CAREERS 360

PREPARATION Series

COMDEK 2025

Preparation Guide

Contents

About

Welcome to the definitive resource for mastering the Consortium of Medical, Engineering, and Dental Colleges of Karnataka (COMEDK) in 2025. Crafted with precision and expertise, this ebook is your companion on the journey to success in one of Karnataka's most esteemed entrance examinations.

COMEDK serves as the gateway to prestigious undergraduate programs in various disciplines across the Karnataka state. As the landscape of education continually evolves, staying ahead with thorough preparation and strategic insights becomes indispensable. This guide is meticulously curated to equip aspiring candidates like you with the tools, knowledge, and confidence needed to excel in COMEDK 2025

What You Will Find in the Ebook:

- **Introduction:** Gain a thorough understanding of COMEDK, its significance, and its role in shaping your academic journey.
- **Exam Dates:** Stay updated with crucial dates and deadlines to ensure you're on track with your preparation schedule.
- **Exam Pattern:** Familiarize yourself with the structure, format, and marking scheme of COMEDK 2025 to formulate an effective exam strategy.
- **Syllabus:** Delve into the detailed syllabus, ensuring you cover all the essential topics and domains as prescribed for COMEDK 2025. Preparation Study Material: Access curated study materials designed to cater specifically to the syllabus and requirements of COMEDK 2025, aiding in comprehensive preparation.
- **Mock Tests:** Put your knowledge to the test with a series of mock tests meticulously crafted to simulate the actual exam environment. These tests are invaluable for assessing your progress, identifying strengths and weaknesses, and fine-tuning your exam-taking strategy. Embark on your COMEDK 2025 preparation journey with confidence, armed with the insights and resources provided in this comprehensive guide.

Let this ebook be your trusted companion as you strive towards achieving your academic aspirations and unlocking the doors to a brighter future.

Happy learning!

Warm regards,
Team Careers360

COMDEK Common Entrance Test

Introduction

COMEDK, the Consortium of Medical, Engineering, and Dental Colleges of Karnataka, annually conducts a computer-based entrance examination for admission to undergraduate courses in engineering, medical, and dental colleges across Karnataka. The exam assesses candidates' knowledge based on the syllabus of their 10+2 or equivalent examinations. Eligibility for the COMEDK exam requires candidates to meet specified academic qualifications. Following qualification, candidates attend mandatory counseling sessions where seats in colleges are allotted based on merit and preferences. Final seat allotment is determined by both merit and the choices made by candidates during counseling sessions.

Exam Dates

The COMDEK 2025 exam date is 10 May 2025.

Exam Pattern

COMEDK Exam Pattern 2025

Particulars	Details
Physics	60
Chemistry	60
Mathematics	60
TOTAL	**60**

Syllabus

Subject	Chapters / Topics
PHYSICS	Units and Measurements
	Motion in a Straight Line
	Motion in a Plane
	Laws of Motion
	Work, Energy, and Power
	System of Particles and Rotational Motion
	Gravitation
	Mechanical Properties of Solids Mechanical Properties of Fluids
	Thermal Properties of Matter
	Thermodynamics
	Kinetic Theory
	Oscillations Waves
	Electric Charges and Fields
	Electrostatic Potential and Capacitance
	Current Electricity
	Moving Charges and Magnetism
	Magnetism and Matter
	Electromagnetic Induction
	Alternating Current
	Electromagnetic Waves
	Ray Optics and Optical Instruments
	Wave Optics
	Dual Nature of Radiation and Matter
	Atoms Nuclei
	Semiconductor Electronics: Materials, Devices and Simple Circuits
CHEMISTRY	Some Basic Concepts of Chemistry
	Structure of Atom
	Classification of Elements and Periodicity in Properties
	Chemical Bonding and Molecular Structure
	Chemical Thermodynamics
	Equilibrium
	Redox Reactions
	Organic Chemistry - Some Basic Principles and Techniques

Subject	Chapters / Topics
CHEMISTRY	Hydrocarbons
	Solutions
	Electrochemistry
	Chemical Kinetics
	The d- and f-Block Elements
	Coordination Compounds
	Haloalkanes and Haloarenes
	Alcohols, Phenols and Ethers
	Aldehydes, Ketones and Carboxylic Acids
	Amines
	Biomolecules
MATHS	Sets Relations & Functions
	Trigonometric Functions
	Complex Numbers and Quadratic Equations
	Linear Inequalities
	Permutations and Combinations
	Binomial Theorem
	Sequence and Series
	Straight Lines
	Conic Sections
	Introduction to Three-dimensional Geometry
	Limits and Derivatives
	Statistics Probability
	Relations and Functions
	Inverse Trigonometric Functions
	Matrices Determinants
	Continuity and Differentiability
	Application of Derivatives
	Integrals Application of Integrals
	Differential Equations
	Vector Algebra
	Three Dimensional Geometry
	Linear Programming
	Probability

Preparation Study Material

Physics

Chapter Name - Physics and Measurement	
Concept Name	**Study link**
Physical Quantity	STUDY HERE
Fundamental And Derived Quantities And Units	STUDY HERE
System Of Unit	STUDY HERE
Practical Units	STUDY HERE
Dimensions Of Physical Quantities	STUDY HERE
Dimensionless Quantities	STUDY HERE
Applications Of Dimensional Analysis	STUDY HERE
Significant Figures	STUDY HERE
Errors Of Measurements	STUDY HERE

Chapter Name - Kinematics	
Concept Name	**Study link**
Kinematics Terminologies	STUDY HERE
Mathematical Tools	STUDY HERE
Scalars And Vectors	STUDY HERE
Vector Addition And Vector Subtraction	STUDY HERE
Multiplication Of Vectors	STUDY HERE
Distance And Displacement	STUDY HERE
Speed And Velocity	STUDY HERE
Acceleration	STUDY HERE
Kinematics Graphs	STUDY HERE
Equation Of Motions	STUDY HERE
Uniform Circular Motion	STUDY HERE
Motion Of Body Under Gravity	STUDY HERE
Projectile Motion	STUDY HERE
Horizontal Projectile Motion	STUDY HERE
Equation Of Path Of A Projectile	STUDY HERE

Projectile On An Inclined Plane	STUDY HERE
Relative Velocity	STUDY HERE
Boat River Problem	STUDY HERE
Rain-man Problem	STUDY HERE

Chapter Name - Laws Of Motion	
Concept Name	**Study link**
Inertia	STUDY HERE
Forces	STUDY HERE
Common Forces In Mechanics	STUDY HERE
Equilibrium Of Concurrent Forces	STUDY HERE
Newton's First Law Of Motion	STUDY HERE
Linear Momentum	STUDY HERE
Newton's Laws Of Motion	STUDY HERE
Acceleration Of Block On Horizontal Smooth Surface	STUDY HERE
Acceleration Of Block On Smooth Inclined Plane	STUDY HERE
Motion Of Bodies In Contact	STUDY HERE
Motion Of Blocks When Connected With String	STUDY HERE
Motion Of Connected Blocks Over Pulley	STUDY HERE
Incline With Mass And Pulley	STUDY HERE
Spring Force	STUDY HERE
Apparent Weight Of Body In A Lift	STUDY HERE
Recoiling Of Gun	STUDY HERE
Rocket Propulsion	STUDY HERE
Friction	STUDY HERE
Kinetic Friction	STUDY HERE
Static Friction	STUDY HERE
Graph Between Applied Force And The Force Of Friction	STUDY HERE
Angle Of Repose	STUDY HERE
Calculation Of Necessary Force In Different Conditions On Rough Surface	STUDY HERE
Acceleration Of Block Against Friction	STUDY HERE
Motion Of Two Bodies One Resting On The Other	STUDY HERE
Motion Of An Insect In The Rough Bowl	STUDY HERE
Minimum Mass Hung From The String To Just Start The Motion	STUDY HERE
Maximum Length Of Hung Chain	STUDY HERE

Coefficient Of Friction Between A Body And Wedge	STUDY HERE
Stopping Of Block Due To Friction	STUDY HERE
Sticking Of A Block With Accelerated Cart	STUDY HERE
Centripetal Force And Centrifugal Force	STUDY HERE
Sticking Of Person With The Wall Of Rotor	STUDY HERE
Skidding Of Vehicle On A Level Road	STUDY HERE
Skidding Of Object On A Rotating Platform	STUDY HERE
Bending A Cyclist	STUDY HERE
Banking Of Road	STUDY HERE
Reaction Of Road On Car	STUDY HERE
Centripetal Force For Non-uniform Circular Motion	STUDY HERE

Chapter Name - Rotational Motion	
Concept Name	**Study link**
Rotational Motion Of Rigid Body	STUDY HERE
Center Of Mass	STUDY HERE
Center Of Mass Of The Uniform Rod	STUDY HERE
Centre Of Mass Of Semicircular Ring	STUDY HERE
Centre Of Mass Of Semicircular Disc	STUDY HERE
Centre Of Mass Of A Triangle	STUDY HERE
Centre Of Mass Of Hollow Hemisphere	STUDY HERE
Centre Of Mass Of Solid Hemisphere	STUDY HERE
Centre Of Mass Of Hollow Cone	STUDY HERE
Centre Of Mass Of A Solid Cone	STUDY HERE
Motion Of The Centre Of Mass	STUDY HERE
Relationship Between Linear And Angular Motion	STUDY HERE
Torque	STUDY HERE
Rotational Equilibrium	STUDY HERE
Moment Of Inertia	STUDY HERE
Moment Of Inertia Of A Rod	STUDY HERE
Moment Of Inertia Of A Rectangular Plate	STUDY HERE
Parallel And Perpendicular Axis Theorem	STUDY HERE
Moment Of Inertia Of A Ring	STUDY HERE
Moment Of Inertia Of A Disc	STUDY HERE
Moment Of Inertia Of Hollow Cylinder	STUDY HERE

Moment Of Inertia Of The Solid Cylinder	STUDY HERE
Moment Of Inertia Of Hollow Sphere	STUDY HERE
Moment Of Inertia Of A Solid Sphere	STUDY HERE
Moment Of Inertia Of Solid Cone	STUDY HERE
Angular Momentum	STUDY HERE
Law Of Conservation Of Angular Momentum	STUDY HERE
Work, Energy And Power For Rotating Body	STUDY HERE
Rigid Bodies: Translational Motion And Rotational Motion	STUDY HERE
Rolling Without Slipping	STUDY HERE
Rolling Without Slipping On An Inclined Plane	STUDY HERE

Chapter Name - Gravitation	
Concept Name	**Study link**
Newton's Law Of Gravitation	STUDY HERE
Acceleration Due To Gravity	STUDY HERE
Mass And Density Of Earth	STUDY HERE
Gravitational Field Intensity	STUDY HERE
Gravitational Potential	STUDY HERE
Gravitational Potential Energy	STUDY HERE
Relation Between Gravitational Field And Potential	STUDY HERE
Work Done Against Gravity	STUDY HERE
Kepler's Laws Of Planetary Motion	STUDY HERE
Escape Velocity	STUDY HERE
Orbital Velocity	STUDY HERE
Orbital Velocity of Satellite	STUDY HERE
Time Period And Energy Of A Satellite	STUDY HERE
Geostationary And Polar Satellites	STUDY HERE
Weightlessness	STUDY HERE

Chapter Name - Properties of Solids and Liquids	
Concept Name	**Study link**
Elasticity	STUDY HERE
Stress And Strain	STUDY HERE
Stress Strain Relationship	STUDY HERE
Hooke's Law	STUDY HERE

Work Done In Stretching A Wire	STUDY HERE
Relation Between Volumetric Strain, Lateral Strain And Poisson’s Ratio	STUDY HERE
Pressure In A Fluid	STUDY HERE
Variation Of Pressure	STUDY HERE
Pascal's Law	STUDY HERE
Variation Of Pressure In An Accelerated Fluid	STUDY HERE
Barometer And Manometer	STUDY HERE
Archimedes Principle	STUDY HERE
Type Of Flow	STUDY HERE
Equation Of Continuity	STUDY HERE
Bernoulli's Theorem	STUDY HERE
Viscosity	STUDY HERE
Stokes' Law And Terminal Velocity	STUDY HERE
Surface Tension	STUDY HERE
Surface Energy	STUDY HERE
Excess Pressure	STUDY HERE
Contact Angle	STUDY HERE
Capillary Action	STUDY HERE
Temperature And Its Scales	STUDY HERE
Thermometer And Its Types	STUDY HERE
Thermal Expansion	STUDY HERE
Thermal Stress And Thermal Strain	STUDY HERE
Thermal Expansion In Liquids And Gases	STUDY HERE
Heat	STUDY HERE
Change Of State	STUDY HERE
Triple Point Of Water	STUDY HERE
Joule's Law Of Heating	STUDY HERE
Calorimetry Principle	STUDY HERE
Heating Curve	STUDY HERE
Heat Transfer	STUDY HERE
Staedy Conduction Heat Transfer	STUDY HERE
Law Of Thermal Conductivity	STUDY HERE
Electrical Analogy For Thermal Conduction	STUDY HERE
Combination Of Metallic Rods	STUDY HERE
Heat Transfer By Convection	STUDY HERE

Heat Transfer By Radiation	STUDY HERE
Black Body Radiation	STUDY HERE
Kirchhoff's Law	STUDY HERE
Wien's Displacement Law	STUDY HERE
Stefan Boltzmann Law	STUDY HERE
Newton's Law Of Cooling	STUDY HERE

Chapter Name - Kinetic Theory of Gases	
Concept Name	**Study link**
States Of Matter	STUDY HERE
Kinetic Theory Of Gases Assumptions	STUDY HERE
The Gas Laws	STUDY HERE
Ideal Gas Equation	STUDY HERE
Real Gas And Equation	STUDY HERE
Pressure Of An Ideal Gas	STUDY HERE
The Maxwell Distribution Laws	STUDY HERE
Mean Free Path	STUDY HERE
Degree Of Freedom	STUDY HERE
Kinetic Energy Of Ideal Gas	STUDY HERE
Specific Heat Of A Gas	STUDY HERE
Mayer's Formula	STUDY HERE

Chapter Name - Thermodynamics	
Concept Name	**Study link**
Introduction To Thermodynamics	STUDY HERE
Thermodynamic State Variables And Equation Of State	STUDY HERE
Thermodynamic Equilibrium	STUDY HERE
Heat, Internal Energy And Work - Thermodynamics	STUDY HERE
First Law Of Thermodynamics	STUDY HERE
Isobaric Process	STUDY HERE
Isochoric Process	STUDY HERE
Isothermal Process	STUDY HERE
Adiabatic Process	STUDY HERE
Polytropic Process	STUDY HERE
Cyclic And Non Cyclic Process	STUDY HERE
Reversible And Irreversible Process	STUDY HERE

Heat Engine	STUDY HERE
Second Law Of Thermodynamics	STUDY HERE
Entropy	STUDY HERE
Carnot Engine	STUDY HERE
Refrigerator Or Heat Pump	STUDY HERE

Chapter Name - Electrostatics	
Concept Name	**Study link**
Electric Charge	STUDY HERE
Electric Charge And Electrification	STUDY HERE
Coulomb's Law	STUDY HERE
Electric Field	STUDY HERE
Electric Field Lines	STUDY HERE
Electric Field Intensity: Continuous Charge Distribution	STUDY HERE
Electric Field Due To A Uniformly Charged Ring	STUDY HERE
Electric Field Of Charged Disk	STUDY HERE
Electric Field Due To An Infinitely Long Charged Wire	STUDY HERE
Motion of charged particle in uniform electric field	STUDY HERE
Electric Dipole	STUDY HERE
Electric Field Of A Dipole	STUDY HERE
Torque On An Electric Dipole In A Uniform Electric Field	STUDY HERE
Electric Flux	STUDY HERE
Electric Flux Through Cone Or Disc	STUDY HERE
Gauss Law And It's Application	STUDY HERE
Electric Potential	STUDY HERE
Relation Between Electric Field And Potential	STUDY HERE
Electric Potential Of Uniformly Charged Ring, Rod, And Disc	STUDY HERE
Potential Due To Hollow Conductiong, Solid Conducting, Hollow Non Conducting	STUDY HERE
Electric Potential Of A Dipole And System Of Charges	STUDY HERE
Equipotential Surface	STUDY HERE
Electrostatic Potential Energy	STUDY HERE
Potential Energy Of A Dipole In An Electric Field	STUDY HERE
Capacitor And Capacitance	STUDY HERE
The Parallel Plate Capacitor	STUDY HERE

Spherical And Cylindrical Capacitors	STUDY HERE
Combination Of Capacitors - Parallel And Series	STUDY HERE
Energy Stored In Capacitor	STUDY HERE
Dielectrics	STUDY HERE

Chapter Name - Oscillations and Waves	
Concept Name	**Study link**
Periodic And Oscillatory Motions	STUDY HERE
Simple Harmonic Motion (S.H.M.) And Its Equation	STUDY HERE
Important Terms In Simple Harmonic Motion	STUDY HERE
Simple Harmonic Motion And Uniform Circular Motion	STUDY HERE
Composition Of Two SHM	STUDY HERE
Energy In Simple Harmonic Motion	STUDY HERE
Oscillations Of A Spring-mass System	STUDY HERE
Oscillation Of Two Particle System	STUDY HERE
Oscillation Of Pendulum	STUDY HERE
Angular Simple Harmonic Motion	STUDY HERE
Physical Pendulum	STUDY HERE
Motion Of A Ball In Tunnel Through The Earth	STUDY HERE
Time Period Of Torsional Pendulum	STUDY HERE
The Oscillation Of Floating Bodies	STUDY HERE
Free, Forced And Damped Oscillation	STUDY HERE
Wave Motion	STUDY HERE
Types Of Wave	STUDY HERE
Travelling Waves	STUDY HERE
Travelling Sine Wave	STUDY HERE
Relation Between Phase Difference And Path Difference	STUDY HERE
Speed Of Transverse Wave On A String	STUDY HERE
Power Transmitted Along The String	STUDY HERE
Interference And Principle Of Superposition	STUDY HERE
Reflection And Transmission Of Waves On A String	STUDY HERE
Standing Wave On A String	STUDY HERE
Sonometer	STUDY HERE
Sound Wave	STUDY HERE
Propagation Of Sound Wave	STUDY HERE

Displacement Wave And Pressure Wave	STUDY HERE
Velocity Of Sound In Different Media	STUDY HERE
Newton's Formula For The Velocity Of Sound In Gas	STUDY HERE
Intensity Of Sound Waves	STUDY HERE
Sound Wave Interference	STUDY HERE
Standing Sound Waves	STUDY HERE
End Correction	STUDY HERE
Resonance Column Method	STUDY HERE
Beats	STUDY HERE
Doppler Effect	STUDY HERE

Chapter Name - Current Electricity	
Concept Name	**Study link**
Electric Current	STUDY HERE
Current Density	STUDY HERE
Drift Velocity	STUDY HERE
Ohms Law	STUDY HERE
Mobility Of Electron	STUDY HERE
Resistance And Resistivity	STUDY HERE
Change Of Resistance In Wires By Stretching	STUDY HERE
Resistor Colour Code	STUDY HERE
Heat And Power Developed In A Resistor	STUDY HERE
Resistors In Series And Parallel Combinations	STUDY HERE
Emf Of A Cell	STUDY HERE
Internal Resistance Of A Cell	STUDY HERE
Current Given By A Cell	STUDY HERE
Grouping Of Cells	STUDY HERE
Charging Of Battery And Discharging Of Battery	STUDY HERE
Kirchhoff First Law	STUDY HERE
Kirchhoff's Second Law	STUDY HERE
Galvanometer	STUDY HERE
Ammeter	STUDY HERE
Voltmeter	STUDY HERE
Wheatstone's Bridge	STUDY HERE
Meter Bridge	STUDY HERE

Potentiometer - Principle And Applications	STUDY HERE
Calculation Of Resistance By Symmetry	STUDY HERE
Charging Of Capacitor And Inductor	STUDY HERE

Chapter Name - Magnetic Effects of Current and Magnetism	
Concept Name	**Study link**
Biot-savart Law	STUDY HERE
Magnetic Field Due To Current In Straight Wire	STUDY HERE
Magnetic Field Due To Circular Current Loop	STUDY HERE
Magnetic Field On The Axis Of Circular Current Loop	STUDY HERE
Ampere's Circuital Law And Its Applications	STUDY HERE
Solenoid	STUDY HERE
Toroid	STUDY HERE
Force On A Moving Charge In Magnetic Field	STUDY HERE
Motion Of A Charged Particle In Uniform Magnetic Field	STUDY HERE
Lorentz Force	STUDY HERE
Cyclotron	STUDY HERE
Force On A Conductor Carrying Current In A Magnetic Field	STUDY HERE
Force Between Two Parallel Current Carrying Conductors	STUDY HERE
Torque On Current Loop And Magnetic Moment Derivation	STUDY HERE
Magnetic Moment Of Revolving Electron And Bohr Magneton	STUDY HERE
Moving Coil Galvanometer	STUDY HERE
Magnetic Field Lines	STUDY HERE
Bar Magnet As An Equivalent Solenoid	STUDY HERE
The Dipole In A Uniform Magnetic Field	STUDY HERE
Gauss Law Of Magnetism	STUDY HERE
Earth's Magnetic Field	STUDY HERE
Magnetization And Magnetic Intensity	STUDY HERE
Magnetic Properties Of Materials	STUDY HERE
Hysteresis Curve	STUDY HERE

Chapter Name - Electromagnetic Induction And Alternating Currents	
Concept Name	**Study link**
Magnetic Flux	STUDY HERE
Faraday's Law Of Induction	STUDY HERE

Lenz's Law	STUDY HERE
Motional Electromotive Force	STUDY HERE
Induced Electric Field	STUDY HERE
Time Varying Magnetic Field	STUDY HERE
Eddy Currents	STUDY HERE
Self Inductance	STUDY HERE
Mutual Inductance	STUDY HERE
Energy Stored In An Inductor	STUDY HERE
AC Generator	STUDY HERE
Average And Rms Value Of Alternating Current And Voltage	STUDY HERE
AC Voltage Applied To A Resistor	STUDY HERE
AC Voltage Applied To An Inductor	STUDY HERE
AC Voltage Applied To A Capacitor	STUDY HERE
Series LR Circuit	STUDY HERE
Series RC Circuit	STUDY HERE
Series LCR Circuit	STUDY HERE
Resonance In Series LCR Circuit	STUDY HERE
Quality Factor In An AC Circuit	STUDY HERE
Power And Power Factor In AC Circuits	STUDY HERE
LC Oscillations	STUDY HERE
Transformers	STUDY HERE

Chapter Name - Electromagnetic Waves	
Concept Name	**Study link**
Displacement Current	STUDY HERE
Maxwell's 4 Equations	STUDY HERE
Nature Of Electromagnetic Waves	STUDY HERE
Energy Density And Intensity Of EM Waves	STUDY HERE
Electromagnetic Spectrum	STUDY HERE

Chapter Name - Optics	
Concept Name	**Study link**
Laws Of Reflection	STUDY HERE
Reflection On A Plane Mirror	STUDY HERE
Object And Image Velocity In Plane Mirror	STUDY HERE

Spherical Mirrors	STUDY HERE
Image Formation By Spherical Mirrors	STUDY HERE
Spherical Mirror Formula And Magnification	STUDY HERE
Refraction Of Light	STUDY HERE
Real Depth And Apparent Depth	STUDY HERE
Total Internal Reflection	STUDY HERE
Refraction Of Light Through Glass Slab	STUDY HERE
Refraction And Dispersion Of Light Through A Prism	STUDY HERE
Refraction At Spherical Surface	STUDY HERE
Concave And Convex Lenses - Image Formation	STUDY HERE
Lens Maker's Formula	STUDY HERE
Power Of Lens And Mirror	STUDY HERE
Magnification In Lenses	STUDY HERE
Relation Between Object And Image Velocity In Lens	STUDY HERE
Compound Lenses	STUDY HERE
Silvering Of Lens	STUDY HERE
Lens Displacement Method	STUDY HERE
Structure And Functions Of Human Eye	STUDY HERE
Simple Microscope	STUDY HERE
Compound Microscope	STUDY HERE
Astronomical Telescope	STUDY HERE
Terrestrial Telescope	STUDY HERE
Wavefronts	STUDY HERE
Huygens Principle	STUDY HERE
Interference Of Light - Condition And Types	STUDY HERE
Young's Double Slit Experiment	STUDY HERE
Optical Path	STUDY HERE
Ydse With Thin Slab	STUDY HERE
Lloyd's Mirror Experiment	STUDY HERE
Fresnel's Biprism	STUDY HERE
Thin Film Interference	STUDY HERE
Diffraction Of Light	STUDY HERE
Fraunhofer Diffraction By A Single Slit	STUDY HERE
Resolving Power Of Microscope And Telescope	STUDY HERE
Polarization Of Light	STUDY HERE

Malus's Law	STUDY HERE
Polarization By Reflection And Brewster's Law	STUDY HERE

Chapter Name - Dual Nature of Matter and Radiation	
Concept Name	**Study link**
Electron Emission	STUDY HERE
Photon Theory Of Light	STUDY HERE
The Photoelectric Effect	STUDY HERE
Einstein's Photoelectric Equation	STUDY HERE
Radiation Pressure	STUDY HERE
Wave Nature Of Matter And De Broglie's Equation	STUDY HERE

Chapter Name - Atoms And Nuclei	
Concept Name	**Study link**
Rutherford's Atomic Model And Limitations	STUDY HERE
Bohr Model Of The Hydrogen Atom	STUDY HERE
Energy Level - Bohr's Atomic Model	STUDY HERE
Line Spectra Of Hydrogen Atom	STUDY HERE
De-broglie's Explanation Of Bohr's Second Postulate	STUDY HERE
Effect Of Nucleus Motion On Energy	STUDY HERE
Atomic Collision	STUDY HERE
Nucleus Structure	STUDY HERE
Binding Energy Per Nucleon	STUDY HERE
Radioactive Decay	STUDY HERE
Law Of Radioactivity Decay	STUDY HERE
Simultaneous And Series Disintegration	STUDY HERE
Nuclear Fission	STUDY HERE
Nuclear Fusion	STUDY HERE
X-rays	STUDY HERE
Continuous X-ray	STUDY HERE
Characteristic X-rays	STUDY HERE
Moseley's Law	STUDY HERE
Bragg's Law	STUDY HERE

Chapter Name - Electronic Devices

Concept Name	Study link
Band Theory Of Solids	STUDY HERE
Types Of Semiconductor: Intrinsic And Extrinsic Semiconductor	STUDY HERE
Electric Conductivity	STUDY HERE
Zener Diode	STUDY HERE
Special Purpose P-N Junction Diodes	STUDY HERE
P-N Junction	STUDY HERE
Semiconductor Diode - Forward Bias And Reverse Bias	STUDY HERE
P-N Junction As A Rectifier	STUDY HERE
Bipolar Junction Transistor (N-P-N And P-N-P Transistor)	STUDY HERE
Transistor As A Device - Switch And Amplifier	STUDY HERE
Logic Gates	STUDY HERE

Chemistry

Chapter Name - Some Basic Concepts In Chemistry	
Concept Name	**Study link**
Nature and Characteristics of Matter	STUDY HERE
Properties of Matter and Their Measurement	STUDY HERE
Uncertainty In Measurement	STUDY HERE
Laws Of Chemical Combination For Elements And Compounds	STUDY HERE
DALTON'S ATOMIC THEORY	STUDY HERE
Atomic Mass And Molecular Mass	STUDY HERE
Mole Concept Basic	STUDY HERE
Percent Composition Formula	STUDY HERE
Empirical and Molecular Formula	STUDY HERE
Stoichiometric Calculations	STUDY HERE
Gravimetric Analysis	STUDY HERE
Molarity And Mole Fraction	STUDY HERE
Law of Equivalence	STUDY HERE
Oleum and its % labeling	STUDY HERE
Some Basic Concept in Chemistry Formula	STUDY HERE

Chapter Name - Atomic Structure	
Concept Name	**Study link**
Thomson atomic model	STUDY HERE
Rutherford atomic model and its limitations	STUDY HERE
Atomic Number(Z), Mass number(A), Isotopes and Isobars	STUDY HERE
Electromagnetic radiation	STUDY HERE
Planck's quantum theory	STUDY HERE
Photoelectric effect	STUDY HERE
Hydrogen Spectrum	STUDY HERE
Bohr's Model Of An Atom	STUDY HERE
Zeeman and Stark's effect.	STUDY HERE
De Broglie Relationship	STUDY HERE
Heisenberg Uncertainty Principle	STUDY HERE
Frequency, Time Period and Angular Frequency	STUDY HERE

Quantum Numbers	STUDY HERE
Shape of Orbitals	STUDY HERE
Radial nodes and planar nodes	STUDY HERE
Hund's Rule, Pauli Exclusion Principle, and the Aufbau Principle	STUDY HERE
Electronic Configuration of Elements	STUDY HERE
Stability Of Orbitals: Half-Filled And Completely-Filled	STUDY HERE

Chapter Name - Classification Of Elements And Periodic Table	
Concept Name	**Study link**
Development Of Modern Periodic Table	STUDY HERE
Mendeleev’s Periodic table	STUDY HERE
Modern periodic table	STUDY HERE
Electronic Configuration in Periods and Groups	STUDY HERE
Nomenclature Of Elements With Atomic Number	STUDY HERE
Classification of Elements and Periodicity in Properties	STUDY HERE
Metals, Non-metals and Metalloids	STUDY HERE
Atomic Size & Atomic Radius	STUDY HERE
Ionisation Enthalpy	STUDY HERE
Electron Gain Enthalpy	STUDY HERE
Electronegativity	STUDY HERE
Physical and chemical properties of elements	STUDY HERE
Modern periodic table trend	STUDY HERE

Chapter Name - Chemical Bonding and Molecular Structure	
Concept Name	**Study link**
Chemical Bonding	STUDY HERE
Lewis Electron Dot Structures	STUDY HERE
Formal Charge And Its Properties	STUDY HERE
Limitations of The Octet Rule	STUDY HERE
Ionic Bond or Electrovalent Bond	STUDY HERE
Lattice Energy	STUDY HERE
Bond Parameters - Bond Order, Angle, Length, and Energy	STUDY HERE
Dragos Rule	STUDY HERE
Resonance Structures	STUDY HERE
Fajan’s Rule	STUDY HERE

Valence Bond Theory	STUDY HERE
Pi Bonds	STUDY HERE
Hybridisation	STUDY HERE
VSEPR Theory	STUDY HERE
Molecular geometry	STUDY HERE
Molecular Orbital Theory	STUDY HERE
Energy Level Diagram	STUDY HERE
Dipole Moment	STUDY HERE
Ionic Bond - Partially Covalent in Nature	STUDY HERE
Van Der Waals Forces	STUDY HERE
Hydrogen Bonding	STUDY HERE

Chapter Name - Chemical Thermodynamics	
Concept Name	**Study link**
Thermodynamics	STUDY HERE
Thermodynamic Property	STUDY HERE
State Functions	STUDY HERE
Reversible, Irreversible, Polytropic Process	STUDY HERE
Zeroth Law Of Thermodynamics	STUDY HERE
Introduction To Heat, Internal Energy And Work	STUDY HERE
Isothermal Expansion of an Ideal Gas	STUDY HERE
Graphical Comparison of Thermodynamic Processes	STUDY HERE
Heat Capacity - Relationship between Cp and Cv	STUDY HERE
Thermochemistry	STUDY HERE
Enthalpy Change	STUDY HERE
Standard Enthalpy Of Formation, Combustion And Bond Dissociation	STUDY HERE
Lattice Enthalpy, Hydration Enthalpy And Enthalpy Of Solution	STUDY HERE
Enthalpy of Neutralization of Strong Acid and Strong Base	STUDY HERE
Ionization And Electron Gain Enthalpy	STUDY HERE
Resonance Energy	STUDY HERE
Kirchoff’s Equation	STUDY HERE
Hess’s Law	STUDY HERE
Born Habers Cycle	STUDY HERE
Bomb Calorimeter	STUDY HERE
Entropy Change	STUDY HERE

Spontaneity in Thermodynamics	STUDY HERE
Gibbs Energy Change And Criteria For Equilibrium	STUDY HERE
Third Law Of Thermodynamics	STUDY HERE
Chemical thermodynamics formula	STUDY HERE

Chapter Name - Equilibrium	
Concept Name	**Study link**
Equilibrium	STUDY HERE
Law Of Mass Action	STUDY HERE
Equilibrium Constant	STUDY HERE
Relation between Kp and Kc	STUDY HERE
Degree of Dissociation	STUDY HERE
Le Chatelier's Principles on Equilibrium	STUDY HERE
Ionic Equilibrium	STUDY HERE
Bronsted Lowry and Lewis Acid-Base theory	STUDY HERE
Ionization Of Acids And Bases	STUDY HERE
Ka and Kb Relationship	STUDY HERE
pH Of Acids And Bases	STUDY HERE
Common ion effect	STUDY HERE
Buffer Solution	STUDY HERE
Salt Hydrolysis	STUDY HERE
Solubility and Solubility Product	STUDY HERE

Chapter Name - Redox Reaction and Electrochemistry	
Concept Name	**Study link**
Oxidation Number	STUDY HERE
Redox Reactions	STUDY HERE
Displacement Reaction	STUDY HERE
Electrochemistry	STUDY HERE
Electrochemical Series	STUDY HERE
Faraday’s Laws of Electrolysis	STUDY HERE
Galvanic Cells	STUDY HERE
Salt Bridge	STUDY HERE
Standard Hydrogen Electrode	STUDY HERE

Gibbs Free Energy of Reaction	STUDY HERE
Nernst Equation	STUDY HERE
Concentration Cells	STUDY HERE
Molar Conductivity	STUDY HERE
Kohlrausch's Law	STUDY HERE
Types Of Battery - Primary cell & Secondary cell	STUDY HERE

Chapter Name - Solutions	
Concept Name	**Study link**
Types of Solutions	STUDY HERE
Expression of Concentration of Solutions	STUDY HERE
Vapour Pressure of solutions	STUDY HERE
Ideal Solution	STUDY HERE
Raoult’s Law	STUDY HERE
Azeotropic Mixture	STUDY HERE
Elevation in Boiling Point	STUDY HERE
Depression in Freezing Point	STUDY HERE
Osmotic Pressure	STUDY HERE
Reverse Osmosis	STUDY HERE
Isotonic, Hypertonic, Hypotonic Solution	STUDY HERE
Van't Hoff Factor and Abnormal Molar Mass	STUDY HERE
Solubility and Henry's Law	STUDY HERE

Chapter Name - Chemical Kinetics	
Concept Name	**Study link**
Rate of Reaction	STUDY HERE
Rate Law	STUDY HERE
Order of Reaction	STUDY HERE
Zero Order Reaction	STUDY HERE
First Order Reaction	STUDY HERE
Second Order Reaction	STUDY HERE
nth Order Reaction	STUDY HERE
Methods of Determining Reaction Order	STUDY HERE
Molecularity of reaction	STUDY HERE
Pseudo First Order Reaction	STUDY HERE

Elementary and Complex Reactions	STUDY HERE
Arrhenius Equation	STUDY HERE

Chapter Name - Some Basic Principles Of Organic Chemistry	
Concept Name	**Study link**
Organic Compounds - Classification Of Organic Compounds	STUDY HERE
Functional Groups	STUDY HERE
Homologous Series	STUDY HERE
IUPAC Nomenclature of Organic Chemistry	STUDY HERE
Isomerism	STUDY HERE
Organic Chemistry	STUDY HERE
Carbocations	STUDY HERE
Carbanions	STUDY HERE
Alkyl Free Radicals	STUDY HERE
Nucleophiles and Electrophiles	STUDY HERE
Inductive Effect	STUDY HERE
Electromeric Effect	STUDY HERE
Mesomeric or Resonance Effect	STUDY HERE
Hyperconjugation	STUDY HERE
Tautomerism	STUDY HERE

Chapter Name - Hydrocarbons	
Concept Name	**Study link**
IUPAC Nomenclature of Alkanes	STUDY HERE
Adsorption and Degree of Unsaturation	STUDY HERE
Preparation of Alkanes	STUDY HERE
Physical Properties of Alkanes	STUDY HERE
Chemical Properties of Alkanes	STUDY HERE
Conformation, Sawhorse and Newman Projections	STUDY HERE
Nomenclature and Isomerism of Alkenes	STUDY HERE
Preparation of Alkenes	STUDY HERE
Chemical Properties of Alkenes	STUDY HERE
Hydroboration and Oxidation	STUDY HERE
Preparation of Alkynes	STUDY HERE
Chemical Properties of Alkynes	STUDY HERE

Aromaticity	STUDY HERE
Reaction of Aromatic Compounds	STUDY HERE
Electrophilic Substitution Reaction	STUDY HERE
Friedel-Crafts Reaction	STUDY HERE
Benzene Reactions - Sulfonation, Nitration and Halogenation	STUDY HERE

Chapter Name - Organic Compounds containing Halogens	
Concept Name	**Study link**
Alkyl Halides	STUDY HERE
Physical & Chemical Properties of Haloalkanes	STUDY HERE
Haloarene	STUDY HERE

Chapter Name - Organic Compounds containing Oxygen	
Concept Name	**Study link**
Preparation of Alcohols	STUDY HERE
Physical and Chemical Properties of Alcohols	STUDY HERE
Preparation of Phenols	STUDY HERE
Physical and Chemical Properties of Phenols	STUDY HERE
Williamson's Ether Synthesis	STUDY HERE
Methods of Preparation of Aldehydes and Ketones	STUDY HERE
Reaction of Aldehydes and Ketones	STUDY HERE
Carboxylic Acids	STUDY HERE

Chapter Name - Organic Compounds Containing Nitrogen	
Concept Name	**Study link**
Preparation of Amines	STUDY HERE
Test for Amines	STUDY HERE
Basicity of Amines	STUDY HERE
Azo-Coupling Reaction	STUDY HERE

Chapter Name - Biomolecules	
Concept Name	**Study link**
Classification of Carbohydrates and its Structure	STUDY HERE
Tests for Carbohydrates	STUDY HERE
Properties of Glucose	STUDY HERE

Amino Acids	STUDY HERE
Proteins	STUDY HERE
Enzymes	STUDY HERE
Vitamins	STUDY HERE

Maths

Chapter Name - Sets, Relations and Functions	
Concept Name	**Study link**
Sets, Roster and Set Builder form of Sets	STUDY HERE
Equal and Equivalent Sets	STUDY HERE
Subsets, Proper Subset, Improper Subset, Intervals	STUDY HERE
Finite set, Infinite set, Singleton set	STUDY HERE
Power set, Universal set	STUDY HERE
Union of sets, Properties of union	STUDY HERE
Intersection of Set, Properties of Intersection	STUDY HERE
Difference of set	STUDY HERE
Complement of a set, Law of Complement, Property of Complement	STUDY HERE
Cardinal number of some sets	STUDY HERE
De-Morgan's Laws	STUDY HERE
Ordered pair, Cartesian product of two sets	STUDY HERE
Relation, Number of relation	STUDY HERE
Domain, Range of Relation	STUDY HERE
Relation, and its types	STUDY HERE
Functions, Image and Pre-image	STUDY HERE
Domain of function, Co-domain, Range of function	STUDY HERE
Inequalities	STUDY HERE
Transcendental function	STUDY HERE
Logarithmic Inequalities	STUDY HERE
Trigonometric Function	STUDY HERE
Inverse Trigonometric Function	STUDY HERE
Modulus Function, Properties of Modulus Function	STUDY HERE
Algebraic function	STUDY HERE
Piecewise function	STUDY HERE
One to One Function	STUDY HERE
Many One Function	STUDY HERE
Onto Function or Surjective	STUDY HERE
Into Function and Bijective function	STUDY HERE
Composition of function: Conditions and Properties	STUDY HERE

Inverse of a function	STUDY HERE
Even and Odd Function	STUDY HERE
Periodic Functions	STUDY HERE
Vertical and Horizontal Transformation	STUDY HERE
Transformations of Functions	STUDY HERE
f(x) = min{g1(x), g2(x).......... } or max{g1(x), g2(x).......... }	STUDY HERE

Chapter Name - Complex Numbers and Quadratic Equations	
Concept Name	**Study link**
Powers of Iota	STUDY HERE
Complex number	STUDY HERE
Algebraic operation on Complex Numbers	STUDY HERE
Multiplication of Complex Numbers	STUDY HERE
Conjugates of Complex Numbers	STUDY HERE
Modulus of complex number and its Properties	STUDY HERE
Argument of complex number	STUDY HERE
Polar form of complex numbers	STUDY HERE
Euler form of complex number	STUDY HERE
Square root of complex numbers	STUDY HERE
De-moivre’s theorem	STUDY HERE
Cube roots of unity	STUDY HERE
nth root of unity	STUDY HERE
Rotation Of Complex Numbers	STUDY HERE
Distance formula and Equation of perpendicular bisector	STUDY HERE
Quadratic Equation	STUDY HERE
Nature Of Roots Depending Upon Coefficients And Discriminant	STUDY HERE
Transformation of Quadratic Equations	STUDY HERE
Condition for common roots	STUDY HERE
Graphical Representation of Quadratic Equation	STUDY HERE
Sign of Quadratic Expression	STUDY HERE
Quadratic Inequalities	STUDY HERE
Quadratic Equation in two Variables	STUDY HERE
Location of Roots	STUDY HERE
Remainder Theorem	STUDY HERE
Rational Inequalities Calculator	STUDY HERE

Irrational equations and Inequalities	STUDY HERE
Exponential Equations In Quadratic Form	STUDY HERE
Logarithmic Equations in Quadratic form	STUDY HERE

Chapter Name - Trigonometry	
Concept Name	**Study link**
Measuring Angles	STUDY HERE
Trigonometric Ratios	STUDY HERE
Trigonometric Identities	STUDY HERE
Sign of Trigonometric Functions	STUDY HERE
Graphs of General Trigonometric Functions	STUDY HERE
Trigonometric Ratios of Allied Angles	STUDY HERE
Trigonometric Ratios of Compound Angles	STUDY HERE
Sum-to-Product and Product-to-Sum Formulas	STUDY HERE
Product To Sum Formulas	STUDY HERE
Double Angle Formulas	STUDY HERE
Triple Angle Identities	STUDY HERE
Half Angle Formula	STUDY HERE
Trigonometric Ratio of Submultiple of an Angle	STUDY HERE
Trigonometric Ratios of Some Specific Angles	STUDY HERE
Summation Of Series In Trigonometry	STUDY HERE
Conditional Trigonometric Identities	STUDY HERE
Maximum and Minimum value of Trigonometric Function	STUDY HERE
General Solution of Trigonometric Equations	STUDY HERE
Simultaneous Trigonometric Equations	STUDY HERE
Trigonometric Equation using Minimum and Maximum value of Function	STUDY HERE
Trigonometric Inequality	STUDY HERE
Law of Sines	STUDY HERE
Law of Cosines	STUDY HERE
Law Of Tangents	STUDY HERE
Projection Formula	STUDY HERE
Semiperimeter and Half Angle Formulae	STUDY HERE
Area of Triangle	STUDY HERE
Circumcircle of a Triangle	STUDY HERE
In-Circle and In-Centre	STUDY HERE

Escribed Circle of Triangle	STUDY HERE
Important Solutions of Triangle Formulas	STUDY HERE
Height and Distance	STUDY HERE
Inverse Trigonometric Functions	STUDY HERE
Domain and Range of Trigonometric Functions	STUDY HERE
Inverse Functions	STUDY HERE
Graph of Inverse Trigonometric Function	STUDY HERE
Trigonometric Ratios of Complementary Angles	STUDY HERE
Sum and difference of angles in terms of arctan	STUDY HERE
Multiple Angles	STUDY HERE

Chapter Name - Matrices and Determinants	
Concept Name	**Study link**
Determine The Order Of Matrix	STUDY HERE
Types of Matrices	STUDY HERE
Triangular matrix (Upper and Lower triangular matrix)	STUDY HERE
Matrix operations	STUDY HERE
Matrix Multiplication	STUDY HERE
Transpose of a Matrix	STUDY HERE
Symmetric Matrix & Skew Symmetric Matrix	STUDY HERE
Conjugate of a Matrix	STUDY HERE
Hermitian matrix	STUDY HERE
Skew Hermitian Matrix	STUDY HERE
Trace of a matrix and properties	STUDY HERE
Orthogonal matrix and Unitary matrix and Idempotent matrix	STUDY HERE
Periodic matrix and Nilpotent matrix and involutory matrix	STUDY HERE
Elementary row operations	STUDY HERE
Singular Matrix	STUDY HERE
Minors And Cofactors	STUDY HERE
Adjoint and Inverse of a Matrix	STUDY HERE
Inverse Matrix	STUDY HERE
Multiplication of Two Determinants	STUDY HERE
Properties of Determinants	STUDY HERE
System of Linear Equations	STUDY HERE
Cramer’s Rule	STUDY HERE

Homogeneous System of Linear Equations	STUDY HERE
Solving Linear Equations Using Matrix	STUDY HERE

Chapter Name - Sequence and Series	
Concept Name	**Study link**
Sequence And Series	STUDY HERE
Arithmetic Progression	STUDY HERE
Sum of n Terms of an AP	STUDY HERE
Arithmetic Mean in AP	STUDY HERE
Geometric Progression	STUDY HERE
Geometric Mean In GP	STUDY HERE
Sum To n Terms Of a GP	STUDY HERE
Harmonic Progression	STUDY HERE
Harmonic Mean in HP	STUDY HERE
Relation between A.M., G.M. and H.M.	STUDY HERE
Arithmetico Geometric Series	STUDY HERE
Sum of an Infinite Arithmetic Geometric Series	STUDY HERE
SUMMATION FORMULA	STUDY HERE
Sum of Common Series	STUDY HERE
If the differences of successive terms of a series are in A.P.and G.P	STUDY HERE
Method of Difference	STUDY HERE
Sum to n Terms of Special Series	STUDY HERE
Application of AM-GM Inequality	STUDY HERE

Chapter Name - Co-ordinate geometry	
Concept Name	**Study link**
Coordinate Axes	STUDY HERE
Distance Between Two Points Formula	STUDY HERE
Section Formula & Conic Sections	STUDY HERE
Centroid, Incentre and Circumcentre and Orthocentre of a Triangle	STUDY HERE
Incentre	STUDY HERE
Circumcentre and Orthocentre	STUDY HERE
Excenters of Triangle	STUDY HERE
Area of Triangle in Coordinate Geometry	STUDY HERE
Locus	STUDY HERE

Transformations of Axes	STUDY HERE
Rotation of Axes About Origin	STUDY HERE
Straight Lines	STUDY HERE
Equation of Straight Line	STUDY HERE
Angle Between Two Lines	STUDY HERE
Position of two points with respect to a line	STUDY HERE
Parallel and Perpendicular Lines	STUDY HERE
Distance of a Point From a Line	STUDY HERE
Point of Intersection Formula	STUDY HERE
Family of Lines	STUDY HERE
Line Equally Inclined with two lines	STUDY HERE
Equation of the Bisectors	STUDY HERE
Foot of Perpendicular and Image	STUDY HERE
Pair of Straight Line	STUDY HERE
Homogeneous Equations in Two Variables	STUDY HERE
Circle	STUDY HERE
Explain parametric equation of a circle.	STUDY HERE
Locus of Mid Point of the Chord of the Circle	STUDY HERE
Diametric Form of a Circle	STUDY HERE
Intercepts Made by Circle on the Axis	STUDY HERE
Position of a Point With Respect to Circle	STUDY HERE
Line and Circle	STUDY HERE
Length of Intercept Cut-Off from a line	STUDY HERE
Equation of the Tangent and Normal to the Circle	STUDY HERE
Power of a point wrt Circle	STUDY HERE
Chord of Contact	STUDY HERE
Pair of Tangent	STUDY HERE
Director Circle	STUDY HERE
Diameter Of A Circle	STUDY HERE
Intersection of Two Circle	STUDY HERE
Common Chord of two Circles	STUDY HERE
Family of Circles	STUDY HERE
Angle of Intersection of Two Circle	STUDY HERE
Radical Axis	STUDY HERE
Conic Sections	STUDY HERE

Parabola	STUDY HERE
Equation of parabola when equation of axis and tangent at vertex and latusrectum are given	STUDY HERE
Position of a Point with Respect to a Parabola	STUDY HERE
Line and a Parabola	STUDY HERE
Tangent to a Parabola	STUDY HERE
Normal in point and Parametric and Slope form of parabola	STUDY HERE
Normal at t1 meets the parabola again at t2	STUDY HERE
Co-normal Points	STUDY HERE
Chord of Contact and Diameter of Parabola	STUDY HERE
Subtangent and Subnormal	STUDY HERE
Some Standard Property of Parabola	STUDY HERE
Ellipse	STUDY HERE
Latus Rectum	STUDY HERE
Parametric equation of Ellipse	STUDY HERE
Horizontal and Vertical Ellipse	STUDY HERE
Position of a point with respect to Ellipse	STUDY HERE
Line and the Ellipse	STUDY HERE
Equation of Tangent to Ellipse	STUDY HERE
Equation of Normal in Point Form and Parametric Form	STUDY HERE
Diameter of Ellipse	STUDY HERE
Director Circle of Ellipse	STUDY HERE
Length of sub-Tangent and Sub-Normal of an Ellipse	STUDY HERE
Hyperbola	STUDY HERE
Position of a point with respect to Hyperbola	STUDY HERE
Line and the Hyperbola	STUDY HERE
Tangents to Hyperbolas	STUDY HERE
Equation of Normal of Hyperbola in Point ,Parametric Form	STUDY HERE
Director Circle of Hyperbola	STUDY HERE
Asymptotes of Hyperbolas	STUDY HERE
Rectangular Hyperbola	STUDY HERE

Chapter Name - Limit , Continuity and Differentiability	
Concept Name	**Study link**
Limit	STUDY HERE
Left-Hand Limits and Right-Hand Limits	STUDY HERE
Algebra of Limits	STUDY HERE

Limit of Indeterminate Form and Algebraic limit	STUDY HERE
Limit of Algebraic function	STUDY HERE
Limit Using Expansion	STUDY HERE
Sandwich Theorem	STUDY HERE
Limits of Trigonometric Functions	STUDY HERE
Exponential and Logarithmic Limits	STUDY HERE
Limits of the form (1 power infinity)	STUDY HERE
L' Hospital's Rule	STUDY HERE
Limit of the form (0 power 0 or infinity power 0)	STUDY HERE
Differentiation	STUDY HERE
Derivative of Polynomials and Trigonometric Functions	STUDY HERE
Differentiation Rules	STUDY HERE
Differentiation of Implicit Function	STUDY HERE
Differentiation of Function in Parametric Form	STUDY HERE
Differentiation of Inverse Trigonometric Function (cos/sine/tan)	STUDY HERE
Differentiation Using Logarithm	STUDY HERE
Differentiation of a Function wrt Another Function and Higher Order derivative of a Function	STUDY HERE
Differentiation of Determinants	STUDY HERE
Differentiation of Function and Relation	STUDY HERE
Differentiation of Inverse Function	STUDY HERE
Continuity and Discontinuity	STUDY HERE
Directional Continuity and Continuity over an Interval	STUDY HERE
Non - Removable, Infinite and Oscillatory Type Discontinuity	STUDY HERE
Continuity and Discontinuity obtained by Algebraic Operations	STUDY HERE
Continuity of Composite Function	STUDY HERE
The Intermediate Value Theorem	STUDY HERE
Differentiability and Existence of Derivative	STUDY HERE
Examining differentiability Using Graph of Function	STUDY HERE
Continuity And Differentiability	STUDY HERE
Differentiability of Composite Function	STUDY HERE
Derivative as Rate Measure	STUDY HERE
Approximations and Errors using Derivatives	STUDY HERE
Tangent to the Curve at a Point	STUDY HERE
Angle of Intersection between Two Curves	STUDY HERE

Length of Tangent and Normal and Subtangent and subnormal	STUDY HERE
Rolle's Theorem	STUDY HERE
Monotonicity and Extremum of Functions	STUDY HERE
Inflection Point	STUDY HERE
Maxima and Minima in Calculus	STUDY HERE
First Derivative Test	STUDY HERE
Application of Monotonicity	STUDY HERE
Nature of Roots of Cubic Polynomial	STUDY HERE
Application of Extremum in Plane Geometry and Solid geometry	STUDY HERE

Chapter Name - Integral Calculus	
Concept Name	**Study link**
Integration as an Inverse Process of Differentiation	STUDY HERE
Integration of Trigonometric Functions	STUDY HERE
Integration by Substitution	STUDY HERE
Fundamental Formulae of Indefinite Integration (Inverse Trigonometric Functions)	STUDY HERE
Integrals of Particular Function	STUDY HERE
Trigonometric Integrals	STUDY HERE
Integration By Parts Formula	STUDY HERE
Integration Using Partial Fraction	STUDY HERE
Integration of Irrational Algebraic Function	STUDY HERE
Reduction Formula	STUDY HERE
Indirect Substitution in Integral	STUDY HERE
Definite Integral - Calculus	STUDY HERE
Definite integral as the limit of a sum	STUDY HERE
Evaluation of Definite Integrals by Substitution	STUDY HERE
Properties of the Definite Integral	STUDY HERE
Piecewise Definite integration	STUDY HERE
Application of Even- Odd Properties in Definite Integration	STUDY HERE
Newton-Leibnitz's Formula	STUDY HERE
Application of Inequality in Definite Integration	STUDY HERE
Area Between Two Curves - Calculus	STUDY HERE

Chapter Name - Differential Equations	
Concept Name	**Study link**

Differential Equation	STUDY HERE
Formation of Differential Equation and Solutions of a Differential Equation	STUDY HERE
Differential equations with variables separable	STUDY HERE
Homogeneous Differential Equation	STUDY HERE
Linear Differential Equation	STUDY HERE
Bernoulli’s Equation	STUDY HERE
Orthogonal Trajectory	STUDY HERE
Exact Differential Equation	STUDY HERE
Application of Differential Equation	STUDY HERE

Chapter Name - Vector Algebra	
Concept Name	**Study link**
3D Geometry	STUDY HERE
Vectors and Scalars	STUDY HERE
Vectors Joining Two Points	STUDY HERE
Types Of Vectors	STUDY HERE
Direction Cosines & Direction Ratios Of A Line	STUDY HERE
Vector Addition and Subtraction	STUDY HERE
Multiplication Of Vectors And Scalar Quantity	STUDY HERE
Section Formula	STUDY HERE
Linear Combination of Vectors	STUDY HERE
Dot Product Of Two Vectors	STUDY HERE
Finding Components of a vector Along and Perpendicular to another Vector	STUDY HERE
Cross product	STUDY HERE
Geometrical Interpretation of Product of Vectors	STUDY HERE
Scalar Triple Product of Vectors	STUDY HERE
Proof of the Vector Triple Product	STUDY HERE
Lagrange's Identity	STUDY HERE

Chapter Name - Three Dimensional Geometry	
Concept Name	**Study link**
Equation Of A Line In Three Dimensions	STUDY HERE
Image of a Point in the given Line	STUDY HERE
Shortest Distance between Two Lines	STUDY HERE
Equation of A Plane In The Normal Form	STUDY HERE

Equation of a plane perpendicular to a given vector and passing through a given point	STUDY HERE
Equation of a plane passing through three non collinear point	STUDY HERE
Equation of a Plane Passing Through a Given Point and Parallel to Two Given Vectors	STUDY HERE
Angle Between Two PlanesAngle Between Two Planes - Planes & Angles	STUDY HERE
Family of Plane	STUDY HERE
Perpendicular Distance Of A Point From A Plane	STUDY HERE
Image of a Point in the Plane	STUDY HERE
Two Sides of a Plane	STUDY HERE
Equation of The Plane Bisecting the Angle Between Two Planes	STUDY HERE
Line of Intersection of Two Plane and Angle Between a Line and a Plane	STUDY HERE
Coplanarity of Two Lines	STUDY HERE
Equation of Sphere	STUDY HERE

Chapter Name - Permutations and Combinations	
Concept Name	**Study link**
Introduction Permutation and Combination	STUDY HERE
Fundamental Principle Of Counting	STUDY HERE
Permutation	STUDY HERE
Permutation Of Objects When Few Are Identical	STUDY HERE
Applications Of Permutation	STUDY HERE
Geometrical Permutations	STUDY HERE
Rank Of A Word In A Dictionary	STUDY HERE
Introduction Of Combinations	STUDY HERE
Applications Of Selections	STUDY HERE
Selection Of Any Number Of Objects	STUDY HERE
Formation of Groups	STUDY HERE
Finding Number Of Solutions Of Equations	STUDY HERE
Distribution Of Things	STUDY HERE
Derangement	STUDY HERE
Permutation Vs Combination	STUDY HERE

Chapter Name - Statistics and Probability	
Concept Name	**Study link**
Introduction	STUDY HERE

Representation of Data	STUDY HERE
Central Tendency	STUDY HERE
Measures of Dispersion	STUDY HERE
Dispersion (Variance and Standard Deviation)	STUDY HERE
Coefficients of Dispersion	STUDY HERE
Some Important Point Regarding Statistics	STUDY HERE
Important Terminologies and Definitions of Probability	STUDY HERE
Algebra of Events	STUDY HERE
Basic Probability Practise Session	STUDY HERE
Set Theoretical Notations of Probability	STUDY HERE
Conditional Probability	STUDY HERE
Multiplication Theorem on Probability	STUDY HERE
Independent Event	STUDY HERE
Total Probability Theorem and Bayes' Theorem	STUDY HERE
Random Variables and its Probability Distributions	STUDY HERE
Bernoulli Trials and Binomial Distribution	STUDY HERE

Chapter Name - Binomial Theorem and its Simple Applications	
Concept Name	**Study link**
Binomial Theorem - Formula, Expansion, Problems and Applications	STUDY HERE
Some Standard Expansions	STUDY HERE
General and Middle Terms	STUDY HERE
Greatest Term (numerically)	STUDY HERE
An Important Theorem	STUDY HERE
Problems on Divisibility	STUDY HERE
Finding last digits	STUDY HERE
Important Result (Comparison)	STUDY HERE
Multinomial Theorem	STUDY HERE
Series Involving Binomial Coefficients	STUDY HERE
Differentiation form of Binomial Coefficients	STUDY HERE
Use of Integration in Binomial	STUDY HERE
Product of two Binomial Coefficients	STUDY HERE
Binomial Inside Binomial	STUDY HERE
Binomial Theorem for any Index	STUDY HERE
Important Results of Binomial Theorem for any Index	STUDY HERE

Mock Test 1

Physics

Q. 1 In a Young's double slit experiment with light of wavelength λ the separation of slits is d and distance of screen is D such that D > > d > > λ. If the Fringe width is β, the distance from point of maximum intensity to the point where intensity falls to half of maximum intensity on either side is :

Option 1:
$\frac{\beta}{2}$

Option 2:
$\frac{\beta}{4}$

Option 3:
$\frac{\beta}{3}$

Option 4:
$\frac{\beta}{6}$

Correct Answer:
$\frac{\beta}{4}$

Solution:
As we learnt in

Malus Law -

$I=I_0 \cdot \cos^2 \theta$

$\theta=$ angle made by E vector with transmission axis.

- wherein

$I=$ Intensity of transmitted light after polarisation .

$I_0=$ Intensity of incident light.

Fringe Width -

$$\beta = \frac{\lambda D}{d}$$

- wherein

$\beta = y_{n+1} - y_n$

$y_{n+1} =$ Distance of $(n+1)^{th}$

Maxima$= (n+1)\frac{\lambda D}{d}$

$y_n =$Distance of n^{th}

maxima $= \frac{n\lambda D}{d}$

$$2I_0 = 4I_0 cos^2\left(\frac{\Delta\phi}{2}\right)$$

Here $\Delta\phi = \frac{\pi}{2}$

$\Delta\phi = \frac{2\pi}{\lambda}\Delta x$ so $\Delta x = \frac{\lambda}{4}$

$\frac{dy}{\Delta} = \frac{\lambda}{4}$ (i)

$\frac{\lambda\Delta}{d} = \beta$ (ii)

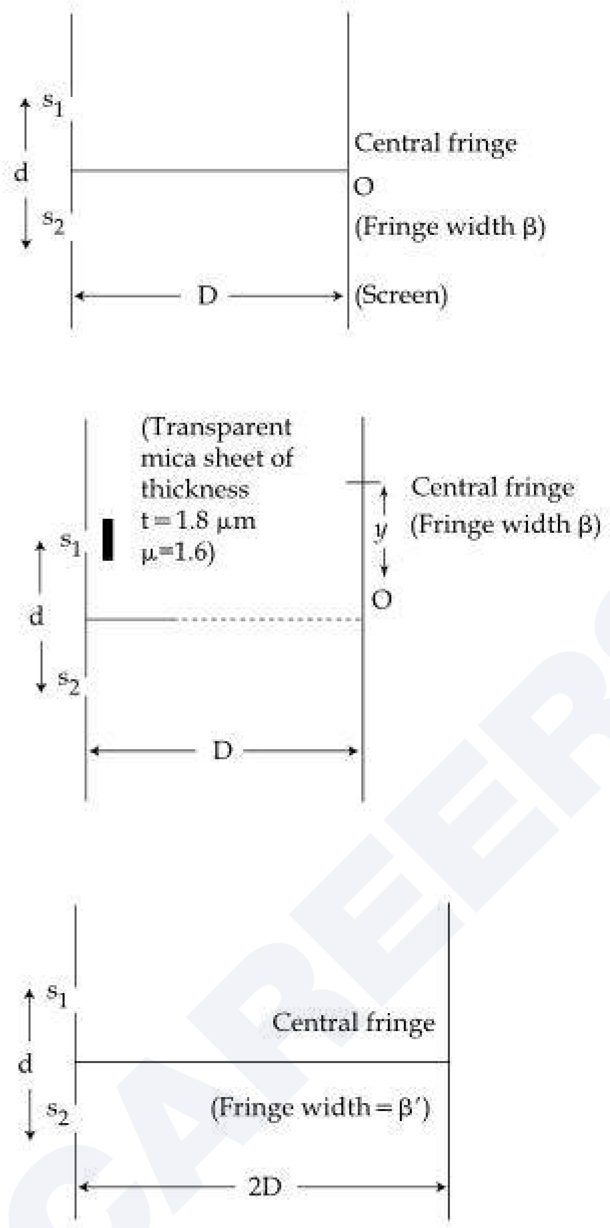
Therefore, from equation (i) and (ii)

$y = \frac{\beta}{4}$

Correct option is 2.

Q. 2 Using monochromatic light of wavelength λ, an experimentalist sets up the Young's double slit experiment in three ways as shown.

If she observes that $y = \beta'$, the wavelength of light used is :



Option 1:
520 nm

Option 2:
540 nm

Option 3:
560 nm

Option 4:
580 nm

Correct Answer:
540 nm

Solution:

Given

$t = 1.8 \times 10^{-6} m$

$\mu = 1.6$

In young's double slit experiment, the fringe width $= \beta = \dfrac{D\lambda}{d}$

The fringe width of the 3^{rd} figure $= \beta' = \frac{2D\lambda}{d}$ [as the distance between screen and slits is $2D$]
In the 2^{nd} figure as there is a material between slit and screen there will be shift of central fringe
The shift $= y = \frac{D(\mu-1)t}{d}$
From the question $y = \beta'$
$\frac{D(\mu-1)t}{d} = \frac{2D\lambda}{d}$
$(\mu - 1)t = 2\lambda$
$\lambda = (\mu - 1)\frac{t}{2}$

Now $\Rightarrow$ $(1.6 - 1) \times 1.8 \times 10^{-6} = 2\lambda$

$\lambda = \dfrac{1.8 \times 10^{-6} \times 0.6}{2} = 540\ nm$

Correct option is 2.

Q. 3 A ray of light is incident from a denser to a rarer medium. The critical angle for total internal reflection is Θ_{iC} and the Brewster's angle of incidence is Θ_{iB}, such that $\sin\Theta_{iC}/\sin\Theta_{iB} = \eta = 1.28$. The relative refractive index of the two media is :

Option 1:
0.2

Option 2:
0.4

Option 3:
0.8

Option 4:
0.9

Correct Answer:
0.8

Solution:
$\sin\theta_{ic} = \frac{\mu_r}{\mu_d}$

μ_r = refractive index of the rarer medium.

μ_d = refractive index of the rarer medium.

In the case of Brewster's angle

$r = 90 - \theta_{iB}$

From Brew's law: $\mu_d . \sin\theta_{iB} = \mu_r . \sin r$

$\frac{\sin\theta_{iB}}{\cos\theta_{iB}} = \frac{\mu_r}{\mu_d}$ or $\tan\theta_{iB} = \frac{\mu_r}{\mu_d}$

$\sin\theta_{iB} = \frac{\mu_r}{\sqrt{\mu_r^2 + \mu_d^2}} \ldots(2)$

$\because \frac{\sin\theta_{ic}}{\sin\theta_{iB}} = 1.28$

$\mu_1^2 + \mu_d^2 = 1.638\mu_d^2$

$or \quad 0.638\mu d^2 = \mu_r^2$

$\frac{\mu_r}{\mu_d} = \sqrt{0.638} = 0.8$

Q. 4 An object is at a distance of 10 cm from a mirror and the image of the object is at a distance of 30 cm from the mirror on the same side as the object. Then nature of the mirror and its focal length is:

Option 1:
Convex, 15cm

Option 2:
Concave, 1.5cm

Option 3:
Convex, 7.5cm

Option 4:
Concave, 7.5cm

Correct Answer:
Concave, 7.5cm

Solution:
As we learn

Sign Convention -

1) All distance are measured from pole.

2) Distance measured in the direction of incident rays are taken as positive.

3) Distance measured in the direction opposite to that of incident rays are taken as negative.

4) Distance above the principal axis as positive and below the principal axis as negative.

u = -10 cm

v = - 30cm from sign convention

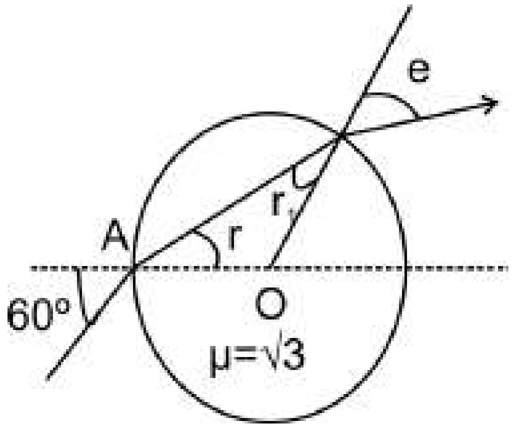
from mirror formula

$$\frac{1}{f}=\frac{1}{v}+\frac{1}{u}=-\frac{1}{30}-\frac{1}{10}=\frac{-4}{30}$$

f = -7.5 cm

since focal length is negative hence its mirror concave

Q. 5 A light ray is incident on a glass sphere of refractive index $\mu = \sqrt{3}$ at an angle of incidence 60^0 as shown the total deviation after two refraction is



Option 1:
30^0

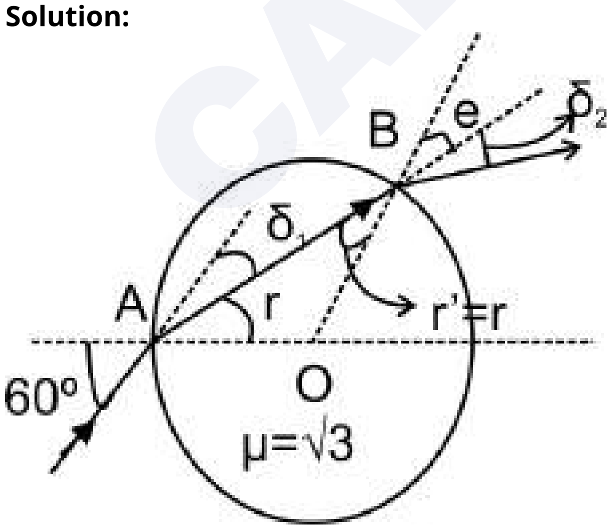
Option 2:
45^0

Option 3:
75^0

Option 4:
60^0

Correct Answer:
60^0

Solution:



At point A:

$$1.\sin 60^0 = \sqrt{3}\sin r$$

$$\Rightarrow r = 30^0$$

from symmetry

$r' = r = 30^0$

Apply snell's law at B

$1.\sin e = \sqrt{3}\sin r' = \frac{\sqrt{3}}{2}$

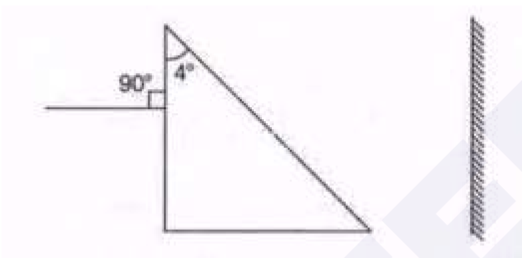
$\Rightarrow e = 60^0$

$\delta_1 = 60^0 - 30^0 = 30^0$

$\delta_2 = e - r^1 = 60^0 - 30^0 = 30^0$

$\therefore$ total deviation $=60^0$

Q. 6 A right angled prism of apex angle 4^0 and r. i. 1.5 is located in front of vertical plane mirror as shown in fig.A horizontal ray of light is falling on the prism. Find the total deviation produced in the light ray at it emerges second time from the prism:



Option 1:
8^0 cw

Option 2:
6^0 cw

Option 3:
180^0 cw

Option 4:
176^0 cw

Correct Answer:
176^0 cw

Solution:
As we learn

Deviation from thin prism -

$\delta = (\mu - 1) A$

- wherein

Applicable when A is very small

(i.e. thin prism)

Deviation produced by prism is

$\delta_1 = (\mu - 1)A = 2cw$

Angle of incidence of mirror is δ_1, so deviation produced by mirror is

$\delta_2 = \pi - 2\delta_1 = 176^0 cw$

deviation produced by the prism for second refraction is

$\delta_3 = 2^0 Acw$

Net deviation is 176^0 cw

Q. 7 The focal length of the objective and the eyepiece of compound microscope are 2cm and 3cm respectively. The distance between the objective and eyepiece is 15 cm. The final image formed by eyepiece is at infinity the distance (in cm) of the object and image produced by the objective, measured from the objective lens are respectively.

