_L^2}$$

$$= I\sqrt{R^2 + \omega^2 L^2}$$

$$= IZ$$

where,

$$Z = \left(R^2 + \omega^2 L^2\right)^{1/2}$$

Here, Z is known as **Impedence** of the circuit.

By using the formula of impedence we can write that -

$$I = \frac{V_0 \sin(\omega t - \phi)}{Z}$$

This is current in steady state which lags behind applied voltage by an angle ϕ.

From here and the above figure, we can write that -

$$tan\phi = \frac{\omega L}{R} = \frac{X_L}{R}$$

Important term -

1. Power factor -

$$\cos\phi = \frac{R}{Z}$$

$R \rightarrow$ resistance

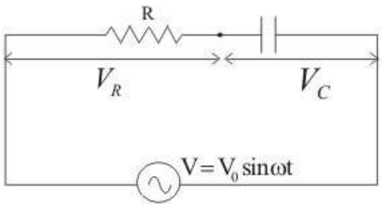
$Z \rightarrow$ impedence

2. Inductive susceptance (S_L) -

It is the reciprocal of reactance.

$$S_L = \frac{1}{X_L} = \frac{1}{2\pi\nu L}$$

6.Series RC circuit -



The above figure shows a circuit containing resisitor and capacitor connected in series through a sinusoidal voltage source of voltage which is given by -

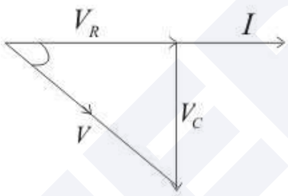
$$V = V_0 \sin(\omega t + \phi)$$

Now, in this case, the voltage across resistor is V_R=IR

And, the voltage across the capacitor is -

$$V_c = \frac{I}{\omega C}$$

As we have studied in the previous concept that the V_R is in phase with current I and V_C lags behind I by a phase angle 90^o



The above figure is the phase diagram of this case. So, the V is the resultant of V_R and V_C. So we can write -

$$V = \sqrt{V_R^2 + V_C^2}$$

$$= i\sqrt{R^2 + \frac{1}{\omega^2 C^2}}$$

$$= iZ$$

where,

$$Z = \sqrt{R^2 + \frac{1}{\omega^2 C^2}}$$

$$i = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + X_c^2}} = \frac{V}{\sqrt{R^2 + \frac{1}{4\pi^2\nu^2 c^2}}}$$

Here, Z is the impedence of this circuit.

Now, from the phasors diagram we can see that the applied voltage lags behind the current by a phase angle ϕ given by -

$$tan\phi = \frac{V_C}{V_R} = \frac{1}{\omega CR}$$

Important points -

1. Capacitive susceptance (S_C) -

$$S_C = \frac{1}{X_c} = \omega C$$

$$\omega C = 2\pi\nu C$$

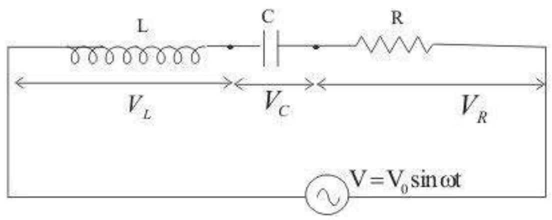
2. Power factor

$$Ratio = \frac{True\ Power}{Apprent\ power}$$

So,

$$\cos\phi = \frac{R}{\sqrt{R^2 + X_c^2}}$$

7.<u>Series LCR circuit-</u>



The Figure given above shows a circuit containing a capacitor, resistor and inductor connected in series through an alternating/sinusoidal voltage source.

As they are in series so the same amount of current will flow in all the three circuit components and for the voltage, the vector sum of potential drop across each component would be equal to the applied voltage.

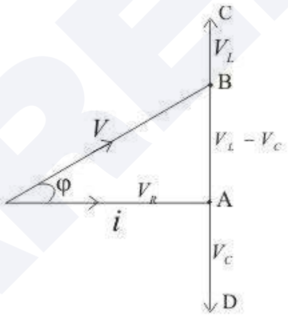
Let 'i' be the amount of current in the circuit at any time and V_L,V_C and V_R the potential drop across L, C and R respectively then

$$v_R = iR \rightarrow \text{ Voltage is in phase with i}$$

$$v_{L=i\omega L} \rightarrow \text{ voltage is leading i by } 90^\circ$$

$$v_c = i/\omega c \rightarrow \text{ voltage is lagging behind i by } 90^\circ$$

By all these we can draw a phasor diagram as shown below -



One thing should be noticed that we have assumed that V_L is greater than V_C which makes i lags behind V. If $V_C > V_L$ then i lead V. So as per our assumption, there resultant will be (V_L -V_C). So, from the above phasor diagram V will represent resultant of vectors V_R and (V_L -V_C). So the equation become -

$$V=\sqrt{V_R^2+(V_L-V_C)^2}$$

$$=i\sqrt{R^2+(X_L-X_C)^2}$$

$$=i\sqrt{R^2+\left(\omega L-\frac{1}{\omega C}\right)^2}$$

$$=iZ$$

where,

$$Z=\sqrt{R^2+\left(\omega L-\frac{1}{\omega C}\right)^2}$$

Here, Z is called the Impedance of this circuit.

Now come to the phase angle. The phase angle for this case is given as -

$$\tan\varphi=\frac{V_L-V_C}{V_R}=\frac{X_L-X_C}{R}=\frac{\omega L-\frac{1}{\omega C}}{R}$$

Now from the equation of the phase angle three cases will arise. These three cases are -

(i) When, $\omega L > \frac{1}{\omega C}$

then, tanφ is positive i.e. φ is positive and voltage leads the current i.

(ii) When $\omega L < \frac{1}{\omega C}$

then, tanφ is negative i.e. φ is negative and voltage lags behind the current i.

(iii) When $\omega L = \frac{1}{\omega C}$,

then tanφ is zero i.e. φ is zero and voltage and current are in phase. **This is called electrical resonance.**

8. Resonance in Series LCR circuit-

When

$$\omega L=\frac{1}{\omega C},$$

then $tan\varphi$ is zero i.e. phase angle (φ) is zero and voltage and current are in phase. We have called it electric resonance. So, if $X_L=X_C$, then the equation of impedence become -

$$Z=\sqrt{R^2+\left(\omega L-\frac{1}{\omega C}\right)^2}=R$$

So, we get minimum value of Z.

In this case impedance is purely resistive and minimum and currents has its maximum value. Now as -

$$\omega L=\frac{1}{\omega C}$$

So,

$$\omega=\frac{1}{\sqrt{LC}}$$

As, $\omega=2\pi f_o$. Where f_o is the frequency of applied voltage.

So,

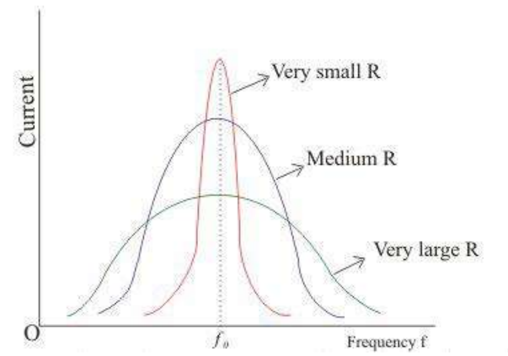
$$f_o=\frac{1}{2\pi\sqrt{LC}}$$

This frequency is called **resonant frequency** of the circuit.

Peak current in this case is given by -

$$i_o = \frac{V_o}{R}$$

We will now discuss about the resonance curve and its nature. We will show the variation in circuit current (peak current i_0) with change in frequency of the applied voltage -



This figure/graph shows the variation of current with the frequency.

Conclusions from the graph -

1. If R has small value, the resonance is sharp which means that if applied frequency is lesser to resonant frequency f_0,the current is high otherwise
2. If R is large, the curve is broad sided which means that those is limited change in current for resonance and non -resonance conditions

Note -

The natural or resonant frequency is Independent from resistance of the circuit.

$$X_L = X_c = \omega_0 L = \frac{1}{\omega_0 c}$$

$$\nu_0 = \frac{1}{2\pi\sqrt{Lc}}(Hz)$$

9.Quality Factor In An AC Circuit

Quality factor-

The quality factor Q is a parameter which is used to describe the sharpness of the resonance curve. So it is defined as the ratio of voltage drop across the inductor or capacitor at resonance to the applied voltage. So,

$$Q = \frac{Voltage\ across\ L\ or\ C\ at\ resonance}{Applied\ voltage}$$

$$Q = \frac{I_v \omega_o L}{I_v R} = \omega_o \frac{L}{R}$$

As we know that, at the resonance -

$$\omega_o = \frac{1}{\sqrt{LC}}$$

So,

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

We can also say that the characteristic of a series resonant circuit is determined by the quality factor (Q - factor) of the circuit. So, if the value of Q-factor is high then the sharpness of the resonant curve is more and vice-versa.

We can also define the Q -factor that is is defined as 2π times the ratio of the energy stored in L or C to the average energy loss per period. So,

$$Q = 2\pi\left[\frac{Maximum\ energy\ stored\ in\ the\ capacitor}{Energy\ loss\ per\ period}\right] \qquad ....(1)$$

Now, the maximum energy stored in the inductor =

$$U = \frac{1}{2}L(I_o)^2$$

Also the energy dissipated per second =

$$P_R = I_{rms}^2 R = \frac{I_o^2 R}{2}$$

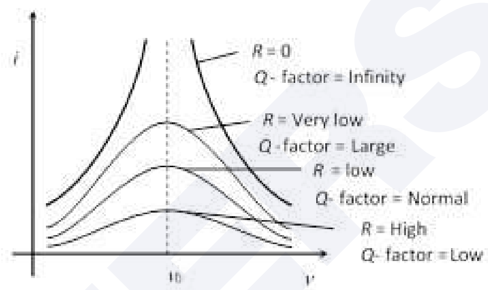
Energy dissipated per time period =

$$U_R = \frac{I_o^2 R}{2} \times T$$

Putting all these in the (1)

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

The Q-factor of the circuit varies inversely as R. Thus, at resonance, the voltage drop across inductance or capacitance is Q-times the applied voltage.



Resonance curve

From the graph we can see that when the Q-factor tends to infinity, then the current become infinite. And as the Q-factor become very low then the amplitude of the current will become very low.

In an ac circuit, If,

$R = 0\ or\ \cos\phi = 0$

$P_{av} = 0$

Important term -

1. Wattless current

In resistance less circuit the power consumed is zero such circuit is called wattless and the current following is called wattless current.

Amplitude of wattless is $I_0 \sin\varphi$

10.<u>Power in an AC circuit-</u>

When the voltage $v = v_m sin\omega t$ applied to a series RLC circuit drives a current in the circuit given by $i = i_m sin(\omega t + \phi)$ where,

$$i_m = \frac{v_m}{Z} \text{ and } \phi = \tan^{-1}\left(\frac{X_C - X_L}{R}\right)$$

So, the instantaneous power is equals to -

$$P_{instantaneous} = vi = (v_m \sin\omega t) \times [i_m \sin(\omega t + \phi)]$$

$$By\ applying\ trigonometric\ application\ \ we\ get,$$

$$P_{instaneous} = \frac{v_m i}{2}[\cos\phi - \cos(2\omega t + \phi)]$$

If we calculate the average power, then the second term of RHS will become zero. Because it is time-dependent and during one complete cycle, the summation will become zero.

$$P_{Average} = \frac{v_m i_m}{2} \cos \phi = \frac{v_m}{\sqrt{2}} \frac{i_m}{\sqrt{2}} \cos \phi$$

$$P_{Average} = VI \cos \phi$$

$$P_{Average} = I^2 Z \cos \phi$$

From the above equation we can see that the average power dissipated depends on the voltage and current and the cosine of the phase angle φ between them. The quantity cosφ is called the **power factor**.

Let us discuss the power factor for various cases -

- Resistive circuit: If in the circuit, only pure R is present, it is called resistive.circuit In that case φ = 0, because cos φ =1. And if the power factor is 1, then there is maximum power dissipation.

- Purely inductive or capacitive circuit: From the previous concept and from the phasor diagram of these cases, we can say that, if the circuit contains only an inductor or capacitor then the phase difference between voltage and current is $\frac{\pi}{2}$. Therefore, cos φ = 0, and no power is dissipated even though a current is flowing in the circuit. This current is referred to as **wattless current**.

- LCR series circuit: As we know that the phase angle in this case is -

$$\varphi = tan^{-1} \frac{X_L - X_C}{R}$$

  So, φ may be non-zero in R-L, R-C or R-L-C. And if it is non-zero, then there must be some power dissipation but that power dissipation is only in resistor.

- Power dissipated at resonance in LCR circuit: As we know that at resonance, $X_C - X_L = 0$, So, phase angle (φ) = 0. Therefore, cosφ = 1. So, $P = I^2 Z = I^2 R$. That is, maximum power is dissipated in a circuit at resonance. The total dissipation is through resistor.

Important point -

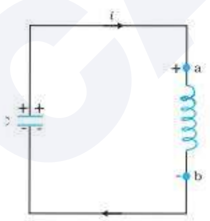
Apparent or virtual power - The product of apparent voltage and apparent current in an electrical circuit. Apparent power be always positive

$$P_{app} = V_{rms} \quad i'_{rms} = \frac{v_0 \quad i'_0}{2}$$

11.LC oscillations:

When a charged capacitor is allowed to discharge through a non-resistance, electrical oscillations of constant amplitude and frequency are produced. These oscillations are called LC-oscillations.

Let a capacitor be charged q_m (at t = 0) and connected to an inductor as shown in Fig.
The moment the circuit is completed, the charge on the capacitor starts decreasing, giving rise to the current in the circuit.



Let q and i be the charge and current in the circuit at time t.

Since $\frac{di}{dt}$ is positive, the induced emf in L will have polarity as shown, i.e., $v_b < v_a$.

According to Kirchhoff's loop rule,

$$\frac{q}{C} - L\frac{\mathrm{d}i}{\mathrm{d}t} = 0$$

i = -(dq/dt) in the present case (as q decreases, I increases).

$$\frac{d^2 q}{dt^2} + \frac{1}{LC} q = 0$$

This equation has the form for

$$\frac{d^2 x}{dt^2} + \omega_0^2 x = 0$$

a simple harmonic oscillator. The charge, therefore, oscillates with a natural frequency.

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

And varies sinusoidally with time as:

$$q = q_m \cos(\omega_0 t + \phi)$$

where q_m is the maximum value of q and φ is a phase constant. Since $q = q_m$ at $t = 0$, we have $\cos\phi = 1$ or $\phi = 0$. Therefore, in the present case

$$q = q_m \cos(\omega_0 t)$$

The current i (= $\frac{di}{dt}$) is given by

$$i = i_m \sin(\omega_0 t)$$

$$\text{where } i_m = \omega_0 q_m$$

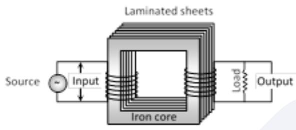
Since there is no current in the circuit; energy in the inductor is zero. Thus, the total energy of LC circuit is

$$U = U_E = \frac{1}{2}\frac{q_m^2}{C}$$

12. Transformers

Transformers

It is a device that raises or lowers the voltage in ac circuits through mutual induction. It consists of two coils wound on the same core. The alternating current passing through the primary creates a continuously changing flux through the core. This changing flux induces an alternating emf in the secondary.



Step-up Transformer: A transformer in which the output (secondary) voltage is greater than its input (primary) voltage is called a step-up transformer.

Step-down Transformer: A transformer in which the output (secondary) voltage is less than its input (primary) voltage is called a step-down transformer

- The transformer works on ac only and never on dc.
- It can increase or decrease either voltage or current but not both simultaneously.
- Transformer does not change the frequency of input ac.
- There is no electrical connection between the winding but they are linked magnetically.
- Effective resistance between the primary and secondary winding is infinite.
- The flux per turn of each coil must be same i.e.

$$\phi_S = \phi_P \quad \text{so,} \quad -\frac{d\phi_P}{dt} = -\frac{d\phi_S}{dt}$$

If N_P=number of turns in primary, N_S = number of turns in secondary,

V_P= applied (input) voltage to primary,

V_S= Voltage across secondary (load voltage or output),

e_P= induced emf in primary ;

e_S= induced emf in secondary,

ϕ= flux linked with primary as well as secondary, current in primary;

i_S= current in secondary (or load current)

As in an ideal transformer, there is no loss of power i.e.$P_{out} = P_{in}$ so, $V_S i_S = V_P i_P$ and $V_P \approx e_P, V_S \approx e_S$.

Hence,

$\frac{e_S}{e_P} = \frac{N_S}{N_P} = \frac{V_S}{V_P} = \frac{i_P}{i_S} = k$,   k = Transformation ratio.

Efficiency of transformer (η): Efficiency is defined as the ratio of output power and input power i.e. η.

For an ideal transformer, $P_{out} = P_{in}$ so η=100.

For practical transformer, $P_{in} = P_{out} + P_{losses}$. Efficiency of a practical transformer lies between 70-90 %.

So $$\eta = \frac{P_{\text{out}}}{(P_{\text{out}} + P_L)} \times 100 = \frac{(P_{\text{in}} - P_L)}{P_{\text{in}}} \times 100$$.

Losses in transformer: In transformers, some power is always lost due to, heating effect, flux leakage eddy currents, hysteresis and humming.

Electromagnetic Waves

Important Formulae

1. Displacement Current-

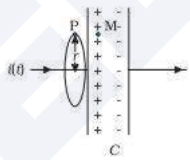
It is a current which produced in the region in which the electric field and hence the electric flux changes with time. As we know an electrical current produces a magnetic field around it. So if there is a change in an electric field, the magnetic field will be produced. This effect explains the existence of radio waves, gamma rays and visible light, as well as all other forms of electromagnetic waves.

The below figure shows a parallel plate capacitor C which is a part of the circuit through which time-dependent current i(t) flows.

Now we have to find the magnetic field at a point such as P, in a region outside the parallel plate capacitor.

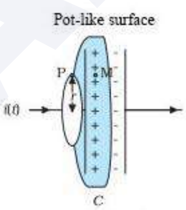
Now for applying Ampere's circuital law, we have to consider a plane circular loop of radius 'r' whose plane is perpendicular to the direction of the current-carrying wire, and we can see that it is centred symmetrically with respect to the wire. As we can see from symmetry the direction of the magnetic field is along the circumference of the circular loop and is the same in magnitude at all points on the loop so that B can be taken outside of the integration and by integrating the loop length, the left side of equation (1) will be equal to B(2π r). So we have

$$B(2\pi r) = \mu_0 i(t)$$

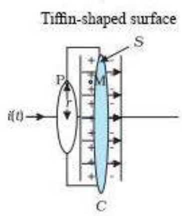


Now we are going to change the surface taken for the Ampere's circuital law such that it has the same boundary.

Let us take two cases, which is shown in the below figure -



(a)



(b)

We can see from both cases that the surface nowhere touches the current.

In case (a), it has a pot-like surface such that its base is between the plates and its mouth has the same surface as we have taken in the earlier case.

Similarly case (b) is a tiffin-like structure. Now if we apply Ampere's circuital law again for both cases, the left side of equation (1) will remain the same but the right side will become zero because there is no current passes through the surface.

But for the same surface and by the same law we are getting different values of the magnetic field which shows that something is incorrect or some term is missing.

Now, what is that missing term??

If we take case (b) again, we can see that an electric field will pass through the surface S.

The magnitude of this electric field is equal to -

$$E = \frac{1}{\varepsilon_0}\frac{Q}{A}$$

Since the electric field is the same over the area and zero outside the plate. So the electric flux will be equal to (By Gauss's Law) -

$$\Phi_E = |\mathbf{E}|A = \frac{1}{\varepsilon_0}\frac{Q}{A}A = \frac{Q}{\varepsilon_0}$$

But if the charge changes with time, the equation can be written as -

$$\frac{d\phi_E}{dt} = \frac{d}{dt}\left(\frac{Q}{\varepsilon_0}\right) = \frac{1}{\varepsilon_0}\frac{dQ}{dt}$$

$$\Rightarrow \varepsilon_0\left(\frac{d\Phi_E}{dt}\right) = i \qquad \text{.................(2)}$$

This is the missing term in Ampere's circuital law.

Now, if we add the total current carried by conductors through the surface, another term which is ε_0 times the rate of change of electric flux through the same surface, the total has the same value of current 'i' for all surfaces.

After doing this B at the point P is non-zero and becomes equal for all the cases, no matter which surface is used for calculating it.

The current carried by conductors due to the flow of charges is called **conduction current**.

The current, given by equation (2), is a new term and is due to the changing electric field. It is, therefore, called **displacement current or Maxwell's displacement current.**

Maxwell generalised the above consequences as follows.

The source of a magnetic field is not just the conduction of electric current due to flowing charges, but also the time rate of change of the electric field. So we can write that -

$$i = i_e + i_d = i_c + \varepsilon_0\frac{d\Phi_E}{dt}$$

where i_c and i_d is the conduction current and displacement current respectively.

This means that outside the capacitor plates, we have only conduction current $i_c = i$, and no displacement current, i.e., $i_d = 0$.

On the other hand, inside the capacitor, there is no conduction current, i.e., $i_c = 0$, and there is only displacement current, so that $i_d = i$.

So, in general, the Ampere's circuital law will be like -

$$\oint \mathbf{B}\cdot d\mathbf{l} = \mu_0 \boldsymbol{i}_e + \mu_0\varepsilon_0\frac{d\phi_E}{dt}$$

This is called the Ampere-Maxwell law.

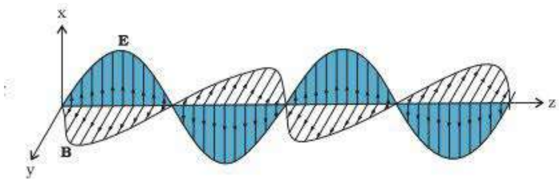
2. Maxwell's equations

The four Maxwell's equations and Lorentz force law together constitute the foundations of classical electromagnetism. The Maxwell's equations are:

1. $\oint \mathbf{E}\cdot d\mathbf{A} = Q/\varepsilon_0$ (Gauss's Law for electricity)
2. $\oint \mathbf{B}\cdot d\mathbf{A} = 0$ (Gauss's Law for magnetism)
3. $\oint \mathbf{E}\cdot d\mathbf{l} = \frac{-d\phi_B}{dt}$ (Faraday's Law)
4. $\oint \mathbf{B}\cdot d\mathbf{l} = \mu_0 i_c + \mu_0\varepsilon_0\frac{d\phi_E}{dt}$ (Ampere-Maxwell Law)

3.Nature of Electromagnetic Waves

From Maxwell's equations, we can observe that electric and magnetic fields in an electromagnetic wave are perpendicular to each other, and to the direction of propagation. Also from our discussion of the displacement current, in that capacitor, the electric field inside the plates of the capacitor is directed perpendicular to the plates. The figure given below shows a typical example of a plane electromagnetic wave propagating along the z direction (the fields are shown as a function of the z coordinate, at a given time t). The electric field E_x is along the x-axis, and varies sinusoidally with z, at a given time. The magnetic field B_y is along the y-axis, and again varies sinusoidally with z.The electric and magnetic fields E_x and B_y are perpendicular to each other, and to the direction z of propagation.



Now from the Lorentz equation -

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

$$E_z = E z_0 \sin(\omega t - ky)$$

$$B_x = B x_0 \sin(\omega t - ky), \text{ where } \frac{\omega}{k} = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

since, $\omega = 2\pi f$, where f is the frequency and $k = \frac{2\pi}{\lambda}$, where λ is the wavelength.

$$\text{Therefore, } \frac{\omega}{k} = \frac{2\pi f}{2\pi/\lambda} = f\lambda$$

But $f\lambda$ gives the velocity of the wave. So, $f\lambda = c = \omega k$. So we can write -

$$c = \frac{\omega}{k} = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

It is also seen from Maxwell's equations that the magnitude of the electric and the magnetic fields in an electromagnetic wave are related as -

$$B_0 = \frac{E_o}{c}$$

In a material medium of permittivity ε and magnetic permeability μ, the velocity of light becomes,

$$v = \frac{1}{\sqrt{\mu \varepsilon}}$$

4. Energy Density and Intensity of EM waves-

Energy Density-

The electric and magnetic fields in a plane electromagnetic wave are given by

$$E = E_0 \sin \omega(t - x/c)$$

$$\text{and, } B = B_0 \sin \omega(t - x/c)$$

In any small volume dV, the energy of the electric field is

$$U_E = \tfrac{1}{2}\varepsilon_0 E^2 dV$$

And the energy of the magnetic field is $U_B = \frac{1}{2\mu_0} B^2 dV$

Thus, the total energy is $U = \frac{1}{2}\varepsilon_0 E^2 dV + \frac{1}{2\mu_0} B^2 dV$

The energy density is $u = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2\mu_o} B^2$

$$u = \tfrac{1}{2}\varepsilon_0 E_0^2 \sin^2 \omega(t - x/c) + \tfrac{1}{2\mu_0} B_0^2 \sin^2 \omega(t - x/c)$$

For taking the average over a long time, the $\sin^2$ terms have an average value of $1/2$

$$\text{So, } u_{av} = \frac{1}{4}\varepsilon_0 E_0^2 + \frac{1}{4\mu_0} B_0^2$$

Now, as we know, $E_0 = cB_0$ and $\mu_0\varepsilon_0 = \frac{1}{c^2}$ so that, $\mu_0 = \frac{1}{\varepsilon_0 c^2}$ and,

$$B_0 = \frac{E_0}{c}\frac{1}{4\mu_0}B_0^2 = \frac{\varepsilon_0 c^2}{4}\left(\frac{E_0}{c}\right)^2 = \frac{1}{4}\varepsilon_0 E_0^2$$

Thus, the electric energy density is equal to the magnetic density in average,

or, $u_{\mathrm{av}} = \frac{1}{4}\varepsilon_0 E_0^2 + \frac{1}{4}\varepsilon_0 E_0^2 = \frac{1}{2}\varepsilon_0 E_0^2$

Also $u_{\mathrm{av}} = \frac{1}{4\mu_0}B_0^2 + \frac{1}{4\mu_0}B_0^2 = \frac{1}{2\mu_0}B_0^2$

Intensity (I): The energy crossing per unit area per unit time, perpendicular to the direction of propagation of EM wave is called intensity.

So,

$$\text{i.e. } I = \frac{\text{Total EM energy}}{\text{Surface area } \times \text{ Time}} = \frac{\text{Total energy density } \times \text{ Volume}}{\text{Surface area } \times \text{ Time}}$$

$$\Rightarrow I = u_{av} \times c = \frac{1}{2}\varepsilon_0 E_0^2 c = \frac{1}{2}\frac{B_0^2}{\mu_0} \cdot c \quad \frac{Watt}{m^2}$$

Momentum: Electromagnetic waves also carries momentum. As we know linear momentum is associated with energy and speed.

So we can write that if wave incident on a completely absorbing surface then momentum delivered will be equal to -

$$p = \frac{u}{c}$$

But if the wave is incident on a totally reflected surface, then the momentum will be equal to -

$$-p = \frac{2u}{c}$$

Poynting vector ($\vec{S}$) : It is defined as the rate of flow of energy crossing a unit area in electromagnetic waves. So,

$$\vec{S} = \frac{1}{\mu_o}(\vec{E} \times \vec{B}) = c^2\varepsilon_0(\vec{E} \times \vec{B})$$

Unit of Poynting vector is Watt/m^2. Now, as we know that in electromagnetic waves, $\vec{E}$ and $\vec{B}$ are perpendicular to each other. So,

$$|\vec{S}| = \frac{1}{\mu_0}EB\sin 90^\circ = \frac{EB}{\mu_0} = \frac{E^2}{\mu C}$$

The importance of the Poynting vector is that the direction of the Poynting vector $\vec{S}$ at any point gives the wave's direction of travel and direction of energy transport to the point.

The average value of the Poynting vector -

$$\vec{S} = \frac{1}{2\mu_0}E_0B_0 = \frac{1}{2}\varepsilon_0 E_0^2 c = \frac{cB_0^2}{2\mu_0}$$

As we can notice that direction of Poynting vector can be given by the vector product so, The direction of $\vec{S}$ does not oscillate but its magnitude varies between zero and a maximum each quarter of the period.

Radiation pressure: It is defined as the momentum imparted per second per unit area on which the light falls

So, for the perfectly absorbing body, we can write in terms of the Poynting vector -

$$P_a = \frac{S}{c}$$

And for perfectly reflecting surface -

$$P_r = \frac{2S}{c}$$

Wave impedance (Z): As the word, impedance tells that obstruction inflowing of something, similarly here, the medium offers hindrance to the propagation of the wave. Such hindrance is called wave impedance and it is given by -

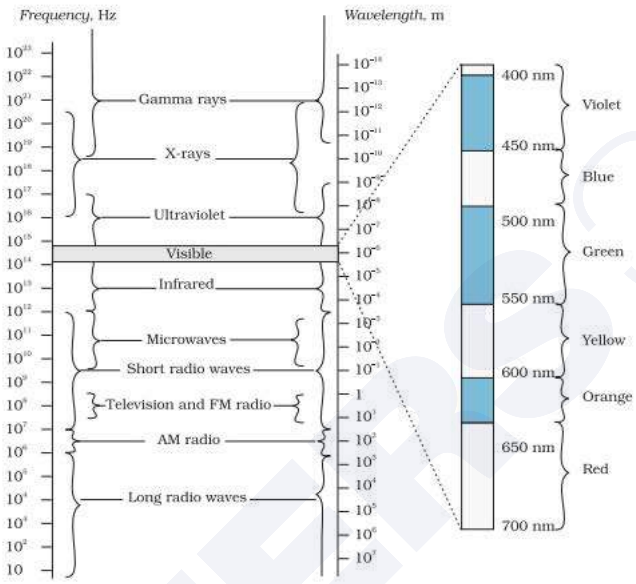
$$Z=\sqrt{\frac{\mu}{\varepsilon}}=\sqrt{\frac{\mu_r}{\varepsilon_r}}\sqrt{\frac{\mu_0}{\varepsilon_0}}$$

For vacuum or free space -

$$Z=\sqrt{\frac{\mu_0}{\varepsilon_0}}=376.6\Omega$$

5.Electromagnetic spectrum-

When we see our surroundings, we see only visible range of electromagnetic waves. So, the only familiar electromagnetic waves were the visible light waves. But, we now know that, electromagnetic waves include visible light waves, X-rays, gamma rays, radio waves, microwaves, ultraviolet and infrared waves. The classification of EM waves according to frequency is the electromagnetic spectrum is shown in the figure given below.



Now we will discuss all these EM waves one by one with the help of the following table -

Type	Wavelength range	Production	Detection
Radio	> 0.1 m	Rapid acceleration and decelerations of electrons in aerials	Receiver's aerials
Microwave	0.1m to 1 mm	Klystron valve or magnetron valve	Point contact diodes
Infra-red	1mm to 700 nm	Vibration of atoms and molecules	Thermopiles Bolometer, Infrared photographic film
Light	700 nm to 400 nm	Electrons in atoms emit light when they move from one energy level to a lower energy level	The eye Photocells Photographic film
Ultraviolet	400 nm to 1nm	Inner shell electrons in atoms moving from one energy level to a lower level	Photocells Photographic film
X-rays	1nm to 10^{-3} nm	X-ray tubes or inner shell electrons	Photographic film Geiger tubes Ionisation chamber
Gamma rays	$<10^{-3}$ nm	Radioactive decay of the nucleus	-do-

Earth's atmosphere -

Earth's atmosphere has the following six layers.

(i) Troposphere:

The troposphere is the innermost layer of Earth's atmosphere. i.e. it is Closest to the surface of the Earth.

It is the thermal classification of the atmosphere."Tropos" means change. This layer gets its name from the weather that is constantly changing. The troposphere is between 8 and 14 kilometers. This layer has the air we breathe and the clouds in the sky.

(ii) Stratosphere:

The stratosphere is located above the troposphere and below the mesosphere.

It extends between 17-50 Km above the earth's surface. The ozone layer is located in the stratosphere.

Ozone layer - It absorbs most of the ultraviolet rays emitted by the sun.

(iii) Mesosphere:

The mesosphere is located above the stratosphere and below the thermosphere.

It is characterized by temperatures that quickly decrease with increasing height. It extends between 50-80 Km.

(iv) Thermosphere:

The thermosphere is located above the mesosphere and below the exosphere.

Based on the vertical temperature profile in the atmosphere, the thermosphere is the highest layer, located above the mesosphere.

In the thermosphere, temperature increases with altitude.

It extends from about 90 km to between 500 and 1,000 km above our planet.

(v) Ionosphere: It starts at about 75 Km and goes up to 650 Km. It contains ions and free electrons. Aurora occurs in the Ionosphere.

(vi) Exosphere - The outermost layer of the earth's atmosphere. (640 Km - 1280 Km)

Point to remember -

1. Polarisation in EM wave - For an EM wave, the direction of polarisation is taken to be the direction of the electric field.

2. Wavelength of EM Wave -

$$\lambda = \frac{\lambda_o}{\mu}$$

λ_o = Wavelength in vacuum

μ = Refractive index of medium

Ray Optics and Optical Instruments

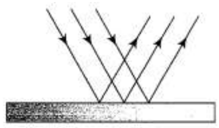
Important Formulae

1. Laws of reflection:

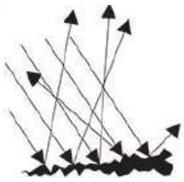
Reflection of light : when ray of light strikes the boundary of two media such as air or glass, a part of light turned back into the same material . This phenomenon is known as "reflection of light".

There are two types of reflection :

1. **Regular reflection:** When the reflection takes place from a perfect plane surface it is called Regular Reflection. In this case, the reflected



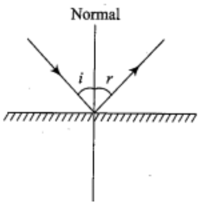
light has large intensity in one direction and negligibly small intensity in other directions.
2. **Diffused reflection:** When light is reflected from a irregular surface we do not get a regular behavior of light. Light reflects from the surface such that a ray incident on the surface is scattered at many angles rather than at just one angle.



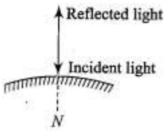
It has been found that rays undergoing reflection follow two laws called the Laws of Reflection:
(i) The incident ray, the reflected ray, and the normal at the point of incidence lie in the same plane. The plane is called the plane of incidence (or plane of reflection).

(ii) The angle of incidence (the angle between normal and the incident ray) and the angle of reflection (the angle between the reflected ray and the normal) are equal, i.e.,

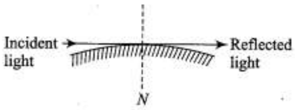
$$\angle i = \angle r$$



Normal Incidence: When a ray incident normally on a plane
$i = r = 0$
$\delta = 180°$



Grazing Incidence: In case light strikes the reflecting surface tangentially



2. Reflection On A Plane Mirror

Object: Objects are sources of light rays that are incident on an optical element.

- **Real object:** An object is real if two or more incident rays actually emanate or seem to emanate from a point.
- **Virtual object:** An object is virtual when two incident rays seem to converge to that point.

Image: An image is the point of convergence or apparent point of divergence of rays after they interact with a given optical element. An object provides rays that will be incident on an optical element. The optical element reflects or refracts the incident light rays which then meet at a point to form an image. As in the case
of objects, images too can be real or virtual.

- **Real Image**: Real images are formed when the reflected or refracted rays actually meet or converge to a point. If a screen is placed at that point, a bright spot will be visible on the screen. Thus, a real image can be captured on a screen.
- **Virtual image:** an optical image formed from the apparent divergence of light rays from a point, as opposed to an image formed from their actual divergence

Image formation by plane mirror:

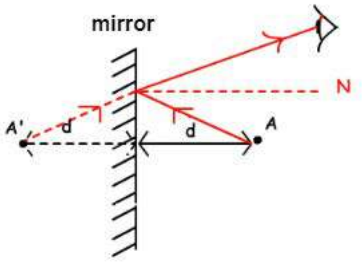
We have to see the rays coming from the object to see it. If the light first hits the mirror and then reflects with the same angle, the extensions of the reflected rays are focused at one point behind the mirror. We see the coming rays as if they are coming from behind the mirror. At a point, A' image of the point is formed and we call this image a virtual image The distance of the image to the mirror is equal to the distance of the object to the mirror.

a) For a point object

Distance of object from mirror = Distance of image from the mirror

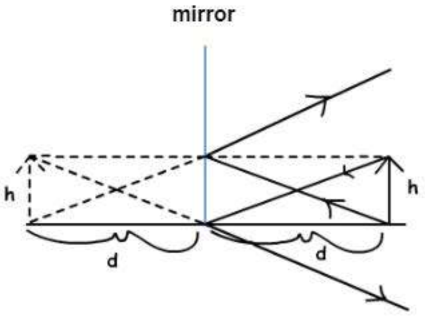
(i) All the incident rays from a point object after reflection from a plane mirror will meet at a single point which is called an image.

(ii) The line joining a point object and its image is normal to the reflecting surface



b) For an extended object :

The size of the image is the same as that of the object. An image of an extended object by a plane mirror is a virtual image. The image will be upright and laterally inverted.



Rotation of plane mirror

i) Mirrror is rotated keeping ray of incidence fixed:

Consider that before the mirror is rotated the angle of incidence and recfection is θ. When the mirror is rotated through and angle of say ϕ in a clockwise direction, then the normal is also rotated by an angle ϕ and thus the new angle of incidence becomes $\theta+\phi$ and thus new angle of reflection will be to $\theta+\phi$ as shown in the below figure.

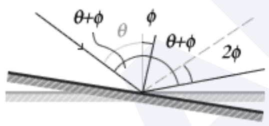
So the angle between the incident ray and new reflected ray is $2(\theta+\phi)........(1)$

Let due to rotation of mirror new reflected ray get deflected by an angle δ in a clockwise direction with respect to the original reflected ray .

So So the angle between the incident ray and the new reflected ray is $2\theta+\delta........(2)$

from the equation (1) and (2) we get $\delta=2\phi$

i.e For fixed incident ray, When the mirror is rotated ϕ in a clockwise direction then reflected ray get deflected by 2ϕ in the clockwise direction



ii) **Mirror is fixed, angle of incident ray changed :**

When the mirror is fixed and the angle of incidence is changed by an angle α in an anti-clockwise direction, Then the angle of reflection will rotate by an angle α in the clockwise direction.

The angle of Deviation -

The angle of Deviation is the angle made by the reflected ray with the direction of the incident ray.

Number of images formed by two plane mirrors:

The number of images formed by two adjacent plane mirrors depends on the angle between the mirror. If θ (in degrees) is angle between the plane mirrors

- If the value of $\frac{360}{\theta}$ is even, then we will use the formula

$$n=\frac{360}{\theta}-1$$

- But If the value of $\frac{360}{\theta}$ is odd, then we have two different cases

1. $$n=\frac{360}{\theta}-1 \quad \ldots(\text{when the object is placed symmetrically})$$

2. $$n=\frac{360}{\theta} \quad \ldots(\text{when the object is placed asymmetrically})$$

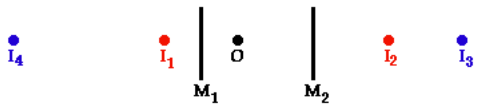
Let us take a few cases(with some conditions) to understand it better -

Case 1: When two mirrors are placed parallel to each other

Number of images formed by using the formula

$$n=\frac{360}{\theta}-1$$

where θ is 0 degrees. So the number of images formed will be infinite.



Case 2: When two mirrors are placed perpendicular to each other

The number of images formed when two mirrors are placed at an angle theta to each other is given by:

$$n=\frac{360}{\theta}-1$$

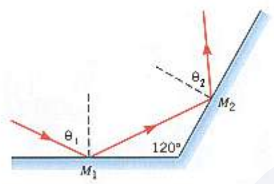
So, here, we have the mirrors placed perpendicular to each other. So, θ= 90 degree

$$n=\frac{360}{90}-1$$

$$n=4-1$$

$$n=3$$

Case 3: When two mirrors are placed at 120 degrees



Here, we have the mirrors placed at an angle of 120 degrees. and object is kept at angle bisector of two mirrors. So, θ = 120 degrees.

$$n=\frac{360}{120}-1$$

$$n=3-1$$

$$n=\mathbf{2}$$

Also when the object is not kept at angle bisector of two mirrors then the number of images formed by two mirrors can be calculated by the formula

$$n=\frac{360}{\theta}$$

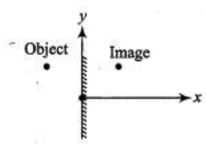
$$n=\frac{360}{120}$$

$$n=\mathbf{3}.$$

3.The relation between the velocity of the object and mirror in-plane mirror:

In case of plane mirror, distance of the object from the mirror is equal to distance of image from the mirror.

i.e Distance of Image formed in the mirror is same as the distance of the object formed the surface of the mirror.



