CAREERS 360

PRACTICE Series

JEE Advanced 2025

Sample Paper

Physics

Q. 1 Six buses each of mass m are connected with each other along a Straight line. The First bus is pulled by a towing truck Which applies a force F on that bus . All the buses move together. Find the ratio of force exerted by second bus on the third bus and force exerted by fourth bus on the fifth bus.(assuming no other forces are acting on the buses in horizontal direction.)

Option 1:
2:1

Option 2:
12:2

Option 3:
1:3

Option 4:
11:2

Correct Answer:
2:1

Solution:
As all the buses are going together we can treat them as a single body to calculate common acceleration.

Let their acceleration be a, by Newton's Second law: $F = (6m)a \Rightarrow a = \frac{F}{6m}$
Let Force by second bus on third bus is $F_{2,\text{ it will pull third, fourth, fifth and sixth}}$ buses.

By Newton's second law:

$$F_2 = (4m)a$$
$$\Rightarrow F_2 = 4m\frac{F}{6m}$$
$$\Rightarrow F_2 = \frac{4F}{6} = \frac{2F}{3}$$

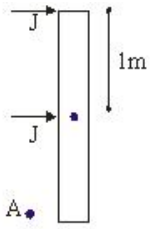
Let force by the fourth bus on the fifth bus is F_4, it will pull fifth and sixth buses.
By Newton's second law:

$$F_4 = (2m)a$$
$$\Rightarrow F_4 = 2m\frac{F}{6m}$$
$$\Rightarrow F_4 = \frac{2F}{6} = \frac{F}{3}$$

Hence their ratio will be : $\frac{F_2}{F_4} = \frac{2}{1}$

Hence, the answer is the option (1).

Q. 2 Two impulses of equal magnitude J = 15 N-s are imparted to a stationary uniform Rod of mass 3 kg and length 2 m as shown. Rod is free to move on a smooth horizontal surface. The magnitude of angular momentum of the rod about point A is a two-digit number. Add both the digits and write your answer.



Correct Answer:
9

Solution:
Velocity of COM,

$V_{cm} = \frac{30}{3} = 10\ \mathrm{m/s}$

Angular velocity,

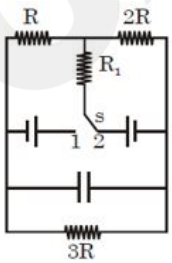
$\omega = \frac{J \times 1}{\frac{m\ell^2}{12}} = \frac{15 \times 12}{3 \times 4} = 15\mathrm{rad/s}$

Angular momentum about point

$$\begin{aligned} \mathrm{A} &= \frac{m\ell^2}{12}\omega(-\hat{k}) + m(1)\,(v_{cm})(-\hat{k}) \\ &= \frac{3 \times 4}{12} \times 15(-\hat{k}) + 3(1)(10)(-\hat{k}) \\ &= -45\hat{k} \end{aligned}$$

Hence, the answer is 9.

Q. 3 When the switch is shifted from 1 to 2 current through R1 remains same in direction but becomes 5 times. The initial charge on capacitor was Q. The Final charge on the capacitor is $\mathrm{Q'}$. $\frac{Q'}{Q}$ is $y \times 10^1$. (Consider only the charge and currents in steady state). Find y?



Option 1:
1

Option 2:
3

Option 3:
5

Option 4:
7

Correct Answer:
1

Solution:
When the switch was at position $12R$ and $3R$ were in series and their combination is in parallel with R.
Let current through R_1 is i_1,

Hence,
$i_{3R} = i_1 \times \frac{1}{6}$
$V_{3R} = i_i \times 3R \times \frac{1}{6} = \frac{i_1 R}{2}$

When the switch was at position 2 R and 3R were in series and their combination was in parallel with 2 R.
Let current through R_1 is i_2 which is 5 times of i_1, hence

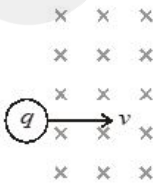
$$i_{3R} = i_2 \times \frac{2}{6}$$
$$V'_{3R} = \frac{i_2}{3} \times 3R = i_2 R$$

The potential through $3R$ and capacitor will be the same hence

$$\frac{Q'}{Q} = \frac{i_2 R}{\frac{i_1 R}{2}} = \frac{i_2}{i_1} \times 2 = 10 = 1 \times 10^1$$

Hence, the answer is the option (1).

Q. 4 A charged particle of charge q and mass m enters a magnetic field B perpendicularly as shown in the figure. The average force on a charged particle by the magnetic field is $\frac{kqvB}{\pi}$. Find k



Correct Answer:
2

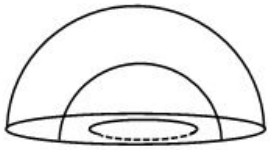
Solution:
This charged particle will perform a circular motion in a magnetic field and come out in a straight line as the field ends. It will complete a half circle in a magnetic field and will come out with the same speed.

Time spent in magnetic field will be $\frac{\pi m}{qB}$
Hence average force on the particle will be

$$F = \frac{\Delta P}{t}$$
$$= \frac{2mv}{\frac{\pi m}{qB}}$$
$$= \frac{2qvB}{\pi}$$

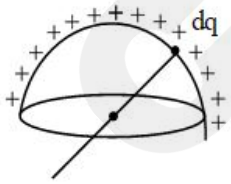
Change in momentum will have a magnitude of 2 mv
Hence, the answer is 2.

Q. 5 N hemispherical shells of the same charge density are kept concentric to each other. Their radii are R, 2R,3R .... and charge density is σ. The potential at the center of the shells is given as $\frac{\sigma R}{y \in_0} N(N+1).Find$ y



Correct Answer:
4

Solution:



$$V_{\text{net}} = V_1 + V_2 + V_3 \ldots$$
$$= K\int \frac{\mathrm{d}q}{R} + K\int \frac{\mathrm{d}q}{R}$$
$$= \frac{KQ_1}{R} + \frac{KQ_2}{2R} + \frac{KQ_3}{3R} \cdots$$
$$V_1 = \frac{1}{4\pi\epsilon_0}\frac{\sigma\pi R^2}{R} = \frac{\sigma R}{2\epsilon_0}$$
$$V_{n\in t} = \frac{\sigma R}{2\epsilon_0} + \frac{\sigma(2R)}{2\epsilon_0} + \frac{\sigma(3R)}{2\epsilon_0} \cdots$$
$$\Rightarrow V_{nst} = \frac{\sigma R}{2\epsilon_0}(1+2+3\ldots)$$
$$\Rightarrow V_{nst} = \frac{\sigma R}{4\epsilon_0}N(N+1)$$

Hence, the correct answer is 4.

Q. 6 A diatomic ideal gas is heated at constant volume until the pressure is doubled and again heated at constant pressure until volume is doubled. The average molar heat capacity for the whole process is $\frac{a}{b}R$. Find the remainder when a is divided by b

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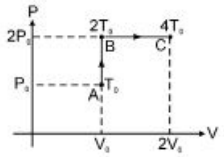
Correct Answer:
1

Solution:
Let initial pressure, volume and temperature be P_0, V_0, T_0 indicated by state A in the P-V diagram. The gas is isochorically taken to state $B\,(2P_0, V_0, 2T_0)$ and then taken from state B to state $\mathrm{C}\,(2P_0, 2V_0, 4T_0)$ isobarically.



Total heat absorbed by 1 mole of gas

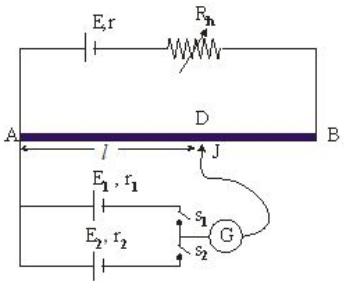
$$\Delta Q = C_v\,(2T_0 - T_0) + C_P\,(4T_0 - 2T_0)$$
$$= \frac{5}{2}RT_0 + \frac{7}{2}R \times 2T_0$$
$$= \frac{19}{2}RT_0$$

Total change in temperature from state A to C is: $\Delta T = 3T_0$

$\therefore$ Molar heat capacity $= \frac{\Delta Q}{\Delta T} = \frac{\frac{19}{2}RT_0}{3T_0} = \frac{19}{6}R$

Hence, the answer is 1.

Q. 7 In a given potentiometer circuit, cell E_1 is balanced at length ℓ_1 and cell E_2 is balanced at length ℓ_2.



a. If only E increases then ℓ_1, ℓ_2 decreases but $\frac{\ell_1}{\ell_2}$ remains the same
b. If only E increases then ℓ_1, ℓ_2 increases but $\frac{\ell_1}{\ell_2}$ decreases
c. If R_h increases then $\ell_1, \ell_2, \frac{\ell_1}{\ell_2}$ increases
d. If R_h increases then ℓ_1, ℓ_2 increases but $\frac{\ell_1}{\ell_2}$ remains the same

Your Answer:

Not Answered

Option 1:
(a) and (b) only

Option 2:
(b) and (d) only

Option 3:
(a) and (c) only

Option 4:
(a) and (d) only

Correct Answer:

(a) and (d) only

Solution:
$E_1 = \lambda\ell_1$ and $E_2 = \lambda\ell_2$ where $\lambda = \left(\frac{E}{r+R_h+R}\right)\frac{R}{\ell}$ and $\frac{E_1}{E_2} = \frac{\ell_1}{\ell_2} \Rightarrow \frac{\ell_1}{\ell_2}$ only depends on ratio $\frac{E_1}{E_2}$.

As E increases, λ increases so ℓ_1 and ℓ_2 decreases
As R_h increases λ decreases so ℓ_1 and ℓ_2 increases.