Option 1:
2.4 and 12

Option 2:
2.4 and 15

Option 3:
2.3 and 3

Option 4:
2.3 and 12

Correct Answer:
2.4 and 12

Solution:
As we learnt

Length of compound microscope -

$L = v_o + u_e$

$v_o =$ Image distance from objective.

$u_e =$ Object distance from eyepiece

f_0 = 2 cm f_e = 3cm

l = 15cm

Final image formed at infinity hence image formed by objective is at focal point of eyepiece

$u_e = f_e = 3cm$

$v_o = l - f_e = 12cm$

For objective:

$v_o = 12cm$ $f_o = 2cm$

$$\frac{1}{v_0} - \frac{1}{u_0} = \frac{1}{f_0}$$

$$\frac{1}{12} - \frac{1}{u_0} = \frac{1}{2}$$

or

$$\frac{1}{u_0} = \frac{1}{12} - \frac{1}{2} = \frac{1-6}{12}$$

or,

$u_0 = -2.4cm$ $v_0 = 12cm$

Q. 8 Planck's constant (*h*), speed of light in vacuum (*c*) and Newton's gravitational constant (*G*) are three fundamental constants. Which of the following combinations of these has the dimension of length?

Option 1:
$$\frac{\sqrt{hG}}{c^{3/2}}$$

Option 2:
$$\frac{\sqrt{hG}}{c^{5/2}}$$

Option 3:
$$\sqrt{\frac{hc}{G}}$$

Option 4:

$$\sqrt{\frac{\mathrm{Gc}}{\mathrm{h}^{3/2}}}$$

Correct Answer:

$$\frac{\sqrt{\mathrm{hG}}}{\mathrm{c}^{3/2}}$$

Solution:

As we learnt in

To convert a physical quantity from one system to other -

$$n_1\left[u_1\right]=n_2\left[u_2\right]$$

- wherein

$$n_2=n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c$$

$$L=h^{1/2}G^{1/2}.c^{-3/2}$$

As a research tool to derive new relation -

Use the method of dimensional analysis between the quantities can be derived

$$\text{-}G=\left[M^{-1}L^3T^{-2}\right]$$

Let Length $[K]\propto[h]^x\,[C]^y\,[G]^z$

$$[.K]=[L]$$

$$[h]=\left[ML^2T^{-1}\right],C=\left[L^1T^{-1}\right]$$

$$G=\left[M^{-1}L^3T^{-2}\right]$$

Comparing Power of both sides

$$[h]^{\frac{1}{2}}\,[c]^{\frac{-3}{2}}[G]^{\frac{1}{2}}$$

Q. 9 A physical quantity of the dimensions of length that can be formed out of c, G and $\frac{e^2}{4\pi\varepsilon_o}$ is (c is velocity of light, G is universal constant of gravitation and e is charge)

Option 1:
$$c^2\left[G\frac{e^2}{4\pi\varepsilon_o}\right]^{\frac{1}{2}}$$

Option 2:
$$\frac{1}{c^2}\left[\frac{e^2}{G4\pi\varepsilon_o}\right]^{\frac{1}{2}}$$

Option 3:
$$\frac{1}{c}G\frac{e^2}{4\pi\varepsilon_o}$$

Option 4:
$$\frac{1}{c^2}\left[G\frac{e^2}{4\pi\varepsilon_o}\right]^{\frac{1}{2}}$$

Correct Answer:
$$\frac{1}{c^2}\left[G\frac{e^2}{4\pi\varepsilon_o}\right]^{\frac{1}{2}}$$

Solution:
As we learnt in

Time period of a simple pendulam -

$$T = Km^a l^b g^0$$

Equating exponents of similar quantities

a=0 b=1/2 c=-1/2

$$\therefore T = 2\pi\sqrt{l/g}$$

- wherein

$$T = time\ period$$

$$l = length$$

$$g = \ acceleration\ due\ to\ gravity$$

$$\frac{e^2}{4\pi\epsilon_0} = [F \times d^2] = ML^3T^{-2}$$

$$G = ML^3T^{-2}$$

$$C = L^{-1}$$

$$l \propto (\frac{e^2}{4\pi\epsilon_0})^P G^q C^r$$

$$[L^1] = [MC^3T^{-2}]^P[M^{-1}L^3T^{-2}]^q[LT^{-1}]^r$$

on comparing both sides we get

$$P = \frac{1}{2}, q = \frac{1}{2}, r = -2$$

$$\therefore l = \frac{1}{C^2}[\frac{Ge^2}{4\pi\epsilon_0}]^{1/2}$$

Q. 10 If the magnitude of sum of two vectors is equal to the magnitude of difference of the two vectors, the angle between these vectors is:

Option 1:
0^o

Option 2:
90^o

Option 3:
45^o

Option 4:
180^o

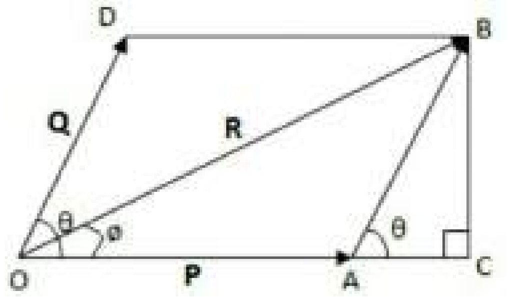
Correct Answer:
90^o

Solution:
As we learnt in

Parallelogram law of vector Addition -

If two vector are represented by both magnitude and direction by two adjacent side of parallelogram taken from same point then their resultant is also represented by both magnitude and direction taken from same point but by diagonal of parallelogram.

- wherein



Represents law of parallelogram vector Addition

$$\left|\vec{A}-\vec{B}\right| = \left|\vec{A}+\vec{B}\right|$$

$$A^2+B^2-2ABcos\theta = A^2+B^2+2ABcos\theta$$

$$4ABcos\theta = 0$$

$$cos\theta = \frac{0}{4AB} = 0 = 90^o$$

Q. 11 The horizontal range and the maximum height of a projectile are equal. The angle of projection of the projectiles is:

Option 1:
$\theta=\tan^{-1}(\frac{1}{4})$

Option 2:
$\theta=\tan^{-1}(4)$

Option 3:
$\theta=tan^{-1}(2)$

Option 4:
$\theta=45^0$

Correct Answer:
$\theta=\tan^{-1}(4)$

Solution:
As we learnt in

Horizontal Range -

Horizontal distance travelled by projectile from the point of projectile to the point on ground where its hits.

$$R = \frac{u^2 \sin 2\Theta}{g}$$

- wherein

Special case of horizontal range

For man horizontal range.

$$\Theta = 45^0$$

$$R_{max} = \frac{u^2 \sin 2(45)}{g} = \frac{u^2 \times 1}{g} = \frac{u^2}{g}$$

R=H

$$\frac{u^2 sin2\theta}{g} = \frac{u^2 sin^2\theta}{2g}$$

$$tan\theta = 4 \qquad => \theta = tan^{-1}4$$

Correct option is 2.

Q. 12 A particle moves so that its position vector is given by $\vec{r} = \cos\omega t\,\hat{x} + \sin\omega t\,\hat{y}$. Where ω is a constant.

Which of the following is true?

Option 1:
Velocity and acceleration both are perpendicular to $\vec{r}$.

Option 2:
Velocity and acceleration both are parallel to $\vec{r}$.

Option 3:
Velocity is perpendicular to $\vec{r}$ and acceleration is directed towards the origin.

Option 4:
Velocity is perpendicular to $\vec{r}$ and acceleration is directed away from the origin.

Correct Answer:
Velocity is perpendicular to $\vec{r}$ and acceleration is directed towards the origin.

Solution:
As we learnt in

Vector or cross product -

Vector or cross product of two vector $\vec{A}$ & $\vec{B}$ written as $A \times B$ is a single vector whose magnitude is equal to product of magnitude of $\vec{A}$ & $\vec{B}$ and the sine of smaller angle Θ between them.

$$\vec{A} \times \vec{B} = A\, B \sin\Theta$$

- wherein

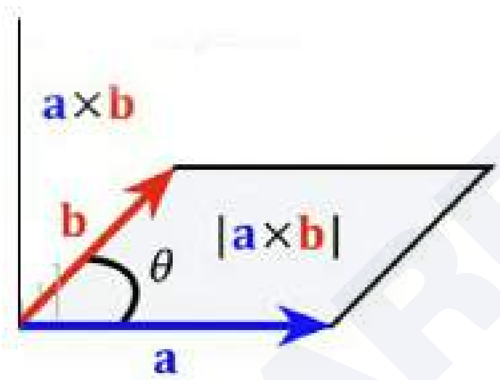


Figure 6 shows representation of vector or cross product of vectors.

shows representation of vector or cross product of vectors

$$\vec{r} = \cos wt\ \hat{i} + \sin wt\ \hat{i}$$

For two perpendicular vectors

$$\vec{V}.\vec{r} = 0$$

$$\frac{dr}{dt} = V \Rightarrow V = -w\left(\sin wt\ \hat{i} - \cos wt\ \hat{i}\right)$$

and $\vec{V}.\vec{r} = 0$

So, velocity is $\perp$ to r and $\vec{a}$ is directed towards the origin.

Correct option is 3.

Q. 13 A projectile is fired from the surface of the earth with a velocity of 5ms-^{1} and angle θ with the horizontal. Another projectile fired from another planet with a velocity of 3 ms^{-1} at the same angle follows a trajectory which is identical with the trajectory of the projectile fired from the earth. The value of the acceleration due to gravity on the planet is (in ms^{-2}) is: (given g = 9.8 ms^{-2})

Option 1:
3.5

Option 2:
5.9

Option 3:
16.3

Option 4:
110.8

Correct Answer:
3.5

Solution:
As we learnt in

Horizontal Range -

Horizontal distance travelled by projectile from the point of projectile to the point on ground where its hits.

$$R = \frac{u^2 \sin 2\Theta}{g}$$

- wherein

Special case of horizontal range

For man horizontal range.

$$\Theta = 45^0$$

$$R_{max} = \frac{u^2 \sin 2(45)}{g} = \frac{u^2 \times 1}{g} = \frac{u^2}{g}$$

Range should be equal

$$\frac{u_e^2 \sin 2\theta}{2g_e} = \frac{u_p^2 \sin 2\theta}{2g_p}$$ (P $\longrightarrow$ plant, e $\longrightarrow$ earth)

$$\frac{5^2 \times \sin 2\theta}{2 \times 9.8} = \frac{3^2 \times \sin 2\theta}{2g_p}$$

$$g_p = \frac{9.8 \times 9}{25} = 3.52 \ m/s^2$$

Correct option is 1.

Q. 14 Out of the following options which one can be used to produce a propagating electromagnetic wave?

Option 1:
A charge moving at constant velocity

Option 2:
A stationary charge

Option 3:
A chargeless particle

Option 4:
An accelerating charge

Correct Answer:
An accelerating charge

Solution:
As we learnt in

Total radiant Flux (Power) -

$$P = \frac{q^2 a^2}{6\pi \epsilon_o c^2}$$

q = Charge

a = Acceleration of particle

c = Speed of light in vacuum

- wherein

The accelerated charge particle produce EM wave. P is total radiant flux emitted by charge at any instant.

Electromagnetic wave can be produced by an accelerating charge.

Q. 15 The transition from the state n = 3 to n = 1 in a hydrogen like atom results in ultraviolet radiation. Infrared radiation will be obtained in the transition from:

Option 1:
$3 \longrightarrow 2$

Option 2:
$4 \longrightarrow 2$

Option 3:
$4 \longrightarrow 3$

Option 4:
$2 \longrightarrow 1$

Correct Answer:
$4 \longrightarrow 3$

Solution:
As we learnt in

Infrared Waves -

Frequency Range 3×10^{11} Hz to 4×10^{14} Hz

- wherein

Wavelength Range 7500 A^o to 1 mm

Energy diffference for infrared radiation will be smaller than those ultraviolet region.

Transition corresponding to Infra red region is:

$$2 \rightarrow 1; \frac{1}{\lambda} = R(1 - \frac{1}{4}) \quad or \quad \lambda = \frac{4}{3R} = 120.3\ nm(UV)$$
$$3 \rightarrow 2; \frac{1}{\lambda} = R\left(\frac{1}{4} - \frac{1}{9}\right) \quad or \quad \lambda = \frac{36}{5R}$$

$$\lambda = \frac{36}{5} \times 91\ nm = 648\ nm(visible\ region)$$
$$4 \rightarrow 2\ \ \frac{1}{\lambda} = R\left(\frac{1}{4} - \frac{1}{16}\right) = R\frac{3}{16}$$
$$\lambda = \frac{16}{3R} = \frac{16}{3R} \times 91\ nm = 480\ nm(visible)$$

$$4 \rightarrow 3;\ \frac{1}{\lambda} = R\left(\frac{1}{9} - \frac{1}{16}\right) = R(\frac{7}{144})$$
$$\lambda = \frac{144}{7R} = \frac{144}{7} \times 91\ nm = 1872\ nm\ IR$$

Correct option is 3.

Q. 16 In an electromagnetic wave in free space the root mean square value of the electric field is $E_{rms} = 6V/m$. The peak value of the magnetic field is:-

Option 1:
$2.83 \times 10^{-8}T$

Option 2:
$0.70 \times 10^{-8}T$

Option 3:
$4.23 \times 10^{-8}T$

Option 4:
$1.41 \times 10^{-8}T$

Correct Answer:
$2.83 \times 10^{-8}T$

Solution:
As we learnt in

Relation between Eo and Bo -

$E_o = c.B_o$

- wherein

E_o= Electric field amplitude

B_o = Magnetic field amplitude

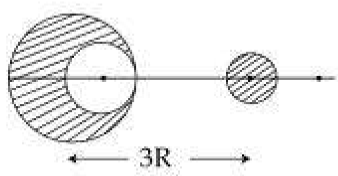
C= Speed of light in vacuum

E_{rms}=6 V/m

peak value $E_o = E_{rms.\sqrt{2}} = (1.41)6\ V/m = 8.46\ V/m$

magneitc field peak value $B_o = \frac{8.46}{3 \times 10^8} T = 2.82 \times 10^{-8} T$

Q. 17 From a sphere of mass M and radius R, a smaller sphere of radius $\frac{R}{2}$ is carved out such that the cavity made in the original sphere is between its centre and the periphery. (See figure). For the configuration in the figure where the distance between the centre of the original sphere and the removed sphere is 3R, the gravitational force between the two spheres is :



Option 1:
$\frac{41GM^2}{3600R^2}$

Option 2:
$\frac{41GM^2}{450R^2}$

Option 3:
$\frac{59GM^2}{450R^2}$

Option 4:
$\frac{GM^2}{225R^2}$

Correct Answer:
$\frac{41GM^2}{3600R^2}$

Solution:
As we discussed in

Newton's Law of Gravitation -

$$F\ \alpha\ \frac{m_1 m_2}{r^2}$$

$$F = \frac{G m_1 m_2}{r^2}$$

$F \rightarrow$ Force

$G \rightarrow$ Gravitational constant

$m_1, m_2 \rightarrow$ Masses

$r \rightarrow$ Distance between masses

- wherein

Force is along the line joining the two masses

Volume of removed sphere

$V_{removed} = \frac{4}{3}\pi(\frac{R}{2})^3 = \frac{4}{3}\pi R^3(\frac{1}{8})$

Volume of the sphere (remaining)

$V_{remain} = \frac{4}{3}\pi R^3 - \frac{4}{3}\pi R^3(\frac{1}{8}) = \frac{4}{3}\pi R^3(\frac{7}{8})$

Therefore the mass of removed sphere and remaining sphere are at respectively $\frac{1}{8}M$ and $\frac{7}{8}M$

$$F_{net} = \frac{GM\frac{M}{8}}{9R^2} - \frac{G\frac{M}{8} \times \frac{1}{8}M}{(\frac{25}{4}R)^2} = \frac{41}{3600}\frac{GM^2}{R^2}$$

Q. 18 When 10^{19} electrons are removed from a neutral metal plate, the electric charge on it is

Option 1:
$-1.6\ C$

Option 2:
$+1.6\,C$

Option 3:
$10^{+19}C$

Option 4:
$10^{-19}C$

Correct Answer:
$+1.6\,C$

Solution:
As we Learned

Electric charge -The loss of electrons gives a Positive charge.

By using $Q = ne \Rightarrow Q = 10^{19} \times 1.6 \times 10^{-19} = +1.6\,C$

Q. 19 When a body is earth connected, electrons from the earth flow into the body. This means the body is

Option 1:
Unchanged

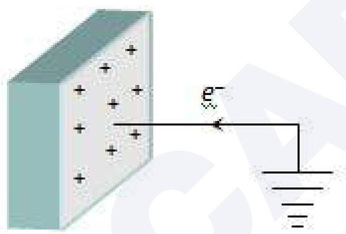
Option 2:
Charged positively

Option 3:
Charged negatively

Option 4:
An insulator

Correct Answer:
Charged positively

Solution:
When a positively charged body is connected to the earth, electrons flow from earth to body and the body becomes neutral.



Q. 20 Two copper balls, each weighing 10g are kept in air 10 *cm* apart. If one electron from every 10^6 atoms is transferred from one ball to the other, the coulomb force between them is (atomic weight of copper is 63.5)

Option 1:
$2.0 \times 10^{10}\,N$

Option 2:
$2.0 \times 10^{4}\,N$

Option 3:
$2.0 \times 10^{8}\,N$

Option 4:

$2.0 \times 10^{6} N$

Correct Answer:

$2.0 \times 10^{8} N$

Solution:

As we learned

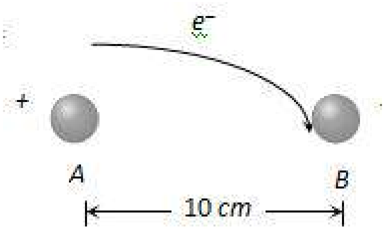
Properties of Charge -

Transferable

- wherein

It can be transferred from one body to another.

Number of atoms in given mass = $\frac{10}{63.5} \times 6.02 \times 10^{23} = 9.48 \times 10^{22}$



Transfer of electron between balls = $\frac{9.48 \times 10^{22}}{10^{6}} = 9.48 \times 10^{16}$

Hence magnitude of charge gained by each ball.

$$Q = 9.48 \times 10^{16} \times 1.6 \times 10^{-19} = 0.015\, C$$

Force of attraction between the balls $F = 9 \times 10^{9} \times \frac{(0.015)^2}{(0.1)^2} = 2 \times 10^{8} N$

Q. 21 The electric field near a conducting surface having a uniform surface charge density σ is given by

Option 1:

$\frac{\sigma}{\varepsilon_0}$ and is parallel to the surface

Option 2:

$\frac{2\sigma}{\varepsilon_0}$ and is parallel to the surface

Option 3:
$\frac{\sigma}{\varepsilon_0}$ and is normal to the surface

Option 4:
$\frac{2\sigma}{\varepsilon_0}$ and is normal to the surface

Correct Answer:
$\frac{\sigma}{\varepsilon_0}$ and is normal to the surface

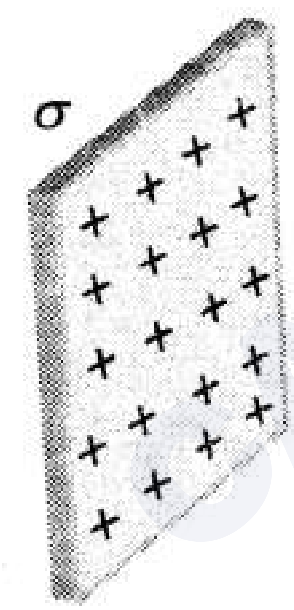
Solution:
As we learned

Surface charge distribution -

(σ) – charge per unit Area

$$\sigma = \frac{Q}{A} = \frac{C}{m^2} = Cm^{-2}$$

- wherein



(Plane sheet, sphere, cylinder etc)

Electric field near the conductor surface is given by $\frac{\sigma}{\varepsilon_0}$ and it is perpendicular to surface.

The correct option is 3.

Q. 22 The current produced in wire when 10^7 electron/sec are flowing in it

Option 1:
$1.6 \times 10^{-25} A$

Option 2:
$1.6 \times 10^{12} A$

Option 3:
$1.6 \times 10^{25} A$

Option 4:
$1.6 \times 10^{-12} A$

Correct Answer:
$1.6 \times 10^{-12} A$

Solution:
$$i = \frac{Q}{t} = \frac{ne}{t} = 10^{7} \times 1.6 \times 10^{-19} = 1.6 \times 10^{-12} \quad A$$

Q. 23 A cylindrical piston of mass M slides smoothly inside a long cylinder closed at one end, enclosing a certain mass of gas. The cylinder is kept with its axis horizontal. If the position is disturbed from its equilibrium position, its oscillates simple harmonically. The period of the oscillation will be

Option 1:
$$T = 2\pi\sqrt{[\frac{Mh}{PA}]}$$

Option 2:
$$T = 2\pi\sqrt{[\frac{MA}{Ph}]}$$

Option 3:
$$T = 2\pi\sqrt{[\frac{M}{PAh}]}$$

Option 4:
$$T = 2\pi\sqrt{MPhA}$$

Correct Answer:
$$T = 2\pi\sqrt{[\frac{Mh}{PA}]}$$

Solution:

Let the piston be displaced through distance x towards left, then volume decreases, pressure increases. If ΔP is increase in pressure and ΔV is decrease in volume, then considering the process to take place gradually (i.e. isothermal)

$$
\begin{aligned}
& P_1 V_1 = P_2 V_2 \\
\Rightarrow\ & PV = (P + \Delta P)(V - \Delta V) \\
\Rightarrow\ & PV = PV + \Delta P V - P \Delta V - \Delta P \Delta V \\
\Rightarrow\ & \Delta P V - P \Delta V = 0 \quad (neglecting\ \Delta P \Delta V) \\
\Rightarrow\ & \Delta P(Ah) = P(Ax) \Rightarrow \Delta P = \frac{P \cdot x}{h}
\end{aligned}
$$

This excess pressure is responsible for providing the restoring force (F) to the piston of mass M.

Hence $F = \Delta P \cdot A = \dfrac{PAx}{h}$

Comparing it with $|F| = kx \Rightarrow k = M\omega^2 = \dfrac{PA}{h}$

$$\Rightarrow \omega = \sqrt{\frac{PA}{Mh}} \Rightarrow T = 2\pi \sqrt{\frac{Mh}{PA}}$$

Q. 24 A simple pendulum has a time period T in vacuum. Its time period when it is completely immersed in a liquid of density one-eight of the density of the material of the bob is

Option 1:

$\sqrt{\frac{7}{8}}T$

Option 2:

$\sqrt{\frac{5}{8}}T$

Option 3:

$\sqrt{\frac{3}{8}}T$

Option 4:

$\sqrt{\frac{8}{7}}T$

Correct Answer:

$\sqrt{\frac{8}{7}}T$

Solution:

In vacuum, $T=2\pi\sqrt{\frac{l}{g}}$

Let V be the volume and p be the density of the mass of the bob.

Net downward force acting on the bob inside the liquid

= weight - upthrust

$$=Vpg-V\frac{p}{8}g=\frac{7}{8}Vpg$$

So, time period of the bob inside the liquid

$$\therefore T_1=2\pi\sqrt{\frac{l}{\frac{7}{8}g}}=2\pi\sqrt{\frac{l}{g}}\times\sqrt{\frac{8}{7}}=\sqrt{\frac{8}{7}}T$$

Q. 25 The charge flowing through a resistance R varies with time t as Q = at - bt^2, where a and b are positive constants. The total heat produced in R is:

Option 1:
$\frac{a^3\mathrm{R}}{6\mathrm{b}}$

Option 2:
$\frac{a^3\mathrm{R}}{3\mathrm{b}}$

Option 3:
$\frac{a^3\mathrm{R}}{2\mathrm{b}}$

Option 4:
$\frac{a^3\mathrm{R}}{\mathrm{b}}$

Correct Answer:
$\frac{a^3\mathrm{R}}{6\mathrm{b}}$

Solution:

$$Q=at-bt^2$$
$$I=dQ/dt=a-2bt$$
$I=0\ for\ t=t_0=\frac{a}{2b}$ i.e, current flows from t=0 → t=t_0

the heat produced is $\int_0^{t_0} i^2Rdt=\frac{a^3R}{6b}$

Q. 26 A potentiometer wire is 100 cm long and a constant potential difference is maintained across it. Two cells are connected in series first to support one another and then in opposite direction. The balance points are obtained at 50 cm and 10 cm from the positive end of the wire in the two cases. The ratio of emf's is :

Option 1:
5 : 1

Option 2:
5 : 4

Option 3:
3 : 4

Option 4:
3 : 2

Correct Answer:
3 : 2

Solution:
Given that,

Length l = 100 cm

The emf when the two cells are connected in series then the balance point is 50 cm.

$$E_1 + E_2 = 50 \qquad \text{....(I)}$$

The emf when the two cells are connected in opposite direction then the balance point is 10 cm.

$$E_1 - E_2 = 10 \qquad \text{....(II)}$$

From equation (I) and (II)

Using elimination method we get,

$$E_1 = 30$$

$$E_2 = 20$$

Now, we will find the ratio of E_1 and E_2

$$\frac{E_1}{E_2} = \frac{3}{2}$$

Q. 27 A potentiometer is an accurate and versatile device to make electrical measurements of E.M.F. because the method involves :

Option 1:
cells

Option 2:
potential gradients

Option 3:
a condition of no current flow through the galvanometer

Option 4:
a combination of cells, galvanometer and resistances

Correct Answer:
a condition of no current flow through the galvanometer

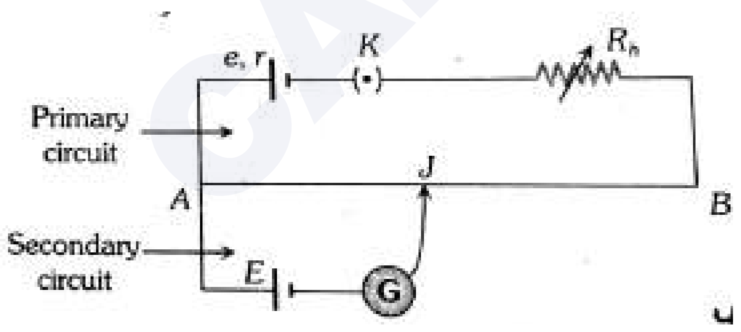
Solution:
As we learnt in

Potentiometer -

It is a device used to measure e.m.f of a given cell and to compare e.m.f's of cells

- wherein

It is also used to measure internal resistance of given cell



Because the method involves a condition of no current flow through the galvanometer the device can be used to measure P.D internal resistance of a cell and compare emf's of two sources.

Q. 28 The resistance of a wire is 'R' ohm. If it is melted and stretched to 'n' times its original length, its new resistance will be :

Option 1:
nR

Option 2:
$\frac{\mathrm{R}}{\mathrm{n}}$

Option 3:
n^2R

Option 4:
$\frac{\mathrm{R}}{\mathrm{n}^2}$

Correct Answer:
n^2R

Solution:
As we learnt in

Resistance formula -

$$R=\rho\frac{l}{A}=\frac{m}{ne^2\tau}\cdot\frac{l}{A}$$

- wherein

$\rho=$ resistivity of material

$n=$ Number of free electrons per unit volume.

$$R=\rho\frac{1}{A}$$

$$R=\frac{\rho l^2}{V}\Rightarrow R\ \propto\ l^2$$

According to the question, $l_2 = nl_1$

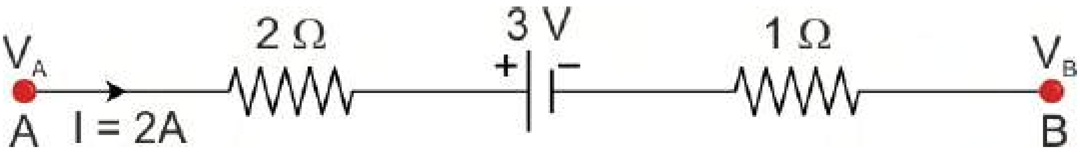
$$\frac{R_2}{R_1}=\frac{n^2{l_1}^2}{{l_1}^2}\Rightarrow\frac{R_2}{R_1}=n^2$$

$$\Rightarrow R_2=n^2R_1$$

$$\because R_1=R\ and\ R_2=R'$$

$$\therefore R'=n^2R$$

Q. 29 The potential difference (V_A - V_B) between the points *A* and *B* in the given figure is



Option 1:
-3 V

Option 2:
+3 V

Option 3:
+6 V

Option 4:
+9 V

Correct Answer:
+9 V

Solution:
As we learnt in

Potential difference -

The voltage accross the terminals of a cell when it is supplying current to external resistance is known as Potential Difference .

- wherein

$V = IR$

$V_{AB} = V_A - V_B$

= 2 × 2+3+1 × 2=9V

Q. 30 A filament bulb (500 W, 100 V) is to be used in a 230 V main supply. When a resistance *R* is connected in series, it works perfectly and the bulb consumes 500 W. The value of *R*

Option 1:
230 Ω

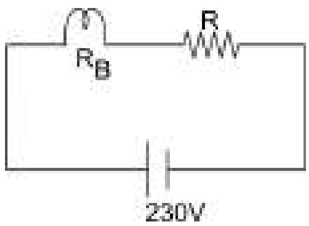
Option 2:
46 Ω

Option 3:
26 Ω

Option 4:
13 Ω

Correct Answer:
26 Ω

Solution:



$$P = VI$$

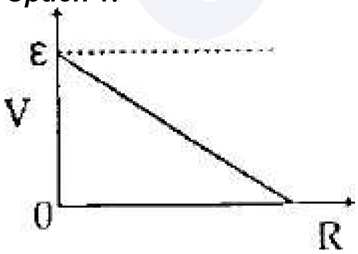
$$I = \frac{P}{V_B} = \frac{500}{100} = 5A$$

$$V_R = IR \quad \Rightarrow (230 - 100) = 5R$$

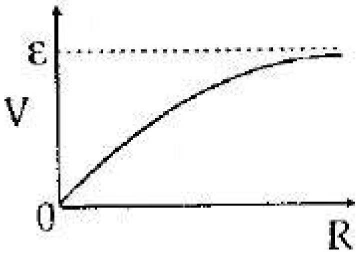
$$R = 26\,\Omega$$

Q. 31 A cell having an emf ε and internal resistance r is connected across a variable external resistance R. As the resistance R is increased, the plot of potential difference V across R is given by:

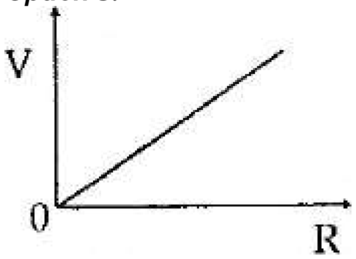
Option 1:



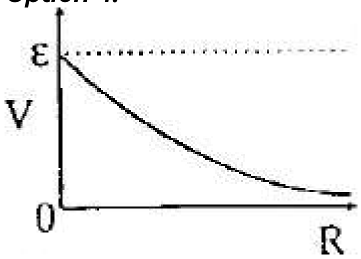
Option 2:



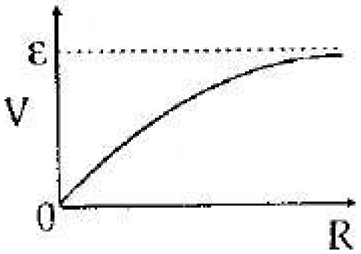
Option 3:



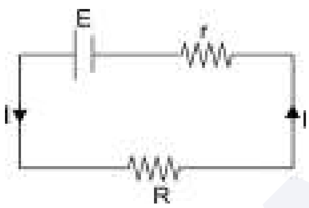
Option 4:



Correct Answer:



Solution:



Current in the circuit $I = \dfrac{E}{R+r}$

The potential difference across R: $V = IR = \left(\dfrac{E}{R+r}\right) R$

When,

$R = 0, \quad V = 0$

$R = \infty, \quad V = E$

Hence the correct option is 2.

Q. 32 If N_0 is the original mass of the substance of half life period $t_{1/2}$ =5 years, then the amount of substance left after 15 years is

Option 1:
$N_0/8$

Option 2:
$N_0/16$

Option 3:
$N_0/2$

Option 4:
$N_0/4$

Correct Answer:
$N_0/8$

Solution:
As we learnt in

Number of nuclei after disintegration -

$N = N_0 e^{-\lambda t}$ or $A = A_0 e^{-\lambda t}$

Number of nucleon activity at a time t is exponentional function

Now,

N_o is the initial amount of substance and N is the amount left after decay.

$$\text{Thus,} \quad N = N_0 \left(\tfrac{1}{2}\right)^n$$
$$n = \text{ number of half-lives } = \frac{t}{t_{1/2}} = \frac{15}{5} = 3$$
$$\text{Therefore,} \quad N = N_0 \left(\tfrac{1}{2}\right)^3 = \frac{N_0}{8}$$

So,

$$N = \frac{N_o}{8}$$

Q. 33 In the nuclear fission reaction,

${}_1^2H + {}_1^3H \rightarrow {}_2^4He + n$ given that the repulsive potential energy between the two nuclei is $\sim 7.7 \times 10^{-14} J$ the temperature at which the gases must be heated to initiate the reaction is nearly

[**Boltzmann's constant** $k = 1.38 \times 10^{-23} J/K$]

Option 1:
$10^7 K$

Option 2:
$10^5 K$

Option 3:
$10^3 K$

Option 4:
$10^9 K$

Correct Answer:
$10^9 K$

Solution:
As we learnt in

Nuclear fission -

e.g. ${}^{236}_{92}U \rightarrow {}^{137}_{53} I + {}^{97}_{39} Y + 2n$

$Q\ value$

$= [(M_U + M_n) - (M_I + M_Y + 2M_n)] .C^2$

- wherein

In nuclear fission neutron trigger the reaction & in the process more than one neutron is released.

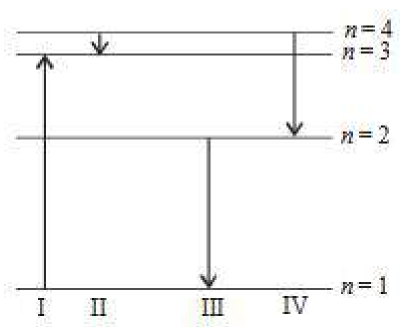
At temperature T, the kinetic energy is $\frac{3}{2}KT$.

$$\frac{3}{2}KT = 7.7 \times 10^{-14}$$

$$T = \frac{7.7 \times 2 \times 10^{-14}}{3 \times 1.38 \times 10^{-23}} = 3.7 \times 10^9 K$$

Correct option is 4.

Q. 34 The diagram shows the energy levels for an electron in a certain atom. Which transition shown represents the emission of a photon with the most energy?



Option 1:
I

Option 2:
II

Option 3:
III

Option 4:
IV

Correct Answer:
III

Solution:
As we learnt in

Energy emitted due to transition of electron -

$$\Delta E = Rhcz^2\left(\frac{1}{{n_f}^2} - \frac{1}{{n_i}^2}\right)$$

$$\frac{1}{\lambda} = Rz^2\left(\frac{-1}{{n_i}^2} + \frac{1}{{n_f}^2}\right)$$

- wherein

$R = Rhydberg\, constant$

$n_i = initial\, state$

$n_f = final\ state$

Highest difference of energy is between n = 1 and n = 3.

According to $\Delta E = E_0 z^2\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right)$. Correct option is III. Since I is absorption of photon.

Correct option is 3.

Q. 35 For which of the following Bohr model is not valid?

Option 1:
Hydrogen atom

Option 2:
singly ionised helium atom (He^{+})

Option 3:
Deuteron atom

Option 4:
singly ionised neon atom (Ne^{+})

Correct Answer:
singly ionised neon atom (Ne^{+})

Solution:
Bohr's theory is applicable to hydrogen-like atoms (single electron system).

But Ne^{+} does not have one electron

So Bohr model is not valid for Ne^{+}

Q. 36 A nucleus disintegrates into two nuclear parts which have their velocities in the ratio 2:1 The ratio of their nuclear sizes will be

Option 1:
$2^{1/3} : 1$

Option 2:
$1 : 3^{1/2}$

Option 3:
$3^{1/2} : 1$

Option 4:
$1 : 2^{1/3}$

Correct Answer:
$1 : 2^{1/3}$

Solution:

From conservation of momentum

$m_1v_1 = m_2v_2$

$\because \quad m \; \alpha \; A$

Hence, $A_1v_1 = A_2v_2$

$$\frac{A_1}{A_2}=\frac{v_2}{v_1}=\frac{1}{2}$$

$$R=R_0A^{1/3}$$

$$\therefore\ \frac{R_1}{R_2}=\left(\frac{A_1}{A_2}\right)^{1/3}=\frac{1}{2^{1/3}}$$

Correct option is 4.

Q. 37 Photoelectric emission occurs only when the incident light has more than a certain minimum:

Option 1:
frequency

Option 2:
power

Option 3:
wavelength

Option 4:
intensity

Correct Answer:
frequency

Solution:
According to Einstein's photoelectric equation

$K_{max} = hv - hv_o$

since K_{max} is + ve, the photoelectric emission occurs only if

$hv > hv_o$ or $v > v_o$

The photoelectric emission occurs only when the incident light has more than a certain minimum frequency. This minimum frequency is called threshold frequency.

Q. 38 Electron used in an electron microscope are accelerated by a voltage of 25 kV. If the voltage is increased to 100 kV then the de-Broglie wavelength associated with the electrons would

Option 1:
increase by 2 times

Option 2:
decrease by 2 times

Option 3:
increase by 4 times

Option 4:
increase by 4 times

Correct Answer:
decrease by 2 times

Solution:
As we learnt in

De - Broglie wavelength with charged particle -

$$\lambda = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2mE}}$$

$$\lambda = \frac{h}{\sqrt{2mqv}}$$

- wherein

$$E \rightarrow kinetic\ energy\ of particle$$

$$q \rightarrow charged\ particle$$

$$\lambda\ =\ \frac{h}{\sqrt{2mev}}\ =>\ \lambda \propto \frac{1}{\sqrt{V}}$$

$$\therefore \frac{\lambda_2}{\lambda_1} = \sqrt{\frac{V_1}{V_2}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$$

$\therefore$ wavelength decreases by a factor of 2.

Q. 39 Two Carnot engines A and B are operated in series. Engine A receives heat from a reservoir at 600 K and rejects heat to a reservoir at temperature T. Engine B receives heat rejected by engine A and in turn rejects it to a reservoir at 100 K. If the efficiencies of the two engines A and B are represented by ηA and ηB, respectively,then what is the value of $\frac{\eta_B}{\eta_A}$? (Work output is same for both)

Option 1:
$\frac{12}{7}$

Option 2:
$\frac{7}{12}$

Option 3:
$\frac{12}{5}$

Option 4:
$\frac{5}{12}$

Correct Answer:
$\frac{12}{7}$

Solution:
As we learnt

Efficiency of a carnot cycle -

$$\eta=\frac{W}{Q_1}=1-\frac{T_2}{T_1}$$

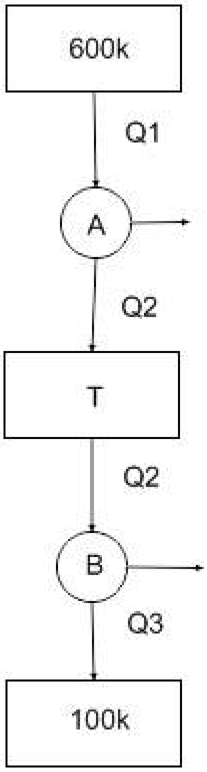
$T_1 \, and \, T_2$ are in kelvin

- wherein

T_1 $=$ Source temperature

T_2 $=$ Sink Temperature

$(T_1 > T_2)$



$$\eta_A = 1 - \frac{T}{600} = \frac{w}{Q_1} \rightarrow (1)$$
$$\eta_B = 1 - \frac{100}{T} = \frac{w}{Q_2} = \frac{w}{Q_1 - w}$$

$$\eta_B = 1 - \frac{100}{T} = \frac{1}{\frac{Q_1}{w} - 1}$$

from eqn1:

$$\eta_B = 1 - \frac{100}{T} = \frac{1}{\frac{1}{1-\frac{T}{600}} - 1} = \frac{1 - \frac{T}{600}}{\frac{T}{600}}$$

$$\eta_B = \frac{600}{T} - 1 = 1 - \frac{100}{T} \ or \ \frac{700}{T} = 2 \ or \ T = 350K$$

$$\frac{\eta_B}{\eta_A} = \frac{1 - \frac{100}{T}}{1 - \frac{T}{600}} = \frac{12}{7}$$

Q. 40 A gas is compressed is isothermally to half its initial volume. The same gas is compressed separately through and adiabatic process until its volume is again reduced to half. Then:

Option 1:
Compressing the gas isothermally will require more work to be done.

Option 2:
Compressing the gas through adiabatic process will require more work to be done.

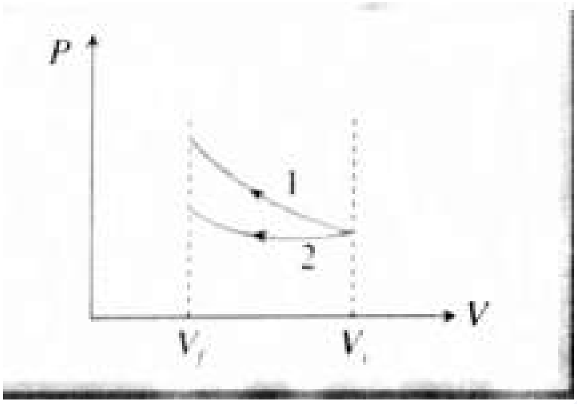
Option 3:
Compressing the gas isothermally or adiabatically will require the same amount of work.

Option 4:
Which of the case (wheather compression through isothermal or through adiabatic process) requires more work will depend upon the atomicty of the gas.

Correct Answer:
Compressing the gas through adiabatic process will require more work to be done.

Solution:
As we learnt in

Comparision between isothermal and adiabatic process in compression -



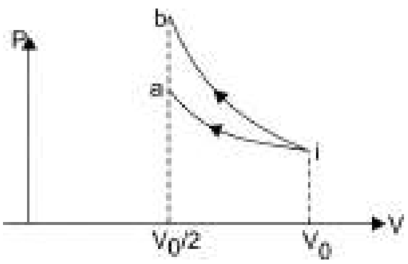
- wherein

$\omega_{adia} > \omega_{isothermal}$

$P_{adia} > P_{isothermal}$

$T_{adia} > T_{isothermal}$

Consider the P-V indicator diagram shown below.



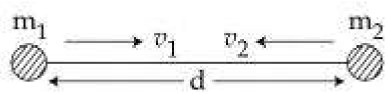
ia $\longrightarrow$ Isothermal process

ib $\longrightarrow$ Adibatic process

Since slope of adiabatic process is greater than slope of isothermal process.

Work done under adiabatic process is greater than work done under isothermal process as area under ib is greater than area under ia.

Q. 41



Two hypothetical planets of masses m_1 and m_2 are at rest when they are infinite distance apart. Because of the gravitational force they move towards each other along the line joining their centres. What is their speed when their separation is 'd' ?

(Speed of m_1 is v_1 and that of m_2 is v_2)

Option 1:
$v_1 = v_2$

Option 2:
$$v_2 = m_1\sqrt{\frac{2G}{d(m_1+m_2)}}$$

Option 3:
$$v_2 = m_2\sqrt{\frac{2G}{d(m_1+m_2)}}$$

Option 4:
$$v_2 = m_1\sqrt{\frac{2G}{m_2}}$$

Correct Answer:
$$v_2 = m_1\sqrt{\frac{2G}{d(m_1+m_2)}}$$

Solution:
Initial energy of the system = 0

Final energy = $\frac{1}{2}M_1V_1^2 + \frac{1}{2}M_2V_2^2 - \frac{GM_1M_2}{d}$

From conservation of energy

$$\frac{1}{2}M_1V_1^2 + \frac{1}{2}M_2V_2^2 = \frac{GM_1M_2}{d} --------1$$

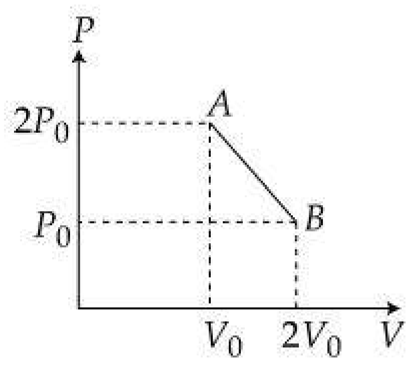
From conservation of Linear Momentum

$$m_1v_1^2 + m_2\left(\frac{-m_1v_1}{m_2}\right)^2 = \frac{2Gm_1m_2}{d}$$

$$\frac{m_1m_2v_1^2 + m_1^2v_1^2}{m_2} = \frac{2Gm_1m_2}{d} = v_1 = m_2\sqrt{\frac{2G}{d(m_1+m_2)}}$$

Similarly $v_2 = m_1\sqrt{\frac{2G}{d(m_1+m_2)}}$

Q. 42 'n' moles of an ideal gas undergoes a process A→B as shown in the figure. The maximum temperature of the gas during the process will be :



Option 1:
$\frac{9P_0V_0}{4nR}$

Option 2:
$\frac{3P_0V_0}{2nR}$

Option 3:
$\frac{9P_0V_0}{2nR}$

Option 4:
$\frac{9P_0V_0}{nR}$

Correct Answer:
$\frac{9P_0V_0}{4nR}$

Solution:
At any point between A & B, we can write relation between P & V by using the equation of the straight line

$$V - V_0 = \frac{2V_0 - V_0}{P_0 - 2P_0}(P - 2P_0)$$

$$V - V_0 = \frac{-V_0}{P_0}(P - 2P_0)$$

$$P\left(\frac{-V_0}{P_0}\right)+2V_0=V-V_0$$

$$P=\frac{-P_0}{V_0}(V-3V_0)$$

From ideal gas equation

PV = nRT

$$\Rightarrow \quad \frac{nRT}{V}=\frac{-P_0}{V_0}(V-3V_0)$$

$$T=\frac{-P_0}{nRV_0}(V^2-3V_0V)$$

For temperature to be maximum at any point $\frac{dT}{dV}=0$

$$\Rightarrow \quad 2V-3V_0=0$$

$$\therefore \quad V=\frac{3V_0}{2}$$

$$\therefore \quad T_{max}=\frac{-P_0}{nRV_0}\left(\frac{9}{4}V_0^2-\frac{9}{2}V_0^2\right)=-\frac{P_0}{nRV_0}\cdot\frac{-9}{4}V_0^2=\frac{9}{4}\frac{P_0V_0}{nR}$$

The correct option is 1.

Q. 43 The molar specific heats of an ideal gas at constant pressure and volume are denoted by C_p and C_v, respectively. If

$\gamma=\frac{C_p}{C_v}$ and **R** is the universal gas constant, then C_v is equal to:

Option 1:
γR

Option 2:
$\frac{1+\gamma}{1-\gamma}$

Option 3:
$\frac{R}{(\gamma-1)}$

Option 4:
$$\frac{(\gamma-1)}{\mathrm{R}}$$

Correct Answer:
$$\frac{\mathrm{R}}{(\gamma-1)}$$

Solution:

$$C_p - C_v = R - - - (1)$$

$$and\ \gamma = \frac{C_p}{C_v} - - - - -(2)$$

Divide equation (1) by C_v we get

$$=> \frac{C_p}{C_v} - 1 = \frac{R}{C_v}$$

$$=> \frac{R}{C_v} = \gamma - 1$$

$$=> C_v = \frac{R}{\gamma - 1}$$

Q. 44 When 100 g of a liquid A at 100°C is added to 50 g of a liquid B at temperature 75°C , the temeprature of the mixture becomes 90°C. The temperature of the mixture (in °C), if 100 g of liquid A at 100°C is added to 50 g of liquid B at 50°C, will be :

Correct Answer: 80

Solution:

Specific Heat -

$$C = \frac{Q}{m.\Delta\theta}$$

- wherein

C = specific heat

$\Delta\theta =$Change in temperature

m = Amount of mass

For (I)

$$100 \times S_A \times (100 - 90) = 50 \times S_B \times (90 - 75)$$

$S_A = \frac{1.5}{2} S_B = \frac{3}{4} S_B \cdots (1)$

now for (II)

$100 \times S_A \times (100 - T) = 50 \times S_B \times (T - 50)$

$\Rightarrow 2S_A (100 - T) = S_B (T - 50) \cdots (2)$

From (1) & (2)

$2 \times \frac{3}{4} \times (100 - T) = (T - 50)$

$3(100 - T) = 2(T - 50)$

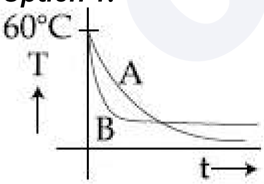
$300 - 3T = 2T - 100$

$400 = 5T$

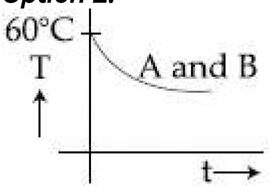
$T = 80°C$

Q. 45 Two identical beakers A and B contain equal volumes of two different liquids at 60^oC each and left to cool down. Liquid in A has density of $8 \times 10^2 kg/m^3$ and specific heat of $2000\ J\ kg^{-1}K^{-1}$ while liquid in B has density of $10^3\ kg\ m^{-3}$ and specific heat of $4000\ J\ kg^{-1}K^{-1}$ which of the following best describes their temperature versus time graph schematically ?(assume the emissivity of both the beakers to be the same)

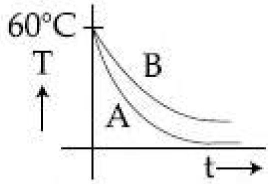
Option 1:



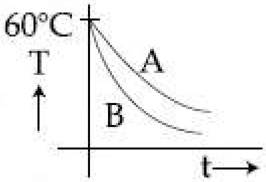
Option 2:



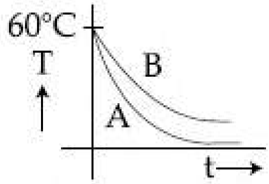
Option 3:



Option 4:



Correct Answer:



Solution:

Newton's Law of Cooling -

$$\frac{d\theta}{dt}\alpha(\theta-\theta_0)$$

- wherein

Temperature difference is not very large.

From given

$\rho_A < \rho_B,$

$\Rightarrow m_A < m_B,$

and $\Rightarrow s_A < s_B,$

By newtons law of cooling

$$\frac{-dT}{dt}=\frac{4\,\sigma\,e\,A\,T_0{}^3(T-T_o)}{ms}$$

$$\Rightarrow\frac{-dT}{dt}\alpha\frac{1}{ms}$$

at $t=0$

$$-(\frac{dT}{dt})_A \alpha \frac{1}{m_A s_A}$$

$$-(\frac{dT}{dt})_B \ \alpha \frac{1}{m_B s_B}$$

and we know

$$m_A s_A < m_B s_B$$

So slope of T v/s t curve for A is more than B.

Q. 46 A magnetic needle of magnetic moment 6.7×10^{-2} Am^2 and moment of inertia 7.5×10^{-6} kg m^2 is performing simple harmonic oscillations in a magnetic field of 0.01 T. Time taken (in seconds) for 10 complete oscillations is :

(give answer till 2 decimal places)

Correct Answer:
6.65

Solution:
As we learned in

The time period of Oscillating Bar Magnet -

$$T = 2\pi\sqrt{\frac{I}{MB}}$$

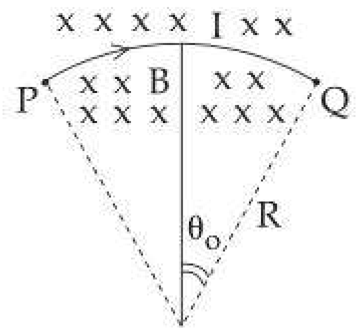
Using

$$I = 7.5 \times 10^{-6} \mathrm{kgm}^2$$
$$M = 6.7 \times 10^{-2} \mathrm{Am}^2$$
$$B = 0.1T$$

we get T = 0.665 s

For 10 oscillations, total time taken = t = 6.65 s

Q. 47 A wire carrying current I is tied between points P and Q and is in the shape of a circular arch of radius R due to a uniform magnetic field B (perpendicular to the plane of the paper, shown by xxx) in the vicinity of the wire. If the wire subtends an angle $2\Theta_0$ at the centre of the circle (of which it forms an arch) then the tension in the wire is :



Option 1:
IBR

Option 2:
$\frac{IBR}{\sin\Theta_0}$

Option 3:
$\frac{IBR}{2\sin\Theta_0}$

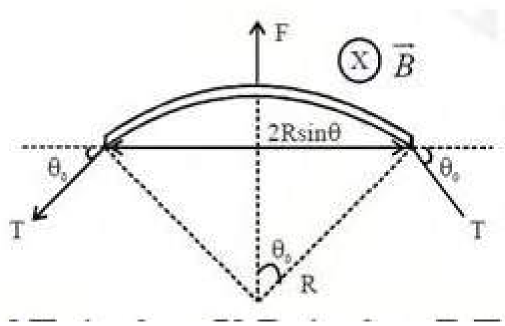
Option 4:
$\frac{IBR\,\Theta_0}{\sin\Theta_0}$

Correct Answer:
IBR

Solution:
As we learned in

Total magnetic force -

$\underset{F}{\rightarrow} = i(\underset{L}{\rightarrow} \times \underset{B}{\rightarrow})$



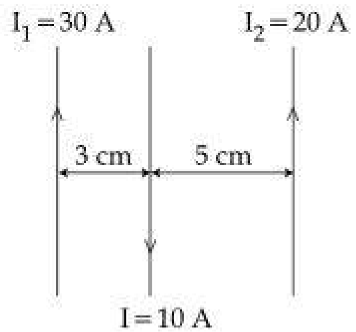
For the arc to be in equilibrium, $F = 2T\sin\theta_0$

$\Rightarrow F = I(2R\sin\theta_0) \times B \therefore$

$2T\sin\theta_0 = I2R\sin\theta_0 \times B$

$\Rightarrow T = IRB$

Q. 48 Three straight parallel current carrying conductors are shown in the figure. The force experienced by the middle conductor of length 25 cm is :



Option 1:
3 x 10^{-4} N toward right

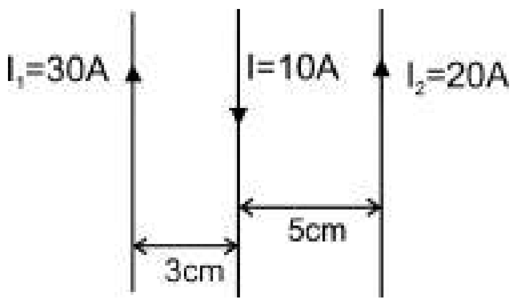
Option 2:
6 x 10^{-4} N toward left

Option 3:
9 x 10^{-4} N toward left

Option 4:
Zero

Correct Answer:
3 x 10^{-4} N toward right

Solution:



Force due to wire one

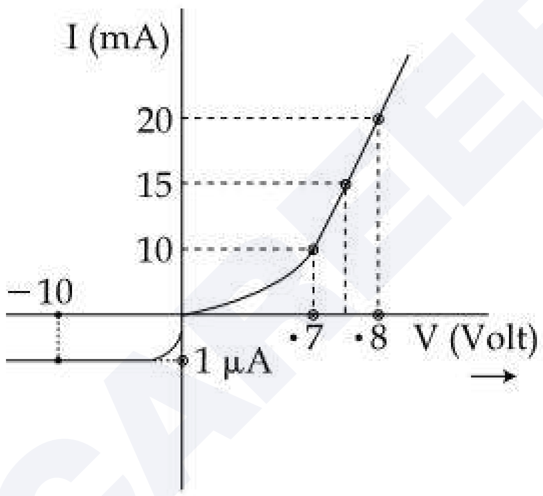
$$F_1 = \frac{\mu_0 I_1 I_2}{2\pi r_1} l = \frac{2 \times 10^{-7} \times 30 \times 10}{3 \times 10^{-2}} \times 25 \times 10^{-2} = 5 \times 10^{-4}$$ towards right

Force due to wire two

$$F_2 = \frac{\mu_0 I I_2}{2\pi r_2} l = \frac{2 \times 10^{-7} \times 20 \times 10}{5 \times 10^{-2}} \times 25 \times 10^{-2} = 2 \times 10^{-4}$$ towards left

Net force = $3 \times 10^{-4}\ towards\ \ right$

Q. 49 The V-I characteristic of a diode is shown in the figure. The ratio of forward to reverse bias resistance is :



Option 1:
10

Option 2:
10^{-6}

Option 3:
10^{6}

Option 4:
100

Correct Answer:
10^{-6}

Solution:

.

Forward Resistance

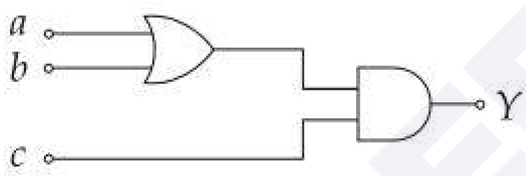
$$R_F = \frac{\Delta V}{\Delta i} = \frac{0.1}{10 \times 10^{-3}} \Omega$$

Reverse bias Resistance $= \frac{\Delta V}{\Delta i} = \frac{10}{10^{-6}} = 10^7 \Omega$

Ratio of forward to reverse bias resistance $= \frac{10}{10^7} = 10^{-6}$

Correct option is 2.

Q. 50 To get an output of 1 from the circuit shown in figure the input must be :



Option 1:
$a = 0, b = 1, c = 0$

Option 2:
$a = 1, b = 0, c = 0$

Option 3:
$a = 1, b = 0, c = 1$

Option 4:
$a = 0, b = 0, c = 1$

Correct Answer:
$a = 1, b = 0, c = 1$

Solution:

To get an output of 1, from the NAND gate

both input should be 1.

$\therefore \quad C = 1$

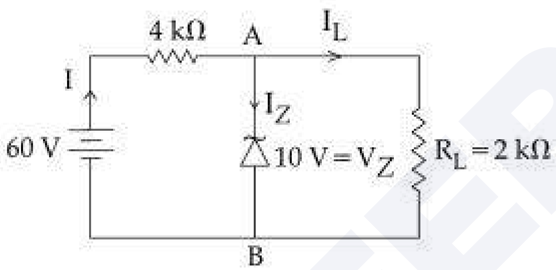
Now Resultant of a & b should be 1.

$$\therefore \left.\begin{array}{c} a=1, b=0 \\ or \\ a=0, b=1 \\ or \\ a=1, b=1 \end{array}\right\} and\ C=1$$

So the Correct option is 3.

Q. 51 A Zener diode is connected to a battery and a load as shown below :

The currents I, I_Z and I_L are respectively



Option 1:
5 mA, 5 mA, 10 mA

Option 2:
15 mA, 7.5 mA, 7.5 mA

Option 3:
12.5 mA, 5 mA, 7.5 mA

Option 4:
12.5 mA, 7.5 mA, 5 mA

Correct Answer:
12.5 mA, 7.5 mA, 5 mA

Solution:
In the given figure

Voltage across R_L= 2KΩ is same as that across zener dioxide i.e 10V

$$\therefore I_L = \frac{V_Z}{R_L} = \frac{10V}{2X10^3} = 5mA$$

Total applied potential = 60 V

$\therefore$ Potential difference across 4KΩ will be 50 V

Current through 4KΩ = $\frac{50V}{4 \times 10^3 \Omega}$

$I = 12.5mA$

$\therefore$ Current through diode

$I_Z = I - I_L = 12.5mA - 5mA = 7.5mA$

Q. 52 A conducting circular loop is placed in a uniform magnetic field 0.04 T with its plane perpendicular to the magnetic field. The radius of the loop starts shrinking at 2 mm/s. The induced emf in the loop when the radius is 2 cm is:

Option 1:
4.8 π μV

Option 2:
0.8 π μV

Option 3:
1.6 π μV

Option 4:
3.2 π μV

Correct Answer:
3.2 π μV

Solution:

Induced emf in the loop is given by $e = -B \cdot \frac{dA}{dt}$ where A is the area of the loop.

So

$$e = -B \cdot \frac{d}{dt}\left(\pi r^2\right) = -B\pi 2r\frac{dr}{dt}$$

$$r = 2\text{cm} = 2 \times 10^{-2}\text{m}$$
$$dr = 2mm = 2 \times 10^{-3}m$$
$$dt = 1s$$
$$e = -0.04 \times 3.14 \times 2 \times 2 \times 10^{-2} \times \frac{2 \times 10^{-3}}{1}V$$
$$e = 0.32\pi \times 10^{-5}V = 3.2\pi \times 10^{-6}V = 3.2\pi\mu V$$

Q. 53 A condenser of capacity C is charged to a potential difference of V_1. The plates of the condenser are then connected to an ideal inductor of inductance L. The current through the inductor when the potential difference across the condenser reduces to V_2 is

Option 1:

$$\left(\frac{C\left(V_1-V_2\right)^2}{L}\right)^{\frac{1}{2}}$$

Option 2:

$$\frac{C\left(V_1^2-V_2^2\right)}{L}$$

Option 3:

$$\frac{C\left(V_1^2-V_2^2\right)}{L}$$

Option 4:

$$\left(\frac{C\left(V_1^2-V_2^2\right)}{L}\right)^{\frac{1}{2}}$$

Correct Answer:

$$\left(\frac{C\left(V_1^2-V_2^2\right)}{L}\right)^{\frac{1}{2}}$$

Solution:

$Here, q_0=CV_1 \quad and \quad q=CV_2$

When a charged capacitor is connected to ideal inductor, the discharge of capacitor is oscillatory.