Hence, from the mirror property:
$x_{\text{im}} = -x_{\text{on}}, y_{\text{im}} = y_{\text{om}}$ and $z_{\text{im}} = z_{\text{om}}$

Here x_{im} means " x" coordinate of image with respect to mirror.

Differentiating w.r.t time, we get,

$v_{(im)x} = -v_{(\text{om})x}; \quad v_{(\text{im})y} = v_{(\text{om})y}; \quad v_{(\text{im})z} = v_{(\text{orn})z}$

Here ,

v_i = velocity of the image with respect to the ground.

v_0 = velocity of the object with respect to the ground.

v_{om} = velocity of the object with respect to the mirror.

v_{im} = velocity of the object with respect to the mirror.

i.e $\vec{v}_{\text{om}} = \vec{v}_{\text{o}} - \vec{v}_{\text{m}}$ and $\vec{v}_{\text{im}} = \vec{v}_{\text{i}} - \vec{v}_{\text{m}}$

For x-axis-

$v_{(im)x} = -v_{(\text{om})x}$

$\Rightarrow \quad v_i - v_{\text{m}} = -(v_{\text{o}} - v_{\text{m}}) \quad (\text{ for } x \text{ -axis })$

- I.e When the object moves with speed v towards (or away) from the plane mirror then image also moves toward (or away) with speed v. But the relative speed of image w.r.t. the object is $2v$.

For y-axis and z-axis

$v_{(\text{im})y} = v_{(\text{om})y}; \quad v_{(\text{im})z} = v_{(\text{om})z}$

| Relative velocity of image w.r.t. mirror | = | Relative velocity of object w.r.t. mirror |

$$\begin{aligned} \text{But} \quad & v_1 - v_{\text{m}} = (v_{\text{o}} - v_{\text{m}}) \\ \text{or} \quad & v_{\text{i}} = v_{\text{o}} \end{aligned} \quad \text{for } y \text{ -and } z \text{ -axis.}$$

Here , v_i = velocity of the image with respect to the ground.

v_0 = velocity of the object with respect to the ground.

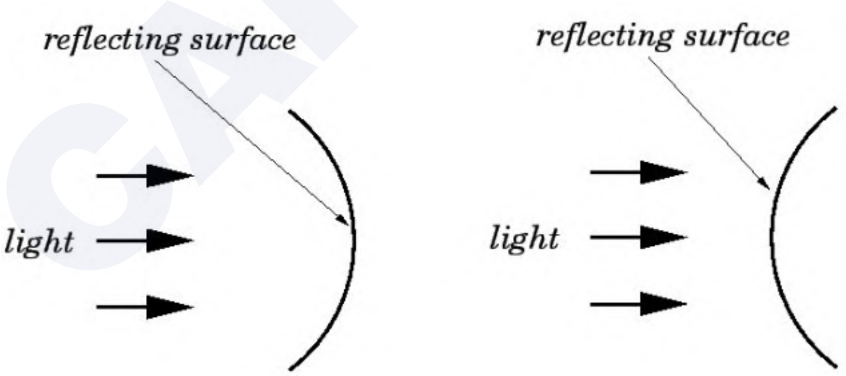
i.e Velocity of the object is equal to the velocity of the image when the object is moving to parallel to the mirror surface.

4. Spherical Mirrors

Spherical mirror-

It is a part of a transparent hollow sphere whose one surface is polished.

There are two types of *spherical mirrors*: concave, and convex.



In the above figure, *A concave (left) and a convex (right) mirror is shown.*

Some important terminology-

- ***Centre of curvature (C)****- The Centre of the sphere of which the mirror is a part is called* ***Centre of curvature.***
- ***Pole (P)****- The geometrical centre of the spherical reflecting surface.*
- **The radius of curvature (R)**- *The* radius *of the sphere of which the mirror is a part is called* the radius of curvature.

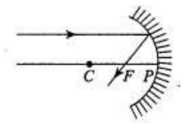
or R=Distance between pole and centre of curvature

$(Note \quad \Rightarrow \quad R_{\text{concave}} = -ve, \quad R_{\text{convex}} = +ve, R_{\text{plane}} = \infty)$

- **Principle axis**- A line passing through P and C is known as the Principle axis.
- **Focus (F)**- When a narrow beam of rays of light, parallel to the principal axis and close to it, is incident on the surface of a mirror, the reflected beam is found to coverage to or appears to diverge from a point on the principal axis. This point is called the focus.

or An image point on the principal axis for which object is at ∞ is called the focus.

C, P, F for a concave mirror are shown in the below figure.



- **Focal Length (f)**- It is the distance between the pole and the principal focus. For spherical mirrors, $f=\frac{R}{2}$

$(i.e \quad f_{\text{concare}} = -ve, f_{\text{convex}} = +ve, f_{\text{plane}} = \infty)$

- **Focal plane**- A plane passing from focus and perpendicular to the principal axis.

5.Image formation by spherical mirrors

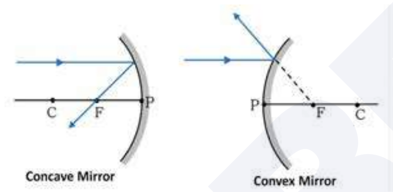
Sign conventions :

- In the cartesian sign convention direction of the incident, the ray is taken as +ve.
- All the measurements are measured from the pole.
- If the incident ray is travelling from left to right the distance, measurement along the right direction will be taken as positive.
- We can treat this direction as +ve x-axis direction and rest can be decided on the basis of graph that we use in mathematics. Like upward direction will be taken as +ve as it is +ve y-axis. And downward as -ve
- Height above the principle axis is taken as positive and below it are taken as negative.
- Angles measured from the normal in anti-clockwise sense are positive, while that in clockwise senses are negative.

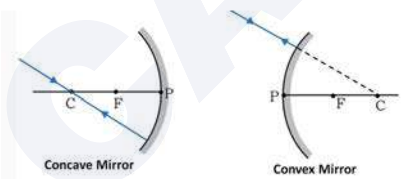
Rules for ray diagrams:

The position of the image formed by spherical mirrors can be found by taking two rays of light coming from a point on the object which intersects each other to form an image. The following are the rules which are used for obtaining images formed by spherical mirrors.

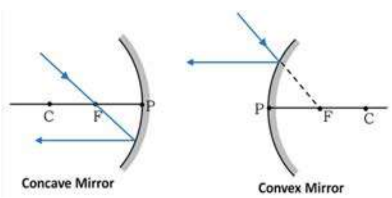
(1). A ray of light that runs parallel to the principal axis, after reflection, passes through the principal focus F of a concave mirror or appears to pass through the principal focus of a convex mirror.



(2). A ray of light passing through the center of curvature in a concave mirror or a ray of light going towards the center of curvature of a convex mirror is reflected back along the same path.



(3). A ray of light passing through the principal focus of a concave mirror or appearing to pass through the principal focus of a convex mirror becomes parallel to the principal axis after reflection.



(4). A ray incident at pole is reflected back making same angle with principle axis.

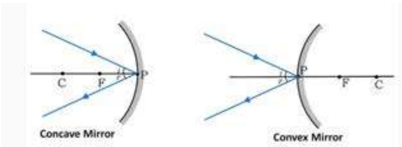
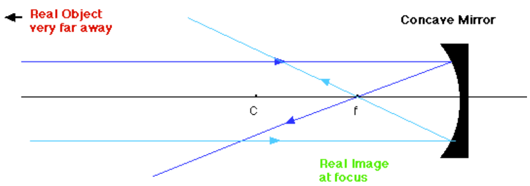
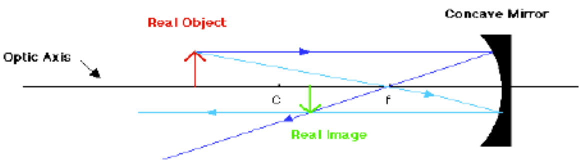


Image formation by concave mirror:

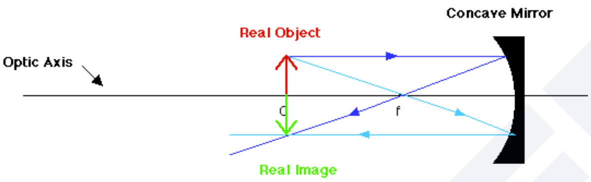
1. For a real object very far away from the mirror, the real image is formed at the focus.



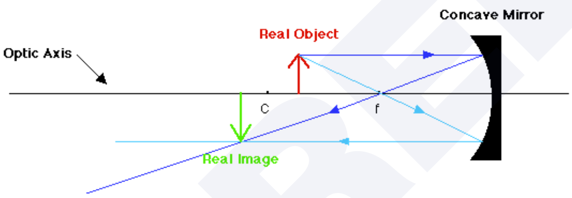
2. For a real object close to the mirror but outside of the center of curvature, the real image is formed between C and f. The image is inverted and smaller than the object.



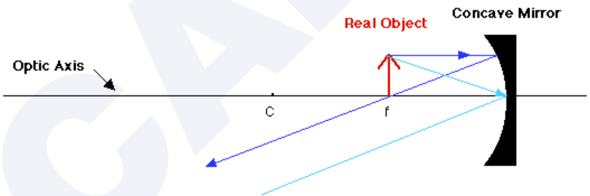
3. For a real object at C, the real image is formed at C. The image is inverted and the same size as the object.



4. For a real object between C and f, a real image is formed outside of C. The image is inverted and larger than the object.



5. For a real object at f, no image is formed. The reflected rays are parallel and never converge.



6. For a real object between f and the mirror, a virtual image is formed behind the mirror. The position of the image is found by tracing the reflected rays back behind the mirror to where they meet. The image is upright and larger than the object.

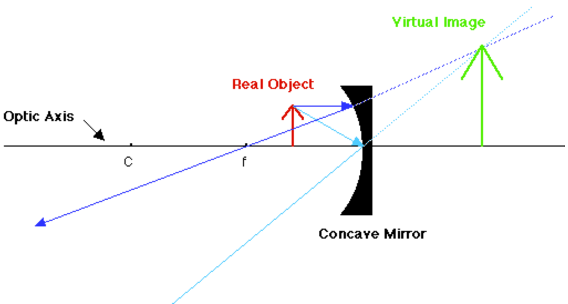
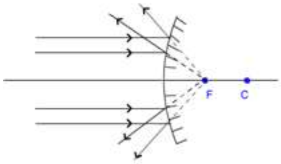


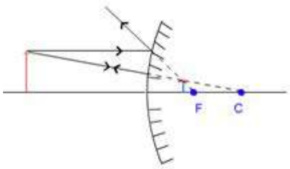
Image formation by convex mirror:

1. When the object is at the infinity, a point sized image is formed at principal focus behind the convex mirror.



Properties of image: Image is highly diminished, virtual and erect.

2. When the object is between infinity and pole of a convex mirror, a diminished, virtual and erect image is formed between pole and focus behind the mirror.



Properties of image: Image is diminished, virtual and erect.

6. Spherical Mirror Formula And Magnification

Mirror formula-

Let the object distance (u), image distance (v) and focal length (f).

Then Mirror formula is given by

$$f=\frac{1}{u}+\frac{1}{v}$$

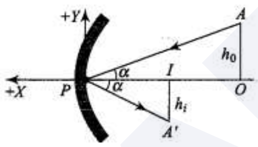
Magnification in Spherical mirrors:-

lateral magnification:

The lateral magnification is defined as the ratio:

$$m_v=\frac{\text{height of image}}{\text{height of object}}=\frac{h_i}{h_0}$$

Consider the extended object OA shown in Figure.



then

$$m_v=\frac{h_i}{h_0}=-\frac{v}{u}$$

magnification formula can be modified as:

$$m=\frac{-v}{u}=\frac{f}{f-u}=\frac{f-v}{f}$$

Longitudinal magnification: When an object lies along the principal axis then its axial magnification 'm' is given by

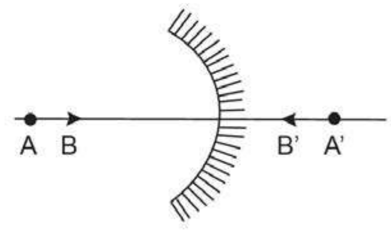
$$m=\frac{I}{O}=\frac{-\left(v_2-v_1\right)}{\left(u_2-u_1\right)}$$

If the object is small,

$$m=-\frac{dv}{du}=\left(\frac{v}{u}\right)^2=\left(\frac{f}{f-u}\right)^2=\left(\frac{f-v}{f}\right)^2$$

Relation between the velocity of object and image in Spherical mirror

Case I: When the object moves along the principal axis



Then

$\frac{dv}{dt} = V_{im}$ = velocity of image w.r.t. miror

$$\Rightarrow \quad V_{im} = -\frac{v^2}{u^2} V_{OM}$$

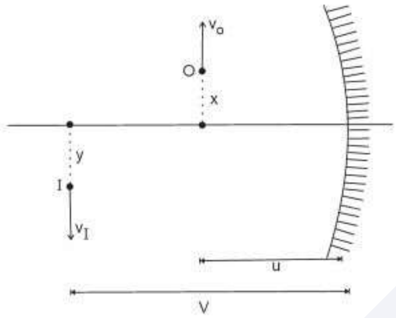
$\frac{du}{dt} = v_{oM}$ = velocity of object w.r.t. mirror

$$V_{im} = -m^2 V_{OM}$$

$$\Longrightarrow \; V_i - V_m = -m^2 (V_O - V_m)$$

Therefore, $\Longrightarrow \; V_i = -m^2 V_O$ when the mirror is at rest along the principal axis.

Case II: When the object moves perpendicular to the principal axis



then

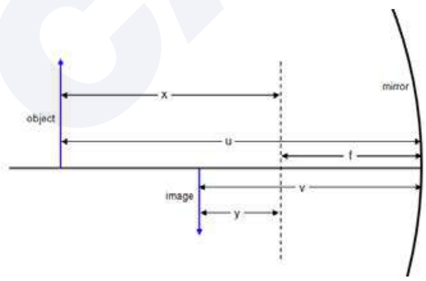
$$\Longrightarrow \; (V_{im})_y = \frac{-v}{u} (V_{om})_y$$

$$\Longrightarrow \; (V_{im}) = m (V_{om})_y$$

Newton's Formula:

As we know that the mirror formula is given as

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$



Let's assume, x = distance of the object from focus

y = distance of the image from focus

Newton's formula is useful for calculating the image position for a curved mirror.

The diagram shows the position of an object and its image formed by a concave mirror.

Let the distances of the object and image from the principal focus of the mirror be x and y respectively.

Then: Object distance (u) = f+x and Image distance (v) = f+y

Using the mirror formula $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$ we have: $\frac{1}{f+x}+\frac{1}{f+y}=\frac{1}{f}$

and simplifying this we get: $\boxed{f^2=xy}$

7.Refraction Of Light

Refraction-

Deviation or bending of light rays from their original path while passing from one medium to another is called refraction. It is due to change in the speed of light as light passes from one medium to another medium. If the light is incident normally then it goes to the second medium without bending, but still, it is called refraction. When a light ray passes from one medium to another such that
it undergoes a change in velocity, refraction takes place. Hence, the wavelength of light changes, but frequency remains the same.

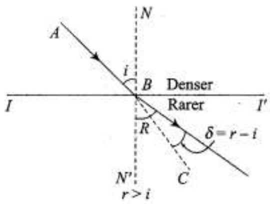
Types of medium:

1. **Rarer medium:** Medium in which the speed of light is more is called optically Rarer medium.
2. **Denser medium:** Medium in which light travels more slowly is called optically denser medium.

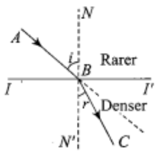
Refractive index: Refractive index of a medium is defined as the factor by which speed of light reduces as compared to the speed of light in vacuum.

$$\mu=\frac{c}{v}=\frac{\text{speed of light in vacuum}}{\text{speed of light in medium}}$$

When light moves from denser to a rarer medium, it bends away from the normal.



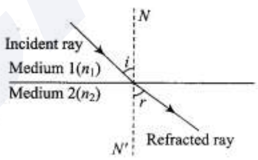
When light moves from rarer to denser medium, it bends towards the normal.



Laws of refraction:

1. The incident ray, the normal to any refracting surface at the point of incidence, and the refracted ray all lie in the same plane called the plane of incidence or plane of refraction.
2. The ratio of sine of angle of incidence to the angle of refraction is always constant.

$$\frac{\sin i}{\sin r}=\text{constant}$$



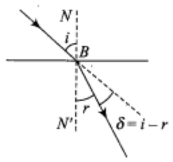
Also, $$\frac{\sin i}{\sin r}=\frac{\mu_2}{\mu_1}=\frac{v_1}{v_2}=\frac{\lambda_1}{\lambda_2}$$

For applying in problems remember
$\mu_1\sin i=\mu_2\sin r$

$\frac{\sin(i)}{\sin(r)}=\mu_{21}$ = refractive index of the second medium with respect to the first medium.

Deviation due to refraction:

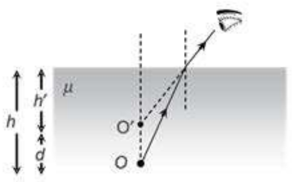
Deviation (δ) of ray incident at $\angle i$ and refracted at $\angle r$ is given by :
$\delta=|i-r|$



8.Real depth and Apparent depth

Case 1. When object is in denser medium and observer is in rarer medium.

If object and observer are situated in different medium then due to refraction, object appears to be displaced from it's real position.



Here O is the real position of the object and O' is the apparent position of the object as seen by the observer. 'h' is the real depth of the object from the surface of the water and h' is the apparent depth of the object. μ_2 is the density of the medium where the object is placed. μ_1 is the density of the rarer medium.

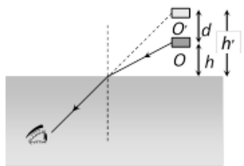
$$\frac{\mu_2}{\mu_1}=\frac{\text{Real depth}}{\text{Apparent depth}}=\frac{h}{h'}.$$

Therefore **Real depth > Apparent depth.**

Apparent shift:

$$d=h-h'=\left(1-\frac{\mu_1}{\mu_2}\right)h$$

Case 2. Object is in rarer medium and observer is in denser medium.



$$\frac{\mu_2}{\mu_1}=\frac{\text{Apparent depth}}{\text{Real depth}}=\frac{h'}{h}.$$

Therefore **apparent depth > real depth.**

Apparent shift:

$$d=\left(\frac{\mu_1}{\mu_2}-1\right)h$$

9.Total Internal Reflection:

When a ray of light goes from denser to rarer medium it bends away from the normal and as the angle of incidence in denser medium increases, the angle of refraction in rarer medium also increases and at a certain angle, angle of refraction becomes 90° this angle of incidence is called critical angle (C).

When Angle of incidence exceeds the critical angle than light ray comes back into the same medium after reflection from interface. This phenomenon is called Total internal reflection (TIR).

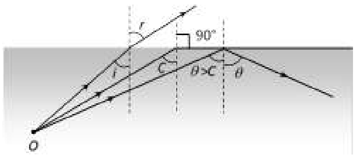
Using snell's law :

$$\mu_2 \sin C=\mu_1 \sin r$$

$$\Longrightarrow \mu_2 \sin C=\mu_1 \quad \text{since, } \sin r=1.$$

$$\Longrightarrow \sin C=\frac{\mu_1}{\mu_2}=\frac{\text{R.I of rarer medium}}{\text{R.I of denser medium}}$$

or $\boxed{\mu=\frac{1}{\sin C}}$ when $\mu_1 = 1$ for air and $\mu_2=\mu$.

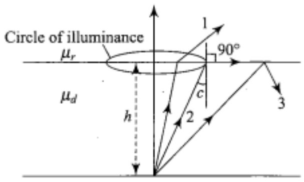


Conditions for TIR :

(i) The ray must travel from denser medium to rarer medium.

(ii) The angle of incidence 'i' must be greater than critical angle 'C' i.e $i > C$.

Circle of illuminance:



From the figure:

$\tan(\theta_c) = \frac{R}{h}$ where R is the radius of C.O.I

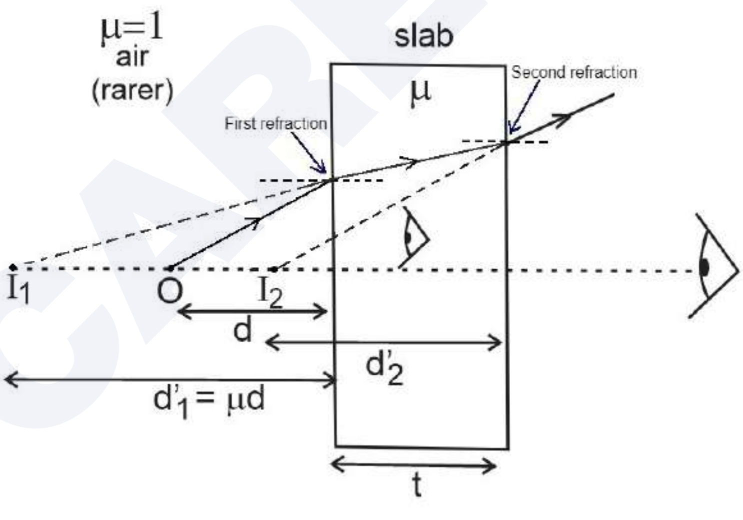
$\implies R = h\tan(\theta_c)$

Also, $\sin(\theta_c) = \frac{1}{\mu}$

Therefore using trigonometry, $\tan(\theta_c) = \frac{1}{\sqrt{\mu^2 - 1}}$

So, the radius of the circle of illuminance, $\boxed{R = \frac{h}{\sqrt{\mu^2 - 1}}}$

10.Refraction Of Light Through Glass Slab



Consider an object O placed at distance d in front of a glass slab of thickness "t" and refractive index μ. The observer is on the other side of the slab. A ray of light from the object first refracts at the surface 1 and then refracts at the surface 2 before reaching the observer as shown in the above figure.

So for the refraction at the surface (1)

$$\text{Apparent depth, } d_1' = \frac{d_{\text{real}}}{n_{\text{relative}}} = \frac{d}{\left(\frac{n_{\text{incident}}}{n_{\text{refraction}}}\right)} = \frac{d}{\frac{1}{\mu}} = d\mu$$

Similarly for the refraction at the surface (2)

$$\text{Apparent depth, } d_2' = \frac{d_{\text{real}}}{n_{\text{relative}}} = \frac{d_1' + t}{\left(\frac{n_{\text{incident}}}{n_{\text{reftaction}}}\right)} = \frac{d_1' + t}{\frac{\mu}{1}} = \frac{d_1\mu + t}{\mu}$$

As you observe, The refracting surfaces of a glass slab are parallel to each other. When a light ray passes through a glass slab it is refracted twice at the two parallel faces and finally emerges out parallel to its incident direction.

i.e. the ray undergoes no deviation ($\delta = 0$).

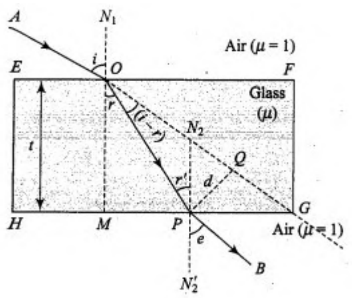
the object appears to be shifted towards the slab by the distance known as apparent shift or Normal shift.

And the apparent shift= OA-I_2A

I.e $\text{Apparant shift } = t\left\{1 - \frac{1}{\mu}\right\}$

If the slab is placed in the medium of the refractive index μ_{sur}

then $\text{Apparant shift } = t\left\{1 - \frac{\mu_{sur}}{\mu}\right\}$



In the above figure Incident, ray AO is an incident on the EF surface of the slab at an angle of incident i, and PB is the emergent ray emerging out of the HG surface of the slab.

for the surface EF

Applying Snell's law at the surface EF and HG

$$\mu_a \sin i = \mu \sin r \quad \text{and} \quad \mu \sin r' = \mu_a \sin e$$
$$Using \quad r' = r \quad \text{and} \quad \mu_a = 1, \text{ we get}$$
$$\sin i = \sin e \text{ or } e = i$$

i.e the emergent ray is parallel to the incident ray.

If PQ is the perpendicular dropped from P on the incident ray produced.

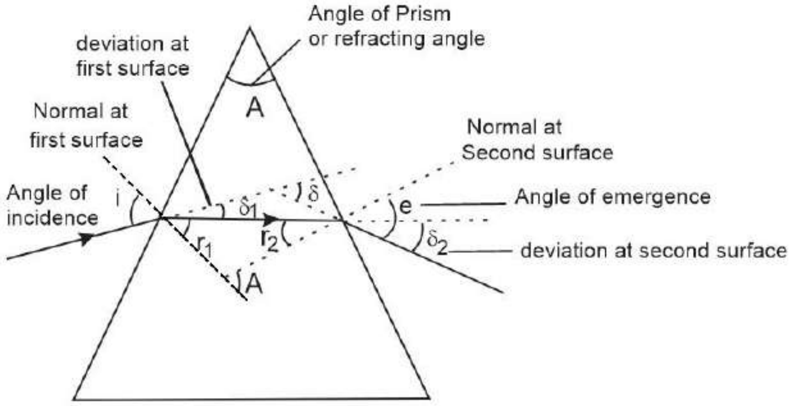
Then PQ=d is known as lateral displacement which is given as

$$d = PQ = OP\sin(i - r) = \frac{OM}{\cos r}\sin(i - r) = \frac{t\sin(i - r)}{\cos r}$$

- $\text{If } i \text{ is very small, } r \text{ is also very small, then } \quad d = \left(1 - \frac{1}{\mu}\right) ti$

11. Refraction And Dispersion Of Light Through A Prism

A prism is a transparent medium whose refracting surfaces are not parallel but are inclined to each other at an angle A which is also known as angle of the prism.



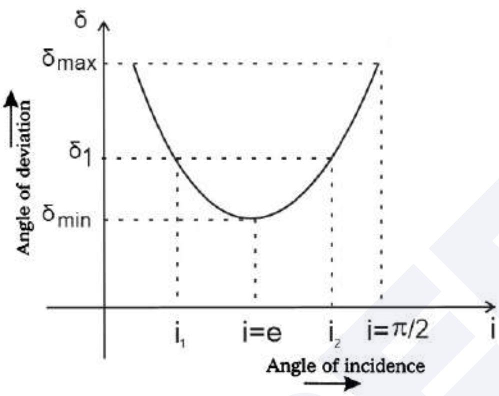
The angle of deviation (δ)-It is the angle between the emergent and the incident ray.

For the above figure $\delta=(i-r_1)+(e-r_2)$ or $\delta=i+e-(r_1+r_2)$

and using $Using \quad A=r_1+r_2 \quad we \quad get \quad \delta=i+e-A$

Note-From the above formula, we can say that if we interchange i and e then also we will get the same value of δ.

The plot of $\delta \quad vs \quad i$



As shown in the above figure The graph is a parabola.

If we vary i between $0^0 \quad to \quad 90^0$

then $for \quad 0<i<e$ the value of δ decreases

and $for \quad e<i<90$ the value of δ increases

And when $i=e \quad then \quad \delta=\delta_{min}$

and when $i=90^0 \quad or \quad e=90^0 \quad then \quad \delta=\delta_{max}$

- Grazing Incidence-When i = 90°, the incident ray grazes along the surface of the prism. This is known as grazing incidence.
- Grazing Emergence- When e = 90°, the emergent ray grazes along the prism surface. This is known as grazing emergence.

This happens when the light ray strikes the second face of the prism at the critical angle for glass - air.

$I.e \quad when \quad r_2=\theta_c \quad then \quad e=90^0$

I.e For the prism of refractive index μ places in the air.

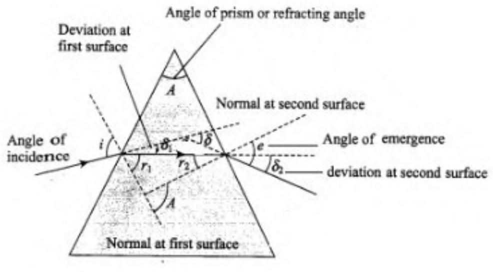
then $i=\sin^{-1}[\sqrt{\mu^2-1}\sin A-\cos A]$ then e = 90°

Refractive index of prism (μ)**in case of minimum deviation condition-**

As we learned The angle of deviation (δ) for the prism is given as $\delta=i+e-A$

and from The plot of $\delta \quad vs \quad i$ we get $i=e \quad then \quad \delta=\delta_{min}$

$$i.e \quad \delta_{min} = i + e - A = i + i - A = 2i - A$$
$$\Rightarrow i = \frac{A + \delta_{min}}{2}$$



For the prism of refractive index μ places in the air.

For the first surface, we can write $1 \times sini = \mu sinr_1$

similarly For the second surface, we can write $\mu sinr_2 = 1 \times sine$

using i=e we get $r_1 = r_2$

$$A = r_1 + r_2 = 2r_1$$
$$\rightarrow r_1 = \frac{A}{2}$$

So $1 \times sini = \mu sinr_1$ will give us

$$\Rightarrow 1 \times sin(\frac{A + \delta_{min}}{2}) = \mu sin(\frac{A}{2})$$
$$\Rightarrow \mu = \frac{sin(\frac{A+\delta_{min}}{2})}{sin(\frac{A}{2})}$$

- For thin films (i.e $A \quad and \quad \delta_{min} \ are \ small$)

Then $sin(\frac{A + \delta_{min}}{2}) = \frac{A + \delta_{min}}{2}$

and $sin(\frac{A}{2}) = \frac{A}{2}$

So we get

$$\mu = \frac{A + \delta_{min}}{A}$$
$$\Rightarrow \delta_{min} = A(\mu - 1)$$

- **Condition of no emergence-**

i.e A ray of light incidence on a prism of angle A & Refractive index μ will not emerge out of a prism

This will happen when $A > 2\theta_c$

where $\theta_c =$ critical angle

Dispersion of light -The splitting of white light into its constituent colors or wavelengths is called dispersion of light.

or

angular splitting of a ray of white light into a number of components and spreading in different directions is called diversion of light.

This phenomenon arises due to the fact that the refractive index varies with wavelength.

When white light is incident on the prism it will split itself into its constituent colors as shown in the below figure.

The deviation is given as $\delta = (\mu - 1)A$

$$Since \ \mu_{violet} > \mu_{red}$$
$$So \quad \delta_{violet} > \delta_{red}$$

- **Angular dispersion (θ)**- Angular separation between extreme colors

i.e $\boldsymbol{\theta = \delta_V - \delta_R = (\mu_V - \mu_R)\, A}$.

It depends upon μ and A.

- **Dispersive power (ω)-** Ratio of angular dispersion to mean deviation.

i.e $\omega = \frac{\delta_v - \delta_r}{\delta}$

where where δ is deviation of mean ray (especially yellow)

$$using \quad \delta_v = (\mu_v - 1) A, \delta_r = (\mu_r - 1) A$$
$$we \ get \quad \omega = \frac{\mu_v - \mu_r}{\mu_y - 1} \quad \text{where} \quad \mu_y = \frac{\mu_v - \mu_r}{2}$$

where

μ_v = Refractive index of violet

$\mu_r =$ Refractive index of red

$\mu_y =$ Refractive index of yellow

Condition for deviation without dispersion-

This means an achromatic combination of two prisms in which net(or) resultant dispersion is 0, but and deviation is produced .

$$\text{For the two prisms,}$$
$$\theta_{net} = 0 \Rightarrow \theta_1 + \theta_2 = 0$$
$$(\mu_v - \mu_r) A + (\mu_v' - \mu_r') A' = 0$$
$$\Rightarrow \quad A' = \frac{(\mu_v - \mu_r) A}{\mu_v' - \mu_r'}$$

where

μ_v = Refractive index of violet (prism 1)

$\mu_r =$ Refractive index of red (prism 1)

μ_v' = Refractive index of violet (prism 2)

$\mu_r' =$ Refractive index of red (prism 2)

Similarly

$$\text{For the two prisms,}$$
$$\theta_{net} = 0 \Rightarrow \theta_1 + \theta_2 = 0$$
$$\omega\delta + \omega'\delta' = 0$$
$$\Rightarrow \delta' = -\delta\frac{\omega}{\omega'}$$
$$\Rightarrow \delta_{net} = \delta + \delta' = \delta\left[1 - \frac{\omega}{\omega'}\right]$$

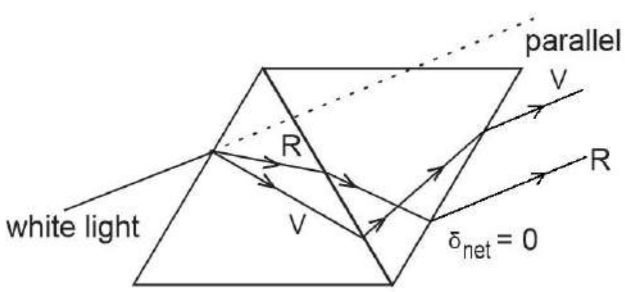
where $\omega \ \ and \ \ \omega'$ are the dispersive powers of the two prisms and their corresponding mean deviations are $\delta \ \ and \ \ \delta'$.

Condition for Dispersion without deviation-

A combination of two prisms in which deviation produced for the mean ray by the first prism is equal and opposite to that produced by the second prism will give a dispersion of light without deviation.

This combination of two prisms is also called a direct vision prism.

i.e $\delta_{net} = 0 \ \ while \ \ \theta_{net} \neq 0$



As shown in the above figure as emergent rays from the second prism is parallel to the incident white ray of prism 1.

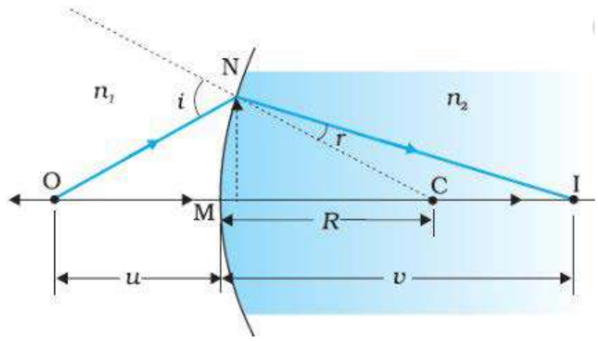
this will give $\delta_{net} = 0$.

$$\text{For zero deviation },$$
$$i.e \quad \delta_{net} = 0 \, (i.e \quad \delta + \delta' = 0)$$
$$\Rightarrow \quad (\mu_y - 1)A + (\mu_y' - 1)\, A' = 0$$
$$\Rightarrow \quad A' = \frac{(\mu_y - 1)A}{(\mu_y' - 1)}$$

and the Angular dispersion is given as

$$\theta_{\text{net}} = \theta_1 + \theta_2 = (\omega\delta + \omega'\delta') = (\omega\delta - \omega'\delta) = \theta\left(1 - \frac{\omega'}{\omega}\right)$$

12. Refraction At Spherical Surface



If an object 0 is placed in front of a curved surface as shown in the above figure, then the Refraction formula is given as

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$

where

n_1=Refractive index of the medium from which light rays are coming (from the object).

n_2=Refractive index of the medium in which light rays are entering.

and $n_1 < n_2$

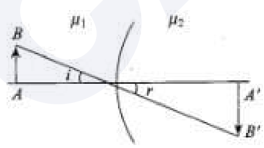
and u = Distance of object, v = Distance of image, R = Radius of curvature

Note -

- use sign convention while solving the problem
- Real image forms on the side of a refracting surface that is opposite to the object, and virtual image forms on the same side as the object.
- Using $R = \infty$ (i.e for plane surface)

we get $\frac{n_2}{v} = \frac{n_1}{u} \Rightarrow \frac{n_2}{n_1} = \frac{v}{u}$

Lateral Magnification For Refracting Spherical Surface-



If an object AB is placed in front of a curved surface as shown in the above figure, then the lateral Magnification formula is given as

$$\text{Lateral magnification, } m = \frac{\text{Image height}}{\text{Object height}} = \frac{-(A'B')}{AB}$$

or $m = -\frac{A'B'}{AB} = -\frac{\mu_1}{\mu_2} \times \frac{v}{u} = -\frac{v/\mu_2}{u/\mu_1}$

where

μ_1=Refractive index of the medium from which light rays are coming (from the object).

μ_2=Refractive index of the medium in which light rays are entering.

and $\mu_1 < \mu_2$

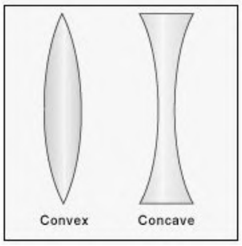
and u = Distance of object, v = Distance of image, R = Radius of curvature

13. Concave And Convex Lenses

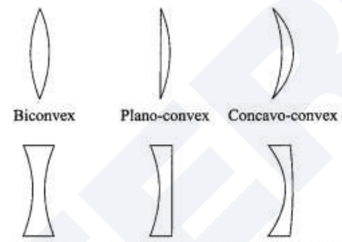
Thin lens-

A lens is a transparent medium bounded by two surfaces which refract the light, such that at least one surface is curved. The curved surface can be cylindrical, spherical etc.

A **thin lens** is called convex if it is thicker in the middle as compared to the ends and it is called concave if it is thicker at the ends as compared to the middle. The figure shows the convex and concave lens -



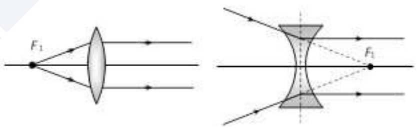
There are few types of concave and convex lens as shown below -



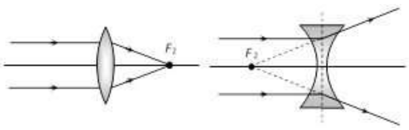
From all the above shapes we can see that there are two surfaces (may be spherical or plane), so there are two centres of curvature C_1 and C_2 and correspondingly two radii of curvature R_1 and R_2. In this case, the **principal axis** is the line joining C_1 and C_2 of the lens and the centre of the thin lens which is on the principal axis, is called the optical centre.

Now as there are two surfaces in the lens so there are two principal focuses for the lens, which are:-

First principal focus(F_1): An object point for which an image is formed at infinity.



Second principal focus(F_2): An image point for an object at infinity.



Note -

1. In this chapter we are mainly concerned with the second principal focus (F_2). So, whenever or wherever we use the term focus, it means the second principal focus.

2. A ray passing through the optical centre proceeds undeviated through the lens.

SIgn convention in the lens - All the distances along the direction of the incident light ray are positive if we measure the distances from the pole of the lens. Also, all the distances above the principal axis are taken as positive and below the principal axis are taken as negative. All these conventions can be seen in the figure given below -

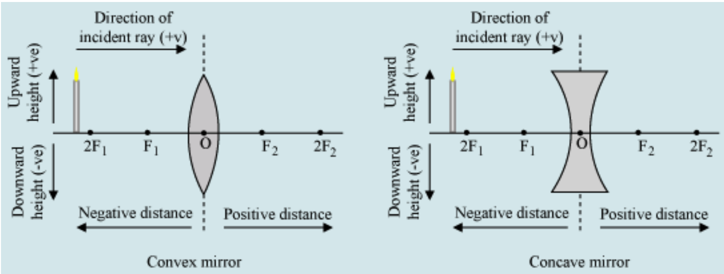
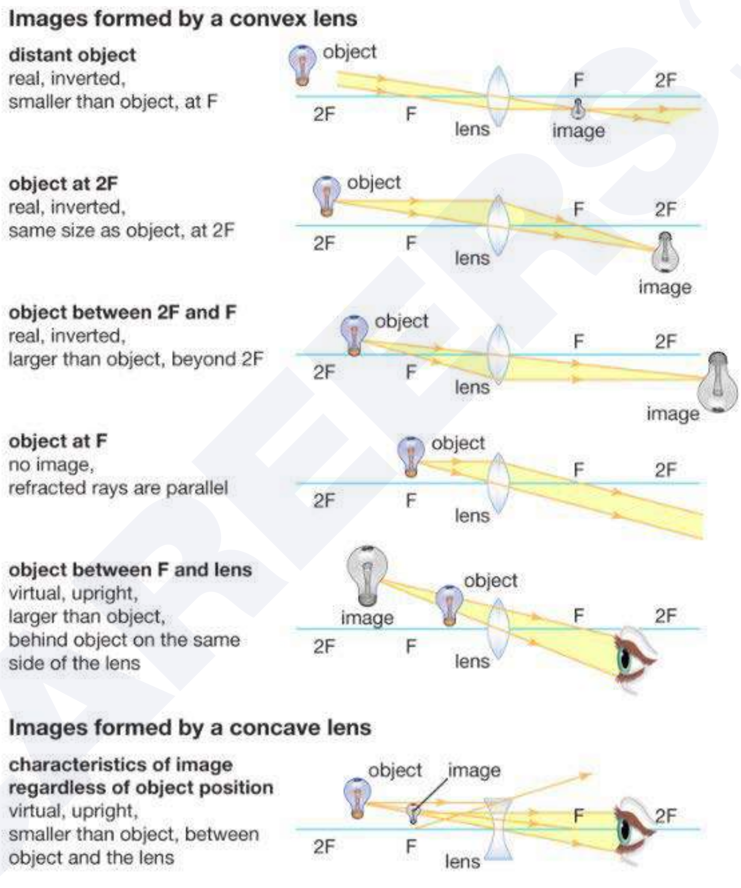


Image formation by lens-

Convex lens -

The figure given below shows the position of image formation for different positions of the object -



So, from this image we can conclude the following table -

For convex lens -

Object location	Image location	Image Nature	Image size
Infinity	At F	Real and Inverted	Diminished
Beyond 2F	Between 2F and F	Real and Inverted	Diminished
Between 2F and F	Beyond 2F	Real and Inverted	Enlarged
At F	At infinity	Real and Inverted	Enlarged
At 2F	At 2F	Real and Inverted	Same size
Between F and 0	On the same side as object	Virtual and Erect	Enlarged

For concave lens -

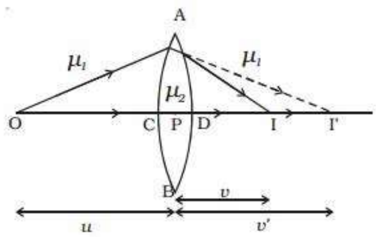
Object location	Image location	Image Nature	Image size
Infinity	At F	Virtual and Erect	Highly Diminished
Beyond infinity and 0	Between F and Optical centre	Virtual and Erect	Diminished

Note - In the table O is the optical center of the lens.

