CAREERS360

PRACTICE Series

JEE Main 2026

Sample Paper

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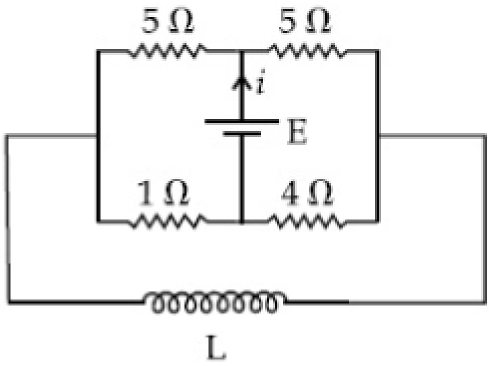
Q. 1 The current (i) at time t=0 and t=∞ respectively for the given circuit is:



Option 1:
$\frac{5E}{18}, \frac{10E}{33}$

Option 2:
$\frac{10E}{33}, \frac{5E}{18}$

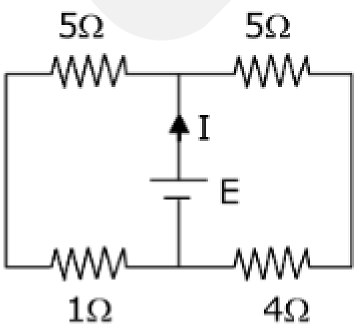
Option 3:
$\frac{5E}{18}, \frac{18E}{33}$

Option 4:
$\frac{18E}{55}, \frac{5E}{18}$

Correct Answer:
$\frac{5E}{18}, \frac{10E}{33}$

Solution:
At t = 0, inductor is open

So the corresponding equivalent circuit is given below

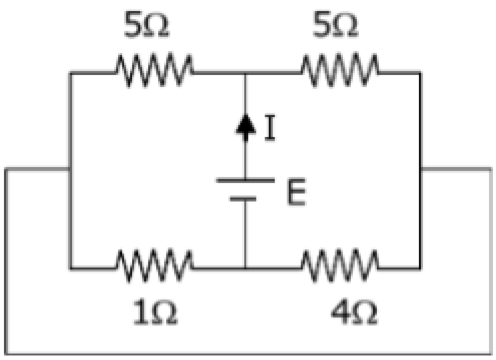


$$R_{eq} = \frac{6 \times 9}{6+9} = \frac{54}{15}$$

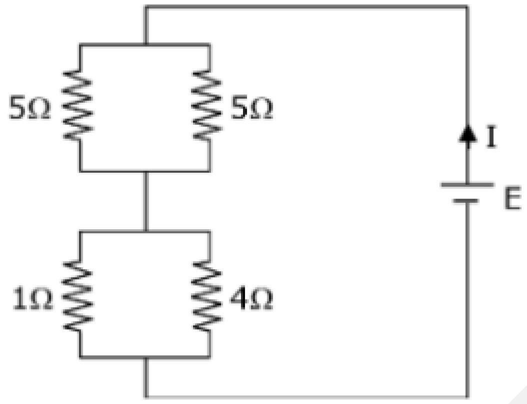
$$I(at\ t=0)=\frac{15E}{54}=\frac{5E}{18}$$

At $t=\infty$, For steady state inductor is replaced by plane wire

So the corresponding equivalent circuit is given below



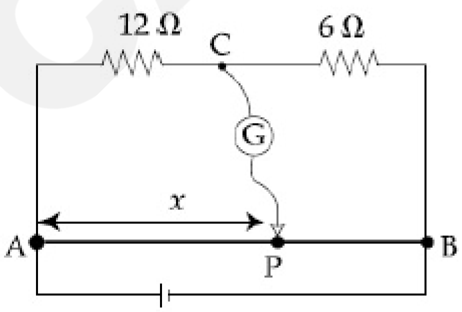
We can reduce the above circuit to the below circuit.



$$\mathrm{R_{eq}}=\left(\frac{1\times4}{1+4}\right)+\left(\frac{5\times5}{5+5}\right)=\frac{4}{5}+\frac{5}{2}=\frac{8+25}{10}=\frac{33}{10}$$

$$\mathrm{I}=\frac{\mathrm{E}}{\mathrm{R_{eq}}}=\frac{10\mathrm{E}}{33}$$

Q. 2 Consider a 72cm long AB as shown in the figure. The galavnometer jockey is placed at P on AB at a distance xcm from A. The galvanometer shows zero deflection.