The charge on capacitor at an instant t is given by, $q=q_0 \sin \omega t$

$$where\ \omega=\frac{1}{\sqrt{LC}}$$

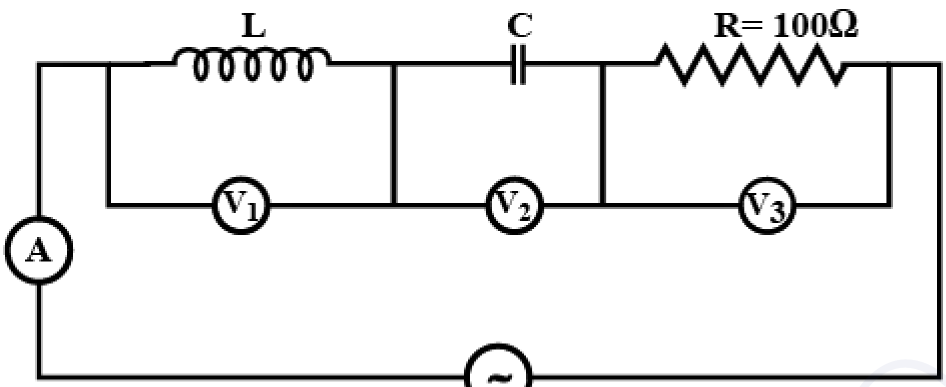
$$Therefore, \sin \omega t=\frac{q}{q_0}=\frac{CV_2}{CV_1}=\frac{V_2}{V_1}$$

Current through inductor is

$$I=\frac{dq}{dt}=\frac{d}{dt}\left(q_0 \sin \omega t\right)=q_0 \omega \cos \omega t=q_0 \omega\left[1-\sin^2 \omega t\right]^{1/2}$$

$$Or, I=CV_1 \times \frac{1}{\sqrt{LC}}\left[1-\left(\frac{V_2}{V_1}\right)^2\right]^{1/2}=\left[\frac{C\left(V_1^2-V_2^2\right)}{L}\right]^{1/2}$$

Q. 54 In the given circuit the reading of voltmeter V_1 and V_2 are 300 volts each. The reading of the voltmeter V_3 and ammeter A are respectively



220V. 50Hz

Option 1:
100 V, 2.0 A

Option 2:
150 V, 2.2 A

Option 3:
220 V, 2.2 A

Option 4:
220 V, 2.0 A

Correct Answer:
220 V, 2.2 A

Solution:
V_1 and V_2 are equal and out of phase with each other and thus get cancelled.
Thus $\mathrm{V}_3=220 \mathrm{~V}$ and $\mathrm{i}=\frac{220}{100}=2.2 \mathrm{~A}$

Q. 55 A person trying to lose weight by burning fat lifts a mass of 10 kg upto a height of 1 m 1000 times. Assume that the potential energy lost each time he lowers the mass is dissipated. How much fat will he use up considering the work done only when the weight is lifted up? Fat supplies 3.8×10^7 J of energy per kg which is converted to mechanical energy with a 20% efficiency rate. Take g = 9.8 ms^{-2} :

Option 1:
2.45×10^{-3} kg

Option 2:
6.45×10^{-3} kg

Option 3:
9.89×10^{-3} kg

Option 4:
12.89×10^{-3} kg

Correct Answer:
12.89×10^{-3} kg

Solution:
As we discussed in

If only conservative forces act on a system, total mechanical energy remains constant -

$$K+U=E\,(constant)$$

$$\Delta K+\Delta U=0$$

$$\Delta K=-\Delta U$$

-

Total work done by the person in lifting the weight = mgh

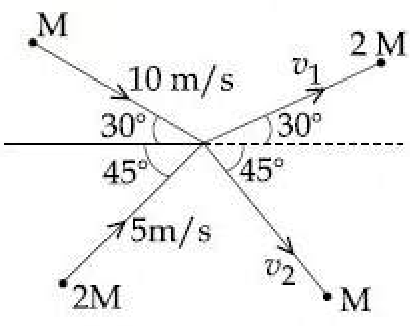
$$=10\times\ 9.8\times\ 1\times\ 1000$$

$$=\ 98\times\ 10^3J$$

Total mechanical energy produced by burning 1 kg fat $=(3.8\times10^7)\times0.20=7.6\times10^6J$

Total fat burn $=\dfrac{98\times10^3}{7.6\times10^6}kg=12.89\times10^{-3}kg$

Q. 56 Two particles, of masses M and 2M, moving as shown, with speeds of 10 m/s and 5 m/s, collide elastically at the origin. After the collision, they move along the indicated directions with speeds v_1 and v_2. The values of v_1 and v_2 are approximately :



Option 1:
6.5 m/s and 6.3 m/s

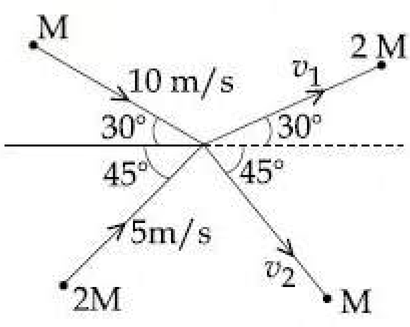
Option 2:
3.2 m/s and 6.3 m/s

Option 3:
6.5 m/s and 3.2 m/s

Option 4:
3.2 m/s and 12.6 m/s

Correct Answer:
6.5 m/s and 6.3 m/s

Solution:



u_1 $\quad u_{1x} = 10 \times \frac{\sqrt{3}}{2}\hat{i}$

$$u_{1y} = 10 \times \frac{1}{2}\hat{j} = -5\hat{j} \qquad M_1 = M$$

u_2 $\quad u_{2x} = \frac{5}{\sqrt{2}}\hat{i}$

$u_{2y} = \frac{5}{\sqrt{2}}\hat{j}$ $\qquad M_2 = 2M$

v_1 $\quad v_{1x} = v_1 \times \frac{\sqrt{3}}{2}\hat{i}$

$v_{1y} = v_1 \times \frac{1}{2}\hat{j}$

v_2 $\quad v_{2x} = v_2 \times \frac{1}{\sqrt{2}}\hat{i}$

$v_{2y} = \frac{v_2}{\sqrt{2}}(-\hat{j})$

$$\Delta P_x = 0 => m \times (\frac{10\sqrt{3}}{2}) + (2m \times \frac{5}{\sqrt{2}}) = 2m(\frac{\sqrt{3}}{2})v_1 + m(\frac{v_2}{\sqrt{2}})$$

$$=> 5\sqrt{3} + 5\sqrt{2} = \sqrt{3}v_1 + \frac{v_2}{\sqrt{2}} \text{..............................(1)}$$

$$\Delta P_y = 0 => -m \times (5) + (2m \times \frac{5}{\sqrt{2}}) = 2m(\frac{v_1}{2}) - m(\frac{v_2}{\sqrt{2}})$$

$$=> 5\sqrt{2} - 5 = v_1 - \frac{v_2}{\sqrt{2}} \text{...(2)}$$

On adding (1) and (2)

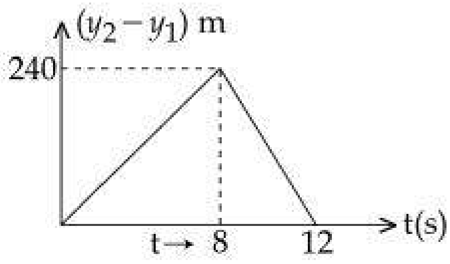
$$5(\sqrt{3} - 1) + 10\sqrt{2} = (\sqrt{3} + 1)v_1$$

$$=> v_1 = \frac{5(\sqrt{3} - 1) + 10\sqrt{2})}{\sqrt{3} + 1} \approx 6.5\ m/s$$

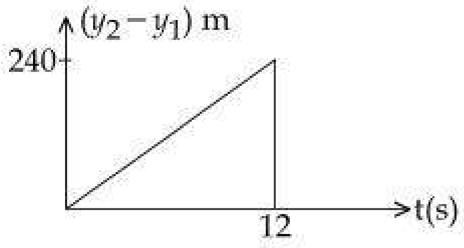
$$=> v_2 \approx 6.3\ m/s$$

Q. 57 Two stones are thrown up simultaneously from the edge of a cliff 240 m high with initial speed of 10 m/s and 40 m/s respectively. Which of the following graph best represents the time variation of relative position of the second stone with respect to the first? (Assume stones do not rebound after hitting the ground and neglect air resistance, take g=10 m/s^2

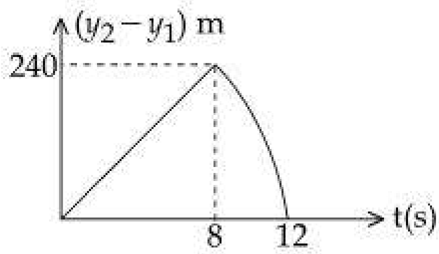
Option 1:



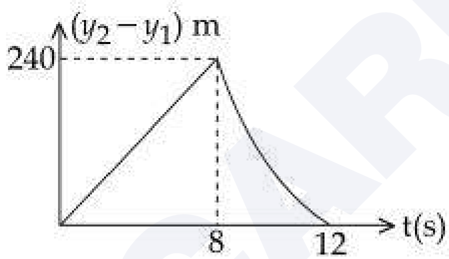
Option 2:



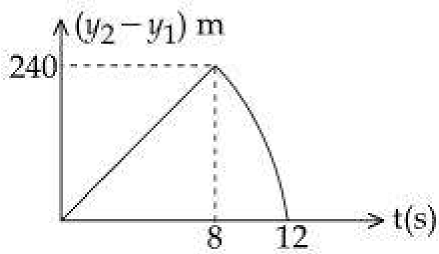
Option 3:



Option 4:



Correct Answer:



Solution:

For the first stone time required to reach the ground is given by

$y = ut - \frac{1}{2}gt^2$

$-240 = 10t - \frac{1}{2} \times 10 \times t^2$

$\therefore -240 = 10t - 5t^2$

$5t^2 - 10t - 240 = 0$

$(t - 8)(t + 6) = 0$

$t = 8\ s$

For the Second stone time required to reach the ground is given by

$y = ut - \frac{1}{2}at^2$

$y = ut - \frac{1}{2}gt^2$

$-240 = 40t - \frac{1}{2} \times 10 \times t^2$

$\therefore 5t^2 - 40t - 240 = 0$

$(t - 12)(t + 8) = 0$

$\therefore t = 12\ s$

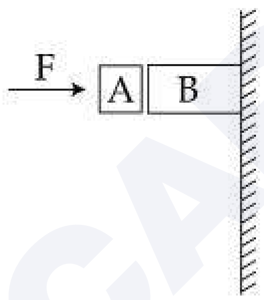
for $0 < t < 8\ sec \rightarrow a_{rel} = 0$

straight line x-t graph

for $8 < t < 12\ sec \rightarrow a_{rel} = -g$

downward parabola

Q. 58



Given in the figure are two blocks A and B of weight 20 N and 100 N, respectively. These are being pressed against a wall by a force F as shown. If the coefficient of friction between the blocks is 0.1 and between block B and the wall is 0.15, the frictional force (in N) applied by the wall on block B is :

Correct Answer:
120

Solution:
Solution :

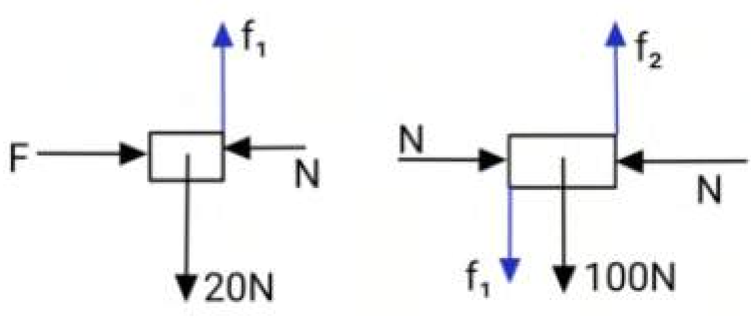
Given :

Weights of blocs $A = 20$ N
Weight of block B=100 N

As the blocks are at rest, both blocks must be in equilibrium.

Let the Friction force between the blocks be $f_1$and between block B and wall be f_2

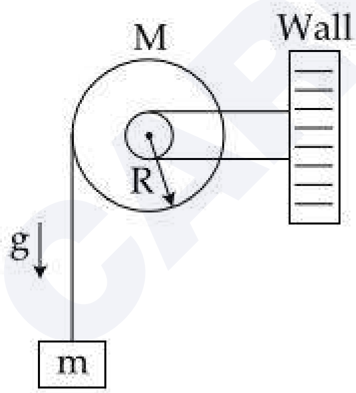
F.B.D of the blocks :



From F.B.D,

$F = N$
$f_1 = 20$
$f_2 = f_1 + 100$
$\Rightarrow f_2 = 20 + 100 = 120N$

Q. 59 A uniform disc of radius R and mass M is free to rotate only about its axis. A string is wrapped over its rim and a body of mass m is tied to the free end of the string as shown in the figure. The body is released from rest. Then the acceleration of the body is :



Option 1:
$$\frac{2mg}{2m + M}$$

Option 2:
$$\frac{2Mg}{2m + M}$$

Option 3:
$$\frac{2Mg}{2M+m}$$

Option 4:
$$\frac{2mg}{2M+m}$$

Correct Answer:
$$\frac{2mg}{2\ m+M}$$

Solution:
$$\mathbf{T}=\mathbf{I}\alpha$$
$$\mathbf{F}\cdot\mathbf{R}=\mathbf{I}\alpha$$
$$\mathbf{mgR}=\left(\frac{\mathbf{MR}^2}{2}+\mathbf{mR}^2\right)\boldsymbol{\alpha}$$
$$2\mathbf{mgR}=(2\mathbf{m}+\mathbf{M})\mathbf{R}^2\times\frac{\mathbf{a}}{\mathbf{R}}$$
$$\mathbf{a}=\frac{2\mathrm{mg}}{2\mathbf{m}+\mathbf{M}}$$

Q. 60 From a solid sphere of mass M and radius R a cube of maximum possible volume is cut. Moment of inertia of cube about an axis passing through its center and perpendicular to one of its faces is :

Option 1:
$$\frac{MR^2}{32\sqrt{2}\pi}$$

Option 2:
$$\frac{MR^2}{16\sqrt{2}\pi}$$

Option 3:
$$\frac{4MR^2}{9\sqrt{3}\pi}$$

Option 4:
$$\frac{4MR^2}{3\sqrt{3}\pi}$$

Correct Answer:
$$\frac{4MR^2}{9\sqrt{3}\pi}$$

Solution:
$$a=\frac{2}{\sqrt{3}}R$$

$$\frac{M}{M'}=\frac{\frac{4}{3}\pi R^3}{a^3}=\frac{\frac{4}{3}\pi R^3}{\left(\frac{2}{\sqrt{3}}R\right)^3}\Rightarrow\frac{M}{M'}=\frac{\frac{4}{3}\pi R^3}{\frac{8}{3\sqrt{3}}R^3}=\frac{4\pi}{3}\times\frac{3\sqrt{3}}{8}$$

$$\frac{M}{M'}=\frac{\sqrt{3}\pi}{2}\Rightarrow M'=\frac{2M}{\sqrt{3}\pi}$$

∴ M.O.I. of the cube about the given axis.

$$I=\frac{M'a^2}{6}=\frac{\frac{2M}{\sqrt{3}\pi}\times\left(\frac{2}{\sqrt{3}}R\right)^2}{6}=\frac{4MR^2}{9\sqrt{3}\pi}$$

Chemistry

Q. 1 In a 0.2 molal aqueous solution of a weak acid HX, the degree of ionization is 0.3 .Taking K_f for water as 1.85 K molal^{-1} , the freezing point of the solution will be nearest to

Option 1:
$-0.480°C$

Option 2:
$-0.360°C$

Option 3:
$-0.260°C$

Option 4:
$+0.480°C$

Correct Answer:
$-0.480°C$

Solution:
Case of dissociation

$$HX \rightleftharpoons H^+ + X^-$$

$$\text{van't Hoff factor (i)} = 1+(n-1)\alpha = 1+(2-1)\,0.3 = 1.3$$

$$\Delta T_f = iK_f \times m$$

$$\Delta T_f = 1.3\times 1.85\times 0.2 = 0.4810$$

Freezing point of solution = -0.4810 ^{o}c.

Correct option is (1)

Q. 2 Two liquids X and Y form an ideal solution. At 300 K, vapour pressure of the solution containing 1 mol of X and 3 mol of Y is 550 mm Hg. At the same temperature, if 1 mol of Y is further added to this solution, vapour pressure of the solution increases by 10 mm Hg. Vapour pressure (in mm Hg) of X and Y in their pure states will be, respectively

Option 1:
200 and 300

Option 2:
300 and 400

Option 3:
400 and 600

Option 4:
500 and 600

Correct Answer:
400 and 600

Solution:

$$P = P_A^0 \chi_A + P_B^0 \chi_B$$

Given that 1 mole of X and 3 moles of Y are present

$$\therefore \chi_A = \frac{1}{4}, \quad \chi_B = \frac{3}{4}$$

$$550 = p_A^0 \times \frac{1}{4} + p_B^0 \times \frac{3}{4} \qquad (1)$$

After adding 1 mole y the vapour pressure is 560

$$560 = p_A^0 \times \frac{1}{5} + p_B^0 \times \frac{4}{5} \qquad (2)$$

By solving equation 1 & 2

$$p_A^0 = 400, \quad p_B^0 = 600$$

Therefore, **option(3) is correct.**

Q. 3 The Van't Hoff factor will be highest for:

Option 1:
Sodium chloride

Option 2:
Magnesium chloride

Option 3:
Sodium phosphate

Option 4:
Urea

Correct Answer:
Sodium phosphate

Solution:
Van't Hoff factor for dissociation of strong electrolyte

$$NaCl \quad n = 2$$

$$CaCl_2 \quad n = 3$$

$$Na_3PO_4 \quad n = 4$$

$$NH_2CONH_2 \quad n = 1$$

Na_3PO_4 gives the maximum ions and hence it shows the highest Van't Hoff factor.

Therefore, **option(3) is correct**

Q. 4 Resistance of 0.2 M solution of an electrolyte is 50 Ω. The specific conductance of the solution is 1.4 S m^{-1}. The resistance of 0.5 M solution of the same electrolyte is 280 Ω. The molar conductivity of 0.5 M solution of the electrolyte in S m^2 mol^{-1} is :

Option 1:
5 x 10^{-4}

Option 2:
5 x 10^{-3}

Option 3:
5 x 10^{3}

Option 4:
5 x 10^{2}

Correct Answer:
5 x 10^{-4}

Solution:
Specific conductance,

$$\sigma = 1.4 Sm^{-1} = 1.4 \times 10^{-2}\ Scm^{-1}$$

Resistivity,

$$\rho = \frac{1}{\kappa} = \frac{1}{1.4 \times 10^{-2}}\ \Omega cm$$

Resistance, R=$\frac{\rho l}{A}$

Now, for a 0.5 M solution, R=280 Ω

$$\kappa = \frac{1}{\rho} = \frac{1}{R} \times \frac{l}{A} = \frac{1}{280} \times 50 \times 1.4 \times 10^{-2}$$

$$= 2.5 \times 10^{-3}\, Scm^{-1}$$

∴ molar conductivity,

$$\mu = \frac{\kappa \times 1000}{c} = \frac{2.5 \times 10^{-3} \times 1000}{0.5}$$

= $5\ \mathrm{Scm^2mol^{-1}}$

$$= 5 \times 10^{-4}\mathrm{Sm^2mol^{-1}}$$

Therefore, the correct option is (1).

Q. 5 The standard reduction potentials for Zn^{2+}/Zn, Ni^{2+} / Ni and Fe^{2+} / Fe are –0.76, –0.23 and –0.44 V respectively.

The reaction X + $Y^{2+} \longrightarrow X^{2+}$ + Y will be spontaneous when :

Option 1:
X = Ni, Y = Fe

Option 2:
X = Ni, Y = Zn

Option 3:
X = Fe, Y = Zn

Option 4:
X = Zn, Y = Ni

Correct Answer:
X = Zn, Y = Ni

Solution:
For a spontaneous reaction, E^o must be positive.

So, E^o = E^o reduced constituent - E^o oxidized constituent

E^o = (-0.23) - (-0.76)

[We get this by maximizing E^oR and minimizing E^oO]

Alternatively, we can also solve this qualitatively. Elements with a low value of SRP are good reducing agents and therefore, can be easily oxidized. Also, elements with a low negative value of SRP are good oxidizing agents and can be reduced easily. Keeping the above in mind, we can clearly see that $\mathrm{X}=\mathrm{Zn}$ and $\mathrm{Y}=\mathrm{Ni}$ and the reaction is:

$$\mathrm{Zn}+\mathrm{Ni}^{2+} \rightarrow \mathrm{Zn}^{2+}+\mathrm{Ni}$$

Therefore, **option(4) is correct**

Q. 6 Aluminium oxide may be electrolysed at 1000°C to furnish aluminium metal (At. Mass = 27 amu,1 Faraday= 96,500 Coulombs) The cathode reaction is

$$Al^{3+}+3e^{-} \rightarrow Al^{0}$$

To prepare 5.12 kg of aluminium metal by this method would require

Option 1:
$5.49 \times 10^{7} \mathrm{C}$ of electricity

Option 2:
$1.83 \times 10^{7} \mathrm{C}$ of electricity

Option 3:
$5.49 \times 10^{4} \mathrm{C}$ of electricity

Option 4:
5.49×10^{1} C of electricity

Correct Answer:
$5.49 \times 10^{7} \mathrm{C}$ of electricity

Solution:

$$\text{Moles of Al}=\frac{5.12 \times 1000}{27}$$

$$\approx 190$$

$$\therefore \text{Moles of } e^{-}=3 \times 190 \therefore \text{Total charge}=3 \times 190 \times 96500$$

$$\simeq 5.49 \times 10^{7} \mathrm{C}$$

Therefore, the correct option is (1).

Q. 7 6.02×10^{20} molecules of urea are present in 100 mL of its solution. The concentration of solution is:

Option 1:
0.1 M

Option 2:
0.02 M

Option 3:
0.01 M

Option 4:
0.001 M

Correct Answer:
0.01 M

Solution:
As we learnt in

$$\text{Molarity} = \frac{\text{Moles of solute}}{\text{Vol. of solution(L)}}$$

no. of moles = $\dfrac{6.02 \times 10^{20}}{6.02 \times 10^{23}}$ mol = 10^{-3} mol

Vol. = 100 ml

Molarity, M = $\dfrac{10^{-3}}{10^{-1}}\ \mathrm{M} = 10^{-2}\mathrm{M} = 0.01\mathrm{M}$

Therefore, **Option(3) is correct.**

Q. 8 In an experiment it showed that 10 ml of 0.05M solution of chloride required 10 mL of 0.1 M solution of $AgNO_3$, which of the following will be the formula of the chloride (X stands for the symbol of the element other than chlorine):

Option 1:
X_2Cl

Option 2:
X_2Cl_2

Option 3:
XCl_2

Option 4:
XCl_4

Correct Answer:
XCl_2

Solution:
Moles = Molarity X volume in litre.

No. of moles of $AgNO_3$=M X V = 0.1 X 10 (In ml) = 0.1 X 10 X 10^{-3} (in L) = 10^{-3} mol

No. of moles the chloride= 0.05 X 10 X 10^{-3} (in L)= 0.5 x 10^{-3} mol.

Suppose the formula for the chloride is XCl_n then moles of chloride ion = n x 0.5 x 10^{-3}

The reaction goes as follows:

$$Ag^+ + Cl^- \rightarrow AgCl$$

Then, going by stoichiometry we get

1 mol of Ag = 1 mol of Cl

so. 10^{-3} mol = n x 0.5 x 10^{-3}

$$n = \frac{1}{0.5} = 2$$

So, the formula is XCl_2

Therefore, the correct option is (3).

Q. 9 The osmotic pressure of solution increases, if

Option 1:
Temperature is decreased

Option 2:
Solution concentration is increased

Option 3:
Number of solute molecules is increased

Option 4:
Volume is increased

Correct Answer:
Number of solute molecules is increased

Solution:
As we learned

Osmotic Pressure -

Osmotic Pressure (π) is excess pressure developed on solution side due to osmosis.

-

As soon as the solute molecules increases the osmotic pressure of solution increase.

Therefore, option (3) is correct.

Q. 10 Which one of the following statements is FALSE?

Option 1:
The correct order of osmotic pressure for 0.01 M aqueous solution of each compound is $BaCl_2$ > KCl > CH_3COOH > sucrose.

Option 2:
The osmotic pressure (π) of a solution is given by the equation $\pi = MRT$ where M is the molarity of the solution.

Option 3:
Raoult's law states that the vapour pressure of a component over a solution is proportional to its mole fraction.

Option 4:
Two sucrose solutions of same molality prepared in different solvents will have the same freezing point depression.

Correct Answer:
Two sucrose solutions of same molality prepared in different solvents will have the same freezing point depression.

Solution:
As we have learnt
Freezing -
Freezing occurs when liquid solvent is in equilibrium with solid solvent. As non volatile solute decreases, the vapour pressure freezing point decreases.

The extent of depression in freezing point varies with the number of solute particles for a fixed solvent only and it is a characteristics feature of the nature of solvent also. So for two different solvents the extent of depression may vary even if number of solute particles be dissolved.

Therefore, **option(4) is correct**

Q. 11 The rise in the boiling point of a solution containing 1.8 gram of glucose in 100 g of a solvent in 0.1^{0}C. The molal elevation constant of the liquid is:

Option 1:
$0.01K/m$

Option 2:
$0.1K/m$

Option 3:
$1K/m$

Option 4:
$10K/m$

Correct Answer:
$1K/m$

Solution:
$\Delta T_b = K_b\, m$

$$K_b = \frac{\Delta T_b}{m} = \frac{0.1 \times 100}{\frac{1.8}{180} \times 1000} = 1K/m$$

Therefore, **option(3) is correct**

Q. 12 The reason for double helical structure of DNA is the operation of :

Option 1:
Electrostatic attractions

Option 2:
van der Waals forces

Option 3:
Dipole - Dipole interactions

Option 4:
Hydrogen bonding

Correct Answer:
Hydrogen bonding

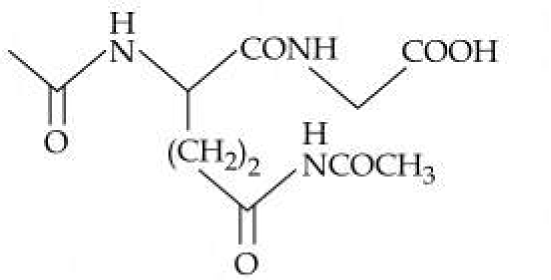
Solution:
As learnt in

The double-helical structure of DNA is the operation of hydrogen bonding.

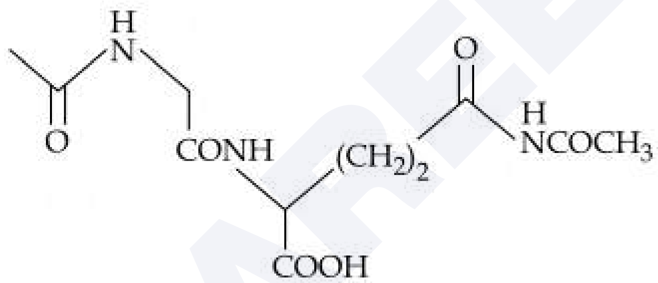
Hence, the option number (4) is correct.

Q. 13 The dipeptide, Gln-Gly, on treatment with CH_3COCl followed by aqueous work up gives :

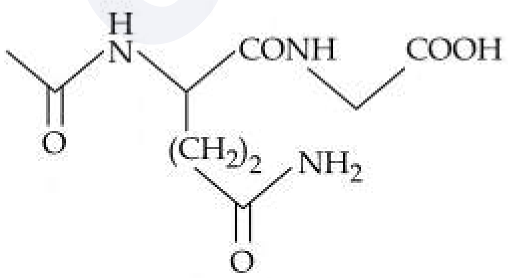
Option 1:



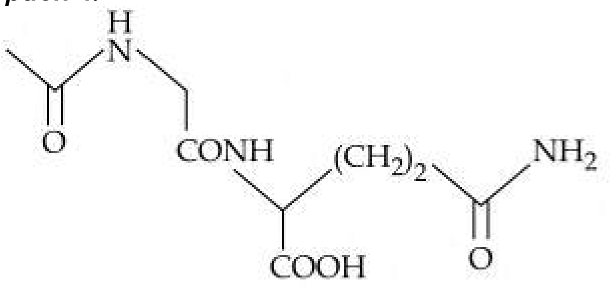
Option 2:



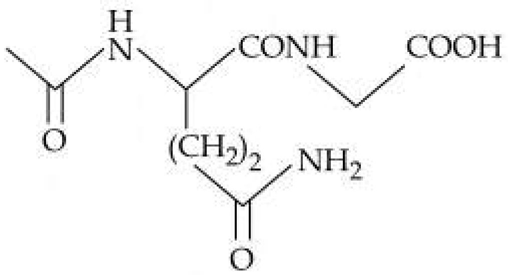
Option 3:



Option 4:

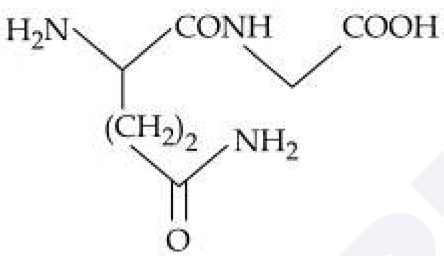


Correct Answer:



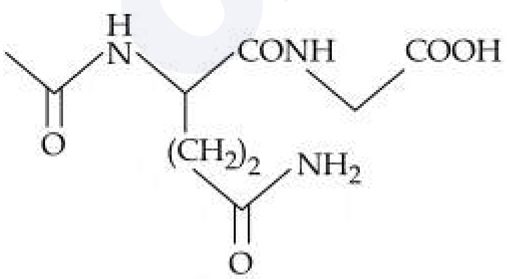
Solution:

The structure of the dipeptide Gln-gly is given as



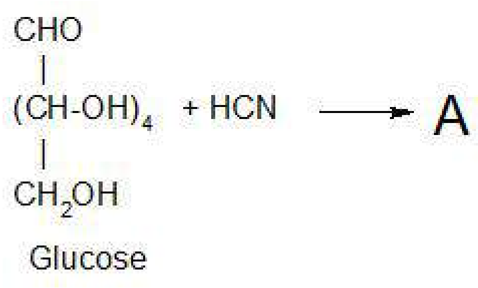
Now, acetylation occurs at the free hydroxy and amino groups. Amides do not undergo acetylation.

So, Gly-Gly on treatment with CH_3COCl followed by hydrolysis gives



Hence, the correct answer is Option (3)

Q. 14



A is :

Option 1:
Glucose cyanohydrin

Option 2:
Glucose oxime

Option 3:
Glucosazone

Option 4:
Gluconic acid

Correct Answer:
Glucose cyanohydrin

Solution:
As we have learnt,

Glucose reacts with Hydrogen cyanide and undergoes nucleophilic addition reaction to form its cyanohydrin.

Hence, the option number (1) is correct.

Q. 15 What type of redox reaction is the following?

$$N_{2(g)} + 3H_{2(g)} \rightarrow 2NH_{3(g)}$$

Option 1:
Decomposition Reaction

Option 2:
Combination Reaction

Option 3:
Displacement Reaction

Option 4:
Disproportionate Reaction

Correct Answer:
Combination Reaction

Solution:
The different types of redox reactions are:

- Decomposition Reaction
- Combination Reaction
- Displacement Reaction
- Disproportionation Reactions

Decomposition Reaction
This is the reaction that involves the breakdown of a compound into different compounds. Some examples of this type of reaction are:

$$2H_2O(l) \xrightarrow{\Delta} 2H_2(g) + O_2(g)$$
$$2KClO_3(s) \xrightarrow{\Delta} 2KCl(s) + 3O_2(g)$$

This must be noted here that all decomposition reactions are not redox reactions. For example, the decomposition of calcium carbonate is not a redox reaction.
$$CaCO_3(s) \xrightarrow{\Delta} CaO(s) + CO_2(g)$$

Combination Reaction
These types of reactions are the opposite of decomposition reactions and hence involve the combination of two compounds to form a single compound. Some examples include:

$$C(s) + O_2(g) \xrightarrow{\Delta} CO_2(g)$$
$$CH_4(g) + 2O_2(g) \xrightarrow{\Delta} CO_2(g)$$

Displacement Reaction
Displacement reactions, also known as replacement reactions, involve compounds and the replacing of elements. They occur as single and double replacement reactions. In other words, in these types of reactions, an atom or an ion in a compound is substituted by another element. The general representation of this reaction is as follows:

$$X + YZ \rightarrow XZ + Y$$

Disproportionation Reactions

Disproportionation reactions are those reactions in which a single element in one oxidation state is simultaneously oxidized and reduced. Some examples include:

$P_4(s) + 3OH^-(aq) + 3H_2O(l) \rightarrow PH_3(g) + 3H_2PO_2^-$
$S_8(s) + 12OH(aq) \rightarrow 4S^{2-}(aq) + 2S_2O_3^2(aq) + 6H_2O(l)$

-

Since the reaction fits the form: $A + B \rightarrow C$ and either A or B is in elemental form so it's a combination reaction.

Therefore, **option(2) is correct**.

Q. 16 If the reaction quotient of the cell

$Zn(s)|Zn^{2+}(x.M)||Ag^+(1.25M)|Ag(s)$ is 6.4×10^{-3}, then what is x?

Option 1:
0.2

Option 2:
0.01

Option 3:
0.1

Option 4:
0.02

Correct Answer:
0.01

Solution:
As we have learned

M(s) is Solid -

[M] = 1

- wherein

$$E_{M^{n+}/M} = E^0_{M^{n+}/M} - \frac{RT}{nf} ln \frac{1}{[M^{n+}]}$$

Reaction is $Zn + 2Ag^+ \rightarrow 2Ag + Zn^{2+}$

$reaction\ quotient\ formula$

$$Q = \frac{[C]^a[D]^b}{[E]^l[F]^m}$$

as [Ag]= [Zn] =1

$$Q = \frac{[Zn^{2+}][Ag]^2}{[Zn][Ag^+]^2} = \frac{[Zn^{2+}]}{[Ag^+]^2}$$

$$6.4 \times 10^{-3} = \frac{x}{(1.25)^2}$$

$$x = 0.01$$

Therefore, **Option(2) is correct**

Q. 17 If the conductivity of the cell is $0.08Sm^{-1}$,then find the resistance of the cell. Cell constant = $40m^{-1}$

Option 1:
3.2

Option 2:
500

Option 3:
0.002

Option 4:
0.3125

Correct Answer:
500

Solution:
We know,

$$R = \frac{\rho l}{A}$$

$$R = \frac{1}{k} \times \frac{l}{A}$$

$$(0.08)^-1 \times 40 = \frac{40}{0.08} = 500$$

Hence, the option number (2) is correct.

Q. 18 Consider the given complexes

(i) $K_3[Cr(CN)_6]$

(ii) $K_4[Fe(CN)_6]$

(iii) $K_3[Fe(CN)_6]$

(iv) $K_2[Ni(CN)_4]$

Out of the following, select the set containing only paramagnetic complexes

Option 1:
(i), (ii)

Option 2:
(ii), (iii)

Option 3:
(i), (iii)

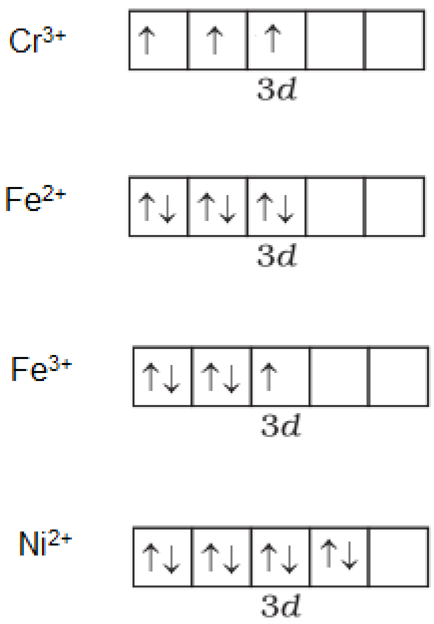
Option 4:
(i), (ii) and (iv)

Correct Answer:
(i), (iii)

Solution:
As we have learnt,

Cyanide ions $(CN)^-$ are strong field ligands and cause pairing of electrons.

The electronic arrangement in the central metal ions in the given complexes with the Strong field ligand is shown below:



Hence, only complexes (i) and (iii) are paramagnetic as they contain unpaired electrons

Therefore, **option (3) is correct.**

Q. 19 In any ferric salt, on adding potassium ferrocyanide, a Prussian blue colour is obtained, which is:

Option 1:
$K_3Fe(CN)_6$

Option 2:
$KFe[Fe(CN)_6]$

Option 3:
$FeSO_4 \cdot Fe(CN)_6$

Option 4:
$Fe_4[Fe(CN)_6]_3$

Correct Answer:
$Fe_4[Fe(CN)_6]_3$

Solution:
As we have learnt,

Potassium Ferrocyanide reacts with ferric chloride to form Prussian's Blue

$$3K_4[Fe(CN)_6] + 4FeCl_3 \rightarrow Fe_4[Fe(CN)_6]_3 + 12KCl$$

Therefore, **option (4) is correct.**

Q. 20 An aqueous solution of potash alum gives

Option 1:
Two types of ions

Option 2:
Only one type of ion

Option 3:
Four types of ions

Option 4:
Three types of ions

Correct Answer:
Three types of ions

Solution:
Potash alum is a mixed salt of K_2SO_4 and $Al_2(SO_4)_3$ and upon dissolution in water, it gives all the three ions from which it is made viz. Al^{3+}, K^+, SO_4^{2-}

Therefore, **option (4) is correct.**

Q. 21 What will be the molarity of a solution containing 5 g of sodium hydroxide in 250 ml solution:

Correct Answer:
0.5

Solution:

$$M = \frac{W}{\text{Mol. Wt.}} \times \frac{1000}{\text{Volume in ml}} = \frac{5 \times 1000}{40 \times 250} = 0.5\ M$$

Q. 22 A $20\ litre$ container at $400K$ contains $CO_2(g)$ at pressure $0.4atm$ and an excess of SrO (neglect the volume of solid SrO). The volume of the containers is now decreased by moving the movable piston fitted in the container. The maximum volume of the container, when pressure of CO_2 attains its maximum value, will be :

(Given that : $\mathrm{SrCO_3(s) \rightleftharpoons SrO(s) + CO2(g),\ K_p = 1.6\,atm}$)

Option 1:
$5\ litre$

Option 2:
$10\ litre$

Option 3:
$4\ litre$

Option 4:
$2\ litre$

Correct Answer:
$5\ litre$

Solution:
$SrCO_3 \rightleftharpoons SrO + CO_2(g), K_p = 1.6\ atm$

$K_p = p_{CO_2}$

Before compression,

$$n_{CO_2} = \frac{pV}{RT} = \frac{0.4 \times 20}{R \times 400} = \frac{8}{R400} = \frac{1}{50R}$$

After Compression

For the maximum Volume of the container, let's assume that $n = constant,\ \ p = 1.6atm$

$$V = \frac{nRT}{p} = \frac{1}{50R} \times \frac{R \times 400}{1.6} = 5L$$

The correct answer is option 1.

Q. 23 The equilibrium constants of the following are:

$$N_2 + 3H_2 \rightleftharpoons 2NH_3 \quad K_1$$

$$N_2 + O_2 \rightleftharpoons 2NO \quad K_2$$

$$H_2 + \frac{1}{2}O_2 \rightarrow H_2O \quad K_3$$

The equilibrium constant (K) of the reaction:

$$2NH_3 + \frac{5}{2}O_2 \overset{K}{\rightleftharpoons} 2NO + 3H_2O$$

Option 1:

$$\frac{K_1 K_3^3}{K_2}$$

Option 2:

$$\frac{K_2 K_3^3}{K_1}$$

Option 3:

$$\frac{K_2 K_3}{K_1}$$

Option 4:

$$\frac{K_3 K_2^3}{K_1}$$

Correct Answer:

$$\frac{K_2 K_3^3}{K_1}$$

Solution:

The equilibrium constant for the reverse reaction is the inverse of the equilibrium constant for the reaction in the forward direction.

$$K_c' = \frac{1}{K_c}$$

K_c' is the equilibrium constant for the reverse direction.

$$N_2 + 3H_2 \rightleftharpoons 2NH_3, \quad K_1$$

$$2NH_3 \rightleftharpoons 3H_2 + N_2, \quad \frac{1}{K_1} \quad \text{--------- (1)}$$

$$N_2 + O_2 \rightleftharpoons 2NO, \quad K_2 \quad \text{......................(2)}$$

$$H_2 + \frac{1}{2}O_2 \rightarrow H_2O, \quad K_3$$

$$3H_2 + \frac{3}{2}O_2 \rightarrow 3H_2O, \quad K_3^3$$ ---------- (3)

adding (1) (2) (3)

$$3H_2 + \frac{3}{2}O_2 \rightleftharpoons 3H_2O, \quad K = \frac{K_2K_3^3}{K_1}$$

The correct answer is option 2.

Q. 24 The K_{sp} of $Ag_2CrO_4, AgCl, AgBr\ and\ AgI$, are respectively, $1.1 \times 10^{-12}, 1.8 \times 10^{-10}, 5.0 \times 10^{-13}, 8.3 \times 10^{-17}$. Which one of the following salts will precipitate last if $AgNO_3$ solution is added to the solution containing equal moles of $NaCl, NaBr, NaI\ and\ Na_2CrO_4$?

Option 1:
$AgBr$

Option 2:
Ag_2CrO_4

Option 3:
AgI

Option 4:
$AgCl$

Correct Answer:
Ag_2CrO_4

Solution:
Let us calculate the solability of each salt

$$Ag_2CrO_4 \rightarrow 2Ag^+ + CrO_4^{2-}$$

1	0	0
1-s	2s	s

$$K_{sp} = \frac{(2s)^2 s}{1-s}, \quad s << 1$$

$$\Rightarrow 1.1 \times 10^{-12} = 4s^3$$

$$\Rightarrow s = 6.5 \times 10^{-5}$$

$$\mathrm{AgCl} \rightarrow \mathrm{Ag}^+ + \mathrm{Cl}^-$$

s s

$1.8 \times 10^{-10} = s^2$

$\Rightarrow s = 1.34 \times 10^{-5}$

$AgBr \rightarrow Ag^+ + Br^-$

s s

$K_{sp} = s^2 = 5 \times 10^{-13}$
$s = 7.1 \times 10^{-7}$

$AgI \rightarrow Ag^+ + I^-$

s s

$K_{sp} = s^2 = 8.3 \times 10^{-17}$
$s = 9 \times 10^{-9}$

Since the solubility of Ag_2CrO_4 is the maximum hence, it will precipitate last.

The correct answer is option 2.

Q. 25 MY and NY_3 two nearly insoluble salts, have the same K_{sp} values of 6.2×10^{-13} at room temperature. Which statement would be true in regard of MY and NY_3?

Option 1:
The molar solubilities of $MY\ and\ NY_3$ in water are indetical.

Option 2:
The molar solubility of in water is less than that of NY_3.

Option 3:
The salts MY and NY_3 are more soluble in $0.5M\ KY$ than in pure water.

Option 4:
The addition of the salt of KY to solution of $MY\ and\ NY_3$ will have no effect on their solubilities.

Correct Answer:
The molar solubility of in water is less than that of NY_3.

Solution:
$\underset{1-s}{MY} \rightleftharpoons \underset{s}{M^+} + \underset{s}{Y^-}$ $K_{sp} = 6.2 \times 10^{-13}$

$\Rightarrow \dfrac{s^2}{1-s} = 6.2 \times 10^{-13}$

$s << 1$

$\therefore s = \sqrt{6.2 \times 10^{-13}} = 7.87 \times 10^{-7}$

Also

$$NY_3 \rightleftharpoons N^{3+} + 3Y^-, \qquad K_{sp} = 6.2 \times 10^{-13}$$
$$1 - s' \qquad s' \quad 3s'$$

$$\Rightarrow K_{sp} = \frac{(3s')(s)}{(1 - s')}$$

$$\Rightarrow 27s'^4 = 6.2 \times 10^{-13}$$

$$\Rightarrow s' = 3.89 \times 10^{-4}$$

Clearly s' > s

The correct answer is option 2.

Q. 26 Apart from +3 oxidation state, other stable oxidation states shared by *f*-block elements is/are:

Option 1:
+2 only

Option 2:
+2 and +4

Option 3:
+4 only

Option 4:
none of these

Correct Answer:
+2 and +4

Solution:
As we have learned

Valence characteristics of f- block elements -

– They are all metals.

– They show variable valency.The +3 is the most important oxidation state.Few elements show +2 and +4 oxidation states.

-

+2 and +4 oxidation state are also shown by *f*-block elements.

Therefore, **Option(2) is correct.**

Q. 27 The complex, $[Cu(H_2O)_6]^{2+}$ has d^9 electronic configuration and has one unpaired electron, which of the following statements are true?

1) The complex is octahedral

2) It is outer sphere complex

3) It is diamagnetic

4) Coordination number of this compound is 6

Option 1:
1,2,3,4

Option 2:
1,2,3

Option 3:
2,3

Option 4:
1,2,4

Correct Answer:
1,2,4

Solution:
As we have learned

Metallic character of transition elements -

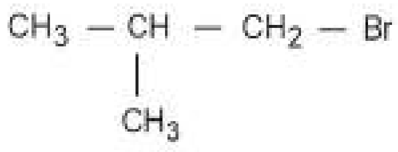
Transition elements have relatively low ionisation energies and have one or two electrons in their outer most energy level (ns^1 or ns^2).As a result, metallic bonds are formed.The unpaired d- electron also result in the formation of the metallic bond.

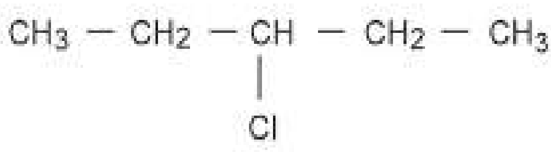
-

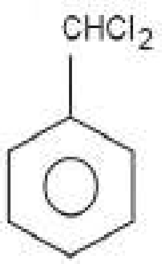
Since there are 6 H_2O molecules attached as the ligand, thus its coordination number is 6 and its geometry is octahedral but since it has 1 unpaired electron thus the complex is paramagnetic.

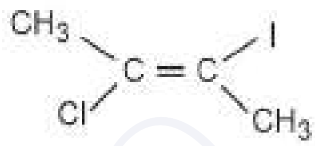
Therefore, **Option(4) is correct,**

Q. 28 Which ones are dihalogen derivatives of alkanes?

a) 

b) 

c) 

d) 

Option 1:
a, b only

Option 2:
a, b and d only

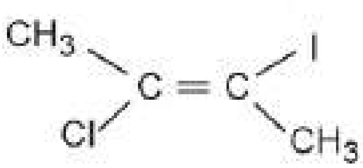
Option 3:
b only

Option 4:
a,b and c only

Correct Answer:
a,b and c only

Solution:
Dihalogen derivatives of hydrocarbons contain two halogen atoms apart from carbon and hydrogen

Here, in



It is Dihalogen derivatives of alkene not alkane.

In (c) Hydrogen is replaced from the alkane group, not from the Benzene group, so it is Dihalogen derivatives of alkane.

a,b and c are Dihalogen derivatives of alkene.

Therefore, **option (4) is correct.**

Q. 29 Which one of the following orders is correct for the bond dissociation enthalpy of halogen molecules?

Option 1:
$I_2 > Br_2 > Cl_2 > F_2$

Option 2:
$Cl_2 > Br_2 > F_2 > I_2$

Option 3:
$Br_2 > I_2 > F_2 > Cl_2$

Option 4:
$F_2 > Cl_2 > Br_2 > I_2$

Correct Answer:
$Cl_2 > Br_2 > F_2 > I_2$

Solution:
Bond dissociation energies of the halogen family decrease down the group as the size of the atom increases. The bond dissociation energy of fluorine, is, however, lower than those of chlorine and bromine because of interelectronic repulsions present in the small atom of fluorine. Thus, bond energy decreases in the order

$$Cl_2 > Br_2 > F_2 > I_2$$

Therefore, **Option(2) is correct.**

Q. 30 In which case change in entropy is negative?

Option 1:
Evapouration of water

Option 2:
Expansion of a gas at constant temperature

Option 3:
Sublimation of solid to gas

Option 4:
$2H_{(g)} \rightarrow H_{2(g)}$

Correct Answer:
$2H_{(g)} \rightarrow H_{2(g)}$

Solution:
$2H_{(g)} \rightarrow H_{2(g)}$

degree of randomness decreases

$\therefore$ entropy decreases and is negative.

Hence, the option number (4) is correct.

Q. 31 U is equal to :

Option 1:
Adiabatic work

Option 2:
Isothermal work

Option 3:
Isochoric work

Option 4:
Isobaric work

Correct Answer:
Adiabatic work

Solution:
Adiabatic Process -

Heat exchange between the system and surroundings is zero.

So,

$\Delta E = q + w$

$q = 0$

$\Delta E = w$

No change in internal energy = Adiabatic work

Ans(1)

Q. 32 XeO_4 molecular is tetrahedral having:

Option 1:
Two $p\pi - d\pi$ bonds

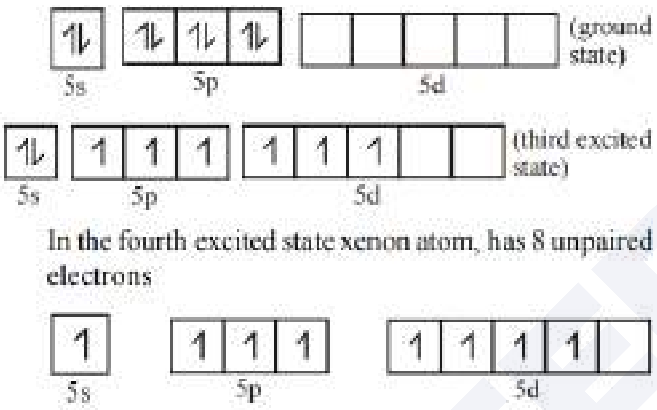
Option 2:
One $p\pi - d\pi$ bond

Option 3:
Four $p\pi - d\pi$ bonds

Option 4:
Three $p\pi - d\pi$ bonds

Correct Answer:
Four $p\pi - d\pi$ bonds

Solution:



One s and three p orbital undergo sp^3 hybridization. Four sp^3 hybrid orbitals form four σ bonds with oxygen atoms. They are σ sp^3 – p. Four pπ – dπ bonds are also formed with oxygen atoms by the unpaired electrons.

Q. 33 Which of the following pairs of compounds is isoelectronic and isostructural?

Option 1:
$BeCl_2$, XeF_2

Option 2:
TeI_2, XeF_2

Option 3:
IBr_2^-, XeF_2

Option 4:
IF_3, XeF_2

Correct Answer:
IBr_2^-, XeF_2

Solution:

Species	Number of electron	Structure
$BeCl_2$ XeF_2	38 72	Linear Linear
TeI_2 XeF_2	158 72	Bent Linear
IBr_2^- XeF_2	124 72	Linear Linear
IF_3 XeF_2	80 72	T-shaped Linear

In this question, in place of isoelectronic there should be same number of valence electron.

Q. 34 The species, having bond angles of 120° is :

Option 1:
PH_3

Option 2:
ClF_3

Option 3:
NCl_3

Option 4:
BCl_3

Correct Answer:
BCl_3

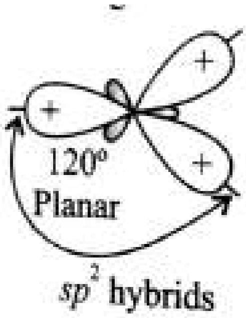
Solution:

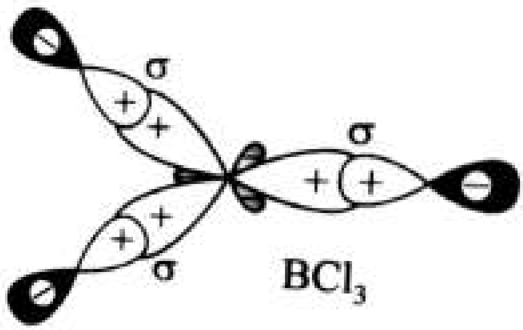
Sp^2 Hybridisation -

When one s - and two p - orbitals of the same shell of an atom mix to form three new equivalent orbitals.The hybridised orbital is called sp^2 orbital.

- wherein

Shape is Trigonal planar





$BCl_3 \rightarrow Trigonal Planar\ Sp^2\ hybridised\ ,\ Bond\ angle = 120^{\circ}$

Q. 35 Which one of the following has highest metallic character?

Option 1:
K

Option 2:
Ni

Option 3:
Al

Option 4:
Ca

Correct Answer:
K

Solution:
As we have learned

Decreasing order of metallic character -

$K > Na > Ba > Ca > Mg > Al > Zn > Fe > Ni > Sn > pb > H > pCu > Hg > Ag > Au > pt$

-

K has highest metallic character among the given elements as it has the highest size among the given elements thus removing the electron from potassium is easiest.

Therefore, **Option(1) is correct.**

Q. 36 Consider the Hunsdiecker reaction given below

$$\mathrm{RCOOAg} + \mathrm{Br_2} \xrightarrow[\text{Reflux}]{\mathrm{CCl_4}} \mathrm{R-Br} + \mathrm{CO_2} + \mathrm{AgBr}$$

What should be the order of reactivity of the above reaction if R is $1^\circ, 2^\circ, 3^\circ$ Carbon.

Option 1:
$1^\circ > 2^\circ > 3^\circ$

Option 2:
$3^\circ > 2^\circ > 1^\circ$

Option 3:
$2^\circ > 3^\circ > 1^\circ$

Option 4:
$2^\circ > 1^\circ > 3^\circ$

Correct Answer:
$1^\circ > 2^\circ > 3^\circ$

Solution:
As we have learnt,

The reaction mechanism involves formation of free radicals and 1^o carbon free radicals will react faster due to lesser steric hinderance followed by 2^o and then 3^o.

Therefore, **option (1) is correct.**

Q. 37 A reaction at 1 bar is non-spontaneous at low temperature but becomes spontaneous at high temperature. Identify the correct statement about the reaction among the following :

Option 1:
Both ΔH and ΔS are negative.

Option 2:
Both ΔH and ΔS are positive.

Option 3:
ΔH is positive while ΔS is negative.

Option 4:
ΔH is negative while ΔS is positive.

Correct Answer:
Both ΔH and ΔS are positive.

Solution:
$\Delta G = \Delta H - T\Delta S$

At low temperature,
$T\Delta S < \Delta H$
$\Delta G > 0$ and the reaction is non-spontaneous.

At high temperature,
$T\Delta S > \Delta H$
$\Delta G < 0$ and the reaction is spontaneous.

Q. 38 The order of reactivity of the given haloalkanes towards nucleophile is:

Option 1:
RI >RBr >RCl

Option 2:
RCl>RBr>RI

Option 3:
RBr>RCl>RI

Option 4:
RBr>RI>RBr

Correct Answer:
RI >RBr >RCl

Solution:
The reactivity for nucleophile will be higher for those which can release H^+ or R^+ as easily as possible. Thus, the correct order is:

RI > RBr > RCl

Therefore, **Option(1) is correct**

Q. 39 The effect of lanthanoid contraction in the lanthanoid series of the element by and large means?

Option 1:
Increase in both atomic and ionic radii.

Option 2:
Decrease in atomic radii and increase in ionic radii.

Option 3:
Decrease in both atomic and ionic radii.

Option 4:
Increase in atomic radii and decrease in ionic radii.

Correct Answer:
Decrease in both atomic and ionic radii.

Solution:
As we have learnt,

As a result of the Lanthanoid contraction, both the atomic radii and ionic radii decrease gradually in the lanthanoid series.

Therefore,option(3) is correct

Q. 40 An octahedral complex of Co^{3+} is diamagnetic. The hybridisation involved in the formation of the complex is :

Option 1:
sp^3d^2

Option 2:
dsp^2

Option 3:
d^2sp^3

Option 4:
dsp^3d

Correct Answer:
d^2sp^3

Solution:
As we discussed in

Hybridisation -

sp3d2 - square bipyramidal or octahedral

d2sp3 - octahedral

sp3 - tetradedral

dsp2 - square planar

- wherein

sp3d2 - outer complex

d2sp3 - inner complex

sp3 - $[Ni(Cl)_4]^{2-}$

dsp2 - $[Pt(CN)_4]^{2-}$

diamagnetic octahedralcomplex of CO^{3+} is d^2sp^3 hybridised.

Q. 41 Consider the coordination compound, $[Co(NH_3)_6]Cl_3$.

In the formation of this complex, the species which acts as the Lewis acid is :

Option 1:
$[Co\,(NH_3)_6]^{3+}$

Option 2:
Cl^-

Option 3:
Co^{3+}

Option 4:
NH_3

Correct Answer:
Co^{3+}

Solution:
As we discussed in

Central atom -

In coordination entity, the atom/ion to which a fixed no of ions/groups are bound in a definite geometrical arrangement around it.

- wherein

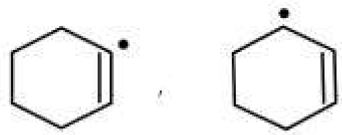
eg: $[CoCl\,(NH_3)_5]^{2+} \rightarrow Co^{3+}$ is called Central metal atom

In $[Co(NH_3)_6]Cl_3,\quad Co^{3+}$ works as lewis acid and NH_3 works as lewis base.

Q. 42 In which of the following pairs A is more stable than B?

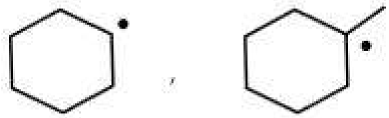
Option 1:

A B



Option 2:

A B



Option 3:

A B

Option 4:

A B

$Ph_3C^{\bullet}$, $(CH_3)_3C^{\bullet}$

Correct Answer:

A B

$Ph_3C^{\bullet}$, $(CH_3)_3C^{\bullet}$

Solution:

$Ph_3C^{\bullet}$ is resonance stabilised while $(CH_3)_3C^{\bullet}$ is stable due to hyperconjugation, Since resonance is dominating over hyperconjugation,

So, $Ph_3C^{\bullet}$ is more stable than $(CH_3)_3C^{\bullet}$.

option(1): B is more stable than A as B involves resonance.

Option(2): B is more stable than A as B has more alpha hydrogen(7 to 4) which implies more number of hyperconjugation structures. This makes B more stable.

Option(3): B is more stable than A as a six-membered ring is more stable than a 3 membered ring due to the lesser ring strain of the six-membered ring.

Option(4): A is more stable than B due to the possibility of resonance with each of the phenyl groups attached to C.B doesn't show any resonance.

Therefore, option (4) is correct.

Q. 43 A first-order reaction is 50 % completed in 20 min at 27^oC and in 5 min at 47^oC. The energy of activation (in kJ/mol) of reaction is:

Correct Answer:
55.14

Solution:
Given,

A first-order reaction is 50 % completed in 20 min at 27^oC and in 5 min at 47^oC.

Case 1 ; T_1= 27^oC = 300 K , $t_{1/2}$ = 20 min

$$k_1 = \frac{\ln 2}{t_{1/2}} = \frac{\ln 2}{20}$$

Case 1 ; T_2= 47^oC = 320 K , $t_{1/2}$ = 5 min

$$k_2 = \frac{\ln 2}{t_{1/2}} = \frac{\ln 2}{5}$$

Now,

Arrhenius Equation -

$$k = Ae^{-E_a/RT}$$

$$\ln k = \ln A - \frac{E_a}{RT}$$

k = Rate constant

So,

$$\frac{k_2}{k_1} = \frac{e^{-E_a/RT_2}}{e^{-E_a/RT_1}}$$

Take both sides (ln)-

$$\ln\frac{k_2}{k_1} = \frac{-E_a}{RT_2} - \frac{-E_a}{RT_1}$$

$$\ln\frac{k_2}{k_1} = \frac{-E_a}{R}\left(\frac{1}{T_2} - \frac{1}{T_1}\right)$$

Put values from the above calculations-

$$\ln\left(\frac{\ln 2/5}{\ln 2/20}\right) = \frac{-E_a}{R}\left(\frac{1}{320} - \frac{1}{300}\right)$$

$$\ln 4 = \frac{-E_a}{R}\left(\frac{-20}{320 \times 300}\right)$$

$$2 \times 2.303 \times 0.3010 = \frac{-E_a}{8.314}\left(\frac{-20}{320 \times 300}\right)$$

$$E_a = 55.14\ \mathrm{kJ/mol}$$

Q. 44 For the first-order reaction:

$$2N_2O_5\,(g) \rightarrow 4NO_2\,(g) + O_2\,(g)$$

(A) The concentration of the reactant decreases exponentially with time.

(B) The half-life of the reaction decreases with increasing temperature.

(C) The half-life of the reaction depends on the initial concentration of the reactant.

(D) The reaction proceeds to 99.6% completion in eight half-life duration.

The correct statements are -

Option 1:
Only A and B

Option 2:
Only B and C

Option 3:
A, B, and D

Option 4:
A, B, C, and D

Correct Answer:
A, B, and D

Solution:
A) The concentration of reactant which is following first-order kinetics always decreases exponentially and becomes zero at infinity.

$$A_t = A_o e^{-kt}$$

B) The half-life of the reaction decreases with increasing temperature

As the temperature increases, the rate constant increases and the half-life decreases since half-life is inversely dependent on the rate constant.

$$t_{1/2} = \frac{\ln 2}{k}$$

$$t_{1/2} \propto \frac{1}{k}$$

K increase on increasing T.

C) The half-life of the reaction **does not depend** on the initial concentration of the reactant.

$$t_{1/2} = \frac{\ln 2}{k}$$

D) The reaction proceeds to 99.6% completion in eight half-life duration

After eight half-lives.

$$A = \frac{A_0}{2^8}$$

$$\% \text{ completion } = \frac{A_o - \frac{A_o}{2^8}}{A_o} \times 100 = 99.6\%$$

So, A, B, and D are correct.

Option 3 is correct.

Q. 45 The half life of a first order reaction varies with temperature according to

Option 1:
$$\ln t_{\frac{1}{2}} = \frac{a}{T} + b$$

Option 2:
$$\ln t_{\frac{1}{2}} = aT + b$$

Option 3:
$$\ln t_{\frac{1}{2}} = \frac{a}{T^2} + b$$

Option 4:
$$\ln t_{\frac{1}{2}} = aT^2 + b$$

Correct Answer:
$$\ln t_{\frac{1}{2}} = \frac{a}{T} + b$$

Solution:
As we learnt for a first order reaction,

$$t_{\frac{1}{2}} = \frac{0.693}{k} \text{ and } k = Ae^{-E_a/RT}$$

$$t_{\frac{1}{2}} = \frac{0.693}{Ae^{-E_a/RT}} \quad \left[\text{Assume} \frac{0.693}{A} = c\right]$$

$$t_{\frac{1}{2}} = ce^{E_a/RT}$$

$$\ln t_{1/2} = \ln c + \frac{E_a}{RT}$$

Now, ln(c) = b and E_a/R = a

So,

$$\ln t_{1/2} = \frac{a}{T} + b$$

Hence, option number (1) is correct.

Q. 46 For a reaction scheme $A \xrightarrow{k_1} B \xrightarrow{k_2} C$, if the rate of formation of B is set to be zero then the concentration of B is given by:

Option 1:
$(k_1 - k_2)[A]$

Option 2:
$k_1 k_2 [A]$

Option 3:
$(k_1 + k_2)[A]$

Option 4:
$\left(\frac{k_1}{k_2}\right)[A]$

Correct Answer:
$\left(\frac{k_1}{k_2}\right)[A]$

Solution:
Reaction:

$$A \xrightarrow[R_1]{k_1} B \xrightarrow[R_2]{k_2} C$$

Given,

$$\frac{d[B]}{dt} = 0$$

Question [B]=?

$R_1 = k_1 [A]$

$R_2 = k_2 [B]$

So, Net rate of formation of

$$B = \frac{d[B]}{dt} = R_1 - R_2$$

$k_1[A] - k_2[B] = 0$

$k_1[A] = k_2[B]$

$$[B] = \left(\frac{k_1}{k_2}\right)[A]$$

Correct Option is 4.

Q. 47 The electron in the hydrogen atom undergoes a transition from higher orbitals to the orbital of radius 211.6 pm. This transition is associated with :

Option 1:
Lyman series

Option 2:
Balmer series

Option 3:
Paschen series

Option 4:
Brackett series

Correct Answer:
Balmer series

Solution:
As we have learned,

For *hydrogen* atom, the radius of nth orbit, $r_n = 52.9\ pm \times n^2$

$211.6\ pm = 52.9\ pm \times n^2$

$n^2 = 4$

$n = 2$

Hence, the transition is from a higher orbit to the second orbit.
This corresponds to the Balmer series.

Hence, the option number (2) is correct.

Q. 48 With respect to the conformers of ethane, which of the following statements is true?

Option 1:
Bond angle remains same but bond length changes

Option 2:
Bond angle changes but bond length remains same

Option 3:
Both bond angle and bond length change

Option 4:
Both bond angles and bond length remains same

Correct Answer:
Both bond angles and bond length remains same

Solution:
= Both bond angles ad bond length remains same

Q. 49 Which of the following pairs of ions are isoelectronic and isostructural?

Option 1:
ClO_3^-, CO_3^{2-}

Option 2:
CO_3^{2-}, NO_3^{2-}

Option 3:
ClO_3^-, SO_3^{2-}

Option 4:
CO_3^{2-}, SO_3^{2-}

Correct Answer:
ClO_3^{-}, SO_3^{2-}

Solution:
No. of electrons in $ClO_3^{-} = 17 + 24 + 1 = 42$

No. of electrons in $SO_3^{2-} = 16 + 24 + 2 = 42$

ClO_3^{-} and SO_3^{2-}, both are sp^3 hybridized and thus have similar structure.

Q. 50 The most suitable method of separation of 1:1 mixture of ortho and para-nitrophenols is :

Option 1:
Sublimation

Option 2:
Chromatography

Option 3:
Crystallisation

Option 4:
Steam distillation

Correct Answer:
Steam distillation

Solution:
The 1:1 mixture of ortho and para nitrophenols is separated by steam distillation

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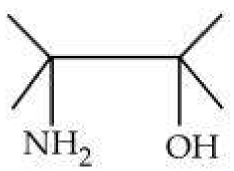
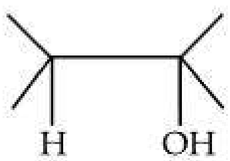
Q. 51 The major product of the reaction

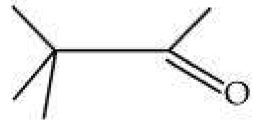


$\xrightarrow{NaNO_2/H_2SO_4}$ is

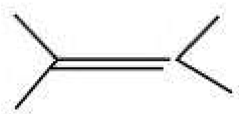
Option 1:



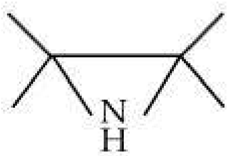
Option 2:



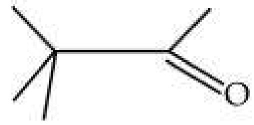
Option 3:



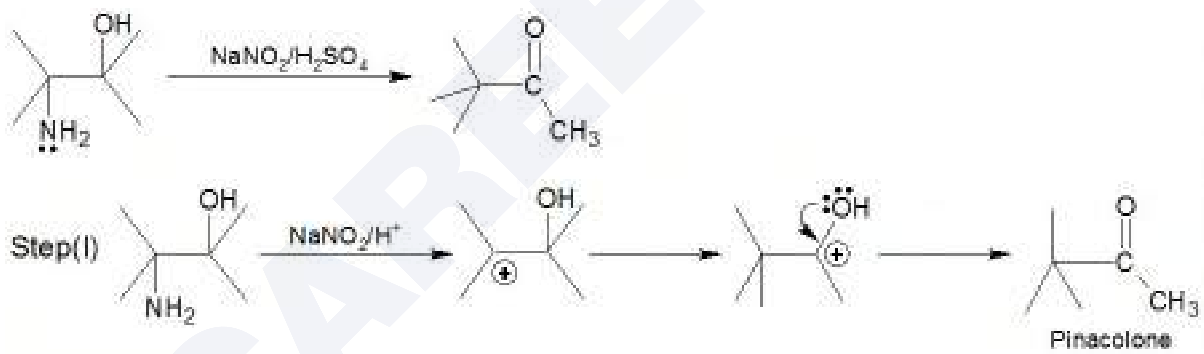
Option 4:



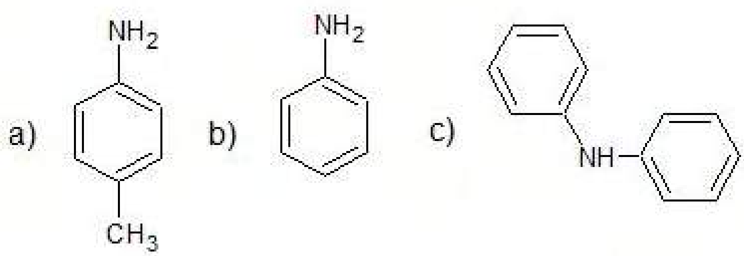
Correct Answer:



Solution:



Q. 52 Arrange the following in the order of basicity :



Option 1:
c > a > b

Option 2:
b > a> c

Option 3:
a > b > c

Option 4:
a > c > b

Correct Answer:
a > b > c

Solution:
All the given bases are aromatic amines having their lone pairs in conjugation with the Benzene ring.

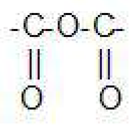
Among the given bases, " a" is most basic because of methyl at the para position and "c" is least basic because it shares lone pair with both phenyl rings.

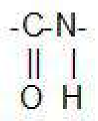
Therefore, the correct order of basic strength of the given aromatic amines follow the order:

(a) > (b) > (c)

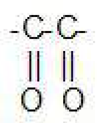
Hence, option number (3) is correct.

Q. 53 Which of the following is a peptide bond ?

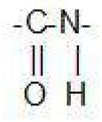
Option 1:


Option 2:


Option 3:


Option 4:


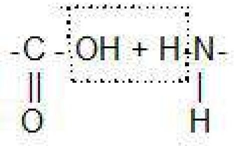
Correct Answer:



Solution:
As we have learnt,

Peptide bond is an amide linkage formed between two amino acids which is formed by condensation reaction.

The bond formed between two amino acids by the elimination of a water molecules is called peptide linkage or bond



Therefore, Option (2) is correct

Q. 54 6.02×10^{20} molecules of urea are present in 100 mL of its solution. The concentration of the solution is:

Option 1:
0.01 M

Option 2:
0.001 M

Option 3:
0.1 M

Option 4:
0.02 M

Correct Answer:
0.01 M

Solution:
As we learnt in

Molarity (M) = (Number of moles of solute)/(volume of solution in litres)

Molarity is defined as the number of moles of the solute in 1 litre of the solution.

No. of molecules (given) = 6.02×10^{20}

No. of moles $= \dfrac{6.02 \times 10^{20}}{6.02 \times 10^{23}} = 10^{-3}\ \mathrm{mol}$

Volume of solution = 100 ml = 0.1 L

Therefore,

$$\text{Molarity} = \frac{\text{no. of moles}}{\text{volume}}$$

$$\text{Molarity} = \frac{10^{-3}}{0.1} = 0.01 \text{ M}$$

Therefore, the correct option is (1).

Q. 55 When 22.4 litres of H_2(g) is mixed with 11.2 litres of Cl_2(g), each at S.T.P, the moles of HCl(g) formed is equal to:

Option 1:
1 mol of HCl (g)

Option 2:
2 mol of HCl (g)

Option 3:
0.5 mol of HCl (g)

Option 4:
1.5 mol of HCl (g)

Correct Answer:
1 mol of HCl (g)

Solution:
As we learnt in

Number of Moles of a gas at STP = given volume of gas / 22.4 litre

Concept of limiting reagent and excess reagent -

The reactant which gets consumed and thus limits the amount of product formed is called the limiting reagent.

$$\begin{array}{lccccc} & H_2(g) & + & Cl_2(g) & \rightarrow & 2HCl(g) \\ \text{initial vol.} & 22.4 \text{ L} & & 11.2 \text{ L} & & 2 \text{ mol} \end{array}$$

Therefore, 22.4 L volume at STP is occupied by $Cl_2 = 1 \text{ mole}$

Therefore, 11.2 L volume will be occupied by,

$$Cl_2 = \frac{1 \times 11.2}{22.4} = 0.5 \text{ mole}$$

$$\text{Thus, } \underset{1 \text{ mole}}{H_2} \text{ (g)} + \underset{0.5 \text{ mole}}{Cl_2} \text{ (g)} \rightarrow 2HCl(g)$$

Since, Cl_2 possesses a minimum number of moles,

thus it is the limiting reagent.