Lens Maker's formula -

Consider a lens having refractive index = μ_2 and the surrounding is having refractive index = μ_1.

Also, let us assume that the lens has two refracting surfaces having radii R_1 and R_2.



Here I' is the intermediate image and I is the final image.

As we have learned the formula of refraction at a single spherical surface. Let us apply this to the surface ACB, we get -

$$\frac{\mu_2}{v_1} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R_1} \ldots (1)$$

Similarly for the second surface ADB-

$$\frac{\mu_1}{v} - \frac{\mu_2}{v_1} = \frac{\mu_1 - \mu_2}{R_2} \ldots (2)$$

Here, v_1 is the position of the image formed by the first surface and the same image will now act as an object for the second surface.

Now adding equations (1) and (2),

$$\frac{\mu_1}{v} - \frac{\mu_1}{u} = (\mu_2 - \mu_1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$\Rightarrow \frac{1}{v} - \frac{1}{u} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

Now we are going to arrange this equation in the desired as -

$$So, put, \quad u = \infty \; and \; v = f$$

we get,

$$\frac{1}{f} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$\mathbf{\frac{1}{f} = (\mu_{relative} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$$

Where,

$$\mu_{\text{relative}} = \frac{\mu_{\text{lens}}}{\mu_{\text{medium}}}$$

There are certain limitations of this lens maker's formula -

- The lens should not be thick so that the space between the two refracting surfaces can be small.
- The medium used on both sides of the lens should always be the same.

Power and lens and mirror-

Power of a lens is defined as the reciprocal of focal length and Power of a mirror is defined as the negative of reciprocal of focal length.

So,

$$P_{Lens} = \frac{1}{f} \quad and \quad P_{Mirror} = \frac{-1}{f}$$

Where, f = focal length

The unit of power is **Diopter, 1D = 1 m^{-1}**

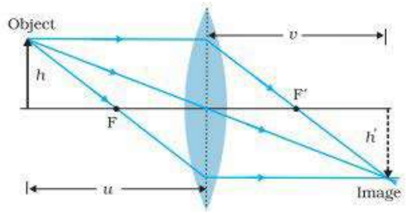
The sign of the focal length will be taken according to the sign convention which we have discussed in the previous concept.

The power of a combination of lenses in contact is the algebraic sum of the powers of individual lenses, so it can be written as -

$$P_{Total} = P_I + P_2 + P_3 + \cdots + P_n$$

Magnification in Lenses-

Magnification produced by a lens is defined as the ratio of the size of the image to that of the object.



So mathematically, magnification can be written as -

$$m = \frac{h'}{h} = \frac{v}{u}$$

Relation between object and image velocity in lens -

Case 1 : When object is moving along the principal axis -

$$\Rightarrow (\vec{v}_{iL})_x = \frac{v^2}{u^2} (\vec{v}_{oL})_x$$

$$\Rightarrow (\vec{v}_{iL})_x = m^2 (\vec{v}_{oL})_x$$

Case 2 : When object is moving perpendicular to the principal axis -

$$\Rightarrow \frac{dh_i}{dt} = \frac{v}{u}\frac{dh_0}{dt} \Rightarrow \left(\vec{V}_{iL}\right)_y = m\left(\vec{V}_{oL}\right)_y$$

Here, m = magnification,

v = Position of image

u = Position of object

$\vec{V}$ = Velocity vector

14. Compound Lenses

Combination of thin lens in contact -

The focal length of the combination of thin lenses is given by

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} \quad \ldots$$

Similarly, the power of the combination of thin lenses is given by

$$P = P_1 + P_2 + P_3 + \ldots$$

In terms of magnification, we can write the net magnification as -

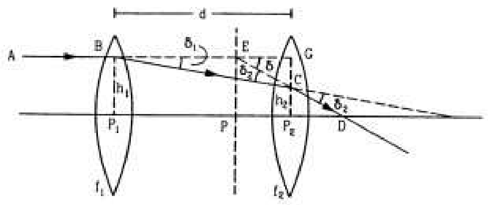
$$m = m_1.m_2.m_3 \ldots$$

The combination of lenses is needed in lenses for cameras, microscopes, telescopes and other optical instruments.

Lenses at a distance -

Consider two thin lenses placed coaxially at a separation d.

The focal length of the equivalent lens is F = PD,



$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

Position of the Equivalent Lens-

$$PP_2 = \left(\frac{dh_1}{f_1}\right)\left(\frac{F}{h_1}\right) = \frac{dF}{f_1}$$

Thus, the equivalent lens is to be placed at a distance $\frac{d.F}{f_1}$ behind the second lens.

Note - Both the above relation are true only for the special case of the parallel incident beam. If the object is at a finite distance, one should not use the above equations.

15. Silvered lens-

Silvering a surface has the effect of converting the lens into a mirror.

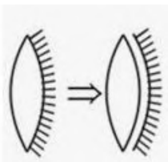
If we silvered a convex lens, then that silvered side will act as a concave mirror and similarly, if we silvered the convex lens then the silvered side will act as a convex mirror.

Our objective is to find the effective focal length of this silvered lens.

Let us take an example of a silvered convex lens as shown in the given figure.

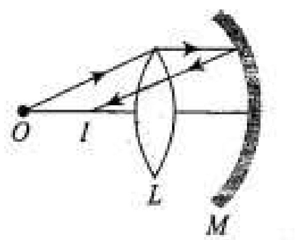
Now we use the principle of superposition to find the focal length of the silvered lens.

See the image given below which shows we are separating the lens and the mirror



In this arrangement, a ray of light is first refraction by lens L, then it is reflected at the curved mirror M, and finally refracted once again at the lens L.

Let the object O be located in front of the lens. Let the image from the lens I_1 be formed at v_1 .



Then, from the lens-makers formula, (Assume the focal length of the lens f_{L1}) we have

$$\frac{1}{v_1} - \frac{1}{u} = \frac{1}{f_{L1}}$$

Now the image I_1 formed by the lens will act as an object for the mirror having focal length f_m

Let I_2 be the image formed by the mirror at a distance of v_2.

Again applying the formula -

$$\frac{1}{v_2} + \frac{1}{v_1} = \frac{1}{f_m}$$

Now, I_2 will be the object for the final refraction at lens L.

If I_3 be the final image formed at v from the center of the lens, then we

$$\frac{1}{v} - \frac{1}{v_2} = \frac{1}{f_{L2}}$$

Now, $f_{L_1} = f_L \quad \text{then} \quad f_{L_2} = f_L$

So the above equation become -

$$\frac{1}{v_1} - \frac{1}{u} = \frac{1}{f_L}$$

$$\frac{1}{v_2} + \frac{1}{v_1} = \frac{1}{f_m}$$

$$\frac{1}{v} - \frac{1}{v_2} = -\frac{1}{f_L}$$

By manipulating the above equation we get,

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f_m} - \frac{2}{f_L}$$

So the equivalent focal length will be equal to -

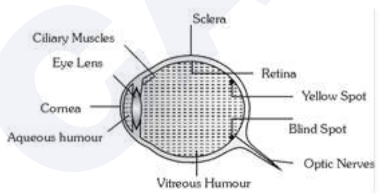
$$\frac{1}{f_e} = \frac{1}{f_m} - \frac{2}{f_L}$$

16.Structure And Functions Of Human Eye

The Eye :

The human eye is one of the most sensitive sense organs of sight which enables us to see the wonderful world of light and color around us. The eye is essentially a closed sphere into which light passes through a lens and strikes a light-sensitive surface.

Structure of the human eye:



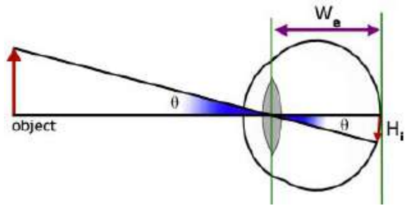
- **Sclera:** It is the outer covering, a protective tough white layer called the sclera (white part of the eye).
- **Cornea:** The front transparent part of the sclera is called cornea. Light enters the eye through the cornea.
- **Iris:** A dark muscular tissue and ring-like structure behind the cornea are known as the iris. The color of iris actually indicates the color of the eye. The iris also helps regulate or adjust exposure by adjusting the iris.
- **Pupil:** A small opening in the iris is known as a pupil. Its size is controlled by the help of iris. It controls the amount of light that enters the eye.
- **Lens:** Behind the pupil, there is a transparent structure called a lens. By the action of ciliary muscles, it changes its shape to focus light on the retina. It becomes thinner to focus distant objects and becomes thicker to focus nearby objects.
- **Retina:** It is a light-sensitive layer that consists of numerous nerve cells. It converts images formed by the lens into electrical impulses. These electrical impulses are then transmitted to the brain through optic nerves.
- **Optic nerves:** Optic nerves are of two types. These include cones and rods.

1. **Cones:** Cones are the nerve cells that are more sensitive to bright light. They help in detailed central and color vision.
2. **Rods:** Rods are the optic nerve cells that are more sensitive to dim lights. They help in peripheral vision.

Functioning of the human eye: Much like the electronic device, the human eye also focuses and lets in light to produce images. So basically, light rays that are deflected from or by distant objects land on the retina after they pass through various mediums like the cornea, crystalline lens, aqueous humour, the lens, and vitreous humour

As the light rays move through the various mediums, they experience refraction of light. The light rays are received and focused on the retina. The retina contains photoreceptor cells called rods and cones and these basically detect the intensity and the frequency of the light. Further, the image that is formed is processed by millions of these cells and they also relay the signal or nerve impulses to the brain via the optic nerve. The image formed is usually inverted but the brain corrects this phenomenon. This process is also similar to that of a convex lens.

Visual Angle: The visual angle of an object is a measure of the size of the object's image on the retina. The visual angle depends on the distance between the object and the observer. Larger distances lead to smaller visual angles. The visual angle also depends on the object's size. Larger objects lead to larger visual angles.



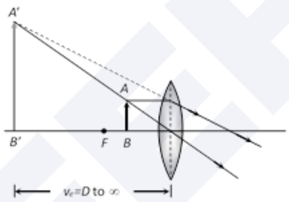
Visual angle $(\phi) = \frac{h}{d}$ where 'h' is the height of the object and d is the distance from the lens.

17. Optical instruments

Magnifying power: It is defined as the ratio between the dimensions of the image and the object.

Magnifying power of an optical instrument is given by,

$$m = \frac{\text{Visual angle with instrument}(\beta)}{\text{Visual angle when object is placed at least distance of distinct vision }(\alpha)}$$



Simple Microscope-

- It is a single convex lens of lesser focal length.
- Also called magnifying glass or reading lens.

Case 1: Magnification, when the final image is formed at D and ∞.

(i.e. m_D and m_∞)

$$m_D = \left(1 + \frac{D}{f}\right)_{max} \text{ and}$$

$$m_\infty = \left(\frac{D}{f}\right)_{min}$$

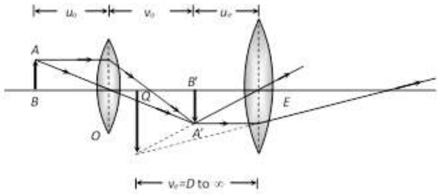
Case 2: If lens is kept at a distance a from the eye then

$$m_D = 1 + \frac{D-a}{f} \text{ and}$$

$$m_\infty = \frac{D-a}{f}.$$

Compound Microscope:

A compound microscope is of two converging lenses called objective and eye lens. It is used to view magnified images of small objects on a glass slide. It can achieve higher levels of magnification than stereo or other low power microscopes and reduce chromatic aberration.



$f_{\text{eyelens}} > f_{\text{objective}}$ and $(diameter)_{\text{eyelens}} > (diameter)_{objective}$
Intermediate image is real and enlarged. The final image is magnified, virtual and inverted.

Here in the diagram

u_o = Distance of object from objective (o),

v_o= Distance of image (A′B′) formed by objective from objective,

u_e= Distance of A′B′ from eye lens,

v_e= Distance of final image from eye lens,

f_o= Focal length of objective,

f_e= Focal length of eye lens.

Case 1: Final image is formed at D : Magnification $m_D = \frac{v_o}{u_o}\left(1 + \frac{D}{f_e}\right)$ and length of the microscope tube (distance between two lenses) is $L_D = v_e + u_e$.
Generally, object is placed very near to the principal focus of the objective hence $u_o \equiv f_o$. The eye piece is also of small focal length and the image formed by the objective is also very near the eyepiece.

So $v_o \equiv L_D$, the length of the tube.
Hence, we can write

$$m_D = \frac{L}{f_o}\left(1 + \frac{D}{f_e}\right).$$

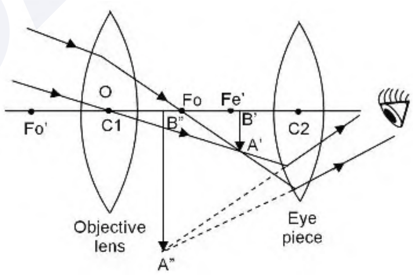
Case 2: Final image is formed at ∞ : Magnification
$m_\infty = \frac{v_0}{u_0} \cdot \frac{D}{f_e}$ and length of tube $L_\infty = v_0 + f_e$

In terms of length $m_\infty = \frac{(L_\infty - f_o - f_e) D}{f_o f_e}$.

- For large magnification of the compound microscope, both f_o and f_e should be small.
- If the length of the tube of microscope increases, then its magnifying power increases.
- The magnifying power of the compound microscope may be expressed as $M = m_o \times m_e$ where m_0 is the magnification of the objective and m_e is magnifying the of eyepiece.

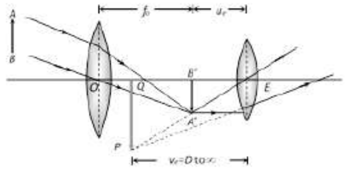
Astronomical Telescope

An astronomical telescope is an optical instrument which is used to see the magnified image of distant heavenly bodies like stars, planets, satellites and galaxies etc. An astronomical telescope works on the principle that when an object to be magnified is placed at a large distance from the objective lens of telescope, a virtual, inverted and magnified image of the object is formed at the least distance of distinct vision from the eye held close to the eye piece.



An astronomical telescope consists of two convex lenses : an objective lens O and an eye piece E. the focal length fo of the objective lens of astronomical telescope is large as compared to the focal length fe of the eye piece. And the aperture of objective lens O is large as compared to that of eye piece, so that it can receive more light from the distant object and form a bright image of the distant object. Both the objective lens and the eye piece are fitted at the free ends of two sliding tubes, at a suitable distance from each other.

The ray diagram to show the working of the astronomical telescope is shown in figure. A parallel beam of light from a heavenly body such as stars, planets or satellites fall on the objective lens of the telescope. The objective lens forms a real,inverted and diminished image A'B' of the heavenly body. This image (A'B') now acts as an object for the eye piece E, whose position is adjusted so that the image lies between the focus fe' and the optical centre C_2 of the eye piece. Now the eye piece forms a virtual, inverted and highly magnified image of object at infinity. When the final image of an object is formed at infinity, the telescope is said to be in 'normal adjustment .



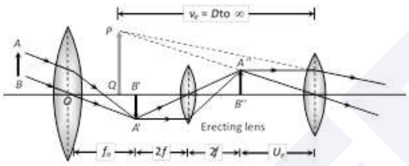
- $f_{\text{objective}} > f_{\text{eyelens}}$ and $d_{\text{objective}} > d_{\text{eye lens}}$
- The intermediate image is real, inverted and small.
- The final image is virtual, inverted and highly magnified.
- Magnification: $m_D = -\frac{f_0}{f_e}\left(1+\frac{f_e}{D}\right)$ and $m_\infty = -\frac{f_o}{f_e}$.
- Length: $L_D = f_0 + u_e$ and $L_\infty = f_0 + f_e$

Terrestrial Telescope

A refracting telescope has inverting lenses or an eyepiece that presents an erect image. A telescope for use on earth rather than for making astronomical observations. Such telescopes contain an additional lens or prism system to produce an erect image.

The erection of an image can be made by introducing a third lens between the objective and the eye-piece of the telescope. This modified telescope is known as the "Terrestrial Telescope" whose magnifying power is just equal to the magnification of an astronomical telescope but it just gives an erect image.

The terrestrial telescope contains three lenses as compared to the astronomical telescope. It is also known as the spyglass. As an astronomical telescope forms an inverted image of the object so, the main difference between the astronomical and terrestrial telescope is the erection of the final image with respect to the object. The third lens of short focal length **f** is placed at **2f** which forms an inverted image of the object. This image serves as the object for the eye piece. The lens placed in the centre of the telescope which actually erects the image is called the Erecting lens. The resolving power of the telescope can be given by the relations as follows:



$$M = -\frac{f_o}{f_e} \times (-1) = \frac{f_o}{f_e}$$

$$L = f_o + f_e + 4f$$

Where,

f_o = Focal length of the objective lens

f_e = Focal length of the eye-piece lens

f = Focal length of the lens placed between objective and eye-piece

- Magnification at D, $m_D = \frac{f_0}{f_e}\left(1+\frac{f_e}{D}\right)$
- Magnification at infinity, $m_\infty = \frac{f_0}{f_e}$.

Wave Optics

Important Formulae

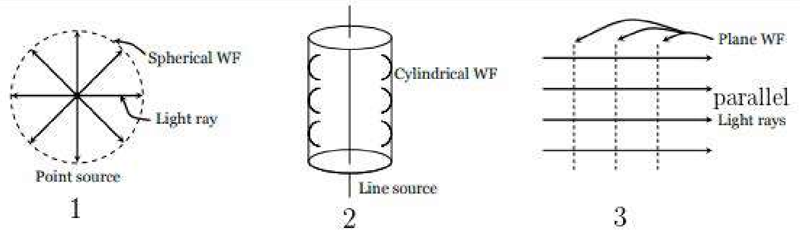
1. Huygens Principle

Light also shows the wave nature. According to Huygens, each point source of light is a centre of disturbance from which waves spread in all directions.

Wavefront-

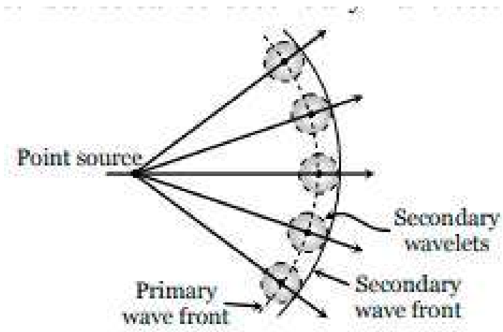
- The locus of all particles in a medium, vibrating in the same phase is called WaveFront (WF).
- The direction of propagation of light is perpendicular to the WF.
- The time taken by the light to travel from one wavefront to another is the same along any ray.
- The phase difference between various particles on the wavefront is zero.
- Various types of wavefront-

1. Spherical WF- For a point source
2. Cylindrical WF- For line source
3. Plane WF- For parallel light rays



Huygens principle-

According to the **Huygens principle** , Every point on the given wavefront acts as a source of a new disturbance called **secondary wavelets.** And a common tangent to these secondary wavelets in the forward direction at any instant gives the new wavefront at that instant as shown in the below figure. This is called **secondary wavefront.**



2.Interference Of Light - Condition And Types

In order to observe interference in light waves, the following conditions must be met:

- The sources must be coherent.
- The source should be monochromatic (that is, of a single wavelength).

Coherent sources-

Two sources are said to be coherent if they produce waves of the same frequency with a constant phase difference.

- The relation between Phase difference $(\Delta\phi)$ and Path difference (Δx)

Phase difference $(\Delta\phi)$**:**

The difference between the phases of two waves at a point is called phase difference.

i.e. if $y_1 = a_1 \sin \omega t$ and $y_2 = a_2 \sin(\omega t + \phi)$ so phase difference $= \phi$

Path difference (Δx):

The difference in path lengths of two waves meeting at a point is called path difference between the waves at that point.

And The relation between Phase difference $(\Delta\phi)$ and Path difference (Δx) is given as

$$\Delta\phi = \frac{2\pi}{\lambda}\Delta x = k\Delta x$$

where $\lambda = wavelength\ of\ waves$

Principle of Super Position-

According to the principle of Super Position of waves, when two or more waves meet at a point, then the resultant wave has a displacement (y) which is the algebraic sum of the displacements (y_1 and y_2) of each wave.

i.e $y = y_1 + y_2$

consider two waves with the equations as

$$y_1 = A_1 \sin(kx - wt)$$
$$y_2 = A_2 \sin(kx - wt + \phi)$$

where ϕ is the phase difference between waves y_1 and y_2.

And According to the principle of Super Position of waves

$$\begin{aligned} y = & y_1 + y_2 = A_1 \sin(kx - wt) + A_2 \sin(kx - wt + \phi) \\ & = A_1 \sin(kx - wt) + A_2[\sin(kx - wt)\cos\phi + \sin\phi\cos(kx - \omega t)] \end{aligned}$$
$$\Rightarrow y = \sin(kx - wt)\left[A_1 + A_2\cos\phi\right] + A_2\sin\phi\cos(kx - wt)\ldots(1)$$

Now let

$$\begin{aligned} & Acos\theta = A_1 + A_2\cos\phi \\ and \quad & Asin\theta = A_2 sin\phi \end{aligned}$$

Putting this in equation (1) we get

$$y = A\sin(kx - \omega t)\cos\theta + A\sin\theta\cos(kx - \omega t)$$

thus we get the equation of the resultant wave as

$$y = A\sin(kx - \omega t + \theta)$$

where A=Resultant amplitude of two waves

and $A = \sqrt{A_1{}^2 + A_2{}^2 + 2A_1A_2\cos\phi}$

and $\theta = \tan^{-1}\left(\frac{A_2\sin\phi}{A_1 + A_2\cos\phi}\right)$

where

$A_1 =$ the amplitude of wave 1

$A_2 =$ the amplitude of wave 2

- $A_{\max} = A_1 + A_2 \quad and \quad A_{\min} = A_1 - A_2$

Resultant Intensity of two waves (I)-

Using $I \quad \alpha \quad A^2$

we get $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$

where

$I_1 =$ The intensity of wave 1

$I_2 =$ The intensity of wave 2

- $I_{\max} = I_1 + I_2 + 2\sqrt{I_1 I_2} \Rightarrow I_{\max} \quad = (\sqrt{I_1} + \sqrt{I_2})^2$
- $I_{\min} = I_1 + I_2 - 2\sqrt{I_1 I_2} \Rightarrow I_{\min} = (\sqrt{I_1} - \sqrt{I_2})^2$
- For identical sources-

  $$I_1 = I_2 = I_0 \Rightarrow I = I_0 + I_0 + 2\sqrt{I_0 I_0}\cos\phi = 4I_0\cos^2\frac{\phi}{2}$$

- $\text{Average intensity}: \ I_{av} = \frac{I_{\max} + I_{\min}}{2} = I_1 + I_2$
- **The ratio of maximum and minimum intensities**

$$\frac{I_{\max}}{I_{\min}} = \left(\frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}}\right)^2 = \left(\frac{\sqrt{I_1/I_2} + 1}{\sqrt{I_1/I_2} - 1}\right)^2 = \left(\frac{a_1 + a_2}{a_1 - a_2}\right)^2 = \left(\frac{a_1/a_2 + 1}{a_1/a_2 - 1}\right)^2$$

or

$$\sqrt{\frac{I_1}{I_2}} = \frac{a_1}{a_2} = \left(\frac{\sqrt{\frac{I_{\max}}{I_{\min}}} + 1}{\sqrt{\frac{I_{\max}}{I_{\min}}} - 1}\right)$$

Interference of Light-

It is of the following two types.

1. Constructive interference-

- When the waves meet a point with the same phase, constructive interference is obtained at that point.

  i.e we will see a bright fringe/spot.

- The phase difference between the waves at the point of observation is $\phi = 0^\circ \text{ or } 2n\pi$
- Path difference between the waves at the point of observation is $\Delta x = n\lambda(i.e.\text{ even multiple of } \lambda/2)$
- The resultant amplitude at the point of observation will be the maximum

$$i.e \quad A_{\max} = a_1 + a_2$$
$$\text{If } a_1 = a_2 = a_0 \Rightarrow A_{\max} = 2a_0$$

- Resultant intensity at the point of observation will be the maximum

$$i.e \quad I_{\max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$$
$$I_{\max} = (\sqrt{I_1} + \sqrt{I_2})^2$$
$$\text{If} \quad I_1 = I_2 = I_0 \Rightarrow I_{\max} = 4I_0$$

2. Destructive interference-

- When the waves meet a point with the opposite phase, Destructive interference is obtained at that point.

  i.e we will see dark fringe/spot.

- The phase difference between the waves at the point of observation is

$$\phi = 180^\circ \text{ or } (2n-1)\pi; n = 1, 2, \ldots$$
$$\text{or } (2n+1)\pi; n = 0, 1, 2 \ldots.$$

- Path difference between the waves at the point of observation is $\Delta x = (2n-1)\frac{\lambda}{2}(\text{ i.e. odd multiple of } \lambda/2)$
- The resultant amplitude at the point of observation will be minimum

$$i.e \quad A_{\min} = A_1 - A_2$$
$$\text{If } A_1 = A_2 \Rightarrow A_{\min} = 0$$

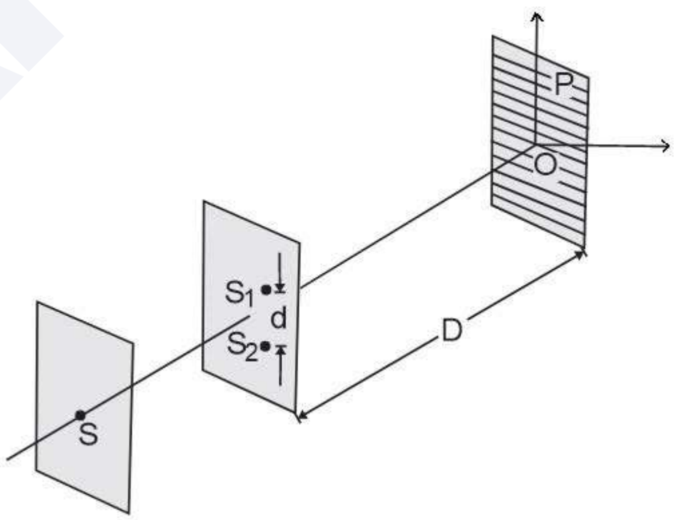
- Resultant intensity at the point of observation will be minimum

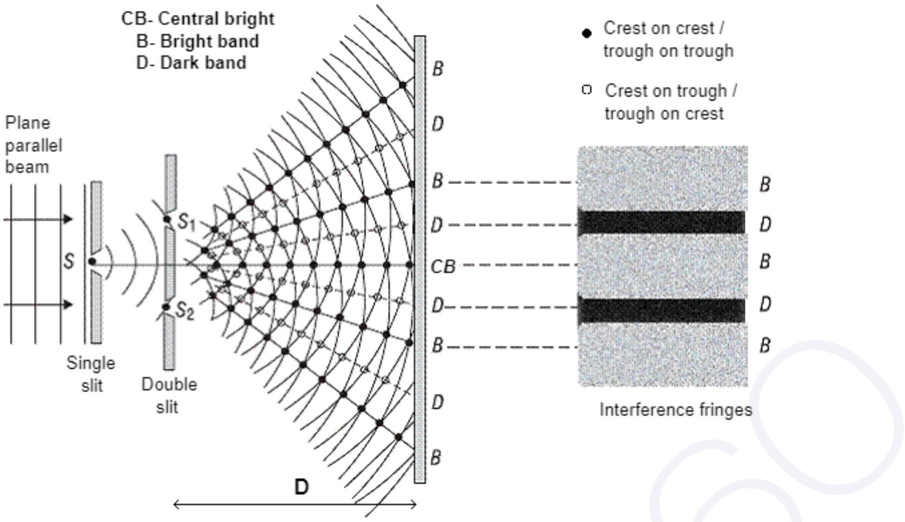
$$I_{\min} = I_1 + I_2 - 2\sqrt{I_1 I_2}$$
$$I_{\min} = (\sqrt{I_1} - \sqrt{I_2})^2$$
$$\text{If } I_1 = I_2 = I_0 \Rightarrow I_{\min} = 0$$

3. Young's Double Slit Experiment

Young's double-slit experiment -

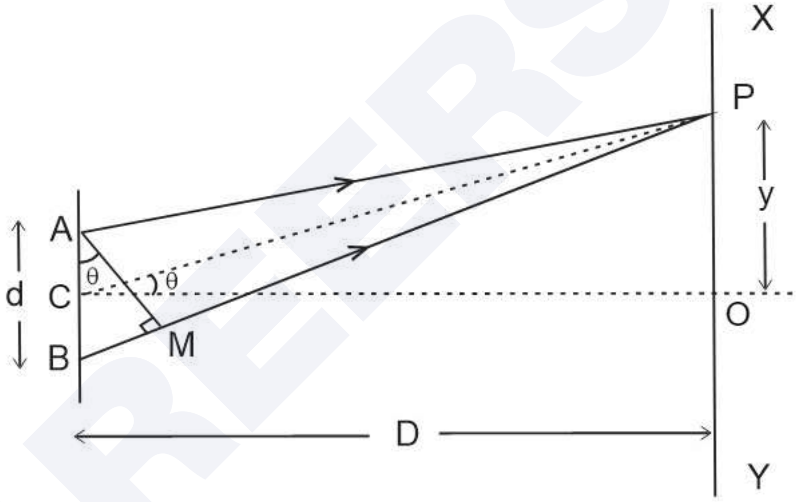
This experiment is performed by British physicist Thomas Young. He used an arrangement as shown below. In this he used a monochromatic source of light S . He made two pinholes S_1 and S_2 (very close to each other) on an opaque screen as shown in the figure Each source can be considered as a source of coherent light source.





So, we can see that the monochromatic light source ‘s’ kept at a considerable distance from two slits s_1 and s_2. The arrangement is such that the S is equidistant from S_1 and S_2. S_1 and S_2 behave as two coherent sources, as both are derived from S.

Let *d* be the distance between two coherent sources A and B having wavelength λ. A screen XY is placed parallel to an opaque screen at a distance D. O is a point on the screen equidistant from A and B. P is a point at a distance *x* from O



From the above figure, we can see that the waves from A and B meet at P. It may be in phase or out of phase depending upon the path difference between the two waves.

Draw AM perpendicular to BP

The path difference $\delta = \mathrm{BP} - \mathrm{AP}$

As we can see that, $\mathrm{AP} = \mathrm{MP}$

$\delta = \mathrm{BP} - \mathrm{AP} = \mathrm{BP} - \mathrm{MP} = \mathrm{BM}$

In right angled ABM, $\mathrm{BM} = \mathrm{d} \sin\theta$ If θ is small,

. $\sin\theta = \theta$

The path difference $\delta = \theta . d$

In right angled triangle COP, $\tan\theta = \mathrm{OP}/\mathrm{CO} = \mathrm{X}/\mathrm{D}$

For small values of θ, $\tan\theta = \theta$

Thus, the path difference $\delta = \mathrm{xd}/\mathrm{D}$

$$So, the\ path\ differnece\ is = \frac{xd}{D}$$

The assumption in this experiment -

1. D> d: Since D > > d, the two light rays are assumed to be parallel.

2. **d/λ >> 1:** Often, d is a fraction of a millimetre and λ is a fraction of a micrometre for visible light.

For Bright Fringes -

By the principle of interference, the condition for constructive interference is the path difference = nλ

$\frac{xd}{D} = n\lambda$

Here, $n = 0, 1, 2 \ldots\ldots$ indicate the order of bright fringes
So, $x = \left(\frac{n\lambda D}{d}\right)$

This equation gives the distance of the n^{th} bright fringe from the point O.

For Dark fringes -

By the principle of interference, the condition for destructive interference is the path difference = $\frac{(2n-1)\lambda}{2}$

Here, n = 1,2,3 … indicates the order of the dark fringes.

So,

$$x = \frac{(2n-1)\lambda D}{2d}$$

The above equation gives the distance of the n^{th} dark fringe from point O.

So, we can say that the alternately dark and bright fringe will be obtained on either side of the central bright fringe.

Band Width (β) -

The distance between any two consecutive bright or dark bands is called bandwidth.

Take the consecutive dark or bright fringe -

$$x_{n+1} - x_n = \frac{(n+1)\lambda D}{d} - \frac{(n)\lambda D}{d}$$

$$x_{n+1} - x_n = \frac{\lambda D}{d}$$

$$\beta = \frac{\lambda D}{d}$$

Angular fringe width -

$$\theta=\frac{\beta}{D}=\frac{\lambda D/d}{D}=\frac{\lambda}{d}$$

The intensity of Fringes In Young's Double Slit Experiment-

For two coherent sources S_1 and S_2, the resultant intensity at point *P* on the screen is given by-

$$I=I_1+I_2+2\sqrt{I_1 I_2}\cos\phi$$

where

$I_1 =$ The intensity of the wave from S_1

$I_2 =$ The intensity of wave S_2

Putting I_1 and $I_2=I_o$ (Because d<<<D)

$$\Rightarrow I=I_0+I_0+2\sqrt{I_0 I_0}\cos\phi=4I_0\cos^2\frac{\phi}{2}$$

So the intensity variation from maximum to minimum depends on the phase difference.

For maximum intensity

The phase difference between the waves at the point of observation is $\phi=0^\circ$ or $2n\pi$.

Path difference between the waves at the point of observation is $\Delta x=n\lambda(i.e.$ even multiple of $\lambda/2)$

Resultant intensity at the point of observation will be the maximum

$$\begin{aligned} i.e\quad I_{\max}&=I_1+I_2+2\sqrt{I_1 I_2}\\ I_{\max}&=(\sqrt{I_1}+\sqrt{I_2})^2\\ \text{If}\quad I_1=I_2&=I_0\Rightarrow I_{\max}=4I_0\end{aligned}$$

For Minimum Intensity -

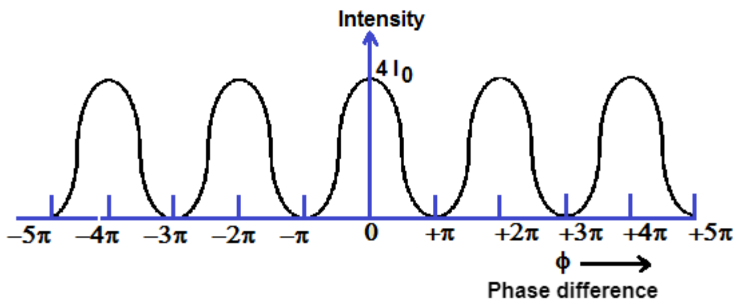
The phase difference between the waves at the point of observation is

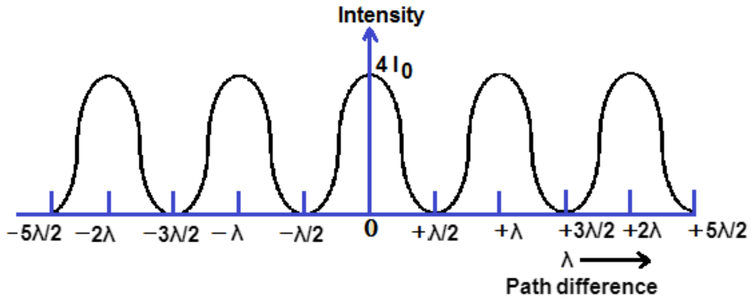
$$\begin{aligned}\phi=180^\circ \text{ or } (2n-1)\pi; n=1,2,\ldots\\ \text{or } (2n+1)\pi; n=0,1,2\ldots.\end{aligned}$$

Path difference between the waves at the point of observation is $\Delta x=(2n-1)\frac{\lambda}{2}($ i.e. odd multiple of $\lambda/2)$

Resultant intensity at the point of observation will be minimum

$$\begin{aligned} I_{\min}&=I_1+I_2-2\sqrt{I_1 I_2}\\ I_{\min}&=(\sqrt{I_1}-\sqrt{I_2})^2\\ \text{If } I_1=I_2=I_0&\Rightarrow I_{\min}=0\end{aligned}$$





Maximum Order of Interference Fringes -

As we know that the position of n^{th} order maxima on the screen is -

$$\frac{n\lambda D}{d}; n = 0, \pm 1, \pm 2, \ldots$$

Value of 'n' cannot be taken as infinitely large, because it violates the assumption of the Young's double slit experiment which means that the θ is small or we can write x<<D. So,

$$\Rightarrow \frac{x}{D} = \frac{n\lambda}{d} << 1$$

So the above formula is only applicable for -

$$n << \frac{d}{\lambda}$$

But when, $n \approx \frac{d}{\lambda}$, which means that the n is comparable with $\frac{d}{\lambda}$. Then the above formula is not applicable, then we have to go with the basic and we will equate path difference as -

$$\Rightarrow d \sin\theta = n\lambda$$

$$\Rightarrow n = \frac{d \sin\theta}{\lambda}$$

$$So,$$
$$n_{\max} = \left[\frac{d}{\lambda}\right]$$

The above represents box function or greatest integer function.

Similarly, the highest order of interference minima

$$n_{\min} = \left[\frac{d}{\lambda} + \frac{1}{2}\right]$$

4. Optical Path

Optical path-

It is defined as the distance travelled by light in a vacuum, at the same time in which it travels a given path length in a medium.

Let light cover distance t in the medium having a refractive index as μ in Time T.

So $T = \frac{t}{v}$ where v=speed of light in the medium

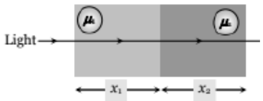
and $\mu = \frac{c}{v}$ where c=speed of light in the vacuum.

So $T = \frac{t\mu}{c}$

So in the same time T distance covered by light in a vacuum is $l = cT = c \times \frac{\mu t}{c} = \mu t$

So the relation between geometrical path (t) and optical path (l) is given as $l = \mu t$

- For two mediums in contact as shown in the below figure



The optical path is equal to $\mu_1 x_1 + \mu_2 x_2$

- **Change in the optical path in a transparent slab**

Consider the following two cases

Case I- A light cover distance $l = AB$

So the optical path in this case is $L_1 = l$

Case II- Now a slab of thickness t is placed between A and B

So the distance travelled in slab=t

and distance travelled in air=$l - t$

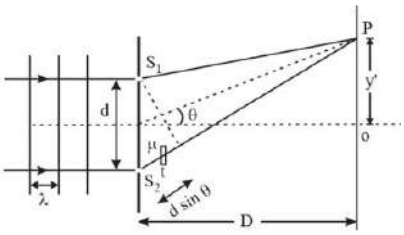
So the optical path in this case is $L_2 = (l - t) + \mu t$

Now change in the optical path = $\Delta x = L_2 - L_1 = [(l - t) + \mu t] - [l] = t(\mu - 1)$

I.e due to the insertion of the slab the optical path is increased by $\Delta x = t(\mu - 1)$

5. YDSE with a thin slab

YDSE with thin slab:



$\Delta x = [(S_2P - t) + \mu t] - S_1P$ or

$\Delta x = (S_2P - S_1P) + (\mu - 1)t$

$$\begin{aligned} &\text{since} \quad S_2P - S_1P = d\sin\theta = d\,(y'/D) \quad \text{(from the fig.)} \\ &\therefore \quad \Delta x = dy'_n/D + (\mu - 1)t \end{aligned}$$

From the nth maxima,

$\Delta x = n\lambda, \therefore n\lambda = dy_n/D + (\mu - 1)t$ or

$$y_n = \frac{n\lambda D}{d} - \frac{(\mu - 1)tD}{d}$$

The position of nth maxima and minima has shifted downward by the same distance which is called $S = y_n - y'_n = (\mu - 1)\frac{tD}{d}$

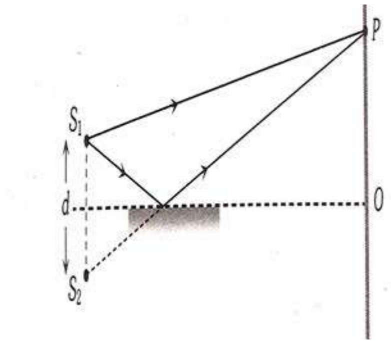
- The distance between two successive maxima or minima remains unchanged. That is, the fringe width remains unchanged by introducing a transparent film.
- The distance of shift is in the direction where the film is introduced. That is, if a film is placed in front of the upper slit the S_1, fringe pattern shifts upwards, if a film is placed in front of the lower slit S_2, the fringe pattern shifts downward.