Hence, the answer is the option (4).

Q. 8 A uniformly charged spherical shell of radius R has charge density $+\sigma \mathrm{C/m^2}$. A point charge q is kept on centre of shell. Select correct alternative (s).

(A) Net electrostatic force on charge q is zero irrespective of its nature and magnitude
(B) Net electrostatic force on one hemispherical part is given as $F = \frac{\sigma^2}{2\epsilon_0}\pi R^2 + \frac{q\sigma}{2\epsilon_0}$
(C) Electric field at centre of shell is zero.
(D) Value of q for which net electrostatic force on one hemispherical part is zero, is given as $q = -2\sigma\pi R^2$

Your Answer:

Not Answered

Option 1:
Options (A) and (B) are correct.

Option 2:
Options (B) and (C) are correct.

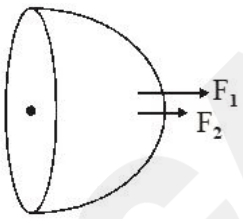
Option 3:
Options (A) and (D) are correct.

Option 4:
Options (A),(C) and (D) are correct.

Correct Answer:

Options (A),(C) and (D) are correct.

Solution:
Electric field at center due to the shell is zero so force on point charge is zero.



Force on one hemispherical part $= F_1 + F_2$
$F_1 \rightarrow$ Force due to other hemispherical part
$F_2 \rightarrow$ Force due to point charge

$F_1 = \left(\frac{\sigma^2}{2\epsilon_0}\right)\pi R^2$

Force on hemisphere due to point charge = force on point charge due to hemisphere

$\Rightarrow F_2 = (q)\left(\frac{\sigma}{4\epsilon_0}\right) = \frac{q\sigma}{4\epsilon_0}$

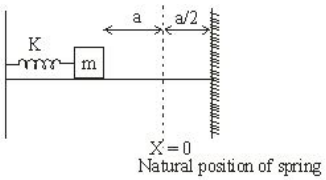
Net Force,

$F = \frac{\sigma^2 \pi R^2}{2\epsilon_0} + \frac{q\sigma}{4\epsilon_0}$

When $\mathrm{F} = 0 \Rightarrow q = -2\sigma\pi R^2$

Hence, the answer is the option (4).

Q. 9 In the given figure, if the block is released from rest and the collision with the wall is elastic, then (i) find out the maximum compression after the first collision with the wall. Also, (ii) what will be the answer if a collision is inelastic with e = ½? Mark the correct answers.



Your Answer:

Not Answered

Option 1:
a

Option 2:
$\frac{a}{2}$

Option 3:
$\frac{\sqrt{7}a}{4}$

Option 4:
Both 1 and 3

Correct Answer:

Both 1 and 3

Solution:
(i) As collision is perfectly elastic, no change in energy takes place after collision. Therefore maximum compression will be 'a' only.
(ii) If collision is inelastic then new amplitude will be calculated by conservation of energy after collision, $\{KE_i$ is kinetic energy after collision}

$$\frac{1}{2}\mathrm{mv_i^2} + \frac{1}{2}\,\mathrm{K}\left(\frac{\mathrm{a}}{2}\right)^2 = 0 + \frac{1}{2}\,\mathrm{K\,A_n^2}$$

v_i will be half of velocity with which block collides

$$\Rightarrow v_i = \frac{1}{2}\frac{\sqrt{3}a\omega}{2}$$

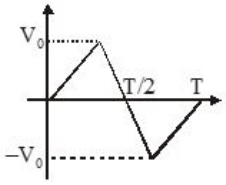
$$\Rightarrow A_n = \frac{\sqrt{7}a}{4}$$

Hence, the answer is the option (4).

Q. 10 For the given cases, select the correct answer:

(i) $V = A\sin\omega t + B\sin\omega t$

(ii)



Your Answer:

Not Answered

Option 1:

(i) $\text{Avg}(V) = 0$

Option 2:

(i) $\text{RMS(V)} = \sqrt{\frac{A^2+B^2}{2}}$

Option 3:

(ii) $\text{Avg}(V) = 0$

Option 4:

(ii) $\text{RMS(V)} = \frac{V_0}{\sqrt{3}}$

Correct Answer:

(i) $\text{Avg}(V) = 0$

(i) $\text{RMS(V)} = \sqrt{\frac{A^2+B^2}{2}}$

(ii) $\text{Avg}(V) = 0$

(ii) $\text{RMS(V)} = \frac{V_0}{\sqrt{3}}$

Solution:

$$\mathrm{Avg(V)}=\frac{\int_0^{\mathrm{T}}\mathrm{Vdt}}{\mathrm{T}},$$

$$\mathrm{RMS(\ V)}=\sqrt{\frac{\int_0^{\mathrm{T}}\ \mathrm{V^2dt}}{\mathrm{T}}}$$

(i) $\mathrm{Avg(V)}=0$

$$\mathrm{RMS(\ V)}=\sqrt{\frac{\mathrm{A^2+B^2}}{2}}$$

(ii) $\mathrm{Avg(V)}=0\{$ area under curve $=0\}$

$$\mathrm{RMS(V)}=\left(\frac{\int_0^{\mathrm{T}/4}4\left(\frac{4\,\mathrm{V_0t}}{\mathrm{T}}\right)^2\mathrm{dt}}{\mathrm{T}}\right)^{1/2}=\frac{\mathrm{V_0}}{\sqrt{3}}$$

Hence, the answer is the option(1), (2), (3) and (4).

Q. 11 A particle is moving on the x-axis such that its coordinate as a function of time is: $x=\frac{t^3}{3}-\frac{5t^2}{2}+6t+7$. Find out the correct option(s):

Option 1:
The particle will turn at t =2 and t = 3 seconds

Option 2:
Displacement in 6 seconds is 18m

Option 3:
both 1 and 2

Option 4:
none of the above

Correct Answer:
both 1 and 2

Solution:
At the turning point velocity becomes zero and changes its sign,

$\mathrm{v}=0\Rightarrow \mathrm{t^2-5t+6}=0\Rightarrow \mathrm{t}=2\ \mathrm{and}\ \mathrm{t}=3$
For, $\mathrm{t}=0$ to $\mathrm{t}=2\Rightarrow$ going right
$t=2$ to $t=3\Rightarrow$ going left.
$t=3$ to $t=\infty\Rightarrow$ going right.
So, $\mathrm{t}=2$ and $\mathrm{t}=3$ are the turning points.
By given formulae,

$x_6=\frac{6^3}{3}-\frac{5\times6^2}{2}+(6\times6)+7; x_6=25; x_0=7$
So displacement, $\mathrm{s}=18\ \mathrm{m}$

Hence, the answer is the option (3).

Q. 12 A container is half filled with water. It is made up of steel and is cylindrical in shape. The relation between α (of steel) and γ (of water) when

Your Answer:

Not Answered

Option 1:
Container remains half-filled on increasing temperature is $\gamma = 3\alpha$

Option 2:
Height of water column remains unchanged is $\gamma = 2\alpha$

Option 3:
Vacant space remains constant is $\gamma = 6\alpha$

Option 4:
All are incorrect

Correct Answer:

All are incorrect

Solution:
(A) We want the container to be half filled that means at all temperatures, the ratio of the volume of water and container remains constant.

$$\frac{\mathrm{V_{water}}}{\mathrm{V_{container}}} = \text{Constant (at all temperatures)}$$
$$\Rightarrow \frac{\mathrm{V/2}}{\mathrm{V}} = \frac{\mathrm{V/2(1+\gamma \Delta T)}}{\mathrm{V(1+3\alpha \Delta\ T)}}$$
$$\Rightarrow \gamma = 3\alpha$$

(B) The height of the water column is the ratio of the volume of water and the area of the base of the container.

$$\frac{V_{\mathrm{Water}}}{A_{\mathrm{Base}}} = \text{constant (at all temperature)}$$
$$\Rightarrow \quad \frac{V/2}{A} = \frac{V/2(1+\gamma \Delta T)}{A(1+2\alpha \Delta T)}$$
$$\Rightarrow \gamma = 2\alpha$$

(C) Vacant space is the difference between the volume of the container and the volume of water so

$$V_{\mathrm{con}} = V_{\mathrm{water}} = \text{constant (at all temperature)}$$
$$\Rightarrow V - \frac{V}{2} = V(1+3\alpha \Delta T) - \frac{V}{2}(1+\gamma \Delta T)$$
$$\Rightarrow \gamma = 6\alpha$$

Hence, the answer is the option (4).

Q. 13 Johnny was whirling his yoyo (0.5 kg) in a horizontal circle of radius 1 m at a height of 5m above the ground. The string of yoyo suddenly breaks and the yoyo finally strikes the ground at a horizontal separation of 20 m. Find out the maximum tension(nearest integer in N) that the yoyo string can bear.

Correct Answer:
200

Solution:
After the string breaks the yoyo will fall under gravity in a parabolic path.
Initial vertical velocity $= 0$

Hence by vertical motion analysis,
The time of flight will be,

$$t = \sqrt{\frac{2H}{g}} \Rightarrow t = \sqrt{\frac{2 \times 5}{10}} = 1s$$

The horizontal distance travelled $= 20\ \text{m}$
Hence speed of yoyo will be $= 20\ \text{ms}^{-1}$
In circular path, $\text{v} = 20\ \text{ms}^{-1}$

Hence centripetal acceleration,

$$a_c = \frac{v^2}{r}$$
$$a_c = 400\ \text{ms}^{-2}$$

The string will be slightly inclined to give centripetal acceleration and balance the weight.

$$T = \sqrt{\left(\frac{mv^2}{r}\right)^2 + (mg)^2}$$
$$T = 0.5\sqrt{(400)^2 + 10^2}$$
$$T = 200$$

This must be the breaking strength of the string.