As per the equation,

$$1 \text{ mole } Cl_2 \equiv 2 \text{ mole HCl}$$

$$\therefore 0.5 \text{ mol } Cl_2 = 2 \times \ 0.5 \text{ mol HCl}$$

Hence, 1.0 mole of HCl(g) is produced by 0.5 mole of Cl_2.

Therefore, the correct option is (1).

Q. 56 1.0 g of magnesium is burnt with 0.56 g O_2 in a closed vessel. Which reactant is left in excess and how much?

(At. wt.Mg = 24; O = 16)

Option 1:
Mg, 0.16 g

Option 2:
O_2 , 0.16 g

Option 3:
Mg, 0.44 g

Option 4:
O_2, 0.28 g

Correct Answer:
Mg, 0.16 g

Solution:
The reaction is as follows:

$$Mg + \frac{1}{2}O_2 \rightarrow MgO$$

moles of $Mg = \frac{1}{24} = 0.04167$

moles of $O_2 = \frac{0.56}{32} = 0.0175$

According to reaction stoichiometry, 1 mole of Mg require 1/2 moles of O_2.

So, $\frac{1}{24}$ moles of Mg require $\frac{1}{48}$ moles of O_2 or 0.02083 moles of O_2

Clearly, O_2 is the limiting reagent.

moles of Mg required by 0.0175 moles of O_2 $= 0.0175 \times 2 = 0.035 \text{ mole}$

$\therefore \text{Excess Mg} = 0.04167 - 0.035 \Rightarrow \text{Excess Mg} = 0.00667 \text{ mole}$

we know 1 mole of Mg = 24 gram

so, 0.00667 mol = 0.00667 X molar mass

= 0.00667 X 24

= 0.16 g

Therefore, the correct option is (1).

Q. 57 What is the mole fraction of the solute in a 1.00m aqueous solution?

Option 1:
0.177

Option 2:
1.770

Option 3:
0.0354

Option 4:
0.0177

Correct Answer:
0.0177

Solution:
As we learnt in $\text{Mole Fraction} = \dfrac{\text{Moles of solute}}{\text{Moles of solute} + \text{Moles of solvent}}$

Given,

It is a 1.0 molal aqueous solution.

If we have 1 mole of solute in 1 molal solution.

Then, wt. of solvent will be = 1 kg

It is an aqueous solution means the solvent is water. $\text{Moles of solvent} = \dfrac{1000}{18} \text{ mol}$

$$\text{Mole fraction of solute} = \frac{1}{1 + \frac{1000}{18}}$$

$$= \frac{18}{1018}$$

$= 0.0177$

Option 4 is correct.

Q. 58 Identify the **wrong** statement in the following :

Option 1:
Amongst isoelectronic species, smaller the positive charge on the cation, smaller is the ionic radius.

Option 2:
Amongst isoelectronic species, greater the negative charge on the anion, larger is the ionic radius.

Option 3:
Atomic radius of the elements increases as one moves down the first group of the periodic table.

Option 4:
Atomic radius of the elements decreases as one moves across from left to right in the 2^{nd} period of the periodic table.

Correct Answer:
Amongst isoelectronic species, smaller the positive charge on the cation, smaller is the ionic radius.

Solution:
The Atomic radius of the elements decreases across a period from left to right due to an increase in effective nuclear charge. On moving down a group, since, the number of shells increases, so atomic radius increases. Amongst isoelectronic species, the ionic radius increases with increases in negative charge or decrease in positive charge.

Therefore, the correct option is (1).

Q. 59 In the following sequence of reactions,

$$CH_3CH_2OH \xrightarrow{P+I_2} A \xrightarrow[ether]{Mg} B \xrightarrow{HCHO} C \xrightarrow{H_2O} D$$

the compound D is

Option 1:
propanal

Option 2:
butanal

Option 3:
n-butyl alcohol

Option 4:
n-propyl alcohol.

Correct Answer:
n-propyl alcohol.

Solution:
As we learnt in

Test for alcohols by Victor - Meyer's test -

Reagents used

$$P + I_2,\ AgNO_2,\ HNO_2,\ NaOH$$

- wherein

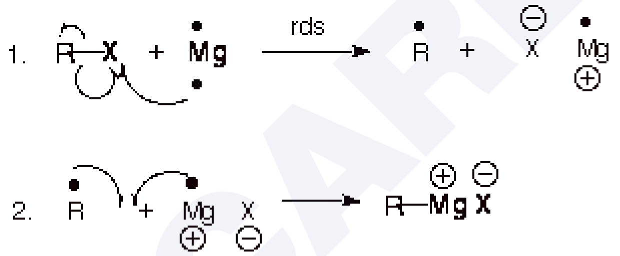
$$1°alcohol \rightarrow red\ color$$

$$2°alcohol \rightarrow blue\ color$$

$$3°alcohol \rightarrow No\ action$$

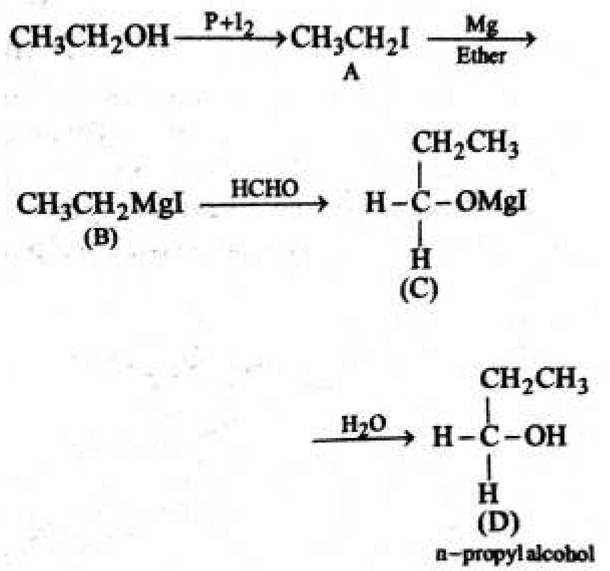
The reaction of Haloalkanes with Mg in the presence of ether - Grignard reagent is obtained

- wherein



$$CH_3CH_2OH \overset{P+I_2}{\rightarrow} CH_3CH_2I \xrightarrow[dry\,ather]{Mg} CH_3CH_2MgI \overset{HCHO}{\rightarrow} CH_3CH_2CH_2OMgI \overset{H_2O}{\rightarrow} CH_3CH_2CH_2OH + Mg(OH)I$$

$$CH_3CH_2CH_2OH\ (n - propylachohol)$$



Hence, the option number (4) is correct.

Q. 60 Phenol, when it first reacts with concentrated sulphuric acid and then with concentrated nitric acid, gives:

Option 1:
nitrobenzene

Option 2:
2, 4, 6-trinitrobenzene

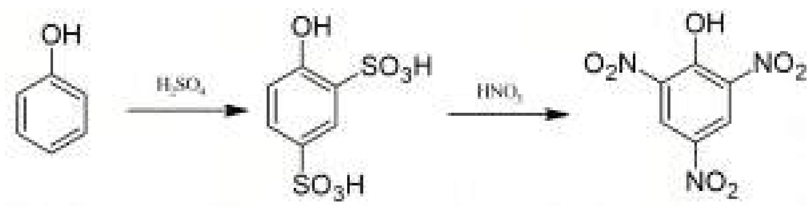
Option 3:
2,4,6- trinitrophenol

Option 4:
p-nitrophenol

Correct Answer:
2,4,6- trinitrophenol

Solution:
Direct nitration of phenol is difficult as it gets oxidised by the nitrating mixture $(\mathrm{conc.\ H_2SO_4 + conc.\ HNO_3})$. However, picric acid is formed as a minor product in the reaction.

To get a better yield of Picric acid, it is prepared by treating phenol first with concentrated sulphuric acid which converts it to phenol-2,4-disulphonic acid, and then with concentrated nitric acid to get 2,4,6-trinitrophenol.



Therefore, **option (3) is correct.**

Maths

Q. 1 Let T_n be the number of all possible triangles formed by joining vertices of an n-sided regular polygon. If $T_{n+1} - T_n = 10$ then the value of n is

Option 1:
8

Option 2:
7

Option 3:
5

Option 4:
10

Correct Answer:
5

Solution:
As we have learnt,

If there are n points in the plane and out of which no three are collinear then, total no. of triangles that can be formed using these n points = $^nC_3$

Now,

$T_n = {}^n C_3 =$ no. of the selection of 3 vertices out of n vertices

$$T_{n+1} - T_n = 10$$

$$\Rightarrow {}^{n+1} C_3 - {}^n C_3 = 10$$

$$\Rightarrow \frac{(n+1)(n)(n-1)}{6} - \frac{n(n-1)(n-2)}{6} = 10$$

$$\Rightarrow n(n-1)\{n+1-n+2\} = 60$$

$$\Rightarrow n^2 - n = 20$$

$$\Rightarrow n^2 - n - 20 = 0$$

$$\therefore n = 5 \ \ or \ \ n = -4$$

Q. 2

$$\lim_{x \to 3} \frac{\sqrt{3x}-3}{\sqrt{2x-4}-\sqrt{2}}$$ is equal to

Option 1:
$\sqrt{3}$

Option 2:
$\frac{1}{\sqrt{2}}$

Option 3:
$\frac{\sqrt{3}}{2}$

Option 4:
$\frac{1}{2\sqrt{2}}$

Correct Answer:
$\frac{1}{\sqrt{2}}$

Solution:
Limit is of 0/0 form. We can rationalize the irrational powers

$$\lim_{x \to 3} \frac{\sqrt{3x}-3}{\sqrt{2x-4}-\sqrt{2}}$$

$$= \lim_{x \to 3} \frac{\sqrt{3x}-3}{\sqrt{2x-4}-\sqrt{2}} \times \frac{\sqrt{2x-4}+\sqrt{2}}{\sqrt{2x-4}+\sqrt{2}} \times \frac{\sqrt{3x}+3}{\sqrt{3x}+3}$$

$$= \lim_{x \to 3} \frac{3x-9}{2x-4-2} \times \frac{\sqrt{2x-4}+\sqrt{2}}{\sqrt{3x}+3}$$

$$= \lim_{x \to 3} \frac{3(x-3)}{2(x-3)} \times \frac{\sqrt{2x-4}+\sqrt{2}}{\sqrt{3x}+3}$$

$$= \frac{3}{2} \cdot \frac{\sqrt{2 \times 3 - 4}+\sqrt{2}}{\sqrt{3 \times 3}+3}$$

$$= \frac{3}{2} \times \frac{\sqrt{2}+\sqrt{2}}{6}$$

$$= \frac{2\sqrt{2}}{4} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$$

Q. 3 The integral $\int \sqrt{1+2cotx(cosecx+cotx)dx}\left(0<x<\frac{\pi}{2}\right)$ is equal to (where C is a constant of integration)

Option 1:
$4\log\left(\sin\frac{x}{2}\right)+C$

Option 2:
$2\log\left(\sin\frac{x}{2}\right)+C$

Option 3:
$2\log\left(\cos\frac{X}{2}\right)+C$

Option 4:
$4\log\left(\cos\frac{x}{2}\right)+C$

Correct Answer:
$2\log\left(\sin\frac{x}{2}\right)+C$

Solution:
As we learned in

Integrals for Trigonometric functions -

$$\frac{\mathrm{d}}{\mathrm{d}x}(-cosx)=sinx$$

$$\therefore\int sinx dx=-cosx+c$$

-

$$\int\sqrt{(1+2\cot x\ cosecx\quad+2\cot^2 x)}\,dx$$

$$=\int\sqrt{(1+\cot^2 x)+2\cot x\ cosecx+\cot^2 x\,dx}$$

$$=\int\sqrt{(cosec^2x+2\cot x\ cosecx+\cot^2 x)dx}$$

$$=\int(cosecx+\cot x)\,dx$$

$= \ln\ |\csc x - \cot x| + \ln|\sin x| + c$
$= \ln|1 - \cos x| + c$
$= \ln|2\sin^2 \frac{x}{2}| + c$
$= \ln|\sin^2 \frac{x}{2}| + \ln 2 + c$
$= 2\ln|\sin \frac{x}{2}| + c_1$

Q. 4 If the equations $x^2 + 2x + 3 = 0\ and\ ax^2 + bx + c = 0, a, b, c\ \epsilon R,$ have a common root, then a : b : c is :

Option 1:
3 : 1 : 2

Option 2:
1 : 2 : 3

Option 3:
3 : 2 : 1

Option 4:
1 : 3 : 2

Correct Answer:
1 : 2 : 3

Solution:
For $x^2 + 2x + 3 = 0$

Discriminant = 4-12 = -8 < 0

So it has 2 imaginary roots. Let these roots be p+iq and p-iq

Now case 1: If the first root is common between the equations, then p+iq is a root of second equation. Now as coefficients of second equation are real, so its other root must be p-iq. So, both roots are common in these 2 equations

Case 2: If the second root is common between the equations, then p-iq is a root of second equation. Now as coefficients of second equation are real, so its other root must be p + iq. So, both roots are common in these 2 equations in this case as well

So, in any case, both roots will be common in these 2 equations

So applying condition for both roots common, a/1 = b/2 = c/3

$\therefore a : b : c = 1 : 2 : 3$

Q. 5 If a point P has co-ordinates (0, -2) and Q is any point on the circle, $x^2 + y^2 - 4x - 2y + 4 = 0$, then the maximum value of PQ is :

Option 1:
$\sqrt{13} + 4$

Option 2:
$\sqrt{13} + 1$

Option 3:
$\sqrt{15} + 1$

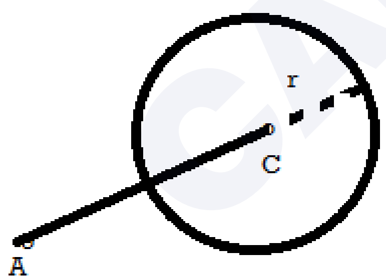
Option 4:
None of these

Correct Answer:
$\sqrt{13} + 1$

Solution:
As we learnt in

Greatest distance of a point A from a circle

$AC + r$ (for both cases P lying inside and outside the circle)



Now,

Q is point on the circle $x^2 + y^2 - 4x - 2y + 4 = 0,$

$r = \sqrt{4 + 1 - 4} = 1$

Centre C (2, 1)

PQ = Maximum distance =PC + r

$= \sqrt{(2-0)^2+(1-(-2))^2}+1$
$= \sqrt{13}+1$

Q. 6 The value of k for which the function

$$f(x) = \begin{cases} \left(\frac{4}{5}\right)^{\frac{\tan 4x}{\tan 5x}}, & 0 < x < \frac{\pi}{2} \\ k + \frac{2}{5}, & x = \frac{\pi}{2} \end{cases}$$

is continuous at, $x = \dfrac{\pi}{2}$ is:

Option 1:
$\frac{17}{20}$

Option 2:
$\frac{2}{5}$

Option 3:
$\frac{3}{5}$

Option 4:
$-\frac{2}{5}$

Correct Answer:
$\frac{3}{5}$

Solution:
As we learnt in

Result of 1 to the power of infinity Form -

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e$$

$$\lim_{x\to 0}\ (1+x)^{\frac{1}{x}} = e$$

$$\lim_{x\to 0}\ (1+\lambda x)^{\frac{1}{x}} = e^{\lambda}$$

$$\lim_{x\to\infty}\left(1+\frac{\lambda}{x}\right)^x = e^\lambda$$

-

$$\lim_{x\to\frac{\pi^-}{2}}\left(\frac{4}{5}\right)^{\frac{\tan 4x}{\tan 5x}}$$

Let $y = \lim_{x\to\frac{\pi^-}{2}}\left(\frac{4}{5}\right)^{\frac{\tan 4x}{\tan 5x}}$

$$log_e^y = \lim_{x\to\frac{\pi^-}{2}}\frac{\tan 4x}{\tan 5x}log\frac{4}{5}$$

Put $x = \frac{\pi}{2} - h$

$$log_e^y = \lim_{h\to 0} -\frac{\tan 4h}{\cot 5h}log\frac{4}{5}$$

$$= \lim_{h\to 0} -\tan 4h \cdot \tan 5h\, log\frac{4}{5}$$

$$= 0$$

$$\therefore y = 1$$

So that,

$$k + \frac{2}{5} = 1$$

$$k = 1 - \frac{2}{5} = \frac{3}{5}$$

Q. 7 The number of real values of λ for which the system of linear equations

$2x+4y-\lambda z=0$

$4x+\lambda y+2z=0$

$\lambda x+2y+2z=0$

has infinitely many solutions, is :

Correct Answer: 1

Solution:
As we learnt in

By using the concept of

Cramer's rule for solving system of linear equations -

When $\Delta = 0$ and $\Delta_1 = \Delta_2 = \Delta_3 = 0$,

then the system of equations has infinite solutions.

- wherein

$a_1x + b_1y + c_1z = d_1$

$a_2x + b_2y + c_2z = d_2$

$a_3x + b_3y + c_3z = d_3$

and

$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$\Delta_1, \Delta_2, \Delta_3$ are obtained by replacing column 1,2,3 of Δ by (d_1, d_2, d_3) column

$$\begin{vmatrix} 2 & 4 & -\lambda \\ 4 & \lambda & 2 \\ \lambda & 2 & 2 \end{vmatrix} = 0$$

$$\Rightarrow 2\begin{vmatrix} \lambda & 2 \\ 2 & 2 \end{vmatrix} - 4\begin{vmatrix} 4 & 2 \\ \lambda & 2 \end{vmatrix} - \lambda\begin{vmatrix} 4 & \lambda \\ \lambda & 2 \end{vmatrix} = 0$$

$$\Rightarrow 2(2\lambda - 4) - 4(8 - 2\lambda) - \lambda(8 - \lambda^2) = 0$$

$$\Rightarrow 4\lambda - 8 - 32 + 8\lambda - 8\lambda + \lambda^3 = 0$$

$$\Rightarrow \lambda^3 + 4\lambda - 40 = 0$$

It will give only one real value of λ

Q. 8 If $y = [x + \sqrt{x^2 - 1}]^{15} + [x - \sqrt{x^2 - 1}]^{15}$ then $(x^2 - 1)\frac{d^2y}{dx^2} + x\frac{dy}{dx}$ is equal to:

Option 1:
125 y

Option 2:
224 y^2

Option 3:
225 y^2

Option 4:
225 y

Correct Answer:
225 y

Solution:
As we learnt in

Differential Equations -

An equation involving independent variable (x), dependent variable (y) and derivative of dependent variable with respect to independent variable

$$\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)$$

- wherein

$$y=\left[x+\sqrt{x^2-1}\right]^{15}+\left[x-\sqrt{x^2-1}\right]^{15}$$

$$\text{Since }\left[x+\sqrt{x^2-1}\right]\left[x-\sqrt{x^2-1}\right]=1$$

So that

$$y=\left[x+\sqrt{x^2-1}\right]^{15}+\left[x+\sqrt{x^2-1}\right]^{-15}$$

$$\frac{dy}{dx}=15\left[x+\sqrt{x^2-1}\right]^{14}\times\left(1+\frac{2x}{2\sqrt{x^2-1}}\right)-15\left[x+\sqrt{x^2-1}\right]^{-16}\left(1+\frac{2x}{2\sqrt{x^2-1}}\right)$$

$$=15\left[x+\sqrt{x^2-1}\right]^{15}\frac{1}{\sqrt{x^2-1}}-15\left[x+\sqrt{x^2-1}\right]^{-15}\frac{1}{\sqrt{x^2-1}}$$

$$\therefore\ \frac{15}{\sqrt{x^2-1}}\left[\left[x+\sqrt{x^2-1}\right]^{15}-\left[x+\sqrt{x^2-1}\right]^{-15}\right]$$

$$\frac{dy}{dx}\sqrt{x^2-1}=15\left[\left(x+\sqrt{x^2-1}\right)^{15}-\left(x+\sqrt{x^2-1}\right)^{-15}\right]$$

$$\frac{d^2y}{dx^2}\sqrt{x^2-1}+\frac{dy}{dx}.\frac{2x}{2\sqrt{x^2-1}}=\frac{225}{\sqrt{x^2-1}}\left[\left(x+\sqrt{x^2-1}\right)^{15}+\left(x+\sqrt{x^2-1}\right)^{-15}\right]$$

$$\therefore (x^2-1)\frac{d^2y}{dx^2}+x\frac{dy}{dx}=225y$$

Q. 9 The integral $\int_{\frac{\pi}{12}}^{\frac{\pi}{4}} \frac{8\cos 2x}{(\tan x+\cot x)^3}dx$ equals :

Option 1:
$\frac{15}{128}$

Option 2:
$\frac{15}{64}$

Option 3:
$\frac{13}{32}$

Option 4:
$\frac{13}{256}$

Correct Answer:
$\frac{15}{128}$

Solution:
As we learnt in

Integrals for Trigonometric functions -

$$\frac{\mathrm{d}}{\mathrm{d}x}(-cosx)=sinx$$

$$\therefore \int sinxdx=-cosx+c$$

-

Integration by substitution -

The functions when on substitution of the variable of integration to some quantity gives any one of standard formulas.

- wherein

Since $\int f(x)dx=\int f(t)dt=\int f(\theta)d\theta$ all variables must be converted into a single variable , $(t \; or \; \theta)$

$$\int_{\frac{\pi}{12}}^{\frac{\pi}{4}} \frac{8\cos 2x}{(\tan x+\cot x)^3}\, dx$$

$$\tan x+\cot x=\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}=\frac{\sin^2 x+\cos^2 x}{\sin x\cos x}=\frac{1}{\sin x\cos x}$$

$$=\int_{\frac{\pi}{12}}^{\frac{\pi}{4}} \frac{8\cos 2x}{\left(\frac{2}{\sin 2x}\right)^3}\, dx$$

$$=\int_{\frac{\pi}{12}}^{\frac{\pi}{4}} \frac{8\cos 2x.\sin^3 2x\, dx}{8}$$

$$\sin 2x=t \quad \Rightarrow 2\cos 2x dx=dt$$

$$=\frac{1}{8}\left[sin^4 2x\right]_{\pi/12}^{\pi/4}$$

$$\Rightarrow \frac{15}{128}$$

Q. 10 The function f defined by

$f(x)=x^3-3x^2+5x+7$ is:

Option 1:
increasing in R.

Option 2:
decreasing in R.

Option 3:
decreasing in (0, ∞) and increasing in (−∞, 0).

Option 4:
increasing in (0, ∞) and decreasing in (−∞, 0).

Correct Answer:
increasing in R.

Solution:

First Derivative Test to Get Extrema -

First Derivative Test to Get Extrema

First find the value of x such that f′(x) = 0, let at x = a, f′(x) = 0

Now, find f″(x) at x = a.

1. If f″(a) < 0, then f(x) is maximum at x = a

2. If f″(a) > 0, then f(x) id minimum at x = a

3. If f″(a) = a

Then, find f‴(x) at x = a.

If f‴(a) ≠ 0, then f(x) has neither maximum nor minimum (inflection point) at x = a.

But, if f‴(a) = 0, then find fourth derivative of f(x) at x = a, i.e. $f^{iv}(x)$ at x = a.

If $f^{iv}(a) < 0$, then f(x) is maximum at x = a and if $f^{iv}(a) > 0$ then f(x) is minimum at x = a and so on.

SUMMARY

	First order derivative test	Second order derivative test	Higher order derivative test
Max	$f'(a)=0$ $f'(x)$ changes sign from + ve to -ve as x crosses a	$f'(a)=0$ $f''(a)<0$	$f'(a)=0$ $f'(a)=0$ $\vdots$ $f^{n-1}(a)=0$ $f^{n}(a)<0$ where n is even (If n is odd, $x=a$ is not an extremum point; it is a point of inflextion)
Min	$f'(a)=0$ $f'(x)$ changes sign from − ve to +ve as x crosses a	$f'(a)=0$ $f''(a)>0$	$f'(a)=0$ $f'(a)=0$ $\vdots$ $f^{n-1}(a)=0$ $f^{n}(a)>0$ where n is even (If n is odd, $x=a$ is not an extremum point; it is a point of inflextion)
	point of inflection		$f''(x)$ change sign at $x=a$

-

$f(x) = x^3 - 3x^2 + 5x + 7$

$f'(x) = 3x^2 - 6x + 5$

$D = (6)^2 - 4 \cdot 3 \cdot 5$

$= 36 - 60 < 0$

So,

$f'(x) > 0$ for $x \epsilon R$

Correct option is 1.

Q. 11 The function f : N → N is defined by

$$f(x) = x - 5\left[\frac{x}{5}\right],$$

where N is the set of natural numbers and $[x]$ denotes the greatest integer less than or equal to x, is :

Option 1:
one-one and onto.

Option 2:
one-one but not onto.

Option 3:
onto but not one-one.

Option 4:
neither one-one nor onto.

Correct Answer:
neither one-one nor onto.

Solution:

$f(x) = x - 5\left[\frac{x}{5}\right]$

Taking x in an interval of five natural numbers, we have the following:

$$f(x)=\begin{cases} x-5(0), & 0\leq x<5 \\ x-5(1), & 5\leq x<10 \\ x-5(2), & 10\leq x<15 \\ x-5(3), & 15\leq x<20 \end{cases}$$

Therefore, here, x ∈N. hence, f(x) is neither a one-one function nor onto function.

Q. 12 If the arithmetic mean of two numbers a and b, a > b > 0, is five times their geometric mean, then $\frac{a+b}{a-b}$ is equal to:

Option 1:
$\frac{\sqrt{6}}{2}$

Option 2:
$\frac{3\sqrt{2}}{4}$

Option 3:
$\frac{7\sqrt{3}}{12}$

Option 4:
$\frac{5\sqrt{6}}{12}$

Correct Answer:
$\frac{5\sqrt{6}}{12}$

Solution:
As learned in

Arithmetic mean of two numbers (AM) -

$$A=\frac{a+b}{2}$$

and

Geometric mean of two numbers (GM) -

$$GM=\sqrt{ab}$$

In this Question,

If $\frac{a+b}{2}=5\sqrt{ab}$

$a+b=10\sqrt{ab}$

We know,

$(a-b)^2=(a+b)^2-4ab$

$=(10\sqrt{ab})^2-4ab=100ab-4ab=96ab$

$a-b=4\sqrt{6}\sqrt{ab}$

Therefore:

$\frac{a+b}{a-b}=\frac{10\sqrt{ab}}{4\sqrt{6}\sqrt{ab}}=5\frac{\sqrt{6}}{12}$

Q. 13 An aeroplane flying at a constant speed, parallel to the horizontal ground, $\sqrt{3}$ km above it, is observed at an elevation of $60°$ from a point on the ground. If, after five seconds, its elevation from the same point, is $30°$, then the speed (in m/s) of the aeroplane, is :

Option 1:
1500

Option 2:
400

Option 3:
750

Option 4:
720

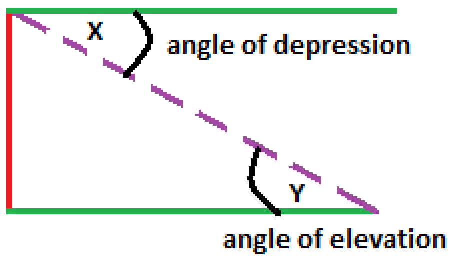
Correct Answer:
400

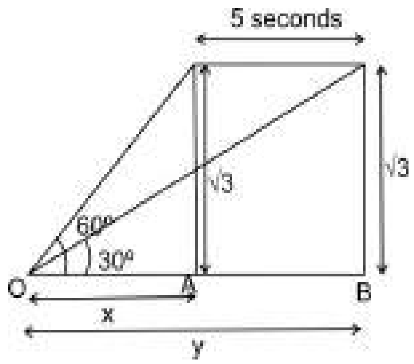
Solution:

Angle of Elevation -

If an object is above the horizontal line from the eye, we have to raise our head to view the object.

- wherein





$$\tan 60^\circ = \frac{\sqrt{3}}{x} = \sqrt{3}$$

$$\Rightarrow x = 1$$

$$\tan 30^\circ = \frac{\sqrt{3}}{y} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow y = 3$$

$$AB = 2\ kms \quad also \quad Time = 5 \ \ seconds$$

$$speed = \frac{distance}{time} = \frac{2 \times 1000}{5} = 400\ m/s$$

Q. 14 The area (in sq. units) of the parallelogram whose diagonals are along the vectors

$$8\hat{i} - 6\hat{j} \ \ and \ \ 3\hat{i} + 4\hat{j} - 12\hat{k},$$

Option 1:
26

Option 2:
65

Option 3:
20

Option 4:
52

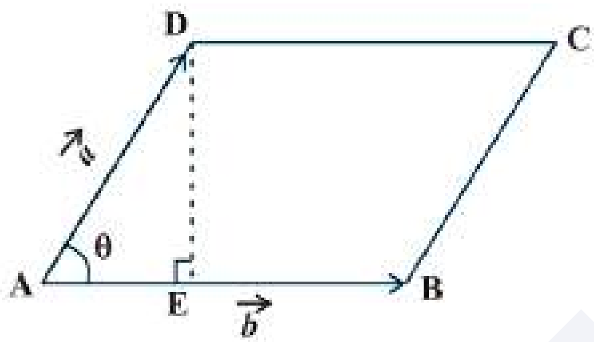
Correct Answer:
65

Solution:

Geometrical Interpretation of Vector product -

Area of Parallelogram

If $\vec{a}$ and $\vec{b}$, are two non-zero, non-parallel vectors represented by AD and AB respectively and let Θ be the angle between them.



$$\text{In } \Delta \text{ ADE}, \quad \sin\theta = \frac{DE}{AD}$$
$$\Rightarrow \quad DE = AD\sin\theta = |\mathbf{\vec{a}}|\sin\theta$$
$$\text{Area of parallelogram ABCD} = \text{AB}\cdot\text{DE}$$
Thus,
$$\text{Area of parallelogram ABCD} = |\vec{b}||\vec{a}|\sin\theta = |\vec{a}\times\vec{b}|$$

NOTE:

1. The area of a parallelogram with diagonals $\vec{\mathbf{d}}_1$ and $\vec{\mathbf{d}}_2$ is $\frac{1}{2}\left|\vec{\mathbf{d}}_1 \times \vec{\mathbf{d}}_2\right|$.

-

As we learnt in

Vector Product of two vectors(cross product) -

If $\vec{a}$ and $\vec{b}$ are two vectors and Θ is the angle between them , then $\vec{a}\times\vec{b} = \left|\vec{a}\,||\,\vec{b}\right| Sin\Theta\hat{n}$

- wherein

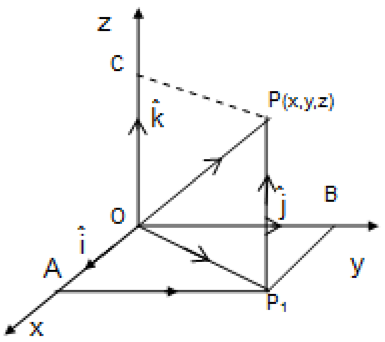
$\hat{n}$ is unit vector perpendicular to both $\vec{a}\ and\ \vec{b}$

and

Position vector of a point -

$$|\ \overrightarrow{OP}\ | = \sqrt{x^2+y^2+z^2} = r$$

- wherein



Area of parallelogram = $\frac{1}{2}\left|\vec{a}\times\vec{b}\right|$

where $\vec{a}\ and\ \vec{b}$ are diagonals

$$=\frac{1}{2}\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 8 & -6 & 0\\ 3 & 4 & -12\end{vmatrix}$$

$$=\left|\frac{1}{2}\left(72\hat{i}+96\hat{j}+50\hat{k}\right)\right|$$

$$=\left|36\hat{i}+48\hat{j}+25\hat{k}\right|$$

magnitude$=\sqrt{36^2+48^2+25^2}$

$$=65$$

Q. 15 The number of ways in which the examiner can assign 30 marks to 8 questions, giving not less than 2 marks to any questions, is :

Option 1:
$^{30}C_7$

Option 2:
$^{21}C_8$

Option 3:
$^{21}C_7$

Option 4:
$^{30}C_8$

Correct Answer:
$^{21}C_7$

Solution:
$\underline{2}\ \ \underline{2}\ \ \underline{2}\ \ \underline{2}\ \ \underline{2}\ \ \underline{2}\ \ \underline{2}\ \ \underline{2}\quad \Rightarrow 30-16=14\ \ \text{Marks}$

Hence,

$n=8$
$r=14$

$\text{no of ways } = {}^{n+r-1}C_r = {}^{8+14-1}C_{14} = {}^{21}C_{14}$

Q. 16 Let f be a polynomial function such that $f(3x)=f'(x)\cdot f''(x)$, for all $x\epsilon R$ Then :

Option 1:
$f(2)+f'(2)=28$

Option 2:
$f''(2)-f'(2)=0$

Option 3:
$f''(2)-f(2)=4$

Option 4:
$f(2)-f'(2)+f'(2)=10$

Correct Answer:
$f''(2)-f'(2)=0$

Solution:

$f(x)$ is given a polynomial function

Let $f(x)$ having n^{th} degree

$f'(x)$ becomes $(n-1)^{th}$ degree

$f''(x)$ becomes $(n-2)^{th}$ degree

Now it is given that

$f(3x) = f'(x) \cdot f''(x)$

If we consider only degree

$$\begin{aligned} n &= (n-1) + (n-2) \\ \Rightarrow n &= 3 \end{aligned}$$

Assume

$f(x) = ax^3 + bx^2 + cx + d$

we get

$f'(x) = 3ax^2 + 2bx + c$ and $f''(x) = 6ax + 2b$

$f(3x) = f'(x) \cdot f''(x)$

$a(3x)^3 + b(3x)^2 + c(3x) + d = (3ax^2 + 2bx + c) \cdot (6ax + 2b)$

$27ax^3 + 9bx^2 + 3cx + d = 18a^2x^3 + 18abx^2 + x(6ac + 4b^2) + 2bc$

Now Comparing coefficient of x^3

$27a = 18a^2$

$a = \dfrac{3}{2}$

Now Comparing coefficient of x^2

$9b = 18ab$

Here a=0 or b=0

a cannot be zero so b=0

Now Comparing coefficient of x

c=0

Similarly d=0

Hence

$$f(x) = \frac{3}{2}x^3$$

$$f'(x) = \frac{3}{2}3x^2 = \frac{9}{2}x^2$$ and

$$f''(x) = \frac{3}{2}6x = 9x$$

$$f''(2) - f'(2) = \frac{9}{2} \times 4 - 9 \times 2 = 0$$

Option 2 is correct

Q. 17 The area (in sq. units) of the region common between the curves, $x^2 + y^2 = 4$ and $y^2 = 3x$, is :

Option 1:
$$\frac{1}{2\sqrt{3}} + \frac{\pi}{3}$$

Option 2:
$$\frac{1}{\sqrt{3}} + \frac{2\pi}{3}$$

Option 3:
$$\frac{1}{2\sqrt{3}} + \frac{2\pi}{\sqrt{3}}$$

Option 4:
$$\frac{1}{\sqrt{3}} + \frac{4\pi}{3}$$

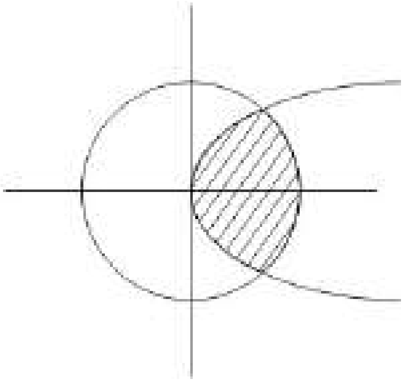
Correct Answer:
$$\frac{1}{\sqrt{3}} + \frac{4\pi}{3}$$

Solution:
Finding point of intersection first

$x^2 + 3x - 4 = 0$

$x^2 + 4x - x - 4 =0$

$x = 1$



and $y = \pm\sqrt{3}$

Using symmetry,

$$Area = 2\left(\int_0^1 \left(\sqrt{3}\sqrt{x}\right) dx + \int_1^2 \left(\sqrt{4-x^2}\right) dx\right)$$

$$= 2\left(\left[2\sqrt{3} \times \frac{x^{\frac{3}{2}}}{3}\right]_0^1 + \left[\frac{x}{2}\sqrt{4-x^2} + \frac{4}{2}\, sin^{-1}\frac{x}{2}\right]_1^2\right)$$

$$= 2\left(\frac{2\sqrt{3}}{3} + 0 + 2\sin^{-1} 1 - \frac{\sqrt{3}}{2} - 2sin^{-1}\frac{1}{2}\right)$$

$$= 2\left(\frac{2}{\sqrt{3}} - \frac{\sqrt{3}}{2} + \pi - \frac{\pi}{3}\right)$$

$$= 2\left(\frac{1}{2\sqrt{3}} + \frac{2\pi}{3}\right)$$

$$= \frac{1}{\sqrt{3}} + \frac{4\pi}{3}$$

Q. 18 The real number k for which the equation, $2x^2 + 3x + k = 0$ has two distinct real roots in [0,1].

Option 1:
does not exist.

Option 2:
lies between 1 and 2 .

Option 3:
lies between 2 and 3 .

Option 4:
lies between -1 and 0 .

Correct Answer:
does not exist.

Solution:
As we have learned

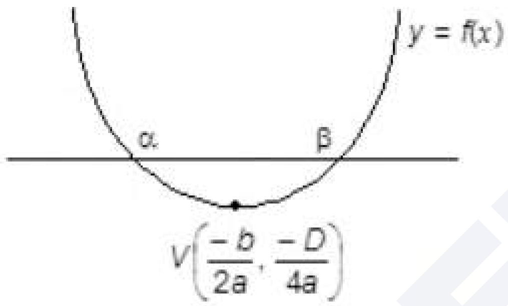
Quadratic Expression Graph when a> 0 & D > 0 -

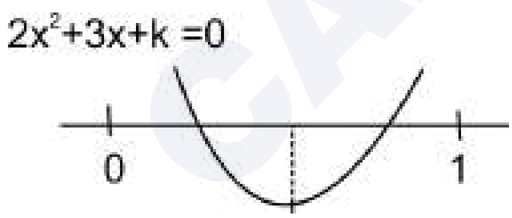
Real and distinct roots of

$f(x) = ax^2 + bx + c$

& $D = b^2 - 4ac$

- wherein





$\frac{-b}{2a} = -3/4$ is the abscissa of vertex

and , it should lie in(0,1) but it's not true

S, no value of 'k' exists

Q. 19 The locus of the point of intersection of the straight lines, $tx-2y-3t=0$ and $x-2ty+3=0$ $(t \in R)$, is

Option 1:
an ellipse with eccentricity $\frac{2}{\sqrt{5}}$

Option 2:
an ellipse with the length of major axis 6

Option 3:
a hyperbola with eccentricity $\sqrt{5}$

Option 4:
a hyperbola with the length of conjugate axis 3

Correct Answer:
a hyperbola with the length of conjugate axis 3

Solution:
As we learnt in

Locus -

Path followed by a point p(x,y) under given condition (s).

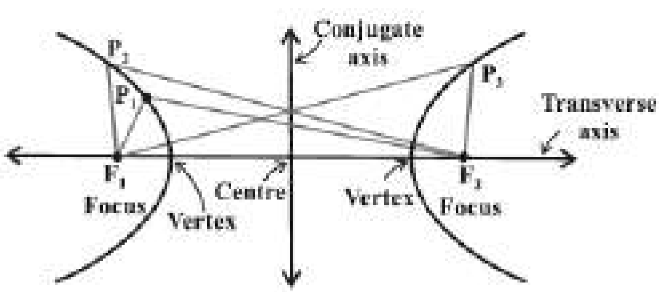
- wherein

It is satisfied by all the points (x,y) on the locus.

Conjugate axis -

The line through the centre and perpendicular to transverse axis.

- wherein



$tx-2y-3t=0$

$x-2ty+3=0(t\epsilon R)$

On solving, $t=\frac{(x+3)}{2y}$ from second equation

Bow from first equation

$x\frac{(x+3)}{2y}-2y-3\frac{(x+3)}{2y}=0$

$x^2+3x-4y^2-3x-9=0$

$x^2-4y^2=9$

$\frac{x^2}{9}-\frac{y^2}{(9/4)}=1$

Here a=3, b=$\frac{3}{2}$

Length of conjugate axis

$=2b=3$

Q. 20 if

$$\lim_{n\to\infty}\frac{1^a+2^a+......+n^a}{(n+1)^{a-1}[(na+1)+(na+2)+.......+(na+n)]}=\frac{1}{60}$$

for some positive real number a, then a is equal to :

Option 1:
7

Option 2:
8

Option 3:
$\frac{15}{2}$

Option 4:
$\frac{17}{2}$

Correct Answer:
7

Solution:

$$\lim_{n\to\infty} \frac{1^a+2^a+3^a+\cdots+n^a}{(n+1)^{a-1}[(na+1)+(na+2)+\cdots+(na+n)]}=\frac{1}{60}$$

$$\Rightarrow \lim_{n\to\infty} \frac{1^a+2^a+\cdots+n^a}{(n+1)^{a-1}\left[n^2a+\frac{n(n+1)}{2}\right]}=\frac{1}{60}$$

$$\Rightarrow \lim_{n\to\infty} \frac{2\left[\sum_{k=1}^{n} k^a\right]}{(n+1)^{a-1}\left[2n^2a+n^2+n\right]}=\frac{1}{60}$$

$$\lim_{n\to\infty} \frac{2\sum_{k=1}^{n}\left(\frac{k}{n}\right)^a}{\frac{(n+1)^{a-1}}{n^{a-1}}\left[\frac{2n^2a+n^2+n}{n}\right]}=\frac{1}{60}$$

$$\lim_{n\to\infty} \frac{2\sum_{k=1}^{n}\left(\frac{k}{n}\right)^a}{\left(1+\frac{1}{n}\right)^{a-1}[2na+n+1]}=\frac{1}{60}$$

$$\lim_{n\to\infty} \frac{\frac{2}{n}\sum_{k=1}^{n}\left(\frac{k}{n}\right)^a}{\left(1+\frac{1}{n}\right)^{a-1}\left[2a+1+\frac{1}{n}\right]}=\frac{1}{60}$$

$$\lim_{n\to\infty} \frac{1}{n}\sum_{1}^{n}\left(\frac{k}{n}\right)^a=\int_0^1 x^a dx \quad [\text{ Derivative as limit of a sum }]$$

$$\therefore \frac{2\int_0^1 x^a dx}{2a+1}=\frac{1}{60}$$

$$\Rightarrow \frac{2\left(x^{a+1}\right)\Big]_0^1}{(a+1)(2a+1)}=\frac{1}{60}$$

$$\Rightarrow \frac{2}{(a+1)(2a+1)}=\frac{1}{60}$$

$$\Rightarrow (a+1)(2a+1)=120$$

$$\Rightarrow 2a^2+3a+1=120$$

$$\Rightarrow 2a^2-3a=119=0$$

$$\Rightarrow a=\frac{-3\pm\sqrt{9+952}}{4}$$

$$=\frac{-3\pm\sqrt{961}}{4}=\frac{-3\pm 31}{4} \quad (\text{ since } a>0)$$

Hence, the value of a is

$$\frac{-3+31}{4}=\frac{28}{4}=7 \Rightarrow a=7$$

Q. 21 Let A be any 3×3 invertible matrix. Then which one of the following is not always true ?

Option 1:
$adj\,(A)=|A|\,.A^{-1}$

Option 2:
$adj\,(adj\,(A))=|A|\,.A$

Option 3:
$adj\,(adj\,(A))=|A|^2\,.\,(adj\,(A))^{-1}$

Option 4:
$adj\,(adj\,(A))=|A|\,.\,(adj\,(A))^{-1}$

Correct Answer:
$adj\,(adj\,(A))=|A|\,.\,(adj\,(A))^{-1}$

Solution:
As we learnt in

Inverse of a matrix -

$$A^{-1}=\frac{1}{|A|}\cdot adj A$$

\-

Option 1: $A^{-1}=\dfrac{adj\,(A)}{|A|}\;(By\ formula)$

Option 2: adj (adj (A)) =$=|A|^{n-2}A$

Put n = 3

$\therefore adj\,(adj\,(A))=|A|^{3-2}A=|A|\,A$

Option 3: and 4

$$\because A(adj A) = |A| I_n$$

$$(adj A)^{-1} = \frac{A}{|A|}$$

$$adj(adj(A)) = |A|^2 (adj(A))^{-1} = |A|^2 \frac{A}{|A|} = |A| \cdot A$$

so option 4 is not always true

Q. 22 The curve satisfying the differential equation, y dx − (x + 3y^2) dy = 0 and passing through the point (1, 1), also passes through the point:

Option 1:
$$\left(\frac{1}{4}, -\frac{1}{2}\right)$$

Option 2:
$$\left(-\frac{1}{3}, \frac{1}{3}\right)$$

Option 3:
$$\left(\frac{1}{3}, -\frac{1}{3}\right)$$

Option 4:
$$\left(\frac{1}{4}, \frac{1}{2}\right)$$

Correct Answer:
$$\left(-\frac{1}{3}, \frac{1}{3}\right)$$

Solution:

$$ydx - \left(x + 3y^2\right) dy = 0$$

$$y\frac{dx}{dy} = x + 3y^2$$

$$\frac{dx}{dy} - \frac{x}{y} = 3y,$$

$P = -\dfrac{1}{y}$ and $Q = 3y$

Let us now find out the integrating factor I.F.,

$$-\int \frac{1}{y} dy = -logy = log\frac{1}{y}$$

$$I.F. = e^{log\frac{1}{y}} = \frac{1}{y}$$

Hence the solution of the equation is

$$x.\frac{1}{y} = \int \frac{1}{y} \times 3ydy = 3y + c$$

$$3y = \frac{x}{y} + C$$

$$\therefore 3 = 1 + C$$

$$\therefore C = 2 \quad [x = 1, y = 1]$$

$$\therefore 3y = \frac{x}{y} + 2$$

Now checking options:

$y = \frac{1}{3}, x = \frac{-1}{3}$ satisfies this equation

Q. 23 If $f\left(\frac{3x-4}{3x+4}\right) = x + 2,\ \ x \neq -\frac{4}{3}$ and $\int f(x)\,dx = A\log|1-x| + Bx + C$ then the ordered pair (A, B) is equal to : (where C is a constant of integration)

Option 1:
$\left(\frac{8}{3}, \frac{2}{3}\right)$

Option 2:
$\left(-\frac{8}{3}, \frac{2}{3}\right)$

Option 3:
$\left(-\frac{8}{3}, -\frac{2}{3}\right)$

Option 4:
$\left(\frac{8}{3}, -\frac{2}{3}\right)$

Correct Answer:
$\left(\frac{8}{3}, \frac{2}{3}\right)$

Solution:
As learnt in concept

Rule of integration by Partial fraction -

Linear and non-repeated:

$$\frac{P(x)}{Q(x)} = \frac{P(x)}{(x-\alpha_1)(x-\alpha_2)\cdots(x-\alpha_n)}$$

Let $\frac{P(x)}{Q(x)} = \frac{A}{(x-\alpha_1)} + \frac{B}{(x-\alpha_2)}\cdots$

Find A, B...

By comparing N^r and $P(x)$

$$f\left(\frac{3x-4}{3x+4}\right) = x+2$$

$$Put\ \frac{3x-4}{3x+4} = y$$

$$=> 3xy + 4y = 3x - 4$$

$$x = \frac{-4\,(y+1)}{3\,(y-1)}$$

$$x = \frac{4\,(1+y)}{3\,(1-y)}$$

$$f\,(y) = \frac{4\,(1+y)}{3\,(1-y)} + 2$$

$$= \frac{10-2y}{3\,(1-y)}$$

$$\therefore f(x) = \frac{2(5-x)}{3\,(1-x)}$$

$$\therefore \frac{2}{3}\int\left(\frac{5-x}{1-x}\right)dx = \frac{2}{3}\int\frac{1-x+4}{(1-x)}dx$$

$$= \frac{2}{3}x + \frac{8}{3}\,ln\,|1-x| + C$$

Q. 24 A tangent to the curve, $y = f(x)\ at\ P(x, y)$

meets x-axis at A and y-axis at B. If AP : BP=1 : 3 and f(1)=1, then the curve also passes through the point :

Option 1:
$\left(\frac{1}{3}, 24\right)$

Option 2:
$\left(\frac{1}{2}, 4\right)$

Option 3:
$\left(2, \frac{1}{8}\right)$

Option 4:
$\left(3, \frac{1}{28}\right)$

Correct Answer:
$\left(2, \frac{1}{8}\right)$

Solution:
As we have learnt in

Selection formula -

$$x = \frac{mx_2 + nx_1}{m+n}$$

$$y = \frac{my_2 + ny_1}{m+n}$$

- wherein

If P(x,y) divides the line joining A(x_1,y_1) and B(x_2,y_2) in ration $m : n$

A slope of a line -

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

- wherein

Slope of line joining A(x_1,y_1) and B(x_2,y_2) .

We have,

$x=\frac{3a}{4}, y=\frac{b}{4} \Rightarrow a=\frac{4x}{3}, b=4y$

Now, Slope m=$\frac{-b}{a}=\frac{-3y}{x}$

Also, slope = $\frac{dy}{dx} \Rightarrow \frac{dy}{dx}=\frac{-3y}{x}$

$\Rightarrow \ln y=-3 \ln x+c$

$\Rightarrow c=0$

So,

$y=\frac{1}{x^3}$

Q. 25 Let f (x)=2^{10}·x + 1 and g(x) = 3^{10}·x−1. If (fog)(x) = x, then x is equal to:

Option 1:
$\frac{3^{10}-1}{3^{10}-2^{-10}}$

Option 2:
$\frac{2^{10}-1}{2^{10}-3^{-10}}$

Option 3:
$\frac{1-3^{-10}}{2^{10}-3^{-10}}$

Option 4:
$\frac{1-2^{-10}}{3^{10}-2^{-10}}$

Correct Answer:
$\frac{1-2^{-10}}{3^{10}-2^{-10}}$

Solution:
Using

$f(x)=2^{10} x+1$

$g(x)=3^{10} x-1$

Then, $fog(x) = f(g(x))$

$$\Rightarrow f\left(3^{10}x - 1\right) = 2^{10}\left(3^{10}x - 1\right) + 1 = x$$

$$\Rightarrow 2^{10} \cdot 3^{10}x - 2^{10} + 1 = x$$

$$\therefore 2^{10} \cdot 3^{10}x - x = 2^{10} - 1$$

$$\therefore x\left(2^{10} \cdot 3^{10} - 1\right) = 2^{10} - 1$$

$$\therefore x = \frac{2^{10} - 1}{2^{10} \cdot 3^{10} - 1}$$

$$\therefore x = \frac{1 - 2^{-10}}{3^{10} - 2^{-10}}$$

Q. 26 If the sum of the first n terms of the series $\sqrt{3} + \sqrt{75} + \sqrt{243} + \sqrt{507} + \ldots\ldots \text{ is } 435\sqrt{3}$, then n equals :

Correct Answer:
15

Solution:
As we learnt in

Sum of n terms of an AP -

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

or

$$S_n = \frac{n}{2}\left[a + l\right]$$

In this Question,

$$\sqrt{3} + \sqrt{75} + \sqrt{243} + \sqrt{507}.... = 435\sqrt{3}$$

$$\Rightarrow \sqrt{3} + \sqrt{25 \times 3} + \sqrt{81 \times 3} + \sqrt{169 \times 3} + ... = 435\sqrt{3}$$

$$\Rightarrow \sqrt{3} + 5\sqrt{3} + 9\sqrt{3} + 13\sqrt{3} + ... = 435\sqrt{3}$$

$$\Rightarrow \sqrt{3}\left[1 + 5 + 9 + 13 + ...\right] = 435\sqrt{3}$$

$$\Rightarrow \sqrt{3} \times \frac{n}{2}\left[2 \times 1 + (n-1) \times 4\right] = 435\sqrt{3}$$

$$\Rightarrow \sqrt{3} \times n\left[2n - 1\right] = 435\sqrt{3}$$

$$\Rightarrow 2n^2 - n - 435 = 0$$

$$\therefore \quad n = \frac{1+\sqrt{3481}}{4} = \frac{1+59}{4} = \frac{60}{4} = 15$$

(Negative value of n is rejected)

Q. 27 The expression $\frac{\tan A}{1-\cot A} + \frac{\cot A}{1-\tan A}$ can be written as:

Option 1:
$secA + cosecA$

Option 2:
$sinA\, cosA + 1$

Option 3:
$secA\, cosecA + 1$

Option 4:
$tanA + cotA$

Correct Answer:
$secA\, cosecA + 1$

Solution:

$$\frac{\tan A}{1-\cot A} + \frac{\cot A}{1-\tan A}$$

$$= \frac{\sin^2 A}{\cos A\,(\sin A - \cos A)} - \frac{cos^2 A}{(\sin A - \cos A)} \times \frac{1}{\sin A}$$

$$= \frac{\sin^3 A - \cos^3 A}{(\sin A \cos A)\ (\sin A - \cos A)}$$

$$= \frac{\sin^2 A + \cos^2 A + \sin A \cos A}{\sin A \cos A}$$

$$= \frac{1+\sin A \cos A}{\sin A \cos A} = \sec A\ cosec\ A + 1$$

Q. 28 If the vector $\vec{b} = 3\hat{j} + 4\hat{k}$ is written as the sum of a vector $\vec{b_1}$, parallel to $\vec{a} = \hat{i} + \hat{j}$, and a vector $\vec{b_2}$ perpendicular to

$\vec{a}$ then $\vec{b_1} \times \vec{b_2}$ is equal to :

Option 1:
$-3\hat{i} + 3\hat{j} - 9\hat{k}$

Option 2:
$6\hat{i} - 6\hat{j} + \frac{9}{2}\hat{k}$

Option 3:
$-6\hat{i} + 6\hat{j} - \frac{9}{2}\hat{k}$

Option 4:
$3\hat{i} - 3\hat{j} + 9\hat{k}$

Correct Answer:
$6\hat{i} - 6\hat{j} + \frac{9}{2}\hat{k}$

Solution:
As we have learned

Collinear Vectors -

Two vectors are said to be collinear if and only if there exists a scalar m such as that $\vec{a} = m\vec{b}$

- wherein

m is a Scalar.

Properties of Scalar Product -

$\vec{a}.\vec{b} = 0$

$\vec{a}\ is\ prependicular\ \vec{b}$

- wherein

Provided that $\vec{a} \neq 0,\ \vec{b} \neq 0$

Vector Product of two vectors -

$$\mathbf{c}=\mathbf{a}\times\mathbf{b}=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\end{vmatrix}$$

- wherein

$$\vec{a}=(a_1\hat{i}+a_2\hat{j}+a_3\hat{k})$$

$$\vec{b}=(b_1\hat{i}+b_2\hat{j}+b_3\hat{k})$$

$$3\hat{j}+4\hat{k}=\vec{b}_1+\vec{b}_2$$
$$\vec{b}_1=\lambda(\hat{i}+\hat{j})$$

$$\vec{b}_2\cdot\vec{a}=0$$

$$(x\hat{i}+y\hat{j}+z\hat{k})(\hat{i}+\hat{j})=0$$

$$x+y=0\Rightarrow x=-y$$

So

$$3\hat{j}+4\hat{k}=\lambda(\hat{i}+\hat{j})+(x\hat{i}-x+z\hat{k})$$

$$0=\lambda+x=0;\lambda-x=3;z=4$$

$$\lambda=3/2$$

$$x=-3/2$$
$$z=4$$

$$\vec{b}_1\times\vec{b}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3/2 & 3/2 & 0\\ -3/2 & 3/2 & 4\end{vmatrix}=6\hat{i}-6\hat{j}+9/2\hat{k}$$

Q. 29 On the sides AB,BC,CA, of $\triangle ABC$ 3,4,5 distibct points (excuding vertices A,B,C) are respectively chosen. the number of trangles that can be constructed using these chosen points as vertices are:

Option 1:
210

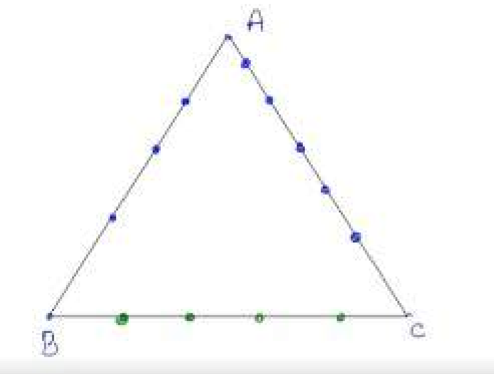
Option 2:
205

Option 3:
215

Option 4:
220

Correct Answer:
205

Solution:



Single triangle needs 3 vertices

Case 1:

One point in each side

$= {}^{3}C_{1} \times {}^{4}C_{1} \times {}^{5}C_{1} = 60$

Case 2:

1 point from side AB & 2 points from BC

$= {}^{3}C_{1} \times {}^{4}C_{2} = 12$

Case 3:

1 point from side BC & 2 points from AC

$= {}^{4}C_{1} \times {}^{5}C_{2} = 40$

Case 4:

1 point from side AC & 2 points from BC

$= {}^{5}C_{1} \times {}^{4}C_{2} = 30$

Case 5:

1 point from side AB & 2 points from AC

$= \ {}^{3}C_1 \times \ {}^{5}C_2 = 30$

Case 6:

1 point from side AC & 2 points from AB

$= \ {}^{5}C_1 \times \ {}^{3}C_2 = 15$

Total Triangle = 60+18+12+40+30+30+15=205

Second Approach

Totel vertices =3+4+5=12

${}^{12}C_{23} - \ {}^{3}C_3 - \ {}^{4}C_3 - \ {}^{5}C_3 = 220 - 1 - 4 - 10 = 205$

Q. 30 $\lim_{x \to \frac{\pi}{2}} \frac{\cot x - \cos x}{(\pi - 2x)^3}$equals:

Option 1:
$\frac{1}{16}$

Option 2:
$\frac{1}{8}$

Option 3:
$\frac{1}{4}$

Option 4:
$\frac{1}{24}$

Correct Answer:
$\frac{1}{16}$

Solution:

$\lim_{x \to \frac{\pi}{2}} \frac{cotx - cosx}{(\pi - 2x)^3}$

Put $x = \frac{\pi}{2} - h$

$$\Rightarrow \lim_{h \to 0} \frac{cot\left(\frac{\pi}{2}-h\right)-cos\left(\frac{\pi}{2}-h\right)}{\left[\pi-2\left(\frac{\pi}{2}-h\right)\right]^3}$$

$$\Rightarrow \lim_{h \to 0} \frac{tanh - sinh}{(\pi-\pi+2h)^3}$$

$$\Rightarrow \lim_{h \to 0} \frac{tanh\,(1-cosh)}{8h^3}$$

$$\Rightarrow \frac{1}{8} \lim_{h \to 0} \frac{tanh}{h} \cdot \frac{2sin^2\frac{h}{2}}{h^2}$$

$$\Rightarrow \frac{1}{8} \lim_{h \to 0} \frac{tanh}{h} \cdot \frac{2sin^2\frac{h}{2}}{4.(\frac{h}{2})^2}$$

$$\Rightarrow \frac{1}{8} \times 1 \times \frac{1}{2} = \frac{1}{16}$$

Q. 31 $\int_1^2 \frac{dx}{(x^2-2x+4)^{\frac{3}{2}}} = \frac{k}{k+5}$,then k is equal to:

Option 1:
1

Option 2:
2

Option 3:
3

Option 4:
4

Correct Answer:
1

Solution:
As learnt in concept

Integration by substitution -

The functions when on substitution of the variable of integration to some quantity gives any one of standard formulas.

- wherein

Since $\int f(x)dx = \int f(t)dt = \int f(\theta)d\theta$ all variables must be converted into single variable , $(t \, or \, \theta)$

$$\int_1^2 \frac{dx}{(x^2-2x+4)\frac{3}{2}}$$

put t = x-1

$$=> \int_0^1 \frac{dt}{\left((t+1)^2-2(t+1)+4\right)^{\frac{3}{2}}}$$

$$=> \int_0^1 \frac{dt}{(t^2+3)^{\frac{3}{2}}}$$

$$t = \sqrt{3}\tan\Theta$$

$$dt = \sqrt{3}\sec^2\Theta \, d\Theta$$

$$=> \int_0^{\tan^{-1}\frac{1}{\sqrt{3}}} \frac{\sqrt{3}\sec^2\theta \, d\theta}{3\sqrt{3}\sec^3\theta}$$

$$=> \frac{1}{3}\int_0^{\tan^{-1}\frac{1}{\sqrt{3}}} \cos\theta \, d\theta$$

$$= \frac{1}{3}[\sin\theta]_0^{\tan^{-1}\frac{1}{\sqrt{3}}}$$

$$= \frac{1}{3} \times \frac{1}{2}$$

$$= \frac{1}{6}$$

$$= \frac{K}{K+5}$$

$$=> K = 1$$

Q. 32 If z is a complex number of unit modulus and argument θ ,then arg $\left(\frac{1+z}{1+\bar{z}}\right)$ equals:

Option 1:

$\pi - \theta$

Option 2:
$-\theta$

Option 3:
$\frac{\pi}{2}-\theta$

Option 4:
θ

Correct Answer:
θ

Solution:
$|z|=1$

Arg (z)= θ

$\Rightarrow z=e^{i\theta}=\cos\theta+i\sin\theta$

So, $\frac{1+z}{1+\bar{z}}=\frac{1+\cos\theta+i\sin\theta}{1+\cos\theta-i\sin\theta}$

$\frac{2\cos^2\theta/2+2i\sin\theta/2\cos\theta/2}{2\cos^2\theta/2-2i\sin\theta/2\cos\theta/2}$

$=\frac{\cos\theta/2+i\sin\theta/2}{\cos\theta/2-i\sin\theta/2}$

$=\frac{e^{i\theta/2}}{e^{-i\theta/2}}=e^{i\theta}$

Thus, arg$\left(\frac{1+z}{1+\bar{z}}\right)=\theta$

Q. 33 If two parallel chords of a circle, having diameter 4 units, lie on the opposite sides of the centre and subtend angles $\cos^{-1}\left(\frac{1}{7}\right)$ and $\sec^{-1}(7)$ at the center respectively, then the distance between these chords, is :

Option 1:
$\frac{4}{\sqrt{7}}$

Option 2:
$\frac{8}{\sqrt{7}}$

Option 3:
$\frac{8}{7}$

Option 4:
$\frac{16}{7}$

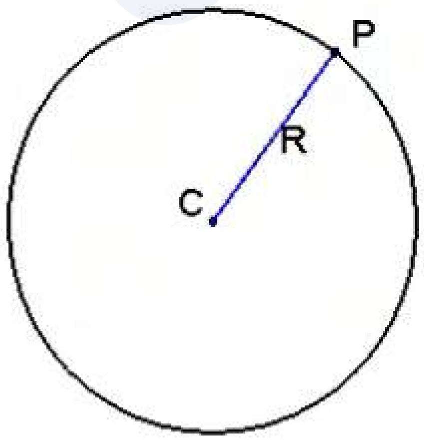
Correct Answer:
$\frac{8}{\sqrt{7}}$

Solution:
As we learnt in

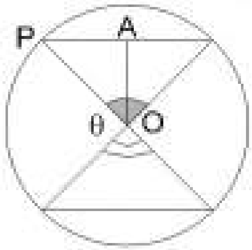
Circle -

A circle is the locus of a moving point such that its distance from a fixed point is constant.

- wherein



and using the concept of chords;



$$\cos\theta = \frac{1}{7}$$

$$2\cos^2\frac{\theta}{2} - 1 = \frac{1}{7}$$

$$2\cos^2\frac{\theta}{2} = \frac{8}{7}$$

$$\Rightarrow \cos\frac{\theta}{2} = \frac{2}{\sqrt{7}}$$

$$\Rightarrow \cos\frac{\theta}{2} = \frac{OA}{r} = \frac{OA}{2}$$

$$OA = \frac{4}{\sqrt{7}};$$

thus distance between them is $2OA = \frac{8}{\sqrt{7}}$

Q. 34 If for $x\ \epsilon \left(o, \frac{1}{4}\right)$, the derivative of

$\tan^{-1}\left(\frac{6x\sqrt{x}}{1-9x^3}\right)$ is $\sqrt{x}\,.g(x)$ then $g(x)$ equals :

Option 1:
$\frac{3x\sqrt{x}}{1-9x^3}$

Option 2:
$\frac{3x}{1-9x^3}$

Option 3:
$\frac{3}{1+9x^3}$

Option 4:
$$\frac{9}{1+9x^3}$$

Correct Answer:
$$\frac{9}{1+9x^3}$$

Solution:
As we learnt in

Inverse Trigonometric Functions -

The functions $\sin^{-1}x, \cos^{-1}x, \tan^{-1}x, \cot^{-1}x, \csc^{-1}x$ and $\sec^{-1}x$ are the inverse trigonometric functions.