6. Lloyd's mirror experiment

In Lloyd's mirror experiment**,** light from a monochromatic slit source reflects from a glass surface at a small angle and appears to come from a virtual source as a result. The reflected light interferes with the direct light from the source, forming interference fringes.

Experimental setup:

A plane glass plate (acting as a miror) is illuminated at almost grazing incidence by a light from a sliftual image S_2 of S_1 is formed closed to S_1 by reflection and these two act as coherent sources. The expression giving the fringe width is the same as for the double silt, but the fringe system differs in one important respect.



The path difference $S_2P - S_1P$ is a whole number of wavelengths, the fringe at P is dark not bright. This is due to $180°$ phase change which occurs when light is reflected from a denser medium. At grazing incidence a fringe is formed at O, where the geometrical path difference between the direct and reflected waves is zero and it follows that it will be dark rather than bright.

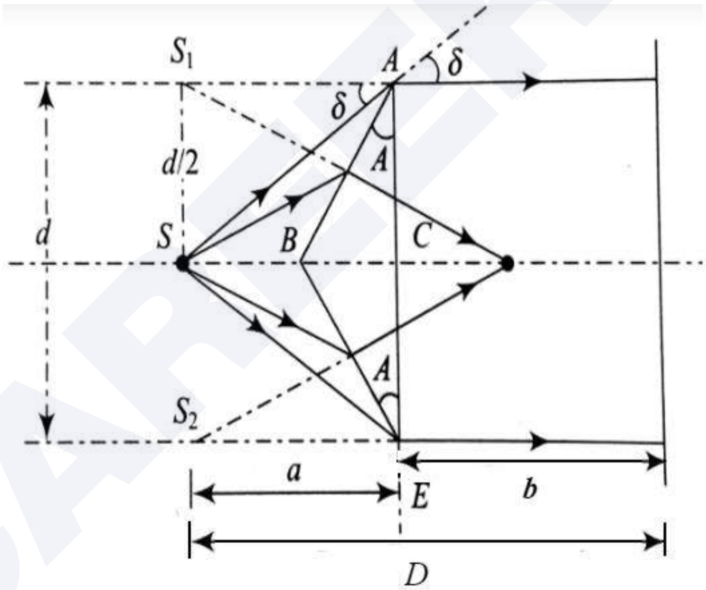
Thus, whenever there exists a phase difference of a π between the two interfering beams of light, conditions of maximas and minimas are interchanged, i.e.,

$\Delta x = n\lambda$ (for minimum intensity) and

$\Delta x = (2n-1)\lambda/2$ (for maximum intensity)

7. Fresnel's Biprism

- It is an optical device for producing interference of light Fresnel's biprism is made by joining base to base two thin prism of very small angle.
- When a monochromatic light source is kept in front of biprism two coherent virtual sources S_1 and S_2 are produced.
- Interference fringes are found on the screen placed behind the biprism interference fringes are formed in the limited region which can be observed with the help eyepiece.
- Fringes are of equal width and its value is $\beta = \frac{\lambda D}{d}$



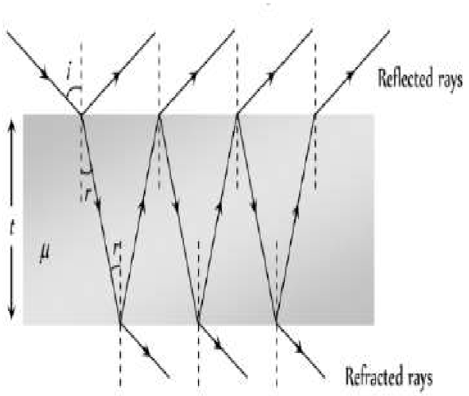
- Let the separation between S_1 and S_2 be d and the distance of slits and the screen from the biprism be a and b respectively i.e. $D = (a+b)$. If the angle of the prism is A and the refractive index is μ then $d = 2a(\mu - 1)A$

$$\therefore \quad \lambda = \frac{\beta[2a(\mu-1)A]}{(a+b)} \quad \Rightarrow \quad \beta = \frac{(a+b)\lambda}{2a(\mu-1)A}$$

8. Thin film interference

Interference effects are commonly observed in thin films when their thickness is comparable to the wavelength of incident light (if it is too thin as compared to the wavelength of light it appears dark and if it is too thick, this will result in uniform illumination of the film). A thin layer of oil on the water surface and soap bubbles show various colours in white light due to the interference of waves reflected from the two surfaces of the film.

In thin films, interference takes place between the waves reflected from its two surfaces and waves refracted through it



- **Interference in reflected light :**

Net path difference between two consecutive waves in the reflected system = $\Delta x = 2\mu t cos r - \frac{\lambda}{2}$

(As the ray suffers reflection at the surface of a denser medium an additional phase difference of π or a path difference of $\frac{\lambda}{2}$ is introduced.)

1. Condition of constructive interference (maximum intensity):

$$\Delta x = n\lambda$$
$$\Rightarrow 2\mu t \cos r + \frac{\lambda}{2} = n\lambda$$
$$\Rightarrow 2\mu t \cos r = (n - \frac{1}{2})\lambda$$

For normal incidence, i.e r=0, so $2\mu t = (2n-1)\frac{\lambda}{2}$

2. Condition of destructive interference (minimum intensity):

$$\Delta x = 2\mu t \cos r = (2n)\frac{\lambda}{2}.$$
$$\text{And For normal incidence } 2\mu t = n\lambda$$

- **Interference in refracted light :**

Net path difference between two consecutive waves in the refracted system =$\Delta x = 2\mu t cos r$

1. Condition of constructive interference (maximum intensity):

$$\Delta x = 2\mu t \cos r = (2n)\frac{\lambda}{2}.$$
$$\text{and For normal incidence } 2\mu t = n\lambda$$

2. Condition of destructive interference (minimum intensity):

$$\Delta x = 2\mu t \cos r = (2n-1)\frac{\lambda}{2}$$

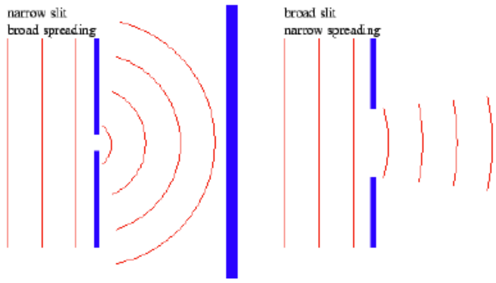
$$For\ normal\ incidence:\ 2\mu t = (2n-1)\frac{\lambda}{2}$$

9. Diffraction Of Light

Diffraction-

The phenomenon of bending of light around the corners of an obstacle of the size of the wavelength of light is called diffraction.

- The phenomenon resulting from the superposition of secondary wavelets originating from different parts of the same wavefront is defined as a diffraction of light.
- Diffraction is the characteristic of all types of waves.
- The wavelength of the wave is directly proportional to its degree of diffraction.



From the above figure, we can say that if the slit width is more then the wave will detract less.

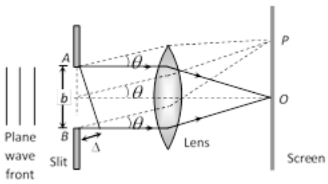
Condition for Diffraction:

- The essential condition for diffraction to occur is that the wavelength of light should be comparable to that of the size of the obstacle.

i.e. If the size of the obstacle is comparable to that of the wavelength of the wave only then we can observe the diffraction phenomena.

Fraunhofer diffraction by a single slit-

let's assume a plane wave front is incident on a slit AB (of width b).



- The diffraction pattern consists of a central bright fringe (central maxima) surrounded by dark and bright lines (called secondary minima and maxima).
- At point O on the screen, the central maxima is obtained. The wavelets originating from points A and B meet in the same phase at this point, hence at O, intensity is maximum

Secondary minima : For obtaining nth secondary minima at P on the screen, path difference between the diffracted waves

$$\Delta x = b\sin\theta = n\lambda$$

1. Angular position of nth secondary minima:

$$\sin\theta \approx \theta = \frac{n\lambda}{b}$$

2. Distance of nth secondary minima from central maxima:

$x_n = D \cdot \theta = \frac{n\lambda D}{b}$ where D = Distance between slit and screen.

$f \approx D$ = Focal length of converging lens.

Secondary maxima : For nth secondary maxima at P on the screen.

Path difference $\Delta x = b\sin\theta = (2n+1)\frac{\lambda}{2}$; where n = 1, 2, 3 .....

(i) Angular position of nth secondary maxima

$$\sin\theta \approx \theta \approx \frac{(2n+1)\lambda}{2b}$$

(ii) Distance of nth secondary maxima from central maxima:

$$x_n = D \cdot \theta = \frac{(2n+1)\lambda D}{2b}$$

Central maxima: The central maxima lie between the first minima on both sides.

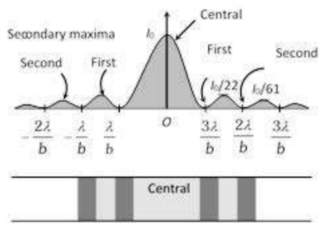
(i) The Angular width d central maxima = $2\theta = \frac{2\lambda}{b}$

(ii) Linear width of central maxima $= 2x = 2D\theta = 2f\theta = \frac{2\lambda f}{b}$

Intensity distribution: if the intensity of the central maxima is I_0 then the intensity of the first and secondary maxima are found to be $\frac{I_0}{22}$ and $\frac{I_0}{61}$. Thus diffraction fringes are of unequal width and unequal intentsities.

(i) The mathematical expression for in intensity distribution on the screen is given by:

$I = I_o\left(\frac{\sin\alpha}{\alpha}\right)^2$ where α is just a convenient connection between the angle θ that locates a point on the viewing screening and light intensity I.



$\phi =$ Phase difference between the top and bottom ray from the slit width b.

Also $\alpha = \frac{1}{2}\phi = \frac{\pi b}{\lambda}\sin\theta$.

(ii) As the slit width increases relative to wavelength the width of the control diffraction maxima decreases; that is, the light undergoes less flaring by the slit. The secondary maxima also decreases in width and becomes weaker.

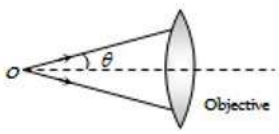
(iii) If b>>λ, the secondary maxima due to the slit disappear; we then no longer have single slit diffraction.

10. Resolving power of optical instruments

Resolving power of optical instruments:

Resolving power of an optical instrument is its ability to resolve or seperate the images of two nearby point objects so that they can be distinctly seen.

1.Resolving power of microscope:



In microscope, the minimum distance between two lines at which they are just distinct is called the Resolving limit (RL) and its reciprocal is called Resolving power (RP)

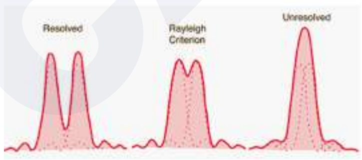
$$\text{R.L.} = \frac{\lambda}{2\mu\sin\theta} \text{ and } R.P. = \frac{2\mu\sin\theta}{\lambda} \Rightarrow R.P. \propto \frac{1}{\lambda}$$

λ =Wavelength of light used to illuminate the object,

μ = Refractive index of the medium between object and objective,

θ = Half angle of the cone of light from the point object

Rayleigh's criterion for the diffraction limit to resolution states that two images are just resolvable when the centre of the diffraction pattern of one is directly over the first minimum of the diffraction pattern of the other. We can use Rayleigh’s criterion to determine the resolving power. The angular separation between two objects must be:

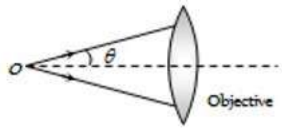


$$\Delta\theta = 1.22\frac{\lambda}{d}$$

Resolving power $= \frac{1}{\Delta\theta} = \frac{d}{1.22\lambda}$

Thus higher the diameter *d*, the better the resolution. The best astronomical optical telescopes have mirror diameters as large as 10m to achieve the best resolution. Also, larger wavelengths reduce the resolving power and consequently, radio and microwave telescopes need larger mirrors.

2. Resolving power of microscope:-



The resolving power of a microscope can be defined as the ability of the microscope to form separate images of two objects placed very close to each other. In a microscope, the minimum distance between two lines at which they are just distinct is called Resolving limit (RL) and its reciprocal is called Resolving power (RP).

$$\text{R.L.} = \frac{\lambda}{2\mu\sin\theta} \text{ and } R.P. = \frac{2\mu\sin\theta}{\lambda} \Rightarrow R.P. \propto \frac{1}{\lambda}$$

λ =Wavelength of light used to illuminate the object,

μ = Refractive index of the medium between object and objective,

θ = Half angle of the cone of light from the point object

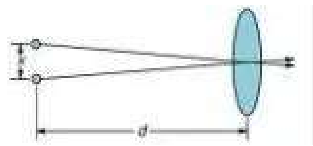
Therefore, from the above expression, we can see that,

- As the R.P is directly proportional to the **refractive index (n)**, So R.P will increases when n increases.
- As the R.P is inversely proportional to the **wavelength (λ)**, So R.P will decreases when λ increases.
- When the **diameter of the objective** is increased, θ increases. Hence, sinθ also increases.

  As the R.P is directly proportional to the sinθ, So R.P will increases when the diameter of objective increases.

- As the R.P is independent of the focal length of the lens, So R.P will remain unchanged when focal length increases.

3.Resolving power of telescope:



In telescopes, very close objects such as binary stars or individual stars of galaxies subtend very small angles on the telescope. To resolve them we need very large apertures. Resolving power of a telescope is defined as the reciprocal of the smallest angle subtended at the objective lens of the telescope by two point objects which can be just distinguished as separate. We can use Rayleigh's to determine the resolving power. The angular separation between two objects must be

$$\Delta\theta = 1.22\frac{\lambda}{d}$$

$$\text{Resolving power } = \frac{1}{\Delta\theta} = \frac{d}{1.22\lambda}$$

where,

λ= Wavelength of light used to illuminate the object,

d= is the critical width of the rectangular slit for just the resolution of two slits or objects.

θ= Half angle of the cone of light from the point object,

Thus higher the diameter *d*, the better the resolution. The best astronomical optical telescopes have mirror diameters as large as 10m to achieve the best resolution. Also, larger wavelengths reduce the resolving power and consequently, radio and microwave telescopes need larger mirrors.

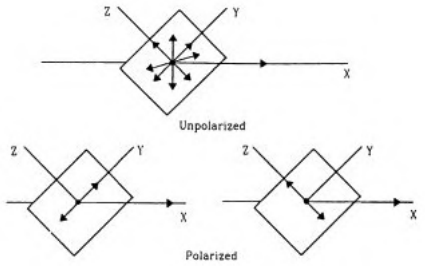
11.Polarization Of Light

Polarization of light-

While writing the equation for light wave,

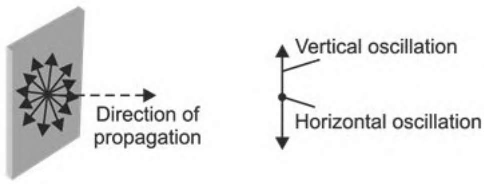
$$E = E_0 \sin\omega(t - x/v)$$

we assumed that the direction of electric field is fixed and the magnitude varies sinusoidally with space and time. So the electric field in a light wave propagating in free space is perpendicular to the direction of propagation. But there may be infinite number of directions perpendicular to the direction of propagation and the eletric field may be along any of these directions. But If the electric field at a point always remains parallel to a fixed direction and remain parallel as the time passes, the light is called linearly polarized along that direction. The same is also called plane polarized light. The plane containing the electric field and the direction of propagation is called the plane of polarization. This can be better understand by the following figure.



Unpolarised light-

In ordinary light (light from bulb, sun, etc.), the electric field vectors are distributed in all directions in a light is called unpolarized light. This resolved into horizontal and vertical component i.e., the electric field.

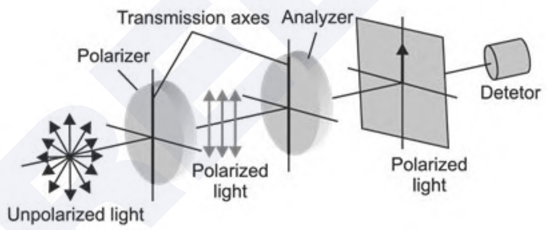


Polarised light -

The phenomenon of limiting the vibration of electric field vector in one direction in a plane perpendicular to the direction of propagation of light wave is called polarization of light.

The plane in which oscillation occurs in the polarised light is called plane of oscillation. The plane perpendicular to the plane of oscillation is called plane of polarization. Light can be polarized by transmitting through certain crystals such as tourmaline or polaroids.

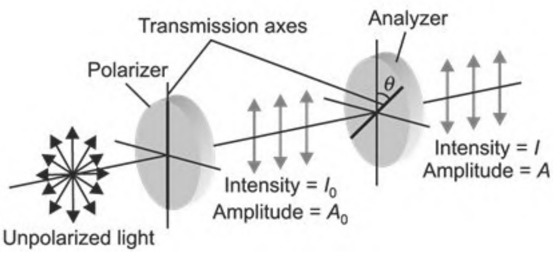
Polaroid
It is a device used to produce the plane polarized light. It is based on the principle of selective absorption. Polaroids allow the light oscillations parallel to the transmission axis pass through them. The crystal or polaroid on which unpolarized light is incident is called polarizer. Crystal or polaroid on which polarized light is incident is called analyzer.



12.Malus's Law

Malus' Law-

This law states that the intensity of the polarized light transmitted through the analyzer varies as the square of the cosine of the angle between the plane of transmission of the analyzer and the plane of the polarizer.



$$I=I_0\cos^2\theta \text{ and } A^2=A_0^2\cos^2\theta \Rightarrow A=A_0\cos\theta$$

$$\text{If } \theta=0^\circ, I=I_0, A=A_0, and \text{ if } \theta=90^\circ, I=0, A=0$$

If I_i = Intensity of unpolarised light. So

$$I_0 = \frac{I_i}{2}$$

i.e. if an unpolarized light is converted into plane polarised light (say by passing it through a Polaroid or a Nicol-prism), its intensity becomes half and

$$I = \frac{I_i}{2} cos^2\theta$$

13. Polarization of light by reflection

Brewster's law-

Brewster discovered that when a beam of unpolarized light is reflected from a transparent medium (refractive index = μ), the reflected light is completely plane polarised at a certain angle of incidence (called the angle of polarisation i.e θ_p).

And also $\mu = \tan\theta_p$.

i.e $For \quad i = \theta_P$

reflected rays will be completely polarised.

$For \quad i < \theta_P \quad or \; i \; > \theta_P$
reflected rays will be partially polarised.

Dual Nature of Matter and Radiation

Important Formulae

1.Electron Emission

Electron Emission-

As we have learned in Chemistry (Atomic structure) that the electrons in the outermost orbit of an atom are at maximum distance from the nucleus and hence most loosely bound to it. These type of electron is called **free electrons**. The free electrons in metals are free to move within the volume of metal even though they do not get ejected out of the surface of metal on their own. The main reason behind this is - whenever an electron tries to leave the surface, the surface acquires a positive charge which pulls back the electron. So for escaping from the surface, an electron has to do a definite amount of work to overcome the force exerted by the opposite charges. To do this work, an external source imaparts minimum energy. This minimum energy is called the **work function** of the metal and is denoted by ϕ_0.

Work function of any particular material is defined as the minimum energy which is required to liberate the most weakly bound surface electrons from that material without giving them any velocity. Since it is energy, but it is generally denoted in electron volt (eV).

$$\mathrm{leV} = \mathrm{le} \times 1\mathrm{V} = \left(1.6 \times 10^{-19}\mathrm{C}\right)(1\mathrm{V}) = 1.6 \times 10^{-19}\mathrm{J}$$

Since the energy of photon is given by - $h\nu$, So the minimum energy i.e., Work function is given by -

$$\phi = h\nu_o = \frac{hc}{\lambda_o}$$

When a free electron gets extra energy i.e., imparted energy $\geq$ work function from an external agent, then it is able to overcome potential barrier and the electron gets ejected out. There are a number of ways in which energy from outside can be supplied and based on these different way, there are different ways in which electron emission can take place. These ways are listed below:

1. **Photoelectric emission:** When electromagnetic radiations of suitable frequency (or wavelength) are incident on a metallic surface, then electrons will be emitted, this phenomenon is known as photoelectric effect.

2. Thermionic emission: In this case, additional energy is given to the electrons to overcome potential barrier in the form of heat by passing current through a filament.

3. Field emission: In this case, metal is placed in a strong electric field due to which the electrons are accelerated to such a speed that the corresponding kinetic energy is sufficient to overcome potential barrier.

4. Secondary emission: It is a process in which the work function is supplied to the free electrons of a metal surface by collisions with fast moving secondary particles like neutrons, beta particles, etc

2.Photon Theory Of Light

Photon theory of light -

According to Eienstein's quantum theory light propagates in the bundles (packets or quanta) of energy, each bundle being called a photon and possessing energy. The energy of one quantum is given by, hν , where h is the Planck's constant and ν is the frequency.

$$E = h\nu = \frac{hc}{\lambda}$$

where c = Speed of light, h = Plank's constant = $6.6 \times 10^{-34} J - \sec$

ν = Frequency in Hz , λ= Wavelength of light.

$$E(eV) = \frac{12400}{\lambda(\mathring{A})}$$ (In the form of eV)

Properties of Photon

1. Photon is a packet of energy (or) particles of light and travels with speed of light in a straight line.
2. Energy of photon is given as $E = h\nu$ and it depends on frequency and it does not change with change in medium.
3. Photons are electrically neutral and not effected by electric and maginetic field.
4. Photons does not exist at rest i.e., it is a moving particle
5. Momentum of photon is given as $p = \frac{h}{\lambda}$
6. All photons of light of a given frequency or wavelength have same energy or momentum irrespective of light intensity
7. Photons can interact with other particles like electrons, which can be seen in Compton effect
8. Photons can be created or destroyed when the radiation is emitted or absorbed i.e no of photons is not conserved during collision
9. The dynamic mass of the photon is m = E/c^2, where E is the energy of the photon
10. During photon-electron collision, the momentum and total energy are conserved
11. Photons do not decay on their own
12. The energy possessed by the photon can be transferred to other particles when it interacts with other particles

After energy now let us discuss the mass of the photon.

Mass of photon :

You will study in the theory of relativity that the rest mass of any body is given by -

$$m_v = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\begin{aligned} \text{Where: } m_v &= \text{ Relativistic mass (kg)} \\ m_o &= \text{ Rest mass (kg)} \\ v &= \text{ velocity } (\text{ms}^{-1}) \\ \text{c} &= \text{ speed of light} = 3 \times 10^8 \text{ms}^{-1} \end{aligned}$$

As the velocity of photon is same as speed of light, so from the above equation we can write that - $m_o = 0$. But it's effective mass is given as -

$$E = mc^2 = h\nu \Rightarrow m = \frac{E}{c^2} = \frac{h\nu}{c^2} = \frac{h}{c\lambda}$$

It is also called as kinetic mass of the photon.

Momentum of the photon -

As the momentum of any body is $= m.v$

Here the velocity = c, i.e., speed of light. So, we can write that -

$$p = m \times c = \frac{E}{c} = \frac{h\nu}{c} = \frac{h}{\lambda}$$

Note -

1. In a photon particle collision, total energy and total momentum will be conserved but the number of photons may be changed.

2. All photons of light of a particular freq. (or) wavelengths have the same energy and momentum whatever may be the intensity.

3.The Photoelectric Effect

Photoelectric effect-

The phenomea of Photoelectric effect was first introduced by Wilhelm Ludwig Franz Hallwachs in 1887 and its experimental verification was confirmed by Heinrich Rudolf Hertz.

They observed that **when a metallic surface is irradiated by monochromatic light of proper frequency, electrons are emitted from it**. This phenomena of ejection of electron is called the Photoelectric effect.

The **photoelectric effect** is the process that involves the release or rejection of electrons from the surface of materials (this material is generally a metal) when light falls on them. This concept that makes us comfortable to understand quantum nature of electron and light.

The electrons ejected during photoelectric effect were called as photoelectrons.There is one condition for photoelectric effect which is very much important that for photoemission to take place, energy of incident light photons should be greater than or equal to the work function of the metal.

Work function (ϕ) is defined as the minimum quantity of energy which is required to remove an electron to infinity from the surface of a given solid, usually a metal.

Now on the basis of work function (ϕ) we can define two related quantity which are Threshold frequency and Threshold wavelength. Now as we know that the energy is of photon is given by -

$$E = h\nu = \frac{hc}{\lambda}$$

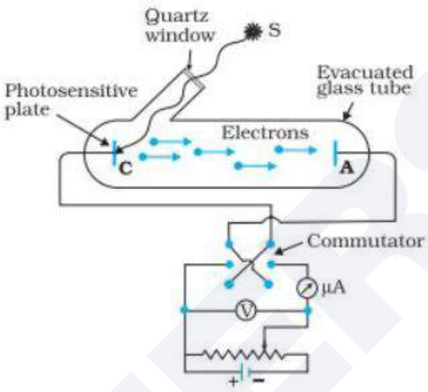
Now the frequency corresponding to the energy equals to work function is called Threshold frequency and similarly the wavelength corresponding to the work function is Threshold wavelength.

$$\phi = h\nu_{th} \quad \nu_{th} = \frac{\phi}{h}$$

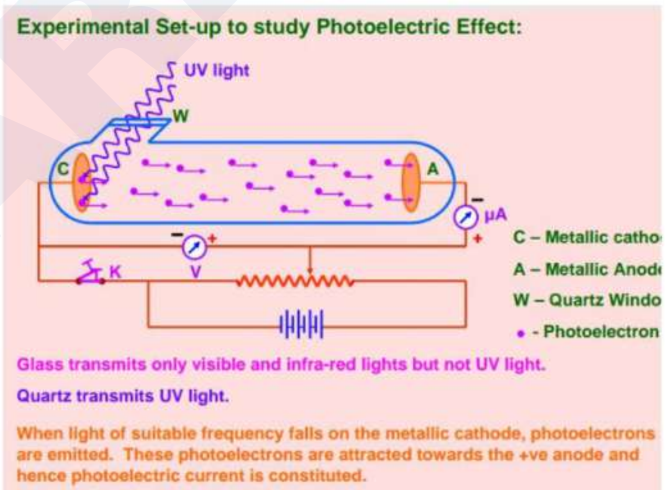
$$Similarly \;\; \phi = \frac{hc}{\lambda_{th}} \quad \lambda_{th} = \frac{hc}{\phi}$$

Now, let us understand with an experiment which was performed by Heinrich Rudolf Hertz.

For this let us consider the given set-up-



In this experiment setup, an evacuated glass tube is there. Two zinc plates C and A are enclosed. Plates A acts as an anode and C acts as a photosensitive plate. Two plates are connected to a battery and ammeter as shown. If the radiation is incident on the plate C through a quartz window, electrons are ejected out of the plate and current flows in the circuit this is known as **photocurrent**. Plate A can be maintained at the desired potential (+ve or – ve) with respect to plate C.



Applications of Photoelectric Effect -

- This phenomenon is used to generate electricity in Solar Panels.
- We come across many sensors in our day-to-day life. A few sensors are also working in the Photoelectric effect.
- It is also used in digital cameras because they have photoelectric sensors.

Note -

- In case of Threshold frequency - If incident frequency $v < \nu_0$. No photoelectron emission. The minimum frequency of incident radiation to eject electron is threshold frequency (ν_0)

- In case of Threshold Wavelength - If $\lambda > \lambda_0$ No photoelectron emission. The maximum wave length of incident raditaion required to eject the electron is Threshold Wavelength (λ_0)
- **Work function**

  $h = Planck's\ constant$

  $\nu_0 = threshold\ frequency$

  Energy used to overcome the surface barrier and come out of metal surface.

  $$\phi = h\nu_0$$

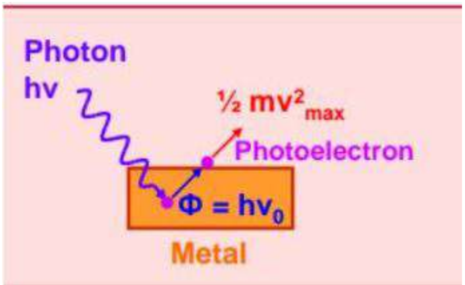
- Kinetic Energy of Photo eletrons

  $m \rightarrow mass\ of\ photoelectron$

  Remaining part of the energy is used in gaining a velocity v to the emitted photoelectron

  $k_{max} = \frac{1}{2} m v_{max}^2$

- Conservation of energy



$h\nu = \phi_0 + \frac{1}{2} m v_{max}^2$

$h\nu = h\nu_0 + \frac{1}{2} m v_{max}^2$

$h(\nu - \nu_0) = \frac{1}{2} m v_{max}^2$

$where, h - Planck's\ constant,\ \nu - Frequency,\ \nu_0 - threshold\ frequency,\ \phi_0 - work\ function$

Graphs related to Photoelectric effect -

1. Stopping potential-

The negative potential of the collector plate at which the photoelectric current becomes zero is called the **stopping potential or cut-off potential**. Stopping potential is the value of retarding potential difference between two plates which is just sufficient to stop the most energetic photoelectrons emitted. It is denoted by "V_o".

We need to equate the maximum kinetic energy Kmax of the photo-electron (having charge e) to the stopping potential V_o

We know that,

Electric potential energy= Potential Difference×Charge

So,

$$U = V_0 \times Q$$

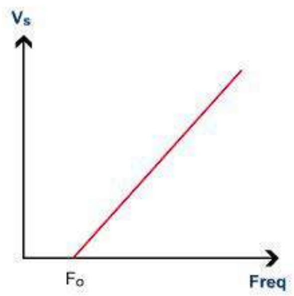
$$U = K_{\max}$$
$$\therefore |V_0 \times e| = K_{\max}$$
$$\Rightarrow K_{\max} = |eV_0|$$

Also

$$h\nu = h\nu_o + K.E._{(max)}$$

Now since $K.E._{max} = eV_o$, So we can write that -

$$eV_s = h(\nu - \nu_o)$$
or
$$V_s = \frac{h}{e}(\nu - \nu_o)$$

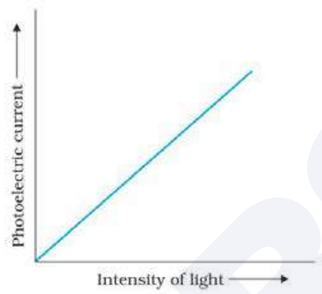


The above graph shows the variation between the stopping potential and frequency.

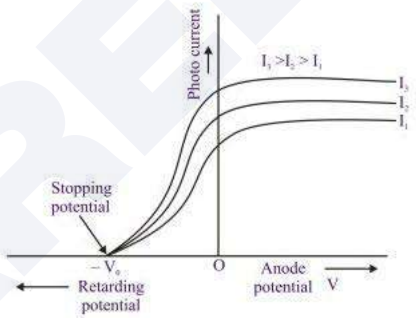
2. Saturation current -

The **photoelectric current** attains a **saturation** value and does not increase further for any increase in the positive potential. It means that this photoelectric current is the saturation current even we are increasing the value of the positive potential.

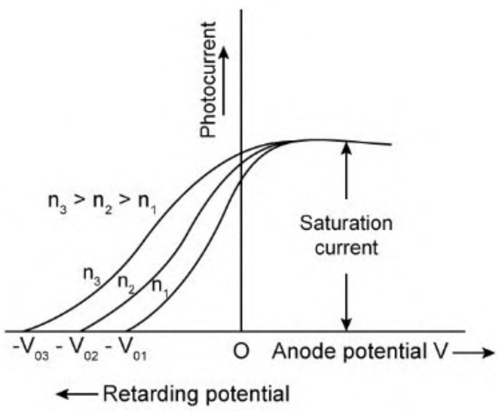
1. Variation of photocurrent with intensity -



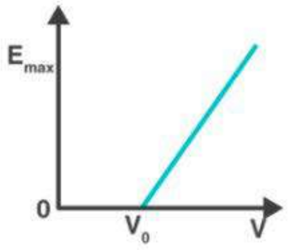
2. Variation of photoelectric current with potential and intensity



3. Effects of frequency of incident light on the stopping potential -



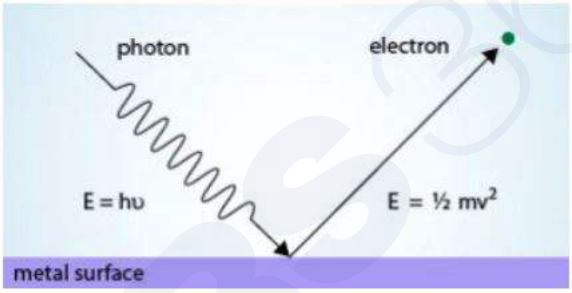
4. Variation of Kinetic energy with frequency



Kinetic energy V/s frequency

4. Einstein's Photoelectric Equation

Einstein's Photoelectric equation-



According to the experiment performed by Albert Einstein, there are some conclusions that those photo-electrons have kinetic energy only. Also, the energy absorbed by the photons is partly used to overcome the force by the metallic surface. Since there is no electric field present outside the metallic surface so there will be only energy present is pure kinetic energy.

So, we have K.E. of the photo-electrons = (Energy obtained from the Photon) – (The energy used to escape the metallic surface)

Here, The energy used to escape the metallic surface is the work function (ϕ).

So **Einstein's Photoelectric equation** can also be written as -

$$\mathrm{K.E.} = \mathrm{hv} - \Phi$$

We can understand the work function more clearly like this -

As we know an electron needs some minimum energy to be extracted from a metallic surface. So from the above equation, if ν = threshold frequency (ν_0) then the electrons gets just enough quantum energy to come out of the metal. It means that the Kinetic Energy of such an electron will be zero. So we can write that -

$$\mathrm{hv}_0 - \Phi = 0 \text{ or } \mathrm{hv}_0 = \Phi$$

This is the relation between the threshold frequency and the work function. We can also change this equation in terms of the threshold wavelength.

5. Radiation Pressure

Photons emitted by a source per second-

Consider a point source of light-emitting photons. And we want to find the number of Photons (n) emitted by this point source per second.

let the wavelength of light emitted by this = λ and

the power of the source as P (in Watt or J/s)

As we know the energy of each photon is given by

$$E = h\nu = \frac{hc}{\lambda} \quad (in\ Joule)$$

where

where c = Speed of light, h = Plank's constant = $6.6 \times 10^{-34} J - \sec$

ν = Frequency in Hz , λ= Wavelength of light.

or we can write the energy of each photon as $E=\frac{12400\ (eV)}{\lambda(A^0)}$

Then (n=the number of photons emitted per second) is given as

$$n=\frac{Power\ of\ source\ (W\ or\ \frac{J}{sec})}{Energy\ of\ each\ photon(J)}=\frac{P}{E}=\frac{P}{\frac{hc}{\lambda}}=\frac{P\lambda}{hc}\ (sec^{-1})$$

The intensity of light (I) :

The intensity of any quantity is defined as that quantity per unit area.

So here, light energy (or radiation) crossing per unit area normally per second is called intensity of light energy (or radiation).

And the intensity **I** is given as

$$I=\frac{E}{At}=\frac{P}{A}\quad\left(where\ \ \frac{E}{t}=P=\text{ radiation power}\right)$$

Its unit is W/m^2 or $\frac{J}{m^2*sec}$

The intensity of light due to a point isotropic source:

An isotropic source means it emits radiation uniformly in all directions.

So The intensity **I** due to a point isotropic source at a distance r from it is given as

$$I=\frac{P}{4\pi r^2}\Rightarrow i.e\ \ I\propto\frac{1}{r^2}$$

Photon Flux-

The photon flux (ϕ) is defined as the number of photons incident on a normal surface per second per unit area.

As we know n (the number of photons emitted per second) is given as

$$n=\frac{Power\ of\ source\ (W\ or\ \frac{J}{sec})}{Energy\ of\ each\ photon(J)}=\frac{P}{E}\ (sec^{-1})$$

Similarly intensity **I** is given as

$$I=\frac{P}{A}$$

So The photon flux (ϕ) is given as ratio of Intensity (I) to Energy of each photon

$$\phi=\frac{Intensity}{Energy\ of\ each\ photon}=\frac{I}{E}=\frac{n}{A}$$

or $\phi=\frac{I}{E}=\frac{I\lambda}{hc}$

- **The photon flux (ϕ) due to a point isotropic source:**

The photon flux (ϕ) due to a point isotropic source at a distance r from it is given as

$$\phi=\frac{number\ of\ photon\ per\ sec}{surface\ area\ of\ sphere\ of\ radius\ r}=\frac{n}{4\pi r^2}$$

Force exerted on a surface due to radiation-

Radiation pressure/force- When photons fall on a surface they exert a pressure/force on the surface. The pressure/force experienced by the surface exposed to the radiation is known as **Radiation pressure/force.**

n=Number of emitted photons per sec is given as $n=\frac{P}{E}=\frac{P}{h\nu}=\frac{P\lambda}{hc}$

where E= The energy of each photon

and Momentum of each photon is given as $p=\frac{E}{c}=\frac{h}{\lambda}$

And we know the force is given as rate of change of momentum.

I.e For each photon $F=\frac{dp}{dt}$

and for n photons per sec $F = n(\Delta p)$

For a black body, we get 100 % absorption or a=1

i.e for this surface 100% of the photon will be absorbed

so $|\Delta p| = |0 - p_i| = \frac{h}{\lambda}$

So Force is given as $F = n(\Delta p) = \frac{P\lambda}{hc} \times \frac{h}{\lambda} = \frac{P}{c}$

where P=Power

As $I = \frac{P}{A} \Rightarrow P = IA$

So Force is given as $F = \frac{P}{c} = \frac{IA}{c}$

and radiation pressure is given as $Pressure = \frac{F}{A} = \frac{I}{c}$

i.e For black body,

$F = \frac{P}{C}$

$Pressure = \frac{I}{c}$

- For perfectly reflecting surface (i.e mirror)

i.e r=1

i.e for this surface 100% of the photon will be reflected

i.e $p_f = -p_i$

So $|\Delta p| = |p_f - p_i| = |-p_i - p_i| = \frac{2h}{\lambda}$

So Force is given as $F = n(\Delta p) = \frac{P\lambda}{hc} \times \frac{2h}{\lambda} = \frac{2P}{c} = \frac{2IA}{c}$

and radiation pressure is given as $Pressure = \frac{F}{A} = \frac{2I}{c}$

- For neither perfectly reflecting nor perfectly absorbing body

i.e body having Absorption coefficient=a and reflection coefficient=r

and we have $a + r = 1$

So Force is given as $F = \frac{aP}{c} + \frac{2Pr}{c} = \frac{P}{c}(a + 2r) = \frac{P}{c}((1 - r) + 2r) = \frac{P}{c}(1 + r)$

and radiation pressure is given as $Pressure = \frac{F}{A} = \frac{P}{Ac}(1 + r) = \frac{I}{c}(1 + r)$

6. Wave nature of matter

As we know light behaves both as a wave and particle. If you are observing phenomena like the interference, diffraction or reflection, you will find that light is a wave. However, if you are looking at phenomena like the photoelectric effect, you will find that light has a particle character.

De Broglie's hypothesis stated that there is symmetry in nature and that if the light behaves as both particles and waves, matter too will have both the particle and wave nature.

i.e if a lightwave can behave as a particle then the particle can also behave as wave.

De Broglie's Equation-

According to De Broglie, A moving material particle can be associated with the wave.

