CAREERS360

PRACTICE Series

CBSE Class 10Th

Mathematics

Standard Answer Key And Solution

SET-1, QP Code- 30/4/2

Section-A

Question number	Correct Option	Question number	Correct Option
1	D	11	C
2	D	12	D
3	C	13	C
4	C	14	C
5	D	15	B
6	D	16	A
7	A	17	A
8	C	18	B
9	C	19	B
10	C	20	D

SECTION - B

21. A) Find the value of x for which

$$(\sin A+\csc A)^2+(\cos A+\sec A)^2=x+\tan^2 A+\cot^2 A$$

Solution:

$$(\sin A+\csc A)^2+(\cos A+\sec A)^2=x+\tan^2 A+\cot^2 A$$

Expanding each term,

$$(\sin A+\csc A)^2=\sin^2 A+2\sin A\csc A+\csc^2 A$$

Since $\sin A\csc A=1$,

$$(\sin A+\csc A)^2=\sin^2 A+2+\csc^2 A$$

Using the identity $\csc^2 A=1+\cot^2 A$.

$$(\sin A+\csc A)^2=\sin^2 A+\cot^2 A+3 \quad -----(1)$$

Similarly,

$$(\cos A+\sec A)^2=\cos^2 A+2\cos A\sec A+\sec^2 A$$

Since $\cos A\sec A=1$,

$$(\cos A+\sec A)^2=\cos^2 A+2+\sec^2 A$$

Using the identity $\sec^2 A=1+\tan^2 A$,

$$(\cos A+\sec A)^2=\cos^2 A+\tan^2 A+3 \quad -----(2)$$

Adding both terms (1) and (2),

$$(\sin A+\csc A)^2+(\cos A+\sec A)^2=\left(\sin^2 A+\cot^2 A+3\right)+\left(\cos^2 A+\tan^2 A+3\right)$$

Using $\sin^2 A + \cos^2 A = 1$,

$(\sin A + \csc A)^2 + (\cos A + \sec A)^2 = 1 + \cot^2 A + 3 + \tan^2 A + 3 = 7 + \tan^2 A + \cot^2 A$

Equating to the given expression.

$x = 7$

Hence, the value of x is 7.

21. (B) Evaluate the following:

$\frac{3\sin 30^\circ - 4\sin^3 30^\circ}{2\sin^2 50^\circ + 2\cos^2 50^\circ}$

Solution:

$\sin 30^\circ = \frac{1}{2}$, so

$3\sin 30^\circ - 4\sin^3 30^\circ = 3 \times \frac{1}{2} - 4 \times \left(\frac{1}{2}\right)^3 = \frac{3}{2} - \frac{4}{8} = \frac{3}{2} - \frac{1}{2} = 1$

Using $\sin^2 A + \cos^2 A = 1$,

$2\sin^2 50^\circ + 2\cos^2 50^\circ = 2\left(\sin^2 50^\circ + \cos^2 50^\circ\right) = 2 \times 1 = 2$

Hence, $\frac{3\sin 30^\circ - 4\sin^3 30^\circ}{2\sin^2 50^\circ + 2\cos^2 50^\circ} = \frac{1}{2}$

22. Saima and Aryaa were born in the month of June in the year 2012. Find the probability that :
(i) they have different dates of birth.
(ii) they have the same date of birth.

Solution:

June has 30 days, so each child can be born on any of these 30 days.
The total number of ways to assign birthdates to both Saima and Aryaa is $30 \times 30 = 900$.
The number of ways they can have the same birthdate is 30 (choosing one day for both).
The probability that they have the same birthdate is

$\frac{30}{900} = \frac{1}{30}$

The probability that they have different birthdates is

$1 - \frac{1}{30} = \frac{29}{30}$

Thus,
(i) Probability that they have different birthdates $= \frac{29}{30}$
(ii) Probability that they have the same birthdate $= \frac{1}{30}$

23. Solve the following system of equations algebraically :

$37x + 63y = 137$
$63x + 37y = 163$

Solution:

Adding both equations:

$$
\begin{aligned}
(37x + 63y) + (63x + 37y) &= 137 + 163 \\
100x + 100y &= 300 \\
x + y &= 3
\end{aligned}
$$

Subtracting the second equation from the first:

$$
\begin{aligned}
(37x + 63y) - (63x + 37y) &= 137 - 163 \\
-26x + 26y &= -26 \\
-x + y &= 1
\end{aligned}
$$

Solving the two equations:

$$
\begin{aligned}
x + y &= 3 \\
-x + y &= 1
\end{aligned}
$$

Adding both equations:

$2y = 4 \Rightarrow y = 2$

Substituting $y = 2$ in $x + y = 3$:

$x + 2 = 3 \Rightarrow x = 1$

Thus, the solution is $x = 1, y = 2$.

24. (A) A 1.5 m tall boy is walking away from the base of a lamp post which is 12 m high, at the speed of 2.5 m/sec. Find the length of his shadow after 3 seconds.

Solution:

Let the height of the lamp post be $h_1 = 12 \text{ m}$ and the height of the boy be $h_2 = 1.5 \text{ m}$.

Let the distance of the boy from the base of the lamp post after 3 seconds be d, and let the length of his shadow be s.