- wherein

If $\sin x = \frac{1}{2}$

then, $x = \sin^{-1}\frac{1}{2}$

Let, $\tan^{-1}\frac{6x\sqrt{x}}{1-9x^3} = \Theta$

$$\tan\Theta = \frac{6x\sqrt{x}}{1-9x^3}$$

Differentiating both sides w.r.t. x we get:

$$\frac{d\Theta}{dx}\sec^2\Theta = \frac{(1-9x^3)\,9\sqrt{x} - 6x^{\frac{3}{2}}(-27x^2)}{(1-9x^3)^2}$$

$$\frac{d\Theta}{dx}\sec^2\Theta = 9\sqrt{x}\ \frac{(1+9x^3)}{(1-9x^3)^2} \qquad \text{................ (1)}$$

Now,

$$\sec^2\theta = 1+\tan^2\theta$$

$$\tan\Theta = \frac{6x\sqrt{x}}{1-9x^2}$$

$$\Rightarrow \sec^2\Theta = 1 + \frac{(6x\sqrt{x})^2}{(1-9x^3)^2}$$

$$= 1 + \frac{36x^3}{(1-9x^3)^2}$$

$$= \frac{1 + 81x^6 + 18x^3}{(1 - 9x^3)^2}$$

$$\sec^2 \Theta = \frac{(1 + 9x^3)^2}{(1 - 9x^3)^2} \qquad \text{.................. (2)}$$

From (1) and (2)

$$\frac{d\Theta}{dx} = \frac{9\sqrt{x}(1 + 9x^3)^2}{(1 - 9x^3)^2} \; \frac{(1 - 9x^3)^2}{(1 + 9x^3)^2}$$

$$\frac{d\Theta}{dx} = \frac{9\sqrt{x}}{1 + 9x^2}$$

On comparison,

$$g(x) = \frac{9}{1 + 9x^3}$$

Q. 35

If $S = \left\{ x \in [0, 2\pi] : \begin{vmatrix} 0 & \cos x & -\sin x \\ \sin x & 0 & \cos x \\ \cos x & \sin x & 0 \end{vmatrix} = 0 \right\}$,

then $\sum_{x \in S} \tan\left(\frac{\pi}{3} + x\right)$ is equal to :

Option 1:
$4 + 2\sqrt{3}$

Option 2:
$-2 + \sqrt{3}$

Option 3:
$-2 - \sqrt{3}$

Option 4:
$-4 - 2\sqrt{3}$

Correct Answer:
$-4 - 2\sqrt{3}$

Solution:

$$\begin{vmatrix} 0 & \cos x & -\sin x \\ \sin x & 0 & \cos x \\ \cos x & \sin x & 0 \end{vmatrix} = 0$$

$\Rightarrow 0(0-\sin x\cos x)-\cos x(0-\ cos^2 x)-\sin x(\sin^2 x)=0$

$\Rightarrow \sin^3 x-\cos^3 x=0$

$\Rightarrow (\sin x-\cos x)(1+\sin x\cos x)=0$

$\Rightarrow \therefore \sin x-\cos x=0$

$\Rightarrow \therefore \sin x-\cos x=0$

$\therefore \tan x=1$

$\therefore x=\frac{\pi}{4}\ or\ \frac{5\pi}{4}$

Now, $\tan(\frac{\pi}{4}+x)=\frac{1+\tan x}{1-\tan x}$

Now Put $x=\frac{\pi}{3}$

$=\frac{1+\tan\frac{\pi}{3}}{1-\tan\frac{\pi}{3}}$

$=\frac{1+\sqrt{3}}{1-\sqrt{3}}$

$=\frac{(1+\sqrt{3})^2}{1-\sqrt{3}}=\frac{1+3+2\sqrt{3}}{-2}=-2-\sqrt{3}$

similarly for $tan(\frac{5\pi}{4}+\frac{\pi}{3})=-2-\sqrt{3}$

$\sum\tan(\frac{\pi}{3}+x)=2\times(-2-\sqrt{3})=-4-2\sqrt{3}$

Q. 36 if $2x=y^{\frac{1}{5}}+y^{-\frac{1}{5}}$ and $(x^2-1)\frac{d^2y}{dx^2}+\lambda x\frac{dy}{dx}+ky=0$

then $\lambda+k$ is equal to :

Correct Answer:
-24

Solution:
As we learnt in

Differential Equations -

An equation involving independent variable (x), dependent variable (y) and derivative of dependent variable with respect to independent variable

$$\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)$$

- wherein

eg:

$$\frac{d^2y}{dx^2}-3\frac{dy}{dx}+5x=0$$

$$2x=y^{\frac{1}{5}}+y^{-\frac{1}{5}}$$

$$=y^{\frac{1}{5}}+\frac{1}{y^{\frac{1}{5}}}$$

$$\therefore 2x=\frac{\left(y^{\frac{1}{5}}\right)^2+1}{y^{\frac{1}{5}}}$$

$$\therefore \left(y^{\frac{1}{5}}\right)^2-2.x.y^{\frac{1}{5}}+x^2=x^2-1$$

$$\therefore \left(x-y^{\frac{1}{5}}\right)^2=x^2-1$$

$$x-y^{\frac{1}{5}}=\sqrt{x^2-1}$$

$$x-\sqrt{x^2-1}=y^{\frac{1}{5}}$$

$$\therefore y=\left(x-\sqrt{x^2-1}\right)^5$$

$$\frac{dy}{dx}=5\left(x-\sqrt{x^2-1}\right)^4\times\left(1-\frac{2x}{2\sqrt{x^2-1}}\right)$$

$$=5\left(x-\sqrt{x^2-1}\right)^4\left(\frac{\sqrt{x^2-1}-x}{\sqrt{x^2-1}}\right)$$

$$\frac{-5.y}{\sqrt{x^2-1}}$$ ....(i)

$$=>\sqrt{x^2-1}\,\frac{dy}{dx}=-5y$$

$$=>\sqrt{x^2-1}\times\frac{d^2y}{dx^2}+\frac{2x}{2\sqrt{x^2-1}}\times\frac{dy}{dx}=-5\times\frac{dy}{dx}$$

$$=>\left(x^2-1\right)\frac{d^2y}{dx^2}+x.\frac{dy}{dx}=-5\sqrt{x^2-1}.\frac{dy}{dx}=-5\times(-5y)=25y=25y$$ from (i)

$$\left(x^2-1\right)\frac{d^2y}{dx^2}+x\frac{dy}{dx}-25y=0$$

$$\therefore \lambda=1, k=-25$$

$$\therefore \lambda+k=-24$$

Q. 37 Let $I_n=\int \tan^n x\, dx \quad (n>1)$

If $I_4+I_6=a\tan^5\ x+bx^5+C$, where C is a constant of integration, then the ordered pair (a, b) is equal to :

Option 1:
$\left(\frac{1}{5},0\right)$

Option 2:
$\left(\frac{1}{5},-1\right)$

Option 3:
$\left(-\frac{1}{5},0\right)$

Option 4:
$\left(-\frac{1}{5},1\right)$

Correct Answer:
$\left(\frac{1}{5},0\right)$

Solution:

$$I_n=\int\left(\tan^n\ x\right)dx$$

$$I_4+I_6=\int\left(\tan^4 x+\tan^6 x\right)dx$$

$$=\int \tan^4 x\left(1+\tan^2 x\right)dx$$

$$=\int \tan^4 x.\sec^2 x\, dx$$

$$=\frac{\tan^5 x}{5}+C$$

Hence, $a = \frac{1}{5}$ and b = 0

Q. 38 The normal to the curve y(x−2)(x−3)=x+6 at the point where the curve intersects the y-axis passes through the point :

Option 1:
$\left(\frac{1}{2}, \frac{1}{2}\right)$

Option 2:
$\left(\frac{1}{2}, -\frac{1}{3}\right)$

Option 3:
$\left(\frac{1}{2}, \frac{1}{3}\right)$

Option 4:
$\left(-\frac{1}{2}, -\frac{1}{2}\right)$

Correct Answer:
$\left(\frac{1}{2}, \frac{1}{2}\right)$

Solution:
As we learnt in

Equation of Normal -

Equation of normal to the curve y = f(x) at the point P(x_1, y_1) on the curve having a slope M_N is

$$(y - y_1) = M_N(x - x_1)$$

$$= \frac{-1}{\frac{dy}{dx}_{(x_1,y_1)}}(x - x_1)$$

-

$$y = \frac{x+6}{x^2 - 5x + 6} \qquad point(0, 1)$$

$$\frac{dy}{dx}=\frac{(x^2-5x+6)\times 1-(x+6)\,(2x-5)}{(x-2)^2\,(x-3)^2}$$

$$=\frac{6+30}{36}=1=MT$$

$$M_N=-1$$

$\therefore$ equation of normal

$$y-1=-1\,(x-0)=-x+0$$

$$\therefore x+y=1$$

Correct option is 1.

Q. 39 The function $f:R\rightarrow\left[-\frac{1}{2},\frac{1}{2}\right]$ defined as $f(x)=\frac{x}{1+x^2}$, is

Option 1:
injective but not surjective

Option 2:
surjective but not injective

Option 3:
neither injective nor surjective

Option 4:
invertible

Correct Answer:
surjective but not injective

Solution:

$$f(x)=\frac{x}{1+x^2}$$

$$f:R\rightarrow\left[-\frac{1}{2},\frac{1}{2}\right]$$

$$f'(x)=\frac{(1+x^2)\times 1-x\times 2x}{(1+x^2)^2}=\frac{1+x^2-2x^2}{(1+x^2)^2}=\frac{1-x^2}{(1+x^2)^2}$$

$\therefore$ So that $\frac{-(x^2-1)}{(x^2+1)^2}$

So that it is not strictly increasing or decreasing function.

So that it is not one-one.

Correct option is 2

Q. 40 The number of ways in which 5 boys and 3 girls can be seated on a round table if a particular boy B_1 and a particular girl G_1 never sit adjacent to each other, is :

Option 1:
$5 \times 6!$

Option 2:
$6 \times 6!$

Option 3:
$7!$

Option 4:
$5 \times 7!$

Correct Answer:
$5 \times 6!$

Solution:
As we learnt in

Rule for Circular Permutations -

(n - 1)! = Clockwise + Anticlockwise arrangement.

Ex. Seating arrangement of persons round a table.

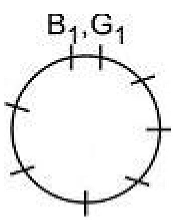
- wherein

DISTINCT arrangement.

Number of ways = Total - when B_1 and G_1 sit together

Total ways to seat 8 people on table = 7!

When B_1 and G_1 sit together $= 6! \times 2!$



Number of ways $= 7! - 2 \times 6! = 6!(7-2) = 5 \times 6!$

Correct option is 1.

Q. 41 ABCD is a trapezium such that AB and CD are parallel and BC$\perp$CD.

If $\angle$ADB =θ, BC= p and CD = q, then AB is equal to:

Option 1:
$$\frac{(p^2+q^2)\sin\Theta}{(p\cos\Theta+q\sin\Theta)^2}$$

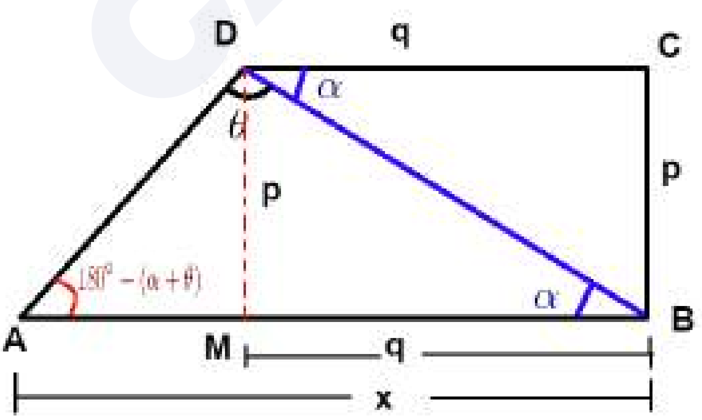
Option 2:
$$\frac{(p^2+q^2)\sin\Theta}{p\cos\Theta+q\sin\Theta}$$

Option 3:
$$\frac{p^2+q^2\cos\Theta}{p\cos\Theta+q\sin\Theta}$$

Option 4:
$$\frac{p^2+q^2}{p^2\cos\Theta+q^2\sin\Theta}$$

Correct Answer:
$$\frac{(p^2+q^2)\sin\Theta}{p\cos\Theta+q\sin\Theta}$$

Solution:



In $\triangle DBM,$

$$\cot\alpha=\frac{q}{p}$$

In $\triangle DAM$,

$$\tan(\pi-\theta-\alpha)=\frac{p}{x-q}$$

$$\tan(\theta+\alpha)=\frac{p}{q-x}$$

$$x=q-p\cot(\theta+\alpha)$$

$$=q-p\left[\frac{\cot\theta\cot\alpha-1}{\cot\alpha+\cot\theta}\right]$$

$$=q-p\frac{(q\cos\theta-p\sin\theta)}{q\sin\theta+p\cos\theta}$$

Thus $AB=\dfrac{(p^2+q^2)\sin\theta}{p\cos\theta+q\sin\theta}$

Q. 42 Let $\vec{a}=2\hat{i}+\hat{j}-2\hat{k}$ and $\vec{b}=\hat{i}+\hat{j}$
Let $\vec{c}$ be a vector such that $|\vec{c}-\vec{a}|=3, |(\vec{a}\times\vec{b})\times\vec{c}|=3$
and the angle between $\vec{c}$ and $\vec{a}\times\vec{b}$ is 30°. Then $\vec{a}.\vec{c}$ is equal to

Option 1:
2

Option 2:
5

Option 3:
$\frac{1}{8}$

Option 4:
$\frac{25}{8}$

Correct Answer:
2

Solution:
As we learnt in

Vector Triple Product -

$$\vec{a}\times\left(\vec{b}\times\vec{c}\right)=x\vec{b}+y\vec{c}$$

- wherein

x and y are scalars.

and

Scalar Product of two vectors -

$\vec{a}.\vec{b} > 0\ an\ acute\ angle$

$\vec{a}.\vec{b} < 0\ an\ obtuse\ angle$

$\vec{a}.\vec{b} = 0\ a\ right\ angle$

- wherein

Θ is the angle between the vectors $\vec{a}\ and\ \vec{b}$

$(\vec{c} - \vec{a}) = 3,\ |(\vec{a} \times \vec{b}) \times \vec{c}| = 3$

Also, $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -2 \\ 1 & 1 & 0 \end{vmatrix}$

$= 2\hat{i} + 2\hat{j} + \hat{k}$

$\left|\left(\vec{a} \times \vec{b}\right) \times \vec{c}\right| = \left|\left(\vec{a} \times \vec{b}\right)\right| |\vec{c}|\ sin30^{\circ}$

$3 = 3 \times |\vec{c}| \times \frac{1}{2}$

$\Rightarrow |\vec{c}| = 2$

Also, $|\vec{c} - \vec{a}| = 3$

So, $|\vec{c}|^2 + |\vec{a}|^2 - 2\vec{c} \cdot \vec{a} = 9$

$4 + 9 - 2\vec{c} \cdot \vec{a} = 9$

$\vec{c} \cdot \vec{a} = 2$

Q. 43 The coordinates of the foot of the perpendicular from the point (1, −2, 1) on the plane containing the lines

$$\frac{x+1}{6}=\frac{y-1}{7}=\frac{z-3}{8} \text{ and}$$

$$\frac{x-1}{3}=\frac{y-2}{5}=\frac{z-3}{7}, \text{ is}$$

Option 1:
(2, −4, 2)

Option 2:
(−1, 2, −1)

Option 3:
(0, 0, 0)

Option 4:
(1, 1, 1)

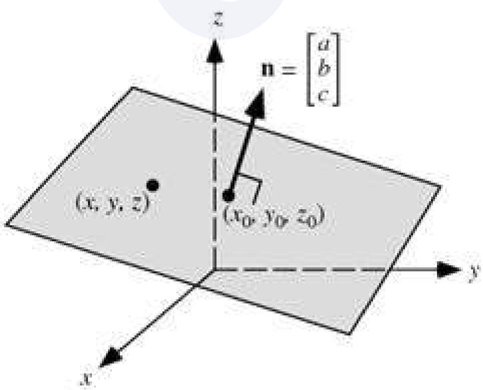
Correct Answer:
(0, 0, 0)

Solution:
As we learnt in

Cartesian equation of plane passing through a given point and normal to a given vector -

$$\left(x-x_0\right)a+\left(y-y_0\right)b+\left(z-z_0\right)c=0$$

- wherein



$$\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$$

$$\vec{a}=x_0\hat{i}+y_0\hat{j}+z_0\hat{k}$$

$\vec{n} = a\hat{i} + b\hat{j} + c\hat{k}$

Putting in

$(\vec{r} - \vec{a}) \cdot \vec{n} = 0$

We get $(x - x_0)\, a + (y - y_0)\, b + (z - z_0)\, c = 0$

First, we find the equation of plane normal vector of plane containing L_1 and L_2 is

$$\vec{x} = \begin{bmatrix} \hat{i} & \hat{t} & \hat{k} \\ 6 & 7 & 8 \\ 3 & 5 & 7 \end{bmatrix}$$

$= 9\hat{i} - 18\hat{j} + 9\hat{k}$

Unit vector $\hat{x} = \dfrac{\hat{i} - 2\hat{j} + \hat{k}}{\sqrt{6}}$

Hence, equation of plane is of the form

$x - 2y + z = k$

It passes through (-1,1,3)

$-1 - 2 + 3 = k \Rightarrow 0$

So plane is $x - 2y + z = 0$

Foot of perpendicular is (0,0,0)

Q. 44 The term independent of x in expansion of $\left(\dfrac{x+1}{x^{2/3} - x^{1/3} + 1} - \dfrac{x-1}{x - x^{1/2}}\right)^{10}$ is :

Correct Answer:
210

Solution:
As we have learned

General Term in the expansion of (x+a)^n -

$T_{r+1} =^n c_r \cdot x^{n-r} \cdot a^r$

Now, $S=\left(\frac{\left(x^{1/3}+1\right)\left(x^{2/3}-x^{1/3}+1\right)}{\left(x^{2/3}-x^{1/3}+1\right)}-\frac{\left(x^{1/2}-1\right)\left(x^{1/2}+1\right)}{x^{1/2}\left(x^{1/2}-1\right)}\right)^{10}$

$=\left(\left(x^{1/3}+1\right)-\left(1+x^{-1/2}\right)\right)^{10}$

$=\left(x^{1/3}-x^{-1/2}\right)^{10}$

$T_{r+1}={}^{10}C_r\left(x^{1/3}\right)^{10-r}\left(-x^{-1/2}\right)^r$

$=(-1)^r.{}^{10}C_r.x^{\frac{10-r}{3}-\frac{r}{2}}$

For term independent of x,

$$\frac{10-r}{3}-\frac{r}{2}=0\Rightarrow 5r=20$$
$$\Rightarrow r=4$$

$So,\ T_{r+1}={}^{10}C_4=210$

Q. 45 if $S=\tan^{-1}\left(\frac{1}{n^2+n+1}\right)+\tan^{-1}\left(\frac{1}{n^2+3n+3}\right)+\ldots$
$+\tan^{-1}\left(\frac{1}{1+(n+19)(n+20)}\right)$, then S is equal to

Option 1:
$$\frac{20}{401+20n}$$

Option 2:
$$\frac{n}{n^2+20n+1}$$

Option 3:
$$\frac{20}{n^2+20n+1}$$

Option 4:
$$\frac{n}{401+20n}$$

Correct Answer:
$$\frac{20}{n^2+20n+1}$$

Solution:

$S=\tan^{-1}\left(\frac{1}{n^2+n+1}\right)+\tan^{-1}\left(\frac{1}{n^2+3n+3}\right)+\ldots+\tan^{-1}\left(\frac{1}{1+(n+19)(n+20)}\right)$

$$
\begin{aligned}
S &= \tan^{-1}\frac{1}{1+n(n+1)}+\tan^{-1}\frac{1}{1+(n+1)(n+2)}+\ldots+\tan^{-1}\frac{1}{1+(n+19)(n+20)} \\
&= \tan^{-1}\frac{n+1-n}{1+n(n+1)}+\tan^{-1}\frac{(n+2)-(n+1)}{1+(n+1)(n+2)}+\ldots++\tan^{-1}\frac{(n+20)-(n+19)}{1+(n+19)(n+20)} \\
&= \left(\tan^{-}(n+1)-\tan^{-1}n\right)+\left(\tan^{-1}(n+2)-\tan^{-1}(n+1)\right)+\ldots \\
&\qquad +\left(\tan^{-1}(n+20)-\tan^{-1}(n+19)\right) \\
&= \tan^{-1}(n+20)-\tan^{-1}n
\end{aligned}
$$

$$S=\tan^{-1}\frac{(n+20)-(n)}{1+n(n+20)}=\tan^{-1}\frac{20}{n^2+20n+1}$$

Q. 46 The objective function of a linear programming model is given as,

maximize, $z=x_1+2x_2$

Subject to constraint 1:$x_1+2x_2\leq 5$

Constraint 2:$x_1+3x_2\leq 9$

What is the objective function value if $(x_1,x_2)=(1,1)$ is used as a possible solution?

Option 1:
3

Option 2:
5

Option 3:
7

Option 4:
None of these

Correct Answer:
3

Solution:

Different Types of Linear Programming Problems -

-

As we learnt in

Corner Point Method -

This method of solving an LPP graphically is based on the principle of extreme points theorem.

-

$z=x_1+2x_2$

$x_1+2x_2 \leq 5$

$x_1+3x_2 \leq 9$

$x_2=4$

$x_1=-3$

at $x_1=1,\ x_2=1$

$\therefore z=1+2\times 1=1+2=3$

Q. 47 The line of intersection of the planes

$\underset{r}{\rightarrow}\cdot\left(3\hat{i}-\hat{j}+\hat{k}\right)=1$ and

$\underset{r}{\rightarrow}\cdot\left(\hat{i}+4\hat{j}-2\hat{k}\right)=2$, is:

Option 1:

$$\frac{x-\frac{4}{7}}{-2}=\frac{y}{7}=\frac{z-\frac{5}{7}}{13}$$

Option 2:

$$\frac{x-\frac{4}{7}}{2}=\frac{y}{-7}=\frac{z+\frac{5}{7}}{13}$$

Option 3:

$$\frac{x-\frac{6}{13}}{2}=\frac{y-\frac{5}{13}}{-7}=\frac{z}{-13}$$

Option 4:

$$\frac{x-\frac{6}{13}}{2}=\frac{y-\frac{5}{13}}{7}=\frac{z}{-13}$$

Correct Answer:

$$\frac{x-\frac{6}{13}}{2}=\frac{y-\frac{5}{13}}{-7}=\frac{z}{-13}$$

Solution:
As we have learned

Equation of line as intersection of two planes -

Let the two intersecting planes be

$ax+by+cz+d=0$ and

$a_1x+b_1y+c_1z+d_1=0$

then the parallel vector of the line formed their intersection can be obtained by

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a & b & c \\ a_1 & b_1 & c_1 \end{vmatrix} = A\hat{i} + B\hat{j} + C\hat{k}(assumed)$$

and points can be obtained by putting $z = 0$ and solving

$ax + by + d = 0$ and

$a_1x + b_1y + d_1 = 0$ say α, β

Now the equation will be

$$\frac{x-\alpha}{A} = \frac{y-\beta}{B} = \frac{z-0}{C}$$

-

$$3x - y + z = 1$$
$$and \;\; x + 4y - 2z = 2$$

putting z = 0

3x-y = 1 and x + 4y = 2

$$\Rightarrow 13x = 6 \Rightarrow x = 6/13$$
$$y = 5/3$$

$$vector \;\; along \;\; line = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 1 \\ 1 & 4 & -2 \end{vmatrix} = -2\hat{i} + 7\hat{j} + 13\hat{k}$$

Q. 48 The sum of the rational terms in the binomial expansion of $\left(2^{\frac{1}{2}} + 3^{\frac{1}{5}}\right)^{10}$ is

Option 1:
25

Option 2:
32

Option 3:
9

Option 4:
41

Correct Answer:
41

Solution:
$$\left(2^{\frac{1}{2}}+3^{\frac{1}{5}}\right)^{10}={}^{10}C_0\left(2^{1/2}\right)^{10}+{}^{10}C_1\left(2^{1/2}\right)^{9}\left(3^{1/5}\right)+\ldots\ldots+{}^{10}C_{10}\left(3^{1/5}\right)^{10}$$

There are only two rational terms - first term and last term.

$$\text{Sum of two rational terms }=(2)^5+(3)^2=32+9=41$$

Q. 49 Let $x\,\epsilon(0,1)$. The set of all x such that $sin^{-1}x>cos^{-1}x$, is the interval :

Option 1:
$\left(\frac{1}{2},\frac{1}{\sqrt{2}}\right)$

Option 2:
$\left(\frac{1}{\sqrt{2}},1\right)$

Option 3:
(1,2)

Option 4:
$\left(0,\frac{\sqrt{3}}{2}\right)$

Correct Answer:
$\left(\frac{1}{\sqrt{2}},1\right)$

Solution:
$$\text{Given }\sin^{-1}x>\cos^{-1}x\text{ where }x\in(0,1)$$
$$\Rightarrow\sin^{-1}x>\frac{\pi}{2}-\sin^{-1}x$$
$$\Rightarrow2\sin^{-1}x>\frac{\pi}{2}\Rightarrow\sin^{-1}x>\frac{\pi}{4}$$
$$\Rightarrow x>\sin\frac{\pi}{4}\Rightarrow x>\frac{1}{\sqrt{2}}$$

Maximum value of $\sin^{-1} x$ is $\frac{\pi}{2}$
So, maximum value of x is 1.
So, $x \in \left(\frac{1}{\sqrt{2}}, 1\right)$

Q. 50 The mean age of 25 teachers in a school is 40 years. A teacher retires at the age of 60 years and a new teacher is appointed in his place. If now the mean age of the teachers in this school is 39 years, then the age (in years) of the newly appointed teacher is :

Option 1:
25

Option 2:
30

Option 3:
35

Option 4:
40

Correct Answer:
35

Solution:
No concept add

mean age $= 40$ years

$\frac{\sum x_i}{25} = 40$ years

= sum of ages (s)

new teacher be of age T let the

now $\frac{S - 60 + T}{25} = 39$

$1000 - 60 + T = 25 \times 39$

$940 + T = 975$

$T = 35\ years$

Q. 51 The last integral value of α of x such that $\frac{x-5}{x^2+5x-14} > 0$ satisfies,

Option 1:
$\alpha^2+3\alpha-4=0$

Option 2:
$\alpha^2-5\alpha+4=0$

Option 3:
$\alpha^2-7\alpha+6=0$

Option 4:
$\alpha^2+5\alpha-6=0$

Correct Answer:
$\alpha^2+5\alpha-6=0$

Solution:
$$\frac{x-5}{x^2+5x-14}>0$$
$$\frac{x-5}{(x-2)(x+7)}>0$$
$$-7<x<2 \quad \text{or} \quad x>5$$

$\alpha=-6$ satisfies

$\alpha^2+5\alpha-6=0$

Q. 52 If the common tangents to the parabola, $x^2=4y$ and the circle, $x^2+y^2=4$ intersect at the point P, then the distance of P from the origin, is :

Option 1:
$\sqrt{2}+1$

Option 2:
$2\sqrt{3+2\sqrt{2}}$

Option 3:
$2\left(\sqrt{2}+1\right)$

Option 4:
$3+2\sqrt{2}$

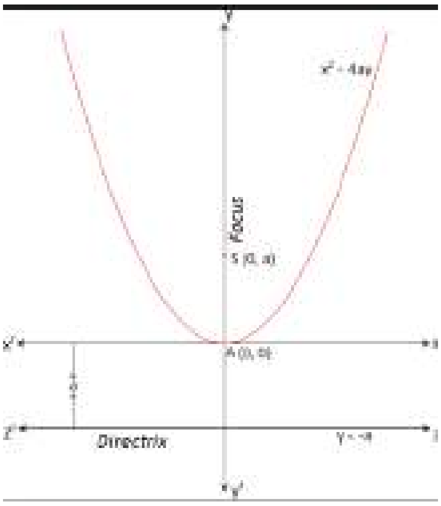
Correct Answer:
$2\sqrt{3+2\sqrt{2}}$

Solution:
As we learnt in

Standard equation of parabola -

$$x^2 = 4ay$$

- wherein

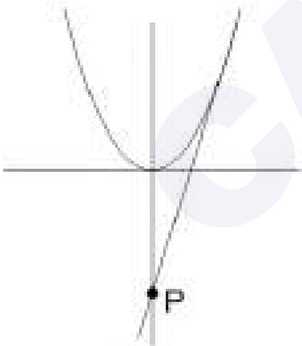


Condition of tangency -

$$c^2 = a^2\ (1 + m^2)$$

- wherein

If $y = mx + c$ is a tangent to the circle $x^2 + y^2 = a^2$



Tangent to $x^2 + y^2 = 4$ is

$$y = mx \pm 2\sqrt{1 + m^2}$$

Also $x^2 = 4y$

$$x^2 = 4mx + 8\sqrt{1 + m^2}$$

If we put D=0

$$m^4 - 4m^2 - 4 = 0$$

$$m^2 = 2 + 2\sqrt{2}$$

$$m^2 = 2(\sqrt{2} + 1)$$

Hence Equation of Tangent is

$$y = mx \pm 2\sqrt{1 + 2(\sqrt{2} + 1)}$$

$$y = mx \pm 2\sqrt{3 + 2\sqrt{2}}$$

As both the tangents intersect at y-axis

Distance from the origin is given by y coordinates or x=0

$$y = 2\sqrt{3 + 2\sqrt{2}}$$

Q. 53 The integral $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{1 + \cos x}$ is equal to :

Correct Answer:
2

Solution:
As learnt in concept

Properties of Definite integration -

$$\int_a^b f(x)\,dx = \int_a^b f(a + b - x)\,dx$$

When $\int_0^b f(x)\,dx = \int_0^b f(b - x)\,dx$

- wherein

Put the $(a + b - x)$ at the place of x in $f(x)$

$$I = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{1 + \cos x}$$

Also, $I = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{1 + \cos(\pi - x)}$

$$= \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{1-\cos x}$$

$$2I = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} dx \left(\frac{1}{1+\cos x} + \frac{1}{1-\cos x}\right)$$

$$2I = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{2}{\sin^2 x} dx$$

$$I = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} cosec^2 x \, dx$$

$$= [-\cot x]_{\frac{\pi}{4}}^{\frac{3\pi}{4}}$$

$$= -((-1)-(1))$$

= 2

Q. 54 The number of solutions of the equation $\sin 2x - 2\cos x + 4\sin x = 4$ in the interval $[0, 5\pi]$ is

Option 1:
3

Option 2:
5

Option 3:
4

Option 4:
6

Correct Answer:
3

Solution:
$\sin 2x - 2\cos x + 4\sin x = 4$
$\Rightarrow 2\sin x \cdot \cos x - 2\cos x + 4\sin x - 4 = 0$
$\Rightarrow (\sin x - 1)(\cos x - 2) = 0$

$\because \cos x - 2 \neq 0, \quad \therefore \sin x = 1$
$\therefore \quad x = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}$

Q. 55 If $Z_1 \neq 0 \ and \ Z_2$ be two complex numbers such that $\dfrac{Z_2}{Z_1}$ is a purely imaginary number , then $\left|\dfrac{2Z_1 + 3Z_2}{2Z_1 - 3Z_2}\right|$ is equal to :

Option 1:
2

Option 2:
5

Option 3:
3

Option 4:
1

Correct Answer:
1

Solution:
$\text{Let } z_1 = 1 + i \text{ and } z_2 = 1 - i$

$$\frac{z_2}{z_1} = \frac{1-i}{1+i} = \frac{(1-i)(1-i)}{(1+i)(1-i)} = -i$$

$$\frac{2z_1 + 3z_2}{2z_1 - 3z_2} = \frac{2 + 3\left(\frac{z_2}{z_1}\right)}{2 - 3\left(\frac{z_2}{z_1}\right)} = \frac{2-3i}{2+3i}$$

$$\left|\frac{2z_1 + 3z_2}{2z_1 - 3z_2}\right| = \left|\frac{2-3i}{2+3i}\right| = \frac{|2-3i|}{|2+3i|}$$

$$\Rightarrow \frac{\sqrt{4+9}}{\sqrt{4+9}} = 1$$

Q. 56 Consider an ellipse, whose centre is at the origin and its major axis is along the x-axis. If its eccentricity is $\frac{3}{5}$ and the distance between its foci is 6, then the area (in sq. units) of the quadrilateral inscribed in the ellipse, with the vertices as the end points of major and minor axes of ellipse, is

Option 1:
8

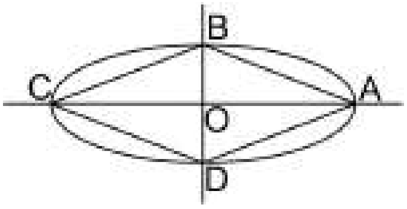
Option 2:
32

Option 3:
80

Option 4:
40

Correct Answer:
40

Solution:



Given

$$e = \frac{3}{5}$$

and

$$2ae = 6 \Rightarrow ae = 3$$

Hence a = 5

$$b^2 = a^2(1 - e^2) \Rightarrow b = 4$$

Area of quadrilateral ABCD = 4 Ar(ΔAOB)

$$= 4 \times \frac{1}{2} \times a \times b$$

$$= 4 \times \frac{1}{2} \times 5 \times 4$$

= 40

Q. 57 The area (in sq. units) of the region

$$\{(x, y) : x \geqslant 0, x + y \leq 3, x^2 \leq 4y \ and \ y \leq 1 + \sqrt{x}\}$$

is :

Option 1:
$\frac{3}{2}$

Option 2:
$\frac{7}{3}$

Option 3:
$\frac{5}{2}$

Option 4:
$\frac{59}{12}$

Correct Answer:
$\frac{5}{2}$

Solution:
As learnt in

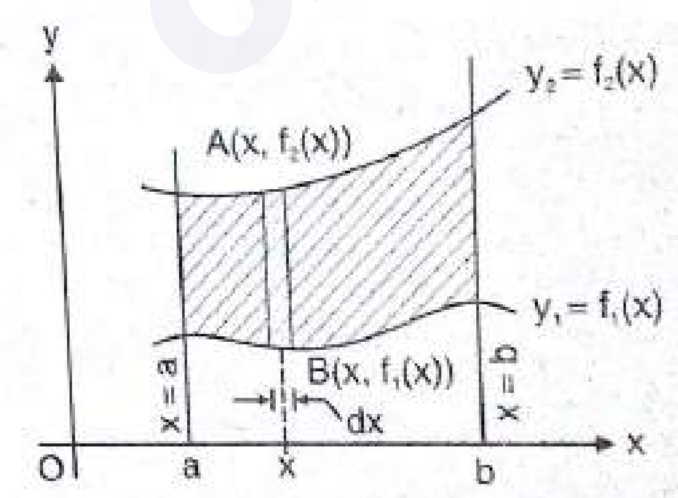
Area along x axis -

Let $y_1 = f_1(x)\, and\, y_2 = f_2(x)$ be two curve then area bounded between the curves and the lines

x = a and x = b is

$$\left| \int_a^b \Delta y \, dx \right| = \left| \int_a^b (y_2 - y_1) \, dx \right|$$

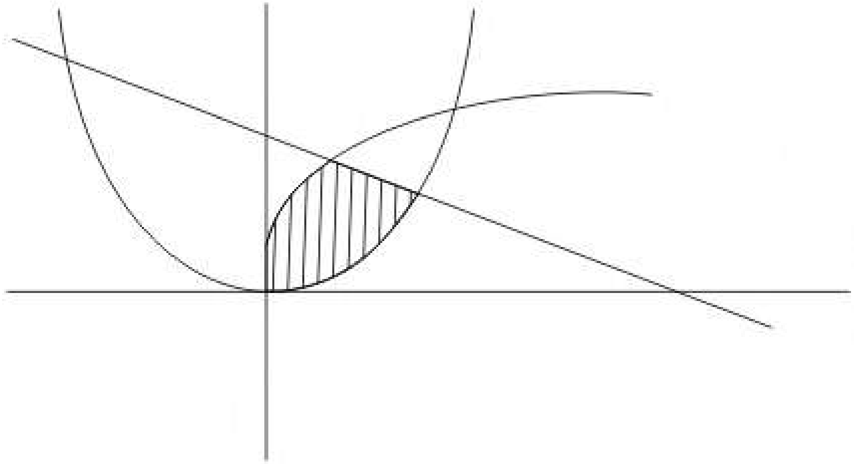
- wherein



Where $\Delta y = f_2\,(x) - f_1(x)$

Point of intersection of

$x + y = 3; y = 1 + \sqrt{x}; x^2 = 4y$



$$Area = \int_0^1 (1+\sqrt{x})\,dx + \int_1^2 (3-x)\,dx - \int_0^2 \left(\frac{x^2}{4}\right) dx$$

$$= \left[x + \frac{2}{3}x^{\frac{3}{2}}\right]_0^1 + \left[3x - \frac{x^2}{2}\right]_1^2 - \left[\frac{x^3}{12}\right]_0^2$$

$$= \left[1 + \frac{2}{3}\right] + 3 - \frac{3}{2} - \frac{8}{12}$$

$$= \frac{5}{3} + \frac{3}{2} - \frac{2}{3}$$

$$= 1 + \frac{3}{2}$$

$$= \frac{5}{2}$$

Q. 58 Let $A = \{\theta : \sin(\theta) = \tan(\theta)\}$ and $B = \{\theta : cos\,(\theta) = 1\}$ be two sets.Then:

Option 1:
$A = B$

Option 2:
$A \not\subseteq B$

Option 3:
$B \not\subseteq A$

Option 4:
$A \subset B \; and \; B - A \neq \phi$

Correct Answer:
$A \not\subset B$

Solution:
Given,

$$A = \{\theta : \sin\theta = \tan\theta\}$$
$$B = \{\theta : \cos\theta = 1\}$$

Now,

$$A = \left\{\theta : \sin\theta = \frac{\sin\theta}{\cos\theta}\right\}$$
$$= \{\theta : \sin\theta(\cos\theta - 1) = 0\}$$
$$= \{\theta = 0, \pi, 2\pi, 3\pi, \ldots.\}$$
For $B : \cos\theta = 1 \Rightarrow \theta = \pi, 2\pi, 4\pi, \ldots\ldots$

This shows that A is not contained in B . i.e.
$A \not\subset B$. but $B \subset A$

Q. 59 A square, of each side 2, lies above the x-axis and has one vertex at the origin. If one of the sides passing through the origin makes an angle 30^0 with the positive direction of the x-axis, then the sum of the x-coordinates of the vertices of the square

Option 1:
$2\sqrt{3} - 1$

Option 2:
$2\sqrt{3} - 2$

Option 3:
$\sqrt{3} - 2$

Option 4:
$\sqrt{3} - 1$

Correct Answer:
$2\sqrt{3} - 2$

Solution:
As we learnt in

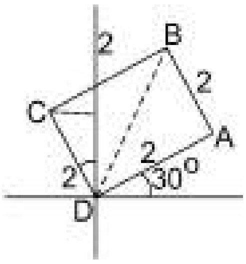
Parametric form -

$$x = x_1 + r\cos\Theta$$

$$y = y_1 + r\sin\Theta$$

- wherein

Where Θ is the inclination of the line and r is the distance between (x, y) and (x_1, y_1)



x-coordinates of A is $2cos30°$

$$= 2 \times \frac{\sqrt{3}}{2} = \sqrt{3}$$

angle of OB with positive x-axis is $30° + 45° = 75°$

Hence x - coordinate of B is $2\sqrt{2}\cos 75°$

$$= 2\sqrt{2}\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right) = (\sqrt{3}-1)$$

x-cordinate of C is -2sin$30°$=-1

Hence Sum=$(\sqrt{3} + \sqrt{3} - 1 + (-1))$

$$= 2\sqrt{3} - 2$$

Q. 60 A line drawn through the point P(4, 7) cuts the circle $x^2+y^2=9$ at the points A and B. Then PA·PB is equal to :

Option 1:
53

Option 2:
56

Option 3:
74

Option 4:
65

Correct Answer:
56

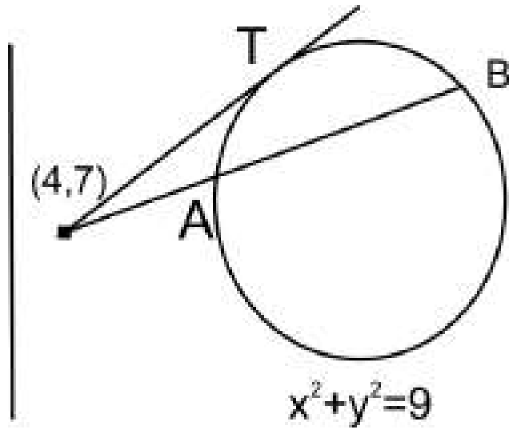
Solution:
As we learnt

Length of a tangent -

$$L=\sqrt{{x_1}^2+{y_1}^2+2gx_1+2fy_1+c}$$

- wherein

Length of tangent from a external point (x_1,y_1) to circle $x^2+y^2+2gx+2fy+c=0$



Since we know,

$PA.PB=PT^2$

$PT=\sqrt{4^2+7^2-9}$

$\Rightarrow PT^2=\left(\sqrt{56}\right)^2=56$

Mock Test 2

Physics

Q. 1 In a Young's double slit experiment, the path difference , at a certain point on the screen between two interfering waves is $\frac{1}{8}th$ of the wavelength. The ratio of the intensity at this point to that at the centre of a bright fringe is close to:

Option 1:
0.80

Option 2:
0.94

Option 3:
0.85

Option 4:
0.74

Correct Answer:
0.85

Solution:
Resultant Intensity of two wave -

$$I=I_1+I_2+2\sqrt{I_1I_2}\cos\theta$$

- wherein

$I_1=$ The intensity of wave 1

$I_2=$ The intensity of wave 2

$\theta=$ Phase difference

$$\Delta x=\frac{\lambda}{8}$$

$$\Delta\phi=\left(\frac{2\pi}{\lambda}\right)\frac{\lambda}{8}=\frac{\pi}{4}$$

$$I=I_1+I_2+2\sqrt{I_1I_2}\cos\theta$$

Putting $I_1\ and\ I_2 = I_o$

we get $\Rightarrow I = I_0 + I_0 + 2\sqrt{I_0 I_0}\cos\phi = 4I_0\cos^2\frac{\phi}{2}$

At the centre $I_c = 4I_0$

and at that point $I = 4I_o\cos^2(\frac{\pi}{8}) = I_c\cos^2(\frac{\pi}{8})$

$\frac{I}{I_c} = \cos^2(\frac{\pi}{8})$

≈ 0.85

Q. 2 In young's double slit experiment, 16 fringes are observed in a certain segment of the screen when light of wavelength 700 nm is used. If the wavelength of light is changed to 400 nm, the number of fringes observed in the same segment of the screen would be:

Option 1:
30

Option 2:
28

Option 3:
18

Option 4:
24

Correct Answer:
28

Solution:

$y = \frac{D\lambda}{d}$

$\text{or } n_1\frac{D\lambda_1}{d} = n_2\frac{D\lambda_2}{d}$

$n_1\lambda_1 = n_2\lambda_2$

$\frac{n_1}{n_2} = \frac{\lambda_2}{\lambda_1}$

$n_2 = n_1\cdot\frac{\lambda_1}{\lambda_2} \Rightarrow 16\times\frac{700}{400} = 28$

Q. 3 An object is placed at a distance of 15cm from a concave mirror of the focal lenth 10cm. The magnification of the image is:

Option 1:
2

Option 2:
-2

Option 3:
0.4

Option 4:
-0.4

Correct Answer:
-2

Solution:
As we learn

Mirror Formula -

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

- wherein

$u =$ Object distance from pole of mirror.

$v =$ Image distance from pole of mirror.

$f =$ focal length of the mirror.

$$\frac{1}{v}+\left(\frac{1}{u}\right)=\frac{1}{f}$$

$$\frac{1}{v}-\frac{1}{15}=\frac{1}{-10}$$

$$v=-30cm$$

$$m=-\frac{v}{4}=\frac{-(-30)}{-15}=-2$$

Q. 4 An object is placed at a distance of 15cm from a concave mirror of focal length 10cm. The magnification of the image is

Option 1:
2

Option 2:
-2

Option 3:
0.4

Option 4:
-0.4

Correct Answer:
-2

Solution:
As we learned

Mirror Formula -

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

- wherein

$u=$ Object distance from pole of mirror.

$v=$ Image distance from pole of mirror.

$f=$ focal length of the mirror.

$$\frac{1}{V}+\frac{1}{-4}=\frac{1}{-f}$$

$$\frac{1}{V}-\frac{1}{15}=\frac{1}{-10}$$

$$\Rightarrow V=-30cm$$

$$m=\frac{-V}{4}=\frac{-(-30)}{-15}=-2$$

Q. 5 Maximum intensity in YOSE is I_1. The intensity at a point on the screen where the phase difference between two interfering beam is $\frac{\pi}{3}$:

Option 1:
0.25 I_1

Option 2:
0.75 I_1

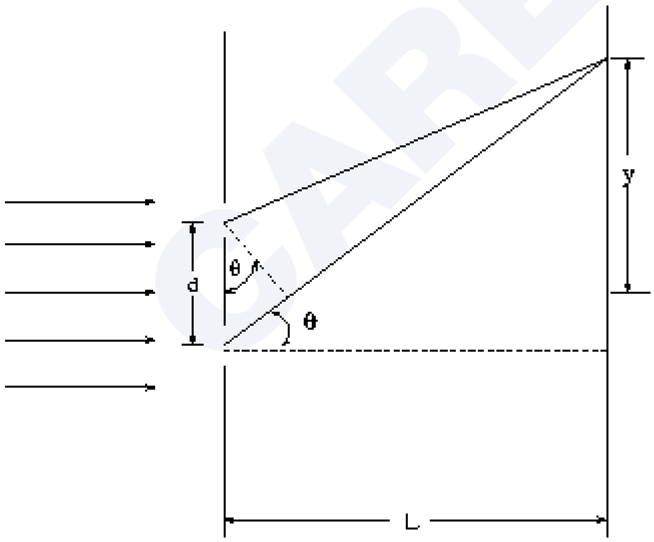
Option 3:
I_1

Option 4:
0.5 I_1

Correct Answer:
0.75 I_1

Solution:
As we learn

Young Double Slit Experiment -



- wherein

$$y=\Delta x\cdot\left(\frac{D}{d}\right)$$

$y=$ Distance of a point on screen from central maxima

$\Delta x=$ Path difference at that point

$$I = I_{max}\cos^2\left(\frac{\phi}{2}\right)$$

$$I = I_1\cos^2\left(\frac{\pi}{6}\right)$$

$$I = I_1 \times \frac{3}{4} = 0.75 I_1$$

Q. 6 Which of the following is true about an object in light rays.

Option 1:
there are two types of objets real object and vitrul object

Option 2:
it may be point object or an extended pbject

Option 3:
both (A) ,and (B)

Option 4:
none of these

Correct Answer:
both (A) ,and (B)

Solution:
as we lern

Object -

A source of light rays that are incident on an optical element. It may be a point object or an extended object .They are of two kind real object & virtual object.

- wherein

There are two types of objects, rel object and virtual object

Q. 7 if keeping the incedence ray fixed ,the mirior rotates by an angle Θ the reflected light will deviate from its original path by an angle

Option 1:
Θ^{o}

Option 2:
0^{0}

Option 3:
$2\Theta^{0}$

Option 4:
$4\Theta^{0}$

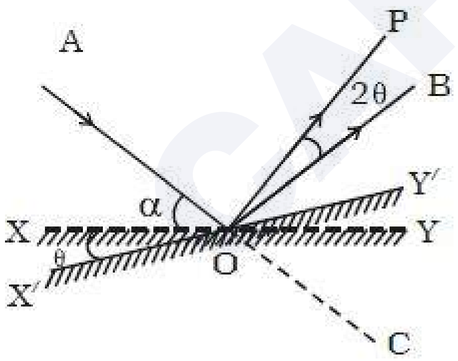
Correct Answer:
$2\Theta^{0}$

Solution:
as we learn

Rotation of Mirror -

Keeping the incidence ray fixed, When mirror is rotated by an angle θ, the reflected ray is rotated by an angle 2θ.

- wherein



The reflected raywill deviate by an angle 2θ

Q. 8 If the dimensions of a physical quantity are given by M^{a} L^{b} T^{c}, then the physical quantity will be:

Option 1:
Velocity if a = 1, b = 0, c = -1

Option 2:
Acceleration if a = 1, b = 1, c = -2

Option 3:
Force if a = 0, b = - 1, c = - 2

Option 4:
Pressure if a = 1, b = - 1, c = - 2

Correct Answer:
Pressure if a = 1, b = - 1, c = - 2

Solution:
$[velocity] = [M^0LT^{-1}]$

and

$[acceleration] = [M^0LT^{-2}]$

and

$[force] = [ma] = [MLT^{-2}]$

and

$[pressure] = [\frac{force}{area}] = \frac{[MLT^{-2}]}{[L^2]} = [ML^{-1}T^{-2}]$

i.e option 4 is correct

Q. 9 The dimensions of $(\mu_o \varepsilon_o)^{-1/2}$ are:

Option 1:
$[L^{-1} T]$

Option 2:
$[L T^{-1}]$

Option 3:
$[L^{1/2} T^{1/2}]$

Option 4:
$[L^{1/2} T^{-1/2}]$

Correct Answer:
[L T^{-1}]

Solution:
As we learnt in

Permeability of free space -

Dimension of permeablity of free space (μ_o)-$M^1L^1T^{-2}A^{-2}$

- wherein

$\frac{newton}{ampere^2}, \frac{henry}{metre}$

$$C = \frac{1}{\sqrt{\mu_0\epsilon_0}}$$

Where, C = Speed of light

C = $\left(\mu_0\epsilon_0\right)^{-1/2}$

$$C = [LT^{-1}]$$

Correct option is 2.

Q. 10 The *x* and *y* coordinates of the particle at any time are *x* = 5t - 2t^2 and *y* = 10t respectively, where *x* and *y* are in meters and t in seconds. The acceleration of the particle at t = 2s is

Option 1:
0

Option 2:
5m /s^2

Option 3:
-4m /s^2

Option 4:
-8m /s^2

Correct Answer:
-4m /s^2

Solution:
As we learnt in

Introduction to Differentiation -

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

- wherein

$$\frac{d}{dx}(x^5) = (n = 5)$$

$$\Rightarrow nx^{n-1}$$

$$\Rightarrow 5x^{5-1}$$

$$\Rightarrow 5x^4$$

x = 5t - $2t^2$ y = 10t

$\frac{dx}{dt} = 5 - 4t$ $\frac{dy}{dt} = 10$

vx=5-4t vy=10

ax=-4 ay=0

Acceleration of particle at $(t = 2) = -4m/s^2$

Correct option is 3.

Q. 11 The speed of a swimmer in still water is 20 m/s . The speed of river water is 10 m/s and is flowing due east . If he is standing on the south bank and wishes to cross he river along the sortest path , the angle at which he should make his strokes w.r.t north is given by :

Option 1:
30 west

Option 2:
$0°$

Option 3:
$60° west$

Option 4:
$45° west$

Correct Answer:
30 west

Solution:
Boat - River Problem -

To cross river in the shortest path

condition (velocity of boat along river flow must be zero)

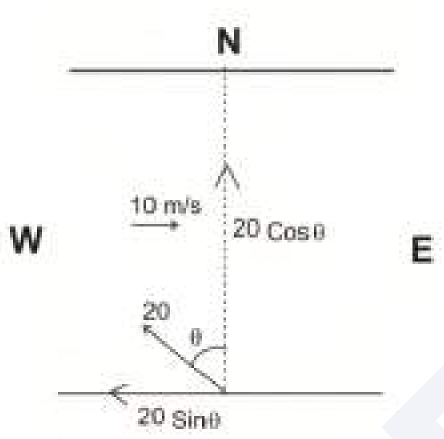
$t=\frac{d}{\sqrt{v^2-u^2}}$

$d=$ width of river

v = Speed of Boat w.r.t. river

u = speed of river

-



$20 \sin \theta=10 \Rightarrow \theta=30^{\circ} west$

Q. 12 Preeti reached the metro station and found that the escalator was not working. She walked up the stationary escalator in time t_1. On other days, if she remains stationary on the moving escalator, then the escalator takes her up in time t_2. The time taken by her to walk up on the moving escalator will be:

Option 1:
$\frac{t_1+t_2}{2}$

Option 2:
$\frac{t_1 t_2}{t_2-t_1}$

Option 3:
$\frac{t_1 t_2}{t_2+t_1}$

Option 4:
$t_1 - t_2$

Correct Answer:
$$\frac{t_1 t_2}{t_2 + t_1}$$

Solution:
$V_1 =$ Velocity of Preeti

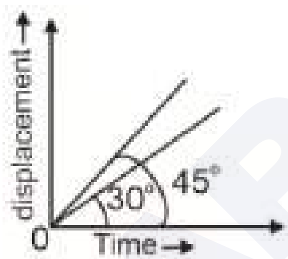
$V_2 \rightarrow$ Velocity of escalator

$l \rightarrow$ distance

$$t = \frac{l}{v_1 + v_2} = \frac{l}{\frac{l}{t_1} + \frac{l}{t_2}}$$

$$= \frac{t_1 t_2}{t_1 + t_2}$$

Q. 13 The displacement time graphs of two moving particles make angles of $30°$ and $45°$ with the x-axis as shown in the figure. the ratios of their respective velocity are



Option 1:
1:1

Option 2:
1:2

Option 3:
$1 : \sqrt{3}$

Option 4:
$\sqrt{3} : 1$

Correct Answer:
$1 : \sqrt{3}$

Solution:

For the displacement-time graph,
Slope = velocity

$$\therefore V_1 = (\text{ slope })_1 = \tan 30^\circ$$
$$V_2 = (\text{ slope })_2 = \tan 45^\circ$$
$$\frac{V_1}{V_2} = \frac{\tan 30^\circ}{\tan 45^\circ} = \frac{1}{\sqrt{3}}$$

Q. 14 Out of below four equations of Maxwell which shows monopole do not exists

Option 1:
$$\oint_s \vec{E} \cdot \overrightarrow{ds} = \frac{q}{\epsilon o}$$

Option 2:
$$\oint_s \vec{B} \cdot \overrightarrow{ds} = 0$$

Option 3:
$$\oint \vec{E} \cdot \overrightarrow{dl} = \frac{d}{dt} \oint \vec{B} \cdot \overrightarrow{ds}$$

Option 4:
$$\oint \vec{B} \cdot \overrightarrow{dl} = \mu o I + \mu o \epsilon o \frac{d}{dt} \oint \vec{E} \cdot \overrightarrow{ds}$$

Correct Answer:
$$\oint_s \vec{B} \cdot \overrightarrow{ds} = 0$$

Solution:
As we learned

Gauss's law for magnetism -

$$\int \underset{B}{\rightarrow} \cdot \underset{ds}{\rightarrow} = 0$$

i.e Total magnetic flux passing through a closed area is equal to zero.

And this Gauss Law for magnetism shows that monopole does not exist

Q. 15 The ratio of amplitude of magnetic field to the amplitude of electric field for an electromagnetic wave propagating in vacuum is equal to:

Option 1:
reciprocal of speed of light in vacuum

Option 2:
the ratio of magnetic permeability to the electric susceptibility of vacuum

Option 3:
unity

Option 4:
the speed of light in vacuum

Correct Answer:
reciprocal of speed of light in vacuum

Solution:
As we learnt in

Relation between Eo and Bo -

$$E_o = c.B_o$$

- wherein

E_o= Electric field amplitude

B_o = Magnetic field amplitude

C= Speed of light in vacuum

$$E_o = c.B_o$$

$$\therefore \frac{B_o}{E_o} = \frac{1}{c}$$

Correct option is 1.

Q. 16 The energy of the em waves is of the order of 15 ke V. To which part of the spectrum does it belong ?

Option 1:
Infra - red rays

Option 2:
Ultraviolet rays

Option 3:
γ - rays

Option 4:
X - rays

Correct Answer:
X - rays

Solution:
As we learnt in

X Rays -

Frequency Range 3×10^{17} Hz to 3×10^{19} Hz

- wherein

Wavelength Range 0.1 A^o to 100 A^o

The wavelength corresponding to 15KeV

$$\lambda = \frac{12400}{15KeV}\mathring{A} = \frac{12.40}{15}\mathring{A} = 0.83\mathring{A}$$

This belong to X-ray region of EM wave.

Q. 17 Energy required to move a body of mass m from an orbit of radius 2R to 3R is

Option 1:
$GMm/12R^2$

Option 2:
$GMm/3R^2$

Option 3:
$GMm/8R$

Option 4:
$GMm/6R$

Correct Answer:
$GMm/6R$

Solution:

E = $(P.E)_{3R} - (P.E)_{2R}$

$$= -\frac{GmM}{3R} - \left(-\frac{GmM}{2R}\right) = +\frac{GmM}{6R}$$

Q. 18 A charge Q is enclosed by a Gaussian spherical surface of radius R. If the radius is doubled, then the outward electric flux will:

Option 1:
Be doubled

Option 2:
Increase four times

Option 3:
Be reduced to half

Option 4:
Remain the same

Correct Answer:
Remain the same

Solution:
As we learnt in

Gauss's Law -

Total flux linked with a closed surface called Gaussian surface.

Formula:

$$\phi = \oint \vec{E} \cdot d\vec{s} = \frac{Q_{enc}}{\epsilon_0}$$

Q_{enc} - charge enclosed by closed surface.

Electric flux through a surface = $\frac{q_{in}}{\epsilon_0}$

since q_{in} is constant, flux will remain same.

Q. 19 Within a spherical charge distribution of charge density ρ(r), N equipotential surfaces of potential V_0, V_0 + V, V_0 + 2V, ... $V_0 + N\Delta V$ $(\Delta V > 0)$, are drawn and have increasing radii r_0, r_1, r_2,..........r_N, respectively. If the difference in the radii of the surfaces is constant for all values of V_0 and ΔV then :

Option 1:
$\rho(r)\alpha r$

Option 2:
$\rho(r) = constant$

Option 3:
$\rho(r)\alpha\frac{1}{r}$

Option 4:
$\rho(r)\alpha\frac{1}{r^2}$

Correct Answer:
$\rho(r)\alpha\frac{1}{r}$

Solution:
As we learnt in

Relation between field and potential -

$$E = \frac{-dV}{dr}$$

- wherein

$\frac{dv}{dr}-$ Potential gradient.

If P lies inside -

$$E_{in} = \frac{1}{4\pi\epsilon_0}\frac{Qr}{R^3} \quad V_{in} = \frac{Q}{4\pi\epsilon_0}\frac{3R^2 - r^2}{2R^3}$$

$$E_{in} = \frac{\rho r}{3\epsilon_0} \qquad V_{in} = \frac{\rho\left(3R^2 - r^2\right)}{6\epsilon_0}$$

We know $E = \frac{-dV}{dr}$

Here Δv and Δr are same for any pair of surfaces.

E= constant

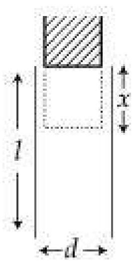
Now, electric field inside the spherical charge distribution

$$E = \frac{\rho}{3\epsilon_0} r$$

E would be constant of $\rho r = constant$

$$\rho(r) \propto \frac{1}{r}$$

Q. 20 A parallel plate capacitor is made of two plates of length *l*, width *w* and separated by distance *d*. A dielectric slab (dielectric constant K) that fits exactly between the plates is held near the edge of the plates. It is pulled into the capacitor by a force $F = -\frac{\partial U}{\partial x}$ where U is the energy of the capacitor when dielectric is inside the capacitor up to distance *x* (See figure). If the charge on the capacitor is Q then the force on the dielectric when it is near the edge is :



Option 1:
$$\frac{Q^2 d}{2wl^2\epsilon_0} K$$

Option 2:
$$\frac{Q^2 w}{2dl^2\epsilon_0}(K-1)$$

Option 3:
$$\frac{Q^2 d}{2wl^2\epsilon_0}(K-1)$$

Option 4:
$$\frac{Q^2 w}{2dl^2\epsilon_0} K$$

Correct Answer:

$$\frac{Q^2 d}{2wl^2\epsilon_0}(K-1)$$

Solution:

$$C = C_1 + C_2 = \frac{K(xw)\varepsilon_0}{d} + \frac{(l-x)w\varepsilon_0}{d}$$

$$C = \frac{w\varepsilon_0}{d} \times (Kx + (l-x))$$

$$U = \frac{1}{2} \times \frac{Q^2}{C} = \frac{Q^2 d}{2w\varepsilon_0(\varepsilon + (k-1)x)}$$

$$\frac{\partial U}{\partial x} = -\frac{dQ^2(K-1)}{2w\varepsilon_0(l+(k-1)x)^2}$$

$$F = -\frac{\partial U}{\partial x} = \frac{Q^2 d(K-1)}{2wl^2\varepsilon_0} \qquad at \quad x = 0$$

Q. 21 The electric field in a region of space is given by, $\vec{E} = E_0\,\hat{i} + 2E_0\,\hat{j}$ where E_0=100 N/C. The flux of this field through a circular surface of radius 0.02 m parallel to the Y-Z plane is nearly :

Option 1:
0.125 Nm^2/C

Option 2:
0.02 Nm^2/C

Option 3:
0.005 Nm^2/C

Option 4:
3.14 Nm^2/C

Correct Answer:
0.125 Nm^2/C

Solution:

$$\underset{E}{\rightarrow} = E_0\hat{i} + 2E_0\hat{J}$$

$$E_0 = 100W/C$$

$$\vec{E} = 100\hat{i} + 200\hat{J}$$

$$A = \pi r^2 = \frac{22}{7} \times 0.02 \times 0.02$$

$$A = 1.25 \times 10^{-3}\hat{i}m^2$$

$\therefore$ New flux $\therefore \phi = EAcos\theta$

$$\phi = (100\hat{i} + 200\hat{J}).1.25 \times 10^{-3}\hat{i}$$

$$\phi = 1.25 \times 10^{-1}Nm^2/c$$

$$= 0.125Nm^2/C$$

Q. 22 A charged particle is suspended in equilibrium in a uniform vertical electric field of intensity 20000 *V/m*. If mass of the particle is $9.6 \times 10^{-16}kg$, the charge on it and excess number of electrons on the particle are respectively $(g = 10\,m/s^2)$

Option 1:
$4.8 \times 10^{-19}C, 3$

Option 2:
$5.8 \times 10^{-19}C, 4$

Option 3:
$3.8 \times 10^{-19}C, 2$

Option 4:
$2.8 \times 10^{-19}C, 1$

Correct Answer:
$4.8 \times 10^{-19}C, 3$

Solution:
Electric field of intensity =20,000 V/m

The mass of particle =9.6 x 10^{-16} kg

For charge in equilibrium, qE=mg

$$q = \frac{mg}{E} = \frac{9.6 \times 10^{-15}}{2 \times 10^4} = 4.8 \times 10^{-19}$$

$$\text{Excess number of electrons on the particle} = \frac{\text{Charge on the particle}}{e} = 3$$

Q. 23 The ratio of densities of nitrogen and oxygen is 14:16. The temperature at which the speed of sound in nitrogen will be same at that in oxygen at $55°C$ is

Option 1:
$35°C$

Option 2:
$48°C$

Option 3:
$65°C$

Option 4:
$14°C$

Correct Answer:
$14°C$

Solution:
As we learnt

$v=\sqrt{\frac{\gamma RT}{M}} \Rightarrow \frac{T_N}{T_O}=\frac{M_N}{M_O}$ (since given the velocities are same)

$$\Rightarrow \frac{T_N}{273+55}=\frac{14}{16}=\frac{7}{8}$$

$$\Rightarrow T_N=287K=14^oC$$

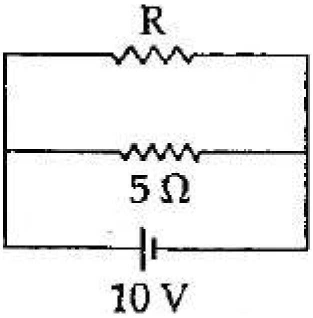
Q. 24 Intensity level of a sound of intensity I is 30 dB. The ratio $\frac{I}{I_0}$ is (where I_0 is the threshold for hearing)

Correct Answer:
1000

Solution:

$$L=10log_{10}\left[\frac{I}{I_o}\right]=30 \Rightarrow \frac{I}{I_o}=10^3$$

Q. 25 The power dissipated in the circuit shown in the figure is 30 Watts. The value of R is :



Option 1:
15Ω

Option 2:
10Ω

Option 3:
30Ω

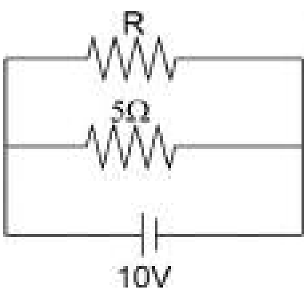
Option 4:
20Ω

Correct Answer:
10Ω

Solution:
As we learnt in

In parallel Grouping -

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}$$



$$R_{eq} = \frac{5R}{5+R}$$

Power $P = \frac{V^2}{R} \;=>\; 30 = \frac{(10^2)}{(\frac{5R}{5+R})}$

150R = 100(5+R) => 150R = 500+100R

$$R = \frac{500}{50} = 10\Omega$$

Q. 26 A milli voltmeter of 25 milli volt range is to be converted into an ammeter of 25 ampere range. The value (in ohm) of necessary shunt will be:

Option 1:
0.001

Option 2:
0.01

Option 3:
1

Option 4:
0.05

Correct Answer:
0.001

Solution:
As we learnt in

Required shunt -

$$s = \frac{i_g G}{(i - i_g)}$$

- wherein

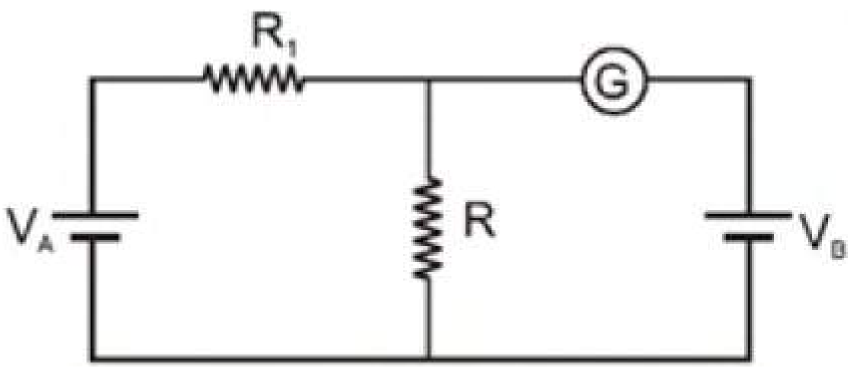
$i_g -$ Current through galvanometer

Full scale deflection current $i_g = \frac{25mV}{G} A$

The value of shunt required for converting it into ammeter of range 25 ampere is

$$s = \frac{i_g G}{i - i_g} \cong \frac{25mv}{25} = 0.001\Omega$$

Q. 27 In the circuit shown the cells A and B have negligible resistances. For $V_A = 12$ V, $R_1 = 500\ \Omega$ and $R = 100\ \Omega$ the galvanometer (G) shows no deflection. The value of V_B is:



Option 1:
4 V

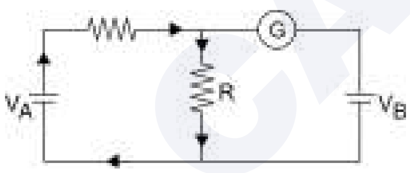
Option 2:
2 V

Option 3:
12 V

Option 4:
6 V

Correct Answer:
2 V

Solution:



$$I = \frac{V_A}{R_1 + R}$$

$$I = \frac{12}{500 + 100} = \frac{12}{600}$$

$$V_B = IR = \frac{12}{600} \times 100 = 2V$$

Q. 28 If voltage across a bulb rated 220 volt - 100 watt drops by 2.5% of its rated value, the percentage of the rated value by which the power would decrease is:

Option 1:
20%

Option 2:
2.5%

Option 3:
5%

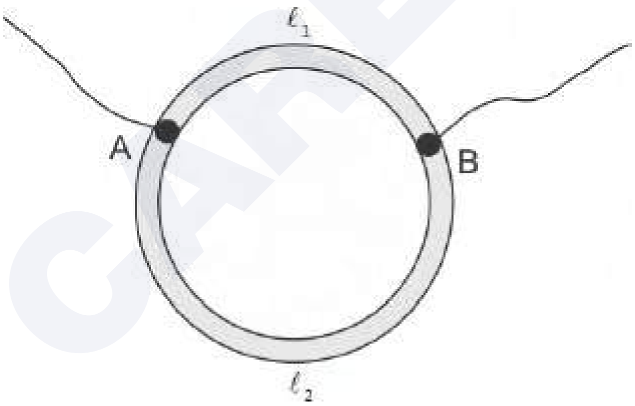
Option 4:
10%

Correct Answer:
5%

Solution:

$$P=\frac{V^2}{R}=\frac{\Delta P}{P}=\frac{2\Delta V}{V}+\frac{\Delta R}{R}$$
$$\frac{\Delta P}{P}=2\times 2.5+0=5\%$$

Q. 29 A ring is made of a wire having a resistance R_0 = 12 Ω.. Find the points A and B as shown in the figure, at which a current carrying conductor should be connected so that the resistance R of the sub circuit between these points is equal to $\frac{8}{3}\Omega$.