The value of x, to the nearest integer, is ____

Correct Answer:
48

Solution:
$\text{In Balanced conditions}$

$$\frac{12}{6}=\frac{x}{72-x}$$

$$x=48cm$$

Q. 3 A liquid of density ρ is coming out of a hose pipe of radius a with horizontal speed v and hits a mesh. 50% of the liquid passes through the mesh unaffected. 25% looses all of its momentum and 25% comes back with the same speed. The resultant pressure (in ρv^2) on the mesh will be:

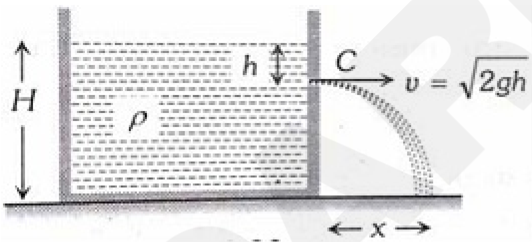
Correct Answer:
0.75

Solution:

Torricelli's Theorem / Velocity of Efflux -

In fluid dynamics relating the speed of fluid flowing out of an orifice.

- wherein



Momentum per second carried by liquid per second

$\rho a V^2$

Net force

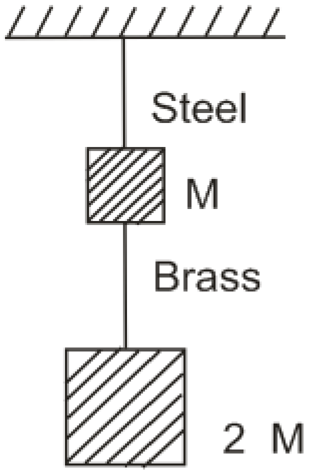
1. due to reflected liquid = $2[\frac{1}{4}\rho a V^2]$

2. due to stopped liquid = $\frac{1}{4}\rho a V^2$

Total force = $\frac{3}{4}\rho a V^2$

$\because P = \frac{3}{4}\rho V^2$

Q. 4 If the ratio of lengths, radii and Young's moduli of steel and brass wires in the figure are a, b and c respectively, then the corresponding ratio of increase in their lengths is :



Option 1:

$\frac{3c}{2ab^2}$

Option 2:

$\frac{2a^2c}{b}$

Option 3:

$\frac{3a}{2b^2c}$

Option 4:

$\frac{3ac}{b^2}$

Correct Answer:

$\frac{3a}{2b^2c}$

Solution:

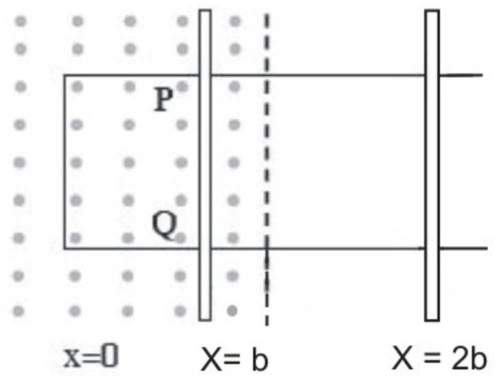
$$T_S = 3mg$$

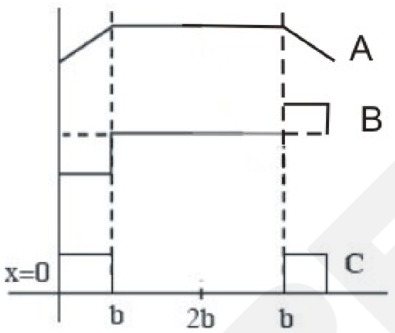
$$\frac{3mg}{A_s} = y_S \times (\Delta l_s)$$

$$\frac{2mg}{A_B} = y_b \times \frac{\Delta l_b}{l_b}$$

$$\frac{\Delta l_s}{\Delta l_b} = \frac{3}{2} \times \frac{l_S}{l_b} \times \frac{A_b}{A_S} \times \frac{y_b}{y_S} = \frac{3}{2} \times \frac{a}{b^2 c}$$

Q. 5 The arm PQ of a rectangular conductor is moving from x = 0 to x = 2b outwards and then inwards from x = 2b to x = 0 as shown in the figure. A uniform magnetic field perpendicular to the plane is acting from x = 0 to x = b. Identify the graph showing the variation of different quantities with distance :





Option 1:
A-Flux, B-Power dissipated, C-EMF

Option 2:
A-Power dissipated, B-Flux, C-EMF

Option 3:
A-Flux, B-EMF, C-Power dissipated

Option 4:
A-EMF, B-Power dissipated, C-Flux

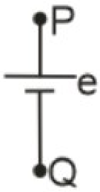
Correct Answer:
A-Flux, B-EMF, C-Power dissipated

Solution:
During motion from $x=0$ to $x=b$, arm PQ cuts the magnetic field lines so there will be induced emf

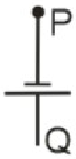
$e=Blv$

Beyond $x=b$ there won't be any induced emf

For right direction motion $(x=0 \text{ to } x=b)$

$e=BlV$ 

For left direction motion $(x=b \text{ to } x=0)$

$e=Blv$ 

$\longrightarrow$ EMF depicted by B in the graph (both positive & negative)

As $flux=\phi=BA$, when arm PQ is moved from $x=0 \text{ to } x=b$, the flux associated with loop increases and then it remains constant for motion between $x=b \text{ to } x=2b$

$\text{Flux} \rightarrow \text{Graph A}$

The correct option is (3).