Hence, The answer is 200.

Q. 14 Two holes are made in the vertical wall of a large open cuboidal tank. The shape of one hole is a square of side length L at a depth h from the top. The shape of the other hole is circular of radius R at a depth of 16 h from the top. When the tank is completely filled with water, the volume flow rate for both holes is the same. If, R is equal to $\frac{L}{k\sqrt{\pi}}$, find the value of k.

Correct Answer:
2.00

Solution:
We know that for a hole at a depth h, the speed of efflux is given as $\sqrt{2gh}$
If the square hole is considered as first hole and the circular hole is considered as second hole,

$A_1 = L^2$ and $A_2 = \pi R^2$
$V_1 = \sqrt{2gh}$ and $V_2 = \sqrt{32gh}$
As the volume flow rate for both is the same,

$Q = A_1V_1 = A_2V_2$
By putting the values,

$L^2 = 4\pi R^2 \Rightarrow R = \frac{L}{2\sqrt{\pi}}$
Hence, the answer is 2.

Q. 15 Let a radioactive substance A be converted into stable element B with half half-life of 1000 years. In an old sample ratio of the number of nuclei of A to the number of nuclei of B is 1: 7. Find the age of the sample in years, if the initial number of nuclei of A and B were the same.

Option 1:
2000

Option 2:
1000

Option 3:
1500

Option 4:
0

Correct Answer:
2000

Solution:

$A \longrightarrow \quad B$

$\text{At time}= \ = 0,$

$N_0 \quad N_0$

$\text{At time } = \text{t},$

$N \quad N_0 + N_0 - N$

$\Rightarrow \quad \frac{NA}{NB} = \frac{N}{2N_0 - N} = \frac{1}{7}$

$N = \frac{N_0}{4} = 2$

$\Rightarrow \quad 2 \text{ half life } = 2000 \text{ years.}$

Hence, the answer is the option (1).

Q. 16 A man of mass 80 kg is standing on the rim of a circular platform rotating about its axis with an angular velocity 1.2rads^{-1}. If the man moves to the centre of the platform, what is the work done (in J) by the man in the process? The mass of the platform is 200 kg and its radius is 2 m .

Correct Answer:
414.72

Solution:
Let the final angular velocity be ω. By conservation of angular momentum about an axis,

$(I_{\text{man}} + I_{\text{disc}})\omega_1 = (0 + I_{\text{disc}})\omega$

Finally, the moment of inertia of man will be zero. Hence to the given data,

$$\left(80 \times 2^2 + \frac{1}{2} \times 200 \times 2^2\right) \times 1.2 = \left(\frac{200}{2} \times 2^2\right)\omega$$

$$\Rightarrow \omega = \frac{10.8}{5}\text{rads}^{-1}$$

The work done by man is equal to the change in the kinetic energy of the system. Hence,

$W = \frac{1}{2}(I_{\text{disc}})\omega^2 - \frac{1}{2}(I_{\text{man}} + I_{\text{disc}})\omega_1^2$

By putting the values we get,

$W = 414.72 \text{ J}$

Hence the answer is 414.72.

Q. 17 A natural satellite of mass 987 kg is revolving in a circular orbit at a distance of 10200 km from the centre of Mars. Another satellite of mass 1319 kg is at a distance of 51000 from the centre of Mars. Their time periods are in the ratio of $\frac{1}{x\sqrt{y}}$. Find $x + y$

Correct Answer:
10

Solution:

The time period does not depend upon the mass of the satellite; it only depends upon the orbital radius.
Using Kepler's law,

$\frac{T_1}{T_2} = \left(\frac{r_1}{r_2}\right)^{3/2} = \left(\frac{10200}{51000}\right)^{3/2} = \left(\frac{1}{5}\right)^{3/2} = \frac{1}{5\sqrt{5}}$
Hence $x = 5$ and $y = 5$ and answer is 10

Hence the answer is 10.

Q. 18 A string fixed at both ends is vibrating in three loops. The length of string is 'l' and amplitude of antinode is 'A'. The amplitude of a particle at a distance $\frac{\ell}{4}$ from one end is $\frac{A}{\sqrt{x}}$. Find x.

Correct Answer:
2

Solution:
There will be a node at any end as the string is fixed at both the ends.

Let x = 0 is a node. Hence amplitude at any point at a distance x is given as:

$A_x = |A\sin(Kx)| \quad \text{where } K = \frac{2\pi}{\lambda}$

As string is vibrating in 3 loops and each loop length is equal to $\frac{\lambda}{2}$, hence

$$\frac{3\lambda}{2} = \ell$$
$$\Rightarrow \quad \lambda = \frac{2\ell}{3}$$
$$\Rightarrow \quad K = \frac{3\pi}{\ell}$$

Therefore, amplitude at distance $\frac{\ell}{4}$ from one end will be

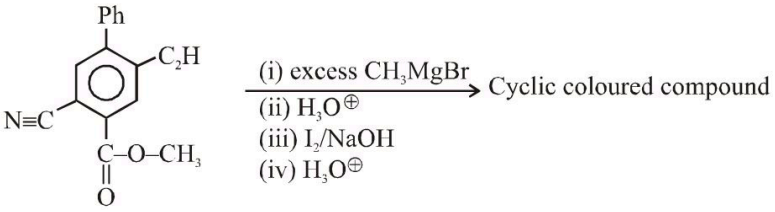
$A_x = A\sin\left(\frac{3\pi}{\ell} \times \frac{\ell}{4}\right) = \frac{A}{\sqrt{2}}$

Hence answer is x=2

Hence, the answer is 2.

Chemistry

Q. 1



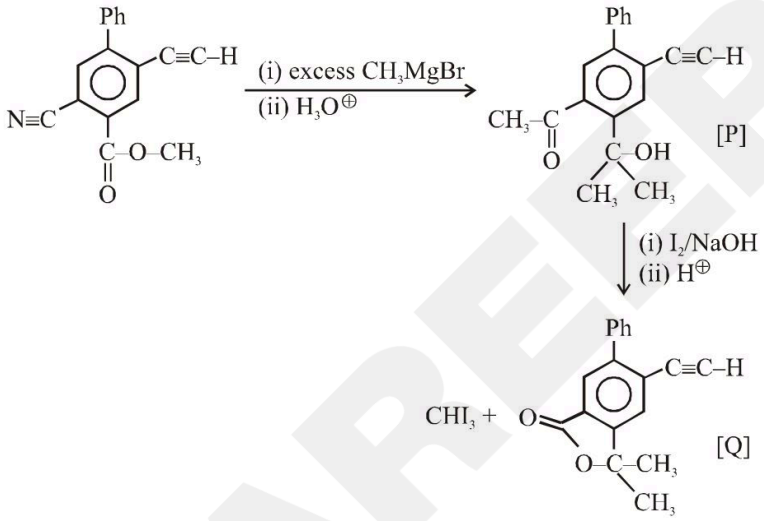
Degree of unsaturation (DU) in cyclic coloured compound is-

If in case the DU is greater than 9, then report the answer as the sum of the digits in the DU

(For example, if the DU is 15 then report the answer as 1+5 i.e. 6)

Correct Answer:
3

Solution:

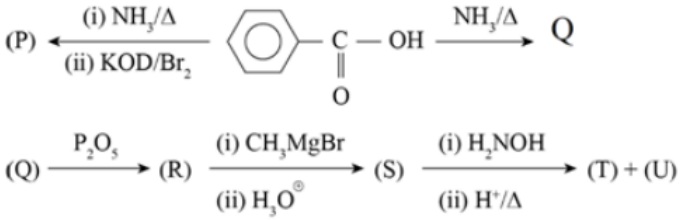


D.U = 12.00

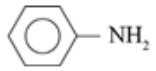
Answer will be 1 + 2 = 3.

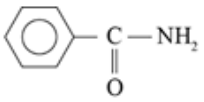
Hence, the answer is 3.

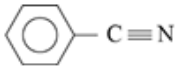
Q. 2

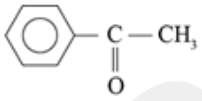


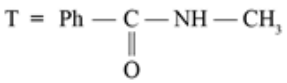
How many of the following representations are correct for the above reaction sequence?

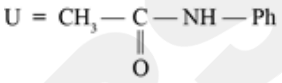
I) P = 

II) Q = 

III) R = 

IV) S = 

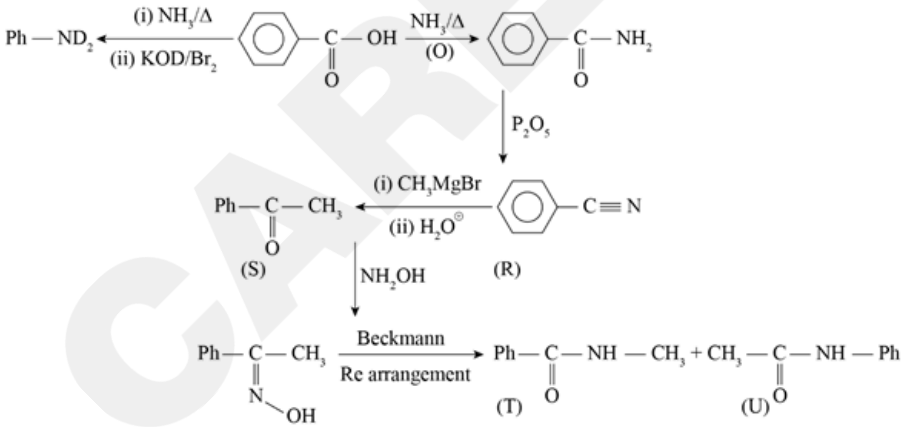
V) 

VI) 

Correct Answer:

5

Solution:



(II), (III), (IV), (V), (VI) are correct.