Since the boy walks at 2.5 m/sec, after 3 seconds:

$d = 3 \times 2.5 = 7.5 \text{ m}$

Using the property of similar triangles,

$\frac{h_1}{d+s} = \frac{h_2}{s}$

Substituting values:

$\frac{12}{7.5+s} = \frac{1.5}{s}$

Cross multiplying:

$$
\begin{aligned}
12s &= 1.5(7.5 + s) \\
12s &= 11.25 + 1.5s \\
12s - 1.5s &= 11.25 \\
10.5s &= 11.25 \\
s &= \frac{11.25}{10.5} = 1.07 \text{ m}
\end{aligned}
$$

Thus, the length of the shadow after 3 seconds is 1.07 m.

24. (B) In parallelogram $ABCD$, side AD is produced to a point E and BE intersects CD at F. Prove that $\triangle ABE \sim \triangle CFB$.

Solution:

In parallelogram $ABCD$, opposite sides are equal:

$AB = CD \quad \text{and} \quad AD = BC$

Since AD is extended to E, and BE intersects CD at F, we need to show similarity between $\triangle ABE$ and $\triangle CFB$.

In $\triangle ABE$ and $\triangle CFB$:
1. Angle $ABE = CFB$ (Vertically opposite angles)
2. Angle $AEB = BFC$ (Corresponding angles, as $AB \| CD$ and transversal BE creates equal angles)

By AA similarity criterion,

$\triangle ABE \sim \triangle CFB$

Thus, it is proved that $\triangle ABE \sim \triangle CFB$.

25. Find the coordinates of the point C which lies on the line AB produced such that $\mathrm{AC} = 2\mathrm{BC}$, where coordinates of points A and B are $(-1, 7)$ and $(4, -3)$ respectively.

Solution:

Using the section formula, the coordinates of a point dividing a line segment AB in the ratio $m : n$ are given by:

$\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$

Here, point C lies on the line AB produced such that $AC = 2BC$, meaning C divides AB externally in the ratio $2 : 1$.

Given:

$A(-1, 7), B(4, -3)$, and ratio $2 : 1$.
Applying the external section formula:

$$x = \frac{2(4) - 1(-1)}{2 - 1} = \frac{8 + 1}{1} = 9$$
$$y = \frac{2(-3) - 1(7)}{2 - 1} = \frac{-6 - 7}{1} = -13$$

Thus, the coordinates of C are $(9, -13)$.

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SECTION - B

21. A) Find the value of x for which

$$(\sin A + \csc A)^2 + (\cos A + \sec A)^2 = x + \tan^2 A + \cot^2 A$$

Solution:

$$(\sin A + \csc A)^2 + (\cos A + \sec A)^2 = x + \tan^2 A + \cot^2 A$$

Expanding each term,

$$(\sin A + \csc A)^2 = \sin^2 A + 2\sin A \csc A + \csc^2 A$$

Since $\sin A \csc A = 1$,

$$(\sin A + \csc A)^2 = \sin^2 A + 2 + \csc^2 A$$

Using the identity $\csc^2 A = 1 + \cot^2 A$.

$$(\sin A + \csc A)^2 = \sin^2 A + \cot^2 A + 3 \quad ------(1)$$

Similarly,

$$(\cos A + \sec A)^2 = \cos^2 A + 2\cos A \sec A + \sec^2 A$$

Since $\cos A \sec A = 1$,

$$(\cos A + \sec A)^2 = \cos^2 A + 2 + \sec^2 A$$

Using the identity $\sec^2 A = 1 + \tan^2 A$,

$$(\cos A + \sec A)^2 = \cos^2 A + \tan^2 A + 3 \quad ------(2)$$

Adding both terms (1) and (2),

$$(\sin A + \csc A)^2 + (\cos A + \sec A)^2 = \left(\sin^2 A + \cot^2 A + 3\right) + \left(\cos^2 A + \tan^2 A + 3\right)$$

Using $\sin^2 A + \cos^2 A = 1$,

$(\sin A + \csc A)^2 + (\cos A + \sec A)^2 = 1 + \cot^2 A + 3 + \tan^2 A + 3 = 7 + \tan^2 A + \cot^2 A$

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Solving the two equations:

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Adding both equations:

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Let the distance of the boy from the base of the lamp post after 3 seconds be d, and let the length of his shadow be s.

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Using the property of similar triangles,

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Substituting values:

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Cross multiplying:

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Given:

$A(-1, 7), B(4, -3)$, and ratio $2 : 1$.
Applying the external section formula:

$$x = \frac{2(4) - 1(-1)}{2-1} = \frac{8+1}{1} = 9$$
$$y = \frac{2(-3) - 1(7)}{2-1} = \frac{-6-7}{1} = -13$$

Thus, the coordinates of C are $(9, -13)$.

SECTION - C

This section consists of 6 questions of 3 marks each.

26. α and β are zeroes of a quadratic polynomial $x^2 - ax - b$. Obtain a quadratic polynomial whose zeroes are $3\alpha + 1$ and $3\beta + 1$.

Solution:

Given that α and β are the zeroes of the polynomial $x^2 - ax - b$, we have:

$$\alpha + \beta = a$$
$$\alpha\beta = -b$$

We need to find a quadratic polynomial whose zeroes are $3\alpha + 1$ and $3\beta + 1$.
Sum of new zeroes:

$$(3\alpha + 1) + (3\beta + 1) = 3(\alpha + \beta) + 2 = 3a + 2$$

Product of new zeroes:

$$\begin{aligned}(3\alpha + 1)(3\beta + 1) &= 9\alpha\beta + 3\alpha + 3\beta + 1 \\ &= 9(-b) + 3(\alpha + \beta) + 1 \\ &= -9b + 3a + 1\end{aligned}$$

The required quadratic polynomial is:

$$x^2 - (3a + 2)x + (-9b + 3a + 1)$$
$$x^2 - (3a + 2)x + (3a - 9b + 1)$$

27. Rectangle $ABCD$ circumscribes a circle of radius 10 cm. Prove that $ABCD$ is a square. Hence, find the perimeter of $ABCD$.