De Broglie proposed that the wavelength λ associated with the moving material particle of momentum p is given as

$$\lambda = \frac{h}{p}$$

where $h = plank's\ constant$ and $h = 6.626 \times 10^{-34}\ Js$

further, we can write De - Broglie wavelength as

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{\sqrt{2mK}}$$

where

$h = plank's\ constant$

$m = mass\ of\ particle$

$v = speed\ of\ the\ particle$

$K = Kinetic\ energy\ of\ particle$

So from De Broglie's Equation, we can conclude that-

- $\lambda\ \alpha\ \frac{1}{m}$

i.e Wavelength associated with a heavier particle is smaller than that with a lighter particle.

- $\lambda\ \alpha \frac{1}{\nu}$

i.e when Particle moves faster, then wavelength will be smaller and vice versa

- if the particle at rest then De - Broglie wavelength will be infinite ($\lambda = \infty$)
- $\lambda\ \alpha\ \frac{1}{p}\ \alpha \frac{1}{v}\ \alpha \frac{1}{\sqrt{K}}$
- De - Broglie wavelength (λ) is independent of charge.

De - Broglie wavelength of Electron-

As **De Broglie's Equation** is given as $\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{\sqrt{2mK}}$

So for an electron having velocity v attained by it when it is accelerated through a potential difference of V.

then (Kinetic energy gain by the electron)=(work is done on an electron by the electric field)

i.e $K = W_E \Rightarrow \frac{1}{2} m_e v^2 = eV$

So **De - Broglie wavelength of Electron** is given as $\lambda_e = \frac{h}{m_e v} = \frac{h}{\sqrt{2m_e K}} = \frac{h}{\sqrt{2m_e(eV)}}$

using $h = 6.626 \times 10^{-34}\ Js$ and $m_e = 9.1 \times 10^{-31}\ kg$ and $e = 1.6 \times 10^{-19}\ C$

we get $\lambda_e = \frac{12.27}{\sqrt{V}} A^\circ$ (i.e answer will be in $A^0 = Angstrom$)

Similarly, we can find De - Broglie wavelength associated with charged particle

De - Broglie wavelength with charged particle-

$$\lambda = \frac{h}{\sqrt{2mK}} = \frac{h}{\sqrt{2mqV}}$$

Where $K \rightarrow kinetic\ energy\ of\ particle$

$q \rightarrow charged\ particle$

$V \rightarrow potential\ diffenence$

- De - Broglie wavelength of the proton

using $m_p = 1.67 \times 10^{-27}\ kg$ and $q_p = e = 1.6 \times 10^{-19}\ C$

we get $\lambda_{proton} = \frac{0.286}{\sqrt{V}} A^\circ$

- De - Broglie wavelength of Deuteron

using $m_D = 2 \times 1.67 \times 10^{-27}\ kg$ and $q_D = e = 1.6 \times 10^{-19}\ C$

we get $\lambda_{deutron} = \frac{0.202}{\sqrt{V}} A^\circ$

- De - Broglie wavelength of an Alpha particle (He^{2+})

using $m_{\alpha^{2+}} = 4 \times 1.67 \times 10^{-27}\ kg$ and $q_{\alpha^{2+}} = 2e = 2 \times 1.6 \times 10^{-19}\ C$

we get $\lambda_{\alpha-partical} = \frac{0.101}{\sqrt{V}} A^\circ$

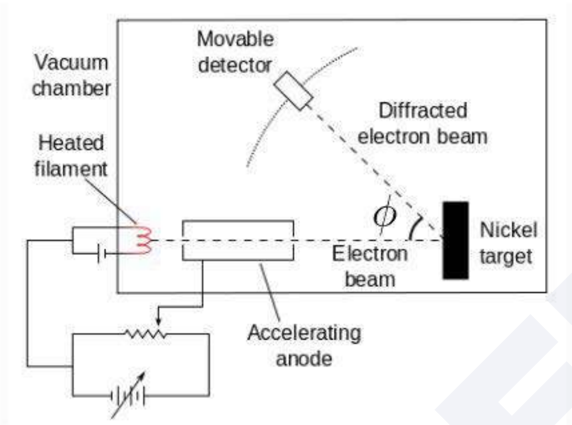
- **Electron microscope-**

An electron microscope is an important application of de-Broglie waves designed to study very minute objects like viruses, microbes and the crystal structure of the solids. In the electron microscope, by selecting a suitable value of potential difference V ,we can have an electron beam of as small wavelength as desired. And this de-Broglie wavelength is calculated by using the formula $\lambda_e = \frac{12.27}{\sqrt{V}} A^\circ$.

7.Davisson-germer Experiment

Davisson and Germer's Experiment, for the first time, proved the wave nature of electrons through electron diffraction and also verified the de Broglie equation.

In this experiment, we will study the scattering of electrons by a Ni crystal.



The experimental setup for the Davisson and Germer experiment is enclosed within a vacuum chamber.

The experimental arrangement of the Davisson Germer experiment consists of the following main parts

- Electron gun: An electron gun comprising of a tungsten filament F was coated with barium oxide and heated through a low-voltage power supply. It emits electrons when heated to a particular temperature. The electrons emitted by the electron gun are again accelerated to a particular velocity.
- Collimator: The accelerator is enclosed within a cylinder perforated with fine holes along its axis, these emitted electrons were made to pass through it. Its function is to render a narrow and straight (collimated) beam of electrons ready for acceleration.
- Target: The target is a Nickel crystal. The beam produced from the cylinder is again made to fall on the surface of a nickel crystal. The crystal is placed such that it can be rotated about a fixed axis. Due to this, the electrons scatter in various directions.
- Detector: A detector is used to capture the scattered electrons from the Ni crystal. The beam of electrons produced has a certain amount of intensity which is measured by the electron detector and after it is connected to a sensitive galvanometer, it is then moved on a circular scale to record the current.

Observations of Davisson Germer experiment-

- The intensity of the scattered electron beam is measured for different values of the angle of scattering (ϕ) by changing the ϕ (angle between the incident and the scattered electron beams).

  These electrons formed a diffraction pattern. Thus the dual nature of matter was verified.

- The energy of the incident beam of electrons can be varied by changing the applied voltage to the electron gun.

Note-Intensity of a scattered beam of electrons is found to be maximum when the angle of scattering is 50^o and the accelerating potential is 54 V.

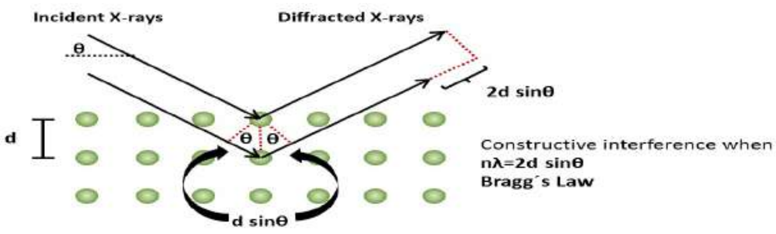
i.e we could see a strong peak in the intensity. This peak was the result of the constructive interference of the scattered electrons.

The intensity of the scattered electrons is not continuous. It shows a maximum and a minimum value corresponding to the maxima and the minima of a diffraction pattern produced by X-rays.

Galvanometer in Davisson Germer Experiment-

The detector is connected to a sensitive galvanometer to measure the small values of current due to a scattered beam of electrons.

- **Bragg’s formula-**



The path difference between electrons scattered from adjacent crystal planes is given by $\Delta x = 2dsin\theta$

and For constructive interference between the two scattered beams

$\Delta x = 2d\sin\Theta = n\lambda$

where $d - distance\ between\ diffracting\ planes$

and θ is the angle between the incident rays and the surface of the crystal

For the above figure $\phi = scattering\ angle$

As $\theta + \phi + \theta = 180^o$

So $\Theta = \frac{180 - \phi}{2}$

The intensity of a scattered beam of electrons is found to be maximum when the angle of scattering is 50^o

So For $\phi = 50^0$ we get $\theta + 50^o + \theta = 180^o$

we get $\theta = 65^o$

Co-relating Davisson Germer experiment and de Broglie relation-

According to de Broglie,

$\lambda_e = \frac{12.27}{\sqrt{V}} A^\circ$

and using V = 54 Volt we get $\lambda_e = 0.167\ nm$

From Bragg's formula, we have $2d\sin\Theta = n\lambda$

The Lattice Spacing in Ni Crystal is given as d=0.092 nm.

And using $n = 1$, $\phi = 50^0\ and\ V = 50\ Volt$

we get $\lambda_e = 0.165\ nm$

Therefore the experimental results are in close agreement with the theoretical values got from the de Broglie equation.

Thus Davisson and Germer's Experiment verify the de Broglie equation.

Frank Hertz Experiment-

This experiment is the first experimental verification of the existence of discrete energy states in atoms.

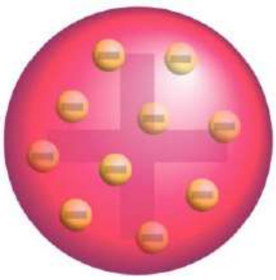
Frank and Hertz proposed that the 4.9 V characteristic of their experiments was due to the ionization of mercury atoms by collisions with the flying electrons emitted at the cathode.

Atoms

Important Formulae

1. Rutherford's Atomic Model And Limitations

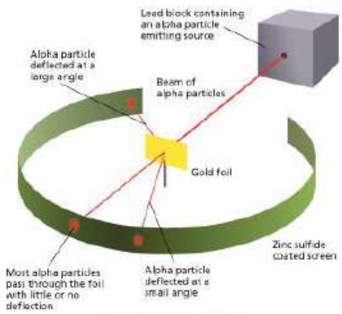
J.J Thomson's model - J. J. Thomson, who discovered the electron in 1897, proposed the plum pudding model of the atom in 1904 before the discovery of the atomic nucleus in order to include the electron in the atomic model. In Thomson's model, the atom is composed of electrons surrounded by a soup of positive charge to balance the electrons' negative charges, like negatively charged "plums" surrounded by positively charged "pudding".



The Rutherford Model (Gold Foil Experiment) :

Rutherford and his colleagues Geiger and Marsden bombarded a thin gold foil of thickness approximately 8.6×10^{-6} cm with a beam of alpha particles in a vacuum. They used gold since it is highly malleable, producing sheets that can be only a few atoms thick, thereby ensuring smooth passage of the alpha particles. A circular screen coated with zinc sulphide surrounded the foil. Since the positively charged alpha particles possess mass and move very fast, it was hypothesized that they would penetrate the thin gold foil and land themselves on the screen, producing fluorescence in the part they struck.

In line with the plum pudding model, since the positive charge of atoms was evenly distributed and too small as compared to that of the alpha particles, the deflection of the particulate matter, if any, was predicted to be less than a small fraction of a degree.



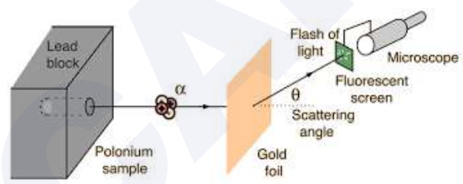
Gold Foil Experiment

Observations :

- Most of the alpha particles behaved as expected, there was a noticeable fraction of particles that got scattered by angles greater than 90 degrees.
- In fact, there were about 1 in every 2000 particles that got scattered by a full 180 degree, that is, they simply retraced their path after hitting the gold foil.

Conclusion: A highly concentrated positive charge at the center of an atom that caused an electrostatic repulsion of the particles strong enough to bounce them back to their source. The particles that got deflected by huge angles passed close to the said concentrated mass. Most of the particles passed undeviated as there was no obstruction to their path, proving that the majority of an atom is empty. Rutherford drew the conclusion that since the dense alpha particles could be deflected by the central core, it shows that almost the entire mass of the atom is concentrated there. Rutherford named it the “nucleus” after performing the experiment in various gases.

Rutherford scattering formula

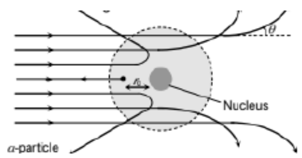


For a detector at a specific angle (θ) with respect to the incident beam, the number of particles per unit area striking the detector is given by the Rutherford formula:

$$N(\theta) \propto \frac{1}{sin^4(\frac{\theta}{2})}$$

<u>Distance of closest approach</u> :

The minimum distance from the nucleus up to which the α - particle approach, is called the distance of closest approach (r_0). At this distance the entire initial kinetic energy has been converted into potential energy so

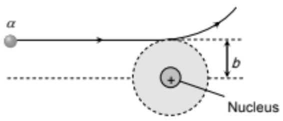


$$(\text{K.E.})_{initial} = K = \frac{1}{2}mv^2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{(Ze)2e}{r_0}$$

$$r_0 = \frac{Ze^2}{mv^2\pi\varepsilon_0} = \frac{4kZe^2}{mv^2} = \frac{2kZe^2}{K}$$

Impact parameter: It is defined as the perpendicular distance of the velocity of the alpha-particle from the centre of the nucleus when it is far away from the atom. The shape of the trajectory of the scattered alpha particle depends on the impact parameter 'b' and the nature of the potential field. Rutherford deduced the following relationship between the impact parameter 'b' and the scattering angle θ. It is given as

$$b = \frac{Ze^2 \cot(\theta/2)}{4\pi\varepsilon_0 \left(\frac{1}{2}mv^2\right)}$$
$$\Rightarrow b \propto \cot(\theta/2)$$



Conclusion and drawback of Rutherford model:

Conclusion:

1. A highly concentrated positive charge at the center of an atom that caused an electrostatic repulsion of the particles strong enough to bounce them back to their source.
2. The particles that got deflected by huge angles passed close to the said concentrated mass (nearly 99.95 %). Most of the particles passed undeviated as there was no obstruction to their path, proving that the majority of an atom is empty.
3. Rutherford drew the conclusion that since the dense alpha particles could be deflected by the central core, it shows that almost the entire mass of the atom is concentrated there. Rutherford named it the “nucleus” after performing the experiment in various gases.

Limitations of Rutherford model:

1. It could not explain stability of an atom because this model does not obey Maxwell law of electrodynamics. According to Maxwell electron should continuous emit radiation and thus and gradually lose energy , so its distance from nucleus should become shorter and finally it should fall into the nucleus.
2. It could not explain line spectra of atom. According to this model the spectrum of atom must be continuous whereas practically it is a line spectrum.
3. It did not explain the distribution of electrons outside the nucleus.

2.Bohr Model Of The Hydrogen Atom

Bohr's Model of hydrogen atom:

Bohr proposed a model for hydrogen atom which is also applicable for some lighter atoms in which a single electron revolves around a stationary nucleus of positive charge Z_e (called hydrogen-like atom)
Bohr's model is based on the following postulates-

(1). Bohr’s first postulate was that an electron in an atom could revolve in certain stable orbits without the emission of radiant energy, contrary to the predictions of electromagnetic theory. **According to this postulate, each atom has certain definite stable states in which it can exist, and each possible state has definite total energy. These are called the stationary states of the atom**

For electrons revolving in a stable orbit, the necessary centripetal force is provided by the coulomb's force

$$\frac{mv_n^2}{r_n} = \frac{kze^2}{r_n^2}$$

(2). Bohr’s second postulate defines these stable orbits. This postulate states that **the electron revolves around the nucleus only in those orbits for which the angular momentum is some integral multiple of $\frac{h}{2\pi}$ where h is the Planck’s constant (= 6.6 × 10^{-34} J s). Thus the angular momentum (L) of the orbiting electron is quantised. That is** $L = mv_nr_n = \frac{nh}{2\pi}; n = 1, 2, 3......\infty$

(3). Bohr’s third postulate incorporated into atomic theory the early quantum concepts that had been developed by Planck and Einstein. **It states that an electron might make a transition from one of its specified non-radiating orbits to another of lower energy. When it does so, a photon is emitted having energy equal to the energy difference between the initial and final states. The frequency of the emitted photon is then given by** $h\nu = E_i - E_f$

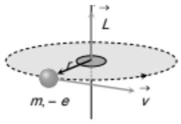
E_i is the energy of the initial state and E_f is the energy of the final state. Also, $E_i > E_f$.

r_n-radius of the nth orbit

v_n- speed of an electron in the nth orbit

Radius of orbit and velocity of the electron-

Radius of the orbit: For an electron around a stationary nucleus the electrostatic force of attraction provides the necessary centripetal force.



ie. $\frac{1}{4\pi\varepsilon_0}\frac{(Ze)e}{r^2}=\frac{mv^2}{r}\quad\cdots$ (i)

also $mvr=\frac{nh}{2\pi}$

From equations (i) and (ii) radius of r orbit

$$r_n=\frac{n^2h^2}{4\pi^2kZme^2}=\frac{n^2h^2\varepsilon_0}{\pi mZe^2}=0.53\frac{n^2}{Z}\mathring{A}\quad\left(k=\frac{1}{4\pi\varepsilon_0}\right)$$

$$\Rightarrow r_n\propto\frac{n^2}{Z}$$

$$\Longrightarrow\ r_n=0.53\frac{n^2}{Z}\mathring{A}$$

Speed of electron:-

From the above relations, the speed of electrons in n^{th} orbit can be calculated as

$$v_n=\frac{2\pi kZe^2}{nh}=\frac{Ze^2}{2\varepsilon_0nh}=\left(\frac{c}{137}\right)\frac{Z}{n}=2.2\times10^6\frac{Z}{n}m/\text{sec}$$

where $(c=\text{ speed of light }=3\times10^8\,\text{m/s})$

3. Energy Level - Bohr's Atomic Model

Energy of electron in nth orbit-

Potential energy: An electron possesses some potential energy because it is found in the field of the nucleus potential energy of an electron in n^{th} orbit of the radius r_n is given by

$$U=k\frac{(Ze)(-e)}{r_n}=-\frac{kZe^2}{r_n}$$

Kinetic energy: Electrons possess kinetic energy because of their motion. Closer orbits have greater kinetic energy than outer ones. As we know $\frac{mv^2}{r_n}=\frac{k(Ze)(e)}{r_n^2}$

$$\text{Kinetic energy } K=\frac{kZe^2}{2r_s}=\frac{|U|}{2}$$

Total energy: Total energy E is the sum of potential energy and kinetic energy ie. $E=K+U$

$$\Rightarrow\quad E=-\frac{kZe^2}{2r_n}\quad\text{also}\quad r_n=\frac{n^2h^2\varepsilon_0}{z\pi me^2}$$

Hence $E=-\left(\frac{me^4}{8\varepsilon_0^2h^2}\right)\frac{z^2}{n^2}=-\left(\frac{me^4}{8\varepsilon_0^2ch^3}\right)ch\frac{z^2}{n^2}$

$$=-R\,\text{ch}\frac{Z^2}{n^2}=-13.6\frac{Z^2}{n^2}eV$$

where $R=\frac{me^4}{8\varepsilon_0^2ch^3}=$ Rydberg's constant $=1.09\times10^7m^{-1}$

The energy level for Hydrogen-

The energy of n[th] level of hydrogen atom (z=1) is given as : $E_n=-\frac{z^213.6}{n^2}eV=-\frac{13.6}{n^2}eV\qquad(\because z=1)$

$$\text{Energy of Ground state}(n=1)=E_1=-\frac{13.6}{1}eV=-13.6eV$$

$$\text{Energy of first excited state}(n=2)=E_2=-\frac{13.6}{4}eV=-3.4eV$$

$$\text{Energy of second excited state}(n=3)=E_3=-\frac{13.6}{9}eV=-1.51eV$$

$$\text{Energy of third excited state}(n=4)=E_4=-\frac{13.6}{16}eV=-0.85eV$$

Binding Energy(B.E.) of n^{th} orbit -The energy required to move an electron from n^{th} orbit to $n=\infty$ is called the Binding energy of n^{th} orbit

OR

The binding energy of n^{th} orbit is the negative of the total energy of that orbit

$$E_{\text{Binding}} = E_{\infty} - E_n = -E_n = \frac{13.6 Z^2}{n^2} \text{eV}$$

Ionization energy (IE): Total energy of a hydrogen atom corresponds to infinite separation between electron and nucleus. Total positive energy implies that the atom is ionized and the electron is in an unbound (isolated) state moving with certain kinetic energy. The minimum energy required to move an electron from the ground state(n=1) to $n=\infty$ is called **the ionization energy of the atom or ion**.

The formula for the ionization energy is -

$$E_{\text{ionisition}} = E_{\infty} - E_1 = -E_1 = 13.6 Z^2 \text{eV}$$

On the basis of ionization energy, we can define the ionization potential also -

Ionization potential (IP): The potential difference through which a free electron must be accelerated from rest such that its kinetic energy becomes equal to ionization energy of the atom is called **the ionization potential of the atom**.

$$V_{\text{ionisation}} = \frac{E_{Ionisation}}{e} = 13.6 Z^2 \text{V}$$

Excitation Energy and Excitation Potential

The process of absorption of energy by an electron so as to raise it from a lower energy level to some higher energy level is called excitation.

Excited state: The states of an atom other than the ground state are called its excited states. Examples are mentioned below -

$$\begin{array}{ll} n=2, & \text{first excited state} \\ n=3, & \text{second excited state} \\ n=4, & \text{third excited state} \\ n=n_0+1, & n_0 \text{ th excited state} \end{array}$$

Excitation energy -

The energy required to move an electron from the ground state of the atom to any other excited state of the atom is called the Excitation energy of that state.

$$E_{excitation} = E_{higher} - E_{lower}$$

Excitation potential can also be defined on the basis of excitation energy. So the excitation potential is the potential difference through which an electron must be accelerated from rest so that its kinetic energy becomes equal to the excitation energy of any state is called the excitation potential of that state.

$$V_{excitation} = \frac{E_{excitation}}{e}$$

4.Line Spectra Of Hydrogen Atom

Line spectra of a Hydrogen atom -

According to Bohr, when an atom makes a transition from a higher energy level to a lower energy level, it emits a photon with energy equal to the energy difference between the initial and final levels. If E_i, the initial energy of the atom before such a transition, E_f is its final energy after the transition, then conservation of energy gives the energy of emitted photon-

$$\begin{aligned} &\text{h}v = \frac{\text{hc}}{\lambda} = E_i - E_f \\ &\frac{hc}{\lambda} = \frac{-13.6}{n_i^2} eV - \frac{-13.6}{n_f^2} eV = 13.6 eV \left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) \\ &Rch = 13.6 eV = 1\text{Rydberg energy} \\ &\Rightarrow \frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) \\ &\text{where } R = Rydberg's\ constant = 1.097 \times 10^7 m^{-1} \end{aligned}$$

For **Hydrogen-like atoms**, the wavelength of an emitted photon during the transition from n_f orbit to n_i orbit is $\frac{1}{\lambda} = Rz^2\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$

Because of this photon, spectra of hydrogen atom will emit and which is studied by various scientists. One such scientist named Balmer found a formula that gives the wavelengths of these lines for all the transitions taking place to the 2^{nd} orbit.

The **Balmer series** is a series of spectral emission lines of the hydrogen atom that result from electron transitions from higher levels down to the energy level with principal quantum number 2

Balmer observed the spectra and found the formula for the visible range spectra which is obtained by Balmer's formula is-

$$\frac{1}{\lambda}=R\left(\frac{1}{2^2}-\frac{1}{n^2}\right)\ldots(1)$$

Here, n=3,4,5,. . . . etc. ,

$R = Rydberg\ constant = 1.097 \times 10^7 \mathrm{m}^{-1}$
and λ is the wavelength of the light photon emitted during the transition.

Since Balmer had found the formula for n = 2, but we can obtain different spectra for different values of n. For n = ∞, we get the smallest wavelength of this series, which is equal to = 3646 $\dot{A}$. We can also obtain the value of wavelength for Balmer's series by putting different values of 'n' in the equation (1). Similarly, we can obtain the wavelength of the different spectra like the Lyman, Paschen series. Since the Balmer series is in the visible range but the Lyman series is in the Ultraviolet range and the Paschen, Brackett, and Pfund are in the Infrared range.

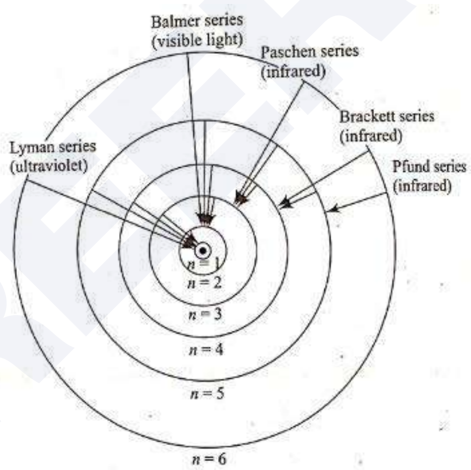
$$\text{Lyman series: } \tfrac{1}{\lambda}=R\left(\tfrac{1}{1^2}-\tfrac{1}{n^2}\right), n=2,3,4,\ldots$$

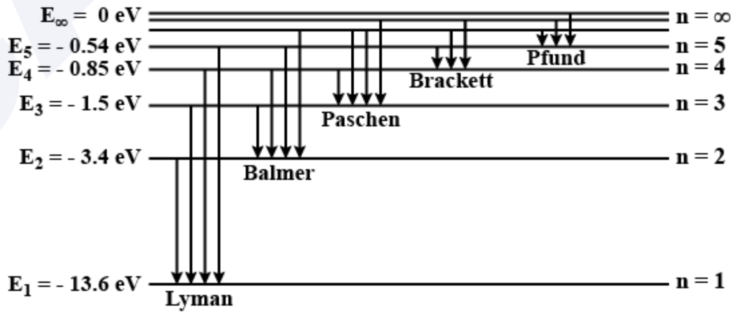
$$\text{Balmer series: } \tfrac{1}{\lambda}=R\left(\tfrac{1}{2^2}-\tfrac{1}{n^2}\right), n=3,4,5,\ldots$$

$$\text{Paschen series: } \tfrac{1}{\lambda}=R\left(\tfrac{1}{3^2}-\tfrac{1}{n^2}\right), n=4,5,6,\ldots$$

$$\text{Brackett series: } \tfrac{1}{\lambda}=R\left(\tfrac{1}{4^2}-\tfrac{1}{n^2}\right), n=5,6,7,\ldots$$

$$\text{Pfund series: } \tfrac{1}{\lambda}=R\left(\tfrac{1}{5^2}-\tfrac{1}{n^2}\right), n=6,7,8$$





Energy level diagram for hydrogen atom

This is for the hydrogen spectrum

5.De-broglie's explanation of Bohr's second postulate

Since the Bohr gave many postulates in his theory, but the second postulate is not very clear and little puzzling. The Scientist De Broglie explained this puzzle very clearly that why the angular momentum of the revolving electron is the integral multiple of the $h/2\pi$. De broglie in his experiment proved that the electron revolving the circular orbit has wave nature also in the last chapter we have seen the experiment performed by the Davison and Germer which proved that the electron shows the wave nature. In analogy to waves travelling on a string, particle waves too can lead to standing waves under resonant conditions. During the chapter Waves and Oscillation, we know that when a string is plucked, a vast number of wavelengths are excited. However only those wavelengths survive which have nodes at the ends and form the standing wave in the string. It means that in a string, standing waves are formed when the total distance travelled by a wave down the string and back is any integral number of wavelengths. Waves with other wavelengths interfere with themselves upon reflection and their amplitudes quickly drop to zero.

For an electron moving in n^{th} circular orbit of radius r_n, the total distance is the circumference of the orbit, $2\pi r_n$.

$$2\pi r_n = n\lambda, \quad n = 1, 2, 3 \ldots$$

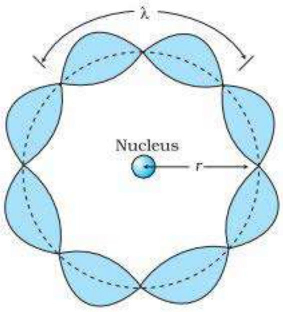


Figure given above illustrates a standing particle wave on a circular orbit for n = 4, i.e., $2\pi r_n = 4\lambda$, where λ is the de Broglie wavelength of the electron moving in n^{th} orbit. From the last chapter we have studied that λ = h/p, where p is the magnitude of the electron's momentum. If the speed of the electron is much less than the speed of light, the momentum is mv_n.

Thus,

$$\lambda = \frac{h}{mv_n}.$$

From the above equation, we have,

$$2\pi r_n = \frac{nh}{mv_n} \quad or, \quad mv_n r_n = \frac{nh}{2\pi}$$

This is the quantum condition proposed by Bohr for the angular momentum of the electron. Thus de Broglie hypothesis provided an explanation for Bohr's second postulate for the quantisation of angular momentum of the orbiting electron.

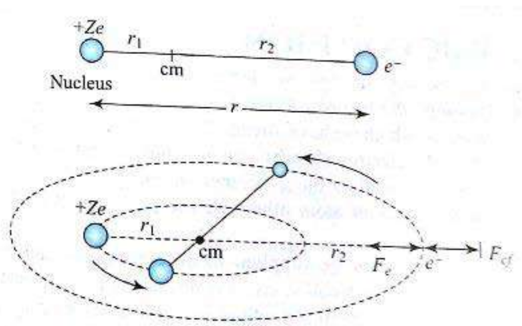
The quantised electron orbits and energy states are due to the wave nature of the electron and only resonant standing waves can persist. Bohr's model, involving a classical trajectory picture (planet-like electron orbiting the nucleus), correctly predicts the gross features of the hydrogenic atoms(Hydrogenic atoms are the atoms consisting of a nucleus with positive charge +Ze and a single electron, where Z is the proton number. Examples are a hydrogen atom, singly ionised helium, doubly ionised lithium, and so forth.), in particular, the frequencies of the radiation emitted or selectively absorbed.

6. Effect of Nucleus motion on Energy-

Till now in Bohr's model, we have assumed that all the mass of the atom is situated at the center of the atom. As the mass of the electron is very much small and negligible as compared to the mass of the nucleus so all the mass is assumed to be concentrated at the center of the nucleus. But actually, centre of mass of the nucleus-electron system is close to nucleus as it is heavy, and to keep the centre of mass at rest, both electron and nucleus revolve around their centre of mass like a double star system as shown in figure. If r is the distance of electron from nucleus, the distances of nucleus and electron from the centre of mass, r_1 and r_2, can be given as -

$$r_1 = \frac{m_e r}{m_N + m_e}$$

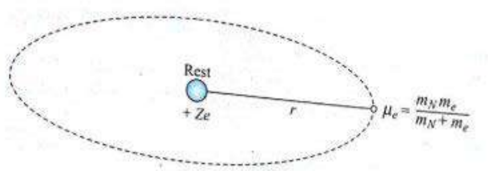
$$\text{and} \quad r_2 = \frac{m_N r}{m_N + m_e}$$



We can see that the atom, nucleus, and electron revolve around their centre of mass in concentric circles of radii r_1 and r_2 to keep the centre of mass at rest. In the above system, we can analyze the motion of electrons with respect to the nucleus by assuming the nucleus to be at rest and the mass of electron replaced by its reduced mass μ_e, given as -

$$\mu_e = \frac{m_N m_e}{m_N + m_e}$$

Now we can change our assumption and the system will look like as shown in the figure with reduced mass -



Now we can derive the equation obtained by Bohr with the reduced mass also -

$$r_n = \frac{n^2 h^2}{4\pi^2 k Z e^2 m_e}$$

Now after replacing the electron mass with its reduced mass, the equation becomes -

$$r_n' = \frac{n^2 h^2}{4\pi^2 k Z e^2 \mu_e} \Rightarrow r_n' = \frac{n^2 h^2 (m_N + m_e)}{4\pi^2 k Z e^2 m_e m_N}$$

$$\text{or} \quad r_n' = r_n \times \frac{m_e}{\mu_e} \Rightarrow r = (0.529\text{A}) \frac{mn^2}{\mu Z}$$

But there will be no effect on the velocity because the term of mass is not present there -

$$v_n = \frac{2\pi k Z e^2}{nh}$$

Similarly for the energy, we can write that -

$$E_n = -\frac{2\pi^2 k^2 Z^2 e^4 m_e}{n^2 h^2}$$

After putting the reduced mass in the equation -

$$E_n' = -\frac{2\pi^2 k^2 Z^2 e^4 m_N m_e}{n^2 h^2 (m_N + m_e)}$$

$$E_n' = E_n \times \frac{\mu_e}{m_e} \Rightarrow E_n = -(13.6\text{eV}) \frac{Z^2}{n^2} \left(\frac{\mu}{m}\right)$$

Thus, we can say that the energy of electrons will be slightly less compared to what we have derived earlier. But for numerical calculations this small change can be neglected unless in a given problem it is asked to consider the effect of motion of nucleus.

7.<u>Atomic Collision-</u>

There are two ways to excite an electron in an atom-

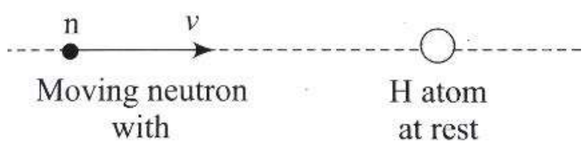
1. By supplying energy to an electron through electromagnetic photons for eg., the Photoelectric effect

2. By the atomic collision, the kinetic energy loss is utilized in the ionization or excitation of the atom.

Now let us understand the atomic collision -

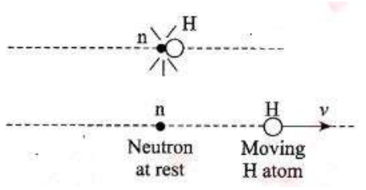
<u>Collision of a Neutron with an atom -</u>

Let us consider an example of a head-on collision of a moving neutron with a stationary hydrogen atom as shown in the figure. Here. for mathematical analysis, let us assume the masses of the neutron and H atom to be the same -



Now there are two cases, the first is a perfect elastic collision and another is a perfectly inelastic collision.

1. Perfect elastic collision -



In this case, since the mass of the neutron and mass of the hydrogen atom, then the hydrogen atom will move with the same speed and kinetic energy which neutron is moving initially.

2. Perfect inelastic collision -

If both have perfect inelastic collision, then both move together. Now by applying conservation of momentum -

$$mv_0 = 2mv_1$$

$$\Rightarrow v_1 = \frac{v}{2}$$

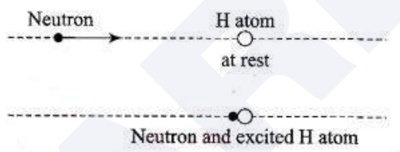
v_o is the initial velocity of the neutron

v_1 is the final combined velocity of the atom and neutron.

Now the difference between initial and final kinetic energy is given as -

$$\begin{aligned}\Delta E &= E_i - E_f \\ &= \frac{1}{2}mv_o^2 - \frac{1}{2}(2m)\left(\frac{v_o}{2}\right)^2 \\ &= \frac{1}{2}mv_o^2 - \frac{1}{4}mv_o^2 \\ &= \frac{1}{4}mv_o^2 = \frac{1}{2}E_i\end{aligned}$$

Thus, half of the initial kinetic energy will be lost in the collision. The energy lost can only be absorbed by the atom involved in the collision and may get excited or ionized by this energy loss which takes place in case of inelastic collision. Here we are not considering the heat energy loss during the collision.



This loss in energy can be absorbed by the H atom only. From the previous concepts, we know that the minimum energy needed by the hydrogen atom to get excited is 10.2 eV for n =1 to n=2. So the minimum energy loss must be equal to 10.2 eV to excite hydrogen atoms. If the loss in energy is more than 10.2 eV then only 10.2 eV is absorbed by the hydrogen atom and the rest of the energy remains in the colliding particles (Neutron and H atom) as the collision is not perfectly inelastic.

8.X-rays

X-rays-

X-rays are highly energetic radiations with very short wavelengths. Their wavelength is even shorter than the ultraviolet radiations and varies between 0.03 and 3 nanometers and some x-rays are as small as a single atom of many elements. X-rays were discovered by Roentgen, who found that a discharge tube(Coolidge's tube), operating at low pressure and high voltage, emitted radiation that caused a fluorescent in the neighbourhood to glow brightly. This indicates that some unknown radiation was responsible for fluorescence. Since the name of these rays was unknown so it is named X-rays.

Properties of X-rays -

- They pass through materials more or less unchanged
- They cannot be refracted
- Electric and magnetic fields do not have any effect on these rays
- These radiations ionize the surrounding air by discharging electrified bodies
- They have a short wavelength varying between 0.1 A° to 1 A°.

- They are produced when a metal anode is bombarded by very high-energy electrons.
- They do not require any medium for propagation
- X-rays cannot be focused on a single point
- These radiations cannot be heard or smelt
- They travel in a straight line in free space
- They cause photoelectric emissions.
- The intensity of X – rays depends on the number of electrons hitting the target.

Application of X-rays -

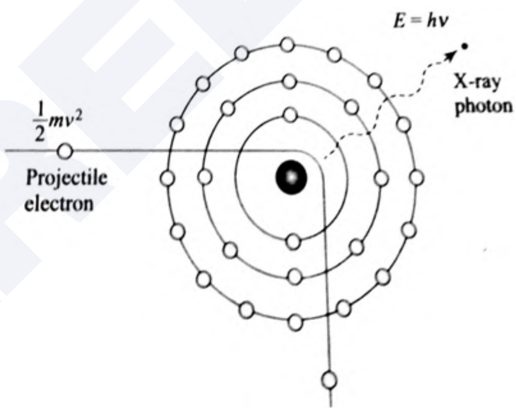
- Medical Science - They are used for medical purposes to detect the breakage in human bones.
- Security - They are used as a scanner to scan the luggage of passengers in airports, rail terminals, and other places.
- Astronomy - It is emitted by celestial objects and is studied to understand the environment.
- Industry - It is widely used to detect defects in welds.
- Restoration - They are used to restore old paintings.

9.Continuous X-ray-

As we know about the phenomenon of visible light, similarly continuous X-ray spectra also contain photons ranging through a lot of wavelengths. From the previous concept, we know that the production of X-rays happens when the target which is made up of an element with a high atomic number is hit by electrons traveling at a high velocity. So out of the total energy, most of the energy applied is wasted by being converted into heat energy in the target material's system. X-rays that have continuously unstable wavelengths are produced due to the loss of energy that the few electrons that were moving fast enough (and penetrated to the interior sections of the atoms of the material being targeted) suffer. Since the attractive pulling forces applied by the nucleus of the target element cause a deceleration of these fast-moving electrons, this decreases the energy of the electron continuously. Due to this, varying frequency of X-rays is emitted continuously due to the retardation of the speed of electrons. The X–rays consist of a continuous range of frequencies up to a maximum frequency $\nu_{\max}$ or minimum wavelength $\lambda_{\min}$. This is called **continuous X – rays**. The minimum wavelength depends on the anode voltage. If V is the potential difference between the anode and the cathode, then -

$$eV = h\nu_{max} = \frac{hc}{\lambda_{min}}$$

To produce the continuous X-ray in the Coolidge tube, an electron is projected toward the anode with an accelerating voltage V. So, the kinetic energy of the projectile electron will be eV. As shown in the figure, it experiences strong electric force toward the nucleus of the atom and due to this strong attraction the velocity of this electron, when it emerges from the atom, will be highly reduced and negligible compared with the initial speed of the projectile electron.



According to the law of conservation of energy, the energy of these electromagnetic radiations will be equal to the decrease in the kinetic energy of the projectile electron.

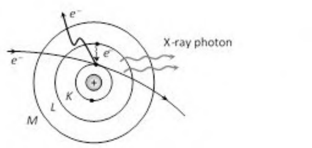
$$eV = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2eV}{m}}$$

But the velocity of the incoming electron will be less as compared to the projectile electron. This difference in kinetic energy will cause the production of X-rays.

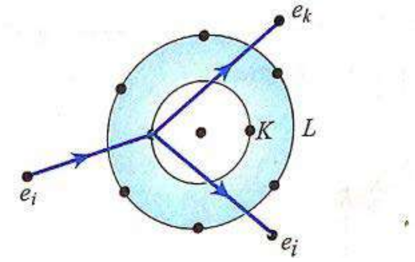
10.Characteristic X-Rays -

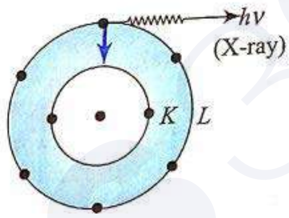
Few of the fast-moving electrons having high velocity penetrate the surface atoms of the target material and knock out the tightly bound electrons even from the innermost shells of the atom. Now when the electron is knocked out, a vacancy is created at that place. To fill this vacancy electrons from higher shells jump to fill the created vacancies, We know that when an electron jumps from a higher energy orbit E_1 to a lower energy orbit E_2, it radiates energy $(E_1 - E_2)$. Thus this energy difference is radiated in the form of X-rays of very small but definite wavelength which depends upon the target material. The X-ray spectrum consists of sharp lines and is called the characteristic X-ray spectrum. These X-rays are called characteristic X-rays because they are characteristic of the element used as the target anode. Characteristic X-rays have a line spectral distribution, unlike continuous X-rays. The wavelength spectrum of the X-frequencies corresponding to these lines is the characteristic of the material or the target, i.e., anode material.