Q. 6 A ball is thrown up with a certain velocity so that it reaches a height $'h'$. Find the ratio of the two different times of the ball reaching $\frac{\text{h}}{3}$ in both the directions.

Option 1:
$\frac{\sqrt{2}-1}{\sqrt{2}+1}$

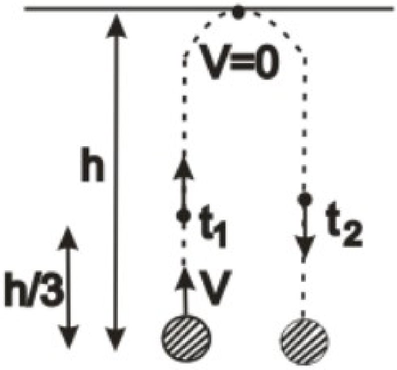
Option 2:
$\frac{1}{3}$

Option 3:
$\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

Option 4:
$$\frac{\sqrt{3}-1}{\sqrt{3}+1}$$

Correct Answer:
$$\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$$

Solution:



$$V = \text{ speed of projection } = \sqrt{2gh}$$

Let at time t_1 and t_2 the ball reaches height h/3.

$$s = ut + \frac{1}{2}at^2$$
$$\left(\frac{h}{3}\right) = \sqrt{2gh} \times t + \frac{1}{2}(-g)t^2$$
$$2h = 6\sqrt{2gh}t - 3gt^2$$
$$3gt^2 - 6\sqrt{2gh}t + 2h = 0$$

$$t = \frac{6\sqrt{2gh} \pm \sqrt{72gh - 4(3g)(2h)}}{2 \times 3g}$$
$$= \frac{6\sqrt{2gh} \pm \sqrt{48gh}}{6g}$$

$$t = \frac{\sqrt{2gh}(6 \pm \sqrt{24})}{6g}$$

$$t_1 = \sqrt{\frac{2h}{9}}\left(1 - \sqrt{\frac{24}{36}}\right)$$
$$t_2 = \sqrt{\frac{2h}{9}}\left(1 + \sqrt{\frac{24}{36}}\right)$$

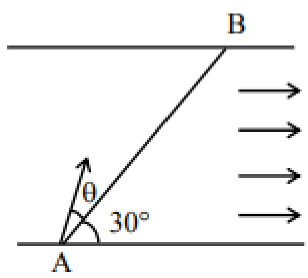
$$\frac{t_1}{t_2} = \frac{1 - \sqrt{\frac{24}{36}}}{1 + \sqrt{\frac{24}{36}}}$$

$$= \frac{1-\sqrt{\frac{2}{3}}}{1+\sqrt{\frac{2}{3}}}$$

$$\Rightarrow \frac{t_1}{t_2} = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+2}$$

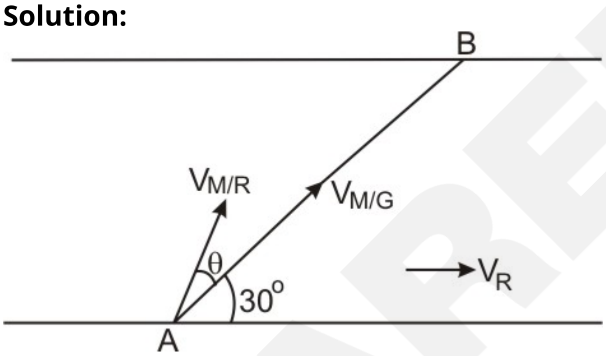
The correct option is (3).

Q. 7 A swimmer wants to cross a river from point A to point B. Line AB makes an angle of 30° with the flow of river. Magnitude of velocity of the swimmer is same as that of the river. The angle θ with the line AB should be ______^{0}, swimmer reaches point B.