Q. 3 How many of the given oxides are amphoteric

$Na_2O, CaO, Al_2O_3, SnO_3, BeO, V_2O_5, CO, ZnO, PbO_2\ N_2O$

Correct Answer:
6

Solution:

Na_2O	Basic
CaO	Basic
Al_2O_3	Amphoteric
SnO_2	Amphoteric
BeO	Amphoteric
V_2O_5	Amphoteric
CO	Neutral
ZnO	Amphoteric
PbO_2	Amphoteric
N_2O	Neutral

Hence, the answer is (6).

Q. 4

The radial wave function for 1s orbital of H-atom is $\mathrm{R(r)}=\frac{1}{\sqrt{\pi}}\left(\frac{1}{\mathrm{a_0}}\right)^{3/2}\mathrm{e}^{-\mathrm{r}/\mathrm{a_0}}$ (where $\mathrm{a_0}=0.529\ \mathrm{A}$)

If the ratio of **radial probability density** of finding electron at $\mathrm{n}=\mathrm{a_0}$ to the radial probability density of finding electron at the nucleus is given as $(x\cdot e^{-y})$

Calculate (y-x)

Correct Answer:
1

Solution:
The probability density is given as $R^2(r)$ Thus, the ratio can be calculated as

$$\frac{\mathrm{R}^2\left(\mathrm{a_0}\right)}{\mathrm{R}^2(0)}=\left(\frac{1}{\pi \mathrm{a}_0^3}\right)\frac{\mathrm{e}^{-2\mathrm{r}/\mathrm{a_0}}}{1\times \mathrm{e}^0}\times\left(\pi \mathrm{a}_0^3\right)=\mathrm{e}^{-2}$$

Given: $=\mathrm{x}\cdot \mathrm{e}^{-\mathrm{y}}=\mathrm{e}^{-2}$

$$\therefore \quad x=1 \quad \& y=2$$
$$\therefore \quad y-x=2-1=1$$

Q. 5 which of the following inferences is correct for the statements given below:

Statement I: ClF_2^+ ion is bent, but ClF_2^- is linear
Statement II: Salt-like KHF_2 is stable, however, $KHCl_2$ is not known.

Statement III: Cyanides are insoluble in water, but alkyl isocyanides are readily soluble in water.

Option 1:
Only statements (I) and (II) are correct.

Option 2:
Statement (II) is incorrect and can be explained based on hydrogen bonding.

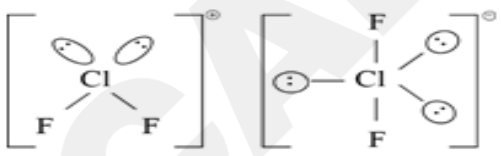
Option 3:
Statement (III) is incorrect and can be explained based on ion-induced dipole interactions.

Option 4:
Both statements (II) and (III) can be explained based on hydrogen bonding.

Correct Answer:
Only statements (I) and (II) are correct.

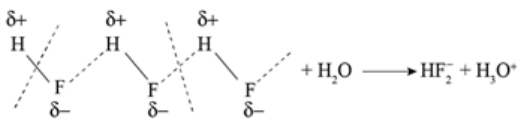
Solution:
Statement I: ClF_2^+: Number of lone pairs on $Cl = 2$
Number of bond pairs on $Cl = 2$
$\therefore$ Hybridization of $Cl = sp^3$
Shape of ClF_2^+ =Bent
ClF_2^-: Number of lone pairs on $Cl = 3$
Number of bond pairs on $Cl = 2$
$\therefore$ Hybridization of $C1 = sp^3d$

Shape of ClF_2^- =Linear



(Bent shape of ClF_2^+) (Linear shape of ClF_2)

Statement II: In an aqueous solution, HF dissociates into HF_2^- ion instead of F^- ion. This occurs due to hydrogen bonding in HF .



On the other hand hydrogen bonding does not occur in aqueous solution of HCl . Thus, $KHCl_2$ does not exist.

Statement III: Lower alkyl cyanides are soluble in water due to formation of hydrogen bonds.

$$R-C^{\delta+}\equiv N^{\delta-}---H^{\delta+}-O-H^{\delta+}---N\equiv C-R$$

Alkyl isocyanides are insoluble in water since the nitrogen atom does not have a lone pair of electrons and hence, cannot form hydrogen bonds.

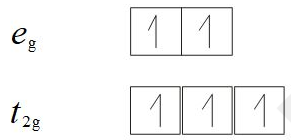
Hence, the answer is the option (1).

Q. 6 For a high spin cationic complex $(Cr(H_2O)_6)^{+x}$, maximum possible value of ' x ' is ' p ' and minimum possible value is ' q ' then. Find the value of $p+q$ will be:

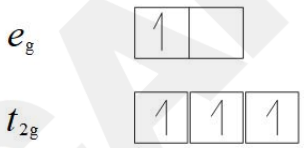
Correct Answer:
3

Solution:
$(Cr(H_2O)_6)^{+x}$ is high spin cationic complex then minimum possible value of ' x ' will be +1

$$Cr^{+1}\Rightarrow [Ar]3d^5$$

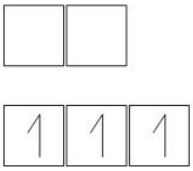


Complex has high spin.
For $x=2 \quad Cr^{+2}\Rightarrow 3\,d^4$



Complex has high spin.

$$x=3 \quad Cr^{+3}\Rightarrow 3\,d^3$$



It will be a d^2sp^3 hybridised complex with its spin not defined
Maximum possible value of ' x ' will be 2
Minimum possible value of ' x ' will be 1
Hence, value of $p+q=3$.

Q. 7 Which of the following statement(s) is true?

Your Answer:

Not Answered

Option 1:
Sucrose has a pyranose and a furanose ring linked together by C_1, C_2- glycosidic linkage

Option 2:
Maltose has one glycosidic linkage and a free hemiacetal end.

Option 3:
Behaviour of hydrolysis product of maltose and sucrose is similar towards plane polarised light

Option 4:
Sucrose is a dextrorotatory sugar.

Correct Answer:

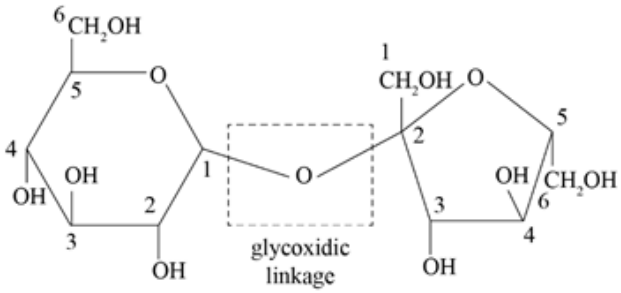
Sucrose has a pyranose and a furanose ring linked together by C_1, C_2- glycosidic linkage

Maltose has one glycosidic linkage and a free hemiacetal end.

Sucrose is a dextrorotatory sugar.

Solution:
Sucrose is dextrorotatory but a non-reducing sugar. It contains a pyranose and a furanose ring linked together by C_1, C_2 - glycoside bond. On hydrolysis, it gives $(+)$ D-glucose and $(-)\mathrm{D}$-fructose. The α is more for later, hence the solution becomes laevorotatory. Maltose is a reducing sugar because of the presence of free hemiacetal group. It contains $\alpha-1,4$-glyco-sidic linkage.

Sucrose:



Sucrose structure has a pyranose and a furanose ring linked together by a glycosidic linkage. Glycosidic linkage exists between C_1
of α-glucose and C_2 of β-fructose.
Sucrose is dextrorotatory but after hydrolysis gives dextrorotatory glucose and laevorotatory fructose and hence will have different behaviour towards plane polarised light.

Maltose is a reducing sugar because of the presence of free hemiacetal group.

Q. 8 Which of the following statement(s) is/are correct for Boric acid?

Your Answer:

Not Answered

Option 1:
It is a Lewis Acid.

Option 2:
All hydrogens are ionizable in water.

Option 3:
It is monobasic in water.

Option 4:
Central atom is sp^3 hybridized.

Correct Answer:

It is a Lewis Acid.

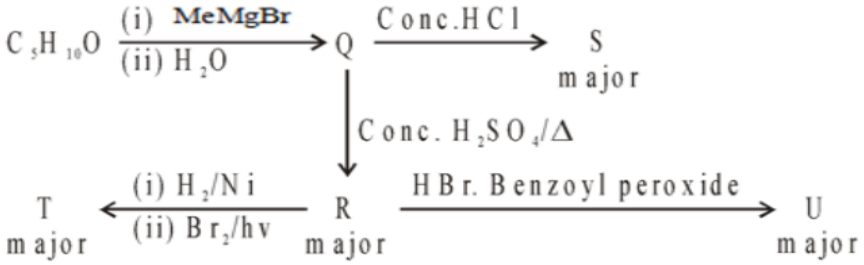
It is monobasic in water.

Solution:
In Boric acid, Boron atom has incomplete octet, it has 6 electrons only, hence it is Lewis acid.
All the hydrogens are non ionizable, it is not Arrhenius acid.
Boron atom is sp 2 hybrid, with three bonds with OH groups, it is trigonal planar molecule.
It is monobasic in water-

$$B(OH)_3 + HOH \longrightarrow [B(OH)_4]^- + H^+$$

Hence, the answer is the options (1 & 3).