Option 1:
$\frac{\ell_1}{\ell_2}=\frac{5}{8}$

Option 2:
$\frac{\ell_1}{\ell_2}=\frac{1}{3}$

Option 3:
$$\frac{\ell_1}{\ell_2}=\frac{3}{8}$$

Option 4:
$$\frac{\ell_1}{\ell_2}=\frac{1}{2}$$

Correct Answer:
$$\frac{\ell_1}{\ell_2}=\frac{1}{2}$$

Solution:
As we learnt in

In series Grouping -

$$R_{eq}=R_1+R_2+R_3+\cdots+R_n$$

- wherein

$R_{eq}-$ Equivalent Resistance

In parallel Grouping -

$$\frac{1}{R_{eq}}=\frac{1}{R_1}+\frac{1}{R_2}+\cdots+\frac{1}{R_n}$$

-

Let x be resistance per unit length of the wire

Upper Portion $R_1=xl_1$

Lower Portion $R_2=xl_2$

Equivalent resistance between A and B is

$$R=\frac{(xl_1)(xl_2)}{xl_2+xl_1}=\frac{xl_1l_2}{l_1+l_2}$$

$$\frac{8}{3}=\frac{xl_1l_2}{l_1+l_2}=>\frac{8}{3}=\frac{xl_1}{(l_1/l_2+1)}$$------------------------(i)

$R_o = xl_1 + xl_2 => 12 = x(l_1 + l_2)$

$12 = xl_2(1 + \frac{l_1}{l_2})$----------------------------------(ii)

$\therefore eqn's(i)and(ii)$

$$\frac{8/3}{12} = \frac{xl_1/l_1/l_1+1}{xl_2(\frac{l_1}{l_2}+1)}$$

or $\frac{8}{36} = \frac{l_1}{l_2(\frac{l_1}{l_2}+1)^2}$

$(\frac{l_1}{l_2}+1)^2\frac{8}{36} = \frac{l_1}{l_2}$

$(\frac{l_1}{l_2}+1)^2\frac{2}{9} = \frac{l_1}{l_2}$

Let $y = \frac{l_1}{l_2}$

$2(y+1)^2 = 9y$

=>$2y^2 + 2 + 4y = 9y$

$2y^2 - 5y + 2 = 0$

$\Rightarrow y = \frac{1}{2} \;\; or \;\; 2$

$\Rightarrow \frac{l_1}{l_2} = \frac{1}{2}$

Q. 30 A wire of resistance 4 Ω is stretched to twice its original length. The resistance of stretched wire would be:

Option 1:
16 Ω

Option 2:
2 Ω

Option 3:
4 Ω

Option 4:
8 Ω

Correct Answer:
16 Ω

Solution:
As we learnt in

Resistance formula -

$$R=\rho \frac{l}{A}=\frac{m}{n e^2 \tau} \cdot \frac{l}{A}$$

- wherein

$\rho=$ resistivity of material

$n=$ Number of free electrons per unit volume.

$$l^{\prime}=2 l$$

Volume of wire remains constant

$$l A=l^{\prime} A^{\prime} \Rightarrow A=\frac{l}{l^{\prime}} A=\frac{l}{2 l} A=\frac{A}{2}$$

New resistance

$$R^{\prime}=\frac{\rho l^{\prime}}{A^{\prime}}=\frac{\rho 2 l}{\left(\frac{A}{2}\right)}=4 \frac{\rho l}{A}$$

$$R^{\prime}=4(4 \Omega)=16 \Omega$$

Q. 31 The internal resistance of a 2.1 V cell which gives a current of 0.2 A through a resistance of 10 Ω is:

Option 1:
1.0 Ω

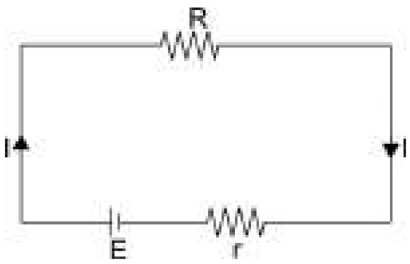
Option 2:
0.2 Ω

Option 3:
0.5 Ω

Option 4:
0.8 Ω

Correct Answer:
0.5 Ω

Solution:



$$I = \frac{E}{R+r}$$

$$IR + Ir = E$$

given

$$R = 10\Omega \qquad r = ?$$

$$E = 2.1V \qquad I = 0.2$$

$$\therefore \quad 0.2 \times 10 + 0.2 \times r = 2.1 \quad => 0.2r = 0.1$$

$$r = \frac{1}{2} = 0.5\Omega$$

Q. 32 Bragg's law for X-rays is -

Option 1:
$d \sin \Theta = 2n\lambda$

Option 2:
$2d \sin \Theta = n\lambda$

Option 3:
$\lambda \sin \Theta = 2d$

Option 4:
$d \sin \Theta = n\lambda$

Correct Answer:
$2d \sin \Theta = n\lambda$

Solution:
Bragg's law -

$$2d \sin \Theta = n\lambda$$

(condition of constructive maxima)

d=distance between parallel lines

λ = wavelength

Θ = angle between light & plane

Q. 33 The half-life period of a radio-active element **X** is same as the mean life time of another radio-active element **Y** . Initially they have the same number of atoms. Then

Option 1:
X and Y decay at same rate always

Option 2:
X will decay faster than **Y**

Option 3:
Y will decay faster than **X**

Option 4:
X and Y have same decay rate initially

Correct Answer:
X will decay faster than **Y**

Solution:
$\tau_{1/2}(i.e\ half\ life\ )of\ X = \tau_{\gamma}(i.e\ mean\ life)\ of\ Y$

So

$$\frac{ln2}{\lambda_X} = \frac{1}{\lambda_Y} \Rightarrow \lambda_X = \lambda_Y ln2$$

$$\lambda_X > \lambda_Y$$

And corresponding Activity is given as

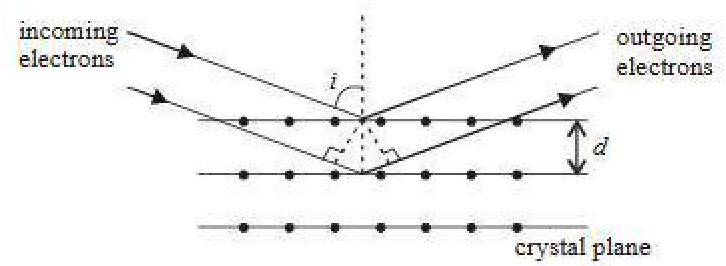
$$\therefore A_X = A_0 e^{-\lambda_X t}; A_Y = A_0 e^{-\lambda_Y t};$$

X will decay faster than Y

The correct option is 2

Q. 34 Question is based on the following paragraph.

Wave property of electrons implies that they will show diffraction effects. Davisson and Germer demonstrated this by diffracting electrons from crystals. The law governing the diffraction from a crystal is obtained by requiring that electron waves reflected from the planes of atoms in a crystal interfere constructively (see figure).



Question : Electrons accelerated by potential V are diffracted from a crystal. If $d=1\,\dot{A}\; and\; i=30^{\circ}, V$ should be about
$(h=6.6\times10^{-34}\;Js, m_e=9.1\times10^{-31}kg, e=1.6\times10^{-19}C)$

Option 1:
50

Option 2:
2000

Option 3:
100

Option 4:
500

Correct Answer:
50

Solution:
As we learnt in

Bragg's formula -

$$2d\sin\Theta=n\lambda$$

- wherein

$$d-distance\;between\;diffracting\;planes$$

Condition of constructive interference is $2d\ sin\theta = n\lambda$

$\theta = 90^o - i = 60^o$

Take n = 1

So $\lambda = 2dsin(60^0)$

& $\lambda = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2meV}}$

$\therefore \quad \frac{h}{\sqrt{2meV}} = \frac{2 \times 1\mathring{A} \times \sqrt{3}}{2}$

Square both side

$\frac{h^2}{2meV} = 3 \times 10^{-20}$

$$V = \frac{(6.62 \times 10^{-34})^2}{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \times 3 \times 10^{-20}} = \frac{43.82 \times 10^{-68}}{87.36 \times 10^{-70}} = 50V$$

Q. 35 X- rays are -

Option 1:
Stream of neutral particles

Option 2:
Stream of negatively charged particles

Option 3:
Stream of protons

Option 4:
electromagnetic radiations of high frequency

Correct Answer:
electromagnetic radiations of high frequency

Solution:
As we learn

Characteristics x-ray -

$\nu(frequency)$

= $Rz^2\left(\frac{1}{{n_1}^2}-\frac{1}{{n_2}^2}\right)$

- wherein

Characteristics x ray can be defined by knocking out of electron from inner orbit of an atom

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Q. 36 Both the nucleus and the atom of some element are in their respective first excited states. They get de-excited by emitting photons of wavelengths λ_N, λ_A, respectively. The ratio λ_N / λ_A is closest to :

Option 1:
10^{-6}

Option 2:
10

Option 3:
10^{-10}

Option 4:
10^{-1}

Correct Answer:
10^{-6}

Solution:
The nucleus emits radiation of order of 0.1 $\mathring{A}$ (like γ rays)

And the energy of γ rays is in order of Mev.

Similary for hydrogen-like atoms

$$E_n = 13.6 * \frac{Z^2}{n^2} eV$$

that energy of hydrogen-like atoms is in order of eV.

or atom emits radiation of order of $10^5 A^0$

ratio $\frac{\lambda_N}{\lambda_a} = \frac{0.1}{10^5} = 10^{-6}$

or $\frac{\lambda_N}{\lambda_a} = \frac{E_a}{E_N} = \frac{1eV}{1MeV} = \frac{1}{10^6} = 10^{-6}$

Q. 37 The momentum of a photon of energy h v will be

Option 1:
$\frac{hv}{c}$

Option 2:
$\frac{c}{hv}$

Option 3:
hv

Option 4:
$\frac{hv}{c^2}$

Correct Answer:
$\frac{hv}{c}$

Solution:
As we learnt in

Kinetic mass of photon -

$$m = \frac{E}{c^2} = \frac{h}{c\lambda}$$

- wherein

$m \rightarrow mass\ of\ photon$

$c \rightarrow speed\ of\ light$

$h \rightarrow plank's\ constant$

$\lambda - wave\ length$

For photon E = pc.

$$\Rightarrow p = \frac{E}{c} = \frac{h\nu}{c}$$

Q. 38 Light of two different frequencies whose photons have energies 1 eV and 2.5eV respectively illuminate a metallic surface whose work function is 0.5 eV successively. Ratio of maximum speeds of emitted electrons will be

Option 1:
1:4

Option 2:
1:2

Option 3:
1:1

Option 4:
1:5

Correct Answer:
1:2

Solution:
As we learnt in

Conservation of energy -

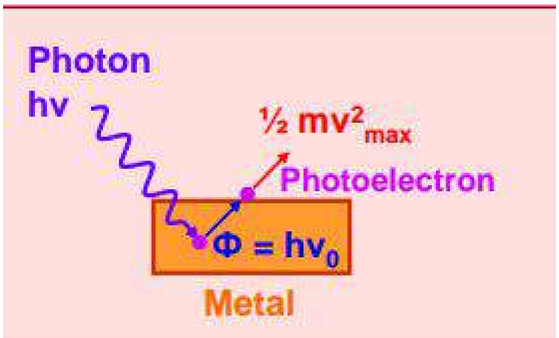
$$h\nu = \phi_0 + \frac{1}{2}mv_{max}^2$$

$$h\nu = h\nu_0 + \frac{1}{2}mv_{max}^2$$

$$h\left(\nu - \nu_0\right) = \frac{1}{2}mv_{max}^2$$

$where, h - Plank's constant\ \nu - Frequency\ \nu_0 - threshold\ frequency\ \phi_0 - work function$

- wherein



$$\frac{1}{2}mV_{max}^2 = h\nu - \phi$$

$$V_{max} = \sqrt{\frac{2(h\nu - \phi)}{m}}$$

$$\frac{V_1}{V_2} = \sqrt{\frac{h\nu_1 - \phi}{h\nu_2 - \phi}} = \sqrt{\frac{1 - 0.5}{2.5 - 0.5}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$$

Q. 39 A gas mixture consists of 2 moles of O_2 and 4 moles of Ar at temperature T. Neglecting all vibrational modes, the total internal energy of the system is

Option 1:
4 RT

Option 2:
15 RT

Option 3:
9 RT

Option 4:
11 RT

Correct Answer:
11 RT

Solution:
As we learnt in

Total internal energy -

$$U = U_K + U_P$$

- wherein

Change in internal energy

$$\Delta U = n\frac{f}{2}R\Delta T$$

(Always)

f is degree of freedom

Internal energy of a system is

$$U=U_1+U_2$$

$$=n_1C_{V1}T+n_2C_{V2}T$$

$$n_1=2, C_{V1}=\frac{fR}{2}=\frac{5R}{2}$$

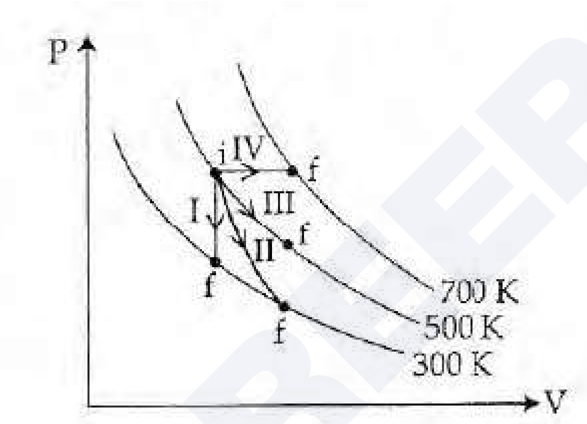
$$n_2=4, C_{V2}=\frac{3R}{2}$$

$$\therefore U=\left(2\times\frac{5R}{2}+4\times\frac{3R}{2}\right)T$$

or $U=11RT$

Correct option is 4.

Q. 40 Thermodynamic processes are indicated in the following diagram.



Match the following

Column-1	**Column-2**
P. Process I	a. Adiabatic
Q. Process II	b. Isobaric
R. Process III	c. Isochoric
S. Process IV	d. Isothermal

Option 1:
P $\longrightarrow$ a, Q $\longrightarrow$ c, R $\longrightarrow$ d, S $\longrightarrow$ b

Option 2:
P $\longrightarrow$ c, Q $\longrightarrow$ a, R $\longrightarrow$ d, S $\longrightarrow$ b

Option 3:
P $\longrightarrow$ c, Q $\longrightarrow$ d, R $\longrightarrow$ b, S $\longrightarrow$ a

Option 4:
P $\longrightarrow$ d, Q $\longrightarrow$ b, R $\longrightarrow$ a, S $\longrightarrow$ c

Correct Answer:
P $\longrightarrow$ c, Q $\longrightarrow$ a, R $\longrightarrow$ d, S $\longrightarrow$ b

Solution:

Path (I) $\rightarrow$ constant volume $\rightarrow$ isochoric
Path (II) $\rightarrow$ $PV^{\gamma} = k$ $\rightarrow$ adiabatic
Path (III) $\rightarrow$ constant temperature $\rightarrow$ Isothermal
Path (IV) $\rightarrow$ constant pressure $\rightarrow$ isobaric

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Q. 41 In a certain region of space, the gravitational field is given by $I = -\left(\frac{K}{R}\right)$ .Taking the reference point to be at $r = r_0$ and potential at the reference point as $V = V_0$.

Then find the potential at a general point ?

Option 1:
$Klog(\frac{r}{r_0}) + V_0$

Option 2:
$Klog(\frac{r_0}{r}) + V_0$

Option 3:
$Klog(\frac{r}{r_0}) - V_0$

Option 4:
$Klog(\frac{r_0}{r}) - V_0$

Correct Answer:
$Klog(\frac{r}{r_0}) + V_0$

Solution:

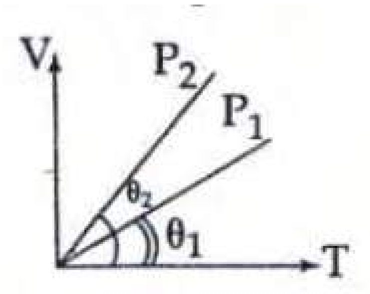
As $I = \frac{-dv}{dr}$ or $dv = -Idr$

$$\int_{v_0}^{v} dv = -\int_{r_0}^{r} \left(\frac{-k}{r}\right) dr$$

$$[V]_{vo}^{v} = K\,[logr]_{ro}^{r} = K\,[logr - logr_o]$$

$$V - V_0 = Klog\frac{r}{r_o} \Rightarrow V = Klog\frac{r}{r_0} + V_o$$

Q. 42 In the given (V - T) diagram, what is the relation between pressures P_1 and P_2 ?



Option 1:
$P_2 > P_1$

Option 2:
$P_2 < P_1$

Option 3:
Cannot be predicted

Option 4:
$P_2 = P_1$

Correct Answer:
$P_2 < P_1$

Solution:
For an ideal gas

$$PV = nRT$$

or $V = (\frac{nR}{P})T$

For V-T graph slope $= \frac{nR}{P}$

From figure, $tan\Theta_2 > tan\Theta_1$

or $\frac{nR}{P_2} > \frac{nR}{P_1}$

or $P_1 > P_2$

Q. 43 In a vessel, the gas is at a pressure P. If the mass of all the molecules is halved and their speed is doubled, then the resultant pressure will be:

Option 1:
4P

Option 2:
2P

Option 3:
P

Option 4:
$\frac{P}{2}$

Correct Answer:
2P

Solution:

Pressure of a gas -

$$P = \frac{1}{3} mn < v^2 >$$

- wherein

m = mass of a molecule

n = number of molecule /volume

$< v^2 >$ = Rms velocity

Pressure of a gas

$$P = \frac{1}{3} mn < v^2 >$$

if mass is halved and speed is doubled pressure becomes 2P

Q. 44 A boy's catapult is made of rubber cord which is $42\ cm$ long, with $6\ mm$ diameter of cross-section and of negligible mass. The boy keeps a stone weighing $0.02\ kg$ on it and stretches the cord by $20\ cm$ by applying a constant force. When released, the stone flies off with a velocity of $20\ ms^{-1}$. Neglect the change in the area of cross-section of the cord while stretched. The young's modulus (Nm^{-2}) of rubber is closest to :

Option 1:
100000000

Option 2:
1000

Option 3:
1000000

Option 4:
10000

Correct Answer:
1000000

Solution:
For string

$$l = 42\ cm$$
$$d = 6\ mm \Rightarrow r = 3\ mm$$
$$A = \pi\ r^2$$
$$\Delta l = 20\ cm$$

for stone

$$m = 0.02\ kg$$
$$V = 20\ m/s$$

So apply energy conservation.

P.E. stored in string = K.E of stone

$$\Rightarrow \frac{1}{2} \times Y \times \left(\frac{\Delta L}{L}\right)^2 \times A \times L = \frac{1}{2}\ mv^2$$

$$\Rightarrow Y = \frac{mv^2 \times L}{(\Delta L)^2 \times A}$$

$$= \frac{2 \times 10^{-2} \times 20 \times 20 \times 42 \times 10^{-2}}{20 \times 20 \times 10^{-4} \times 3.14 \times 9 \times 10^{-6}}$$

$$Y \approx 10^6\ N/m^2$$

Q. 45 A small spherical droplet of density d is floating exactly half immersed in a liquid of density p and surface tension T. The radius of the droplet is (take note that the surface tension applies an upward force on the droplet) :

Option 1:

$$r=\sqrt{\frac{2T}{3(d+p)g}}$$

Option 2:

$$r=\sqrt{\frac{T}{(d+p)g}}$$

Option 3:

$$r=\sqrt{\frac{T}{(d-p)g}}$$

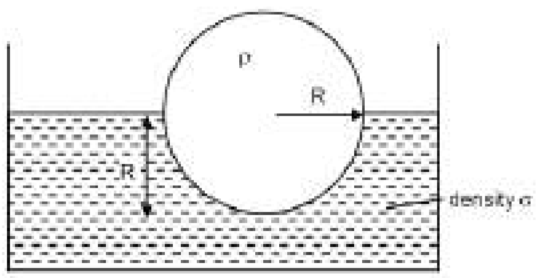
Option 4:

$$r=\sqrt{\frac{3T}{(2d-p)g}}$$

Correct Answer:

$$r=\sqrt{\frac{3T}{(2d-p)g}}$$

Solution:



$$dVg=\rho\left(\frac{V}{2}\right)g+T(2\pi r)\Rightarrow d\cdot\frac{4}{3}\pi r^3g=\rho\cdot\frac{2}{3}\pi r^3g+2\pi rT$$

$$\Rightarrow\frac{2}{3}r^2g(2d-\rho)=2T$$

$$\Rightarrow r = \sqrt{\frac{3T}{(2d - \rho)g}}$$

Q. 46 Needles N_1,N_2,and N_3 are made of a ferromagnetic, a paramagnetic and a diamagnetic substance respectively. A magnet when brought close to them will

Option 1:
attract all three of them

Option 2:
attract N_1 and N_2 strongly but repel N_3

Option 3:
attract N_1 strongly, N_2 weakly and repel N_3 weakly

Option 4:
attract N_1 strongly, but repel N_2 and N_3 weakly.

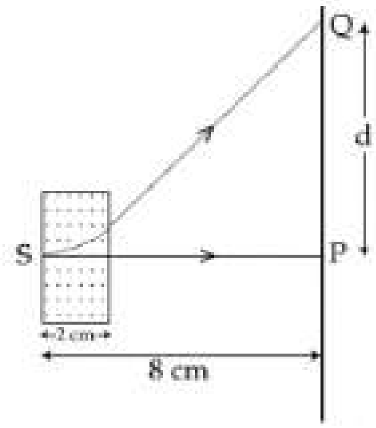
Correct Answer:
attract N_1 strongly, N_2 weakly and repel N_3 weakly

Solution:
Ferromagnetic substance magnetizes strongly in the direction of magnetic field N_1 is made up of this material paramagnetic substance magnetizes weakly in the direction of magnetic field. N_2 is made of this material. Diamagnetic substance magnetizes weakly in the opposite direction of magnetic field. N_3 is made of this material.

$\therefore$ Magnet will attack N_1 strongly, N_2 weakly and it will repel N_3 weakly.

Q. 47 An electron, moving along the x-axis with an initial energy of 100 eV, enters a region of the magnetic field $\vec{B}=(1.5\times10^{-3}T)\hat{k}$ at S(See figure). The field extends between x=0 and x=2 cm. The electron is detected at the point Q on a screen placed 8cm away from point S. The distance d between P and Q (on the screen) is: (in cms)

(electron's charge $=1.6\times10^{-19}C$, mass of electron$=1.9\times10^{-31}kg)$



Correct Answer:
12.87

Solution:

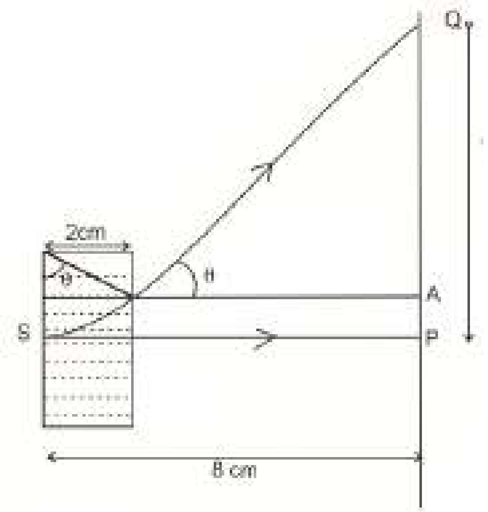
The radius of charged particles -

$$r=\frac{mv}{qB}=\frac{P}{qB}=\frac{\sqrt{2mK}}{qB}$$

m=mass of particle

P=momentum of particle

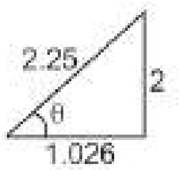
k= kinetic energy of the particle



$$R=\frac{mv}{qB}=\frac{\sqrt{2m(kE)}}{qB}$$

$$R=\frac{\sqrt{2\times 9.1\times 10^{-31}\times(100\times 1.6\times 10^{-19})}}{1.6\times 10^{-19}\times 1.5\times 10^{-3}}$$

R = 2.25 cm



$$\sin\theta=\frac{2}{R}=\frac{2}{2.25}$$

here $d=PA+AQ$

$$AP=R(1-\cos\theta)=2.25(1-\cos\theta)$$

$$AP\approx 1.22$$

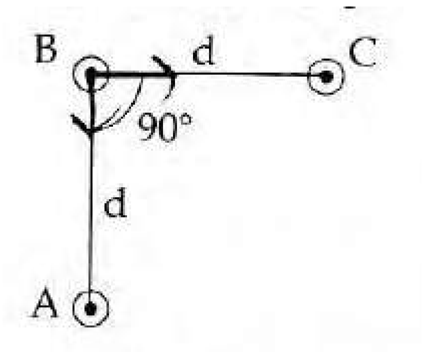
Similarly $\tan\theta=\dfrac{AQ}{8-2}=\dfrac{AQ}{6}$

$$AQ=6\tan\theta=11.7$$

so $d=AQ+PA$

$$d\approx 12.87cm$$

Q. 48 An arrangement of three parallel straight wires placed perpendicular to plane of paper carrying same current 'I' along the same direction is shown in Fig. Magnitude of force per unit length on the middle wire ' B' is given by :



Option 1:
$\frac{\mu_o i^2}{2\pi d}$

Option 2:
$\frac{2\mu_o i^2}{\pi d}$

Option 3:
$\frac{\sqrt{2}\mu_o i^2}{\pi d}$

Option 4:
$\frac{\mu_o i^2}{\sqrt{2}\pi d}$

Correct Answer:
$\frac{\mu_o i^2}{\sqrt{2}\pi d}$

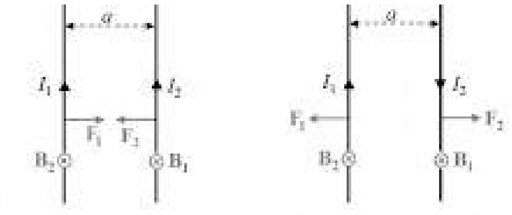
Solution:
As we learnt in

Force between two parallel current carrying conductors -

$$F = \frac{\mu}{4\pi}\frac{2I_{1I_2}}{a}l$$

$$\frac{F}{l} = \frac{\mu o}{4\pi}\frac{2I_{1I_2}}{a}$$

- wherein



I_1 and I_2 current carrying two parallel wires

a-seperation between two wires

Force between wires A and B = Force between B and C

$$F_{BC} = F_{AB} = \frac{\mu_0 I^2 l}{2\pi d}$$

$\vec{F_{AB}} \perp \vec{F_{BC}}$, net force on wire B

$$F_{net} = \sqrt{2} F_{BC} = \frac{\sqrt{2}\mu_0 I^2 l}{2\pi d} \Rightarrow \frac{F_{net}}{l} = \frac{\mu_0 I^2}{\sqrt{2}\pi d}$$

Q. 49 For CE transistor amplifier, the audio signal voltage across the collector resistance of $2 \mathrm{k}\Omega$ is 4 V. If the current amplification factor of the transistor is 100 and the base resistance is $1 \mathrm{k}\Omega$, then the input signal voltage is

Option 1:
10 mV

Option 2:
20 mV

Option 3:
30 mV

Option 4:
15 mV

Correct Answer:
20 mV

Solution:
As we learnt in

Transistor -

Three layered semiconducting device .

NPN or PNP

- wherein

1. Emitter is heavily doped
2. collector is moderately doped .
3. Base is lightly doped & very thin

$V_0 = 4V$

$R_L = 2K\Omega$

$B = 100$

$R_{in} = 1K\Omega$

$\frac{V_0}{V_i} = \beta.\frac{R_L}{R_{in}}$

$\Rightarrow \frac{4}{V_{in}} = 100 \times \frac{2K\Omega}{1K\Omega} = 200$

$\Rightarrow V_{in} = \frac{1}{50}V = 20mV$

Q. 50 The increase in the width of the depeltion region in a p-n junction diode is due to:

Option 1:
forward bias only

Option 2:
reverse bias only

Option 3:
both forward bias reverse bias

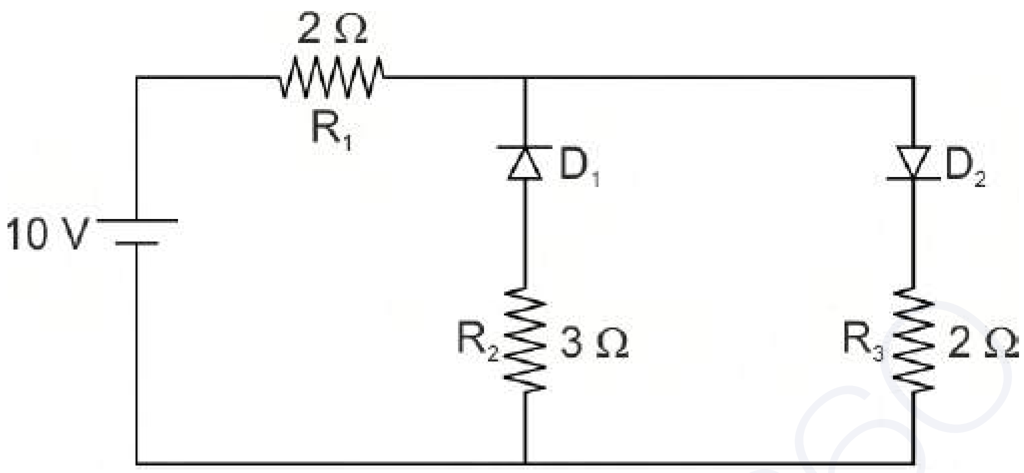
Option 4:
increase in forward current

Correct Answer:
reverse bias only

Solution:
In reverse biasing, the positive terminal of the battery is connected to the n-type whereas the negative terminal is connected to the p-type junction. So the positive terminal tend to pull the electrons (near to the depletion layer) in n-type towards itself whereas the negative terminal pulls the holes towards itself

which results in increase in the width of depletion layer.

Q. 51 The given circuit has two ideal diodes connected as shown in the figure below. The current flowing through the resistance R_1 will be



Option 1:
2.5 A

Option 2:
10.0 A

Option 3:
1.43 A

Option 4:
3.13 A

Correct Answer:
2.5 A

Solution:
As we learnt in

P -N junction as diode -

It is a one way device. It offers a low resistance when forward biased and high resistance when reverse biased.

- wherein

R = 0, Forward

R $\rightarrow$ ∞ Reverse

In the given figure D_1 is in Reverse biased mode. hence no current will flow through it while D_2 is in forward biased mode hence will offer Zero resistance

$\therefore R_{eff} = 2\Omega + 2\Omega = 4\Omega$

$$I = \frac{10V}{4\Omega} = 2.5A$$

Correct Option is 1.

Q. 52 A 220-volt input is supplied to a transformer. The output circuit draws a current of 2.0 ampere at 440 volts. If the efficiency of the transformer is 80%, the current drawn by the primary windings of the transformer is

Option 1:
5.0 ampere

Option 2:
3.6 ampere

Option 3:
2.8 ampere

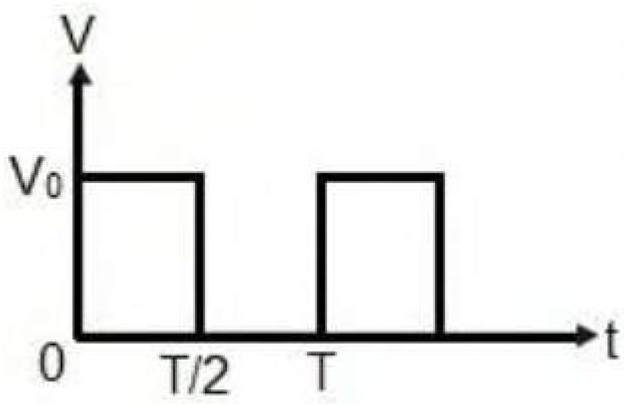
Option 4:
2.5 ampere

Correct Answer:
5.0 ampere

Solution:

$$\eta = \frac{V_s I_s}{\mathrm{V_p I_p}} = 0.8 \Rightarrow I_p = \frac{\mathrm{V_s I_s}}{\mathrm{V_p}\eta} = \frac{440 \times 2}{220 \times .8} = 5 \mathrm{~A}$$

Q. 53 The r.m.s. value of potential difference V shown in the figure is



Option 1:

$$\frac{\mathrm{V_0}}{2}$$

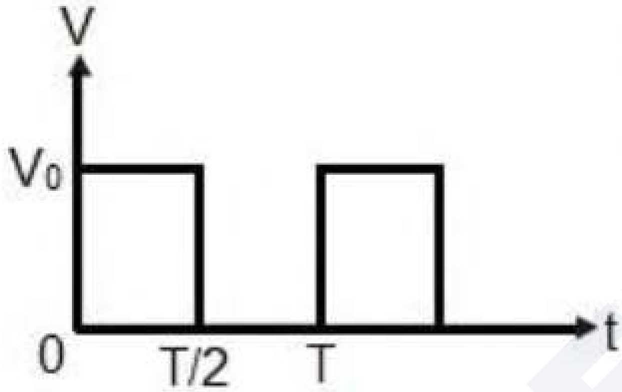
Option 2:
$\frac{V_0}{\sqrt{3}}$

Option 3:
V_0

Option 4:
$\frac{V_0}{\sqrt{2}}$

Correct Answer:
$\frac{V_0}{\sqrt{2}}$

Solution:



$V = V_0 \text{ for } 0 \leq t \leq \frac{T}{2}$
$V = 0 \text{ for } \frac{T}{2} \leq t \leq T$

$$V_{rms} = \left[\frac{\int_0^T V^2 dt}{\int_0^T dt}\right]^{1/2} = \left[\frac{\int_0^T V_0^2 dt + \int_{T/2}^T (0) dt}{\int_0^T dt}\right]^{1/2}$$

$$= \left[\frac{V_0^2}{T}[t]_0^{T/2}\right]^{1/2} = \left[\frac{V_0^2}{T}\left(\frac{T}{2}\right)\right]^{1/2} = \left[\frac{V_0^2}{2}\right]^{1/2}$$

$$V_{rms} = \frac{V_0}{\sqrt{2}}$$

Q. 54 A coil has resistance 30 ohm and inductive reactance 20 ohm at 50 Hz frequency. If an ac source of 200 volt, 100 Hz is connected across the coil, the current in the coil will be

Option 1:
$\frac{20}{\sqrt{13}} \mathrm{A}$

Option 2:
2.0 A

Option 3:
4.0 A

Option 4:
8.0 A

Correct Answer:
4.0 A

Solution:
$X_L = 2\pi f L$

$X_L \alpha f$

$\frac{X_{L_2}}{X_{L_1}} = \frac{f_2}{f_1} \Rightarrow X_{L_2} = 40\Omega$

$R = 30\Omega$

$Z = \sqrt{(30)^2 + (40)^2} = 50\Omega$

$I = \frac{V}{Z} = \frac{200}{50} = 4A$

Q. 55 A particle of mass m is dropped from a height h above the ground. At the same time another particle of the same mass is thrown vertically upwards from the ground with a speed of $\sqrt{2gh}$. If they collide head-on completely inelastically, the time taken for the combined mass to reach the ground, in units of $\sqrt{\frac{h}{g}}$ is :

Option 1:
$\sqrt{\frac{1}{2}}$

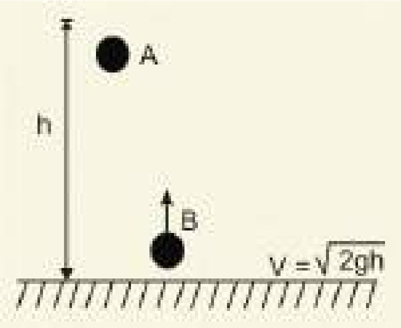
Option 2:
$\frac{1}{2}$

Option 3:
$\sqrt{\frac{3}{2}}$

Option 4:
$\sqrt{\frac{3}{4}}$

Correct Answer:
$\sqrt{\frac{3}{2}}$

Solution:



time for collision -

$$t_1 = \frac{h}{\sqrt{2gh}}$$

After t1 -

$$V_A = 0 - gt_1 = -\sqrt{\frac{gh}{2}}$$

and $V_B = \sqrt{2gh} - gt_1 = \sqrt{gh}\left[\sqrt{2} - \frac{1}{\sqrt{2}}\right]$

at the time of collision

$$\vec{P}_i = \vec{P}_f$$
$$\Rightarrow m\vec{V}_A + m\vec{V}_B = 2m\vec{V}_f$$
$$\Rightarrow -\sqrt{\tfrac{gh}{2}} + \sqrt{gh}\left[\sqrt{2} - \tfrac{1}{\sqrt{2}}\right] = 2\overline{V}_f$$
$$V_f = 0$$

and height from ground

$$= h - \frac{1}{2}gt_1^2 = h - \frac{h}{4} = \frac{3h}{4}$$

so time

$$= \sqrt{2 \times \frac{\left(\frac{3h}{4}\right)}{g}} = \sqrt{\frac{3h}{2g}}$$

Hence the correct option is (3).

Q. 56 A constant power delivering machine has towed a box, which was initially at rest , along a horizontal straight line. The distance moved by the box in time 't' is proportional to :

Option 1:
$t^{2/3}$

Option 2:
t

Option 3:
$t^{3/2}$

Option 4:
$t^{1/2}$

Correct Answer:
$t^{3/2}$

Solution:
$P = C$

$FV = C$

$M\frac{dV}{dt}V = C$

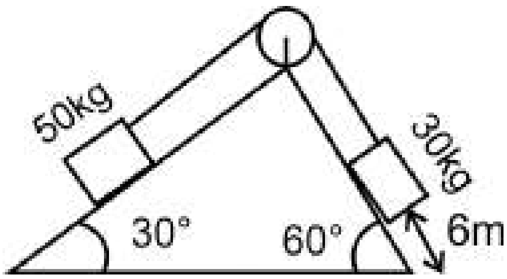
$\frac{V^2}{2} \propto t$

$V \propto t^{1/2}$

$\frac{dx}{dt} \propto t^{1/2}$

$x \propto t^{3/2}$

Q. 57 Two blocks of masses 50 Kg and 30 Kg connected by a massless string pass over a tight frictionless, pulley and rest on two smooth planes inclined at angles 30° and respectively with horizontal as shown in the figure. If the system is released from rest then find the time taken by 30 Kg block to reach the ground



Option 1:
20 sec

Option 2:
30 sec

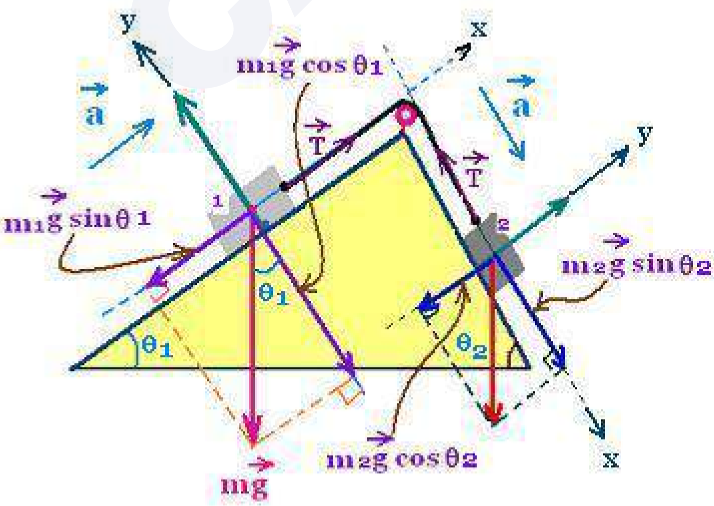
Option 3:
10 sec

Option 4:
50 sec

Correct Answer:
10 sec

Solution:
As we have learned

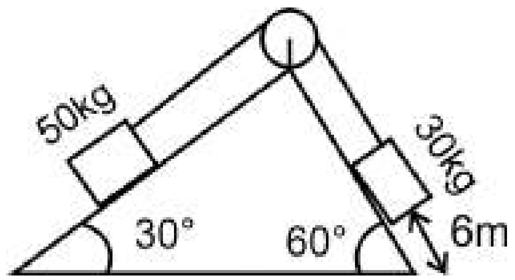
Double inclined plane with different angles -



- wherein

$$a=\frac{(m_2\,sin\theta_2-m_1\,sin\theta_1)g}{m_1+m_2}$$

$$T=\frac{m_1m_2(sin\theta_1+sin\theta_2)\;g}{m_1+m_2}$$



$$T-50g\sin 30=50a....(1)$$
$$30g\sin 60-T=30a.....(2)$$

Adding (1) and (2)

$$a=\frac{30g\sin 60-50g\sin 30}{80}=0.12m/s$$

Now

$$s=ut+1/2at^2$$
$$6=(1/2)\times 0.12\times t^2$$
$$t^2=\frac{6\times 2}{0.12}=100$$
$$t=10sec$$

Q. 58

A man weighs 80 kg. He stands on a weighing scale in a lift which is moving upwards with a uniform acceleration of 5m/s2. What would be the reading on the scale. (g=10m/s2)

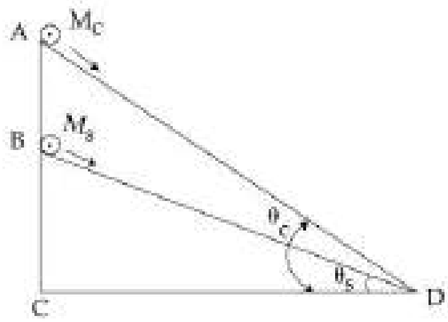
Correct Answer:
1200

Solution :
Reading of weighing scale = m(g+a) = 80(10 + 5) = 1200 N

Q. 59 A cylinder of mass M_c and sphere of mass M_s are placed at points A and B of two inclines, respectively.

(See Figure). If they roll on the incline without slipping such that their accelerations are the same, then

The ratio $\frac{\sin\theta_c}{\sin\theta_s}$ is



Option 1:
$\sqrt{\frac{8}{7}}$

Option 2:
$\sqrt{\frac{15}{14}}$

Option 3:
$\frac{8}{7}$

Option 4:
$\frac{15}{14}$

Correct Answer:
$\frac{15}{14}$

Solution:
Acceleration along inclined plane

$$a = \frac{g\sin\theta}{1+\frac{K^2}{R^2}}$$

For sphere

$$K^2 = 2/5R^2 \Rightarrow a_s = \frac{g\sin\theta_s}{1+2/5}$$

$$a_s = (5/7)g\sin\theta_s$$

For cylinder

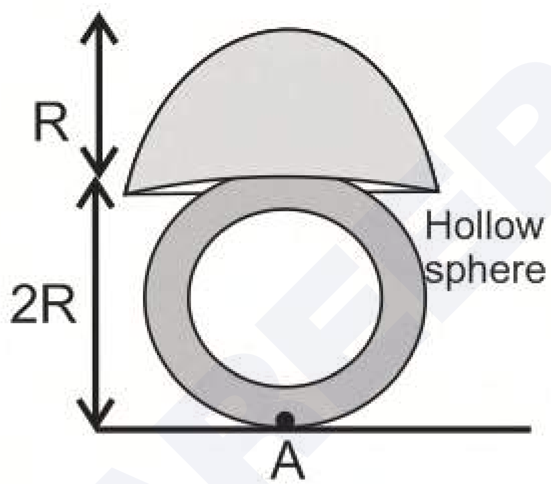
$$K^2 = 1/2R^2 \Rightarrow a_c = \frac{g\sin\theta_c}{1+1/2}$$

$$a_c = 2/3g\sin\theta_c$$

$$a_s = a_c \Rightarrow 5/7(g\sin\theta_s) = 2/3(g\sin\theta_c)$$

$$\frac{\sin\theta_c}{\sin\theta_s} = 15/14$$

Q. 60 A hollow hemisphere of mass m is placed on a hollow sphere of mass m. The center of mass of given arrangement from point A is at the height



Option 1:
$\frac{7R}{2}$

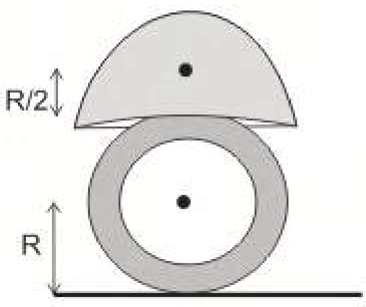
Option 2:
$\frac{7R}{4}$

Option 3:
$\frac{5R}{2}$

Option 4:
$\frac{5R}{4}$

Correct Answer:
$\frac{7R}{4}$

Solution:



$$y_{cm} = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2}$$

$$= \frac{mR + m(2R + \frac{R}{2})}{2m}$$

$$\frac{7R}{4}$$

Chemistry

Q. 1 A solution has a 1 : 4 mole ratio of pentane to hexane. The vapour pressure of the pure hydrocarbons at 20°C are 440 mm Hg for pentane and 120 mm Hg for hexane. The mole fraction of pentane in the vapour phase would be:

Correct Answer:
0.478

Solution:

$$\mathrm{P_p = P_p^0 x_p} = 440 \times \frac{1}{5} = 88$$

$$\mathrm{P_h = P_h^0 x_h} = 120 \times \frac{4}{5} = 96$$

$$\mathrm{P_T = P_p + P_h} = 88 + 96 = 184$$

$$\mathrm{P_p^0 x_p = y_p P_T \Rightarrow y_p} = \frac{88}{184}$$

$y_P = 0.478$

Q. 2 Benzene and toluene form nearly ideal solutions. At 20°C, the vapour pressure of benzene is 75 torr and that of toluene is 22 torr. The partial vapour pressure of benzene at 20°C for a solution containing 78g of benzene and 46g of toluene in torr is:

Correct Answer:
50

Solution:

$$P_B = P_B^0 X_B$$

$$\therefore P_B = \frac{\frac{78}{78}}{\frac{78}{78} + \frac{46}{92}} \times 75$$

$$\therefore P_B = 50\ torr$$

Q. 3 Which of the following mixture form maximum boiling azeotrope?

Option 1:
$H_2O + Benzene$

Option 2:
Benzene + Toluene

Option 3:
C_2H_5OH

Option 4:
$H_2O + HNO_3$

Correct Answer:
$H_2O + HNO_3$

Solution:
Solution showing negative deviation from Raoult's law form maximum boiling azeotropes. The mixture of H_2O and HNO_3 form non-ideal solution which shows negative deviation form Raoult's law.

Therefore, **option(4) is correct**

Q. 4 Which of the following is true about chemical cells?

Option 1:
Convert chemical energy to electrical energy

Option 2:
most batteries are chemical cells

Option 3:
i and ii

Option 4:
none

Correct Answer:
i and ii

Solution:
As we learnt

Chemical Cells -

The cells in which electrical energy is produced from the energy change accompanying chemical reactions or a physical process are known as chemical cells.

-

Chemical cells convert chemical energy to electrical energy

Hence, the option number (3) is correct.

Q. 5 What is the value of the equilibrium constant for a reaction when $\Delta G^0 = -15.38 kj$?

Option 1:
$10^{2.695}$

Option 2:
$e^{2.695}$

Option 3:
$e^{1.738}$

Option 4:
$10^{1.738}$

Correct Answer:
$10^{2.695}$

Solution:
As we have learned

Standard Gibbs Energy -

$$\Delta_r G^0 = -RT\ ln\ k$$

- wherein

K= equilibrium constant of the reaction

$$\Delta G^0 = -RTlnK$$

$$-15.38 = -8.314 * 298 * 2.303logK$$

$$K = 10^{2.695}$$

Q. 6 Iodine reacts with concentrated HNO_3 to yield Y along with other products. The oxidation state of iodine in Y, is :

Option 1:
5

Option 2:
3

Option 3:
1

Option 4:
7

Correct Answer:
5

Solution:
Oxidation state of oxygen family -

Oxygen shows -2,+2 and -1

oxidation states other elements show +2,+4 and +6 oxidation states

-

Rules for Oxidation Number -

The oxidation number of oxygen in most compounds is -2. There are two exceptions. In peroxide (H_2O_2; Na_2O_2), the oxidation number is -1.

- wherein

As we have learned in oxidation number the reaction is

The reaction occurs as follows:

$$I_2 + 10HNO_3 \rightarrow 2HIO_3 + 10NO_2 + 4H_2O$$

Y here is HIO_3 and the oxidation state of iodine in HIO_3 is given below:

$$1 + x + (3 \times (-2)) = 0$$

$$x = 6 - 1 = 5$$

Therefore, **Option(1) is correct**

Q. 7 20.0 g of a magnesium carbonate sample decomposes on heating to give carbon dioxide and 8.0 g magnesium oxide. What will be the percentage purity of magnesium carbonate in the sample ?

(At. Wt. : Mg = 24)

Option 1:
75

Option 2:
96

Option 3:
60

Option 4:
84

Correct Answer:
84

Solution:
As we learnt in

Number of Moles = given mass of substance/ molar mass of the substance

$$MgCO_3(s) \rightarrow MgO(s) + CO_2(g)$$

$$\text{Moles of } MgCO_3 = \frac{\text{mass in grams}}{\text{molar mass}} \text{Mole of } MgCO_3 = \frac{20}{84} = 0.238 \text{ mol}$$

By the stoichiometry of the reaction above, we know that 1 mol of the $MgCO_3$ gives 1 mol of MgO.

moles of MgO obtained

$$= \frac{8}{40} = 0.2$$

Moles of MgO that should have been obtained in a pure sample = 0.238

$\therefore$ moles of $MgCO_3$ in the sample

= moles of MgO in the sample

= 0.2

$$\therefore \% \text{ Purity} = \frac{0.2}{0.238} = 84\%$$

Q. 8 The number of water molecules is maximum in :

Option 1:
18 molecules of water

Option 2:
1.8 gram of water

Option 3:
18 gram of water

Option 4:
18 moles of water

Correct Answer:
18 moles of water

Solution:
Mole Concept -

one mole = 6.0221367×10^{23} = N_A

Let us check the number of molecules in each option.

18 molecules of water have 18 molecules

1.8 grams of water has $\frac{1.8}{18} \times \mathrm{N_A} = 0.1\ \mathrm{N_A}$ molecules

18 grams of water has $\frac{18}{18} \mathrm{N_A} = \mathrm{N_A}$ molecules

18 moles of water has 18 N_A molecules

Therefore, the correct option is (4).

Q. 9 On increasing the temperature, the value of K_H will

Option 1:
Increase

Option 2:
Decrease

Option 3:
Remain Same

Option 4:
Either Increase or Decrease

Correct Answer:
Increase

Solution:
Henry's constant depends on temperature and nature of gas.

K_H increases on increasing temperature.

Therefore, **option (1) is correct**

Q. 10 On increasing pressure of a gas, solubility of gas in liquid:

Option 1:
Decreases

Option 2:
Increases

Option 3:
First Increases then decreases

Option 4:
None

Correct Answer:
Increases

Solution:
As we have learnt,
Solubility of Gases on Temperature and Pressure -
As $P_{gas} \uparrow \ solubility \uparrow \ and\ as\ temperature \uparrow \ solubility \downarrow \ because\ K_H \uparrow$

On increasing pressure, the collision of gas molecules on the liquid surface increases thus solubility of the gas in liquid increases.

Therefore, **option(2) is correct**

Q. 11 The $\mathrm{K_H}$ value of $\mathrm{N_2}$, He, $\mathrm{H_2}$ & $\mathrm{O_2}$ are 80, 145, 75, and 35 Kbar respectively, the least soluble gas in water is:

Option 1:
$\mathrm{N_2}$

Option 2:
$\mathrm{H_2}$

Option 3:
He

Option 4:
$\mathrm{O_2}$

Correct Answer:
He

Solution:
Gases having more K_H values will have less solubility.

Among the given gases 'He' has the highest K_H value.

Therefore, **option(3) is correct**

Q. 12 Which of the following forces stabilize the tertiary structure of proteins i.e. further folding of the secondary structure?

Option 1:
disulphide linkages

Option 2:
hydrogen bond

Option 3:
electrostatic forces of attraction

Option 4:
all of these

Correct Answer:
all of these

Solution:
As we have learnt,

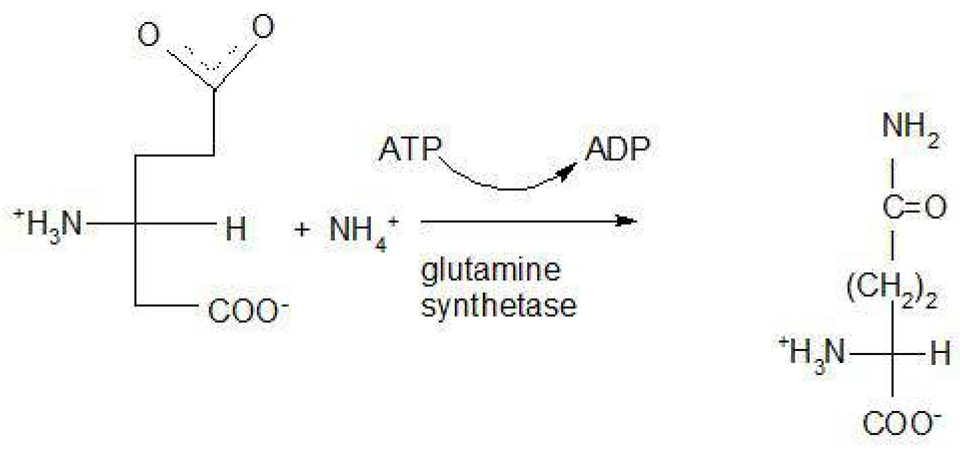
Tertiary proteins are obtained when polypeptides undergo further folding and coiling of secondary protein

Structure is stabilized by :

1. Ionic bond
2. Disulphide bond
3. $H-$ bond
4. Vander waals forces
5. Electrostatic forces

Therefore, **Option(4) is correct.**

Q. 13



The above reaction is catalysed by which class of enzymes ?

Option 1:
Ligases

Option 2:
Hydrolases

Option 3:
Lyases

Option 4:
Oxidareductases

Correct Answer:
Ligases

Solution:
As we have learned

Ligase -

Enzymes are responsible for linking together of two molecules into one at the expense of energy.

- wherein

Eg. Synthetase

Ligases remove the elements of water from two fundamental groups to form a single bond.

Hence, the option number (1) is correct.

Q. 14 Which of the following are water-soluble vitamins?

a) Vitamin A

b) Vitamin B

c) Ascorbic acid

d) Pyridoxine

e) Vitamin D

Option 1:
c,d

Option 2:
a,b,e

Option 3:
b,c

Option 4:
c,e

Correct Answer:
b,c

Solution:
As we have learned

Water-soluble vitamins -

They are soluble in water but not in fats or oils.

Vitamin B groups, Vitamin C are water-soluble vitamins.

Pyridoxine (Vitamin B) and ascorbic acid (Vitamin C) are water-soluble.

Hence, the option number (3) is correct.

Q. 15 For the cell $Zn(s)\left|Zn^{2+}(aq)\right|\left|M^{x+}(aq)\right|M(s)$, different half cells and their standard electrode potential are given below :

$M^{x+}(aq)/M(s)$	$Au^{3+}(aq)/Au(s)$	$Ag^{+}(aq)/Ag(s)$	$Fe^{3+}(aq)/Fe^{2+}(aq)$	$Fe^{2+}(aq)/Fe(s)$
$E^{\circ}_{M^{x+}/M}/(V)$	1.40	0.80	0.77	-0.44

If $E^0_{zn^{2+}/zn} = -0.76V$, which cathode will give a maximun value of E^0_{cell} per electron transfered?

Option 1:
Au^{3+}/Au

Option 2:
Fe^{3+}/Fe^{2+}

Option 3:
Fe^{2+}/Fe

Option 4:
Ag^{+}/Ag

Correct Answer:
Ag^{+}/Ag

Solution:
We know that

$$\Delta_r G^{\ominus} = \Delta_1 G^{\ominus} + \Delta_2 G^{\ominus}$$

And

$$\Delta_r G^{\ominus} = -nFE^{\ominus}_{(cell)}$$

from above

$$nE^{o}_{(cell)} = n_1 E^{o}_1 + n_2 E^{o}_2$$

For Ag^{2+}/Ag

$$Zn(s) + 2Ag^{+} \rightarrow Zn^{2+} + 2Ag$$

Electron transfer :

n (reaction) = 2 , n_1(Zn) = 2, n_2(Ag) = 1

and

$\mathrm{Given}\ \mathrm{E}^{\mathrm{o}}_{\mathrm{Zn^{2+}/Zn}} = -0.76$

$\mathrm{So,}\ \mathrm{E}^{\mathrm{o}}_{\mathrm{Zn/Zn^{2+}}} = 0.76$

After putting the value:

$2 \times \mathrm{E}^{\mathrm{o}}_{(\mathrm{cell})} = 2 \times (0.76) + 1 \times 0.80$

$2 \times \mathrm{E}^{\mathrm{o}}_{(\mathrm{cell})} = 2.32$

$\mathrm{E}^{\mathrm{o}}_{(\mathrm{cell})} = 1.16$

After calculating the other 1.16 will be the maximum.

Therefore,option(4) is correct.

Q. 16 The standard Gibbs energy for the given cell reaction in $KJ\ mol^{-1}$ at $298\ K$ is :

$Zn\,(s) + Cu^{2+}\,(aq) \rightarrow Zn^{2+}\,(aq) + Cu\,(s)\,,$

$E^0 = 2\ V\ at\ 298\ K$

(Faraday's constant, $F = 96000\ C\ mol^{-1}$)

Option 1:
-384

Option 2:
384

Option 3:
192

Option 4:
-192

Correct Answer:
-384

Solution:
$\Delta G^{\circ} = -nFE^{\circ}_{cell} = -2 \times 96500 \times 2.0$

$$\Delta G^{\circ} = -386 \times 10^{3}$$

$$\Delta G^{\circ} = -386\ Kj/mol$$

$$\Delta G^{\circ} \simeq -384\ Kj/mol$$

Hence, **the option number (1) is correct.**

Q. 17 An example of a disproportionation reaction is :

Option 1:
$2CuBr \rightarrow CuBr_2 + Cu$

Option 2:
$2KMnO_4 \rightarrow K_2MnO_4 + MnO_2 + O_2$

Option 3:
$2MnO_4^- + 10I^- + 16H^+ \rightarrow 2Mn^{2+} + 5I_2 + 8H_2O$

Option 4:
$2NaBr + CI_2 \rightarrow 2NaCI + Br_2$

Correct Answer:
$2CuBr \rightarrow CuBr_2 + Cu$

Solution:
Disproportionation reactions are those reactions where oxidation & reduction occur on the same species, simultaneously

$$2CuBr \rightarrow CuBr_2 + Cu$$

the oxidation state of Cu : $+1$ $+2$ 0

in KMnO4 reaction, Mn is going +7 to +6 and +4, only reduction.

In MnO^{4-} and I^- reaction, Mn is going to +7 to +2 and I is going to -1 to zero, both reduction and oxidation but with different atoms.

In NaBr and Cl_2 reaction, both reduction and oxidation but with different atoms.

Therefore, option (1) is correct

Q. 18 $[Co_2(CO)_8]$ displays :

Option 1:
one Co−Co bond, six terminal CO and two bridging CO

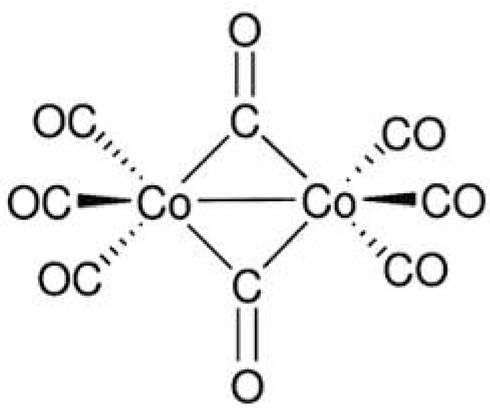
Option 2:
one Co−Co bond, four terminal CO and four bridging CO

Option 3:
no Co−Co bond, six terminal CO and two bridging CO

Option 4:
no Co−Co bond, four terminal CO and four bridging CO

Correct Answer:
one Co−Co bond, six terminal CO and two bridging CO

Solution:
The structure of $[Co_2(CO)_8]$



In $[CO_2(CO_8)]$ one Co-Co bond, six terminal bond and two bridge CO are present

Q. 19 In the extraction of which of the following, complex ion is formed:

Option 1:
Cu

Option 2:
Ag

Option 3:
Fe

Option 4:
Na

Correct Answer:
Ag

Solution:
The extraction of Silver is done using the McArthur Forest Cyanide method.

$$Ag_2S + 4NaCN \rightleftharpoons 2Na[Ag(CN)_2] + Na_2S$$

Sodium dicyano argentate

$$2Na[Ag(CN)_2] + Zn \rightarrow Na_2[Zn(CN)_4] + 2Ag$$

Sodium tetracyano zincate

Therefore, **option (2) is correct.**

Q. 20 Types of isomerism shown by $[\mathrm{Cr(NH_3)_5NO_2]Cl_2}$

Option 1:
Optical

Option 2:
Co-ordination

Option 3:
Geometrical

Option 4:
Linkage

Correct Answer:
Linkage

Solution:
As we have learnt,

$\mathrm{NO_2}$ is an ambidentate ligand and can be attached from both the N site as well as the O site in the form of $(-\mathrm{NO_2})$ and $(-\mathrm{ONO})$ respectively.

Hence, the given complex can show Linkage isomerism

Therefore, **option (4) is correct.**

Q. 21 The freezing point of a diluted milk sample is found to be -0.2^0C, while it should have been -0.5^0C for pure milk. How much water has been added to pure milk to make the diluted sample ?

Option 1:
2 cups of water to 3 cups of pure milk.

Option 2:
3 cups of water to 2 cups of pure milk.

Option 3:
1 cup of water to 3 cups of pure milk.

Option 4:
1 cup of water to 2 cups of pure milk.

Correct Answer:
3 cups of water to 2 cups of pure milk.

Solution:
Freezing point of milk $= -0.5^0C \because \Delta T_f = 0.5^0C$

Freezing point of milk (diluted)$= -0.2^0C \because \Delta T_f = 0.2^0C$

$$\frac{(\Delta T_f)\, i}{(\Delta T_f)\, ii} = \frac{0.5}{0.2} = \frac{K_f m}{K_f m} = \frac{x\,(mole) \times weight\,(2)}{weight_{(1)\times} x\,(mole)}$$

$$W_2 = \frac{5}{2} W_1$$

Q. 22 The percentage of pyridine (C_5H_5N) that forms pyridinium ion ($C_5H_5N{*}H$) in a 0.10 *M* aqueous pyridine solution (K_b for $C_5H_5N = 1.7 \times 10^{-9}$) is:

Option 1:
0.0060%

Option 2:
0.013%

Option 3:
0.77%

Option 4:
1.6%

Correct Answer:
0.013%

Solution:

$$C_5H_5N + H_2O \rightleftharpoons C_5H_5\overset{\oplus}{N}H + \overset{-}{O}H$$

C	—	O	O
$C-C$	—	$C\alpha$	$C\alpha$

$$\mathrm{Kb}=\frac{\mathrm{C}\alpha^2}{1-\alpha}(1>>\alpha)$$

$$\mathrm{Kb}=\mathrm{C}\alpha^2$$

$$\alpha=\sqrt{\frac{\mathrm{Kb}}{\mathrm{C}}}=\sqrt{\frac{1.7\times10^{-9}}{0.1}}$$

$$\alpha=0.013\times10^{-2}$$

% pyridine that form pyridinium

$$\mathrm{ion}=\frac{\mathrm{C}\alpha}{\mathrm{C}}\times100=0.013\%$$

Correct answer is option is **(2)**

Q. 23 The solubility of $AgCl(s)$ with solubility product in $0.1MNaCl$ solution would be:

Option 1:
$1.26\times10^{-5}M$

Option 2:
$1.6\times10^{-9}M$

Option 3:
$1.6\times10^{-11}M$

Option 4:
zero

Correct Answer:
$1.6\times10^{-9}M$

Solution:
$AgCl\,(s)\rightleftharpoons Ag^{+}+Cl^{-}\ \ ,K_{sp}=1.6\times10^{-10}$

Now $[Cl^{-}]=0.1M$, as $NaCl$ is a strong electrolyte.
$$[Ag^{+}]=s\quad[Cl^{-}]=0.1$$
$$s\times0.1=1.6\times10^{-10}as\ \ s<<1$$
$$s=1.6\times10^{-9}$$

The correct answer is option 2

Q. 24 Which of the following fluoro-compounds is most likely to behave as a Lewis base?

Option 1:
BF_3

Option 2:
PF_3

Option 3:
CF_4

Option 4:
SiF_4

Correct Answer:
PF_3

Solution:
A lewis base is a compound that can donate a lone pair of electrons while a lewis acid accepts a lone pair of electrons.

PF_3 behave like lewis's base because it has one pair of electrons, while BF_3 can accept lone pair of an electron in $2p$ and SiF_4 in vacant $3d$ orbitals and act as lewis acid.

CF_4 has complete octet and acts as none.

Henec, the correct answer is option 2.

Q. 25 Given that the equilibrium constant for the reaction

$$2SO_2(g) + O_2(g) \rightarrow 2SO_3(g)$$

has a value of 278 at a particular temperature. What is the value of the equilibrium constant for the following reaction at the same temperature?

$$SO_3(g) \rightarrow SO_2(g) + 1/2O_2(g)$$

Option 1:
3.6×10^{-3}

Option 2:
6.0×10^{-2}

Option 3:
1.3×10^{-5}

Option 4:
1.8×10^{-3}

Correct Answer:
6.0×10^{-2}

Solution:
The equilibrium constant for the reverse reaction is the inverse of the equilibrium constant for the reaction in the forward direction.

$$K_c' = \frac{1}{K_c}$$

K_c' is the equilibrium constant for the reverse direction.

$$2SO_2(g) + O_2(g) \rightleftharpoons 2SO_3(g), \quad K = 278$$

$\therefore$ for $SO_3(g) \rightleftharpoons SO_2(g) + \frac{1}{2}O_2(g), \quad K' = \left(\frac{1}{K}\right)^{1/2}$

$$K' = \left(\frac{1}{278}\right)^{1/2} = 6 \times 10^{-2}$$

Therefore, **Option(2) is correct.**

Q. 26 Which of the following is Turnbull's blue?

Option 1:
Ferricyanide

Option 2:
Ferrous ferricyanide

Option 3:
Ferrous cyanide

Option 4:
Ferri ferrocyanide

Correct Answer:
Ferrous ferricyanide

Solution:
Turnbull's blue has the formula $Fe_3[Fe(CN)_6]_2$

It is named as Ferrous ferricyanide

Hence, option number (2) is correct.

Q. 27 Which of the following can be detected by the borax bead test ?

Option 1:
Ni^+

Option 2:
Co^{2+}

Option 3:
Pb^{2+}

Option 4:
Both a and b

Correct Answer:
Both a and b

Solution:
Borax bead -

Used for the detection of coloured basic radicals in borax bead test.

$$NiO + B_2O_3 \rightarrow Ni(BO_2)_2$$

$$CoO + B_2O_3 \rightarrow Co(BO_2)_2$$

Therefore,**option (4) is correct.**

Q. 28 The major product of reaction of alcoholic silver nitrite with ethyl bromide is:

Option 1:
Ethane

Option 2:
Ethene

Option 3:
Ethylalcohol

Option 4:
Nitroethane

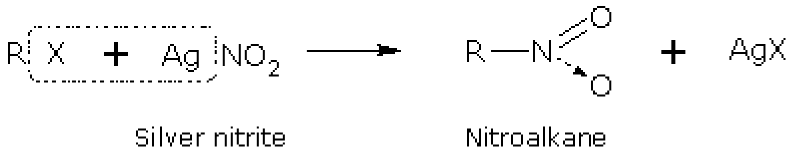
Correct Answer:
Nitroethane

Solution:

Reaction of alkyl halide with AgNO_{2} -

Alkyl halides when treated with AgNO2 in aqueous ethanol, Nitro alkane is obtained as a product.

- wherein



Therefore, **Option(4) is correct**

Q. 29 For the reaction, A(g) + B(g) → C(g) + D(g), H^0 and S^0 are, respectively, −29.8 kJ mol^{-1} and −0.100 kJ K^{-1} mol^{-1} at 298 K. The equilibrium constant for the reaction at 298 K is :

Option 1:
1.0×10^{-10}

Option 2:
1.0×10^{10}

Option 3:
10

Option 4:
1

Correct Answer:
1

Solution:
$$\Delta G^{\circ} = \Delta H^{\circ} - T.\Delta S^{\circ}$$
$$= -29.8 + 298 \times (0.1)$$
$$= -29.8 + 29.8$$
$$\because \Delta G^{\circ} = 0$$

$$\text{apply relation between}$$
$$\Delta G^{\circ} = -RT \ln K_{eq}$$
$$\therefore K_{eq} = 1$$

Q. 30 The heats of combustion of carbon and carbon monoxide are −393.5 and −283.5 kJ mol^{-1}, respectively. The heat of formation (in kJ) of carbon monoxide per mole is:

Correct Answer:
-110

Solution:
$$\mathrm{C_s + O_{2g} \rightarrow CO_{2g} \quad \Delta H = -393.5 KJmol^{-1}}$$

$$CO_{(g)} + \frac{1}{2}O_{2\,(g)} \rightarrow CO_{2\,(g)} \quad \Delta H = -283.5 kJmol^{-1}$$

$$C_{(s)} + \frac{1}{2}O_{2\ (g)} \rightarrow CO_{(g)}$$

$$\therefore \Delta H = -393.5 + 283.5$$

$$= -110.0\ kJmol^{-1}$$

Q. 31 The heat of atomization of methane and ethane are 360 kJ/mol and 620 kJ/mol, respectively. The longest wavelength of light capable of breaking the C - C bond is

$$\left(\text{Avogadro number} = 6.02 \times 10^{23}, h = 6.62 \times 10^{-34}\,\text{Js}\right):$$

Option 1:
$1.49 \times 10^3 nm$

Option 2:
$2.48 \times 10^3 nm$

Option 3:
$2.48 \times 10^4 nm$

Option 4:
$1.49 \times 10^4 nm$

Correct Answer:
$1.49 \times 10^3 nm$

Solution:
The heat of the atomization of methane is 360 kJ/mol.
This corresponds to the breaking of four C−H bonds.

Hence, the heat corresponding to the breaking of one C−H bond is

$$\frac{360}{4} = 90\ \text{kJmol}$$

The heat of atomization of ethane is 620 kJ/mol.

This corresponds to the breaking of one C−C bond and six C−H bonds.