Correct Answer:
30

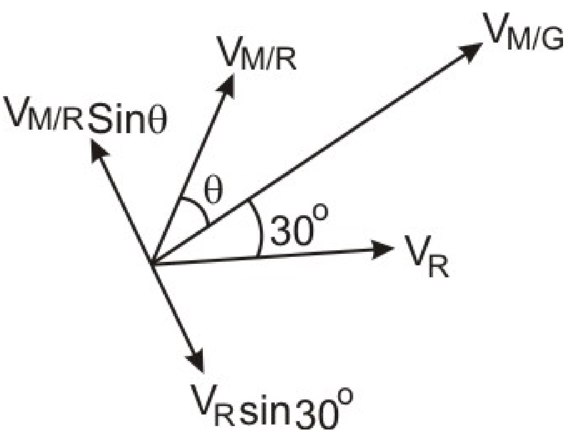
Solution:



$\bar{V}_{M/G} = \bar{V}_{M/R} + \bar{V}_R$

$\bar{V}_{M/G} \rightarrow Velocity\ of\ swimmer\ with\ ground$

$V_{M/G} must\ be\ along\ AB$ to reach pt. B



$$V_{M/G} = V_{M/R}\cos\theta + V_R\cos 30^\circ$$

$$V_{M/R}\sin\theta = V_R\sin(30^\circ)$$
$$V_{M/R} = V_R(\text{Given})$$
$$\sin\theta = \sin 30^\circ$$
$$\theta = 30^\circ$$

Q. 8 A nucleus of mass M emits γ-ray photon of frequency $'\nu'$. The loss of internal energy by the nucleus is :
[Take 'c' as the speed of electromagnetic wave]

Option 1:
$h\nu$

Option 2:
0

Option 3:
$h\nu\left[1-\frac{h\nu}{2Mc^2}\right]$

Option 4:
$h\nu\left[1+\frac{h\nu}{2Mc^2}\right]$

Correct Answer:
$h\nu\left[1+\frac{h\nu}{2Mc^2}\right]$

Solution:
When the nucleus emits γ- ray there will be recoiling of the nucleus.

$p_2 \rightarrow \text{momentum of } \gamma - ray(photon)$
$p_1 \rightarrow \text{Momentum of Nucleus}$

By momentum conservation,

$$0 = p_1 + p_2$$

$$0 = Mv + \frac{h}{\lambda}$$

$$v = \frac{-h}{\lambda M} = \frac{-h\nu}{Mc}$$

Here negative sign implies the opposite direction motion of the nucleus (recoiling) when γ-ray emission takes place.

Internal energy lost by the nucleus is given to the nucleus and x-ray.

$$\therefore \text{Internal energy lost} = \text{KE}_{\text{Nucleus}} + \text{E}_{\gamma\text{-ray}}$$

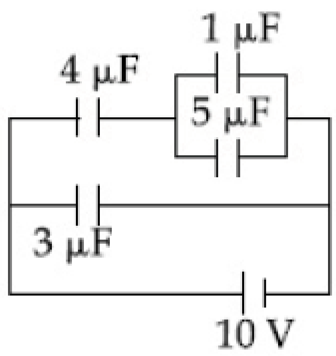
$$= \frac{1}{2}Mv^2 + hv$$

$$= \frac{1}{2}M\frac{h^2v^2}{M^2c^2} + hv$$

$$= hv\left[1 + \left(\frac{h^2}{2Mc^2}\right)\right]$$

Hence, the correct option is (4).

Q. 9 In the given circuit, the charge on $4\mu F$ the capacitor will be:



Option 1:
5.4 μC

Option 2:
9.6 μC

Option 3:
13.4 μC

Option 4:
24 μC

Correct Answer:
24 μC

Solution:

$$\frac{1}{C_{er}} = \frac{1}{4} + \frac{1}{5+1} = \frac{1}{4} + \frac{1}{6} = \frac{10}{24} = \frac{5}{12}$$

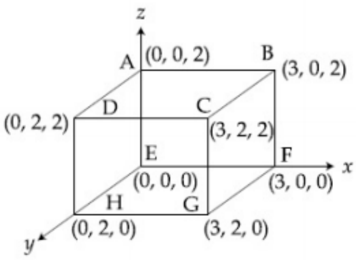
$$C_{er} = \frac{12}{5}$$

$$Q = CV$$

$$\frac{12}{5} \times 10 = 24\mu C$$

Correct option is 4.

Q. 10 An electric field $\hat{E} = 4\,x\hat{i} - (y^2+1)\hat{j}\ N/C$ passes through the box shown in figure. The flux of the electric field through surfaces ABCD and BCGF are marked as F_I and F_{II} respectively. The difference between (F_I-F_{II}) is (in Nm^2/C)______.