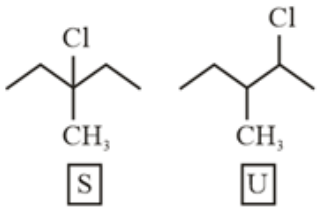
Q. 9 In given set of reaction choose the correct option(s) for the following set of reaction



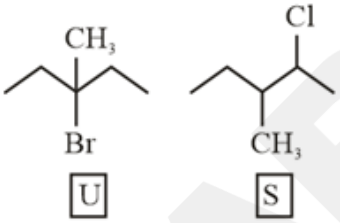
Your Answer:

Not Answered

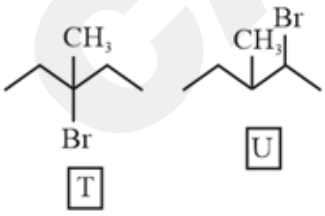
Option 1:



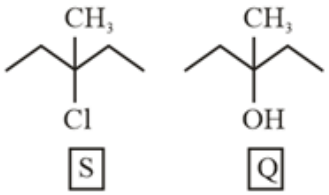
Option 2:



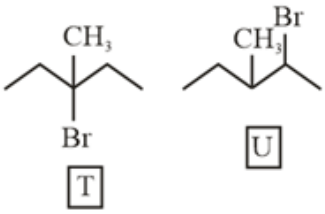
Option 3:

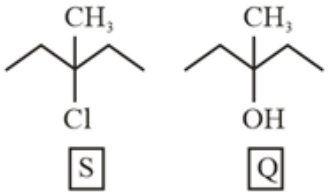


Option 4:

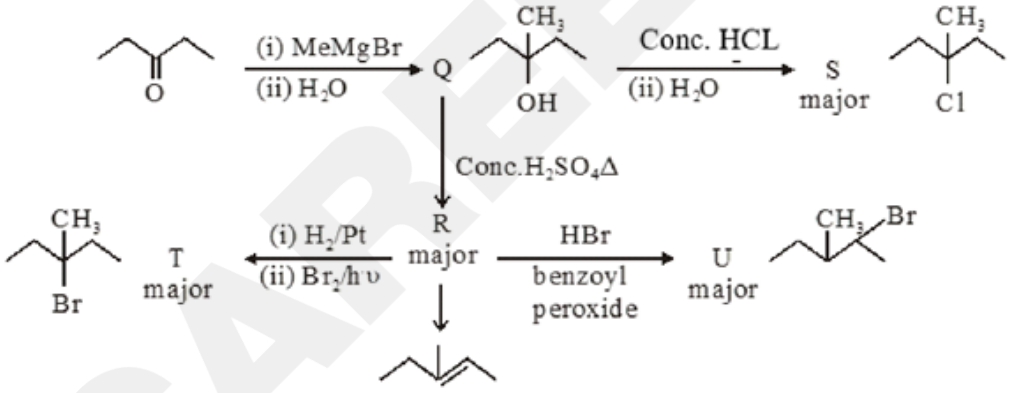


Correct Answer:





Solution:



Hence, the option 3,4 are correct

Q. 10 Which of the following statement(s) is/are not correct?

Your Answer:

Not Answered

Option 1:

All alkali metals form oxides on burning in air.

Option 2:
The solubility of chlorides and hydroxides of alkaline earth metals in water increase on descending the group.

Option 3:
$Be(OH)_2$ is basic in nature.

Option 4:
The tendency of complex formation of alkaline earth metals increases down the group.

Correct Answer:

$Be(OH)_2$ is basic in nature.

The tendency of complex formation of alkaline earth metals increases down the group.

Solution:
Except $Be(OH)_2$, all other hydroxides of alkaline earth metals are basic in nature.

$Be(OH)_2$, is an amphoteric oxide.

The tendency to form complexes decreases down the group.

Q. 11 Which of the statement(s) is/are correct?

Your Answer:

Not Answered

Option 1:
Hydrolysis constant of acetate ion is larger than that of cyanide ion.

Option 2:
The degree of dissociation of water increases with increase in temperature

Option 3:
$pK_w^\circ = pH + pOH$ holds good in any aqueous solution at all temperatures.

Option 4:
The solution of NaCl is acidic in nature.

Correct Answer:

The degree of dissociation of water increases with increase in temperature

$pK_w^\circ = pH + pOH$ holds good in any aqueous solution at all temperatures.

Solution:
(A) $K_a(HCN) < K_a(CH_3COOH)$. Since $K_h = \frac{K_w}{K_a}$, K_h for CN^-

ions is larger than CH_3COO^- ions.

(B) The degree of dissociation of water increases with increase in temperature - Correct

(C) $pK_w^\circ = pH + pOH$ holds good in any aqueous solution at all temperatures - Correct

(D) The solution of NaCl is neutral as the ions Na^+ and Cl^- are obtained from strong base and strong acid respectively.

Hence, the answer is the option (2, 3).

Q. 12 In the reaction,

$$Br-(CH_2)_{12}-C\equiv CH \xrightarrow{NaNH_2} (X) \xrightarrow[\text{catlyst}]{\text{Lindar's}} (Y)$$

which of the following statement(s) is/are correct?

Your Answer:

Not Answered

Option 1:
Formation of product (X) involves an acid-base reaction.

Option 2:
The product (X) is $H_2N-(CH_2)_{12}-C\equiv C^-Na^+$

Option 3:
The product (Y) exhibits geometrical isomerism.

Option 4:
The product (Y) exists in a cyclic form.

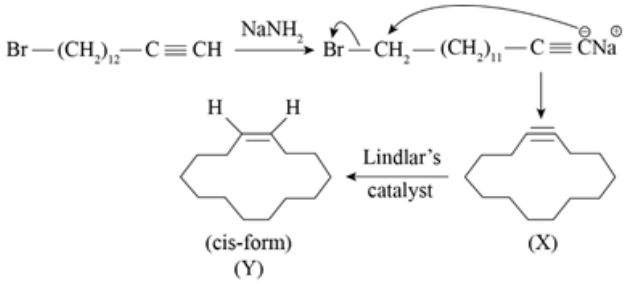
Correct Answer:

Formation of product (X) involves an acid-base reaction.

The product (Y) exhibits geometrical isomerism.

The product (Y) exists in a cyclic form.

Solution:
The given reaction occurs as follows:



The first step involves an acid base reaction.

Hence, the answer is the option (1),(3),(4).

Q. 13 If 18 g of an alkaline earth metal is exposed to nitrogen gas with desired conditions, it forms 25 g of its nitride. Find out the atomic weight of this metal.

Correct Answer:
24

Solution:
$3M + \mathrm{N}_2 \rightarrow M_3\,\mathrm{N}_2$

Let atomic weight of metal $= \mathrm{a}$
Hence $(3a + 28)\mathrm{g}$ nitride contains metal $= (3a)\mathrm{g}$

$\therefore 14.8\ \mathrm{g\ nitride\ contains\ metal} = \frac{3a}{3a+28} \times 25 = 18$

Therefore, $\mathrm{a} = 24$

Hence , the answer is (24).

Q. 14 Consider the redox reaction in acidic medium: $\mathrm{Mn}^{2+} + \mathrm{BiO}_3^- \rightarrow \mathrm{MnO}_4^- + \mathrm{Bi}^{3+}$ The number of moles of acid that are required for the change from BiO_3^- to Bi^{3+} per mole of BiO_3^- is

Correct Answer:
2.80

Solution:
In this reaction, BiO_3^- is reduced by accepting electrons which are donated by Mn^{2+} that is getting oxidized. Balancing the redox equation, we get:

$$\left[Mn^{2+} + 4H_2O \rightarrow MnO_4^- + 8H^+ + 5e^-\right] \times 2$$
$$\left[BiO_3^- + 6H^+ + 2e^- \rightarrow Bi^{3+} + 3H_2O\right] \times 5$$

Hence, the answer is option (1).

$$2Mn^{2+} + 5BiO_3^- + 14H^+ \rightarrow 2MnO_4^- + 5Bi^{3+} + 7H_2O$$

From the balanced equation, it is clear that total of 10 electrons are involved in the reaction, and 2.8 mol of acid are required by per mole of BiO_3^-.

Q. 15 Find the ratio of the weight of solvent to solute in a solution ifit is given that the vapour pressure of the solution of a non-volatile solute B in a solvent A is 90% of the vapour pressure of the solvent at the same temperature. It is given that the molecular weight of the solvent is 0.5 times the molecular weight of the solute.

Correct Answer:
4.5

Solution:
According to the question,

$$P = 0.90P^0$$

According to Raoult's law,

$$P = P^0\chi$$
$$\text{Given } M_A = 0.5M_B$$
$$0.90P^0 = P^0\left(\frac{\frac{W_A}{M_A}}{\frac{W_A}{M_A} + \frac{W_B}{M_B}}\right)$$
$$M_A = \text{ molecular weight of solvent}$$
$$M_B = \text{ molecular weight of solute}$$
$$0.90 = \frac{\frac{W_A}{0.5M_B}}{\frac{W_A}{0.5M_B} + \frac{W_B}{M_B}}$$
$$W_A = \text{ gram weight of solvent}$$
$$W_B = \text{ gram weight of solute}$$
$$\text{On solving, } \frac{W_A}{W_B} = 4.5$$

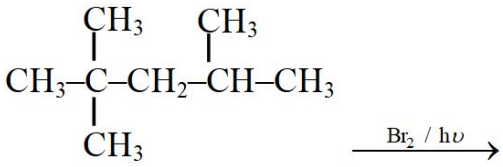
Hence, the answer is the option (1)

Q. 16 For the coagulation of 200 mL of As_2S_3 solution, 10 mL of 1 M NaCl is required. What is the coagulating value of NaCl ?