When the atoms of the target material are bombarded with high-energy electrons (or hard X-rays), which possess enough energy to penetrate into the atom, they knock out the electron of the inner shell (say K shell, n=1). When an electron is missing in the K shell, an electron from the next upper shell makes a quantum jump to fill the vacancy in the K shell. In the transition process, the electron radiates energy whose frequency lies in the X-ray region. The frequency of emitted radiation (i.e., of the photon) is given by -

$$v = RZ_e^2\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)$$

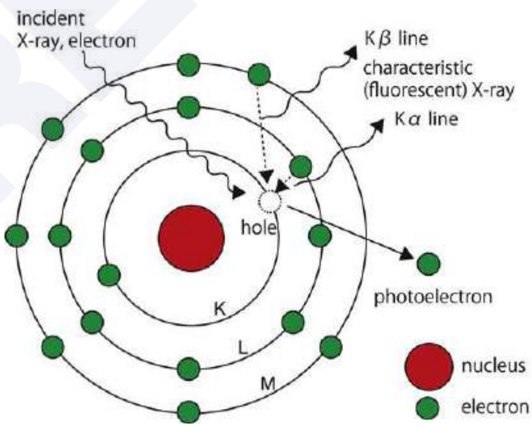




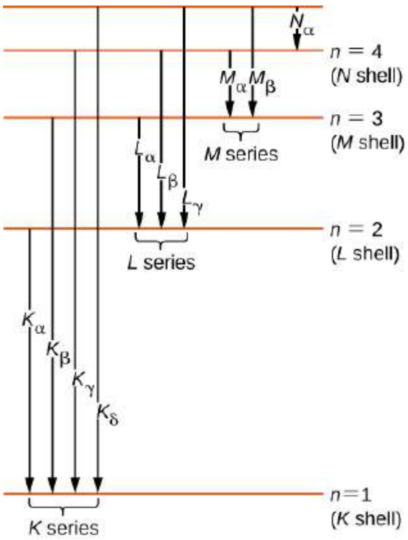
Another vacancy is now created in the L shell which is again filled up by another electron jump from one of the upper shell M which results in the emission of another photon, but of different X-ray frequency. This transition continues till outer shells are reached, thus, resulting in the continues till outer shells are reached, thus, resulting in the emission of a series of spectral lines. The transitions of electrons from various outer shells to the innermost K shell produce a group of X-ray lines called as K -series. These radiations are most energetic and most penetrating. K-series is further divided into $K_\alpha, K_\beta, K_\gamma, \cdots$ depending upon the outer shell from which the transition is made (see figure).

Incident electron is also known as a **projectile electron**

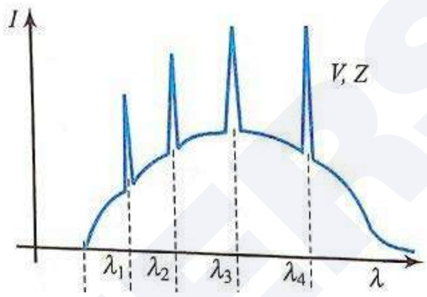
Emitted electron is known as **photo-electron / orbital electron**



Similarly, the rest of the series can be shown as below -



Now notice the graph shown below and the sharp peaks obtained in the graph are known as characteristic X-rays because they are characteristic of the target material. The characteristic wavelengths of the material having atomic number Z are called characteristic X-rays and the spectrum obtained is called a characteristic spectrum. If a target material of atomic number Z' is used, then peaks are shifted.



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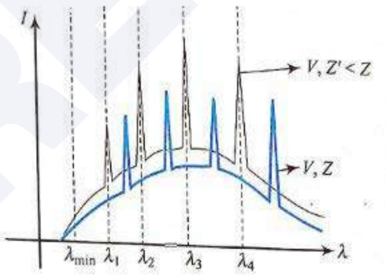
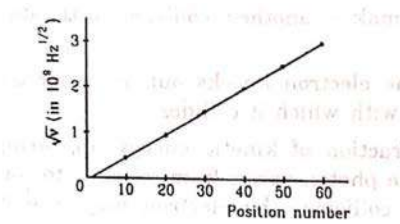


X-ray absorption-

The intensity of X-rays at any point may be defined as the energy falling per second per unit area held perpendicular to the direction of energy flow. Let I_0 be the intensity of incident beam and I be the intensity of beam after penetrating a thickness x of a material, then $I = I_0 e^{-\mu x}$, where μ is the coefficient of absorption or absorption coefficient of the material. The absorption coefficient depends upon the wavelength of X-rays, the density of the material, and the atomic number of material. The elements of high atomic mass and high density absorb X-rays to a higher degree.

11.Moseley's Law-

During the time when the periodic table is arranged with atomic weight, Moseley measured the frequency of characteristic X-rays from a large number of elements and plotted the square root of the frequency against its position number in the periodic table. He discovered that the plot is very close to a straight line. A portion of Moseley's plot is shown in figure where $\sqrt{\nu}$ of K_α X-rays is plotted against the position number. From this linear relation, Moseley concluded that there must be a fundamental property of the atom which increases by regular steps as one moves from one element to the other. This quantity was later identified to be the number of protons in the nucleus and was referred to as the atomic number.

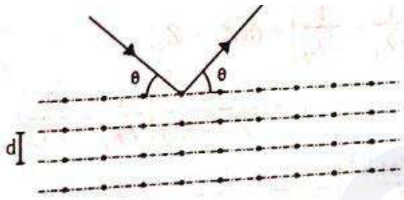


The frequency ν of a characteristic X-ray of an element is related to its atomic number Z by

$$\sqrt{\nu} = a(Z - b)$$

where a and b are constants called proportionality and screening (or shielding) constants. For K series, $a = \sqrt{\frac{3Rc}{4}}$ and that of b is 1. Here R is Rydberg's constant and c is speed of light (as in Bohr's model).

12. Bragg's law -



X-ray is used in measuring the interplanar spacing 'd' and several information about the structure of the solid can be obtained. This phenomenon can be understood by Bragg's law.

X-rays are diffracted by different atoms and the diffracted rays interfere. In certain directions, the interference is constructive and we obtain strong reflected X-rays. The analysis shows that there will be a strong reflected X-ray beam only if -

$$2dsin\theta = n\lambda$$

where n is an integer. For monochromatic X-rays, λ is fixed and there are some specific angles $\theta_1, \theta_2, \theta_3, \ldots$ etc. corresponding to $n = 1, 2, 3, \ldots$ etc. in equation given above. Thus, if the X-rays are incident at one of these angles, they are reflected; otherwise, they are absorbed.

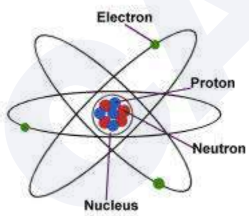
Nuclei

Important Formulae

1. Nucleus Structure

An atom is the basic unit of matter.

The atom consists of a central core called 'nucleus' and the electrons revolve around it in nearly circular orbits as shown in the below figure.



The nucleus of an atom consists of neutrons and protons, collectively referred to as nucleons. The neutron carries no electrical charge and has a mass slightly larger than that of a proton.

Constituents of the nucleus (Nucleons)-

(a) Protons:-
Mass of proton, $m_p = 1.6726 \times 10^{-27}$ kg
Charge of proton $= 1.602 \times 10^{-19}$ C
(b) Neutron:-
Mass of neutron, $m_n = 1.6749 \times 10^{-27}$ kg

The proton is the main part of an atom and carries a positive charge. The number of protons and neutrons is usually the same except in the case of the hydrogen atom which contains a single proton that exists on its own.

The number of protons in a nucleus (called the atomic number or proton number) is represented by the symbol Z.

The number of neutrons (neutron number) is represented by N.

The total number of neutrons and protons in a nucleus is called its mass number and it is represented by N.

And we have A= Z+ N.

Size of the nucleus-

- Nuclear radius - The radius r of the nucleus depends upon the atomic mass A of the element as

$R = R_0 . A^{1/3}$ where R_0=Constant =1.2 fm and A=Mass number of the nucleus

- Nuclear volume: The volume of the nucleus is given by $V = \frac{4}{3}\pi r^3 = \frac{4}{3}R_0^3 A$ i.e $V \ \alpha \ A$
- Nuclear density: Mass per unit volume of a nucleus is called nuclear density. And it is given as

$$\rho = \frac{M}{V}$$

$$\rho = \frac{(A).m_p}{\frac{4\pi}{3}R^3}$$

$$\rho = \frac{A.m_p}{\frac{4\pi}{3}.R_0^3.A} = \frac{3m_p}{4\pi R_0^3}$$

Density is constant for all the nuclei. It is independent of size and mass numbers.

2. Binding Energy Per Nucleon

Energy mass equivalence-

Einstein showed from his theory of special relativity that it is necessary to treat mass as another form of energy.

Einstein showed that mass is another form of energy and one can convert mass into other forms of energy, say kinetic energy and vice-versa.

For this Einstein gave the famous mass-energy equivalence relation

$E = mc^2$ where c is the velocity of light in vacuum and $c = 3 \times 10^8 \ m/s$

or we can say $\Delta E = \Delta mc^2$

where Δm = mass defect and ΔE = energy released

Note:

- For mass defect equal to 1 amu, the energy released is $\Delta E = (\Delta m)c^2 = (1\text{amu}) \times (3 \times 10^8)^2 = 931.5\text{MeV}$
- A small amount of mass corresponds to a large amount of energy. Energy associated with the rest mass of an object is said to be its **rest mass energy**.

Rest mass of an electron (m_e)	9.1×10^{-31} kg
	5.485×10^{-4} amu
Rest mass of a proton (m_p)	1.6726×10^{-27} kg
	1.00727 amu
	1836.15 m_e
Rest mass of a neutron (m_n)	1.6749×10^{-27} kg
	1.0086 amu
Energy equivalence of rest mass of an electron	0.51 MeV

Energy equivalence of rest mass of a proton	938.27 MeV
Energy equivalence of rest mass of a neutron	939.56 MeV

It is very useful for calculating energy emitted in the nuclear process.

Mass defect-

It is found that the mass of a nucleus is always less than the sum of the masses of its constituent nucleons in a free state.

This difference in masses is called the mass defect.

Hence mass defect is given as

Δm= Sum of masses of nucleons– Mass of the nucleus

$$\Delta m=[Z m_p+(A-Z) m_n]-M$$

where

$m_p=$ Mass of proton, $m_n=$ Mass of each neutron,
$M=$ Mass of nucleus, $Z=$ Atomic number, $A=$ Mass number,

Note- The mass of a typical nucleus is about 1% less than the sum of masses of nucleons.

Packing fraction -

Mass defect per atomic mass number is called packing fraction.

The packing fraction measures the stability of a nucleus. The smaller the value of the packing fraction, the larger is the stability of the nucleus.

$$\text{Packing fraction }(f)=\frac{\Delta m}{A}=\frac{M-(Z m_p+(A-Z) m_n)}{A}$$

$m_p=$ Mass of proton, $m_n=$ Mass of each neutron,
$M=$ Mass of nucleus, $Z=$ Atomic number, $A=$ Mass number,

- Packing Fraction can have positive, negative, or zero values.
- The zero value of the packing fraction is found in monoisotopic elements where the isotopic mass is equal to the mass number. For ${}_8O^{16}$, $f \rightarrow zero$
- The negative value of the packing fraction indicates that there is a mass defect, hence binding energy. Such nuclei are stable.
- Positive values of the Packing fraction are unstable when undergoing fission and fusion processes.

Nuclear binding energy (B.E)-

The neutrons and protons in a stable nucleus are held together by nuclear forces and energy is needed to pull them infinitely apart. This energy is called the binding energy of the nucleus.

OR

The amount of energy released when nucleons come together to form a nucleus is called the binding energy of the nucleus.

OR

The binding energy of a nucleus may be defined as the energy equivalent to the mass defect of the nucleus.

If Δm is a mass defect then according to Einstein's mass-energy relation

then $$\begin{aligned} & Binding\ energy=\Delta m \times c^2=[\{m_p Z+m_n(A-Z)\}-M] \times c^2\ J \\ & Binding\ energy=\Delta m \times 931.5\ MeV \end{aligned}$$

Binding Energy Per Nucleon-

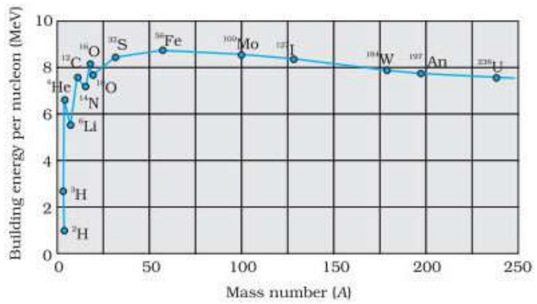
A more useful measure of the binding between protons and neutrons is the binding energy per nucleon or E_{bn}. It is the ratio of the binding energy of a nucleus to the number of nucleons in the nucleus:

$$\mathrm{E_{bn}}=\frac{\mathrm{E_b}}{\mathrm{A}} \quad \text{or} \quad \mathrm{E_{bn}}=\frac{\Delta \mathrm{Mc}^2}{\mathrm{A}}$$

where, A = Number of Nucleons.

We can define binding energy per nucleon theoretically as the average energy per nucleon needed to separate a nucleus into its individual nucleons.

Let’s look at a plot of the binding energy per nucleon versus the mass number for a large number of nuclei:



The following main features of the plot are:

1. The maximum binding energy per nucleon is around 8.75 MeV for mass number (A) = 56.
2. The minimum binding energy per nucleon is around 7.6 MeV for mass number (A) = 238.
3. The binding energy per nucleon, E_{bn}, is practically constant, i.e. practically independent of the atomic number for nuclei of middle mass number (30 < A < 170).
4. E_{bn} is lower for both light nuclei (A<30) and heavy nuclei (A>170).

We can draw some conclusions from these four observations:

Conclusion 1

The force is attractive and sufficiently strong to produce a binding energy of a few MeV per nucleon.

Conclusion 2

- E_{bn} is nearly constant in the range 30 < A < 170 because the nuclear force is short-ranged. Consider a particular nucleon inside a sufficiently large nucleus. It will be under the influence of only some of its neighbours, which come within the range of the nuclear force.
- This means that all nucleons beyond the range of the nuclear force form N_A will have no influence on the binding energy of N_A. So, we can conclude that if a nucleon has ‘p’ neighbours within the range of the nuclear force, then its binding energy is proportional to ‘p’.
- If we increase A by adding nucleons they will not change the binding energy of a nucleon inside. Since most of the nucleons in a large nucleus reside inside it and not on the surface, the change in binding energy per nucleon would be small.
- The binding energy per nucleon is a constant and is equal to pk, where k is a constant having the dimensions of energy. Also, the property that a given nucleon influences only nucleons close to it is also referred to as the saturation property of the nuclear force.

Conclusion 3

A very heavy nucleus, say A = 240, has lower binding energy per nucleon compared to that of a nucleus with A = 120. Thus if a nucleus A = 240 breaks into two A = 120 nuclei, nucleons get more tightly bound. Also, in the process energy is released. This concept is used in Nuclear Fission.

Conclusion 4

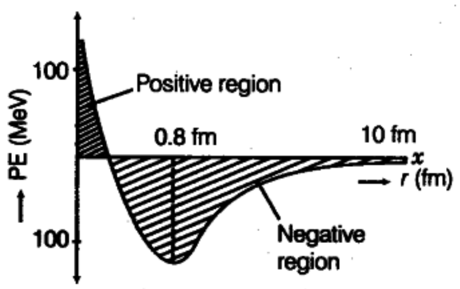
Now consider two very light nuclei with A < 10. If these two nuclei were to join to form a heavier nucleus, then the binding energy per nucleon of the fused and heavier nucleus is more than the E_{bn} of the lighter nuclei. So, the nucleons are more tightly bound post-fusion. Again energy would be released in such a process of fusion. This is the energy source of sun,

Nuclear Force-

Coulomb force is a force that determines the motion of atomic electrons. In the average mass nuclei, the binding energy per nucleon is approximately 8 MeV, This is much larger than the binding energy in atoms. Hence, the nuclear force required to bind a nucleus together must be very strong and of a different type. It must be strong enough to overcome the repulsion between the (positively charged) protons and to bind both protons and neutrons into the tiny nuclear volume.

Le's look at the features of this force also called the nuclear binding force which is obtained from many experiments which were performed between 1930 and 1950.

1. The nuclear force is much stronger than the Coulomb force acting between charges or the gravitational forces between masses. This is because the nuclear force needs to overpower the Coulomb repulsive force between the like-charged protons inside the nucleus. Hence, the nuclear force > the Coulomb force. Also, the gravitational force is much weaker than the Coulomb force.
2. The nuclear force between two nucleons falls rapidly to zero as their distance is more than a few femtometres. This leads to a saturation of forces in a medium or large-sized nucleus, which is the reason for the constancy of the binding energy per nucleon. Also, if the distance falls below 0.7fm, then this force becomes repulsive. A rough plot of the potential energy between two nucleons as a function of distance is shown below.



The potential energy of two nucleons is a function of the distance between them.

$$\text{If distance } > r_\alpha, \text{ then nuclear force } = \text{ attractive}$$
$$\text{If distance } < r_\alpha \text{ the nuclear force } = \text{ repulsive}$$

3. The nuclear force between neutron-neutron, proton-neutron and proton-proton is approximately the same. The nuclear force does not depend on the electric charge.

Nuclear Stability -

Nuclear Stability is a concept that helps to identify the stability of an isotope. The two main factors that determine nuclear stability are the neutron/proton ratio (neutron to proton ratio.) and the total number of nucleons in the nucleus.

I. Neutron/proton ratio-

Stable nuclei with atomic numbers up to about 20 have an n/p ratio of about 1/1.

Above Z = 20, the number of neutrons always exceeds the number of protons in stable isotopes. The stable nuclei are located in the pink band known as the belt of stability. The belt of stability ends at lead-208.

II. NUMBER OF NUCLEONS-

No nucleus higher than lead-208 is stable. That's because, although the nuclear strong force is about 100 times as strong as the electrostatic repulsions, it operates over only very short distances. When a nucleus reaches a certain size, the strong force is no longer able to hold the nucleus together.

3. Radioactive Decay-

Radioactivity(I)

A. H. Becquerel, discovered radioactivity purely by accident.The radioactivity is a nuclear phenomenon in which an unstable nucleus undergoes a decay.

Three types of radioactive decay occur in nature :

(i) α-decay in which a helium nucleus ${}_4^2\mathrm{He}$ is emitted;

(ii) β-decay in which electrons or positrons (positrons is the particles with the same mass as electrons but with a charge exactly opposite to that of the electron) are emitted;

(iii) γ-decay in which high frequency and energy (hundreds of keV or more) photons are emitted. Each of these decay will be considered in subsequent sub-sections.

1.Alpha Radioactive Decay-

An alpha particle is a helium nucleus, which means that it can be represented as ${}^4{}_2\mathrm{He}$. So, whenever a nucleus goes through alpha decay, it gets transformed into a different nucleus by emitting an alpha particle.

Let us take an example when ${}^{238}{}_{92}\mathrm{U}$ undergoes alpha-decay, it gets transformed into ${}^{234}{}_{90}\mathrm{Th}$, which is shown as -

$${}_{92}^{238}\mathrm{U} \rightarrow {}_{90}^{234}\mathrm{Th} + {}_2^4\mathrm{He}$$

Now we have seen that ${}^4{}_2\mathrm{He}$ contains two protons and two neutrons. Hence when the alpha particle gets emitted, the mass number of the emitting nucleus reduces by four and similarly the atomic number reduces by two. Therefore, in general, we can write that ${}_Z^A\mathrm{X}$ nucleus to ${}_{Z-2}^{A-4}\mathrm{X}$ nucleus is expressed as follows,

$${}_Z^A\mathrm{X} \rightarrow {}_{Z-2}^{A-4}\mathrm{X} + {}_2^4\mathrm{He}$$

where ${}_Z^A\mathrm{X}$ is the parent nucleus and ${}_{Z-2}^{A-4}\mathrm{X}$ is the daughter nucleus. One very important point to be noted is that the alpha decay of ${}^{238}{}_{92}\mathrm{U}$ can occur without an external source of energy. The reason behind this is that the total mass of the decay products i.e., (${}^{234}{}_{90}\mathrm{Th}$ and ${}^4{}_2\mathrm{He}$) < the mass of the original ${}^{238}{}_{92}\mathrm{U}$.

Or, we can say that the total mass-energy of the decay products is less than that of the original nuclide.

This gives rise to a new term called '**Q value** of the process' or '**Disintegration energy**' which is the difference between the initial and final mass energy of the decay products. So, for alpha decay, the Q value is expressed as -

$$\mathrm{Q} = (\mathrm{m_X} - \mathrm{m_Y} - \mathrm{m_{He}})\,\mathrm{c}^2$$

This disintegration energy is shared between the daughter nucleus, ${}_{Z-2}^{A-4}\mathrm{X}$ and the alpha particle both and for ${}^4{}_2\mathrm{He}$ it is in the form of kinetic energy.

Note-

Alpha decay obeys the radioactive laws.

Nuclear Reactions-

The process by which the identity of a nucleus is changed when it is bombarded by an energetic particle is called a nuclear reaction.

The general expression for the nuclear reaction is as follows.

$X + Y \rightarrow Z + Q$

where X and Y are known as reactants and Z is known as products.

and Q is the energy of the nuclear reaction (i.e. Q value)

Q value-

The energy absorbed or released during the nuclear reaction is known as the Q-value of the nuclear reaction.

$Q \text{ -value } = (Mass\ of\ reactants - mass\ of\ products)c^2\ Joules$

for the below reaction

$X + Y \rightarrow Z + Q$

$Q = (M_x + M_y - M_z)C^2$

where

M_x and M_y are mass of reactant

M_z is mass of product

- If Q < 0, The nuclear reaction is known as endothermic. (The energy is absorbed in the reaction)
- If Q> 0, The nuclear reaction is known as exothermic (The energy is released in the reaction)

Law of conservation in nuclear reactions-

The following quantities are conserved in nuclear reactions

- The mass number (A)
- The charge number (Z)
- Linear momentum/angular momentum
- Total energy

2.Beta radioactive decay:-

Beta decay happens when a nucleus decays spontaneously by emitting an electron or a positron. Like alpha decay, it is also a spontaneous process, and it is also having definite disintegration energy and half-life as well. It is also following the radioactive laws. A Beta decay can either be a beta minus or a beta plus decay.

In a Beta minus (β^-) **decay** , as the name indicates an electron is emitted by the nucleus.

Let us take an example,

$$^{32}_{15}\mathrm{P} \rightarrow ^{32}_{16}\mathrm{S} + \mathrm{e}^- + \mathrm{v}^-$$

where, v^- is an antineutrino,

Note- Property of neutrino

1. Zero electric charge
2. It's mass much less than the mass of the electron
3. Very weak matter making it quite difficult to detect

i.e For

- β Minus - decay

$^A_Z X \rightarrow ^A_{Z+1} Y + e^- + \nu^- + Q\ value$

$Q\ value = [M_X - M_Y]\, c^2$

In a Beta plus decay, as the name indicates that it is a positron is emitted by the nucleus. Let us take an example,

$$^{22}_{11}\mathrm{Na} \rightarrow ^{22}_{10}\mathrm{Ne} + \mathrm{e}^+ + \mathrm{v}$$

where v is a neutrino, which is a neutral particle with negligible or no mass. The neutrinos and antineutrinos are emitted from the nucleus along with the positron or electron during the beta decay process. Neutrinos interact very weakly with matter.

i.e For

- β plus decay

$${}_Z^AX \rightarrow {}_{Z-1}^{A}Y + e^+ + \nu + Q\ value$$

$$Q\ value = [M_X - M_Y - 2M_e]\,c^2$$

Further, in a Beta minus decay, a neutron transforms into a proton (inside the nucleus):

$$\mathrm{n} \rightarrow \mathrm{p} + \mathrm{e}^- + \mathrm{v}^-$$

Also, in a Beta plus decay, a proton transforms into a neutron within the nucleus:

$$p \rightarrow n + e^+ + v$$

Here, the point to be noted is that the mass number (A) of the emitting nuclide does not change. From the above equation, either a neutron transforms into a proton or vice versa.

3. Gamma Radioactive Decay-

We know that atoms have energy levels. Like the atom, a nucleus has energy levels too. When a nucleus is in an excited state then to make itself stable, transition occurs to a lower energy state by emitting an electromagnetic radiation. The energy difference between the states in a nucleus is in MeV. Hence, the photons emitted by the nuclei have MeV energies and called **Gamma rays**.

As we have seen after an alpha or beta emission, most radionuclides leave the daughter nucleus in an excited state. To reach the ground state, this daughter nucleus emits one or multiple gamma rays.

Let us take an example,

${}^{60}{}_{27}Co$ undergoes a beta decay and then transforms into ${}^{60}{}_{28}Ni$. Then this ${}^{60}{}_{28}Ni$ becomes the daughter nucleus. ${}^{60}{}_{28}Ni$ is in its excited state and this excited nucleus reaches the ground state by the emission of two gamma rays having energies of 1.17 MeV and 1.33 MeV.

4. Law Of Radioactivity Decay

Radioactivity-

The phenomenon by virtue of which a substance, spontaneously, disintegrates by emitting certain radiations is called radioactivity.

Activity (A)-

Activity is measured in terms of disintegration per second.

$$A = -\frac{dN}{dt}$$

Units of radioactivity:-

Its SI unit is 'Bq (Becquerel)'.

Curie (Ci):- Radioactivity of a substance is said to be one curie if its atoms disintegrate at the rate of 3.7×10^{10} disintegrations per second. I.e $1\mathrm{Ci} = 3.7\times10^{10}\mathrm{Bq} = 37\mathrm{GBq}$

Rutherford (Rd):- Radioactivity of a substance is said to be 1 Rutherford if its atoms disintegrate at the rate of 10^6 disintegrations per second.

The relation between Curie and Rutherford- 1 C = 3.7×10^4 Rd

Laws of radioactivity-

Radioactivity is due to the disintegration of a nucleus. The disintegration is accompanied by the emission of energy in terms of α, β and γ-rays either single or all at a time. The rate of disintegration is not affected by external conditions like temperature and pressure etc.

According to **Laws of radioactivity** the rate of the disintegration of the radioactive substance, at any instant, is directly proportional to the number of atoms present at that instant.

i.e $-\frac{dN}{dt} = \lambda N$

where λ= disintegration constant or radioactive decay constant

- Number of nuclei after the disintegration (N)

  $$N = N_0e^{-\lambda t}$$

where N_0 is the number of radioactive nuclei in the sample at t=0.

Similarly, Activity of a radioactive sample at time t

$$A = A_0e^{-\lambda t}$$

where A_0 is the Activity of a radioactive sample at time t =0

- **Half-life ($T_{1/2}$)-**

The half-life of a radioactive substance is defined as the time during which the number of atoms of the substance is reduced to half their original value.

$$T_{1/2} = \frac{0.693}{\lambda}$$

Thus, the half-life of a radioactive substance is inversely proportional to its radioactive decay constant.

- **Number of nuclei in terms of half-life-**

$$N = \frac{N_0}{2^{t/T_{1/2}}}$$

Note- It is a very useful formula to determine the number of nuclei after the disintegration in terms of half-life

- **Mean or Average life (** T_{mean}**)**

Definition: The arithmetic mean of the lives of all the atoms is known as the **mean life or average life** of the radioactive substance.

T_{mean} = sum of lives of all atoms / total number of atoms

Let |dN| be the number of nuclei decaying between t, t + dt; the modulus sign is required to ensure that it is positive.

$dN = -\lambda N_0 e^{-\lambda t} dt$

and $|dN| = \lambda N_0 e^{-\lambda t} dt$

$$T_{mean} = \frac{\int_0^\infty t\,|dN|}{\int_0^\infty |dN|} = \frac{\frac{1}{\lambda^2}}{\frac{1}{\lambda}} = \frac{1}{\lambda}$$

$$\Rightarrow T_{mean} = \frac{1}{\lambda}$$

The average life of a radioactive substance is equal to the reciprocal of its radioactive decay constant.

The average life of a radioactive substance is also defined as the time in which the number of nuclei reduces to $\left(\frac{1}{e}\right)$ part of the initial number of nuclei.

The relation between $T_{1/2}$ and T_{mean}:-

$\Rightarrow T_{1/2} = (0.693)\, T_{mean}$

OR

Half-life = (0.693)Mean life

5. Simultaneous And Series Disintegration

- **Simultaneous decay-**

Due to radioactive disintegration, a radio nuclide transforms into its daughter nucleus. Depending on the nuclear structure and its unstability, a parent nucleus may undergo either $\alpha-$ or $\beta-$ emission.

Sometimes a parent nucleus may undergo both types of emission simultaneously.

If an element decays to different daughter nuclei with different decay constants $\lambda_1,\ \lambda_2,\ \lambda_3, \ldots$ etc. for each decay mode, then the effective decay constant of the parent nuclei can be given as

$$\lambda_{eff} = \lambda_1 + \lambda_2 + \lambda_3, \ldots$$

Similarly, For a radioactive element with decay constant λ which decays by both α and β decays given that the probability for an α emission is P_1 and that for β emission is P_2 the decay constant of the element can be split for individual decay modes. Like in this case the decay constants for α and β decay separately can be given as

$$\lambda_\alpha = P_1\lambda$$
$$\lambda_\beta = P_2\lambda$$

- **Series decay-**

Accumulation of Radioactive element in Radioactive series-

A radioactive element decays into its daughter nuclei until a stable element appears. Consider a radioactive series-

$$A_1 \xrightarrow{\lambda_1} A_2 \xrightarrow{\lambda_2} A_3 \xrightarrow{\lambda_3} \ldots$$

A radioactive element A_1 disintegrates to form another radioactive element A_2 which in turn disintegrates to another element A_3 and so on. Such decays are called **Series or Successive Disintegration.**

Here, the rate of disintegration of A_1 = Rate of formation of A_2

$$\frac{-dN_{A1}}{dt} = \frac{dN_{A2}}{dt} = \lambda_1 N_{A1}$$

$$\frac{-dN_{A2}}{dt} = \frac{dN_{A3}}{dt} = \lambda_2 N_{A2}$$

$$\frac{dN_{A1}}{dt} = -\lambda_1 N_{A1}$$

$$\frac{dN_{A2}}{dt} = -\lambda_2 N_{A2}$$

Therefore, net formation of A_2 = Rate of disintegration of A_1 - Rate of disintegration of A_2

= $\lambda_1 N_{A1} - \lambda_2 N_{A2}$

If the rate of disintegration of A_1 becomes equal to the Rate of disintegration of A_2, then it is called Radioactive equilibrium. So the equation becomes -

$$\Rightarrow \frac{\lambda_1}{\lambda_2} = \frac{N_{A2}}{N_{A1}} = \frac{T_{avg2}}{T_{avg1}} = \frac{(T_{\frac{1}{2}})_2}{(T_{\frac{1}{2}})_1}$$

6. Nuclear Fission and Nuclear Fusion

Nuclear fission-

The process of splitting of a heavy nucleus into two lighter nuclei of comparable masses (after bombardment with energetic particles) with liberation of energy.

For example, on bombarding a uranium target, the nucleus broke into two nearly equal fragments and released a great amount of energy -

$${}_0^1\text{n} + {}_{92}^{235}\text{U} \rightarrow {}_{92}^{236}\text{U} \rightarrow {}_{56}^{144}\text{Ba} + {}_{36}^{89}\text{Kr} + 3{}_0^1\text{n}$$

Fission does not always produce Barium and Krypton. Here is another example:

$${}_0^1\text{n} + {}_{92}^{235}\text{U} \rightarrow {}_{92}^{236}\text{U} \rightarrow {}_{51}^{133}\text{Sb} + {}_{41}^{99}\text{Nb} + 4{}_0^1\text{n}$$

All the fragmented nuclei produced in fission are neutron-rich and unstable. Also, they are radioactive and emit beta particles until they reach a stable end-product. So under favourable conditions, the neutron produced can cause further fission of other nuclei, producing a large number of neutrons, Thus a chain of nuclear fissions is established which continues until the whole Uranium is consumed.

Nuclear fusion-

In nuclear fusion, two (or) more than two lighter nuclei combine/fuse to form a larger nucleus. In this process, energy is released.

Some examples of nuclear fusion:

$${}_1^1\text{H} + {}_1^1\text{H} \rightarrow {}_1^2\text{H} + {}_{+1}^{0}\text{e} + \text{v} + 0.42\text{MeV}$$

Here two protons combine to form a deuteron and a positron releasing 0.42 MeV of energy.

$${}_1^2\text{H} + {}_1^2\text{H} \rightarrow {}_2^3\text{He} + \text{n} + 3.27\text{MeV}$$

Here two deuterons combine to form the light isotope of Helium releasing 3.27 MeV of energy.

$${}_1^2\text{H} + {}_1^2\text{H} \rightarrow {}_1^3\text{H} + {}_1^1\text{H} + 4.03\text{MeV}$$

In this case, two deuterons combine to form a triton and a proton releasing 4.03 MeV of energy.

Here mass of a single nucleus so formed is less than the sum of the mass of the parent nuclei. And this mass difference appears in the form of the release of energy.

The conditions required for Nuclear fusion -

For the fusion to occur, two nuclei must come close enough so that attractive short-range nuclear force is able to affect them. But since both are positively charged particles, they experience coulomb's repulsion force. Therefore they must have enough energy to overcome this repulsion. For this, a high pressure of 10^6 atm and a temperature of 10^9 K is required.

When the fusion is achieved by raising the temperature of the system, so that particles have enough kinetic energy to overcome the coulomb's repulsion force, it is called thermonuclear fusion.

Electronic devices

Important Formulae

1.Band Theory Of Solids

Band Theory of solids-

On the basis of conductivity-

On the basis of the relative values of electrical conductivity (σ) or resistivity, the solids are broadly classified as:

- **Conductor-**

The materials which easily allow the flow of electric current through them are called conductors. Metals such as copper, silver, iron, aluminum, etc. are good conductors of electricity.

They possess very low resistivity (or high conductivity).

$i.e\ \rho \sim 10^{-2}-10^{-8} \Omega m$
$\sigma \sim 10^{2}-10^{8} Sm^{-1}$

- **Insulator-**

The materials which do not allow the flow of electric current through them are called insulators. Insulators are also called as poor conductors of electricity. Rubber, wood, diamond, plastic are some examples of insulators.

They have high resistivity (or low conductivity).

$i.e\ \rho \sim 10^{11}-10^{19} \Omega \mathrm{m}$
$\sigma \sim 10^{-11}-10^{-19} \mathrm{Sm}^{-1}$

- **Semiconductors-**

They have resistivity or conductivity intermediate to metals and insulators.

$i.e\ \ \rho \sim 10^{-5}-10^{6} \Omega \mathrm{m}$
$\sigma \sim 10^{5}-10^{-6} \mathrm{Sm}^{-1}$

Note- At 0 K, it behaves like an insulator (Si, Ge)

Semiconductors are further divided as follows

(i) Elemental semiconductors: Si and Ge

(ii) Compound semiconductors: Examples are:

• Inorganic: CdS, GaAs, CdSe, InP, etc.

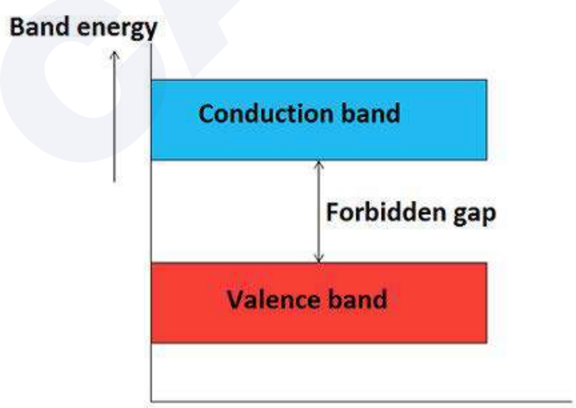
• Organic: anthracene, doped pthalocyanines, etc.

• Organic polymers: polypyrrole, polyaniline, polythiophene, etc

Materials are also classified on the basis of energy bands.

Energy Band Theory-

There is a number of energy bands in solids but three of them are very important which are shown in the below figure.



- Valence band-

The energy band which is formed by grouping the range of energy levels of the valence electrons or outermost orbit electrons is called a valence band. The valence band is present below the conduction band as shown in the above figure.

So Electrons in the valence band have lower energy than the electrons in the conduction band.

The electrons present in the valence band are loosely bound to the nucleus of an atom.

- Conduction band-

The energy band which is formed by grouping the range of energy levels of the free electrons is called a conduction band.

Generally, the conduction band is empty but when external energy has applied the electrons in the valence band then electrons jump into the conduction band and become free electrons.

Electrons in the conduction band have higher energy than the electrons in the valence band.

The conduction band electrons are not bound to the nucleus of the atom.

- Forbidden band or forbidden gap-

The energy band which is present between the valence band and conduction band by separating these two energy bands is called a forbidden band.

In solids, electrons cannot stay in a forbidden band because there is no allowed energy state in this region.

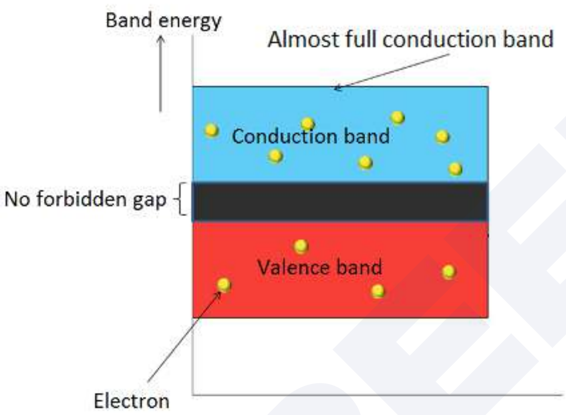
The energy associated with the forbidden band is called the energy gap and it is measured in unit electron volt (eV).

The applied external energy in the form of heat or light must be equal to the forbidden gap in order to push an electron from valence band to the conduction band.

Classification of materials based on a forbidden gap-

- **Conductors-**

In a conductor, the valence band and conduction band overlap each other as shown in the below figure. Therefore, there is no forbidden gap in a conductor.



A small amount of applied external energy provides enough energy for the valence band electrons to move into the conduction band.

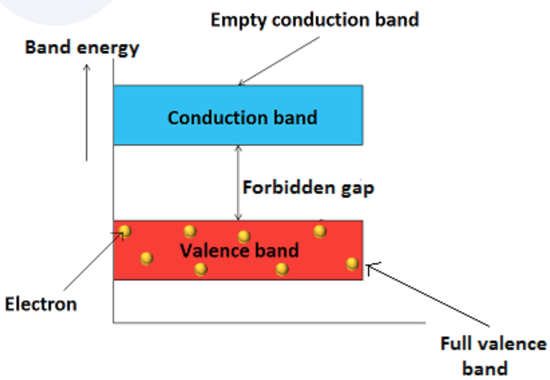
When valence band electrons move to conduction band they become free electrons.

In conductors, a large number of electrons are present in the conduction band at room temperature and these electrons move freely by carrying the electric current from one point to another.

- **Insulators**

The forbidden gap between the valence band and conduction band is very large in insulators as shown in the below figure.

Note- When the energy gap between the valence band and conduction band is more than 3 eV. then the material is insulators.

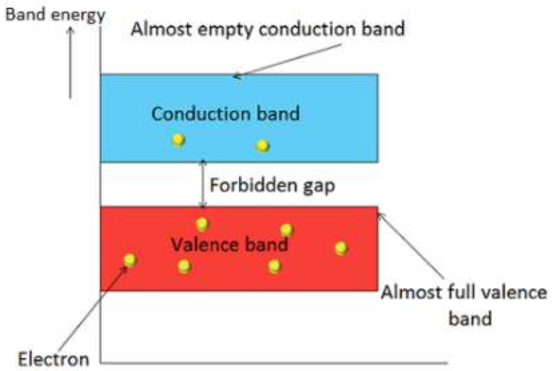


Normally, in insulators, the valence band is fully occupied with electrons due to the sharing of outer most orbit electrons with the neighboring atoms. While no electrons are present in the conduction band. Free electrons density is negligible in the case of the insulator. The electrons in the valence band cannot move by themself because they are locked up between the atoms.

- **Semiconductors**

In semiconductors, the forbidden gap between the valence band and conduction band is very small as shown in the below figure.

Note- When the Energy gap between the valence band and conduction band is less than 3 e.v. Then the material is a semiconductor.