Correct Answer:
48

Solution:

Flux via ABCD

$$\phi_1 = \int \vec{E} \cdot d\vec{A} = 0$$

Flux via BCEF

$$\phi_2 = \int \bar{E} \cdot d\bar{A}$$

$$\phi_2 = \bar{E} \cdot \bar{A}$$

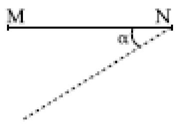
$$= (4x\hat{i} - (y^2+1)\hat{j}) \cdot 4\hat{i}$$

$$= 16x, x = 3$$

$$\phi_2 = 48\frac{N-m^2}{C}$$

$$|\phi_1 - \phi_2| = 48\frac{N - m^2}{C}$$

Q. 11 A thin rod MN, free to rotate in the vertical plane about the fixed end N, is held horizontal. When the end M is released the speed of this end, when the rod makes an angle α with the horizontal, will be proportional to : (see figure)



Option 1:
$\sqrt{sin \alpha}$

Option 2:
$sin\alpha$

Option 3:
$\sqrt{cos\ \alpha}$

Option 4:
$cos\alpha$

Correct Answer:
$\sqrt{sin \alpha}$

Solution:
As we have learnt

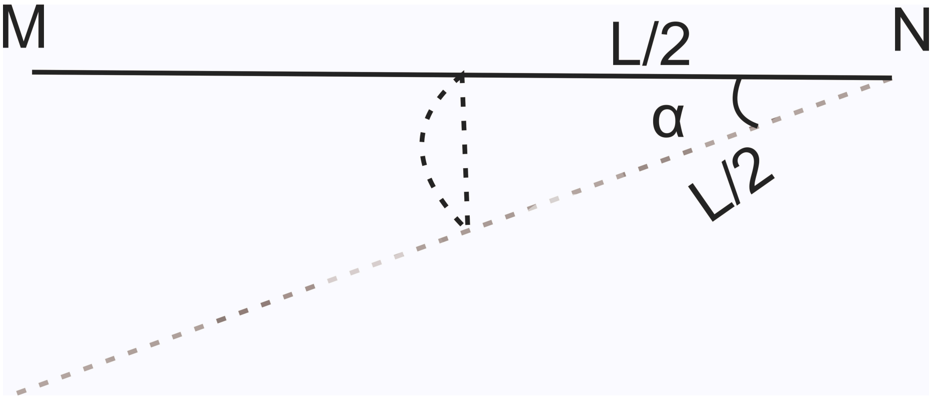
Kinetic energy of rotation -

$$K = \frac{1}{2} I w^2$$

- wherein

I = moment of inertia about axis of rotation

w = angular velocity



From energy conservation,

$$\frac{mg.L}{2} sin\alpha = \frac{1}{2}\left(\frac{mL^2}{3}\right).\omega^2$$

$\omega^2 = \frac{3g}{l} sin\alpha$ or $\omega = \sqrt{\frac{3g}{l}}\sqrt{sin\alpha}$

speed of end m$= \omega l$

$= \sqrt{3gl}\sqrt{sin\alpha}$

$\therefore v\ \alpha \sqrt{sin\ \alpha}$

Q. 12 A cylindrical wire of mass $(0.4 \pm 0.01)\, g$ has length $(8 \pm 0.04)\, cm$ and radius $(6 \pm 0.03)\, mm$. The maximum error in its density will be:

Option 1:
4%

Option 2:
1%

Option 3:
3.5%

Option 4:
5%

Correct Answer:
4%

Solution:

Cylindrical wire $m=(0.4 \pm 0.01)\, g$
$\ell=(8 \pm 0.04) \mathrm{cm}$

$\mathrm{r}=(6 \pm 0.03) \mathrm{mm}$

Density, $\rho=\frac{m}{\pi r^2 \ell} \Rightarrow \rho r^2 \ell m^{-1}=\frac{1}{\pi}=$ const.

Differentiating after taking logs on both sides

$\frac{\mathrm{d} \rho}{\rho}+\frac{2 \mathrm{dr}}{\mathrm{r}}+\frac{\mathrm{d} \ell}{\ell}-\frac{\mathrm{dm}}{\mathrm{m}}=0$

$\frac{\Delta \rho}{\rho}=\frac{\Delta \mathrm{m}}{\mathrm{m}}-\frac{\Delta \ell}{\ell}-\frac{2 \Delta \mathrm{r}}{\mathrm{r}}$

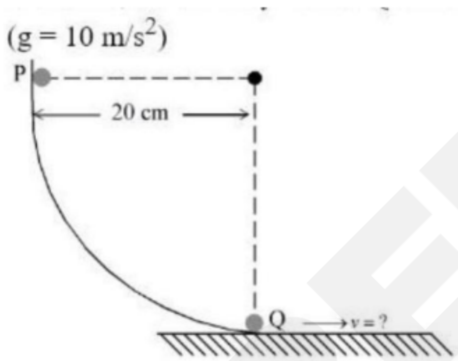
$\left(\frac{\Delta \rho}{\rho}\right)_{\max }=\left[\frac{0.01}{0.4}+\frac{0.04}{8}+2\left(\frac{0.03}{6}\right)\right]$

$\left(\frac{\Delta \rho}{\rho}\right)_{\max }=0.04$

Percentage error $=0.04 \times 100=4 \%$

Hence, the answer is the option (1).