Correct Answer:
50

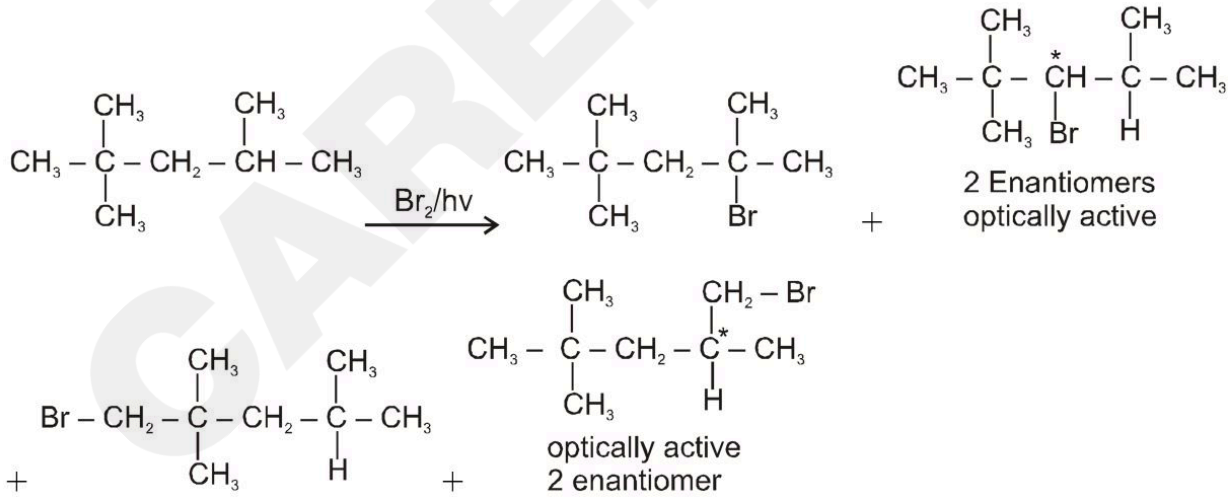
Solution:
Coagulating value = number of millimoles of solute needed for coagulation of 1 litre of solution.
10 mL of 1 M NaCl contains $NaCl = 10 \times 1 = 10$ millimoles
200 ml of As_2S_3 required NaCl for the coagulation $= 10$ millimoles
$\therefore$ 1000ml of As_2S_3 required NaCl for the coagulation $= 10 \times \frac{1000}{200} = 50$ millimoles

Q. 17 For the given reaction, how many monohalo products are optically active (all isomers)?

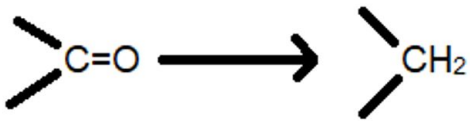


Correct Answer:
4

Solution:



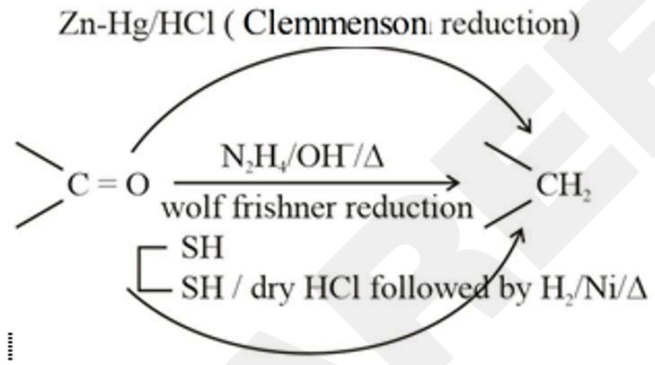
Q. 18 How many of the following reagents can be used to get following conversion \



(A) B_2H_6 |THF $_{\text{followed by}}$ H_2O|ACOH
(B) Hydrazine / base / Δ
(C) $NaBH_4$
(D) $Zn-Hg/HCl$
(E) Red P/I_2
(F) $LiAlH_4$
(G) $HS-CH_2-CH_2-SH/dry HCl$; Followed by $H_2/Ni/\Delta$
(H) dil H_2SO_4
(I) $I_2/NaOH_{\text{followed by Red}}$ P and I_2

Correct Answer:
3

Solution:



Maths

Q. 1 If $= \sum_{r=1}^{24} \left(\omega^r + \frac{1}{\omega^r}\right)^2$, where ω is complex cube root of unity, then value of $\frac{k}{6}$ is

Option 1:
8

Option 2:
6

Option 3:
0

Option 4:
-1

Correct Answer:
8

Solution:

$\omega = e^{i\pi/3}, \omega^r + \frac{1}{\omega^t} = 2\cos\frac{r\pi}{3}$

$k = \sum_{r=1}^{24}\left(\omega^2 + \frac{1}{\omega^r}\right)^2 = \sum_{r=1}^{24} 2^2\cos^2\frac{r\pi}{3}$

$= 2\sum_{r=1}^{24}\left(1+\cos\frac{2r\pi}{3}\right)$

$= 2\left[24 + \sum_{r=1}^{24}\cos\frac{2\pi r}{3}\right]$

$= 2\left[24 + \frac{\cos\left(\frac{2\pi}{3}+\frac{23\pi}{3}\right)\sin\frac{24\pi}{3}}{\sin\frac{\pi}{3}}\right]$

$= 48$

$\therefore \frac{k}{6} = \frac{48}{6} = 8$

Hence, the answer is the option (1).

Q. 2 Die ' A ' has 3 face marked with 4,2 faces marked with 5 and 1 face marked with 6 . Die ' B ' has 3 face marked 1,2 faces marked with 2 and 1 face marked 3 . If both dice are rolled together, $P(E)$ is probability of getting sum ' x ' of number shown by dice. Then value of x when $P(E)$ is maximum

Correct Answer:
6

Solution:

x can be $5, 6, 7, 8, 9$

number of ways in which sum of 5, 6, 7, 8,9 can occur are the coefficient of x^5, x^6, x^7, x^8, x^9 in

$\left(3x^4 + 2x^5 + x^6\right)\left(3x + 2x^2 + x^3\right) = x^9 + 4x^8 + 10x^7 + 12x^6 + 9x^5$

As greatest coefficient is 12 occur with x^6. So $P(E)$ is maximum when $x = 6$

Hence, The answer is 6.

Q. 3 If a hyperbola $xy - 2x - y - 2 = 0$ cuts a circle $x^2 + y^2 + 2gx + 2fy + c = 0$ at points $((-1, 0), (2, 6), (5, 3)$ and $(3, 4)$, then the value of $|f + g|$ is

Option 1:
8

Option 2:
-

Option 3:
-

Option 4:
-

Correct Answer:
8

Solution:
$xy-2x-y-2=0 \quad \Rightarrow \quad (x-1)(y-2)=4_{\text{is a rectangular hyperbola.}}$

When a circle cuts a rectangular hyperbola, the AM of points of intersection is the mid-point of the centers of the hyperbola and circle.
So,

$$\frac{-1+2+3+5}{4}=\frac{-g+1}{2} \Rightarrow \mathrm{g}=\frac{-7}{2}$$
$$\frac{0+3+4+6}{4}=\frac{-\mathrm{f}+2}{2} \Rightarrow \mathrm{f}=\frac{-9}{2}$$

Hence, the answer is 8

Q. 4 Let $\mathbf{a}_{\mathrm{r}}=\mathbf{r}^{10}\mathrm{C}_{\mathrm{r}}$ and $\mathrm{b}_{\mathrm{r}}=(10-\mathrm{r})^{10}\mathrm{C}_{\mathrm{r}}$ and matrix $\mathrm{A}_{\mathrm{r}}=\begin{bmatrix} a_r & 0 \\ 0 & b_r \end{bmatrix}$ and $\mathrm{A}=\sum_{\mathrm{r}=1}^{9}\mathrm{A}_{\mathrm{r}}$, then sum of digits of $\mathrm{tr}(A)$ is

Correct Answer:
5

Solution:

$$A=\begin{bmatrix} \sum_{r=1}^{9}\mathrm{r}^{10}\mathrm{C}_{\mathrm{r}} & 0 \\ 0 & \sum_{\mathrm{r}=1}^{9}(10-\mathrm{r})^{10}\mathrm{C}_{\mathrm{r}} \end{bmatrix}$$
$$=\begin{bmatrix} \sum_{\mathrm{r}=1}^{9}\mathrm{r}^{10}\mathrm{C}_{\mathrm{r}} & 0 \\ 0 & \sum_{\mathrm{r}=1}^{9}(10-\mathrm{r})^{10}\mathrm{C}_{10-\mathrm{r}} \end{bmatrix}$$
$$=\begin{bmatrix} 10\left(2^9-1\right) & 0 \\ 0 & 10\left(2^9-1\right) \end{bmatrix}$$

$\mathrm{tr}(\mathrm{A})=20\left(2^9-1\right)=10220$

Hence, the answer is 5

Q. 5 If $(x) = \frac{3^{2x-1}}{3^{2x-1}+1}$ and $f\left(\frac{1}{2001}\right) + f\left(\frac{2}{2001}\right) + \ldots + f\left(\frac{2000}{2001}\right) = a^b$ then $a - b$ is

Option 1:
7

Option 2:
10

Option 3:
6

Option 4:
8

Correct Answer:
7

Solution:
$f(x) = \frac{3^{2x-1}}{3^{2x-1}+1}$

$f(1-t) = \frac{3^{2(1-t)-1}}{3^{2(1-t)-1}+1}$

$f(1-t) = \frac{3^{1-2t}}{3^{1-2t}+1}$

$f(x) + f(1-x) = 1$

$S = f\left(\frac{1}{2001}\right) + f\left(\frac{2}{2001}\right) + \cdots + f\left(\frac{2000}{2001}\right)$

$f\left(\frac{1}{2001}\right) + f\left(\frac{2000}{2001}\right) = 1$

$f\left(\frac{2}{2001}\right) + f\left(\frac{1999}{2001}\right) = 1$

$S = 1000 \times 1 = 1000$

We are given $S = a^b$. Since $S = 1000$, write $1000 = 10^3$, where $a = 10$ and $b = 3$.