Solution:

Since the rectangle $ABCD$ circumscribes a circle, it means the circle is inscribed in the rectangle.

A necessary condition for a rectangle to have an inscribed circle is that the sum of its opposite sides must be equal, i.e.,

$$AB + CD = AD + BC$$

Since in a rectangle, opposite sides are equal, this simplifies to:

$$2AB = 2AD$$
$$AB = AD$$

This means the length and breadth of the rectangle are equal, which implies that $ABCD$ is a square.
Now, let the side length of the square be s.
Since the circle is inscribed, its diameter is equal to the side length of the square. The given radius of the circle is 10 cm, so the diameter is:

$$2 \times 10 = 20 \text{ cm}$$

Thus, the side length of the square is:

$$s = 20 \text{ cm}$$

The perimeter of square $ABCD$ is:

$$4 \times s = 4 \times 20 = 80 \text{ cm}$$

Thus, $ABCD$ is a square, and its perimeter is 80 cm.

28. (A) Prove that $\sqrt{2}$ is an irrational number.

Solution:

Let us assume, for contradiction, that $\sqrt{2}$ is a rational number. This means it can be expressed in the form:

$\sqrt{2}=\frac{p}{q}$

where p and q are integers with no common factors other than 1 (i.e., $\frac{p}{q}$ is in its simplest form) and $q \neq 0$.

Squaring both sides:

$2=\frac{p^2}{q^2}$

Multiplying both sides by q^2 :

$p^2=2q^2$

This shows that p^2 is divisible by 2, which implies that p itself is divisible by 2 (since the square of an odd number is always odd).

Let $p=2k$ for some integer k. Substituting this in:

$(2k)^2=2q^2$
$4k^2=2q^2$

Dividing both sides by 2 :

$2k^2=q^2$

This shows that q^2 is also divisible by 2, which implies that q is also divisible by 2.
Since both p and q are divisible by 2, they have a common factor of 2, which contradicts our assumption that $\frac{p}{q}$ is in its simplest form.
Thus, our initial assumption that $\sqrt{2}$ is rational is incorrect.
Therefore, $\sqrt{2}$ is an irrational number.

28. (B) Let x and y be two distinct prime numbers and $p=x^2y^3, q=xy^4, r=x^5y^2$. Find the HCF (p,q,r) and $\text{LCM}(p,q,r)$. Further check if $HCF(p,q,r) \times LCM(p,q,r)=p \times q \times r$ or not.

Solution:

The HCF is obtained by taking the lowest power of each prime factor in p,q,r. The prime factor powers are:
- $p=x^2y^3$
- $q=xy^4$
- $r=x^5y^2$

The lowest powers of x and y are $\min(2,1,5)=1$ and $\min(3,4,2)=2$.

$\text{HCF}(p,q,r)=x^1y^2=xy^2$

The LCM is obtained by taking the highest power of each prime factor. The highest powers are $\max(2,1,5)=5$ and $\max(3,4,2)=4$.

$\text{LCM}(p,q,r)=x^5y^4$

To verify the condition:

$$\begin{aligned}\text{HCF}(p,q,r)\times\text{LCM}(p,q,r)&=xy^2\times x^5y^4=x^{1+5}y^{2+4}=x^6y^6\\ p\times q\times r&=\left(x^2y^3\right)\times\left(xy^4\right)\times\left(x^5y^2\right)=x^{2+1+5}y^{3+4+2}=x^8y^9\end{aligned}$$

Since $x^6y^6\neq x^8y^9$, the given condition is not satisfied.

29. The two angles of a given triangle are in the ratio $2:3$. The third angle is greater than the largest angle by 10°. Find all the angles of the triangle algebraically as a system of linear equations.

Solution:

Let the two angles of the triangle be $2x$ and $3x$. The third angle is given as 10° more than the largest angle, so it is $3x+10^\circ$.

Since the sum of the angles in a triangle is always 180°, we get the equation:

$$\begin{aligned}2x+3x+(3x+10)&=180\\ 8x+10&=180\\ 8x&=170\\ x&=21.25\end{aligned}$$

Substituting x into the expressions for the angles:

$$\begin{aligned}2x&=2(21.25)=42.5^\circ\\ 3x&=3(21.25)=63.75^\circ\\ \text{Third angle }&=3x+10=63.75+10=73.75^\circ\end{aligned}$$

Thus, the three angles of the triangle are $42.5^\circ, 63.75^\circ, 73.75^\circ$.

30. $\mathrm{P}(x,y), \mathrm{Q}(-2,-3)$ and $\mathrm{R}(2,3)$ are the vertices of a right triangle PQR right angled at P . Find the relationship between x and y. Hence, find all possible values of x for which $y=2$.