The heat corresponding to the breaking of on C-C bond will be 620−6(90)=80 kJ/mol
This corresponds to 1 mole. For one single bond, the required energy that can be obtained by dividing with Avogadro's number is given by

$$\text{E} = \frac{80 \times 1000}{6.02 \times 10^{23}}$$

The energy is related to wavelength by relation

$$E = \frac{hc}{\lambda}$$

Substitute values in the above expression.

$$\frac{80 \times 1000}{6.02 \times 10^{23}} = \frac{6.62 \times 10^{-34} \times 3 \times 10^{8}}{\lambda}$$

$$\text{Hence, } \lambda = 1.49 \times 10^{-6} \quad \text{m} = 1.49 \times 10^{3} \quad \text{nm}$$

Q. 32 Which one of the following pairs of species have the same bond order?

Option 1:
CO, NO

Option 2:
O_2, NO^+

Option 3:
CN^-, CO

Option 4:
N_2, O_2^-

Correct Answer:
CN^-, CO

Solution:
Bond Order -

Bond order is defined as one half the difference between the number of electrons present in the bonding and the antibonding orbitals.

- wherein

$$Bond\,Order = \frac{N_b - N_a}{2}$$

$$\text{Bond order} = \frac{1}{2}(N_b - N_a)$$

O_2 : N_b = 10, N_a = 6, Bond order = 2

NO^+ : N_b = 10, N_a = 4, Bond order = 3

CN^- : N_b = 10, Na = 4, Bond order = 3

CO : N_b = 10, Na = 4, Bond order = 3

N_2 : N_b = 10, Na = 4, Bond order = 3

O^-_2 : N_b = 10, Na = 7, Bond order = 1.5

NO : N_b = 10, Na = 5, Bond order = 2.5

Q. 33 Pick out the correct statement with respect to $[Mn(CN)_6]^{3-}$:

Option 1:
It is sp^3d^2 hybridised and octahedral

Option 2:
It is sp^3d^2 hybridised and tetrahedral

Option 3:
It is d^2sp^3 hybridised and octahedral

Option 4:
It is dsp^2 hybridised and square planar

Correct Answer:
It is d^2sp^3 hybridised and octahedral

Solution:
Shape of molecules -

S. No.	Number of electron pairs	Arrangement of electron pairs	Molecular geometry	Examples
1.	2	180° Linear	B—A—B Linear	$BeCl_2$, $HgCl_2$
2.	3	120° Trigonal planar	Trigonal planar	BF_3
3.	4	109.5° Tetrahedral	Tetrahedral	CH_4, $NH_4^{\oplus}$
4.	5	90° 120° Trigonal bipyramidal	Trigonal bipyramidal	PCl_5
5.	6	90° 90° Octahedral	Octahedral	SF_6

-

$[Mn(CN)_6]^{3-} is\ d^2sp^3$hybridised and it is octahedral.

Q. 34 Consider the molecules CH_4, NH_3 and H_2O. Which of the given statements is false?

Option 1:
The H - C - H bond angle in CH_4, the H - N - H bond angle in NH_3, and the H - O - H bond angle in H_2O are all greater than 90°.

Option 2:
The H - O - H bond angle in H_2O is larger than the H - C - H bond angle in CH_4.

Option 3:
The H - O - H bond angle in H_2O is smaller than the H - N - H bond angle in NH_3.

Option 4:
The H - C - H bond angle in CH_4 is larger than the H - N - H bond angle in NH_3.

Correct Answer:
The H - O - H bond angle in H_2O is larger than the H - C - H bond angle in CH_4.

Solution:
VSEPR Theory -

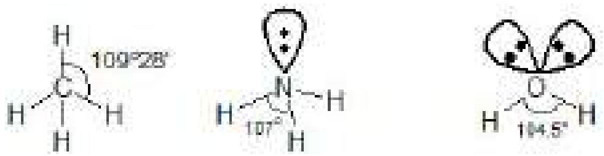
1. The shape of the molecule is determined by repulsions between all of the electron pair present in valence shell.

2. Order of repulsion

$$lone\,pair - Lone\,pair > Lone\,pair - Bond\,pair > Bond\,pair - bond\,pair$$

3 Repulsion among the bond pair is directly proportional to the bond order and electronegativity difference between the central atom and the other atom.

-



From the above fugure it is clear that H—O—H bond angle in H_2O is smaller than H—C—H bond angles in CH_4.

Therefore, **Option(2) is correct.**

Q. 35 Of the following outer electronic configurations of atoms, the highest oxidation state is achieved by which one of them?

Option 1:
$(n-1)d^8ns^2$

Option 2:
$(n-1)d^5ns^1$

Option 3:
$(n-1)d^3ns^2$

Option 4:
$(n-1)d^5ns^2$

Correct Answer:
$(n-1)d^5ns^2$

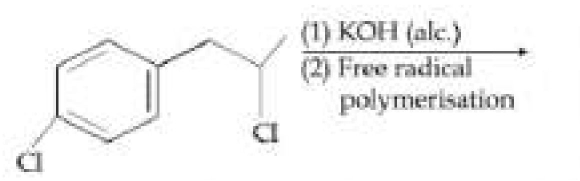
Solution:
$(n-1)d^8ns^2\ can\ achieve\ +40.5\ eg.\ Ni$

$(n-1)d^5ns^1 - - -'' - - - +60.5\ eg\ C_s$

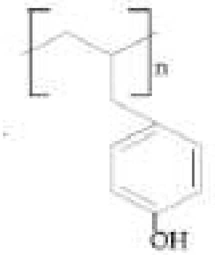
$(n-1)d^3ns^2 ---'' ---+50.5\ eg\ V$

$(n-1)d^5ns^2 ---'' ---+70.5\ eg\ Mn$

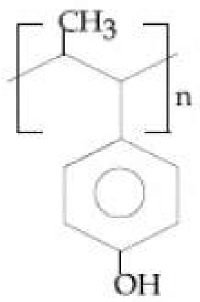
Q. 36 The major product of the following reaction is :



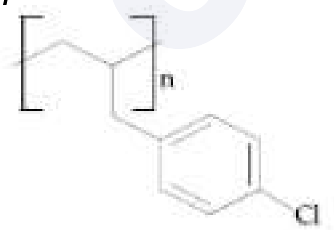
Option 1:



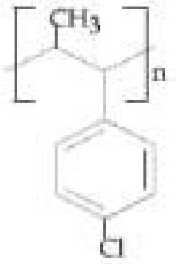
Option 2:



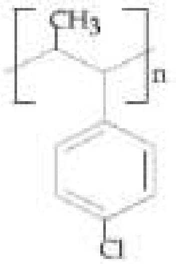
Option 3:



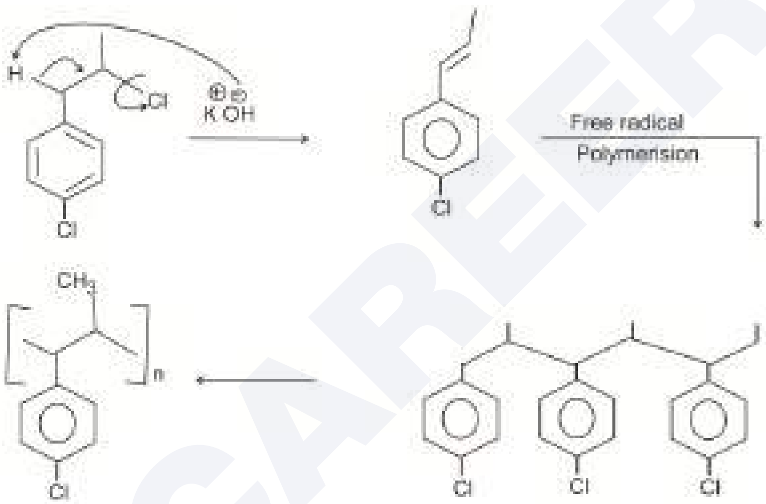
Option 4:



Correct Answer:



Solution:
The given substrate undergoes dehydrohalogenation to form an alkene and the subsequent polymerisation reaction occurs as:



It is to be mentioned that the chlorine attached to the Benzene ring will not undergo substitution with KOH very easily due to partial double bond character between the benzene ring and the chloro group.

Hence, the correct answer is Option (4)

Q. 37 The following reaction is performed at 298 K .

$$2NO(g) + O_2(g) \rightleftharpoons 2NO_2(g)$$

The standard free energy of formation of NO(g) is 86.6 kJ/mol at 298 K.

What is the standard free energy of formation of NO_2(g) at 298 K ? $\left(K_p = 1.6 \times 10^{12}\right)$

Option 1:
$R(298)\ ln(1.6 \times 10^{12}) - 86600$

Option 2:
$86600 + R(298)\ ln(1.6 \times 10^{12})$

Option 3:
$86600 - \frac{ln(1.6 \times 10^{12})}{R(298)}$

Option 4:
$0.5\left[2 \times 86,600 - R(298) ln(1.6 \times 10^{12})\right]$

Correct Answer:
$0.5\left[2 \times 86,600 - R(298) ln(1.6 \times 10^{12})\right]$

Solution:
As we learnt in

$2NO(g) + O_2(g) \leftrightharpoons 2NO_2(g)$

Given that $\Delta G_{f^0}(NO) = 86.6kjmol^{-1}$

and we know that $\Delta G_{f^\circ}(O_2) = 0$

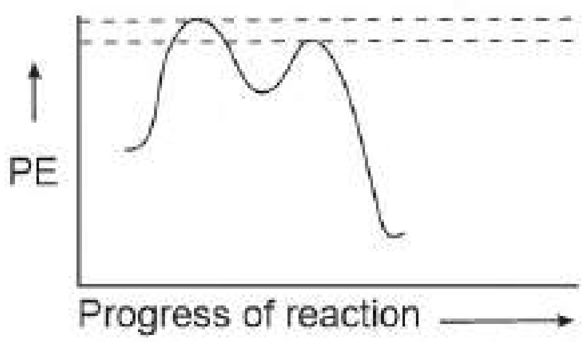
$K = 1.6 \times 10^{12}$

$\Delta - RTlnk = 2\Delta G_{f^\circ}(NO_2) - [2 \times 86600 + 0]$

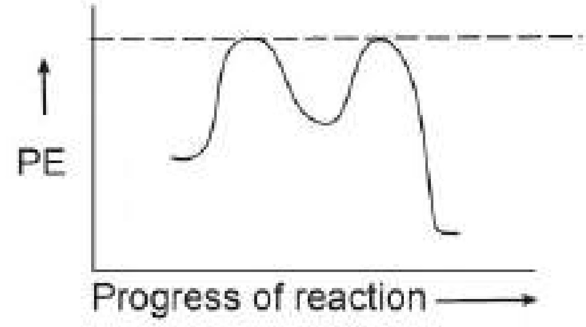
$\Delta G_{f^\circ}(NO_2) = 0.5\left[2 \times 86600 - R(298) ln(1.6X10^{12})\right]$

Q. 38 Which of the following potential energy (PE) diagram represents the SN_1 reaction?

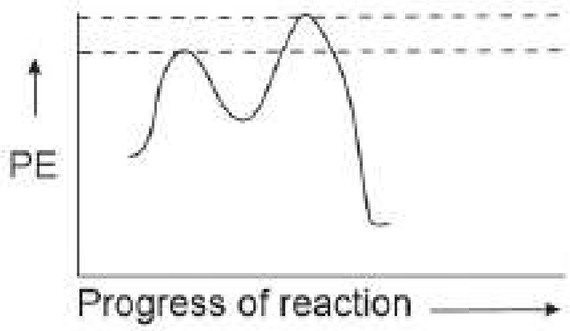
Option 1:



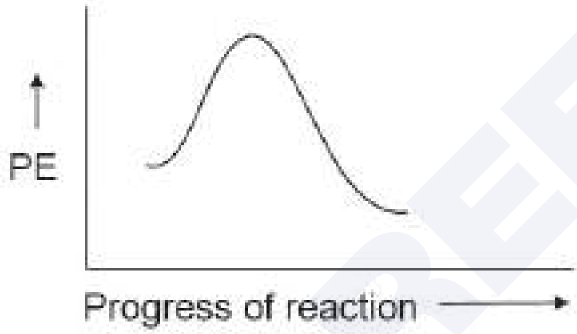
Option 2:



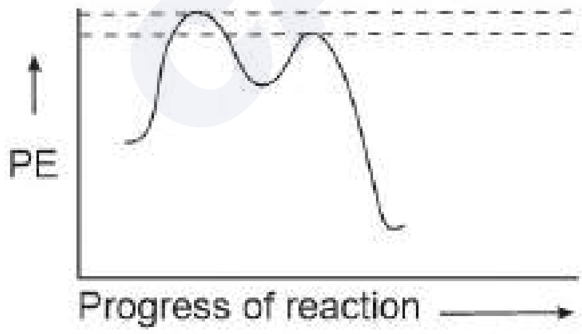
Option 3:



Option 4:

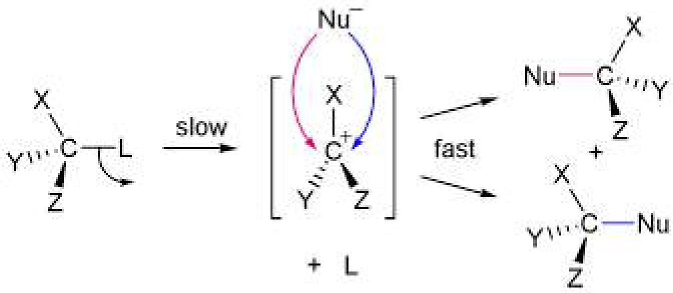


Correct Answer:



Solution:
As we have learnt,

SN_1 reaction occurs in two steps. The first step is the formation of the Carbocation while the second step is the attack of the nucleophile to form the final product.



Since the formation of the Carbocation is the slow step in the reaction, the energy profile diagram will have a higher peak for the formation of the carbocation. The second peak represents the activation energy required for the nucleophile to bond with the Carbocation which will have a lower peak than the slowest step.

Therefore, **option (1) is correct.**

Q. 39 The number of d- electrons retained in Fe^{2+} $(At.\ no.\ Fe = 26)$ ions is

Option 1:
3

Option 2:
4

Option 3:
5

Option 4:
6

Correct Answer:
6

Solution:
As we learnt in

Electronic Configuration -

The 'd' block elements follow the general configuration $(n-1)d^{1-10}(ns^{1-2})$.

-

$$Fe^{2+} \Rightarrow 1s^2,\ 2s^2,\ 2p^6,\ 3s^2,\ 3p^6,\ 3d^6$$

Q. 40 Which of the following exhibits only +3 oxidation state?

Option 1:
Th

Option 2:
Ac

Option 3:
Pa

Option 4:
U

Correct Answer:
Ac

Solution:
Electronic configuration - Actinoids follow the general configuration $5f^{1-14}6d^{0-1}7s^2$.

Due to the electronic configuration $[Rn]6d^17s^2$ of Actinium, it only shows a +3 oxidation state because, after the loss of 3 electrons, actinium acquires noble gas electronic configuration.

The correct answer is option 2.

Q. 41 Which of the statements is not true?

Option 1:
On passing H_2S through acidified $K_2Cr_2O_7$ solution, a milky colour is observed.

Option 2:
$Na_2Cr_2O_7$ is preferred over $K_2Cr_2O_7$ in volumetric analysis.

Option 3:
$K_2Cr_2O_7$ solution in acidic medium is orange.

Option 4:
$K_2Cr_2O_7$ solution becomes yellow on increasing pH beyond 7.

Correct Answer:
$Na_2Cr_2O_7$ is preferred over $K_2Cr_2O_7$ in volumetric analysis.

Solution:
Due to hygroscopic nature of $Na_2Cr_2O_7$

Correct Answer is **Option 2**

Q. 42 When $CH_2 = CH - COOH$ is reduced with $LiAlH_4$, the compound obtained will be

Option 1:
$CH_3 - CH_2 - COOH$

Option 2:
$CH_2 = CH - CH_2OH$

Option 3:
$CH_3 - CH_2 - CH_2OH$

Option 4:
$CH_3 - CH_2 - CHO$

Correct Answer:
$CH_2 = CH - CH_2OH$

Solution:
$LiAlH_4$ is a strong reducing agent, it reduces carboxylic group into primary alcoholic group without affecting the double bond in the compound

$$\mathrm{CH_2 = CH - COOH \xrightarrow{LiAlH_4} CH_2 = CH - CH_2OH}$$

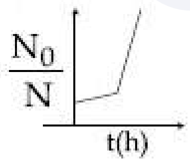
Therefore, **option (2) is correct.**

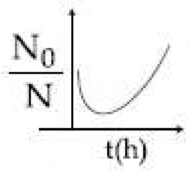
Q. 43 A bacterial infection in an internal wound grows as $N'(t) = N_o exp(t)$,

where the time t is in hours. A dose of antibiotic, taken orally, needs 1 hour to reach the wound.

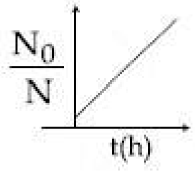
Once it reaches there, the bacterial population goes down as $\frac{dN}{dt} = -5N^2$.

What will be the plot of $\frac{N_o}{N}$ vs. t after 1 hour ?

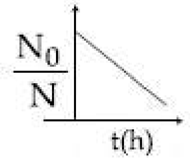
Option 1:


Option 2:


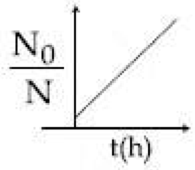
Option 3:



Option 4:



Correct Answer:



Solution:

From t hour => $N'(t) = N_o e^t$

After 1 hour, it is given : $\frac{dN}{dt} = -5N^2$

$\therefore$ at t = 1 hour , $N' = eN_o$

Now,

$$N^{-2} dN = -5dt$$

After 1 hour the graph will be from 1 hour to t hour and N' from eN_o to N.

$$\int_{eN_o}^{N} N^{-2} dN = -5 \int_{1}^{t} dt$$

$$\frac{1}{N} - \frac{1}{eN_o} = 5(t-1)$$

$$\frac{N_o}{N} = 5N_o(t-1) + \frac{1}{e}$$

$$\frac{N_o}{N} = 5N_o t + (\frac{1}{e} - 5N_o)$$

This equation is similar to the straight-line equation (Y=mx+C)

So, the curve will be a straight line and the slope will be positive.

$\therefore$ Option (3) is correct.

Q. 44 A first order reaction has a rate constant, $k = 5.5 \times 10^{-14} s^{-1}$, calculate the Half life of reaction?

Option 1:
1.26×10^{13} s

Option 2:
0.693×10^{14} s

Option 3:
6.93×10^{14} s

Option 4:
12.6×10^{14} s

Correct Answer:
1.26×10^{13} s

Solution:
As we have learnt,

Half life for a first order reaction is given as:

$$t_{1/2} = \frac{0.693}{k} = \frac{0.693}{5.5 \times 10^{-14} s^{-1}} = 1.26 \times 10^{13} \text{ s}$$

Therefore, **option(1) is correct**

Q. 45 For a first order reaction, calculate the ratio between the time taken to complete 3/4 th of the reaction and time taken to complete half of the reaction?

Option 1:
4/3

Option 2:
8/3

Option 3:
2

Option 4:
1

Correct Answer:
2

Solution:

$$t_{1/2}=\frac{0.69}{k},\quad t_{3/4}=t_{75\%}$$
$$t_{3/4}=\frac{2.303}{k}\log\frac{a}{\left(a-\frac{3a}{4}\right)}$$
$$=\frac{2.303}{k}\log 4$$

$$=\frac{2.303}{k}\times 2\times 0.3010=\frac{0.69\times 2}{k}$$
$$\frac{t_{3/4}}{t_{1/2}}=\frac{0.69\times 2}{k}\times\frac{k}{0.69}$$
$$\frac{t_{3/4}}{t_{1/2}}=2$$

Therefore, **option(3) is correct**

Q. 46 The half life of a first order reaction is 60 min. How long (in min.) will it take to consume 90% of the reaction?

Correct Answer:
200

Solution:
For the first-order reaction,
$$k=\frac{0.693}{t_{1/2}}=\frac{0.693}{60}=11.55\times 10^{-3}\text{min}^{-1}$$
Applying first order kinetic equation,

$$t=\frac{2.303}{k}\log\frac{a}{(a-x)}$$

$$\text{Given: } a=100, x=90,\ \text{i.e., } (a-x)=(100-90)=10$$

$$\text{Hence, } t=\frac{2.303}{11.5\times 10^{-3}}\log 10=200\text{ min}$$

Q. 47 Which one is the wrong statement?

Option 1:
de-Broglie's wavelength is given by $\lambda=\frac{\mathrm{h}}{\mathrm{m}v}$, where m = mass of the particle, v = group velocity of the particle.

Option 2:
The uncertainty principle is
$$\Delta E \times \Delta t \geq \frac{h}{4\pi}$$

Option 3:
Half-filled and fully filled orbitals have greater stability due to greater exchange energy, greater symmetry and more balanced arrangement.

Option 4:
The energy of 2s orbital is less than the energy of 2p orbital in case of Hydrogen like atoms.

Correct Answer:
The energy of 2s orbital is less than the energy of 2p orbital in case of Hydrogen like atoms.

Solution:
As we learned in

The principal quantum number determines the size and to large extent the energy of the orbital.

For a hydrogen-like atom, the energy of the 2s orbital is the **same** as the energy of the 2p orbital.

Therefore, the correct option is (4).

Q. 48 In which pair of ions do both the species contain $S - S$ bond?

Option 1:
$S_2O_7^{2-}, S_2O_3^{2-}$

Option 2:
$S_4O_6^{2-}, S_2O_3^{2-}$

Option 3:
$S_2O_7^{2-}, S_2O_8^{2-}$

Option 4:
$S_4O_6^{2-}, S_2O_7^{2-}$

Q. 49 The ease of adsorption of the hydrated alkali metal ions on an ion- exchange resins follows the order:

Option 1:
$Li^+ < K^+ < Na^+ < Rb^+$

Option 2:
$Rb^+ < K^+ < Na^+ < Li^+$

Option 3:
$K^+ < Na^+ < Rb^+ < Li^+$

Option 4:
$Na^+ < Li^+ < K^+ < Rb^+$

Correct Answer:
$Rb^+ < K^+ < Na^+ < Li^+$

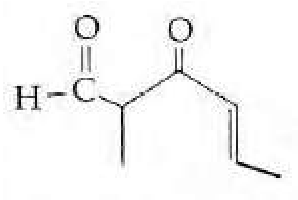
Solution:
Ease of adsorption of the hydrated alkali metal ions on an ion-exchange resin decreases as the size of alkali metal ion increases.

Since the order of the size of alkali metal ion follows the order- $Rb^+ > K^+ > Na^+ > Li^+$

Therefore, the ease of adsorption follows the order-

$Rb^+ < K^+ < Na^+ < Li^+$

Q. 50 The IUPAC name of the compound



is ______.

Option 1:
3-keto-2-methylhex-4-enal

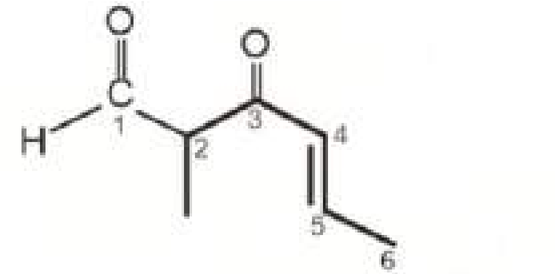
Option 2:
5-formylhex-2-en-3-one

Option 3:
5-methyl-4-oxohex-2-en-5-al

Option 4:
3-keto-2-methylhex-5-enal

Correct Answer:
3-keto-2-methylhex-4-enal

Solution:



3 - keto - 2 - methylhex - 4 enal

Q. 51 Reaction of benzene diazonium with A will give us benzene. A is

Option 1:
Fluroboric acid

Option 2:
Hypophosphorus acid

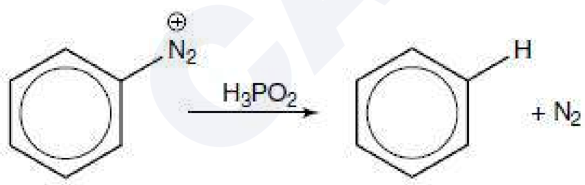
Option 3:
Potassium halide

Option 4:
water

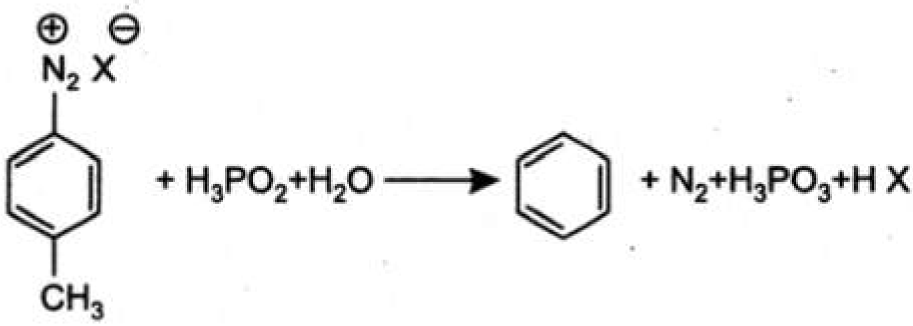
Correct Answer:
Hypophosphorus acid

Solution:
As we have learned

Benzene can be obtained by reacting benzene diazonium with H_3PO_2



Hence, the option number (2) is correct.

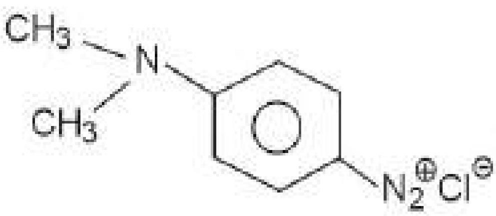


Therefore, **Option(2) is correct.**

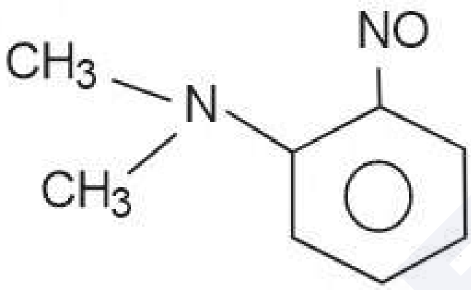
Q. 52 Consider the reaction given below

The major product (A) obtained in the reaction is

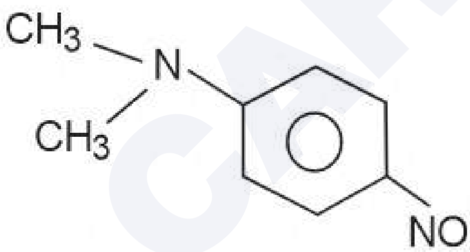
Option 1:



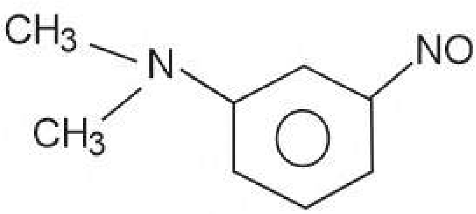
Option 2:



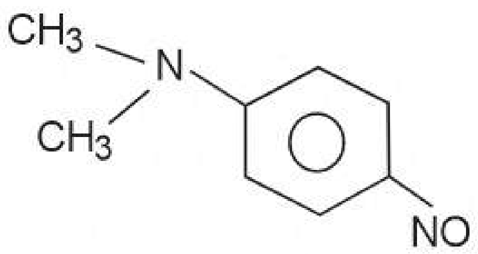
Option 3:



Option 4:



Correct Answer:

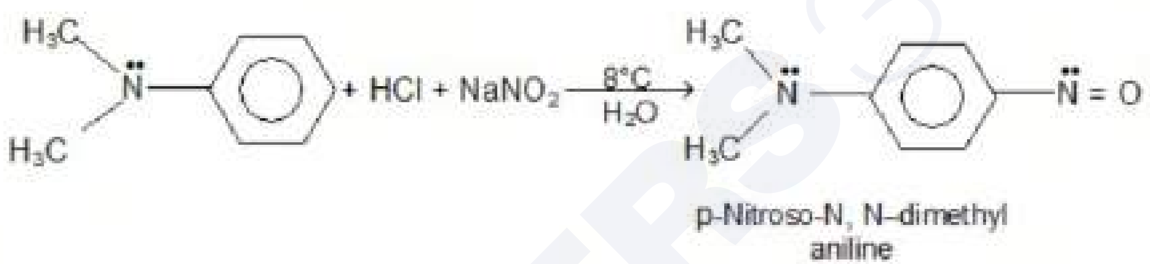


Solution:

The reaction of tertiary arylamines with HNO_2 -

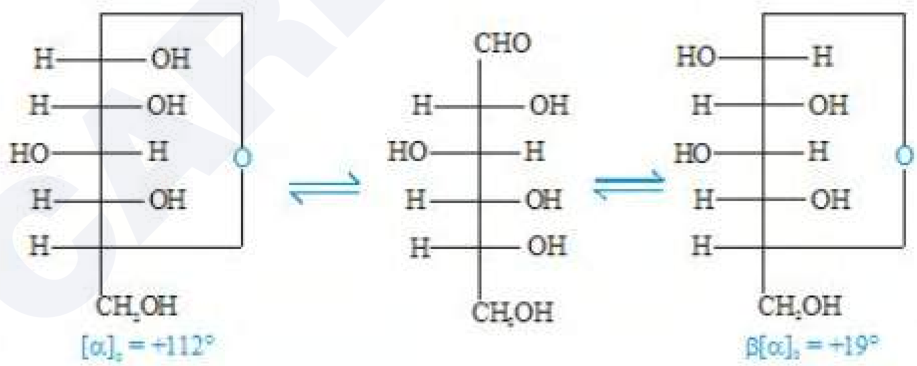
Tertiary arylamines react with nitrous acid to form p - Nitroso aromatic compounds. Electrophilic aromatic substitution occurs on the benzene ring in which a Nitroso group is substituted on the Benzene ring.

The reaction is given below



Therefore, **Option(3) is correct.**

Q. 53



The above process in which α and β form remain in equilibrium with acyclic form and a change in optical rotation is observed which is called as -

Option 1:
Mutarotation

Option 2:
Epimerisation

Option 3:
Condensation

Option 4:
Inversion

Correct Answer:
Mutarotation

Solution:
When either of the two forms of glucose ($\alpha-\mathrm{D}-\mathrm{Glucose}$ and $\beta-\mathrm{D}-\mathrm{Glucose}$) is dissolved in water , there is spontaneous change in specific rotation till the equilibrium value of $+52\cdot5^{\circ}$

The above phenomenon is called as mutarotation.

Hence, the correct answer is Option (1)

Q. 54 What is the mole fraction of the solute in a 1.00 m aqueous solution ?

Option 1:
0.177

Option 2:
1.770

Option 3:
0.0354

Option 4:
0.0177

Correct Answer:
0.0177

Solution:
As we learned in

$$\text{Molality}=\frac{\text{Moles of solute}}{\text{Mass of solution(kg)}}$$

$$\text{No. moles of water in 1000g}=\frac{1000}{8}=55.5$$

$$\text{No. of moles of solute}=1$$

$$\text{Mole fraction of solute}=\frac{1}{1+55.5}=0.0177$$

Q. 55 If Avogadro number N_A, is changed from 6.022 x 10^{23} mol^{-1} to 6.022 x 10^{20} mol^{-1}, this would change :

Option 1:
the definition of mass in units of grams.

Option 2:
the mass of one mole of carbon.

Option 3:
the ratio of chemical species to each other in a balanced equation.

Option 4:
the ratio of elements to each other in a compund.

Correct Answer:
the mass of one mole of carbon.

Solution:
As we learnt in

One mole is the amount of a substance that contains as many particles or entities as there are atoms in exactly 12 g (or 0.012 kg) of the 12C isotope.

1 mole of substance = 6.0221367 X 10^{23} **units** of that substance (such as atoms, molecules, or ions). The number 6.0221367 X 10^{23} is known as Avogadro's number or Avogadro's constant.

1 mol of carbon has mass = 12 g

6.022×10^{23} atoms of carbon mass = 12 g

So if now, 1 mole = 6.022×10^{20}

then 6.022×10^{20} atoms of carbon mass = $\frac{12g}{1000}$

$\Rightarrow$ 1 mol of carbon atom mass = 0.012 g

Hence, option number (2) is correct.

Q. 56 10 g of hydrogen and 64 g of oxygen were filled in a steel vessel and exploded. Amount of water produced in this reaction will be -

Option 1:
1 mol

Option 2:
2 mol

Option 3:
3 mol

Option 4:
4 mol

Correct Answer:
4 mol

Solution:
The reaction will be-

$$2H_2(g) + O_2(g) \rightarrow 2H_2O(l)$$

Two moles of H_2 and one mole of O_2 will produce two moles of water.

$$\text{No. of moles in 10g of } H_2 = \frac{\text{given mass of } H_2}{\text{molar mass of } H_2} = \frac{10}{2} = 5 \text{ moles}$$

$$\text{No. of moles in 64g of } O_2 = \frac{\text{given mass of } O_2}{\text{molar mass of } O_2} = \frac{64}{32} = 2 \text{ moles}$$

5 Moles of H_2 can produce 5 moles of water while 2 moles of O_2 can produce 4 moles of water.

Thus, O_2 is the limiting reactant and 4 moles H_2O will be produced.

Therefore, the correct option is (4).

Q. 57 25.3 g of sodium carbonate (Na_2CO_3) is dissolved in enough water to make 250 ml of solution. If sodium carbonate dissociates completely, the molar concentration of sodium ion (Na^+) and carbonate ion $\left(CO_3^{2-}\right)$ are respectively (Molar mass of Na_2CO_3 = 106 g mol^{-1}

Option 1:
0.477 M and 0.477 M

Option 2:
0.955 M and 1.910 M

Option 3:
1.910 M and 0.955 M

Option 4:
1.90 M and 1.910 M

Correct Answer:
1.910 M and 0.955 M

Solution:

$$\text{Number of moles of } Na_2CO_3 = \frac{25.3}{106} = 0.239 \text{ moles}$$

$$\text{Molarity of solution } = \frac{0.239}{0.25} = 0.956 \text{ mol/litre}$$

$$Na_2CO_3 \longrightarrow 2Na^+ + CO_3^{2-}$$

$$\therefore \text{ Concentration of } CO_3^{2-} = 0.956\text{M}$$

$$\text{Concentration of } Na^+ = 2 \times 0.956 = 1.912\text{M}$$

Therefore, the correct option is (3).

Q. 58 Which of the following orders of ionic radii is correctly represented ?

Option 1:
$H^- > H^+ > H$

Option 2:
$Na^+ > F^- > O^{2-}$

Option 3:
$O^{2-} > F^- > Na^+$

Option 4:
$Al^{3+} > Mg^{2+} > N^{3-}$

Correct Answer:
$O^{2-} > F^- > Na^+$

Solution:
As we learnt in

Size of isoelectronic species -

Smaller the value of z/e, the larger the size of that species. Smaller z means the effective nuclear charge is small hence size is large.

Cations lose electrons and are smaller in size than the parent atom, whereas anions add electrons and are larger in size than the parent atom. Hence the order is $H^- > H > H^+$

For isoelectronic species, the ionic radii decrease with an increase in atomic number, i.e., nuclear charge. Hence the correct orders are as given below:

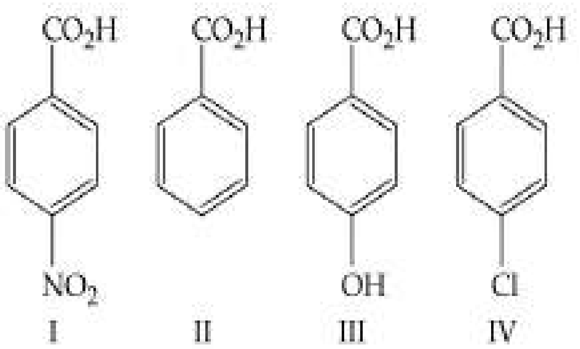
$O^{2-} > F^- > Na^+$

$N^{3-} > Mg^{2+} > Al^{3+}$

$O^{2-} > F^- > Na^+$

Therefore, the correct option is (3).

Q. 59 The increasing order of the acidity of the following carboxylic acids is:



Option 1:
I < III < II < IV

Option 2:
IV < II < III < I

Option 3:
II < IV < III < I

Option 4:
III < II < IV < I

Correct Answer:
III < II < IV < I

Solution:
As we learnt in

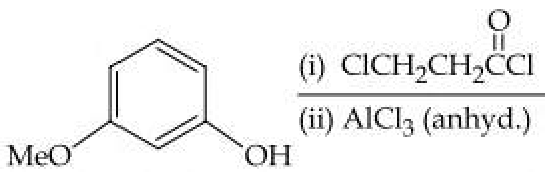
Acidic strength of carboxylic acid -

-NO_2 and -Cl are electron withdrawing groups while -OH is electron donating group.

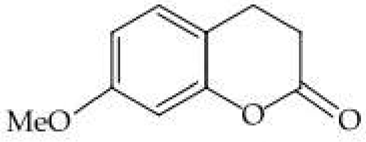
Ring deactivating groups increases the acidic character whereas ring activating groups decreases the acidic strength.

Therefore, **Option(4) is correct.**

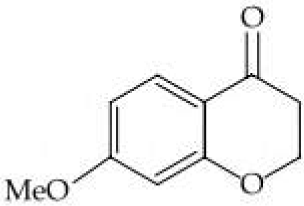
Q. 60 The major product of the following reaction is :



Option 1:

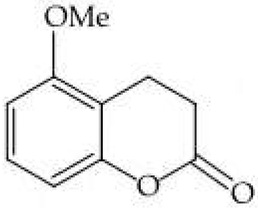


Option 2:

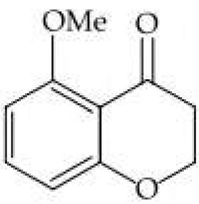


OMe

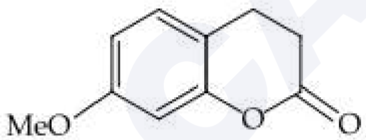
Option 3:



Option 4:

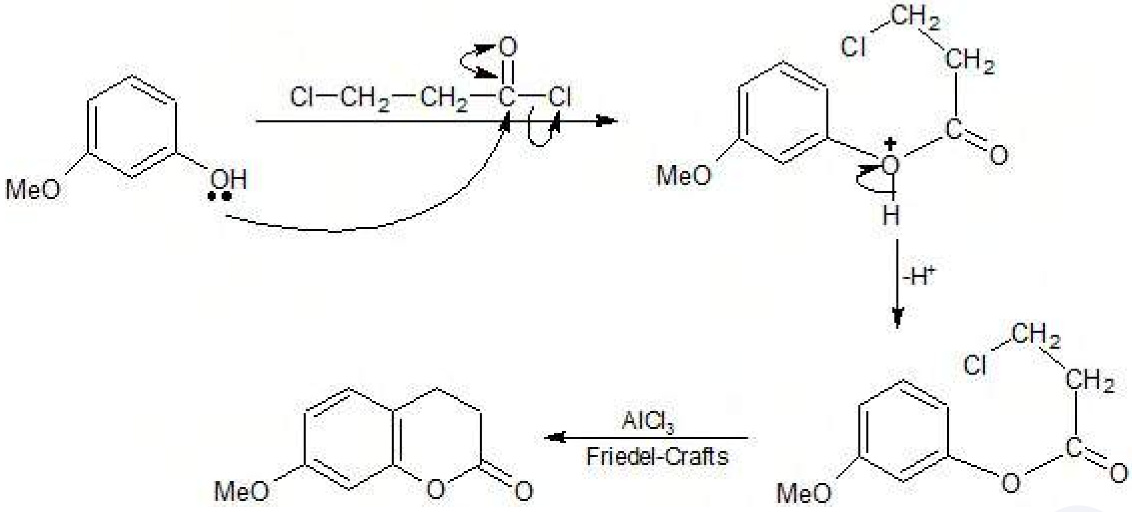


Correct Answer:



Solution:

Between OH and OMe, OH is more attacking species due to the formation of phenoxide ion by OH group.



Hence, the option number (1) is correct.

Maths

Q. 1 5- digit numbers are to be formed using 2,3,5,7,9 without repeating the digits.If p be the number of such numbers that exceed 20000 and q be the number of those that lie between 30000 and 90000 then p:q is :

Option 1:
$6:5$

Option 2:
$3:2$

Option 3:
$4:3$

Option 4:
$5:3$

Correct Answer:
$5:3$

Solution:
Number greater than 20000 or p

1st place can be placed with 5 possibilities and second with 4 possibilities and so on

So $\underline{5} \times \underline{4} \times \underline{3} \times \underline{2} \times \underline{1} = 5! = P$

And

Number greater than 30000 and less than 90000 or q

1st place can be placed with 3 possibilities(3,5,7) and second with 4 possibilities and so on

So $\underline{3} \times \underline{4} \times \underline{3} \times \underline{2} \times \underline{1} = 5! = Q$

$$\frac{P}{Q} = \frac{5!}{3 \times 4!} \Rightarrow \frac{5 \times 4!}{3 \times 4!} = \frac{5}{3}$$

Q. 2 Twenty meters of wire is available for fencing off a flower-bed in the form of a circular sector. Then the maximum area (in sq. m) of the flower-bed, is :

Correct Answer:
25

Solution:
As we have learned

Method for maxima or minima -

By second derivative method :

$$Step\ 1.\ \ find\ values\ of\ x\ for\ \frac{dy}{dx} = 0$$

$$Step\ \ 2.\ \ x = x_{\circ}\ \ is\ a\ point\ of\ local\ maximum\ if$$
$$f''(x) < 0\ \ and\ local\ minimum\ if\ f''(x) > 0$$

- wherein

$$Where\ \ y = f(x)$$

$$\frac{dy}{dx} = f'(x)$$

So,

$$P = 2r + r\theta = 20.......(1)$$
$$A = \frac{1}{2}r^2\theta$$

So ,

$$\frac{dA}{d\theta} = \frac{d}{d\theta}\left(\frac{1}{2}r^2\theta\right) = \frac{d}{d\theta}\left(\frac{1}{2}\left(\frac{20}{2+\theta}\right)^2\theta\right)$$

$$\left(from(1), r = \frac{20}{2+\theta}\right)$$

$$\frac{dA}{d\theta} = 200\frac{d}{d\theta}\left(\frac{\theta}{(2+\theta)^2}\right) = 200 \times \frac{(2+\theta)^2 - 2\theta(2+\theta)}{(2+\theta)^4} = 0$$

$$\therefore (2+\theta)^2 - 2\theta(2+\theta) = 0$$
$$\Rightarrow (2+\theta)(2-\theta) = 0$$
$$\rightarrow \theta = \pm 2$$
$$\Rightarrow r = 5$$

Hence $A_{max} = \frac{1}{2}(5)^2(2) = 25$

Q. 3 If $\int \frac{dx}{\cos^3 x\sqrt{2\sin 2x}} = (tanx)^A + C(tanx)^B + k,$

where *k* is a constant of integration, then A + B + C equals :

Option 1:
$\frac{21}{5}$

Option 2:
$\frac{16}{5}$

Option 3:
$\frac{7}{10}$

Option 4:
$\frac{27}{10}$

Correct Answer:
$\frac{16}{5}$

Solution:
As learnt in concept

Integration by substitution -

The functions when on substitution of the variable of integration to some quantity gives any one of standard formulas.

- wherein

Since $\int f(x)dx = \int f(t)dt = \int f(\theta)d\theta$ all variables must be converted into single variable , $(t\, or\, \theta)$

$$\int \frac{dx}{\cos^3 x\sqrt{2\sin x\ \cos x \times 2}}$$

$$= \frac{1}{2}\int \frac{\sec^4 x dx}{\sqrt{tan x}}$$

$$= \frac{1}{2}\int \frac{(1+\tan^2 x)\sec^2 x\ dx}{\sqrt{\tan x}}$$

$$= \frac{1}{2}\int (\tan x)^{\frac{-1}{2}}\sec^2 x\ dx + \frac{1}{2}\int (\tan x)^{\frac{3}{2}}\sec^2 x\ dx$$

$$= \frac{1}{2}\frac{(\tan x)^{\frac{1}{2}}}{\frac{1}{2}} + \frac{1}{2}\frac{(\tan x)^{\frac{5}{2}}}{\frac{5}{2}} + C$$

$$A = \frac{1}{2}; B = \frac{5}{2}; C = \frac{1}{5}$$

$$A + B + C = 3 + \frac{1}{5}$$

$$= \frac{16}{5}$$

Q. 4

Let $a = Im\left(\frac{1+z^2}{2iz}\right)$,where z is any non-zero complex number.

The set $A = \{a : |z| = 1 and z \neq \pm 1\}$ is equal to:

Option 1:
(-1,1)

Option 2:
[-1,1]

Option 3:
[0,1)

Option 4:
(-1,0]

Correct Answer:
(-1,1)

Solution:
Let $z=x+iy \Rightarrow z^2=x^2-y^2+2ixy$

Now,

$$\begin{aligned}\frac{1+z^2}{2iz} &= \frac{1+x^2-y^2+2ixy}{2i(x+iy)} = \frac{(x^2-y^2+1)+2ixy}{2ix-2y} \\ &= \frac{(x^2-y^2+1)+2ixy}{-2y+2ix} \times \frac{-2y-2ix}{-2y-2ix} \\ &= \frac{y\left(x^2+y^2-1\right)+x\left(x^2+y^2+1\right)i}{2\left(x^2+y^2\right)}\end{aligned}$$

$$a=\frac{x\left(x^2+y^2+1\right)}{2\left(x^2+y^2\right)}$$

$$\begin{aligned}&\text{Since, } |z|=1 \Rightarrow \sqrt{x^2+y^2}=1 \\ &\Rightarrow x^2+y^2=1 \\ &\therefore a=\frac{x(1+1)}{2\times 1}=x \\ &\text{Also } z\neq 1 \Rightarrow x+iy\neq 1 \\ &\therefore \mathrm{A}=(-1,1)\end{aligned}$$

Q. 5 The eccentricity of an ellipse having centre at the origin, axes along the co-ordinate axes and passing through the points (4, −1) and (−2, 2) is :

Option 1:
$\frac{1}{2}$

Option 2:
$\frac{2}{\sqrt{5}}$

Option 3:
$\frac{\sqrt{3}}{2}$

Option 4:
$\frac{\sqrt{3}}{4}$

Correct Answer:
$\frac{\sqrt{3}}{2}$

Solution:
As we learnt in

Eccentricity -

$$e=\sqrt{1-\frac{b^2}{a^2}}$$

- wherein

For the ellipse

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

It passes through (4,-1) and (-2,2)

$$\frac{16}{a^2}+\frac{1}{b^2}=1$$

and $\frac{4}{a^2}+\frac{4}{b^2}=1 \quad \} \times 4$

$$\frac{15}{b^2}=3 => b^2=5$$

$$\frac{4}{a^2}+\frac{4}{5}=1$$

$$a^2=20$$

$$e=\sqrt{1-\frac{b^2}{a^2}}=\sqrt{1-\frac{5}{20}}=\frac{\sqrt{3}}{2}$$

Q. 6

If $\lim\limits_{x\rightarrow\infty}\left(1+\frac{a}{x}-\frac{4}{x^2}\right)^{2x}=e^3$, then '*a*' is equal to :

Option 1:
2

Option 2:
$\frac{3}{2}$

Option 3:
$\frac{2}{3}$

Option 4:
$\frac{1}{2}$

Correct Answer:
$\frac{3}{2}$

Solution:
As we learnt in

1 to the power of infinity Form -

$$Let \quad \lim_{x\to a} f(x)^{g(x)} \quad where$$

$$f(a)=1 \quad and \quad g(a)=\infty$$

$$e^{\lim_{x\to a}(f(x)-1)g(x)}$$

-

$$\lim_{x\to\infty}\left(1+\frac{a}{x}-\frac{4}{x^2}\right)^{2x}=e^3$$

It is 1^∞ form.

$$\therefore \lim_{x\to\infty}\left(1+\frac{a}{x}-\frac{4}{x^2}-1\right)\times 2x$$

$$\therefore \lim_{x\to\infty}\left(\frac{a}{x}-\frac{4}{x^2}\right)\times 2x$$

$$\therefore \lim_{x\to\infty}\left(a-\frac{4}{x}\right)\cdot 2$$

$$\therefore 2a$$

$$\therefore e^{2a}=e^3$$

$$\therefore a=\frac{3}{2}$$

Q. 7 For two 3 × 3 matrices A and B, let A + B = 2B' and 3A + 2B = I_3, where B' is the transpose of B and I_3 is 3 × 3 identity matrix. Then :

Option 1:
5A + 10B = $2I_3$

Option 2:
10A + 5B = $3I_3$

Option 3:
$B + 2A = I_3$

Option 4:
$3A + 6B = 2I_3$

Correct Answer:
$10A + 5B = 3I_3$

Solution:
Given A + B = 2B' ....(1)

Taking transpose of both the sides

(A + B)' = (2B')'

A' + B' = 2(B')'

A' + B' = 2B ....(2)

Also given, 3A + 2B = I ....(3)

Taking transpose of both the sides

3A' + 2B' = I ....(4)

(Note: Transpose of I is I itself)

Now from these 4 equations we need to get a relation in A and B by eliminating A' and B'

Let us first eliminate B'

From (4): 3A' + A + B = I (Using (1)) ....(5)

And 2 x (2):

2A' + 2B' = 4B

2A' + A + B = 4B

2A' + A - 3B = 0 .....(6)

From (5) and (6) we can eliminate A' as well

From (5): 3A' = I - A - B .....(7)

From (6): 2A' = 3B - A .....(8)

2x(7) - 3x(8): 0 = 2I + A - 11B .....(9)

From (9) and (3): A = B = I/5

For option (B) : 10A + 5B = 2I + I = 3I

Hence option (B) is correct

Q. 8 If $(2+\sin x)\frac{dy}{dx}+(y+1)\cos x=0$

and y(0)=1,then $y\left(\frac{\pi}{2}\right)$ is equal to :

Option 1:
$-\frac{2}{3}$

Option 2:
$-\frac{1}{3}$

Option 3:
$\frac{4}{3}$

Option 4:
$\frac{1}{3}$

Correct Answer:
$\frac{1}{3}$

Solution:
As we learnt in

Solution of Differential Equation -

$$\frac{\mathrm{d}y}{\mathrm{d}x}=f\left(ax+by+c\right)$$

put

$$Z=ax+by+c$$

- wherein

Equation with convert to

$$\int \frac{dz}{bf\,(z)+a}=x+c$$

$$(2+\sin x)\,\frac{dy}{dx}+(y+1)\cos x=0$$

$$=>\frac{dy}{dx}=-\,(y+1)\,.\frac{\cos x}{2+\sin x}$$

$$=>-\frac{dy}{y+1}=\frac{\cos x}{2+\sin x}dx$$

$$\therefore-\int\frac{dy}{y+1}=\int\frac{\cos x}{2+\sin x}dx$$

$$=>-log(y+1)=log(2+\sin x)+C$$

$$=>put\ x=0,\ y=1$$

$$-log2=log(2+\sin x)+C=+log2+C$$

$$\therefore C=-2log2=log\frac{1}{4}$$

$$\therefore-log(y+1)=log(2+\sin x)+log\frac{1}{4}$$

Now put

$$x=\frac{\pi}{2}$$

$$\therefore-log(y+1)=log(2+1)+log\frac{1}{4}=log\frac{3}{4}$$

$$\therefore y+1=\frac{4}{3}$$

$$=>y=\frac{1}{3}$$

Q. 9 If $2\int_0^1 \tan^{-1}x dx=\int_0^1 \cot^{-1}(1-x+x^2)dx, then \int_0^1 \tan^{-1}(1-x+x^2)dx$

is equal to :

Option 1:
log4

Option 2:
$\frac{\pi}{2}+log2$

Option 3:
log2

Option 4:
$\frac{\pi}{2}-log4$

Correct Answer:
log2

Solution:
As learnt in concept

Integration By PARTS -

Let u and v are two functions then

$$\int u\cdot vdx=u\int vdx-\int\left(\frac{du}{dx}\int vdx\right)dx$$

- wherein

Where u is the Ist function v is he IInd function

$$2\int_0^1 \tan^{-1} xdx=\int_0^1 \cot^{-1}\left(1-x+x^2\right)dx,$$

$$2\int \tan^{-1} x\,dx=\int_0^1 \frac{\pi}{2}-\tan^{-1}\left(1-x+x^2\right)dx$$

$$\int_0^1 \tan^{-1}\left(1-x+x^2\right)dx=\frac{\pi}{2}-2\int_0^1 \tan^{-1} x\,dx$$

$$\int_0^1 \tan^{-1}\left(1-x+x^2\right)dx$$

$$=\frac{\pi}{2}-2\left[x\tan^{-1} x-\frac{1}{2}\,log\left(1+x^2\right)\right]_0^1$$

$$=\frac{\pi}{2}-2\left[\frac{\pi}{4}-\frac{1}{2}\,log\,2-0\right]$$

$$=log2$$

Q. 10 If the function $f(x)=\begin{cases}-x, & x<1\\ a+\cos^{-1}(x+b), & 1\le x\le 2\end{cases}$ is differentiable at x = 1, then

$\frac{a}{b}$ is equal to :

Option 1:
$\frac{\pi-2}{2}$

Option 2:
$\frac{-\pi-2}{2}$

Option 3:
$\frac{\pi+2}{2}$

Option 4:
$-1-\cos^{-1}(2)$

Correct Answer:
$\frac{\pi+2}{2}$

Solution:
As we learnt in

Rule for continuous -

A function is continuous at x = a if and only if

$$L=R=V$$

L.H.L R.H.L value at x = a.

- wherein

Where

$$L=\lim_{x\to a^-} f(x)$$

$$R=\lim_{x\to a^+} f(x)$$

$$V_I=\lim_{x\to a} f(x)$$

$$f\left(x\right)=\begin{cases}-x & x<1\\ a+cos^{-1}\left(x+b\right) & 1\leq x\leq 2\end{cases}$$

$$f1\left(x\right)=\begin{cases}0-\frac{-1}{\frac{1}{\sqrt{1-(x+b)^2}}}\end{cases}$$

$$\therefore -1=-\frac{1}{\sqrt{1-\left(x+b\right)^2}}$$

$$-1=-\frac{1}{\sqrt{1-\left(1+b\right)^2}}$$

$\therefore$ 1+b=0 b=-1

Now f(x) will be continuous at x=1

$$\therefore -1=a+cos^{-1}(1-1)=a+cos^{-1}0=a+\frac{\pi}{2}$$

$$\therefore\ a=-1-\frac{\pi}{2}$$

$$\frac{a}{b}=\frac{-1-\frac{\pi}{2}}{-1}=1+\frac{\pi}{2}$$

Q. 11 For, $x\epsilon R, x\neq 0, x\neq 1$ let $f_0(x)=\frac{1}{1-x}$ and $f_{n+1}(x)=f_0(f_n(x)),\ n=0,1,2,3.....$ then the value of $f_{100}\left(3\right)+f_1\left(\frac{2}{3}\right)+f_2\left(\frac{3}{2}\right)$ is approximately equals to:

Correct Answer:
1.67

Solution:

$$f_0(x)=\frac{1}{1-x}$$

$$f_{n+1}(x)=f_0(f_n(x))$$

Put n = 0

$$\therefore\quad f_1(x)=f_0(f_0(x))=\frac{1}{1-\frac{1}{1-x}}=\frac{1-x}{1-x-1}=\frac{1-x}{-x}=\left(\frac{x-1}{x}\right)$$

$$\therefore\quad f_1(x)=\frac{x-1}{x}=1-\frac{1}{x}$$

Put n = 1

$$\therefore \quad f_2(x) = f_0(f_1(x)) = f_0\left(\frac{x-1}{x}\right) = \frac{1}{1-\frac{x-1}{x}} = x$$

Put n = 2

$$f_3(x) = \frac{1}{1-x}$$ (same as $f_0(x)$)

So at n = 0, 3, 6, ...96,99;

$$f_n(x) = \left(\frac{1}{1-x}\right)$$

at n = 1, 4, 7, ...97,100,103;

$$f_n(x) = 1 - \frac{1}{x}$$

and n = 2, 5, 8,............98,101,104 ; $f_n(x) = x$

$$\therefore \quad f_{100}(3) = 1 - \frac{1}{x} = 1 - \frac{1}{3} = \frac{2}{3}$$

$$\text{and } f_1(2/3) = 1 - \frac{1}{2/3} = 1 - \frac{3}{2} = -0.5$$

$$\text{and } f_2(3/2) = x = 3/2 = 1.5$$

$$f_{100}(3) + f_1\left(\frac{2}{3}\right) + f_2\left(\frac{3}{2}\right) = 2/3 + 1.5 - 0.5 = 5/3$$

Q. 12 Let $S_n = \frac{1}{1^3} + \frac{1+2}{1^3+2^3} + \frac{1+2+3}{1^3+2^3+3^3} \ldots\ldots + \frac{1+2 \ldots\ldots + n}{1^3+2^3 \ldots\ldots + n^3}$.

If 100 S_n = n, then n is equal to :

Correct Answer:
199

Solution:

$$\frac{1}{1^3} + \frac{1+2}{1^3+2^3} + \frac{1+2+3}{1^3+2^3+3^3} + --- + \frac{1+2+3+--+n}{1^3+2^3+--+n^3}$$

Its T_n term is $\frac{1+2+3+--n}{1^3+2^3+3^3+--n^3}$

$$= \frac{\left(\frac{n(n+1)}{2}\right)}{\left[\frac{n(n+1)}{2}\right]^2}$$

$$= \frac{1}{\frac{n(n+1)}{2}}$$

$$= \frac{2}{n\,(n+1)}$$

$$T_n = 2\left[\frac{(n+1)-n}{n\,(n+1)}\right]$$

$$= 2\left[\frac{1}{n} - \frac{1}{n+1}\right]$$

$$T_1 = 2\left[\frac{1}{1} - \frac{1}{2}\right]$$

$$T_2 = 2\left[\frac{1}{2} - \frac{1}{3}\right]$$

.....

$$T_n = 2\left[\frac{1}{n} - \frac{1}{n+1}\right]$$

Adding all these

$$S_n = 2\left[1 - \frac{1}{n+1}\right]$$

$$= 2\left(\frac{n+1-1}{n+1}\right)$$

$$= \left(\frac{2n}{n+1}\right)$$

Now $100S_n = \frac{2n}{n+1} \times 100 = n(given)$

$\therefore\ n+1 = 200$

$n = 199$

Q. 13 A value of x for which $\sin(\cot^{-1}(1+x)) = \cos(\tan^{-1} x)$, is :

Option 1:
$-\frac{1}{2}$

Option 2:
1

Option 3:
0

Option 4:
$\frac{1}{2}$

Correct Answer:
$-\frac{1}{2}$

Solution:
We have given

$$\sin(\cot^{-1}(1+x)) = \cos(\tan^{-1} x)$$

$$\sin\left(\sin^{-1}\left(\frac{1}{\sqrt{2+2x+x^2}}\right)\right) = \cos\left(\cos^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)\right)$$
$$\Rightarrow 2+2x+x^2 = 1+x^2$$
$$\Rightarrow 1+2x = 0$$
$$\Rightarrow x = -\frac{1}{2}$$

Q. 14 In a triangle ABC, right angled at the vertex A, if the position vectors of A, B and C are respectively $3\hat{i}+\hat{j}-\hat{k}, -\hat{i}+3\hat{j}+p\hat{k}\ and\ 5\hat{i}+q\hat{j}-4\hat{k},$

then the point (p, q) lies on a line :

Option 1:
parallel to x-axis.

Option 2:
parallel to y-axis.

Option 3:
making an acute angle with the positive direction of x-axis.

Option 4:
making an obtuse angle with the positive direction of x-axis

Correct Answer:
making an acute angle with the positive direction of x-axis.

Solution:
As we learnt in

Scalar Product of two vectors -

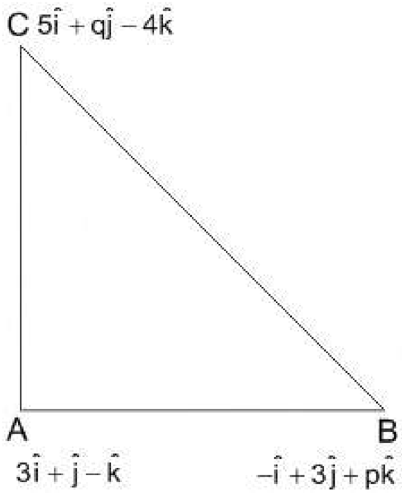
$\vec{a}.\vec{b} > 0\ an\ acute\ angle$

$\vec{a}.\vec{b} < 0\ an\ obtuse\ angle$

$\vec{a}.\vec{b} = 0\ a\ right\ angle$

- wherein

Θ is the angle between the vectors $\vec{a}\ and\ \vec{b}$



$\overrightarrow{AB} = 4\hat{i} - 2\hat{j} - (p+1)\hat{k}$

$\overrightarrow{AC} = 2\hat{i} + (q-1)\hat{j} - 3\hat{k}$

$\overrightarrow{AB}.\overrightarrow{AC} = 0$

$8 - 2q + 2 + 3p + 3 = 0$

$3p - 2q + 13 = 0$

Replace (p,q) with (x,y)

3x-2y+13=0

$Slope = \frac{3}{2}$

Acute angle with +x-axis

Q. 15 A committee of 4 person is to be formed from 2 ladies , 2 old men and 4 young men such that it includes at least 1 lady, at least 1 old man and at most 2 young men . Then the total number of ways in which this committee can be formed is :

Option 1:
40

Option 2:
41

Option 3:
16

Option 4:
32

Correct Answer:
41

Solution:
$2 \text{ lady} + 2 \text{ old men} + 4 \text{ young men}$

Case 1

0 young men 2 old men and 2 lady

Case 2

1 young men 1 old men and 2 lady

Case 3

1 young men 2 old men and 1 lady

Case 4

2 young men 1 old men and 1 lady

Hence

$(0\,YM + 2L + 2\,OM) + (1\,YM + 1L + 2\,OM) + (1\,YM + 1L + \ 1OM) + (2\,YM + 1L + 1\,OM)$

$= ({}^4C_0 \cdot {}^2C_2 \cdot {}^2C_2) + ({}^4C_1 \cdot {}^2C_1 \cdot {}^2C_2) + ({}^4C_1 \cdot {}^2C_2 \cdot {}^2C_1) + \left({}^4C_2 \cdot {}^2C_1 \cdot {}^2C_1\right)$

$= 1 \cdot 1 \cdot 1 + 4 \cdot 2 \cdot 1 + 4 \cdot 1 \cdot 2 + 6 \cdot 2 \cdot 2$

$= 1 + 8 + 8 + 24 = 41$

Q. 16 The minimum distance of a point on the curve $y = x^2 - 4$ from the origin is :

Option 1:
$\frac{\sqrt{19}}{2}$

Option 2:
$\sqrt{\frac{15}{2}}$

Option 3:
$\frac{\sqrt{15}}{2}$

Option 4:
$\sqrt{\frac{19}{2}}$

Correct Answer:
$\frac{\sqrt{15}}{2}$

Solution:
As learnt in concept @ZWYX and use of maxima/minima

Let point at minimum distance from O is

$$(h, h^2-4)$$
$$\therefore OP^2 = h^2 + (h^2-4)^2$$
$$\frac{d(OP^2)}{dh} == 2h + 2(h^2-4)2h = 0$$
$$\Rightarrow h = \pm\sqrt{\frac{7}{2}}, 0$$
$$\left(\frac{d^2(OP^2)}{dh^2}\right)_{at\sqrt{\frac{7}{2}}} > 0$$
$$\therefore OP\ is\ min\ at\ h = \pm\sqrt{\frac{7}{2}}$$
$$OP_{min} = \sqrt{\frac{7}{2} + \left(\frac{7}{2} - 4\right)^2} = \frac{\sqrt{15}}{2}$$

Q. 17 The area (in sq. units) of the region described by A = {(*x*, *y*)|$y \geq x^2 - 5x + 4$, $x + y \geq 1$, $y \leq 0$} is :

Option 1:
$\frac{7}{2}$

Option 2:
$\frac{19}{6}$

Option 3:
$\frac{13}{6}$

Option 4:
$\frac{17}{6}$

Correct Answer:
$\frac{19}{6}$

Solution:
As learnt in concept

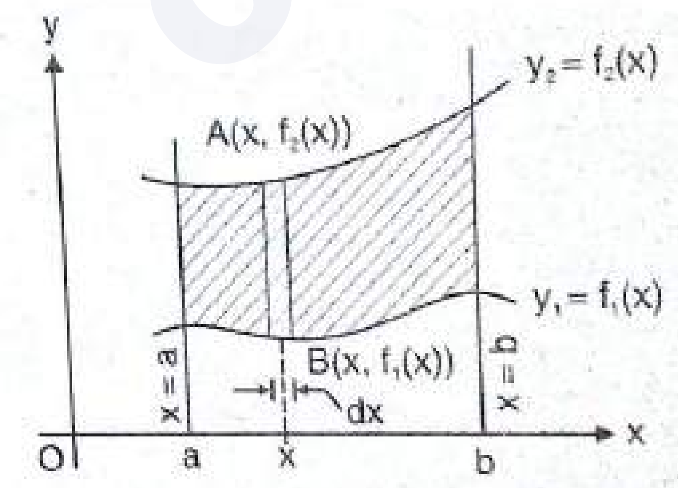
Area along x axis -

Let $y_1 = f_1(x)\ and\ y_2 = f_2(x)$ be two curve then area bounded between the curves and the lines

x = a and x = b is

$$\left|\int_a^b \Delta y\, dx\right| = \left|\int_a^b (y_2 - y_1)\, dx\right|$$

- wherein



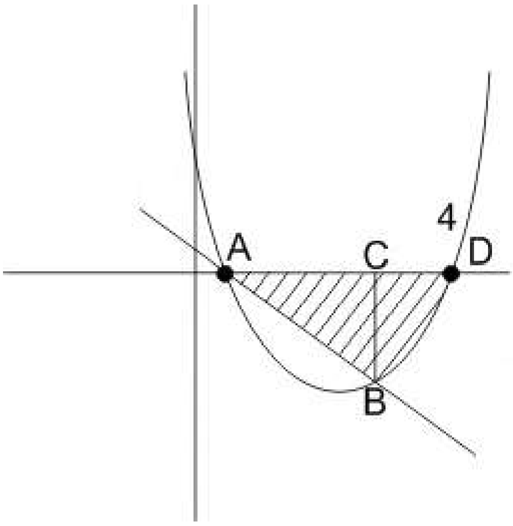
Where $\Delta y = f_2\,(x) - f_1(x)$

Point of intersection of

y= x^2-5x+4 and x+y=1 are x=1,3

at x=1, y=0;

x=3, y= -2



Req Area $= Area\ \Delta ABC + \left|\int_3^4 (x^2-5x+4)\,dx\right|$

$$= \frac{1}{2}\times 2\times 2 + \left[\frac{x^3}{3}-\frac{5x^2}{2}+4x\right]_3^4$$

$$= \frac{19}{6}$$

Q. 18 The values of $'a'$ for which one root of the equation $x^2-(a+1)x+a^2+a-8=0$ exceeds 2 and the other is lesser than 2, are given by :

Option 1:
$3 < a < 10$

Option 2:
$a \geq 10$

Option 3:
$-2 < a < 3$

Option 4:
$a \leq -2$

Correct Answer:
$-2 < a < 3$

Solution:

$x^2 - (a+1)x + a^2 + a - 8 = 0$

Since roots are different, therefore $D > 0$

$\Rightarrow (a+1)^2 - 4\left(a^2 + a - 8\right) > 0$

$\Rightarrow (a-3)(3a+11) < 0$

There are two cases that arise

Case 1 a - 3 > 0 and 3a + 11 < 0

$\Rightarrow a > 3 \text{ and } a < -\dfrac{11}{3}$

Hence, no solution in this case

Case 2 a - 3 < 0 and 3a + 11 > 0

$\Rightarrow a < 3 \text{ and } a > -\dfrac{11}{3}$

$\therefore -\dfrac{11}{3} < a < 3 \Rightarrow -2 < a < 3$

Q. 19 If y = mx + c is the normal at a point (in first quadrant) on the parabola y^2 = 8x whose focal distance is 8 units, then |c| is equal to

Option 1:

$2\sqrt{3}$

Option 2:

$8\sqrt{3}$

Option 3:

$10\sqrt{3}$

Option 4:

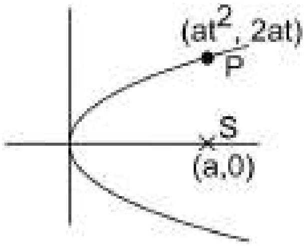
$16\sqrt{3}$

Correct Answer:

$10\sqrt{3}$

Solution:

Here $y^2 = 4ax\ ;\ a = 2$



We know that for $y^2 = 4ax$,

Focal distance = a + x-coordinate of the point

Thus 8 = 2 + at^2

2t^2 = 6

$t = \sqrt{3}$

Now

$Equation\ of\ normal\ \ is\ \ y + tx = 2at + at^3$

Put value $a = 2, t = \sqrt{3}$

$y = -\sqrt{3}x + 2(2)\sqrt{3} + 2(\sqrt{3})^3$

Comparing with y = mx + c

$|c| = 4\sqrt{3} + 6\sqrt{3} = 10\sqrt{3}$

Q. 20 If $f(x)$ is a differentiable function in the interval (0, ∞) such that $f(1)$ = 1 and $\lim_{t \to x} \frac{t^2 f(x) - x^2 f(t)}{t - x} = 1,$ for each $x > 0$, then $f(3/2)$ is equal to :

Option 1:
$\frac{13}{6}$

Option 2:
$\frac{23}{18}$

Option 3:
$\frac{25}{9}$

Option 4:
$\frac{31}{18}$

Correct Answer:
$\frac{31}{18}$

Solution:
As we learnt in

L - Hospital Rule -

$$In\ the\ form\ of\ \ \frac{0}{0}\ \ and\ \ \frac{\infty}{\infty}\ \ we\ differentiate\ \ \frac{N^r}{D^r}\ \ separately.$$

$$\Rightarrow \lim_{x\rightarrow a} \frac{f(x)}{g(x)} = \lim_{x\rightarrow a} \frac{f'(x)}{g'(x)}$$

- wherein

$$\lim_{x\rightarrow a} \frac{\frac{d}{dx}f(x)}{\frac{d}{dx}g(x)}$$

$$Where\ \ f(x)\ \ and\ \ g(x)=0$$

$$\lim_{t\rightarrow x} \frac{t^2f(x)-x^2f(t)}{t-x}=1$$

$$\lim_{t\rightarrow x} \frac{2tf(x)-x^2f'(t)}{1}=1$$

$$\therefore 2xf(x)-x^2f'(x)=1$$

Now, let y=f(x)

$$2xy-x^2\frac{dy}{dx}=1$$

$$\Rightarrow x^2\frac{dy}{dx}-2xy=-1$$

$$\Rightarrow \frac{dy}{dx}-\frac{2}{x}y=-\frac{1}{x^2}$$

$$P=-\frac{2}{x}\ and\ Q=-\frac{1}{x^2}$$

$$\therefore \int Pdx=-2\int \frac{dx}{x}=-2logx=log\frac{1}{x^2}$$

$\therefore$ If $e^{log\frac{1}{x^2}}=\frac{1}{x^2}$

$$\therefore\ Solution\ is$$

$$y \cdot \frac{1}{x^2} = \int -\frac{1}{x^2} \times \frac{1}{x^2}\ dx = \int \frac{1}{x^4} dx$$

$$= -\int x^{-4} dx = \frac{-x^{-4+1}}{-4+1} + C$$

$$\frac{y}{x^2} = \frac{-x^{-3}}{-3} + C = C + \frac{1}{3x^3}$$

Put, x=1, y=1

$$\frac{1}{1} = C + \frac{1}{3}$$

$$\therefore C = 1 - \frac{1}{3} = \frac{2}{3}$$

$$\frac{y}{x^2} = \frac{1}{3x^3} + \frac{2}{3}$$

$$\therefore y = \frac{1}{3x} + \frac{2x^2}{3}$$

Put, $x = \frac{3}{2}$

$$y = \frac{1}{3 \times \frac{3}{2}} + \frac{2}{3} \times \frac{9}{4}$$

$$= \frac{2}{9} + \frac{3}{2} = \frac{4+27}{18} = \frac{31}{18}$$

Q. 21 If $x = a, y = b, z = c$ is a solution of the system of linear equations

$x + 8y + 7z = 0$

$9x + 2y + 3z = 0$

$x + y + z = 0$

such that the point (a, b, c) lies on the plane $x + 2y + z = 6$, then $2a + b + c$ equals :

Correct Answer: 1

Solution:

As we learnt in

Cramer's rule for solving system of linear equations -

When $\Delta = 0$ and $\Delta_1 = \Delta_2 = \Delta_3 = 0$,

then the system of equations has infinite solutions.