$a - b = 10 - 3 = 7$

Hence, the answer is the option (1).

Q. 6 If $[x]$ denotes greatest integer part of x , then value of $\left[\int_2^3 \frac{dx}{\sqrt{x^3+1}}\right]$ is

Correct Answer:
0

Solution:

$$\frac{1}{(1+x)^2} \leq \frac{1}{\sqrt{x^3+1}} \leq \frac{1}{1+x} \text{ for } x \in [2,3]$$

$$\Rightarrow \quad \int_2^3 \frac{1}{(1+x)^2}dx \leq \int_2^3 \frac{dx}{\sqrt{x^3+1}} \leq \int_2^3 \frac{dx}{1+x}$$

$$\Rightarrow \quad \frac{1}{12} \leq \int_2^3 \frac{dx}{\sqrt{x^3+1}} \leq \ln\frac{4}{3}$$

$$\Rightarrow \quad \frac{1}{12} \leq \int_2^3 \frac{dx}{\sqrt{x^3+1}} < 1$$

$$\Rightarrow \quad \left[\int_2^3 \frac{dx}{\sqrt{x^3+1}}\right] = 0$$

Hence, The answer is 0.

Q. 7 If $f(x) = \begin{cases} \max\left\{\sqrt{9-x^2}, \sqrt{1+x^2}\right\}, & -3 \leq x \leq 0 \\ \min\left\{\sqrt{9-x^2}, \sqrt{1+x^2}\right\}, & 0 < x \leq 3 \end{cases}$, then $\mathrm{f(x)}$ has

Your Answer:

Not Answered

Option 1:
A point of discontinuity at x=0

Option 2:
A point of maxima at x=-2 and point of minima at x=2

Option 3:
A point of minima at x=-2 and a point of maxima at x=2

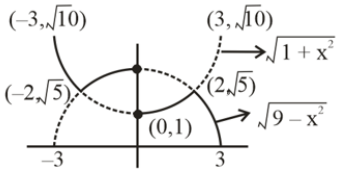
Option 4:
No turning point

Correct Answer:

A point of discontinuity at x=0

A point of minima at x=-2 and a point of maxima at x=2

Solution:
Answer (A, C)



From the graph, it is clear that $f(x)$ is not continuous at x = 0

at x = -2, minima is there

at x = 2, maxima is there

Hence, the answer is the option (1) and (3).

Q. 8 The distance of a variable plane from the origin is p (constant). If the plane cuts coordinate axes in D, E, and F then

Your Answer:

Not Answered

Option 1:
Locus of centroid of tetrahedron ODEF is $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\frac{16}{p^2}$

Option 2:
The Locus of the centroid of tetrahedron ODEF is
$\frac{1}{\mathrm{x}^2}+\frac{1}{\mathrm{y}^2}+\frac{1}{\mathrm{z}^2}=\frac{4}{\mathrm{p}^2}$

Option 3:
Parametric equation of the centroid of tetrahedron is $\left(\frac{\mathrm{p}}{4\cos\alpha\cos\beta},\frac{\mathrm{p}}{4\cos\alpha\sin\beta},\frac{\mathrm{p}}{4\sin\alpha}\right)$

Option 4:
none of these

Correct Answer:

Locus of centroid of tetrahedron ODEF is $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\frac{16}{p^2}$

Parametric equation of the centroid of tetrahedron is $\left(\frac{\mathrm{p}}{4\cos\alpha\cos\beta},\frac{\mathrm{p}}{4\cos\alpha\sin\beta},\frac{\mathrm{p}}{4\sin\alpha}\right)$

Solution:
Equation of plane $\mathrm{lx}+\mathrm{my}+\mathrm{nz}=\mathrm{p}$ where $\mathrm{l},\mathrm{m},\mathrm{n}$ are direction cosines of normal of plane

$\mathrm{D}\left(\frac{\mathrm{p}}{\ell}, 0, 0\right), \mathrm{E}\left(0, \frac{\mathrm{P}}{\mathrm{m}}, 0\right), \mathrm{F}\left(0, 0, \frac{\mathrm{p}}{\mathrm{n}}\right)$

$\text{centroid } \left(\left(\frac{\mathrm{p}}{4\ell}, \frac{\mathrm{p}}{4\,\mathrm{m}}, \frac{\mathrm{p}}{4\mathrm{n}}\right) = (\mathrm{x}, \mathrm{y}, \mathrm{z})\right)$

$\text{As } l^2 + m^2 + n^2 = 1 \Rightarrow \frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{16}{p^2}$

Also, $\left(\frac{\mathrm{p}}{4\cos\alpha\cos\beta}, \frac{\mathrm{p}}{4\cos\alpha\sin\beta}, \frac{\mathrm{p}}{4\sin\alpha}\right)$ satisfy the equation of locus of centroid.

Hence, the correct options are (1) and (3).

Q. 9 If $\vec{a}, \vec{b}, \vec{c}$ are linearly independent vectors such that $\vec{r} = (\vec{d} \times \vec{c}) \times (\vec{a} \times \vec{b}) + (\vec{d} \times \vec{a}) \times (\vec{b} \times \vec{c}) + (\vec{d} \times \vec{b}) \times (\vec{c} \times \vec{a})$ then

Your Answer:

Not Answered

Option 1:

$\vec{d}(\vec{a} \times \overrightarrow{\mathbf{r}}) = 0$

Option 2:

$\vec{r}$ is collinear with $\vec{d}$

Option 3:

$\vec{r}$ is a linear combination of $\vec{a}, \vec{b}$ and $\vec{c}$

Option 4:

If $\vec{c} \cdot \overrightarrow{\mathrm{m}} = 0 \cdot |\overrightarrow{\mathrm{m}}| = |\overrightarrow{\mathrm{c}}|, [(\overrightarrow{\mathrm{b}} + \overrightarrow{\mathrm{c}}) \times (\overrightarrow{\mathrm{b}} + \overrightarrow{\mathrm{m}})] \times (\overrightarrow{\mathrm{c}} + \overrightarrow{\mathrm{m}}) \cdot \overrightarrow{\mathrm{c}} = [\overrightarrow{\mathrm{bdc}}]|\overrightarrow{\mathrm{d}}|^2$, where $\vec{m} = \frac{\vec{r}}{-2[\vec{a}\vec{b}\vec{c}]}$

Correct Answer:

$\vec{d}(\vec{a} \times \overrightarrow{\mathbf{r}}) = 0$

$\vec{r}$ is collinear with $\vec{d}$

$\vec{r}$ is a linear combination of $\vec{a}, \vec{b}$ and $\vec{c}$

If $\vec{c} \cdot \overrightarrow{\mathrm{m}} = 0 \cdot |\overrightarrow{\mathrm{m}}| = |\overrightarrow{\mathrm{c}}|, [(\overrightarrow{\mathrm{b}} + \overrightarrow{\mathrm{c}}) \times (\overrightarrow{\mathrm{b}} + \overrightarrow{\mathrm{m}})] \times (\overrightarrow{\mathrm{c}} + \overrightarrow{\mathrm{m}}) \cdot \overrightarrow{\mathrm{c}} = [\overrightarrow{\mathrm{bdc}}]|\overrightarrow{\mathrm{d}}|^2$, where $\vec{m} = \frac{\vec{r}}{-2[\vec{a}\vec{b}\vec{c}]}$

Solution:

Since $\vec{a}, \vec{b}, \vec{c}$ they are linearly independent, $[\vec{a}\vec{b}\vec{c}] \neq 0$

$\vec{d}$ can be written as a linear combination of $\vec{a}, \vec{b}$ and $\vec{c}$

$\overrightarrow{d} = \frac{[\vec{d}\,\vec{b}\,\vec{c}]\vec{a}+[\vec{d}\,\vec{c}\,\vec{a}]\vec{b}+[\vec{d}\,\vec{a}\,\vec{b}]\vec{c}}{[\vec{a}\,\vec{b}\,\vec{c}]}$

Now

$\vec{r} = [\vec{d}\vec{b}\vec{a}]\vec{c} - [\vec{d}\vec{b}\vec{c}]\vec{a} + [\vec{d}\vec{c}\vec{b}]\vec{a} - [\vec{d}\vec{c}\vec{a}]\vec{b} + [\vec{d}\vec{a}\vec{c}]\vec{b} - [\vec{d}\vec{a}\vec{b}]\vec{c}$

$= 2([\vec{d}\vec{c}\vec{b}]\vec{a} + [\vec{d}\vec{a}\vec{c}]\vec{b} + [\vec{d}\vec{b}\vec{a}]\vec{c})$

$= -2[\vec{a}\vec{b}\vec{c}]d$

$(\vec{b}+\vec{c}) \times (\vec{b}+\vec{d}) \times (\vec{c}+\vec{d}) \cdot \vec{c} = ((\vec{c} \times \vec{b} + \vec{c} \times \vec{d} + \vec{b} \times \vec{d})) \times ([\vec{c}+\vec{d}]) \cdot \vec{c}$

$= ([\vec{c}\vec{b}\vec{d}]\vec{c} - [\vec{b}\vec{d}\vec{c}]\vec{d}) \cdot \vec{c}$

$= [\vec{b}\vec{d}\vec{c}]|\vec{d}|^2$

Hence, the answer is the option (1),(2),(3),(4).