Solution:

Since $\triangle PQR$ is right-angled at P, the product of the slopes of PQ and PR should be -1.
The slope of PQ is

$\frac{y+3}{x+2}$

The slope of PR is

$\frac{y-3}{x-2}$

By the perpendicularity condition,

$$\left(\frac{y+3}{x+2}\right)\times\left(\frac{y-3}{x-2}\right)=-1$$

Using the identity $(a+b)(a-b)=a^2-b^2$.

$$\frac{y^2-9}{x^2-4}=-1$$

Multiplying both sides by x^2-4,

$y^2 - 9 = -x^2 + 4$

Rearranging,

$x^2 + y^2 = 13$

This is the required relationship between x and y.
For $y = 2$,

$$\begin{aligned} x^2 + 2^2 &= 13 \\ x^2 + 4 &= 13 \\ x^2 &= 9 \\ x &= \pm 3 \end{aligned}$$

Thus, the possible values of x for $y = 2$ are $x = 3$ and $x = -3$.

31. (A) Prove that $\frac{\cos A + \sin A - 1}{\cos A - \sin A + 1} = \operatorname{cosec} A - \cot A$

Solution:

We need to prove the identity:

$\frac{\cos A + \sin A - 1}{\cos A - \sin A + 1} = \csc A - \cot A$

Rewriting the right-hand side:

$\csc A - \cot A = \frac{1}{\sin A} - \frac{\cos A}{\sin A} = \frac{1 - \cos A}{\sin A}$

This means we need to prove:

$\frac{\cos A + \sin A - 1}{\cos A - \sin A + 1} = \frac{1 - \cos A}{\sin A}$

Cross multiplying:

$(\cos A + \sin A - 1) \sin A = (1 - \cos A)(\cos A - \sin A + 1)$

Expanding both sides:

$\cos A \sin A + \sin^2 A - \sin A = (1 - \cos A)(\cos A - \sin A + 1)$

Expanding $(1 - \cos A)(\cos A - \sin A + 1)$:

$$\begin{aligned} & 1(\cos A - \sin A + 1) - \cos A(\cos A - \sin A + 1) \\ = & (\cos A - \sin A + 1) - \left(\cos^2 A - \cos A \sin A + \cos A\right) \\ = & \cos A - \sin A + 1 - \cos^2 A + \cos A \sin A - \cos A \\ = & 1 - \cos^2 A - \sin A + \cos A \sin A \end{aligned}$$

Since $1 - \cos^2 A = \sin^2 A$, we get:

$\sin^2 A - \sin A + \cos A \sin A$

Both sides simplify to:

$\cos A \sin A + \sin^2 A - \sin A = \sin^2 A - \sin A + \cos A \sin A$

Since both sides are equal, the given equation is proven:

$\frac{\cos A+\sin A-1}{\cos A-\sin A+1}=\csc A-\cot A$

Hence Proved

31. (B) If $\cot \theta+\cos \theta=p$ and $\cot \theta-\cos \theta=q$. prove that $p^2-q^2=4 \sqrt{p q}$

Solution:

We are given the following equations:

$\cot \theta+\cos \theta=p$
$\cot \theta-\cos \theta=q$

We are tasked with proving that:

$p^2-q^2=4 \sqrt{p q}$

Subtract the second equation from the first:

$(\cot \theta+\cos \theta)-(\cot \theta-\cos \theta)=p-q$

Simplifying the left-hand side:

$2 \cos \theta=p-q$

This gives:

$\cos \theta=\frac{p-q}{2}$

Now, add the first and second equations:

$(\cot \theta+\cos \theta)+(\cot \theta-\cos \theta)=p+q$

Simplifying the left-hand side:

$2 \cot \theta=p+q$

This gives:

$\cot \theta=\frac{p+q}{2}$

Next, recall the identity $\cot \theta=\frac{\cos \theta}{\sin \theta}$. Substituting the expressions for $\cot \theta$ and $\cos \theta$ into this equation:

$\frac{p+q}{2}=\frac{\frac{p-q}{2}}{\sin \theta}$

Simplifying:

$\frac{p+q}{2}=\frac{p-q}{2 \sin \theta}$

Multiplying both sides by 2 :

$p+q=\frac{p-q}{\sin \theta}$

Solving for $\sin \theta$:

$\sin \theta = \frac{p-q}{p+q}$

Now, using the identity $\cot^2 \theta + 1 = \csc^2 \theta$, we get

$\left(\frac{p+q}{2}\right)^2 + 1 = \frac{1}{\sin^2 \theta}$

Substitute the expression for $\sin \theta$:

$\left(\frac{p+q}{2}\right)^2 + 1 = \frac{(p+q)^2}{(p-q)^2}$

Now, compute $p^2 - q^2$:

$p^2 - q^2 = (p+q)(p-q)$

Thus, the required identity is proven:

$p^2 - q^2 = 4\sqrt{pq}$

Hence Proved

Section D

This section consists of 4 questions of 5 marks.

32. The following table shows the number of traffic challans issued in the month of April by the traffic police:

Number of Challans	Number of Days
$0-10$	5
$10-20$	3
$20-30$	10
$30-40$	9
$40-50$	2
$50-60$	1
Total	30

Find the 'mean' and 'mode' of the above data.

Solution:

Mean:

The mean of a frequency distribution can be calculated using the formula:

$$\text{Mean } = \frac{\sum(f_i \cdot x_i)}{N}$$

Where:
- f_i is the frequency (the number of days in each class).
- x_i is the midpoint of each class interval,
- N is the total number of days.