Q. 13 As per the given figure, a small ball P slides down the quadrant of a circle and hits the other ball Q of equal mass which is initially at rest. Neglecting the effect of friction and assume the collision to be elastic, the velocity of ball Q after collision will be :



Option 1:
$0\, m/s$

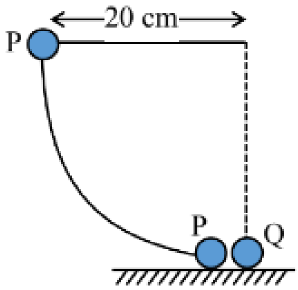
Option 2:
$4\, m/s$

Option 3:
$2\, m/s$

Option 4:
$0.25\, m/s$

Correct Answer:
$2\, m/s$

Solution:



Energy conservation for P

$$\mathrm{mgh}=\frac{1}{2}\mathrm{mV}^2$$
$$\mathrm{V}=\sqrt{2\mathrm{gh}}$$
$$\mathrm{V}=\sqrt{2\times 10\times 0.2}$$
$$\mathrm{V}=2\ \mathrm{m/sec}$$

Now collision between P and Q is elastic and both have same mass then P will transfer all velocity to then Q. So velocity Q will be $2\ \mathrm{m/sec}$

Q. 14 The minimum and maximum distances of a planet revolving around the Sun are x_1 and x_2. If the minimum speed of the planet on its trajectory is v_0 then its maximum speed will be :

Option 1:
$\frac{\mathrm{v}_0\mathrm{x}_1^2}{\mathrm{x}_2^2}$

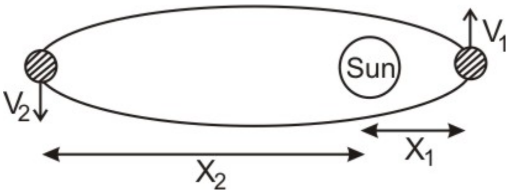
Option 2:
$\frac{\mathrm{v}_0\mathrm{x}_2^2}{\mathrm{x}_1^2}$

Option 3:
$\frac{v_0x_1}{x_2}$

Option 4:
$\frac{v_0x_2}{x_1}$

Correct Answer:
$\frac{v_0x_2}{x_1}$

Solution:



In a given orbit , L = mvr = constant

$\therefore mv_1x_1 = mv_2x_2$

$v_2 \rightarrow Minimum\, speed$

(the speed at aphelion)

$\therefore v_2 = v_0$

$v_1 = v_{maximum} = \frac{v_2x_2}{x_1}$

$v_{max} = \frac{v_0x_2}{x_1}$

(at perihelion)

The correct option is (4)

Q. 15 The mean intensity of radiation on the surface of the sun is about 10^8 W/m^2. The rms value of the corresponding magnetic field is closest to:

Option 1:
1 T

Option 2:
10^{-4} T

Option 3:
10^2 T

Option 4:
10^{-2} T

Correct Answer:
10^{-4} T

Solution:

Intensity of EM wave -

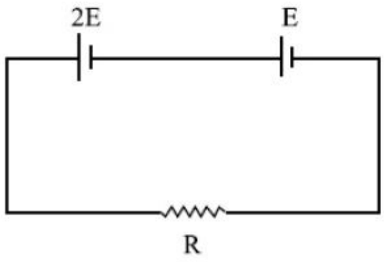
$$I = \frac{1}{2}\epsilon_o E_o^2 c$$

Given: I = 10^8 w/m^2

$$I = \epsilon_0 C E_{rms}^2 \quad \& \quad E_{rms} = cB_{rms}$$
$$I = \varepsilon_0 \quad C^3 B_{rms}^2$$
$$B_{rms} = \sqrt{\frac{I}{\epsilon_0 C^3}}$$
$$B_{rms} = \sqrt{\frac{10^8}{8.85\times10^{-12}(3\times10^8)^3}}$$
$$B_{rms} \approx 10^{-4}$$

Hence, the correct option is 2.

Q. 16 Two cells of emf 2E and E with internal resistance $r_1 \text{ and } r_2$ respectively are connected in series to an external resistor R (see figure). The value of R, at which the potential difference across the terminals of the first cell becomes zero is



Option 1:
$\frac{r_1}{2} + r_2$

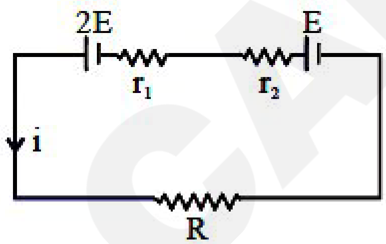
Option 2:
$r_1 - r_2$

Option 3:
$\frac{r_1}{2} - r_2$

Option 4:
$r_1 + r_2$

Correct Answer:
$\frac{r_1}{2} - r_2$

Solution:



$i = \frac{3E}{R+r_1+r_2}$

$TPD = 2E - ir_1 = 0$

$2E = ir_1$

$2E = \frac{3E \times r_1}{R+r_1+r_2}$

$2R + 2r_1 + 2r_2 = 3r_1$

$R = \frac{r_1}{2} - r_2$

Q. 17 The length of a metal wire is ℓ_1, when the tension in it is T_1 and is ℓ_2 when the tension is T_2. The natural length of the wire is :

Option 1:
$\sqrt{\ell_1 \ell_2}$

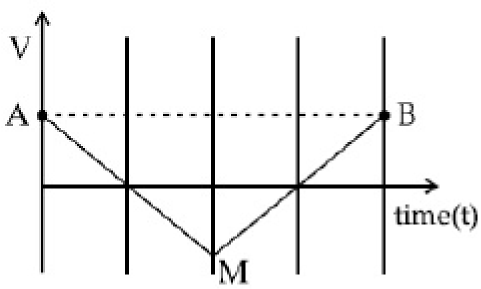
Option 2:
$\frac{\ell_1 T_2 - \ell_2 T_1}{T_2 - T_1}$

Option 3:
$\frac{\ell_1 T_2 + \ell_2 T_1}{T_2 + T_1}$

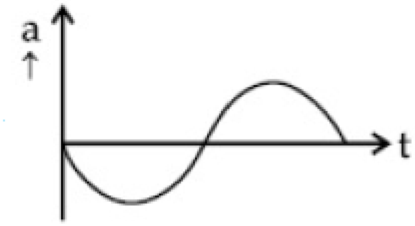
Option 4:
$\frac{\ell_1 + \ell_2}{2}$

Correct Answer:
$\frac{\ell_1 T_2 - \ell_2 T_1}{T_2 - T_1}$

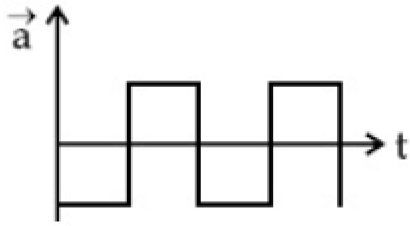
Q. 18 If the velocity-time graph has the shape AMB, what would be the shape of the corresponding acceleration -time graph??



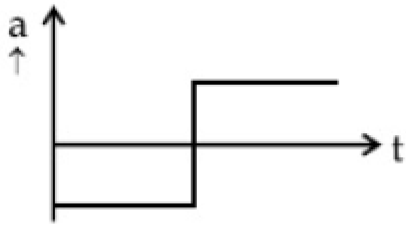
Option 1:



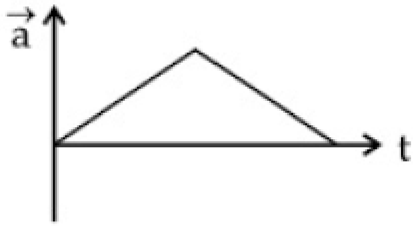
Option 2:



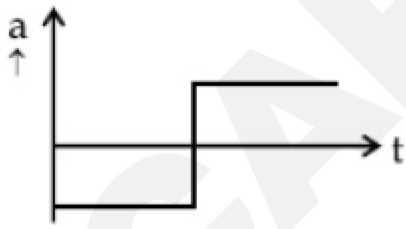
Option 3:



Option 4:

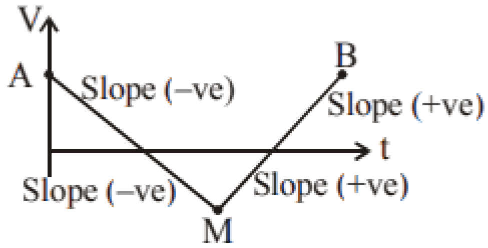


Correct Answer:

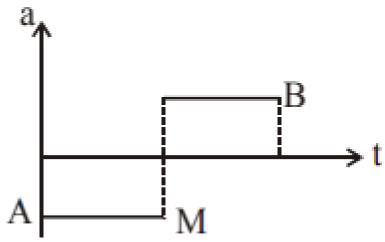


Solution:

Slope of v-t graph gives acceleration



$\Rightarrow$ Acceleration will be



Q. 19 A He^{+} ion is in its first excited state. Its ionization energy (in eV) is :

Correct Answer:
13.6

Solution:
Ionization energy -

The energy required to move an electron from the ground state to $n=\infty$

$$E_{ion}=E_{\infty}-E_{n}$$

$$=13.6\frac{Z^{2}}{n^{2}}$$

we know that,

$$E_{n}=-13.6\left(\frac{Z^{2}}{n^{2}}\right)$$

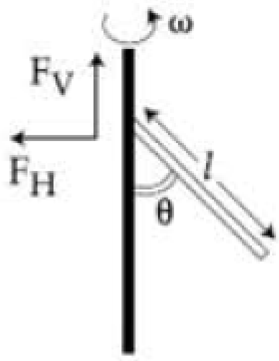
in the first excited state for He^{+}

$$\Rightarrow n=2, Z=2$$

$$E=-13.6\times\left(\frac{2^{2}}{2^{2}}\right)=-13.6\,eV$$

Hence, the answer is the option (1).