- wherein

$a_1x + b_1y + c_1z = d_1$

$a_2x + b_2y + c_2z = d_2$

$a_3x + b_3y + c_3z = d_3$

and

$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$\Delta_1, \Delta_2, \Delta_3$ are obtained by replacing column 1,2,3 of Δ by (d_1, d_2, d_3) column

$$a + 8b + 7c = 0$$
$$9a + 2b + 3c = 0$$
$$a + b + c = 0$$

and $a + 2b + c = 6$ given

solve for a,b, and c

so that

$$a = 1$$
$$b = 6$$
$$c = -7$$

$$\therefore 2a + b + c = 2 \times 1 + 6 - 7$$
$$= 2 + 6 - 7$$
$$= 8 - 7 = 1$$

Q. 22 The order and the degree of the differential equation of all ellipses with centre at the origin, major axis along x-axis and
eccentricity \frac{\sqrt{3}}{2} are, respectively :

Option 1:
2, 2

Option 2:
1, 1

Option 3:
2, 1

Option 4:
1, 2

Correct Answer:
1, 1

Solution:
As we learned in

Order of a Differential Equation -

The order of a differential equation is the order of highest order occurring in a differential equation

- wherein

order of

$$\frac{d^2y}{dx^2}+5=0$$

is 2.

Let the equation of ellipse is

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \quad but\ 1-\frac{b^2}{a^2}=\frac{3}{4}$$

$$\therefore \frac{x^2}{4b^2}+\frac{y^2}{b^2}=1 \quad \therefore \frac{b^2}{a^2}=\frac{1}{4}$$

$$\therefore x^2+4y^2=4b^2 \quad \therefore a^2=4b^2$$

$$\therefore 2x+8y.\frac{dy}{dx}=0$$

$$\therefore x+4y.\frac{dy}{dx}=0$$

$$order=1$$
$$degree=1$$

Q. 23 The integral $\int \frac{dx}{(1+\sqrt{x})\sqrt{x-x^2}}$

is equal to: (where C is a constant of integration.)

Option 1:
$-2\sqrt{\frac{1+\sqrt{x}}{1-\sqrt{x}}}+C$

Option 2:
$-2\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}+C$

Option 3:
$-\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}+C$

Option 4:
$2\sqrt{\frac{1+\sqrt{x}}{1-\sqrt{x}}}+C$

Correct Answer:
$-2\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}+C$

Solution:

$$\mathrm{I}=\int \frac{\mathrm{dx}}{(1+\sqrt{\mathrm{x}})\sqrt{\mathrm{x}-\mathrm{x}^2}}$$

$$\text{put } \mathrm{x}=\cos^2\theta$$

$$\mathrm{dx}=-2\cos\theta\sin\theta\,\mathrm{d}\theta$$

$$I=\int \frac{-2\sin\theta\cos\theta d\theta}{(1+\cos\theta)\cos\theta\sin\theta}$$

$$=-2\int \frac{d\theta}{2\cos^2\theta/2}$$

$$=-\int \sec^2\left(\frac{\theta}{2}\right)\mathrm{d}\theta$$

$$=-2\tan\theta/2+c$$

as

$$\tan\left(\frac{\alpha}{2}\right)=\pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}$$

$\because \cos^2\theta = x \Rightarrow \cos\theta = \sqrt{x}$

$$I = -2\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}} + c$$

Q. 24 If *m* and *M* are the minimum and the maximum values of

$$4 + \frac{1}{2}\sin^2 2x - 2\cos^4 x\,, x \epsilon R,$$

then *M*−*m* is equal to :

Option 1:
$\frac{15}{4}$

Option 2:
$\frac{9}{4}$

Option 3:
$\frac{7}{4}$

Option 4:
$\frac{1}{4}$

Correct Answer:
$\frac{1}{4}$

Solution:
As we learnt in

Method for maxima or minima -

First and second derivative method :

$$Step\ 1.\ \ find\, values\, of\ x\ for\ \frac{dy}{dx} = 0$$

$$Step\ 2.\ \ x = x_\circ\ \ is\, a\, point\, of\, local\, maximum\, if\ \ f'(x) > 0$$
$$and\, local\, minimum\, if\ \ f'(x) < 0.$$

$$Step\ \ 3.\ \ \ x = x_\circ\ \ is\, a\, point\, of\, local\, mizimum\, if$$
$$f''(x) < 0\ \ and\, local\, minimum\, if\ \ f''(x) > 0$$

- wherein

$Where\ \ y=f(x)$

$\frac{dy}{dx}=f'(x)$

Let, $y=4+\frac{1}{2}sin^2 2x-2cos^4 x$

$\frac{dy}{dx}=0+\frac{1}{2}\cdot 2sin2x\cdot cos2x\times 2+8cos^3x\cdot sinx$

$=sin4x+8cos^3x\ \ sinx$

$\Rightarrow\ sin2x\left(4cos^2x-1\right)$

$0=sin2x\left(4cos^2x-1\right)$ So that $M=\frac{17}{4}$

$\therefore 2x=n\pi$ m = 4

$\therefore x=\frac{\pi}{2}$

and $cosx=\pm\frac{1}{2}$

$\therefore M-m=\frac{1}{4}$

Correct option is 4.

Q. 25 If $f(x)+2f\left(\frac{1}{x}\right)=3x, x\neq 0$, and $S=\{x\,\epsilon\,R:\ f(x)=f(-x)\}$; then S:

Option 1:
is an empty set

Option 2:
contains exactly one element

Option 3:
contains exactly two elements

Option 4:
contains more than two elements

Correct Answer:
contains exactly two elements

Solution:

$$f(x)+2f\left(\frac{1}{x}\right)=3x$$

Put $\frac{1}{x}$ at the place of

$$f\left(\frac{1}{x}\right)+2f(x)=\frac{3}{x} \qquad \text{(i)}$$

$$2f\left(\frac{1}{x}\right)+f(x)=3x \qquad \text{(ii)}$$

Multiplying (i) by 2

$$2f\left(\frac{1}{x}\right)+4f(x)=\frac{6}{x}$$

$$2f\left(\frac{1}{x}\right)+f(x)=3x$$

$$3f(x)=\frac{6}{x}-3x$$

$$f(x)=\frac{2}{x}-x$$

and $$f(-x)=\frac{2}{-x}+x$$

$$\therefore \quad \frac{2}{x}-x=-\frac{2}{x}+x$$

$$\Rightarrow \quad \frac{4}{x}-2x=0$$

$$\Rightarrow \quad \frac{4-2x^2}{x}=0$$

$$\Rightarrow \quad 4=2x^2$$

$$\Rightarrow \quad x^2=2$$

$$x=\pm\sqrt{2},\ x\neq 0$$

Correct option is 3.

Q. 26 For any three positive real numbers a, b and c, $9(25a^2+b^2)+25(c^2-3ac)=15b(3a+c)$. Then:

Option 1:
b, c and a are in A.P.

Option 2:
a, b and c are in A.P.

Option 3:
a, b and c are in G.P.

Option 4:
b, c and a are in G.P.

Correct Answer:
b, c and a are in A.P.

Solution:
$$9\left(25a^2+b^2\right)+25\left(c^2-3ac\right)=15b\left(3a+c\right)$$

$$=225a^2+9b^2+25c^2-75ac=45ab+15bc$$

$$225a^2+9b^2+25c^2-45ab-15bc-75ac=0$$

multiply and divide by 2

$$\frac{1}{2}\left[450a^2+18b^2+50c^2-90ab-30bc-150ac\right]=0$$

$225a^2+9b^2+25c^2+225a^2+9b^2+25c^2-2(15a)(3b)-2(3b)(5c)-2(5c)(15a) = 0$

$$\therefore (15a-3b)^2+(3b-5c)^2+(5c-15a)^2=0$$

So that 15a - 3b = 0 and 3b - 5c = 0

So that

$$15a=3b=5c=K$$

$$\therefore\ a=\frac{K}{15}$$

$$b=\frac{K}{3}$$

$$C=\frac{K}{5}$$

Now

$$2\times b=\frac{2K}{3}$$

$$And\ a+c=\frac{K}{15}+\frac{K}{5}$$

$$=\frac{(1+3)\,k}{15}$$

$$=\left(\frac{4K}{15}\right)$$

$\therefore$ a, b,c are not in A.P.

Now

$2c=2k/5$

And a + b = 2k/5

So b, c, a are in A.P.

Q. 27 The number of solutions of the equation, $\sin^{-1}x=2\tan^{-1}x$ (in principal values)is:

Option 1:
1

Option 2:
4

Option 3:
2

Option 4:
3

Correct Answer:
3

Solution:
Given equation is

$$\sin^{-1}\mathbf{x}=2\tan^{-1}\mathbf{x}$$
$$\sin^{-1}\mathbf{x}=\sin^{-1}\left(\frac{2\mathrm{x}}{1+\mathrm{x}^2}\right)$$
$$\mathrm{x}=\frac{2\mathrm{x}}{1+\mathrm{x}^2}$$
$$\mathrm{x}\left(1+\mathrm{x}^2\right)=2\mathrm{x}$$
$$\mathrm{x}+\mathrm{x}^3=2\mathrm{x}$$
$$\mathrm{x}^3-\mathrm{x}=0$$
$$\mathrm{x}=0,\ \mathrm{x}=1,\ \text{or } \mathrm{x}=-1$$

So, the number of solutions is 3.

Q. 28 If $\hat{a}$, $\hat{b}$ and $\hat{c}$ are unit vectors satisfying $\hat{a}-\sqrt{3}\hat{b}+\hat{c}=\vec{0}$, then the angle between the vectors $\hat{a}$ and $\hat{c}$ is :

Option 1:
$\frac{\pi}{4}$

Option 2:
$\frac{\pi}{3}$

Option 3:
$\frac{\pi}{6}$

Option 4:
$\frac{\pi}{2}$

Correct Answer:
$\frac{\pi}{3}$

Solution:

$$\text{Let angle between } \hat{a} \text{ and } \hat{c} \text{ be } \theta .$$
$$\text{Now, } \hat{a}-\sqrt{3}\hat{b}+\hat{c}=\overrightarrow{0}$$
$$\Rightarrow (\hat{a}+\hat{c})=\sqrt{3}\hat{b}$$
$$\Rightarrow (\hat{a}+\hat{c})\cdot(\hat{a}+\hat{c})=3(\hat{b}\cdot\hat{b})$$
$$\Rightarrow \hat{a}\cdot\hat{a}+\hat{a}\cdot\hat{c}+\hat{c}\cdot\hat{a}+\hat{c}\hat{c}=3\times 1$$
$$\Rightarrow 1+2\cos\theta+1=3$$
$$\Rightarrow \cos\theta=\frac{1}{2}\Rightarrow\theta=\frac{\pi}{3}$$

Q. 29 The sum of the digits in the unit's place of all the 4-digit numbers formed by using the numbers 3, 4, 5 and 6, without repetition, is :

Option 1:
432

Option 2:
108

Option 3:
36

Option 4:
18

Correct Answer:
108

Solution:
No. of 4 digits number formed = 4 !

Now each digit comes equal no. of times at unit place = 3!

Each digit appears at unit place

So sum = 3! (3+4+5+6)

= 6 x 18

= 108

Q. 30 $\lim_{x \to 0} \frac{(1-\cos 2x)^2}{2x\ \tan x - x \tan 2x} is$

Option 1:
−2

Option 2:
$-\frac{1}{2}$

Option 3:
$\frac{1}{2}$

Option 4:
2

Correct Answer:
−2

Solution:

$$\lim_{x \to 0} \frac{(1-\cos 2x)^2}{2x\ \tan x - x \tan 2x}$$

$$\lim_{x \to 0} \frac{4\sin^4 x}{x\,(2\tan x - \tan 2x)}$$

$$\because 1-\cos 2x = 2\sin^2 x$$

$$\lim_{x \to 0} 4\left(\frac{\sin x}{x}\right)^4 \frac{x^3}{(2\tan x - \tan 2x)} = \lim_{x \to 0} \frac{4x^3}{(2\tan x - \tan 2x)}$$

$$\because \lim_{x \to 0} \frac{\sin x}{x} = 1$$

Now use series expansion

$$\lim_{x \to 0} \frac{4x^3}{(2\tan x - \tan 2x)}$$

$$\lim_{x \to 0} \frac{4x^2}{2\left(x + \frac{x^3}{3} + \frac{2x^5}{15} + \ldots\right) - \left(2x + \frac{(2x)^3}{3} + 2\cdot\frac{(2x)^5}{15} + \ldots\right)}$$

$$\lim_{x \to 0} \frac{4x^3}{2x + \frac{2x^3}{3} + \frac{4x^5}{15} + \ldots - 2x - \frac{8x^3}{3} - \frac{64x^5}{15} - \ldots}$$

$$\lim_{x \to 0} \frac{4}{\frac{2}{3} + \frac{4x^2}{15} + \ldots - \frac{8}{3} - \frac{64x^2}{15} - \ldots}$$

$$\lim_{x \to 0} \frac{4}{\frac{2}{3} - \frac{8}{3}} = -2$$

Q. 31 The value of the integral

$$\int_4^{10} \frac{[x^2]\, dx}{[x^2 - 28x + 196] + [x^2]}, where\ [x]$$

denotes the greatest integer less than or equal to x, is :

Option 1:
6

Option 2:
3

Option 3:
7

Option 4:
$\frac{1}{3}$

Correct Answer:
3

Solution:

$$I=\int_4^{10}\frac{[x^2]}{[x^2-28x+196]+[x^2]}dx$$

$$I=\int_4^{10}\frac{[x^2]}{[(x-14)^2]+[x^2]}dx$$

Using Property $\int_a^b f(x)=\int_a^b f(a+b-x)$

$$I=\int_4^{10}\frac{[(x-14)^2]}{[x^2]+[(x-14)^2]}dx$$

$$2I=\int_4^{10}\frac{[x^2]+[(x-14)^2]}{[x^2]+[(x-14)^2]}dx$$

$$2I=\int_4^{10}1\cdot dx=x|_4^{10}=10-4=6$$

$$I=3$$

Q. 32 If p and q are non-zero real numbers and $\alpha^3+\beta^3=-p$, $\alpha\beta=q$, then a quadratic equation whose roots are $\frac{\alpha^2}{\beta},\frac{\beta^2}{\alpha}$ is :

Option 1:
$px^2-qx+p^2=0$

Option 2:
$qx^2+px+q^2=0$

Option 3:
$px^2+qx+p^2=0$

Option 4:
$qx^2-px+q^2=0$

Correct Answer:
$qx^2+px+q^2=0$

Solution:
Given $\alpha^3+\beta^3=-p$ and $\alpha\beta=q$

Let $\frac{\alpha^2}{\beta}$ and $\frac{\beta^2}{\alpha}$ be the root of required quadratic equation.

$$\text{So, sum of roots} = \frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = \frac{\alpha^3+\beta^3}{\alpha\beta} = \frac{-p}{q}$$

$$\text{and product of roots} = \frac{\alpha^2}{\beta} \times \frac{\beta^2}{\alpha} = \alpha\beta = q$$

Hence, the required quadratic equation is

$$x^2 - \left(\frac{-p}{q}\right)x + q = 0$$

$$\Rightarrow x^2 + \frac{p}{q}x + q = 0$$

$$\Rightarrow qx^2 + px + q^2 = 0$$

Q. 33 The two adjacent sides of a cyclic quadrilateral are 2 and 5 and the angle between them is 60^0. If the area of the quadrilateral is $4\sqrt{3}$ then the perimeter of the quadrilateral is :

Option 1:
12.5

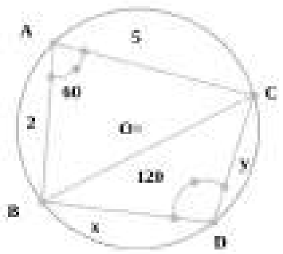
Option 2:
13.2

Option 3:
12

Option 4:
13

Correct Answer:
12

Solution:
As we have Learnt



Using concept ABDC, **Cyclic quadrilateral (opp. angles are supplementary),** and concept of properties of triangle

Area of $\Delta ABC = \frac{1}{2} \times 2.5 sin60^{\circ}$

$\Delta ABC = 5\frac{\sqrt{3}}{2}$

Area of $\Delta BCD = \frac{1}{2}xy\,sin120^\circ = \frac{1}{2}xy \times \frac{\sqrt{3}}{2}$

now,

$ar(\Delta ABC) + ar(\Delta BCD) = ar(ABDC)$

$5\frac{\sqrt{3}}{2} + \frac{1}{2}xy \times \frac{\sqrt{3}}{2} = 4\sqrt{3}$

$xy = 6$

$cos60^\circ = \frac{5^2 + 2^2 - d^2}{2 \times 5 \times 2}$ (Using cosine's rule)

$d = \sqrt{2^2 + 5^2 - 2.25cos60^\circ}$

$d^2 = 19$

and

$cos120^\circ = \frac{x^2 + y^2 - d^2}{2 \times x \times y}$ (Using cosine's rule)

$d^2 = x^2 + y^2 + xy$

$xy = 6$

Also

$$\begin{aligned}
&19 = x^2 + y^2 + xy \\
&19 + xy = x^2 + y^2 + 2xy \\
&25 = (x + y)^2 \\
&x + y = 5
\end{aligned}$$

Hence, Perimeter =(x+y)+5+2=5+7=12

Q. 34 Let a, b ∈ **R**, (a ≠ 0). If the function f defined as

$$f(x)=\begin{cases}\dfrac{2x^2}{a}, & 0\le x<1\\ a, & 1\le x<\sqrt{2}\\ \dfrac{2b^2-4b}{x^3}, & \sqrt{2}\le x<\infty\end{cases}$$

is continuous in the interval [0, ∞), then an ordered pair (*a, b*) is :

Option 1:
$\left(\sqrt{2}, 1-\sqrt{3}\right)$

Option 2:
$\left(-\sqrt{2}, 1+\sqrt{3}\right)$

Option 3:
$\left(\sqrt{2}, -1+\sqrt{3}\right)$

Option 4:
$\left(-\sqrt{2}, 1-\sqrt{3}\right)$

Correct Answer:
$\left(\sqrt{2}, 1-\sqrt{3}\right)$

Solution:
Given function is

$$f(x)=\begin{cases}\frac{2x^2}{a}, & 0\le x<1\\ a, & 1\le x<\sqrt{2}\\ \frac{2b^2-4b}{x^3}, & \sqrt{2}\le x<\infty\end{cases}$$

since the function is continuous

$$\Rightarrow \text{ continuous at } x=1 \text{ and } x=\sqrt{2}$$
$$\operatorname{Lim}_{x\to1^-} f(x)=\operatorname{Lim}_{x\to1^+} f(x)=f(1)$$
$$\Rightarrow \frac{2}{a}=a\Rightarrow a^2=2$$

and $\text{Lim}_{x\to\sqrt{2}} f(x) = \text{Lim}_{x\to\sqrt{2}} f(x) = f(\sqrt{2})$

$\Rightarrow a = \dfrac{2b^2 - 4b}{2\sqrt{2}}$

$\Rightarrow b^2 - 2b = \sqrt{2}a$

If $a = \sqrt{2}$ then $b^2 - 2b - 2 = 0 \Rightarrow b = 1 \pm \sqrt{3}$

If $a = -\sqrt{2}$ then $b^2 - 2b + 2 = 0 \Rightarrow b$ is imaginary which is not possible

$\Rightarrow (a, b) = (\sqrt{2}, 1 + \sqrt{3})$ or $(\sqrt{2}, 1 - \sqrt{3})$

Q. 35

If $A = \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}$, then adj $\left(3A^2 + 12A\right)$ is equal to :

Option 1:

$\begin{bmatrix} 51 & 63 \\ 84 & 72 \end{bmatrix}$

Option 2:

$\begin{bmatrix} 51 & 84 \\ 63 & 72 \end{bmatrix}$

Option 3:

$\begin{bmatrix} 72 & -63 \\ -84 & 51 \end{bmatrix}$

Option 4:

$\begin{bmatrix} 72 & -84 \\ -63 & 51 \end{bmatrix}$

Correct Answer:

$\begin{bmatrix} 51 & 63 \\ 84 & 72 \end{bmatrix}$

Solution:

As we leant in

Multiplication of matrices -

$$\begin{pmatrix} a11 & a12 & a13 \\ a21 & a22 & a23 \\ a31 & a32 & a33 \end{pmatrix} \begin{pmatrix} b11 & b12 & b13 \\ b21 & b22 & b23 \\ b31 & b32 & b33 \end{pmatrix} =$$

$$\begin{pmatrix} a11.b11 + a12.b21 + a13.b31 & a11.b12 + a12.b22 + a13.b32 & a11.b13 + a12.b23 + a13.b33 \\ a21.b11 + a22.b21 + a23.b31 & a21.b12 + a22.b22 + a23.b32 & a21.b13 + a22.b23 + a23.b33 \\ a31.b11 + a32.b21 + a33.b31 & a31.b12 + a32.b22 + a33.b32 & a31.b13 + a32.b23 + a33.b33 \end{pmatrix}$$

-

Adjoint of a square matrix -

Transpose of the matrix of co-factors of elements of A is called the adjoint of A

- wherein

$$\operatorname{adj}(\mathbf{A}) = \mathbf{C}^{\mathsf{T}} = \begin{pmatrix} +\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} & -\begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} & +\begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix} \\ -\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} & +\begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} & -\begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix} \\ +\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} & -\begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix} & +\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \end{pmatrix}$$

$$A = \begin{bmatrix} 2 & -4 \\ -4 & 1 \end{bmatrix}$$

$$\therefore A^2 = \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix} \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix} = \begin{bmatrix} 16 & -9 \\ -12 & 13 \end{bmatrix}$$

$$3A^2 + 12A = \begin{bmatrix} 48 & -27 \\ -36 & 39 \end{bmatrix} + \begin{bmatrix} 24 & -36 \\ -48 & 12 \end{bmatrix}$$

$$= \begin{bmatrix} 72 & -63 \\ -84 & 51 \end{bmatrix}$$

adj A= Transpose of cofactors

so that $\begin{bmatrix} 51 & 63 \\ 84 & 72 \end{bmatrix}$

Q. 36 The solution of the differential equation

$$\frac{dy}{dx}+\frac{y}{2}\sec x=\frac{\tan x}{2y}, where\, 0\leq x<\frac{x}{2},$$

and y(0)=1, is given by :

Option 1:
$$y=1-\frac{x}{\sec x+\tan x}$$

Option 2:
$$y^2=1+\frac{x}{\sec x+\tan x}$$

Option 3:
$$y^2=1-\frac{x}{\sec x+\tan x}$$

Option 4:
$$y=1+\frac{x}{\sec x+\tan x}$$

Correct Answer:
$$y^2=1-\frac{x}{\sec x+\tan x}$$

Solution:
Given differential equation is

$$\frac{dy}{dx}+\frac{y}{2}\sec x=\frac{\tan x}{2y}, where\, 0\leq x<\frac{x}{2},$$

$$2y\frac{dy}{dx}+y^2\sec x=\tan x$$

$$\text{put } y^2=t \quad\Rightarrow\quad 2\text{y}\,\frac{dy}{dx}=\frac{dt}{dx}$$

$$\frac{dt}{dx}+t\sec x=\tan x$$

$$\text{I.F. }=e^{\int\sec x dx}=e^{(n(\sec x+\tan x)}=\sec x+\tan x$$

$$t.(\sec x+\tan x)=\int(\sec x+\tan x)\tan x dx$$

$$=\int\sec x\tan x dx+\int\tan^2 x dx$$

$$y^2(\sec x+\tan x)=\sec x+\tan x-x+c$$

$$y(0)=1 \Rightarrow \quad c=0$$

$$\Rightarrow \quad y^2=1-\frac{x}{\sec x+\tan x}$$

Q. 37 For $x \epsilon R, x \neq 0$, if $y(x)$ is a differentiable function such that

$$x \int_1^x y(t)\, dt = (x+1) \int_1^x t\, y(t)\, dt,$$

then y(x)
equals : (where C is a constant.)

Option 1:
$\frac{C}{x} e^{-\frac{1}{x}}$

Option 2:
$\frac{C}{x^2} e^{-\frac{1}{x}}$

Option 3:
$\frac{C}{x^3} e^{-\frac{1}{x}}$

Option 4:
$C x^3 e^{\frac{1}{x}}$

Correct Answer:
$\frac{C}{x^3} e^{-\frac{1}{x}}$

Solution:

$$x \int_1^x y(t)\, dt = (x+1) \int_1^x t\, y(t)\, dt,$$

differentiate above equation

$$xy(x)+\int_1^x y(t)dt=(x+1)xy(x)+\int_1^x ty(t)dt$$
$$\int_1^x y(t)dt=x^2y(x)+\int_1^x ty(t)dt$$

Again differentiate

$$y(x) = 2xy(x) + x^2y'(x) + xy(x)$$
$$y = 3xy + x^2\frac{dy}{dx}$$
$$y(1-3x) = x^2\frac{dy}{dx}$$
$$\frac{(1-3x)}{x^2}dx = \frac{dy}{y}$$

Solve the differential equation

$$-\frac{1}{x} - 3\ln x = \ell ny + \ln c$$
$$-\frac{1}{x} = \ln x^3y + \ln c$$
$$x^3yc = e^{-1/x}$$
$$y = \frac{c}{x^3}e^{-1/x}$$

Q. 38 Let $f(x) = \sin^4 x + \cos^4 x$. Then f is an increasing function in the interval :

Option 1:

$\left]0, \frac{\pi}{4}\right[$

Option 2:

$\left]\frac{\pi}{4}, \frac{\pi}{2}\right[$

Option 3:

$\left]\frac{\pi}{2}, \frac{5\pi}{8}\right[$

Option 4:

$\left]\frac{5\pi}{8}, \frac{3\pi}{4}\right[$

Correct Answer:

$\left]\frac{\pi}{4}, \frac{\pi}{2}\right[$

Solution:

$$f(x) = \sin^4 x + \cos^4 x$$
$$f'(x) = 4\sin^3 x\cos x - 4\cos^3 x\sin x$$
$$f'(x) = 4\sin x\cos x\left(\sin^2 x - \cos^2 x\right)$$
$$f'(x) - 2\sin 2x \cdot \cos 2x$$
$$f'(x) - \sin 4x > 0$$
$$f'(x) = \sin 4x < 0$$
$$\Rightarrow \pi < 4x < 2\pi$$
$$\frac{\pi}{4} < x < \frac{\pi}{2}$$

Q. 39 For $x \epsilon R, \; f(x) = |\log 2 - \sin x| \;\; and \;\; g(x) = f(f(x))$, then

Option 1:
$g \;\; is \;\; not \;\; differentiable \;\; at \;\; x = 0$

Option 2:
$g'(0) = \cos(\log 2)$

Option 3:
$g'(0) = -\cos(\log 2)$

Option 4:
$g \;\; is \;\; differentiable \;\; at \;\; x = 0 \;\; and \;\; g'(0) = -\sin cos(\log 2)$

Correct Answer:
$g'(0) = \cos(\log 2)$

Solution:
As we learnt in

COMPOSITION OF FUNCTIONS -

Let f: A → B and g: B → C be two functions.
composition of f and g, denoted by g o f, then g o f (x) = g (f (x)), ∀ x ∈ A.

-

$$f(x) = |\log 2 - \sin x|$$

$$g(x) = f(f(x) = f(|\log 2 - \sin x|) = |\log 2 - \sin x|(\log 2 - \sin x)||$$

$$g'(x) = 0 - \cos(\log 2 - \sin x)(-\cos x) = \cos x\cos(\log 2 - \sin x)$$

$$g'(0) = \cos 0 \times \cos(\log 2 - \sin x) = 1 \times \cos(\log 2 - 0) = \cos\log 2$$

Correct option is 2.

Q. 40 Let $a, b, c, \epsilon R$ if $f\left(x\right)=ax^2+bx+c$ is such that a+b+c=3 and

$f(x+y)=f(x)+f(y)+xy\,\forall\, x, y\,\epsilon\, R\, then$

$\sum_{n=1}^{10} f(n)$ is equal to :

Correct Answer:
330

Solution:
As we learnt in

Summation of series of natural numbers -

$$\sum_{k=1}^{n} K=\frac{1}{2}n\left(n+1\right)$$

and

$$\sum_{k=1}^{n} K^2=\frac{1}{6}n\left(n+1\right)\left(2n+1\right)$$

Now,

f(x) = ax^2 + bx + c

a + b + c = 3

f(x+y) = f(x) + f(y)+xy

Then $\sum_{n=1}^{10} f(n)$

f(1) = a + b + c =3

∴ f(1) =3

put x = 1, y = 1

f(2) = f(1) + f(1) + 1 $\times$ 1

=2f(1)+1

=2 $\times$ 3+1=7

put x=1, y = 2

f(3)= f(1)+f(2)+2 $\times$1

=3+7+2

=12

Similarly f(4) = 18 (By putting x = 2 and y = 2)

$\therefore$ f(1)+f(2)+f(3)+ f(4)+.....------= 3+7+12+18---

Now, as differences of cosecutive terms are 4, 5, 6, ... which is an AP, so

Let $T_n = An^2+Bn+C$

put n =1: 3 = A+B+C

At n =2: 7 = 4A+2B+C

At n =3: 12 = 9A+3B+C

Subtracting these equations

4 = 3A+B and 5 = 5A+B

1= 2A

$\therefore A = \frac{1}{2}$

$\therefore$ B+C = $3 - \frac{1}{2} = \frac{5}{2}$

$\therefore$ 2B+C

$= 7 - 4 \times \frac{1}{2}$

=7-2 = 5

$B = \frac{5}{2}$

C=0

So $T_n = \frac{n^2}{2} + \frac{5n}{2}$

$$S_n = \sum T_n = \frac{1}{2} \cdot \frac{n(n+1)(2n+1)}{6} + \frac{5}{2} \times \frac{n(n+1)}{2}$$

$$= \frac{n(n+1)(2n+1)}{12} + \frac{5n(n+1)}{4}$$

put n =10

$$S_{10} = \frac{10 \times 11 \times 21}{12} + \frac{50 \times 11}{4}$$

$$= \frac{55 \times 7}{2} + \frac{25 \times 11}{2}$$

$$= \frac{385 + 275}{2}$$

$$= \frac{660}{2}$$

= 330

Q. 41 **Statement 1:** The number of common solutions of the trigonometric equations $2sin^2\theta - cos2\theta = 0$ and $2cos^2\theta - 3sin\theta = 0$ in the interval $[0, 2\pi]$ is two :

Statement 2 : The number of solutions of the equation, $2cos^2\theta - 3sin\theta = 0$ in the interval $[0, \pi]$ is two.

Option 1:
Statement 1 is true ; Statement 2 is true ; Statement 2 is a correct explanation for Statement 1

Option 2:
Statement 1 is true; Statement 2 is true; Statement 2 is not a correct explanation for Statement 1

Option 3:
Statement 1 is false; Statement 2 is true.

Option 4:
Statement 1 is true ; Statement 2 is false.

Correct Answer:
Statement 1 is true; Statement 2 is true; Statement 2 is not a correct explanation for Statement 1

Solution:

$2\sin^2\theta - \cos 2\theta = 0$
$\Rightarrow 2\sin^2\theta - \left(1 - 2\sin^2\theta\right) = 0$
$\Rightarrow 2\sin^2\theta - 1 + 2\sin^2\theta = 0$
$\Rightarrow 4\sin^2\theta = 1 \Rightarrow \sin\theta = \pm\frac{1}{2}$
$\therefore \theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

Now $2\cos^2\theta - 3\sin\theta = 0$
$\Rightarrow 2\left(1 - \sin^2\theta\right) - 3\sin\theta = 0$
$\Rightarrow -2\sin^2\theta - 3\sin\theta + 2 = 0$
$\Rightarrow -2\sin^2\theta - 4\sin\theta + \sin\theta + 2 = 0$
$\Rightarrow 2\sin^2\theta - \sin\theta + 4\sin\theta - 2 = 0$
$\Rightarrow \sin\theta(2\sin\theta - 1) + 2(2\sin\theta - 1) = 0$
$\Rightarrow \sin\theta = \frac{1}{2}, -2$
But $\sin\theta = -2$, is not possible
$\therefore \quad \sin\theta = \frac{1}{2}, \quad \Rightarrow \quad \theta = \frac{\pi}{6}, \frac{5\pi}{6}$

Hence, there are two common solutions, there each of the statement- 1 and 2 are true but statement- 2 is not a correct explanation for statement-1.

Q. 42 Let $\vec{a}\ and\ \vec{b}$ be two unit vectors such that $\left|\vec{a} + \vec{b}\right| = \sqrt{3}$ if $\vec{c} = \vec{a} + 2\vec{b} + 3\left(\vec{a} \times \vec{b}\right)$ then $2\left|\vec{c}\right|$ is equal to:

Option 1:
$\sqrt{55}$

Option 2:
$\sqrt{51}$

Option 3:
$\sqrt{43}$

Option 4:
$\sqrt{37}$

Correct Answer:
$\sqrt{55}$

Solution:
As we learnt in

Vector Product of two vectors(cross product) -

If $\vec{a}$ and $\vec{b}$ are two vectors and Θ is the angle between them , then $\vec{a}\times\vec{b}=\left|\vec{a}\right|\left|\vec{b}\right| Sin\Theta\hat{n}$

- wherein

$\hat{n}$ is unit vector perpendicular to both $\vec{a}\; and\, \vec{b}$

and

Scalar Product of two vectors (dot product) -

$\vec{a}\vec{b}=|a|\;|b|\;Cos\theta$

- wherein

Θ is the angle between the vectors $\vec{a}\; and\, \vec{b}$

$$\left|\vec{a}+\vec{b}\right|=\sqrt{3}$$

Squaring both sides

$$|\vec{a}|^2+\left|\vec{b}\right|^2+2\,|\vec{a}|\,\left|\vec{b}\right|\,cos\theta=3$$

$$cos\theta=\frac{1}{2}\;\Rightarrow\theta=\frac{\pi}{3}$$

$$\vec{c}=\vec{a}+2\vec{b}+3\,|\vec{a}|\,\left|\vec{b}\right|\,sin\frac{\pi}{3}\hat{n}$$

$$\vec{c}=\vec{a}+2\vec{b}+\frac{3\sqrt{3}}{2}\hat{n}$$

$$\vec{c}\cdot\vec{c}=\left(\vec{a}+2\vec{b}+\frac{3\sqrt{3}}{2}\hat{n}\right)\cdot\left(\vec{a}+2\vec{b}+\frac{3\sqrt{3}}{2}\hat{n}\right)$$

$$|\vec{c}|^2=|\vec{a}|^2+4\left|\vec{b}\right|^2+\frac{27}{4}+4\left(\vec{a}\cdot\vec{b}\right)$$

$$|\vec{c}|^2=1+4+\frac{27}{4}+4\times1\times1\times\frac{1}{2}$$

$$|\vec{c}|^2=7+\frac{27}{4}=\frac{55}{4}$$

$$|\vec{c}|^2=\frac{\sqrt{55}}{2}$$

$$2\,|\vec{c}|=\sqrt{55}$$

Q. 43 If a variable plane, at a distance of 3 units from the origin, intersects the coordinate axes at A, B and C, then the locus of the centroid of ABC is :

Option 1:
$$\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=1$$

Option 2:
$$\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=3$$

Option 3:
$$\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\frac{1}{9}$$

Option 4:
$$\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=9$$

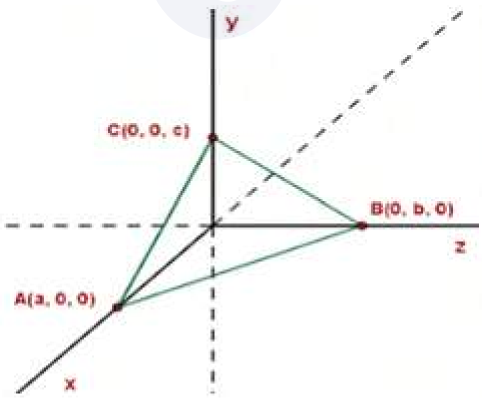
Correct Answer:
$$\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=1$$

Solution:
As we learnt

Intercept form -

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$$

- wherein

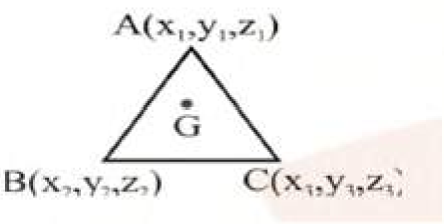


Let the plane cuts the coordinate axis at $A\left(a,0,0\right),B\left(0,b,0\right)\ and\ C\left(0,0,c\right)$

Centroid of triangle -

$$\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right)$$

- wherein



Distance of a point from plane (Cartesian form) -

The length of perpendicular from $P(x_1, y_1, z_1)$ to the plane

$ax+by+cz+d=0$ is given by $\dfrac{[ax_1+by_1+cz_1+d]}{\left|\sqrt{a^2+b^2+c^2}\right|}$

-

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$$

where, $\dfrac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}}=3$

$$\Rightarrow \frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}=\frac{1}{9}$$

$$G=(h_3k,l)=\left(\frac{a}{3},\frac{b}{3},\frac{c}{3}\right)$$

So, $\dfrac{1}{9h^2}+\dfrac{1}{9k^2}+\dfrac{1}{9l^2}=\dfrac{1}{9}$

$$\Rightarrow \frac{1}{h^2}+\frac{1}{k^2}+\frac{1}{l^2}=1$$

$$\Rightarrow \frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=1$$

Q. 44 If for positive integers $r > 1, n > 2$, the coefficients of the $(3r)^{th}$ and $(r+2)^{th}$ powers of x in the expansion of $(1+x)^{2n}$ are equal,then n is equal to:

Option 1:
$2r+1$

Option 2:
$2r-1$

Option 3:
$3r$

Option 4:
$r+1$

Correct Answer:
$2r+1$

Solution:
General term of the expansion $(p+q)^n$ is
$T_{r+1} = {}^nC_r p^{n-r} q^r \quad \text{for } r = 0, 1, \ldots, n$

$(1+x)^{2n} = 1 + {}^{2n}\mathrm{C}_1 x + {}^{2n}\mathrm{C}_2 x^2 + \ldots + {}^{2n}\mathrm{C}_r x^r + {}^{2n}\mathrm{C}_{r+1} x^{r+1} + \ldots + {}^{2n}\mathrm{C}_{2n} x^{2n}$
As given ${}^{2n}\mathrm{C}_{r+2} = {}^{2n}\mathrm{C}_{3r}$

$\because {}^n\mathrm{C}_x = {}^n\mathrm{C}_y \Rightarrow x + y = n$

$\therefore r + 2 + 3r = 2n$
$\Rightarrow 2n = 4r + 2$
$\Rightarrow n = 2r + 1$

Q. 45 A bird is sitting on the top of a vertical pole 20 m high and its elevation from a point O on the ground is 45^0. It flies off horizontally straight away from the point O.After one second, the elevation of the bird from O is reduced to 30^0. Then the speed (in m/s) of the bird is :

Option 1:
$20\sqrt{2}$

Option 2:
$20\left(\sqrt{3}-1\right)$

Option 3:
$40\left(\sqrt{2}-1\right)$

Option 4:
$40\left(\sqrt{3}-\sqrt{2}\right)$

Correct Answer:
$20\left(\sqrt{3}-1\right)$

Solution:
As we learnt in

Height and Distances -

The height or length of an object or the distance between two distant objects can be determined with the help of trigonometric ratios.

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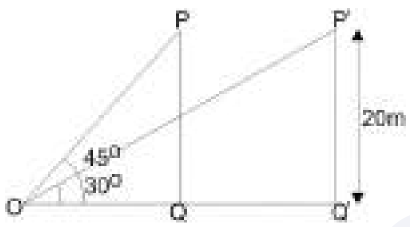


$$In\ \Delta PQQ\ , tan45^\circ = \frac{PQ}{OQ}$$

$$\Rightarrow OQ = 20m$$

$$In\ \Delta P'Q'O,\ tan30^\circ = \frac{20}{OQ'} \Rightarrow OQ' = 20\sqrt{3}$$

$$Thus\ QQ' = 20\sqrt{3} - 20 = 20(\sqrt{3} - 1)$$

$$Thus\ speed = distance/time = \frac{20\left(\sqrt{3}-1\right)}{1} = 20\left(\sqrt{3}-1\right)\ m/sec$$

Q. 46 Following is an infeasible solution for the linear programming model given below:

Maximize, $Z = x_1 + 2x_2$

Subject to constraint $1 : x_1 + x_2 \leq 5$

Constraint 2 $2 : x_1 + 3x_2 \leq 9$

Option 1:
$(x_1, x_2) = (1, 3)$

Option 2:
$(x_1, x_2) = (1, 1)$

Option 3:
$(x_1, x_2) = (3, 1)$

Option 4:
$None\ of\ the\ above$

Correct Answer:
$(x_1, x_2) = (1, 3)$

Solution:

Different Types of Linear Programming Problems -

-

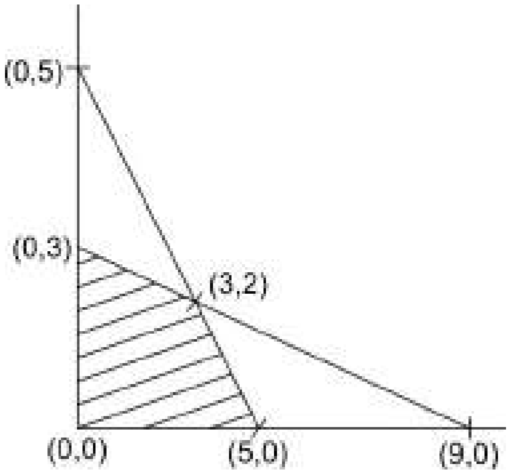
As we learnt in

Corner Point Method -

This method of solving a LPP graphically is based on the principle of extreme points theorem.

-

$Z = x_1 + 2x_2$



$x_1 + x_2 \leq 5$

$x_1 + 3x_2 \leq 9$

$x_1 = 3$

$x_2 = 2$

$$z = 1 + 6 = 7 \ at(1,3)$$
$$= 1 + 2 = 3 \ at(1,1)$$
$$= 3 + 2 = 5 \ at(3,1)$$

At (1,3), it is maximum

Q. 47 If the line $\frac{x-3}{1} = \frac{y+2}{-1} = \frac{z+\lambda}{-2}$ lies in the plane,

2x−4y+3z=2,

then the shortest distance between this line and the line

$\frac{x-1}{12} = \frac{y}{9} = \frac{z}{4}$ is

Option 1:
2

Option 2:
1

Option 3:
0

Option 4:
3

Correct Answer:
0

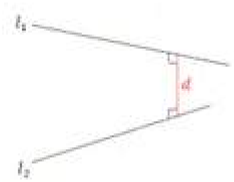
Solution:
As we have learned

Shortest distance between two skew lines (vector form) -

Shortest distance between $\vec{r}=\vec{a}+\lambda\vec{b}\ and\ \vec{r}=\vec{a_1}+\mu\vec{b}$ is given by

$$\left|\frac{\left(\vec{b}\times\vec{b_1}\right)\cdot(\vec{a}-\vec{a_1})}{\left|\vec{b}\times\vec{b_1}\right|}\right|$$

- wherein



shortest distance is among the line which is perpendicular to both

$$LM=\vec{b}\times\vec{b_1}$$

shortest distance will be projection of PQ = $\vec{a}-\vec{a_1}$ on LM

$$(3,-2,-\lambda)\ \ lies\ \ on\ \ plane$$

So, $\begin{array}{l} 6+8-3\lambda=2 \\ \Rightarrow 3\lambda=12 \\ \lambda=4 \end{array}$

Where $\vec{b}\times\vec{c}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -1 & -2\\ 12 & 9 & 4\end{vmatrix}=14\hat{i}-28\hat{j}+21\hat{k}$

So distance = 0

Q. 48 If the 7th term in the binomial expansion of $\left(\frac{3}{\sqrt[3]{84}}+\sqrt{3}\ln x\right)^9, x>0$, is equal to 729, then x can be :

Option 1:
e^2

Option 2:
e

Option 3:
$\frac{e}{2}$

Option 4:
$2e$

Correct Answer:
e

Solution:
Given expansion is

$$\left(\frac{3}{\sqrt[3]{84}}+\sqrt{3}\ln x\right)^9$$

We have
$\mathrm{T}_{r+1}={}^n\mathrm{C}_r(x)^{n-r}a^r$ for $(x+a)^n$

According to the question

$$T_7={}^9\mathrm{C}_6\left(\frac{3}{\sqrt[3]{84}}\right)^3\cdot(\sqrt{3}\ln x)^6=729$$

$$\Rightarrow 3^6=84\times\frac{3^3}{84}\times 3^3\times(6\ln x)$$
$$\Rightarrow(\ln x)^6=1\Rightarrow(\ln x)^6=(\ln e)^6$$
$$\Rightarrow x=\mathrm{e}$$

Q. 49 The number of values of α in $[0,2\pi]$ for which $2\sin^3\alpha-7\sin^2\alpha+7\sin\alpha=2,\ \ is:$

Correct Answer:
3

Solution:
As we learnt in

Trigonometric Equations -

The equations involving trigonometric function of unknown angles are known as trigonometric equations.

- wherein

e.g. $\cos^2\Theta - 4\cos\Theta = 1$

$$2\sin^3\alpha - 7\sin^2\alpha + 7sin\alpha = 2$$

$$\Rightarrow (2sin^2\alpha - 5sin\alpha + 2)\,(sin\alpha - 1) = 0$$

$$\Rightarrow sin\alpha = 1 \;\; or \;\; sin\alpha = \frac{5 \pm \sqrt{25-16}}{4} = \frac{8}{4}$$

Hence, possible solutions are $sin\alpha = 1 \; or \; \frac{1}{2}$

Hence solutions are $\frac{\pi}{6}, \frac{\pi}{2} \; and \; \frac{3\pi}{6}$

Q. 50 Three people P, Q and R independently try to hit a target. If the probabilities of their hitting the target are $\frac{3}{4}, \frac{1}{2}$ and $\frac{5}{8}$ respectively, then the probability that the target is hit by P or Q but not by R is :

Option 1:
$\frac{21}{64}$

Option 2:
$\frac{9}{64}$

Option 3:
$\frac{15}{64}$

Option 4:
$\frac{39}{64}$

Correct Answer:
$\frac{21}{64}$

Solution:
As we learnt in

Addition Theorem of Probability -

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

in general:

$$P(A_1 \cup A_2 \cup A_3 \cdots A_n) = \sum_{i=1}^{n} P(A_i) - \sum_{i<j}^{n} P(A_i \cap A_j) + \sum_{i<j<k}^{n} P(A_i \cap A_j \cap A_k) -$$
$$\cdots - (-1)^{n-1} P(A_1 \cap A_2 \cap A_3 \cdots \cap A_n)$$

-

Independent events -

Two or more events are said to be independent if occurrence or non-occurrence of any of them does not affect the probability of occurrence of or non - occurrence of other events.

-

$$P(P) = \frac{3}{4}; P(Q) = \frac{1}{2}; P(R) = \frac{5}{8}$$

$$P(P\ or\ Q) = P(P) + P(Q) - P(P \cap Q)$$

$$= \frac{3}{4} + \frac{1}{2} - \frac{3}{8}$$

$$= \frac{7}{8}$$

$$P(\bar{R}) = \frac{3}{8}$$

P (P or Q not R) =$\frac{7}{8} \times \frac{3}{8} = \frac{21}{64}$

Q. 51 Let z satisfy $|z| = 1$ and $z = 1 - \bar{z}$.

Statement I : z is a real number.

Statement II: Principal argument of z is $\frac{\pi}{3}$.

Option 1:
Statement I is true;Statement II is true;Statement II is a correct explanation for statement I.

Option 2:
Statement I is false; Statement II is true .

Option 3:
Statement I is true;Statement II is false.

Option 4:
Statement I is true;Statement II is true;Statement II is not a correct explanation for statement I.

Correct Answer:
Statement I is false; Statement II is true .

Solution:
$$\text{Let } z = x + iy,\ \bar{z} = x - iy$$
$$\text{Now, } z = 1 - \bar{z}$$
$$\Rightarrow x + iy = 1 - (x - iy)$$
$$\Rightarrow 2x = 1 \Rightarrow x = \frac{1}{2}$$
$$\text{Now, } |z| = 1 \Rightarrow x^2 + y^2 = 1 \Rightarrow y^2 = 1 - x^2$$
$$\Rightarrow y = \pm\frac{\sqrt{3}}{2}$$
$$\text{Now, } \tan\theta = \frac{y}{x} \quad (\theta \text{ is the argument })$$
$$\tan\theta = \frac{\sqrt{3}/2}{1/2} = \sqrt{3}$$
$$\theta = \tan^{-1}\sqrt{3} = \frac{\pi}{3}$$

Hence, z is not a real number
So, statement-1 is false and 2 is true.

Q. 52 Let k be an integer such that the triangle with vertices (k, −3k), (5, k) and (−k, 2) has area 28 sq. units. Then the value of k is

Option 1:
3

Option 2:
1

Option 3:
2

Option 4:
None of these

Correct Answer:
2

Solution:
Area = 28 sq units

$$Area = \frac{1}{2}\left|k(k-2)+5(2+3k)-k(-3k-k)\right| = 28$$

$$|5k^2+13k+10| = 56$$

$$5k^2+13k+10=56 \quad or \quad 5k^2+13k+10=-56$$

$5k^2+13k-46=0$ or $5k^2+13k+66=0$

On solving k= 2 is the only integral solution

Q. 53 The integral $\int \frac{2x^{12}+5x^9}{\left(x^5+x^3+1\right)^3} dx$ is equal to:

Option 1:

$$\frac{-x^5}{\left(x^5+x^3+1\right)^2}+C$$

Option 2:

$$\frac{x^{10}}{2\left(x^5+x^3+1\right)^2}+C$$

Option 3:

$$\frac{x^5}{2\left(x^5+x^3+1\right)^2}+C$$

Option 4:

$$\frac{-x^{10}}{2\left(x^5+x^3+1\right)^2}+C$$

Correct Answer:

$$\frac{x^{10}}{2\left(x^5+x^3+1\right)^2}+C$$

Solution:
As learnt in concept

Integration by substitution -

The functions when on substitution of the variable of integration to some quantity gives any one of standard formulas.

- wherein

Since $\int f(x)dx = \int f(t)dt = \int f(\theta)d\theta$ all variables must be converted into single variable , $(t \ or \ \theta)$

$$\int \frac{2x^{12}+5x^9}{(x^5+x^3+1)^3}dx$$

$$= \int \frac{\left(\frac{2}{x^3}+\frac{5}{x^6}\right)}{\left(1+\frac{1}{x^2}+\frac{1}{x^5}\right)^3}dx$$

$$Put \ 1+\frac{1}{x^2}+\frac{1}{x^5}=t$$

Differentiating, $\left(\frac{-2}{x^3}-\frac{5}{x^6}\right)dx = dt$

$$-\left(\frac{2}{x^3}+\frac{5}{x^6}\right)dx = dt$$

$$=> \int \frac{-dt}{t^3} = \frac{t^{-2}}{2}+C$$

$$= \frac{x^{10}}{2\left(x^5+x^3+1\right)^2}+C$$

Q. 54 if $\frac{cos\theta}{1-sin\theta} = \frac{p}{q} \ (p \neq q \neq 0), \ \ then \ \ cot\left(\frac{\pi}{4}+\frac{\theta}{2}\right)$ is equal to :

Option 1:
$\sqrt{\frac{p}{q}}$

Option 2:
$\frac{q}{p}$

Option 3:
$\sqrt{pq}$

Option 4:
pq

Correct Answer:
$\frac{q}{p}$

Solution:

$$Given,\ \frac{cos\theta}{1-sin\theta}=\frac{p}{q}$$

$$\frac{cos^2(\theta/2)-sin^2(\theta/2)}{cos^2(\theta/2)+sin^2(\theta/2)-2sin(\theta/2)cos(\theta/2)}=\frac{p}{q}$$

$$\frac{(cos(\theta/2)+sin(\theta/2))(cos(\theta/2)-sin(\theta/2))}{(cos(\theta/2)-sin(\theta/2))^2}=\frac{p}{q}$$

$$\frac{(cos(\theta/2)+sin(\theta/2))}{(cos(\theta/2)-sin(\theta/2))}=\frac{p}{q}$$

Dividing numerator and denominator by $cos(\theta/2)$

$$\frac{(1+tan(\theta/2))}{(1-tan(\theta/2))}=\frac{p}{q}$$

$$tan(\pi/4+\theta/2)=p/q$$

Q. 55 If a complex number z satisfies the equation $z+\sqrt{2}\,|z+1|+i=0$, then $|z|$ is equal to :

Option 1:
2

Option 2:
$\sqrt{3}$

Option 3:
$\sqrt{5}$

Option 4:
1

Correct Answer:
$\sqrt{5}$

Solution:
Given equation is

$$z+\sqrt{2}|z+1|+i=0$$

put z = x + iy in the above equation

$$(x+iy)+\sqrt{2}|x+iy+1|+i=0$$
$$\Rightarrow x+iy+\sqrt{2}\left[\sqrt{(x+1)^2+y^2}\right]+i=0$$

Now, equating real and imaginary part, we get

$$x+\sqrt{2}\sqrt{(x+1)^2+y^2}=0 \text{ and}$$
$$y+1=0 \Rightarrow y=-1$$
$$\Rightarrow x+\sqrt{2}\sqrt{(x+1)^2+(-1)^2}=0 \quad (\because y=-1)$$
$$\Rightarrow \sqrt{2}\sqrt{(x+1)^2+1}=-x$$
$$\Rightarrow 2\left[(x+1)^2+1\right]=x^2$$
$$\Rightarrow x^2+4x+4=0$$
$$\Rightarrow x=-2$$

z = -2 - i

$$|z|=\sqrt{(-2)^2+(-1)^2}=\sqrt{5}$$

Q. 56 The radius of a circle, having minimum area, which touches the curve $y=4-x^2$

and the lines, $y=|x|$ is:

Option 1:
$2\left(\sqrt{2}-1\right)$

Option 2:
$4\left(\sqrt{2}-1\right)$

Option 3:
$4\left(\sqrt{2}+1\right)$

Option 4:
$2\left(\sqrt{2}+1\right)$

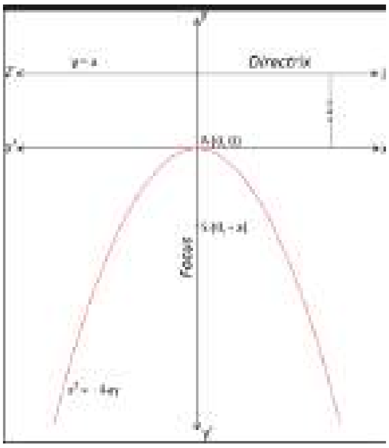
Correct Answer:
$4\left(\sqrt{2}-1\right)$

Solution:
As we learnt in

Standard equation of parabola -

$$x^2=-4ay$$

- wherein



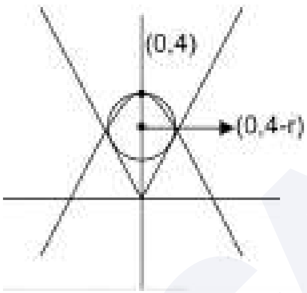
and concept

Perpendicular distance of a point from a line -

$$\rho=\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$$

- wherein

ρ is the distance from the line $ax+by+c=0$.



This figure shows the circle with least area satisfying the conditions.

Circle touches y=x;

In the figure, centre (x,y) of the circle is (0,4-r)

Now using distance formula to determine the radius of the circle,

$$\therefore\left|\frac{0-(4-r)}{\sqrt{2}}\right|=r(distance\ of\ point\ from\ a\ line)$$

$$4-r=\sqrt{2}r$$

$$r=\frac{4}{\sqrt{2}+1}$$

On rationalising,

$\Rightarrow r = 4(\sqrt{2} - 1)$

Q. 57 $\lim_{n \to \infty} \left(\frac{(n+1)(n+2).....3n}{n^{2n}} \right)^{\frac{1}{n}}$ is equal to

Option 1:
$\frac{18}{e^4}$

Option 2:
$\frac{27}{e^2}$

Option 3:
$\frac{9}{e^2}$

Option 4:
$3 \log 3 - 2$

Correct Answer:
$\frac{27}{e^2}$

Solution:
As we learnt

Walli's Method -

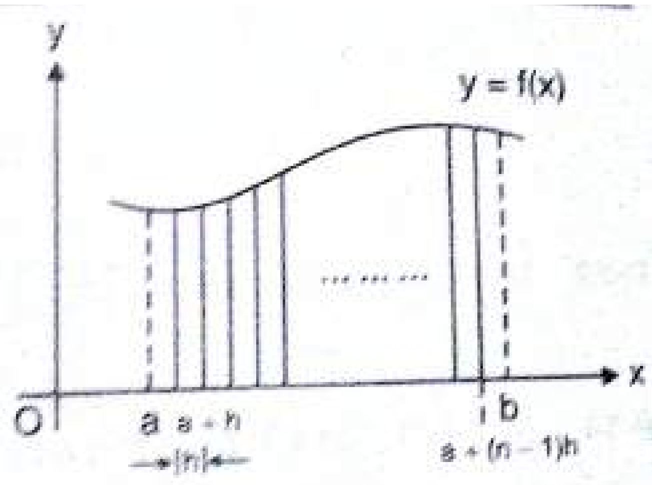
Definite integral by first principle

$$\int_a^b f(x)dx = (b-a) \lim_{n \to \infty} \frac{1}{n} [f(a) + f(a+h) + f(a+2h)....]$$

where

$$h = \frac{b-a}{n}$$

- wherein



$$y=\lim_{n\to\infty}\left[\frac{(n+1)(n+2)\ldots(n+2n)}{n^{2n}}\right]^{1/n}$$

$$=\lim_{n\to\infty}\left[\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right)\ldots\left(1+\frac{2n}{n}\right)\right]^{1/n}$$

$$\Rightarrow \log y=\lim_{n\to\infty}\frac{1}{n}\left[\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right)\ldots\left(1+\frac{2n}{n}\right)\right]$$

$$=\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{2n}\log\left(1+\frac{r}{n}\right)$$

$$=\int_0^2\log(1+x)\,dx$$

$$=[\log(1+x).x]-\int_0^2\frac{1}{1+x}.xdx$$

$$=[x\log(1+x)-x+\log(1+x)]_0^2$$

$$=2\log 3-2+\log 2$$

$$\Rightarrow y=\frac{27}{e^2}$$

Q. 58 Two ships A and B are sailing straight away from a fixed point O along routes such that $\angle$ AOB is always 120^0. At a certain instance, OA = 8 km, OB = 6 km and the ship A is sailing at the rate of 20 km/hr while the ship B sailing at the rate of 30 km/hr. Then the distance between A and B is changing at the rate (in km/hr) :

Option 1:
$\frac{260}{\sqrt{37}}$

Option 2:
$\frac{260}{37}$

Option 3:
$\frac{80}{\sqrt{37}}$

Option 4:
$\frac{80}{37}$

Correct Answer:
$\frac{260}{\sqrt{37}}$

Solution:
As we learnt in

Height and Distances -

The height or length of an object or the distance between two distant objects can be determined with the help of trigonometric ratios.

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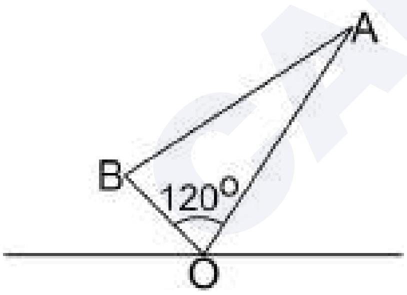

$$In \triangle OAB,\ AB^2 = OA^2 + OB^2 - 2OA.OB.\cos 120^\circ$$

$$= OA^2 + OB^2 - 2OA.OB\left(-\frac{1}{2}\right)$$

$$= OA^2 + OB^2 + OA \cdot OB$$

Now, $\frac{d(AB)^2}{dt} = \frac{d(OA^2 + OB^2 + OA.OB)}{dt}$ [Differenting both side by t]

$$\Rightarrow 2AB \cdot \frac{d(AB)}{dt} = 2OA \cdot \frac{d(OA)}{dt} + 2OB \cdot \frac{d(OB)}{dt} + OA \cdot \frac{d(OB)}{dt} + OB \cdot \frac{d(OA)}{dt} \quad \cdots\cdot(1)$$

Now, at the given instant, OA=8, OB=6 $\frac{d(OA)}{dt} = 20$ .... (2)

$$\frac{d(OB)}{dt} = 30$$

$Now, AB = \sqrt{36+64+48} = \sqrt{148} = 2\sqrt{37}$ .... (2)

Using (1) and (2),

$$\Rightarrow 2 \cdot\ 2\sqrt{37} \cdot \frac{d(AB)}{dt} = 2 \cdot 8 \cdot 20 + 2 \cdot 6 \cdot 30 + 8 \cdot 30 + 6 \cdot 20$$

$$\frac{d(AB)}{dt} = \frac{160+180+120+60}{2\sqrt{37}} = \frac{260}{\sqrt{37}}$$

Q. 59 The eccentricity of an ellipse whose centre is at the origin is $\frac{1}{2}$. If one of its directrices is x = −4, then the equation of the normal to it at $\left(1, \frac{3}{2}\right)$ is:

Option 1:
$4x - 2y = 1$

Option 2:
$4x + 2y = 7$

Option 3:
$x + 2y = 4$

Option 4:
$2y - x = 2$

Correct Answer:
$4x - 2y = 1$

Solution:

We have $e = \frac{1}{2}$

Directrix: x = - 4

$$-\frac{a}{e}=-4 \Rightarrow a=4e$$

So a = 2

Now, $b^2=a^2(1-e^2)=3$

Hence, the equation of the ellipse is

$$\frac{x^2}{4}+\frac{y^2}{3}=1$$

Using point form of normal, the equation of normal at $(1, \frac{3}{2})$ is

$$\frac{a^2x}{x_1}-\frac{b^2y}{y_1}=a^2-b^2$$

$$\frac{4x}{1}-\frac{3y.2}{3}=4-3$$

$$4x-2y=1$$

Q. 60 A hyperbola passes through the point $P\left(\sqrt{2}, \sqrt{3}\right)$ and has foci at (±2, 0).

Then the tangent to this hyperbola at P also passes through the point

Option 1:
$\left(2\sqrt{2}, 3\sqrt{3}\right)$

Option 2:
$\left(\sqrt{3}, \sqrt{2}\right)$

Option 3:
$\left(-\sqrt{2}, -\sqrt{3}\right)$

Option 4:
$\left(3\sqrt{2}, 2\sqrt{3}\right)$

Correct Answer:
$\left(2\sqrt{2}, 3\sqrt{3}\right)$

Solution:
As foci lie on x-axis and mid-point of foci is th eorigin, so the equation of hyperbola can be assumed to be $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

Focus is $(\pm 2, 0)$ $\Rightarrow ae=2;$

Also we know, $b^2 = a^2(e^2 - 1) \Rightarrow b^2 = a^2e^2 - a^2 \Rightarrow a^2 + b^2 = 4$

Hyperbola passes through $(\sqrt{2}, \sqrt{3})$, so

$$\frac{2}{a^2} - \frac{3}{b^2} = 1$$

Solving these 2 equations we get

$$a^2 = 1, b^2 = 3$$

Equation of hyperbola is $\frac{x^2}{1} - \frac{y^2}{3} = 1$

Equation of tangent at P using point form is $\sqrt{2}x - \frac{\sqrt{3}y}{3} = 1$

Out of the given options $(2\sqrt{2}, 3\sqrt{3})$ satisfies it.