First, we find the midpoints (x_i) of each class interval:
- For $0-10$, midpoint $x_1 = \frac{0+10}{2} = 5$

- For $10-20$, midpoint $x_2=\frac{10+20}{2}=15$
- For $20-30$, midpoint $x_3=\frac{20+30}{2}=25$
- For $30-40$, midpoint $x_4=\frac{30+40}{2}=35$
- For $40-50$, midpoint $x_5=\frac{40+50}{2}=45$
- For $50-60$, midpoint $x_6=\frac{50+60}{2}=55$

Now, we calculate $f_i \cdot x_i$ for each class interval:
- $f_1 \cdot x_1=5 \times 5=25$
- $f_2 \cdot x_2=3 \times 15=45$
- $f_3 \cdot x_3=10 \times 25=250$
- $f_4 \cdot x_4=9 \times 35=315$
- $f_5 \cdot x_5=2 \times 45=90$
- $f_6 \cdot x_6=1 \times 55=55$

Now, sum these values:

$$\sum\left(f_i \cdot x_i\right)=25+45+250+315+90+55=780$$

The total number of days, N, is 30. Therefore, the mean is:

$$\text { Mean }=\frac{780}{30}=26$$

Mode:

The mode of a frequency distribution is the class interval with the highest frequency. In this case:
- The highest frequency is 10 in the class interval $20-30$.

Thus, the mode is the class interval $20-30$. However, to calculate the mode more precisely, we use the following formula:

$$\text { Mode }=L+\left(\frac{f_1-f_0}{2 f_1-f_0-f_2}\right) \cdot h$$

Where:
- L is the lower boundary of the modal class (20 in this case).
- f_1 is the frequency of the modal class (10).
- f_0 is the frequency of the class before the modal class (3).
- f_2 is the frequency of the class after the modal class (9).
- h is the class width (10).

Substitute the values:

$$\text { Mode }=20+\left(\frac{10-3}{2 \times 10-3-9}\right) \cdot 10$$

Simplify:

$$\text { Mode }=20+\left(\frac{7}{20-12}\right) \cdot 10=20+\left(\frac{7}{8}\right) \cdot 10=20+\frac{70}{8}=20+8.75=28.75$$

Hence, the mean is 26 and mode is 28.75.

33. (A) The sides of a right triangle are such that the longest side is 4 m more than the shortest side and the third side is 2 m less than the longest side. Find the length of each side of the triangle. Also, find the difference between the numerical values of the area and the perimeter of the given triangle.

Solution:

Let the sides of the right triangle be:
- The shortest side $= x$ meters,
- The longest side (hypotenuse) $= x+4$ meters,
- The third side $= x+2$ meters.

Using the Pythagorean theorem:

$$x^2+(x+2)^2=(x+4)^2$$

Expanding both sides:

$$x^2+\left(x^2+4x+4\right)=\left(x^2+8x+16\right)$$

Simplifying:

$$x^2+x^2+4x+4=x^2+8x+16$$

Combining like terms:

$$2x^2+4x+4=x^2+8x+16$$

Bringing all terms to one side:

$$2x^2+4x+4-x^2-8x-16=0$$

Simplifying:

$$x^2-4x-12=0$$

Using the quadratic formula to solve for x :

$$x=\frac{-(-4) \pm \sqrt{(-4)^2-4(1)(-12)}}{2(1)}$$
$$x=\frac{4 \pm \sqrt{16+48}}{2}$$
$$x=\frac{4 \pm \sqrt{64}}{2}$$
$$x=\frac{4 \pm 8}{2}$$

Thus, we have two possible values for x :

$$x=\frac{4+8}{2}=6 \quad \text{or} \quad x=\frac{4-8}{2}=-2$$

Since the side length cannot be negative, $x=6$.

Now, the lengths of the sides are:
- The shortest side $=x=6$ meters,
- The longest side $=x+4=6+4=10$ meters,
- The third side $=x+2=6+2=8$ meters.

The area of the triangle is:

$$\text{Area }=\frac{1}{2} \times 6 \times 8=24 \text{ square meters}$$

The perimeter of the triangle is:

$$\text{Perimeter } = 6+8+10=24 \text{ meters}$$

The difference between the area and the perimeter is:

$$\text{Difference } = 24-24=0$$

Thus, the lengths of the sides are 6 meters, 8 meters, and 10 meters, and the difference between the area and the perimeter is 0.

33. (B) Express the equation $\frac{x-2}{x-3}+\frac{x-4}{x-5}=\frac{10}{3}(x \neq 3,5)$ as a quadratic equation in standard form. Hence, find the roots of the equation so formed.

Solution:

The given equation is:

$\frac{x-2}{x-3}+\frac{x-4}{x-5}=\frac{10}{3}$

To combine the left-hand side, we need a common denominator, which is $(x-3)(x-5)$. Rewriting the fractions with this common denominator:

$\frac{(x-2)(x-5)}{(x-3)(x-5)}+\frac{(x-4)(x-3)}{(x-3)(x-5)}=\frac{10}{3}$

Simplifying the numerators:

$(x-2)(x-5)=x^2-7x+10$ and $(x-4)(x-3)=x^2-7x+12$

Now, adding these:

$\left(x^2-7x+10\right)+\left(x^2-7x+12\right)=2x^2-14x+22$

Thus, the equation becomes:

$\frac{2x^2-14x+22}{(x-3)(x-5)}=\frac{10}{3}$

Cross-multiply:

$3\left(2x^2-14x+22\right)=10(x-3)(x-5)$

Expanding both sides:

$$3\left(2x^2-14x+22\right)=6x^2-42x+66$$
$$10(x-3)(x-5)=10\left(x^2-8x+15\right)=10x^2-80x+150$$

Equating both sides:

$6x^2-42x+66=10x^2-80x+150$

Rearranging the terms:

$6x^2-42x+66-10x^2+80x-150=0$

Simplifying:

$-4x^2 + 38x - 84 = 0$

Multiply through by -1 :

$4x^2 - 38x + 84 = 0$

Now, apply the quadratic formula:

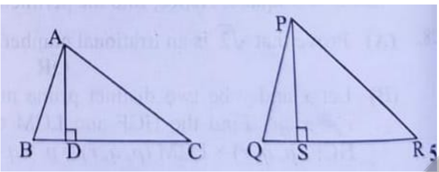
$$x = \frac{-(-38) \pm \sqrt{(-38)^2 - 4(4)(84)}}{2(4)}$$
$$x = \frac{38 \pm \sqrt{1444 - 1344}}{8}$$
$$x = \frac{38 \pm \sqrt{100}}{8}$$
$$x = \frac{38 \pm 10}{8}$$

The solutions are:

$$x = \frac{38+10}{8} = \frac{48}{8} = 6$$
$$x = \frac{38-10}{8} = \frac{28}{8} = 3.5$$

Thus, the roots of the equation are 6 and 3.5.

34. (A) The corresponding sides of $\triangle ABC$ and $\triangle PQR$ are in the ratio 3:5. AD $\perp$ BC and PS $\perp$ QR, as shown in the adjoining figures.



(i) Prove that $\triangle$ADC $\sim$ $\triangle$PSR.
(ii) If $AD = 4$ cm, find the length of PS.
(iii) Find $ar(\triangle ADC) : ar(\triangle PQR)$.

Solution:

Given:
- Corresponding sides of triangles $\triangle ABC$ and $\triangle PQR$ are in the ratio $3:5$.
- AD $\perp$ BC and PS $\perp$ QR, meaning AD and PS are altitudes of triangles $\triangle ABC$ and $\triangle PQR$. respectively.

(i) Prove that $\triangle ADC \sim \triangle PSR$

Since AD $\perp$ BC and PS $\perp$ QR, both $\triangle ADC$ and $\triangle PSR$ have a right angle at D and S respectively. The angles at $\angle ADC$ and $\angle PSR$ are both right angles.

We are given that the corresponding sides of the triangles are in the ratio $3:5$, meaning:

$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} = \frac{3}{5}$

The triangles $\triangle ABC$ and $\triangle PQR$ are similar by the AA (Angle-Angle) criterion since:
- $\angle ABC = \angle PQR$ (corresponding angles).
- $\angle ACB = \angle PRQ$ (corresponding angles).
- $\angle ADC = \angle PSR = 90^\circ$ (both are right angles).

Thus, by the property of similar triangles, the ratio of corresponding sides is the same for both triangles, and we can conclude that:

$\triangle ADC \sim \triangle PSR$

(ii) If $AD = 4$ cm, find the length of PS

Since $\triangle ADC \sim \triangle PSR$, the corresponding sides of these triangles are proportional. Therefore, the ratio of the corresponding altitudes AD and PS will also be the same as the ratio of the corresponding sides.

Thus:

$\frac{AD}{PS} = \frac{AB}{PQ} = \frac{3}{5}$

We are given that $AD = 4$ cm, so:

$\frac{4}{PS} = \frac{3}{5}$

Now, solve for PS :

$PS = \frac{4\times 5}{3} = \frac{20}{3}$ cm

Thus, the length of PS is $\frac{20}{3}$ cm.

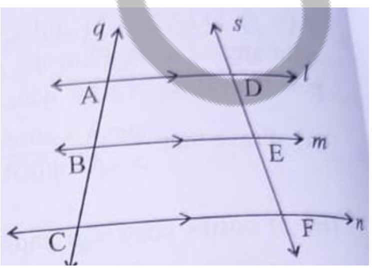
(iii) Find $\frac{\operatorname{ar}(\triangle ADC)}{\operatorname{ar}(\triangle PQR)}$

Since $\triangle ADC \sim \triangle PSR$, the ratio of their areas is the square of the ratio of their corresponding sides. Therefore, the ratio of the areas is:

$\frac{\operatorname{ar}(\triangle ADC)}{\operatorname{ar}(\triangle PSR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{3}{5}\right)^2 = \frac{9}{25}$

Thus, the ratio of the areas is $\frac{9}{25}$.

34. (B) State basic proportionality theorem. Use it to prove the following: If three parallel lines I, m, n are intersected by transversals q and s as shown in the adjoining figure, then $\frac{AB}{BC} = \frac{DE}{EF}$.



Solution:

The Basic Proportionality Theorem states that if a line is drawn parallel to one side of a triangle and intersects the other two sides at distinct points, then it divides those sides in the same ratio.

In the given figure, three parallel lines $\mathrm{I}, \mathrm{m}, \mathrm{n}$ are intersected by two transversals q and s at points A, B, C and D, E, F, respectively.