At low temperature, the valence band is completely occupied with electrons and the conduction band is empty because the electrons in the valence band do not have enough energy to move into the conduction band. Therefore, semiconductor behaves as an insulator at low temperature.

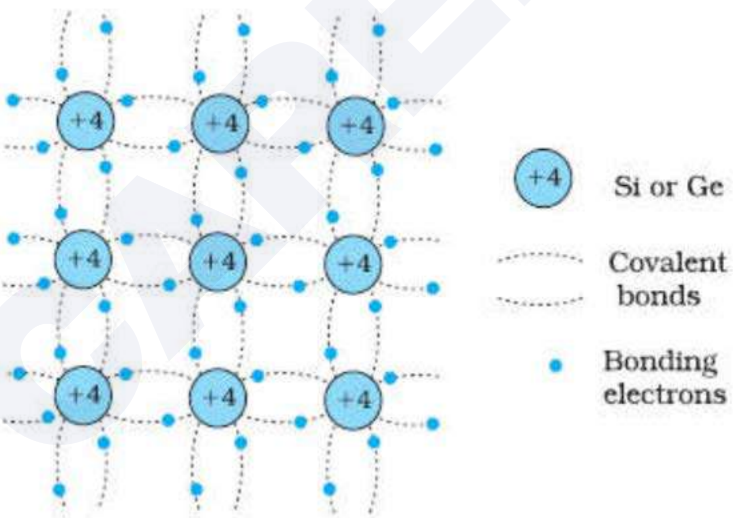
However, at room temperature, some of the electrons in valence band gains enough energy in the form of heat and moves into the conduction band. When the temperature increases, then the number of valence band electrons moving into the conduction band also increases. So free electron density in the conduction band increases. This shows that the electrical conductivity of the semiconductor increases with an increase in temperature.

2.Types Of Semiconductor: Intrinsic And Extrinsic Semiconductor

Intrinsic semiconductor-

It is a pure semiconductor. Silicon and germanium are the most common examples of intrinsic semiconductors. Both these semiconductors are most frequently used in the manufacturing of transistors, diodes and other electronic components.

Both Si and Ge have four valence electrons. In its crystalline structure, every Si or Ge atom tends to share one of its four valence electrons with each of its four nearest neighbour atoms, and also to take a share of one electron from each such neighbour as shown in the below figure. This shared pair of the electron is called a Covalent bond or a Valence bond.



The above figure shows the structure with all bonds intact (i.e no bonds are broken). This is possible only at low temperatures.

As the temperature increases, more thermal energy becomes available to these electrons and some of these electrons may break–away from the conduction band becoming the free electron and creating a vacancy in the bond. This vacancy with an effective positive electronic charge is called a hole.

In intrinsic semiconductors, the number of free electrons (n_e) is equal to the number of holes (n_h)

i.e $n_e = n_h = n_i$ where n_i is called intrinsic carrier concentration.

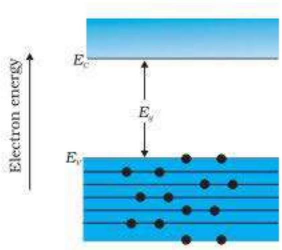
Semiconductors possess the unique property in which, apart from electrons, the holes also move.

The free-electron moves completely independently as a conduction electron and gives rise to an electron current, I_e under an applied electric field. while Under an electric field, these holes move towards the negative potential generating hole current (I_h).

Hence, the total current (I) is given as $I = I_e + I_h$

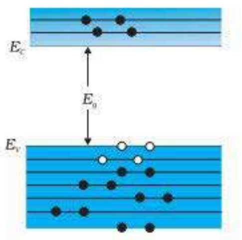
And apart from the process of generation of conduction electrons and holes, a simultaneous process of recombination occurs in which the electrons recombine with the holes. At equilibrium, the rate of generation is equal to the rate of recombination of charge carriers.

An intrinsic semiconductor will behave like an insulator at T = 0 K. As shown in the below figure., at T = 0 K, the electrons stay in the valence band and there is no movement to the conduction band.



An intrinsic semiconductor at $T = 0$ K behaves like insulator.

When the temperature increases, at T > 0K, some electrons get excited. These electrons jump from the valence to the conduction band as shown in the below figure.



The conductivity of an intrinsic semiconductor at room temperature is very low. As such, no important electronic devices can be developed using these semiconductors. Hence there is a necessity of improving their conductivity. This can be done by making use of impurities. because when a small amount of a suitable impurity is added to the pure semiconductor, the conductivity of the semiconductor is increased manifold

Extrinsic semiconductors-

An extrinsic semiconductor is a semiconductor doped by a specific impurity which is able to deeply modify its electrical properties, making it suitable for electronic applications. The deliberate addition of a desirable impurity is called doping and the impurity atoms are called dopants. Another term for Extrinsic semiconductors is 'Doped Semiconductor'.

The size of the dopant and Semiconductor atoms should be the same, for making sure that the amount of impurity added should not change the lattice structure of the Semiconductor.

Following types of dopants used in doping the tetravalent (valency 4) Si or Ge:

(i) Pentavalent (valency 5); like Arsenic (As), Antimony (Sb), Phosphorous (P), etc.

This will give n-type semiconductor

(ii) Trivalent (valency 3); like Indium (In), Boron (B), Aluminium (Al), etc.

This will give p-type semiconductor

N-type semiconductor-

When a pentavalent impurity is added to an intrinsic or pure semiconductor (silicon or germanium), then it is said to be an n-type semiconductor. Pentavalent impurities such as phosphorus, arsenic, antimony, etc are called donor impurity.

The four valence electrons of each phosphorus atom form 4 covalent bonds with the 4 neighboring silicon atoms.

The free-electron (fifth valence electron) of the phosphorus atom does not involve in the formation of covalent bonds.

This shows that each phosphorus atom donates one free electron. Therefore, all the pentavalent impurities are called donors.

So, there is a donor energy level between the valence band and conduction band. Just below the conduction band.

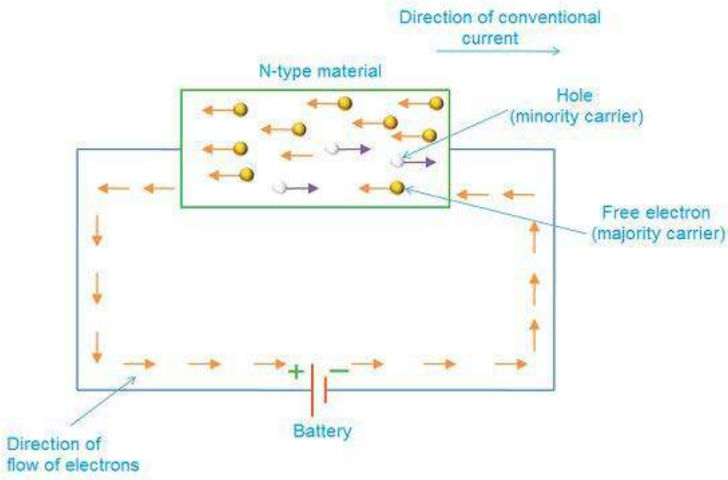
The number of free electrons depends on the amount of impurity (phosphorus) added to the silicon.

Charge on n-type semiconductor-

Even though n-type semiconductor has a large number of free electrons, but the total electric charge of n-type semiconductor is neutral.

Conduction in n-type semiconductor-

When voltage is applied to n-type semiconductors as shown in the below figure; then the free electrons move towards the positive terminal of the applied voltage. Similarly, holes move towards the negative terminal of the applied voltage.



In an n-type semiconductor, conduction is mainly because of the motion of free electrons.

because In an n-type semiconductor, the population of free electrons is more whereas the population of holes is less (i.e $n_e >> n_h$)

In an n-type semiconductor, free electrons are called majority carriers and holes are called minority carriers.

P-type semiconductor-

When the trivalent impurity is added to an intrinsic semiconductor (Si and Ge), then it is said to be a p-type semiconductor. Trivalent impurities such as Boron (B), Gallium (G), Indium(In), Aluminium(Al), etc are called acceptor impurity.

The three valence electrons of each boron atom form 3 covalent bonds with the 3 neighboring silicon atoms.

For the fourth covalent bond, only silicon atom contributes one valence electron. Thus, the fourth covalent bond is incomplete with the shortage of one electron. and This missing electron is called a hole.

This shows each boron atom accepts one electron to fill the hole. Therefore, all the trivalent impurities are called acceptors.

So there is an acceptor energy level just above the valence band.

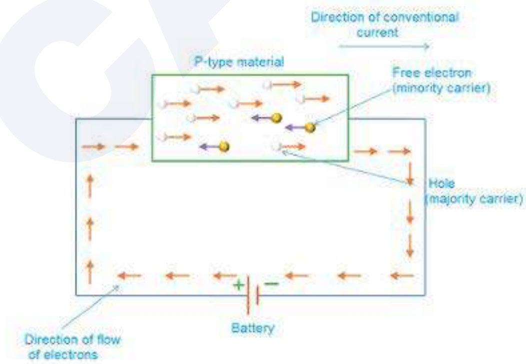
A small addition of impurity (boron) provides millions of holes.

Charge on the p-type semiconductor-

Even though p-type semiconductor has a large number of holes, but the total electric charge of p-type semiconductors is neutral.

Conduction in p-type semiconductor-

When voltage is applied to p-type semiconductor as shown in the below figure; then the free electrons move towards the positive terminal of the applied voltage. Similarly, holes move towards the negative terminal of the applied voltage.



In a p-type semiconductor, conduction is mainly because of the motion of holes in the valence band.

because In a p-type semiconductor, the population of free electrons is less whereas the population of holes is more (i.e $n_h >> n_e$)

In a p-type semiconductor, holes are called majority carriers and free electrons are called minority carriers.

- **Number of electrons or holes-**

The electron and hole concentration in a semiconductor in thermal equilibrium are related as:

$n_e \times n_h = n_i^2$

On the increasing temperature, the number of currents carriers increases.

And the relation is given as $n_e = n_h = AT^{\frac{3}{2}} e^{-\frac{E_g}{2KT}}$

where

E_g = Energy gap

K = Boltzmann Constant

T = Temperature in kelvin

3.Electric Conductivity

Electrical Conductivity (σ)-

The semiconductor conducts electricity with the help of these two types of electricity or charge carriers (i.e electrons and holes).

These holes and electrons move in the opposite direction. The electrons always tend to move in opposite direction to the applied electric field.

Let the mobility of the hole in the crystal is μ_h and the mobility of electron in the same crystal is μ_e

The current density due to drift of holes is given by,

$J_h = en_h v_h = en_h \mu_h E$

And The current density due to the drift of electrons is given by,

$J_e = \mathrm{en_e v}_e = \mathrm{en_e}\,\mu_e E$

hence resultant current density would be

$$J = J_h + J_e = en_h v_h + en_e v_e = en_h \mu_h E + en_e \mu_e E = (n_h \mu_h + n_e \mu_e)\, eE$$
$$and \quad J = \sigma E$$

So, the general equation for conductivity is given as

$\sigma = e\,(n_e \mu_e + n_h \mu_h)$

where

$n_e = electron\ density$

$n_h = hole\ density$

$\mu_e = mobility\ of\ electron$

$\mu_h = mobility\ of\ holes$

For intrinsic semiconductors (no impurities)-

As the number of electrons will be equal to the number of holes.

i.e $n_e = n_h = n_i$

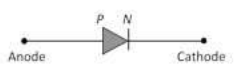
$\sigma = n_i e(\mu_e + \mu_h)$

4.P-N Junction

P-N Junction-

A p-n junction is the basic building block of many semiconductor devices like diodes, transistors,etc.

When a P-type semiconductor is suitably joined to an N-type semiconductor, then the resulting arrangement is called P-N junction or P-N junction diode.



Formation of a p-n Junction

Let’s imagine thin p-type silicon (p-Si) semiconductor wafer. Now, we have to add a pentavalent impurity to it. This result in a small part of the p-Si wafer converting into an n-Si. Due to this two-region will get created one is the wafer containing a p-region and another is an n-region with a metallurgical junction between the two. There are two important processes that take place during the formation of a p-n Junction:

1. Diffusion
2. Drift

As we have learned that in an n-type semiconductor, the concentration of electrons is more than that of holes similarly in a p-type semiconductor, the concentration of holes is more than that of electrons.

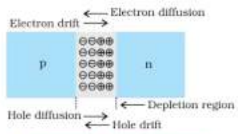
When a p-n junction is being formed, diffusion of holes starts from the p-side to the n-side (p→n) while diffusion of electrons occurs from the n-side to the p-side (n→p). The reason behind this diffusion is the concentration gradient across p and n sides. Due to this, a diffusion current generates across the junction. Let’s discuss both the scenarios one by one -

Electron diffusion from n→p -

Electron diffusion leaves a positive charge (ionized donor) on the n-side. This positive charge is bonded to the surrounding atoms and is not moveable. As diffusion is going on, more electrons start diffusing to the p-side, and a layer of positive charge on the n-side of the junction is formed.

Hole Diffuses from p→n

Hole diffusion leaves a negative charge on the p-side. As the diffusion proceeds, holes start diffusing to the n-side, a layer of negative charge on the p-side of the junction is formed. Both the phenomena i.e., diffusion of electrons and holes across the junction depletes the region of its free charges, these space charge regions together are called the depletion region.



p-n junction formation process.

This process is shown in the above figure. The thickness of the depletion region is very small and its thickness is around one-tenth of a micrometre. Since there is an electric field which is directed from the p-side to the n-side of the junction. Due to this electric field electrons moves from the p-side to the n-side and holes from the n-side to the p-side. This motion of charged carriers due to the electric field is called **drift**. From this we can conclude that the drift current direction is opposite to the direction to the diffusion current. This is also seen in the figure given above.

The Last Stages of Formation of a p-n Junction

When the diffusion starts, the diffusion current is large as compare to the drift current. As diffusion process continues, the space-charge regions on either side of the junction start extending. Due to this the electric field gets strengthen and same with the drift current. This process will continues till diffusion current = drift current. This is how a p-n junction is formed.

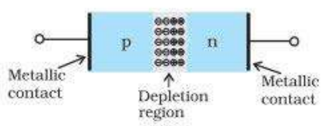
Barrier Potential

In the state of equilibrium, there will be no current in a p-n junction. Due to increase in potential difference across the junction of the two regions due to the loss of electrons by the n-region and the subsequent gain by the p-region. This potential opposes the further flow of carriers to maintain the state of equilibrium. This potential is called **Barrier potential.**

5.Semiconductor Diode - Forward Bias And Reverse Bias

Semiconductor diode-

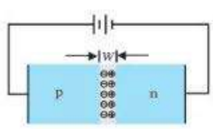
If a p-n junction has metallic contacts at both the ends for application of external voltage. This is called a **semiconductor diode**.



The symbolical representation of a semiconductor diode is shown below -

In the figure given above, the arrow indicates the direction of current when the diode is under forward bias. One should note here that the equilibrium barrier potential can be altered. This can be done by applying an external voltage across the diode. Depending on how this voltage is applied, the diode is a forward-bias or a reverse-bias diode.

1.P-N junction diode under forward bias-



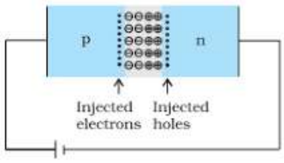
In the figure given above, we can see that an external voltage is applied across the semiconductor diode where the p-side of the diode is connected to the positive terminal and the n-side is connected to the negative terminal of the battery. This type of arrangement for the diode is forward biased.

Formation of a Forward Bias Diode-

As the depletion region have no charge so the resistance is very high there so the applied voltage drops primarily across this region. The drop in voltage across the p and n side of the junction is relatively negligible. And the direction of the applied voltage (V) being opposite to that of the built-in potential (V_o) due to which the depletion layer's width decreases and the barrier height reduce.

If the applied voltage is small, then the barrier potential is reduced marginally only below the equilibrium value. Then only small number of carriers crossing the junction, so the current is small. Similarly for a significantly high value of voltage, more carriers have the energy to cross the junction so, the current will be high.

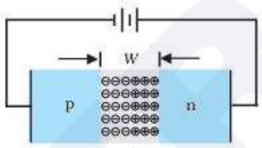
One should also note that when the voltage is applied, some electrons cross to the p-side and some holes cross to the n-side. Under forward bias, this process is the minority charge injection process. Hence, the minority charge concentration which is electrons on the p-side are a minority and holes on the n-side are a minority, is significantly higher at the junction boundary.



Due to this concentration gradient, the injected electron diffuse from the junction-end to the far-end of the p-side. Similarly, injected holes diffuse to the far end of the n-side. This gives rise to current too.

The total diode forward current = Hole diffusion current + Electron diffusion current (mA)

2. P-N junction diode under reverse bias-



In the figure given above, we can see that an external voltage is applied across the diode. We can see that the n-side of the diode connects to the positive terminal and the p-side connects to the negative terminal of the battery. This type of arrangement in diode is a reverse-bias diode.

Formation of a Reverse Bias Diode

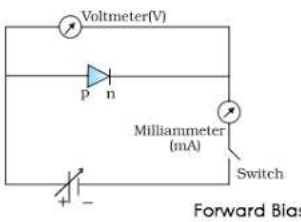
As the depletion region have no charges, so the resistance will be very high, as a result the applied voltage drops primarily across this region. Also the drop in voltage across the p and n side of the junction is relatively negligible. Now here the direction of the applied voltage (V) being the same as that of the built-in potential (V_0) (Opposite to the forward bias), because of this the depletion layer's width widens and the barrier height also increases. This decreases the flow of electron to the p-side and holes to the n-side. So, the diffusion current decreases to a great extent.

Because of the direction of the electric field, the electrons in the p-side and holes in the n-side are drive to their majority zones, if they come close to the junction. This will produces the drift current. The drift current is usually of a few microAmprere. This current is very low even in the forward-biased diode as compared to the current due to the injected carriers.

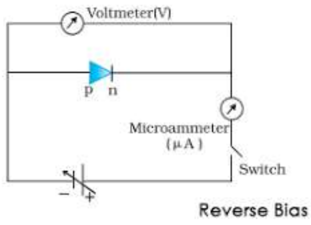
Critical Value of Reverse Bias Voltage -

A small amount of voltage applied to the diode is sufficient to sweep the minority charge carrier to the far side of the junction. This diode reverses current which is not dependent on the voltage but on the concentration of the minority charge carriers on both sides of the junction. However, the current is independent up to a critical value of reverse bias voltage which is the Breakdown Voltage (V_{br}). When the voltage applied crosses breakdown voltage i.e., V_{br}, even a small change in the bias voltage causes a huge change in current. There is an upper limit of current for every diode, beyond which the diode gets destroyed due to overheating. This is the rated value of current.

Experimental Study of the V-I characteristics of a Semiconductor Diode



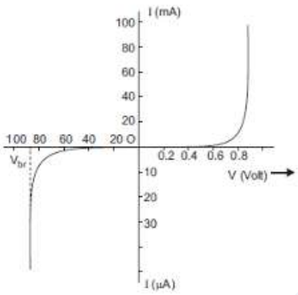
Forward Bias



V-I Characteristics of Diode

The figure shows a diode connected in reverse bias. The battery connects to the diode through a potentiometer by which we can change the voltage for the sake of the experiment. A microammeter is also used (since the expected current is in milliAmpere) measures the current.

Here is the result of the experiment



As we can see in the graph above, in the forward biased diode, initially when the current increases almost negligibly till a certain value is reached. After that, the current increases exponentially even for a small increase in diode bias voltage. This voltage is called as **threshold voltage**. (Its value is approximately ~0.7 V for silicon diode and ~ 0.2 V for germanium diode)

In the reverse biased diode, the current is very small and almost remains constant with a change in bias voltage. It is called as Reverse saturation current. It is observed that in some cases, beyond the breakdown voltage, the current increases suddenly.

Hence, from this experiment and the given graph, we can conclude that the p-n junction diode allows the flow of current only in one direction, i.e. forward-bias, which means that the forward bias resistance is lower than the reverse bias resistance.

Extra edge -

1. **P -N junction as diode**

R = 0, Forward

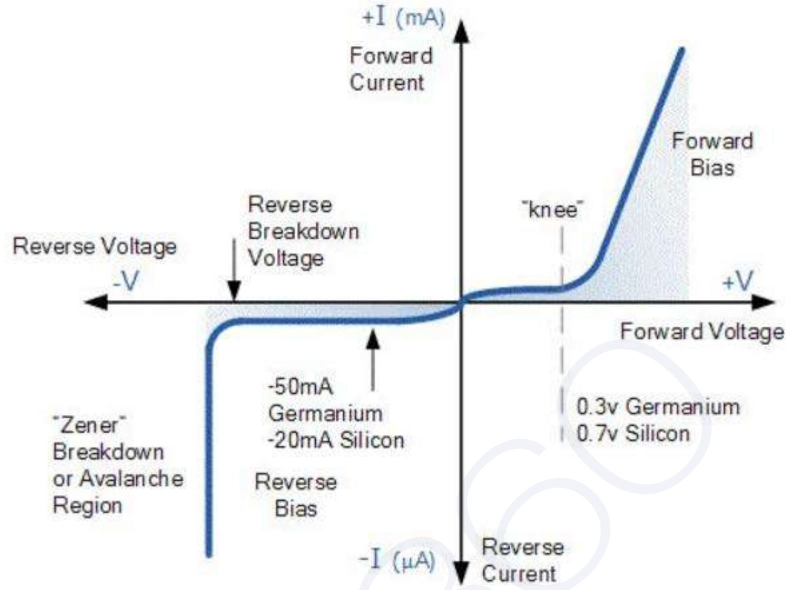
R $\rightarrow \infty$ Reverse

It is a one way device. It offers a low resistance when forward biased and high resistance when reverse biased.

2. **Dynamic Resistance**

Since slope of potential vs current graph is non uniform hence resistance keep changing .

$$R_d = \frac{dv}{di}$$



3. **Knee voltage of P-N junction**

Knee voltage for Ge is 0.3 V

Knee voltage for Si is 0.7 V

It is defined as that forward voltage at which the current through the junction starts rising rapidly with increase in voltage .

4. **Relation between current I and Voltage V**

K = Boltzmann constant

I_0 = reverse saturation current

In forward bias

$e^{\frac{ev}{KT}} >> 1$

Then forward biasing current is

$I = I_0 \cdot e^{\frac{ev}{KT}}$

$I = I_0 \left(e^{\frac{ev}{KT}} - 1\right)$

6.P-N Junction As A Rectifier

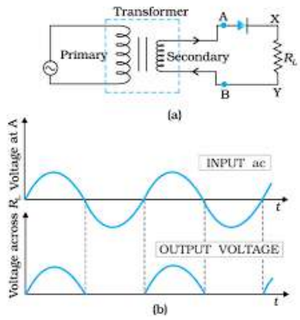
Application of junction diode as a rectifier-

A rectifier converts Alternating Current into Direct Current. Sometimes we have an AC power point but need to connect a device that requires a DC.

The Volt-Ampere characteristics of a junction diode give a reason for how current is passed through the diode only when it is forward biased. So, when an alternating voltage is applied across a junction diode, then the current will flow only in the part where it is forward biased. Due to this property junction diode is used to rectify alternating voltage/ current. The circuit used for this purpose is a Rectifier.

Based on the usage a junction diode can be used as a rectifier in two ways:

1.Half-wave Rectifier-

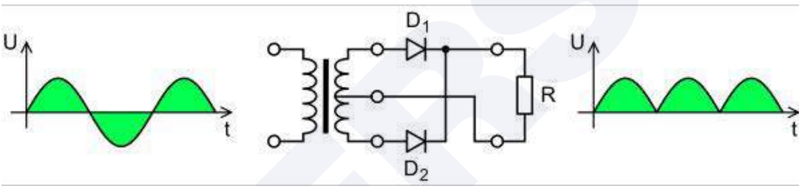


Look at the diagram given above. An alternating voltage is applied across a junction diode which is connected to a load in a series connection. In this case, only during those half cycles of the AC input when the diode is forward biased, a voltage will appear across the load. This type of circuit, which rectifies only one half of the input current is a **Half-wave Rectifier**.

The alternating current is supplied at points A and B. During the alternating cycle, when the voltage at point A is positive, the diode is forward biased. This will happen when the diode conducts. On the other hand, the diode is reverse biased when the voltage at point A is negative, and it doesn't conduct. Generally, for all practical purposes, the reverse saturation current can be considered zero since it is negligible.

Hence, we will get output voltage only through one half of the input cycle. Also, there will be no current available in the other half. Hence, the output still varies between positive to zero but the negative cycle is cut off and the output voltage is said to be rectified.

2.Full-wave Rectifier-



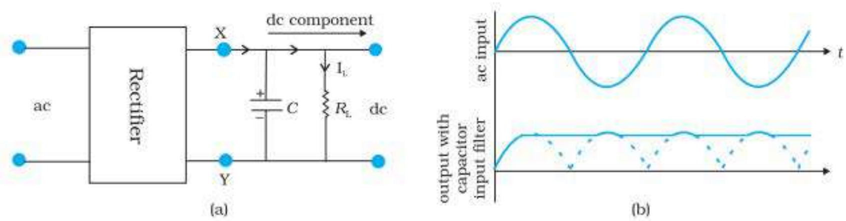
Look at the figure given above. In the circuit given above, two junction diodes are connected to a load. In this circuit both positive and negative halves of the AC cycle will come out. Hence, it is a Full-wave Rectifier. In this circuit, the p-sides of both the diodes are connected to the input while the n-sides are connected together and connected to the load. To complete the circuit load is connected to the mid-point of the transformer. Since, this mid-point of connection is also called Center tap and because of this the transformer is called Center tap transformer.

Here two diodes are connected, one diode rectifies the voltage for one half of the cycle while the other diode rectifies it for the other half. Therefore, the output between centre-tap of the transformer and their common terminals becomes a full-wave rectifier output. Let's see how this works-

If the voltage at point A is positive, then that at point B is negative. In this case, the diode D_1 is forward biased while D_2 is negatively biased. So, D_1 conducts while D_2 blocks the current. So, during the positive half of the input AC cycle, we will get output current. Afterwards, the voltage at point A becomes negative and that at point B becomes positive. In this case, D_2 conducts while D_1 blocks the current. So, we will get an output current in the negative half of the input AC cycle too. So this circuit rectifies both the halves of the input voltage, that's why it is called Full-wave Rectifier. But, one thing should be noted that the output is pulsating and not steady. So to derive a steady DC output there is need of capacitor across the output terminals (parallel to the load).

Role of a Capacitor

The role of the capacitor is to filters out the AC ripple and provides pure DC output. Let us discuss how it works:



We can see in the circuit given above, a capacitor is connected parallel to the load. The capacitor gets charged when the voltage across the capacitor rises and It discharges only when a load is connected to it and the voltage across it falls. As the AC cycle changes and the second diode kicks in, the capacitor charges again to its peak value and then again discharged due to the presence of the load.

One should note that the rate of discharge depends on the inverse product of Capacitor and Resistance (or load). To increase the discharge time and get a steady DC output, we should connect large capacitors. The idea is to obtain an output voltage close to the peak voltage of the rectified current.

7.Zener Diode

Zener diode-

It is invented by C. Zener.

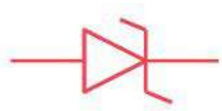
A Zener diode is a p-n junction semiconductor device designed to operate in the reverse breakdown region.

It is a highly doped p-n junction which is not damaged by high reverse current. It can operate continuously, without being damaged in the region of reverse background voltage. It forms a very thin depletion region and an extremely high electric field across the junction even for a small reverse bias voltage (~5 V).

In the forward bias, the Zener diode acts as an ordinary diode.

Symbol of Zener diode-

The symbol of the Zener diode is shown in the below figure.



VI characteristics of Zener diode-

Zener Breakdown-

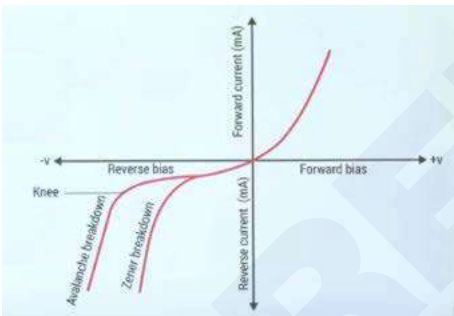
When a reverse bias is increased the electric field at the junction also increases. At some stage, the electric field becomes so high that it breaks the covalent bonds creating electron, hole pairs. Thus a large number of carriers are generated. This causes a large current to flow. This mechanism is known as Zener breakdown.

Avalanche breakdown-
At high reverse voltage, due to the high electric field, the minority charge carriers, while crossing the junction acquire very high velocities.

These by collision breaks down the covalent bonds, generating more carriers. A chain reaction is established, giving rise to high current.

This mechanism is known as **Avalanche** breakdown.

The VI characteristics of a Zener diode are shown in the below figure.



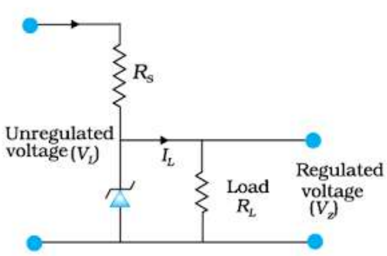
When forward-biased voltage is applied to the Zener diode, it works like a normal diode.

When reverse-biased voltage is applied to a Zener diode, it allows only a small amount of leakage current until the voltage is less than Zener voltage (V_z). As the reverse bias voltage (V) reaches the breakdown voltage of the Zener diode (Vz), there is a large change in current. Also, note that for a negligible change in the reverse bias voltage, a large change in current is produced.

Zener Diode as a Voltage Regulator-

A Zener diode is used to get constant DC voltage from a DC unregulated output of a rectifier.

The circuit diagram of a voltage regulator using a Zener diode is shown in the below figure.



Here the unregulated DC output of a rectifier is connected to Zener diode through a series of resistance (R_S) such that the Zener diode is reverse biased. Let's see how it works

If the input voltage increases, the current through R_S and Zener diode also increases. This increases the voltage drop across R_S.

But the voltage across the Zener diode does not change, because, in the breakdown region, Zener voltage remains constant despite the change in current.

Similarly, if the input voltage decreases, the current through R_S and Zener diode also decreases. This decreases the voltage drop across R_S . But the voltage across the Zener diode does not change.

Hence, a change of voltage drop across the R_S does not change the voltage across the Zener diode.

Hence, Zener diode acts as a voltage regulator.

8.Special Purpose P-N Junction Diodes

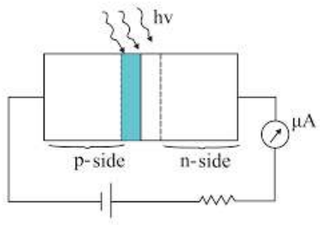
Devices in which carriers are generated by photons are called optoelectronic devices.

The following are important examples of optoelectronic devices.

1. Photodiodes to detect optical signals
2. Light Emitting Diodes (LEDs)
3. Solar cells

- **Photodiode-**

A photodiode is a p-n junction that consumes light energy to generate electric current. It is operated under reverse bias.



Reverse bias means that the p-side of the photodiode is connected to the negative terminal of the battery and the n-side is connected to the positive terminal of the battery.

It is also sometimes referred to as a photo-detector, photo-sensor, or light detector.

Suppose an optical photon of frequency ν is incident on a semiconductor, such that its energy is greater than the bandgap of the semiconductor (i.e.$h\nu > E_g$). This photon will excite an electron from the valence band to the conduction band leaving a vacancy or hole in the valence band. This increases the conductivity of the semiconductor.

And by measuring the change in the conductance (or resistance) of the semiconductor, one can measure the intensity of the optical signal. Thus photodiode can be used as a photodetector to detect optical signals.

- **Light-emitting diode (LED)-**

A LED is a heavily doped p-n junction which under forward bias emits spontaneous radiation.

(Note-When the diode is forward biased electrons move from n to p-side and holes move from p to n-side.)

LED is covered in a capsule with a transparent cover allowing the emitted light to come out.

LED'S are made of GaAsp, Gap, etc.

The V-I characteristics of a LED is similar to that of a Si junction diode. But the threshold voltages are much higher and slightly different for each color. The reverse breakdown voltages of LEDs are very low, typically around 5V.

The intensity of the emitted light increases as this input current increases and reached a maximum value. After this, an increase in forward bias input current leads to a decrease in light intensity.

LEDs that can emit red, yellow, orange, green and blue light are commercially available.

LEDs are used extensively in remote controls, optical communication, etc.

LEDs have the following important advantages over conventional incandescent low power lamps:

(i) Low operational voltage and less power.

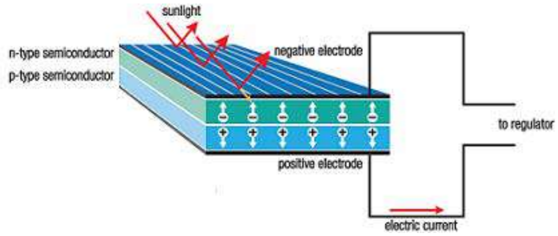
(ii) Fast on-off switching capability.

(iii) Long life.

- **Solar cell-**

A solar cell is a p-n junction that generates emf when solar radiation falls on the p-n junction. i.e It converts solar energy into electrical energy.

The working principle is similar to the photodiode except that no external bias is applied and the junction area is much larger to enable solar radiation incidence. One of the semiconductor regions is made so thin that the light incident on it reaches the P-N-junction and gets absorbed.



The important criteria for the selection of a material for solar cell fabrication are

(i) bandgap (~1.0 to 1.8 eV)- Semiconductors with band gap close to 1.5 eV are ideal materials for solar cell fabrication.

(ii) high optical absorption ($\sim 10^4\ cm^{-1}$),

(iii) electrical conductivity,

(iv) availability of the raw material, and

(v) cost

So Solar cells are made with semiconductors like Si , GaAs , CdTe , etc.

9.Bipolar Junction Transistor (N-P-N And P-N-P Transistor)

Bipolar Junction Transistor (BJT)-

it is a three-terminal electronic device that amplifies the flow of current. It has three doped regions (emitter, base, and collector) forming two p-n junctions between them.

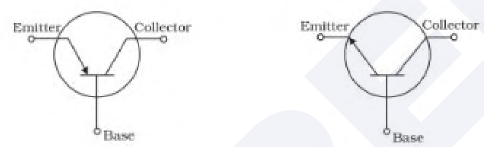
Based on their construction, Bipolar junction transistors are classified into two types as

1) N-P-N transistor

2) P-N-P transistor-

All three segments (emitter, base, and collector) of a transistor have different thickness and their doping levels are also different.

The schematic symbols of both these transistors are given in the below figure.



p-n-p transistor   n-p-n transistor

In the schematic symbols, as shown in the above figure, the arrowhead shows the direction of conventional current in the transistor.

The three segments of a transistor-

- Emitter- This segment is on one side of the transistor. It is of moderate size and heavily doped causing it to supply a large number of carriers for the flow of current.
- Base: This is the central segment. It is very thin and lightly doped.
- Collector: This segment collects a major portion of the majority carriers supplied by the emitter. The collector side is moderately doped and larger in size as compared to the emitter.

Working of a junction transistor-

In the case of a junction transistor, the depletion regions are formed at the emitter-base junction and the base-collector junction.

A junction transistor works as an amplifier when

1) The emitter-base junction is forward biased and

2) The base-collector junction is reverse biased.

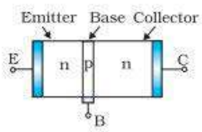
Some Terminologies-

- V_{EB}=The voltage between emitter and base
- V_{CB}=The voltage between collector and base
- V_{EE} = Power supply connected between the emitter and base.
- V_{CC} = Power supply connected between the collector and base

1. N-P-N transistor-

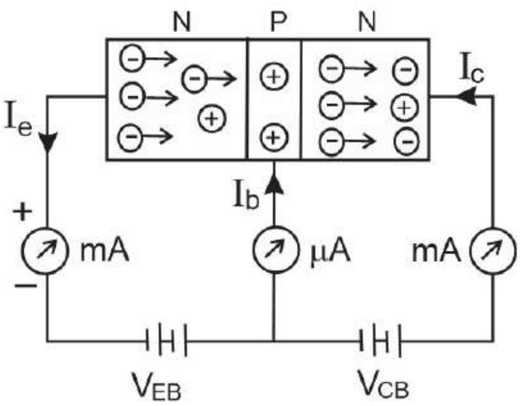
In this, a p-type semiconductor (base) separates two segments of the n-type semiconductor (emitter and collector) as shown in the below figure. In an NPN transistor electrons are the majority charge carriers and flow from emitter to base.

n-p-n transistor

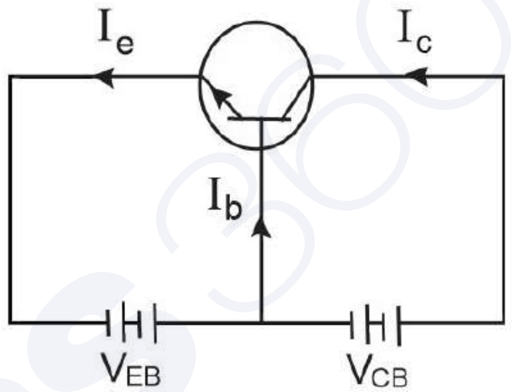


- **Working of an N-P-N Transistor -**

Circuit diagram of NPN transistor



OR

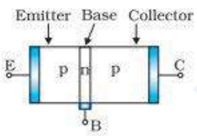


In an NPN transistor, 5% emitter electrons combine with the holes in the base region resulting in a small base current.
The remaining 95 % of electrons enter the collector region. i.e $I_e > I_c$

And according to Kirchhoff's law $I_e = I_b + I_c$

2.P-N-P Transistor-

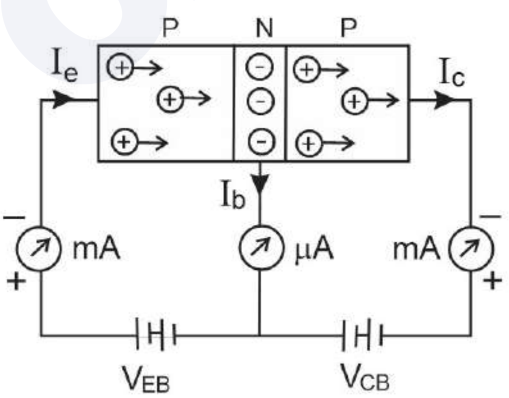
In this, an n-type semiconductor (base) separates two segments of the p-type semiconductor (emitter and collector).as shown in the below figure. In PNP transistor holes are majority charge carriers and flow from emitter to base.



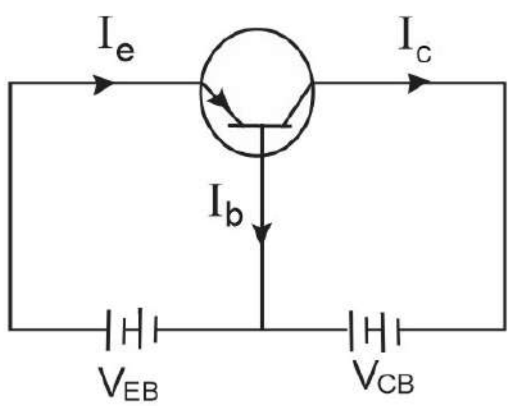
p-n-p transistor

- **Working of a P-N-P Transistor -**

Circuit diagram of PNP transistor



OR



In PNP transistor 5% emitter holes combine with the electrons in the base region resulting in a small base current. Remaining 95% holes enter the collector region. i.e $I_e > I_c$

And according to Kirchhoff's law $I_e = I_b + I_c$

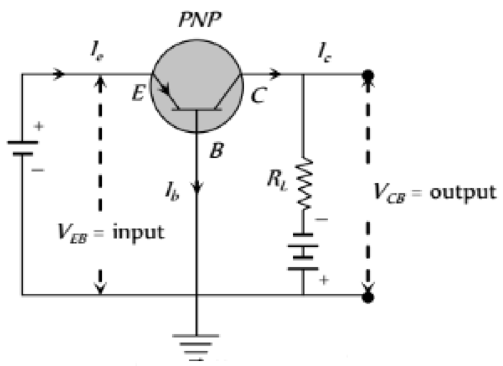
Basic transistor circuit configurations-

A transistor can be connected in a circuit in the following three different configurations.

Common base (CB), Common emitter (CE) and Common collector (CC) configuration.

(1) CB configurations-

In these configurations, Base is common to both emitter and collector as shown in the below figure.