Q. 10 Let PQR be a triangle with fixed base $QR = 1$ and vertex 'P' is variable. If one base angle is double of the other base angle then, locus of 'P' is conic having

Your Answer:

Not Answered

Option 1:
Length of Latus rectum 2

Option 2:
Length of Latus rectum 4

Option 3:
Eccentricity is 4

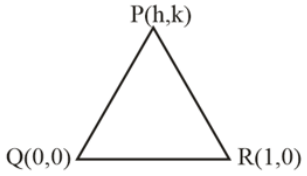
Option 4:
Eccentricity is 2

Correct Answer:

Length of Latus rectum 2

Eccentricity is 2

Solution:



$$\angle Q = 2\angle R$$
$$\tan Q = \frac{2\tan R}{1-\tan^2 R}$$
$$\Rightarrow \quad \frac{k}{h} = \frac{\frac{2k}{1-h}}{1-\frac{k^2}{(1-h)^2}}$$
$$\Rightarrow \quad \frac{\left(h-\frac{2}{1}\right)^2}{\frac{1}{y}} - \frac{k^2}{\frac{1}{3}} = 1$$
$$e = 2, LR = 2$$

Hence, the answer is the Option (1) and (4)

Q. 11 A function $f : \left[0, \frac{\pi}{2}\right] \to R \quad f(0) = 0$
$\int \cos x e^{f(x)} \left(x - 4\sin x \cos x - x\tan^2 x\right)dx = \cos x \cos 2x e^{f(x)} + c$. If $g(x) = \frac{x}{1+f(x)}$, then

Your Answer:

Not Answered

Option 1:
$f(x) = 1$ has no solution

Option 2:
$g(x)$ has one local extremum in $\left(0, \frac{\pi}{2}\right)$

Option 3:
$g(x)$ has exactly one local minimum in $\left(0, \frac{\pi}{2}\right)$

Option 4:
$\left(\frac{d}{dx}(f(x))\right)^{-1}$ at $x = \frac{\pi}{4}$ is $\frac{2}{\pi+2}$

Correct Answer:

$g(x)$ has one local extremum in $\left(0, \frac{\pi}{2}\right)$

$\left(\frac{d}{dx}(f(x))\right)^{-1}$ at $x = \frac{\pi}{4}$ is $\frac{2}{\pi+2}$

Solution:
$$f(x) = x\tan x : g(x) = \frac{x}{1 + x\tan x}$$
$$g'(x) = \frac{1 + x\tan x - x\left(x\sec^2 x + \tan x\right)}{(1 + x\tan x)^2}$$
$$= \frac{(\cos x - x)(\cos x + x)}{\cos^2 x(1 - x\tan x)^2}$$

For original point

$g'(x) = 0 \quad \text{at } x = \alpha$

$\Rightarrow \quad g'(x) > 0, x \in (0, \alpha)$

$g'(x) < 0, x \in \left(\alpha, \frac{\pi}{2}\right)$

$$\frac{1}{\frac{d}{dx}(f(x))_{x=\frac{\pi}{4}}} = \frac{1}{x\sec^2 x + \tan x}$$

$$= \frac{2}{\pi + 2}$$

Hence, the answer is the option (2) and (4).

Q. 12 If $\left(1 + x + x^2\right)^n = a_0 + a_1 x + a_2 x^2 + \ldots + a_{2n} x^{2n}$, when n is a odd integer. If $S_1 = \sum_{r=0} a_{4r}$, $S_2 = \sum_{r=0} a_{4r+1}$, $S_3 = \sum_{r=0} a_{4r+2}$ $S_4 = \sum_{r=0} a_{4r+3}$, then

Your Answer:

Not Answered

Option 1:
$S_1 = S_3$

Option 2:
$S_2 = S_4$

Option 3:
$S_2 + S_4 = 0$

Option 4:
Either $S_1 = S_2 = S_3$ or $S_1 = S_3 = S_4$

Correct Answer:

$S_1 = S_3$

Solution:
$\left(1 + x + x^2\right)^n = a_0 + a_1 x + a_2 x^2 + \ldots\ldots + a_{2n} x^{2n}$

Put $x = 1$ and $x = -1$ then add and subtract obtained equation to get $\Rightarrow \quad a_1 + a_3 + a_5 + \ldots \cdot \frac{3^n - 1}{2}$

Put $x = i$ in given expression, If $n = 4m + 1$

$\Rightarrow \quad a_1 - a_3 + a_5 - a_7 + \ldots = 1 \qquad S_2 = \frac{3^n + 1}{4}$

$\Rightarrow a_0 - a_2 + a_4 - a_6 + \ldots\ldots = 0 \quad \text{So,} \quad S_4 = \frac{3^n - 3}{4}$

Similarly if $n = 2m + 3$

$S_2 = \frac{3^n-3}{4}, S_4 = \frac{3^n+1}{4}$

Hence, the answer is the Option (1).

Q. 13 4 different dice are rolled together, and number appearing on top listed. Then total number of outcomes when the largest number appearing in the list is not 5,

Correct Answer:

927

Solution:

$$\text{Number of ways} = \text{Total case} - (\text{when highest number is } 5)$$

$$6^4 - (5^4 - 4^4)$$

$$= 927$$

Hence, The answer is 927.

Q. 14 Five digit number are formed using the digit 0 to 9, without repetition. If x is the number of number when successive digit are in decreasing order and y, when successive digits are in ascending order, then x – y is

Correct Answer:

126

Solution:

$x = {}^{10}C_5, y = {}^9C_5$

Because after selection of digits, they can be arranged in one way only.

Hence, The answer is 126.

Q. 15 26 cards from a pack of 52 card are lost. From the remaining 26 cards, 3 cards are drawn. The probability that all 3 are king is $\frac{1}{a \cdot b \cdot c}$, when a, b, c are prime numbers then a+b+c

Correct Answer:

155

Solution:

As 3 kings are to be drawn, hence in 26 lost cards either one king can lost or none of the king.

$$P = \frac{^{48}C_{26}}{^{52}C_{26}} \times \frac{^{4}C_{3}}{^{26}C_{3}} + \frac{^{4}C_{1}{}^{48}C_{25}}{^{52}C_{26}} \times \frac{^{3}C_{3}}{^{26}C_{3}}$$

$$= \frac{23}{13.17.26.49} + \frac{26}{13.17.26.49}$$

$$\text{So } = \frac{1}{13.17.25}$$

Hence, The answer is 155.

Q. 16 $y = C_1e^{ax} + C_2e^{bx} + C_3e^{cx}$, where C_1, C_2, C_3 are arbitrary constants and a, b, c are roots of $t^3 - 7t + 6 = 0$. If given curve y , also satisfy $y''' - 7y'' + ky = 0$, then k is equal to

Correct Answer:
6

Solution:
Put y''', y'' and y in given DE to get $k = 6$

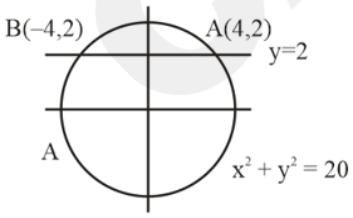
Hence, The answer is 6.

Q. 17 θ is angle between the curve $x^2 + y^2 = 20$ and $y = 1 + [|\sin x| + |\cos x|]$, where $[.]denotes$ greatest integer function. Then value of $|\cot\theta|$ is

Correct Answer:
0.5

Solution:
As $1 \le |\sin x| + |\cos x| \le \sqrt{2}$
So, $[|\sin x| + |\cos x|] = 1$
So, $y = 2$
Point of intersection will be $A(4, 2)$ and $B(-4, 2)$. Slope of tangent at point A is $\frac{-x}{y}$
$\therefore$ Slope of tangent is -2 and 2

So, $\tan\theta = 2 \Rightarrow \cot\theta = \frac{1}{2}$



Hence, The answer is 0.5.

Q. 18 A continuous function $f:[0,1] \rightarrow R$ which satisfy the equation $\mathrm{yf(x)+xf(y)} \leq 1$ where $x, y \in [0,1]$. If maximum value of $\int_0^1 \mathrm{f(x)dx}$ is l, then $\frac{l}{\pi}$ is

Correct Answer:
0.25

Solution:
Put $x = \sin\theta$

$\int_0^1 f(x)dx = \int_0^{\pi/2} f(\sin\theta)\cos\theta d\theta \quad ....1$

Put $x = \cos\theta$

$\int_0^1 f(x)dx = \int_0^{\pi/2} f(\cos\theta)\sin\theta d\theta \quad .....2$

On adding equations 1 and 2

$$2\int_0^1 \mathrm{f(x)} = \int_0^{\pi/2} (\mathrm{f}(\sin\theta)\cos\theta + \mathrm{f}(\cos\theta)\sin\theta)\mathrm{d}\theta \leq \int_0^{\pi/2} 1.\,\mathrm{dx}$$

$$\Rightarrow \int_0^1 \mathrm{f(x)} \leq \frac{\pi}{4} \quad \Rightarrow \quad \ell = \frac{\pi}{4}$$

Hence, The answer is 0.25.