In $\triangle ABC$, since m is parallel to n, it divides AB and BC in the same ratio:

$\frac{AB}{BC} = \frac{x}{y}$

In $\triangle DEF$, since m is parallel to n, it divides DE and EF in the same ratio:

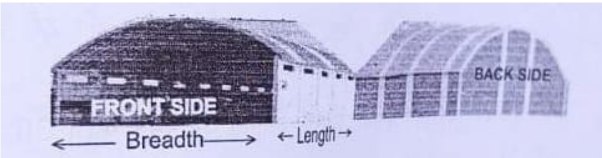
$\frac{DE}{EF} = \frac{x}{y}$

Since both ratios are equal,

$\frac{AB}{BC} = \frac{DE}{EF}$

Thus, the given statement is proved.

35. In order to provide shelter to flood victims, a shed was constructed using tin sheets, which is in the form of a cuboid surmounted by a half-cylinder as shown below.



The length, breadth, and height of the cuboidal portion are $10 \mathrm{~m}$, $7 \mathrm{~m}$, and 3 m, respectively. The diameter of the cylindrical portion is 7 m. Find the cost of the tin sheets required to make the shed at the rate of ₹ 70 per square meter, given that the shed is open from the front side and closed from the back side.

Solution:

The shed consists of a cuboidal portion with a length of 10 m , a breadth of 7 m , and a height of 3 m , along with a half-cylinder on top that has a diameter of 7 m , giving it a radius of 3.5 m . Since the shed is open from the front side and closed from the back side, the required tin sheet area includes the curved surface area of the half-cylinder, the lateral surface area of the cuboid, and the back semicircular face.

The curved surface area of a full cylinder is given by $2\pi rh$. Since only half of the cylinder is present, the curved surface area is π rh. Substituting the values,

$$\begin{aligned} & = \pi \times 3.5 \times 10 \\ & = 35\pi \\ & = 109.9 \mathrm{~m}^2 \end{aligned}$$

The lateral surface area of the cuboid consists of two vertical side walls and the back wall. The area of the two side walls is

$$\begin{aligned} & = 2 \times (3 \times 10) \\ & = 60 \mathrm{~m}^2 \end{aligned}$$

The back wall consists of a semicircular face with an area of $\left(\frac{1}{2}\right) \mathrm{mr}^2$.

$= (\frac{1}{2}) \times \pi \times (3.5)^2$
$= (\frac{1}{2}) \times \pi \times 12.25$
$= 19.23 \text{ m}^2$

Adding this to the lateral surface area of the cuboid,

$= 60 + 19.23$
$= 79.23 \text{ m}^2$

Adding the curved surface area of the half-cylinder and the lateral surface area of the cuboid,

$= 109.9 + 79.23$
$= 189.13 \text{ m}^2$

The cost per square meter is $Rs. 70$. Multiplying the total area by the rate per square meter,

$= 189.13 \times 70$
$= 13,239.10$

The total cost of the tin sheets required is ₹ $13,239.10$.

SECTION - E

This section consists of 3 case-study based questions for 4 marks each.

36. Cable cars at hill stations are one of the major tourist attractions.
On a hill station, the length of the cable car ride from the base point to the topmost point on the hill is 5000 m.
Poles are installed at equal intervals on the way to provide support to the cables on which cable cars move.

The distance of the first pole from the base point is 200 m, and subsequent poles are installed at equal intervals of 150 m, with the distance of the last pole from the top being 300 m.

Based on the above information, answer the following questions:
1. Find the distance of the 10th pole from the base.
2. Find the distance between the 15th pole and 25th pole?

Solution:

1) The given information states that the first pole is at 200 m, and subsequent poles are installed at equal intervals of 150 m.

The position of the poles follows an arithmetic progression where:
First term (a) $= 200$ m
Common difference (d) $= 150$ m
For the 10th pole, the position is given by the formula for the nth term of an arithmetic sequence:

$$Tn = a + (n - 1) * d$$

Substituting the values,

$T10 = 200 + (10 - 1) * 150$
$= 200 + 1350$
$= 1550 \text{ m}$

Thus, the 10th pole is 1550 m from the base.

2) To find the distance between the 15th pole and the 25th pole, we calculate their respective positions:

$\mathrm{T}15 = 200 + (15 - 1) * 150$
$= 200 + 2100$
$= 2300 \text{ m}$
$\mathrm{T}25 = 200 + (25 - 1) * 150$
$= 200 + 3600$
$= 3800 \text{ m}$

The distance between the 15th pole and the 25th pole is:

$= 3800 - 2300$
$= 1500 \text{ m}$

Thus, the distance between the 15th and 25th pole is 1550 m.

37. A drone was used to facilitate the movement of an ambulance on the straight highway to a point P on the ground where there was an accident. The ambulance was travelling at a speed of 60 km/h. The drone stopped at a point Q 100 m vertically above point P. The angle of depression of the ambulance was found to be 30° at a particular instant. Based on the above information, answer the following questions:

(ii) Find the distance between the ambulance and the site of the accident (P) at the particular instant. (Use $\sqrt{3} = 1.73$)
(iii) (a) Find the time (in seconds) in which the angle of depression changes from 30° to 45°.
OR
(iii) (b) How long (in seconds) will the ambulance take to reach point P from a point T on the highway such that the angle of depression of the ambulance at T is 60° from the drone?