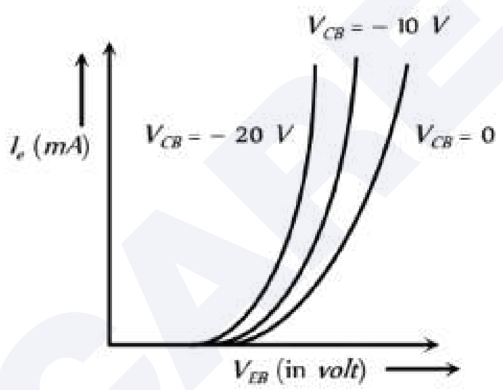
For the above figure Input current = I_e and Input voltage = V_{EB}

whereas Output voltage = V_{CB} and Output current = I_c

With a small increase in emitter-base voltage V_{EB}, the emitter current I_e increases rapidly due to small input resistance.

- **Input characteristics-**

If V_{CB}= constant, the curve between I_e and V_{EB} (as shown in the below figure) is known as input characteristics. It is also known as emitter characteristics.



And the Dynamic input resistance of a transistor is given by

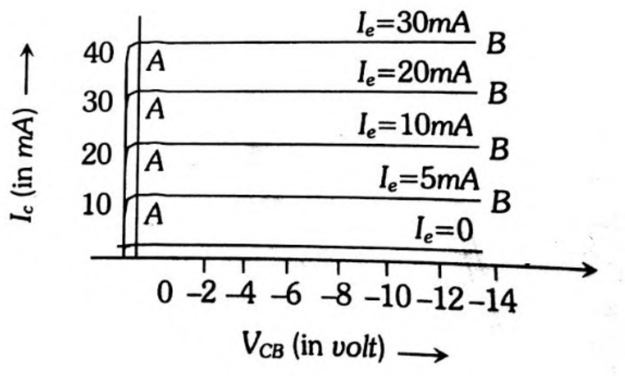
$$R_i = \left(\frac{\Delta V_{EB}}{\Delta I_e}\right)_{V_{CB}=\text{constant}}$$

and

R_i is of the order of 100 Ω.

Output characteristics-

Taking the emitter current i_e constant, the curve is drawn between I_C and V_{CB} (as shown in the below figure) are known as output characteristics of CB configuration.



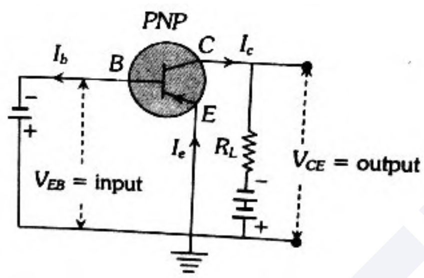
And the Dynamic output resistance of a transistor is given by

$$R_0 = \left(\frac{\Delta V_{CB}}{\Delta I_C}\right)_{i_e=\text{constant}}$$

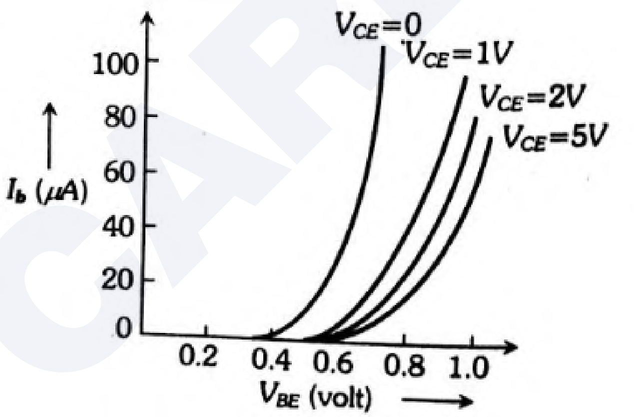
(2) CE configurations-

In these configurations, Emitter is common to both base and collector as shown in the below figure.

The graphs between voltages and currents when emitter of a transistor is common to input and output circuits are known as CE characteristics of a transistor.



Input characteristics: The input characteristic curve is drawn between base current I_b and emitter-base voltage V_{EB} , at constant collector-emitter voltage V_{CE}(as shown in the below figure).

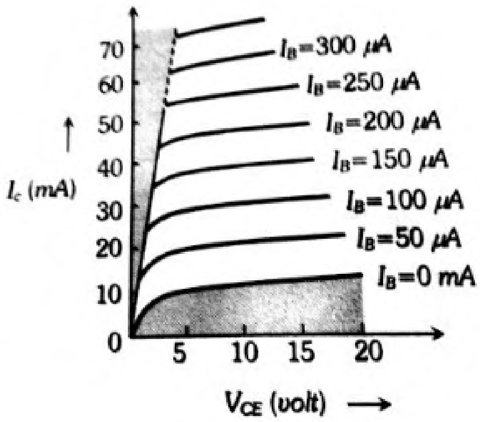


And the Dynamic input resistance of a transistor is given by

$$R_i = \left(\frac{\Delta V_{BE}}{\Delta I_B}\right)_{V_{CE}=\text{constant}}$$

Output characteristics-

Variation of collector current I_C with V_{CE} can be noticed for V_{CE} between 0 to 1 V only (as shown in the below figure).



The value of V_{CE} up to which the I_C changes with V_{CE} is called **knee voltage**.And The transistor is operated in the

the region above knee voltage.

And the Dynamic output resistance of a transistor is given by

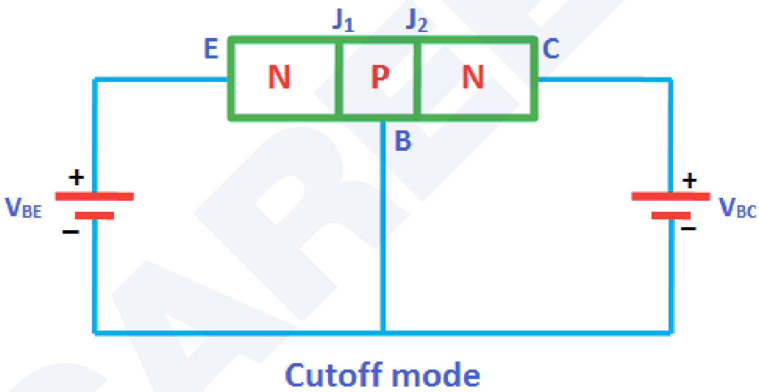
$$R_0=\left(\frac{\Delta V_{CE}}{\Delta I_C}\right)_{i_B=\text{constant}}$$

10.Transistor As A Device - Switch And Amplifier

- **BJT operation modes-**

1.Cutoff mode- In the cutoff mode, both the junctions of the transistor are reverse biased.

As in reverse bias condition, no current flows through the device. Hence, no current flows through the transistor. Therefore, the transistor is in off state and acts as an open switch. The cutoff mode of the transistor is used in switching operation for switch OFF application.

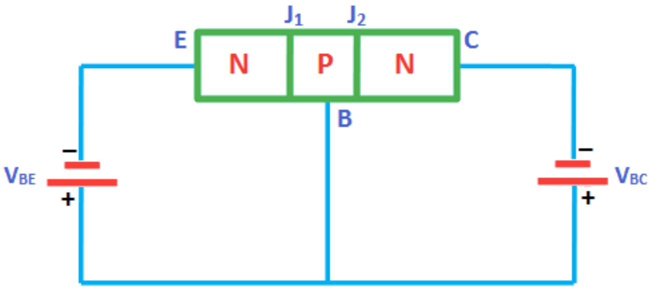


2. Saturation mode-In the saturation mode, both the junctions of the transistor are forward biased.

As in forward bias condition, current flows through the device. Hence, electric current flows through the transistor.

And in this mode Maximum collector current flows and the transistor acts as a closed switch.

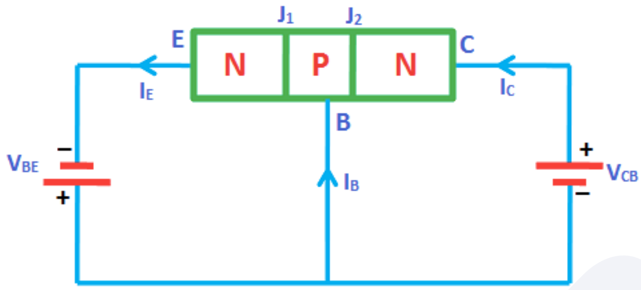
The saturation mode of the transistor is used in switching operation for switch ON application.



Saturation mode

So we can say that by operating the transistor in saturation and cutoff region, we can use the transistor as an ON/OFF switch.

3.Active mode- In the active mode, one junction (emitter to base) is forward biased and another junction (collector to base) is reverse biased. The active mode of operation is used for the amplification of current.



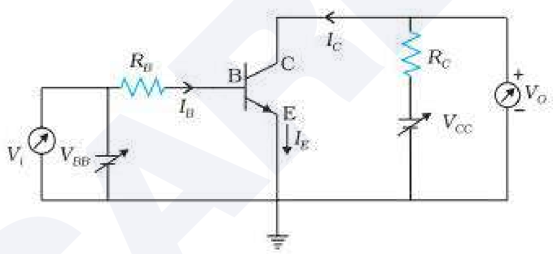
Active mode

The transistor can be used as a device application like a switch, amplifier, etc depending on parameters like the configuration used (namely CB, CC, and CE), the biasing of the E-B and B-C junction and the operation region namely cutoff, active region, and saturation.

- **Transistor as a switch-**

When a junction transistor is used in the cutoff or saturation state, it acts as a switch.

For the base-biased transistor in CE configuration shown in the below figure lets try to understand the operation of a transistor as a switch.

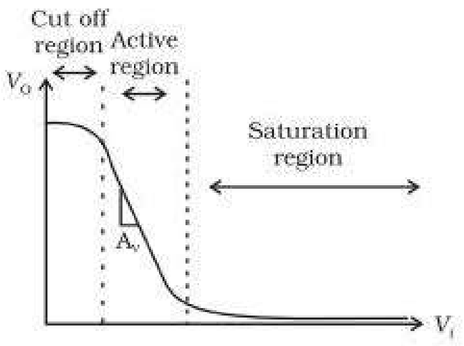


Applying Kirchhoff's voltage rule to the input and output sides of this circuit, we get

$$V_i = V_{BB} = I_B R_B + V_{BE} \ldots (1) \text{ and}$$
$$V_0 = V_{CE} = V_{CC} - I_C R_C \ldots (2)$$

where V_i *and* V_0 are dc input and output voltage respectively.

The plot of V_o vs. V_i (as shown in the below figure) is *called the transfer characteristics of a base-biased transistor.*



From the above graph, we can conclude that

1. If V_i is low and unable to forward-bias the transistor, then V_o is high (=V_{CC}).
2. If V_i is high enough to move the transistor into saturation, then V_o is very low (~0).

And when a transistor is not conducting, it is **switched off**. On the other hand, when it is in the saturation state, it is **switched on**.

If some low and high states are defined below and above certain voltage levels (i.e cutoff and saturation levels of the transistor).

then we can say for a low input switches the transistor off and a high input switches it on.

And thus transistors can operate as a switch.

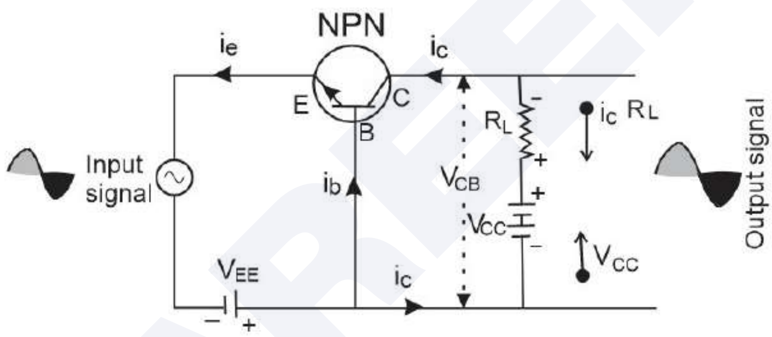
- **Junction Transistor as an Amplifier-**

A device that increases the amplitude of the input signal is called an amplifier.

The transistor can be used as an amplifier in the following three configurations

(i) CB amplifier (ii) CE amplifier (iii) CC amplifier

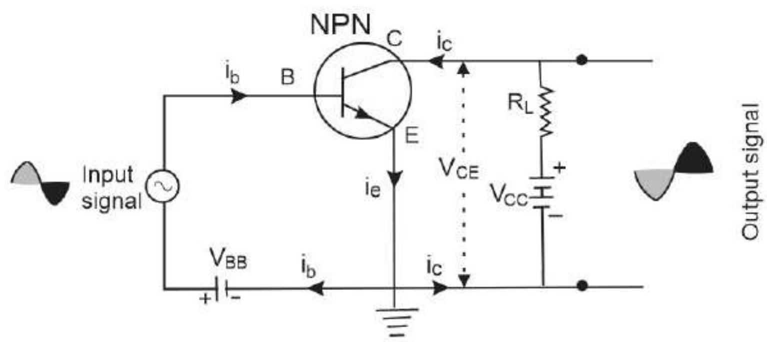
1. NPN transistor as CB amplifier



(i) $i_e = i_b + i_C; \quad i_b = 5\% \;\; of \;\; i_e \;\; and \;\; i_C = 95\% \;\; of \;\; i_e$
(ii) $V_{EE} < V_{CC}$
(iii) Net collector voltage $V_{CB} = V_{CC} - i_c R_L$

(iv) Input and output signals are in the same phase

2. NPN transistor as CE amplifier



(i) $i_e = i_b + i_c; i_b = 5\% \;\; of \;\; i_e \;\; and \;\; i_c = 95\% \;\; of \;\; i_e$
(ii) $V_{CC} > V_{BB}$

(iii) Net collector voltage $V_{CE} = V_{CC} - i_c R_L$
(iv) Input and output signals are 180° out of phase.

- **Different Gains in CE/CB Amplifiers**

(1) Transistor as CB amplifier-

$$\text{(i) ac current gain } \alpha_{\text{ac}} = \frac{\text{Small change in collector current } (\Delta i_c)}{\text{Small change in emitter current } (\Delta i_e)} \quad V_{CE} = \text{ constant}$$

$$\text{(ii) dc current gain } \alpha_{dc} \text{ (or } \alpha) = \frac{\text{Collector current } (i_c)}{\text{Emitter current } (i_e)}$$
valve of α_{dc} lies between 0.95 to 0.99

$$\text{(iii) Voltage gain } A_v = \frac{\text{Change in output voltage } (\Delta V_o)}{\text{Change in input voltage } (\Delta V_i)}$$
$\Rightarrow A_v = \alpha_{\text{ac}} \times$ Resistance gain

$$\text{(iv) Power gain } = \frac{\text{Change in output power } (\Delta P_o)}{\text{Change in input power } (\Delta P_c)}$$

$\Rightarrow$ Power gain $= \alpha_{\text{ac}}^2 \times$ Resistance gain

(2) Transistor as CE amplifier

$$\text{(i) ac current gain } \beta_{ac} = \left(\frac{\Delta i_c}{\Delta i_b}\right) \quad V_{CE} = \text{ constant}$$

$$\text{(ii) } dc \text{ current gain } \beta_{dc} = \frac{i_c}{i_b}$$

$$\text{(iii) Voltage gain : } A_v = \frac{\Delta V_o}{\Delta V_i} = \beta_{ac} \times \text{ Resistance gain}$$

$$\text{(iv) Power gain } = \frac{\Delta P_o}{\Delta P_i} = \beta_{ac}^2 \times \text{ Resistance gain}$$

- **Relation between** α *and* β

$$\beta = \frac{\alpha}{1-\alpha} \text{ or } \alpha = \frac{\beta}{1+\beta}$$

11.Logic Gates

Logic Gates-

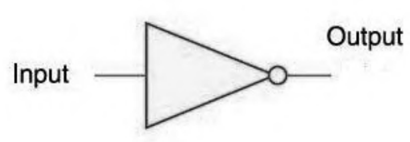
In our day to day life,we come across many digital electronic devices. But do you know, for digital devices to function the way they do, a logic needs to be established between the input and output voltages. This is done by using a gate or a digital circuit that follows the logical relationship. They are called logic gates because they control the flow of information based on a certain logic.

Symbols are given to each logic gate and each logic gate has a truth table which displays all possible input-output combinations. So the truth tables help understand the behaviour of the logic gates. All these gates are made using semiconductor devices. The five most commonly used logic gates are:

- NOT
- AND
- OR
- NAND
- NOR

NOT Gate -

A NOT gate is also known as an inverter because it simply inverts the input signal. It is a simple gate with one input and one output. So, the output is ‘0’ when the input is ‘1’ and vice-versa.



A is input

Y is output

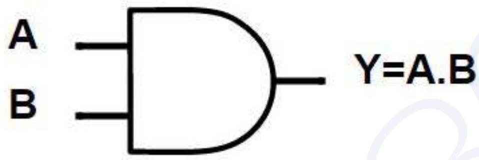
$Y = \bar{A}$

The truth table for a NOT gate is as follows:

Input	Output
A	Y
0	1
1	0

AND Gate-

An AND gate has two or more inputs and a single output. In this gate, the output is 1(High) only when all the inputs are 1(High). The most commonly used symbol for an AND gate is as follows:



A and B are input

Y is output

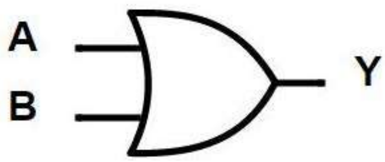
$$Y = A \cdot B$$

And the truth table for the AND gate is as follows

Input		Output
A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

OR Gate-

Like AND Gate, OR gate has also two or more inputs and one output. For this Gate, the logic is that the output would be 1 when at least one of the inputs is 1. It means when the output is high when any of the input is high. The commonly used symbol for an OR gate is as follows:



A and B are input

Y is output

Relation between input and output

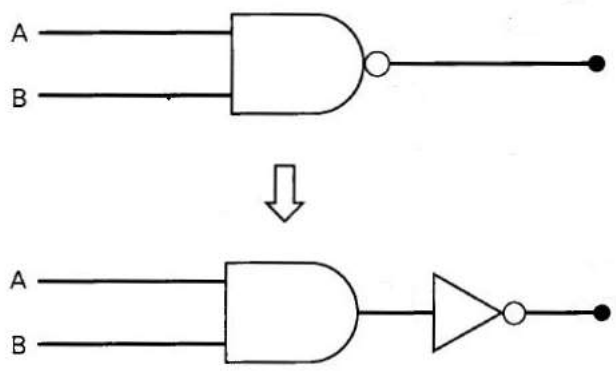
$$Y = A + B$$

And, the truth table for an OR gate is as follows:

Input		Output
A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

NAND Gate-

A NAND gate is an arrangement of AND gate followed by a NOT gate. The output is 1 only when all inputs are NOT 1 Or the output is high when at least one of them is low. These are also called **Universal gates**. The commonly used symbol for a NAND gate is as follows:



$$Y = \overline{A \cdot B}$$

A and B are input

Y is output

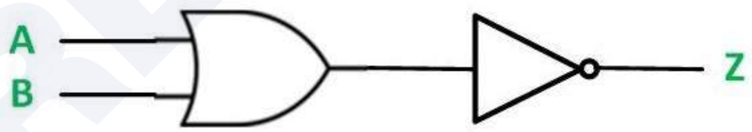
NOT + AND gate

And, the truth table for a NAND gate is as follows:

Input		Output
A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

NOR Gate-

Like NAND Gate, NOR gate is also an arrangement of an OR gate followed by a NOT gate. In this the output is 1(High) only when all inputs are 0(Low). These are also called **Universal gates.** The commonly used symbol for a NOR gate is as follows:



$$Y = \overline{A + B}$$

A and B are input

Y is output

NOT + OR Gate

And the truth table for a NOR gate is as follows:

Input		Output
A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

D'morgan's Theorem -

A and B are input.

1) $\overline{A + B} = \bar{A} \cdot \bar{B}$

2) $\overline{A \cdot B} = \bar{A} + \bar{B}$

3) $\overline{\bar{A} + \bar{B}} = A \cdot B$

4) $\overline{\bar{A} \cdot \bar{B}} = A + B$

Some Important relation -

$A + A = A$

$A \cdot A = A$

$A + 1 = 1$

$A \cdot 1 = 1$

$A \cdot 0 = 0$

$A + 0 = A$

Communication Systems

Important Formulae

1.Basic elements of a Communication System:

Basic elements of a Communication System:

Information: The idea or message that is to be conveyed is known as information. The message may be individual one or a set of messages.

Signal : A single valued function of time (that conveys the information).

Transmitter : A device which make an incoming message signal suitable for transmission through a channel and subsequent reception.

Transducer: A device that convert one form of energy into another.

Repeater: It is used to extend the range of signal. Combination of receiver and a transmitter.

Amplifier: It boosts the power of modulated signal.

Antenna: Signal is radiated in the space with the help of an antenna.

Noise : An unwanted signal that tend to disturb the transmission and processing of the message signal. Source of noise may be located within the system or out of the system.

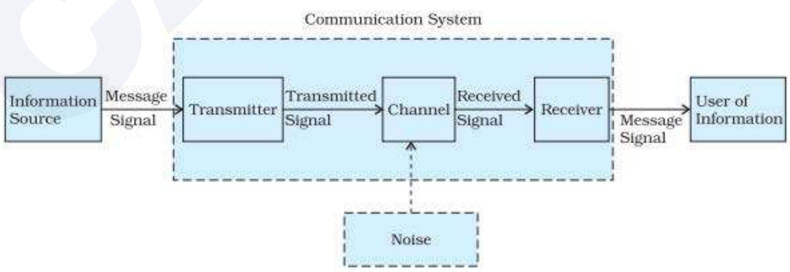
Receiver: The device which extract the desired message signal from the received signal at the channel output.

Amplification: It is the process of increasing the amplitude of a signal using an electronic circuit called the amplifier. Amplification is necessary to compensate for the attenuation of the signal in communication systems. The energy needed for additional signal strength is obtained from a DC power source. Amplification is done at a place between the source and the destination wherever signal strength becomes weaker than the required strength.

Attenuation: The loss of strength of a signal while propagating through a medium is known as attenuation.

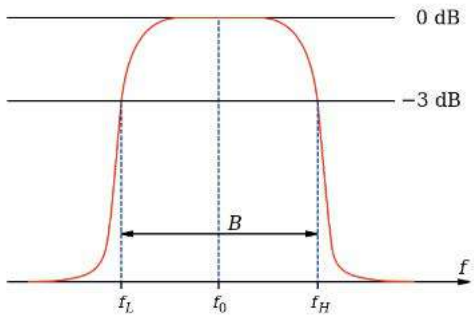
Modulation: It is the process carried out at transmitter in which the low frequency message signal is superimposed on a high frequency carrier signal.

Demodulation: The process of retrieval of information from the carrier wave at the receiver. Reverse process of modulation.



Bandwidth of Signals:

The bandwidth of a signal is defined as the difference between the upper and lower frequencies of a signal generated. As seen from the representation below, Bandwidth (B) of the signal is equal to the difference between the higher or upper-frequency (fH) and the lower frequency (fL). It is measured in terms of Hertz(Hz) i.e. the unit of frequency.



For example, Whenever we tune into a radio we find various stations at varying particular frequencies. The bandwidth of FM radio is 200 KHz from 88.1 MHz to 101.1 MHz for most places. As you tune, the radio you find various stations at various frequencies.

For speech signals, a frequency range of 300 Hz to 3100 Hz is considered adequate. Therefore speech signal requires a bandwidth of 2800 Hz (3100 Hz – 300 Hz) for commercial telephonic communication. To transmit music, an approximate bandwidth of 20 kHz is required because of the high frequencies produced by the musical instruments. The audible range of frequencies extends from 20 Hz to 20 kHz. Video signals for the transmission of pictures require about 4.2 MHz of bandwidth. A TV signal contains both voice and picture and is usually allocated 6 MHz of bandwidth for transmission.

Bandwidth of Transmission Medium

A transmission medium is a material substance (solid, liquid, gas, or plasma) that can propagate energy waves. For example, the transmission medium for sounds is usually air, but solids and liquids may also act as transmission media for sound. The absence of a material medium in a vacuum may also constitute a transmission medium for electromagnetic waves such as light and radio waves. While material substance is not required for electromagnetic waves to propagate, such waves are usually affected by the transmission media they pass through, for instance by absorption or by reflection or refraction at the interfaces between media.

Coaxial cable is a widely used wire medium, which offers a bandwidth of approximately 750 MHz. Such cables are normally operated below 18 GHz. Communication through free space using radio waves takes place over a very wide range of frequencies: from a few hundred kHz to a few GHz

Spectrum Allocations:

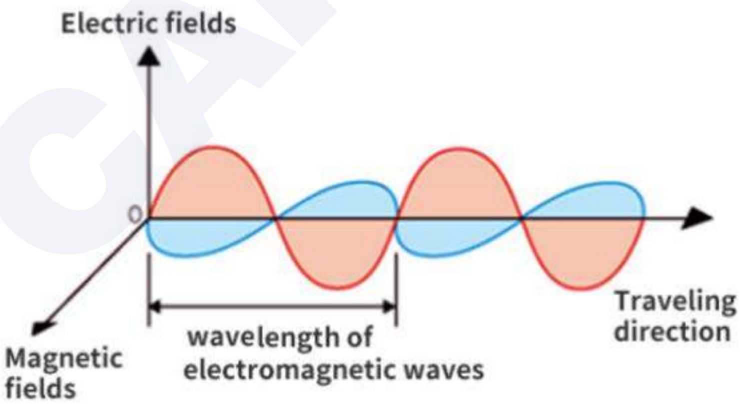
A spectrum is a large bandwidth of frequencies. Cellular or digital methods use this bandwidth for communication. These allocations have arrived with the help of international plans and policies. Often they require an upgrade of existing systems and technologies.

For example: 4G communication is for cellular devices accessible. The upcoming years will see the introduction of the 5G spectrum as well. With such huge bandwidths, easy, reliable and ultra-fast data transmissions are possible regularly.

2. Propagation of Electromagnetic Waves

Electromagnetic Waves are basically defined as superimposed oscillations of an Electric and Magnetic Field in space with their direction of propagation perpendicular to both of them. Electromagnetic waves are oscillations produced due to the crossing over of an electric and a magnetic field.

The direction of the propagation of such waves is perpendicular to the direction of the force of either of these fields as shown in the diagram below.



In communication using radio waves, an antenna at the transmitter radiates Electromagnetic waves (em waves), which travel through space and reach the receiving antenna at the other end. As the em wave travels away from the transmitter, the strength of the wave keeps on decreasing. There are several factors which can influence the propagation of em waves and the path they follow.

Ground wave: These waves are used for a low-frequency range transmission, mostly less than 1 MHz. This type of propagation employs the use of large antenna order which is equivalent to the wavelength of the waves and uses the ground or Troposphere for its propagation. Signals over large distances are not sent using this method. It causes severe attenuation which increases with the increased frequency of the waves.

SkyWave: Used for the propagation of EM waves with a frequency range of 3 – 30 MHz. They are present in the ionosphere region of charged ions about 60 to 300 km from the earth's surface. These ions provide a reflecting medium to the radio or communication waves within a particular frequency range. We use this property of the ionosphere for long-distance transmission of the waves without much attenuation and loss of signal strength.

Another thing to consider is the angle of the emission of these waves from the ground. The transmitter emits the EM Waves at a critical angle to ensure total reflection to the ground just like the total internal reflection of optic waves otherwise the waves may escape into space. Skip Distance is the distance between the 2 points between which the wave transmission happens.

Space Wave: It is used for a line of Sight communication also known as LoS. Space satellite communication and very high-frequency waves use this method of propagation. It involves sending a signal in a straight line from the transmitter to the receiver. One must ensure that for very large distances, the height of the tower used for transmission is high enough to prevent waves from touching the earth's curvature thus preventing attenuation and loss of signal strength.

The important relationship for determining the height of the antennas and their corresponding distance of transmission is given by:

$$d_M = \sqrt{2Rh_T} + \sqrt{2Rh_R}$$

where,

d_m=distance between 2 antennas;

R=Radius of earth=6400 km

h_T = Height of transmission antenna;

h_R = Height of receiver antenna

Another important relation for determining the range of transmission(Dt) for a given antenna of height H_t is:

$$d_T = \sqrt{2Rh_T}$$

3. Need Of Modulation In Communication Systems

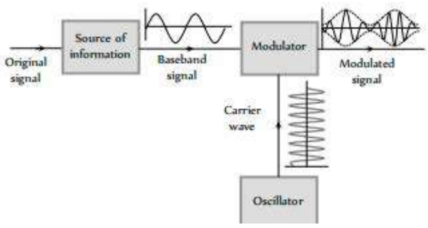
Modulation and its Necessity

The purpose of a communication system is to transmit information or message signals. Message signals are also called baseband signals, which essentially designate the band of frequencies representing the original signal, as delivered by the source of information.

Digital and analogue signals to be transmitted are usually of low frequency and hence cannot be transmitted as such. These signals require some carrier to be transported. These carriers are known as carrier waves or high-frequency signals. The process of placement of a low-frequency (LF) signal over a high-frequency (HF) signal is known as modulation.

Need for modulation: The sound wave (20 Hz to 20 kHz) cannot be transmitted directly from one place to another for the following reasons :

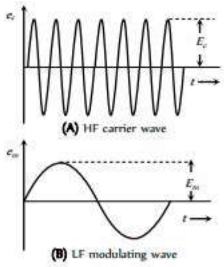
1. **Size of the antenna or aerial**: For efficient radiation and reception, the height of transmitting and receiving antennas should be comparable to a quarter of the wavelength of the frequency used. For 15 kHz it is 5000 m (too large) and for 1 MHz it is 75 m.
2. **Effective power radiated by an antenna**: The power radiated is proportional to $\left(\frac{l}{\lambda}\right)^2$. It shows that for the same antenna length, the power radiated increases with decreasing λ i.e., increasing frequency. Hence, the effective power radiated by a long-wavelength baseband signal would be small. For a good transmission, we need high powers and hence this also points out the need to use the high-frequency transmission.
3. **Detecting signals**: All audible signals are in the range of 20 Hz to 20 kHz so the signals from all sources remain heavily mixed up in the air. It will be very difficult to differentiate or detect the broadcast signal at the receiving station. Thus modulation is necessary for a low-frequency signal. When it is to be sent to a distant place so that the information may not get erased in the way itself as well as for the proper identification of a signal and to keep the height of the antenna small too.



4. Amplitude Modulation

Amplitude Modulation-

The process of changing the amplitude of a carrier wave in accordance with the amplitude of the audio frequency (AF) signal is known as amplitude modulation (AM). Carrier wave remains unchanged in AM frequency. The amplitude of a modulated wave is varied in accordance with the amplitude of the modulating wave.



Modulation index: The ratio of change of amplitude of the carrier wave to the amplitude of the original carrier wave is called the modulation factor or degree of modulation or modulation index (m).

$$\mu_a = \frac{\text{Change in amplitude of carrier wave}}{\text{Amplitude of original carrier wave}} = \frac{E_m}{E_c}$$

where $\mu_a = \frac{E_m}{E_c} = \frac{E_{\max} - E_{\min}}{E_{\max} + E_{\min}}$

If a carrier wave is modulated by several sine waves the total modulated index m is given by

$$m_t = \sqrt{m_1^2 + m_2^2 + m_3^2 + \ldots\ldots.}$$

Voluage equation for AM wave:

Suppose voltage equations for carrier wave and modulating wave are $e_c = E_c \cos \omega_c t$ and $e_m = E_m \sin \omega_m t = m E_c \sin \omega_m t$
where,

e_c= Instantaneous voltage of carrier wave,

E_c= Amplitude of the carrier wave,

$\omega_c = 2\pi f_c =$ Angular velocity at the carrier frequency f_c

$e_m =$The instantaneous voltage of modulating.

$E_m =$The amplitude of the modulating wave,

$\omega_m = 2\pi f_m =$ Angular velocity of modulating frequency 'f'

The voltage equation for AM wave is

$$e = E \sin \omega_c t = (E_c + e_m) \sin \omega_c t = (E_c + e_m \sin \omega_m t) \sin \omega_c t$$

$$= E_c \sin \omega_c t + \frac{m_a E_c}{2} \cos(\omega_c - \omega_m) t - \frac{m_a E_c}{2} \cos(\omega_c + \omega_m) t$$

The above AM wave indicated that the AM wave is equivalent to the summation of three sinusoidal waves, one having amplitude 'E' and the other two having amplitude $\frac{m_a E_c}{2}$.

Sideband frequencies: The AM wave contains three frequencies, f_c, $(f_c + f_m)$ and $(f_c - f_m)$. f_c is called carrier frequency, $(f_c + f_m)$ and $(f_c - f_m)$ are called sideband frequencies.

$(f_c + f_m)$: Upper sideband (USB) frequency

$(f_c - f_m)$: Lower sideband (LSB) frequency

In general sideband frequencies are close to the carrier frequency.

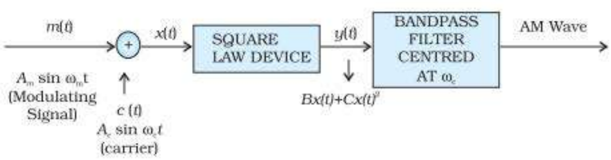
Bandwidth: The two sidebands lie on either side of the carrier frequency at equal frequency interval f_m.

So, bandwidth =

5. Production Of Amplitude Modulated Wave

Production of Amplitude Modulated Wave-

Any signal that is generated from a source and needs to be sent over large distances from the source to the receiver, needs to be modified. This can be done by superimposition with a carrier signal to ensure the signal can be transmitted in a suitable bandwidth. Amplitude modulation can be produced by a variety of methods. A conceptually simple method is shown in the block diagram below

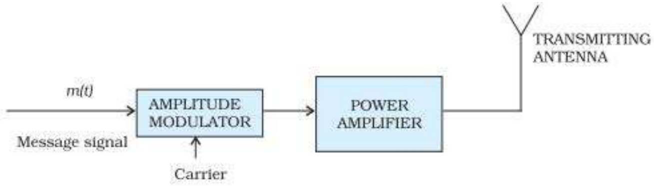


Here the modulating signal is added to the carrier signal to produce the signal x (t). This signal is passed through a square-law device which is a non-linear device which produces an output.

where B and C are constants.

This square waveform passes through a bandpass filter. The bandpass filter is a device which filters out the noise that is the unwanted frequencies. For example, if the frequencies of the system differ from those including 'ω' and ω±ω', then the bandpass filter automatically rejects them.

Yet, the process is incomplete. The modulated signal generated is quite weak and cannot sustain attenuation over a large distance. This demands strengthening the signal. We get this by amplification of the signal using an amplifier diode. The quality of the signal does not change only its strength increases by the amplifier which forms the second last part of the circuit.



Finally, the amplified and modulated signal goes to a transmitter or antenna for radiation at a particular bandwidth frequency. This antenna transmits the signal over large distances using radiation. But this alone does not ensure the signal will reach its destination.

Power in AM waves

If V_{rms} is root mean square value

and R = Resistance

then Power dissipated in any circuit.

So **Carrier Power** will be given as

The amplitude of the carrier wave

R = Resistance

Similarly, the **Total Power of sidebands** will be given as

Where

modulation index

The amplitude of carrier waves

R = resistance

And this gives the **Total power of the AM wave** as

where

modulation index

The amplitude of carrier waves

R = Resistance

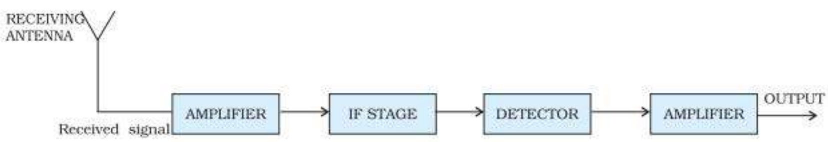
Note-maximum power in the AM wave without distortion Occurs when

I.e

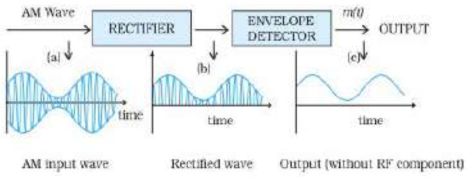
6. Detection Of Amplitude Modulated Wave

Detection of Amplitude Modulated Wave-

The transmitted message gets attenuated in propagating through the channel. The receiving antenna is, therefore, to be followed by an amplifier and a detector. In addition, to facilitate further processing, the carrier frequency is usually changed to a lower frequency by what is called an intermediate frequency (IF) stage preceding the detection. The detected signal may not be strong enough to be made use of and hence is required to be amplified. A block diagram of a typical receiver is shown in fig. below



Detection is the process of recovering the modulating signal from the modulated carrier wave. We just saw that the modulated carrier wave contains the frequencies   In order to obtain the original message signal m(t) of angular frequency , a simple method is shown in the form of a block diagram below-



The modulated signal of the form given in the above figure (a) is passed through a rectifier to produce the output shown in (b). This envelope of a signal (b) is the message signal. In order to retrieve m(t), the signal is passed through an envelope detector.

So the detector actually removes these frequencies from the signal using diodes for an analog signal or uses digital means to obtain the natural frequency of the signal. Thus the detector generates the original frequency of the signal.

An important point to note is that in the above process, a simple RC circuit can be additionally used along with the detector to generate the original frequency of the signal. This is known as a Detector Envelope which can be used to differentiate the incoming signal from the IF stage signal.

Limitation of amplitude modulation-

(1) Noisy reception

(2) Low efficiency

(3) Small operating range

(4) Poor audio quality

Some types of amplitude modulation-

- Pulse amplitude modulation (PAM)-The amplitude of the pulse varies in accordance with the modulating signal
- Pulse width modulation (PWM)-The pulse duration varies in accordance with the modulating signal.
- Pulse position modulation (PPM)-The position of the pulses of the carrier wave train is varied in accordance with the instantaneous value of the modulating signal.

Frequency modulation-

- Frequency modulation deviation The amount by which carrier frequency is varied from its unmodulated value.

The deviation is proportional to the instantaneous value of the modulating voltage.

- Value of frequency deviation =

$E_m =$ modulating amplitude

- The modulation index of frequency modulation-

It is defined as the ratio of maximum frequency deviation to the modulating frequency.

Experimental skills

Important Formulae

1. To Measure The Diameter Of Small Spherical Cylindrical Body Using Vernier Callipers

A Vernier calliper is an instrument that measures internal or external dimensions and distances. It allows you to take more precise measurements than you could with regular rulers.

Important terminologies -

LEAST COUNT AND ZERO ERROR

The magnitude of the smallest measurement that can be measured by an instrument accurately is called its least count (L.C.).

The difference between one main scale division (M.S.D.) and one vernier scale division is called the least count.

i.e. L.C. = One M.S.D. – One V.S.D.

ZERO ERROR

If there is no object between the jaws (i.e., jaws are in contact), the vernier should give zero reading. But due to some extra material on the jaws, even if there is no object between the jaws, it gives some excess reading. This excess reading is called zero error.

Zero correction: Zero correction is an invert of zero error.

Zero correction = – (Zero error)

Actual reading = observed reading – excess reading (zero error)
= observed reading + zero correction

Theory

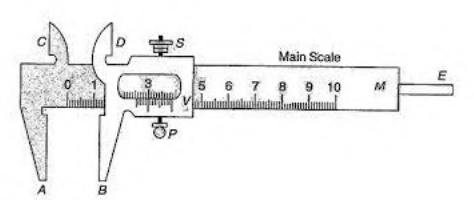
If with the body between the jaws, the zero of the vernier scale lies ahead of the Nth division of the main scale, then the main scale reading (M.S.R.) = N

If with division of the vernier scale coincides with any division of the main scale, then the vernier scale reading (V.S.R.)

= n × (L.C.) (Here, L.C. is the least count of vernier callipers)

= n × (V.C.) (Here, V.C. is vernier constant of vernier callipers)

And total reading, T.R. = M.S.R. + V.S.R. = N + n × (V.C.)



Calculating the Vernier constant (least count) of the Vernier Callipers:

1 M.S.D. = 1 mm

10 vernier scale divisions = 9 main scale divisions

i.e. 10 V.S.D. = 9 M.S.D.

1 V.S.D. = M.S.D.

Vernier Constant (V.C) = 1 M.S.D. - 1 V.S.D. = 1 M.S.D. M.S.D.

2. To measure the thickness of the given sheet using a screw gauge

Theory

1. If we place the sheet between plane faces A and B, the edge of the cap lies ahead of the Mb division of the linear scale. Then, linear scale reading (L.S.R.) =N.
If the nth division of the circular scale lies over the reference line.
Then, circular scale reading (C.S.R) = n x (L.C.) (Here, L.C. is the least count of screw gauge)

Total reading (TR) = LS.R. + C.S.R. =N+n (L . C .)

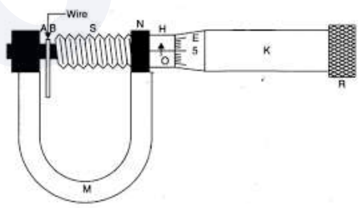


Fig. Screw gauge measuring diameter of the wire.

Calculations -

Total reading = M.S.R + C.S.R

=

L.C = least count