Solution:

(ii) The distance between the ambulance and the site of the accident P :

Using $\tan 30^\circ = \frac{QP}{PB}$.

$$\frac{1}{\sqrt{3}} = \frac{100}{PB}$$
$PB = 100\sqrt{3}$

Using $\sqrt{3} = 1.73$,

$PB = 100 \times 1.73 = \frac{173}{1} \text{ m} \quad \text{or} \quad 173 \text{ m}$

(iii) (a) When the angle of depression changes from 30° to 45° :

For 45° :

$$\tan 45^\circ = \frac{QP}{P'B}$$
$$1 = \frac{100}{P'B}$$
$$P'B = 100 \text{ m}$$

The initial distance $PB = 173$ m, new distance $P'B = 100$ m.
Distance covered by the ambulance:

$173 - 100 = 73$ m

Speed of the ambulance:

$60 \text{ km/h} = \frac{60 \times 1000}{3600} = \frac{50}{3} \text{ m/s}$

Time taken:

$$\text{Time } = \frac{73}{\frac{50}{3}}$$
$$= \frac{73 \times 3}{50} = \frac{219}{50} \text{ seconds or } 4\frac{19}{50} \text{ seconds.}$$

(iii) (a) When the angle of depression changes from 30° to 45° :

For 45° :

$$\tan 45^\circ = \frac{QP}{P'B}$$
$$1 = \frac{100}{P'B}$$
$$P'B = 100 \text{ m}$$

The initial distance $PB = 173$ m, new distance $P'B = 100$ m.
Distance covered by the ambulance:

$173 - 100 = 73$ m

Speed of the ambulance:

$60 \text{ km/h} = \frac{60 \times 1000}{3600} = \frac{50}{3} \text{ mm/s}$

Time taken:

$$\text{Time } = \frac{73}{\frac{50}{3}}$$
$$= \frac{73 \times 3}{50} = \frac{219}{50} \text{ seconds or } 4\frac{19}{50} \text{ seconds.}$$

(iii) (b) When the angle of depression is 60° at T :

$$\tan 60^\circ = \frac{QP}{TB}$$

$$\sqrt{3} = \frac{100}{TB}$$

$$TB = \frac{100}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$TB = \frac{100\sqrt{3}}{3}$$

Using $\sqrt{3} = 1.73$,

$TB = \frac{100\times1.73}{3} = \frac{173}{3}$ m or $57\frac{2}{3}$ m.

Distance covered by the ambulance:

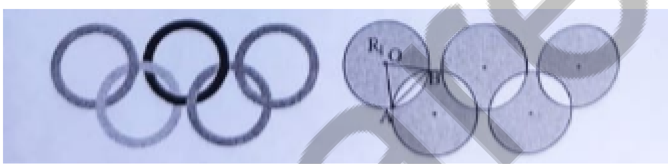
$173 - \frac{173}{3} = \frac{519}{3} - \frac{173}{3} = \frac{346}{3}$ m or $115\frac{1}{3}$ m

Time taken:

$$\text{Time} = \frac{\frac{346}{3}}{\frac{50}{3}}$$

$$= \frac{346}{3} \times \frac{3}{50} = \frac{1038}{150} = \frac{173}{25} \text{ seconds or } 6\frac{23}{25} \text{ seconds.}$$

38. The Olympic symbol comprising five interlocking rings represents the union of the five continents of the world and the meeting of athletes from all over the world at the Olympic Games. In order to spread awareness about the Olympic Games, students of Class X took part in activities organized by the school. One such group of students made 5 circular rings in the school lawn with the help of ropes. Each circular ring required 44 m of rope. Also, in the shaded regions as shown in the figure, students made rangoli showcasing various sports and games. It is given that $\triangle OAB$ is an equilateral triangle and all unshaded regions are congruent.



Based on the above information, answer the following questions:
(i) Find the radius of each circular ring.
(ii) What is the measure of $\angle AOB$?
(iii) (a) Find the area of the shaded region R1.
OR
(iii) (b) Find the length of rope around the unshaded regions.

Solution:

(i) The circumference of each circular ring is given as 44 m. Using the formula for the circumference of a circle:

$$2\pi r = 44$$
$$r = \frac{44}{2\pi}$$
$$r = \frac{44}{2 \times 3.14} = \frac{44}{6.28} \approx 7 \text{ m}$$

So, the radius of each circular ring is 7 meters.

(ii) Since $\triangle OAB$ is an equilateral triangle, all its angles are:

$\angle AOB = 60^\circ$

(iii) (a) To find the area of the shaded region R_1, we use the formula for the area of a sector:

$$\text{Area of sector } = \frac{\theta}{360^\circ} \times \pi r^2$$
$$= \frac{60}{360} \times 3.14 \times 7^2$$
$$= \frac{1}{6} \times 3.14 \times 49$$
$$= \frac{3.14 \times 49}{6} = \frac{153.86}{6} \approx 25.64 \text{ m}^2$$

So, the area of the shaded region R_1 is 25.64 m^2.

(iii) (b) The length of the rope around the unshaded regions consists of three circular arcs, each forming a 60° sector of the ring.

$$\text{Arc length } = \frac{\theta}{360^\circ} \times 2\pi r$$
$$= \frac{60}{360} \times 2 \times 3.14 \times 7$$
$$= \frac{1}{6} \times 2 \times 3.14 \times 7$$
$$= \frac{2 \times 3.14 \times 7}{6} = \frac{43.96}{6} \approx 7.33 \text{ m}$$

Since there are three such arcs, the total length of rope around the unshaded regions is:

$3 \times 7.33 = 21.99 \text{ m} \approx 22 \text{ m}$