Q. 20



A uniform rod of length $'l'$ is pivoted at one of its ends on a vertical shaft of negligible radius. When the shaft rotates at angular speed ω the rod makes an angle θ with it (see figure).To find θ equate the rate of change of angular momentum (direction going into the paper) $\frac{ml^2}{12}\omega^2 \sin\theta\cos\theta$ about the centre of mass (CM) to the torque provided by the horizontal and vertical forces F_H and F_V about the CM. The value of θ is then such that :

Option 1:
$\cos\theta = \frac{2g}{3l\omega^2}$

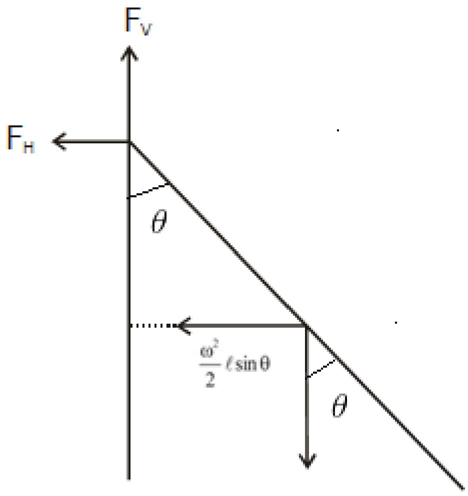
Option 2:
$\cos\theta = \frac{g}{2l\omega^2}$

Option 3:
$\cos\theta = \frac{g}{l\omega^2}$

Option 4:
$\cos\theta = \frac{3g}{2l\omega^2}$

Correct Answer:
$\cos\theta = \frac{3g}{2l\omega^2}$

Solution:



$F_V = mg$
$F_H = m\omega^2 \frac{\ell}{2}\sin\theta$

$\therefore \tau_{net}$ about COM $= F_v \cdot \frac{\ell}{2}\sin\theta - F_H(\frac{\ell}{2}\cos\theta) = \frac{m\ell^2}{12}\omega^2\sin\theta\cos\theta$

$\Rightarrow mg\left(\frac{\ell}{2}\sin\theta\right) - m\omega^2\left(\frac{\ell}{2}\sin\theta\right)\left(\frac{\ell}{2}\cos\theta\right) = m\omega^2\left(\frac{\ell^2}{12}\right)(\sin\theta)(\cos\theta)$

$\Rightarrow \frac{g\ell}{2} - \frac{\omega^2\ell^2}{4}\cos\theta = \frac{\ell^2}{12}\omega^2\cos\theta$

$\Rightarrow \frac{g\ell}{2} = \omega^2\ell^2\cos\theta\left(\frac{1}{12} + \frac{1}{4}\right)$

$\Rightarrow \frac{g\ell}{2} = \frac{\omega^2\ell^2\cos\theta}{3}$

$\Rightarrow \cos\theta = \frac{3g}{2\omega^2\ell}$

Q. 21 A linearly polarized electromagnetic wave in a vacuum is $E = 3.1\cos\left[(1.8)z - \left(5.4\times10^6\right)t\right]\hat{i}N/C$ is incident normally on a perfectly reflecting wall $z = a$. Choose the correct option

Option 1:
The wavelength is $5.4\ m$

Option 2:
The frequency of electromagnetic wave is $54\times10^4\ Hz$

Option 3:
The transmitted wave will be $3.1\cos\left[(1.8)z - \left(5.4\times10^6\right)t\right]\hat{i}N/C$

Option 4:
The reflected wave will be $3.1 \cos \left[(1.8) \mathrm{z}+\left(5.4 \times 10^{6}\right) \mathrm{t}\right] \hat{\mathrm{i}} \mathrm{N} / \mathrm{C}$

Correct Answer:
The reflected wave will be $3.1 \cos \left[(1.8) \mathrm{z}+\left(5.4 \times 10^{6}\right) \mathrm{t}\right] \hat{\mathrm{i}} \mathrm{N} / \mathrm{C}$

Solution:
$\vec{E}=3 \cdot 1 \cos \left(1 \cdot 8 z-5 \cdot 4 \times 10^{6} t\right) \hat{i}$
$i . e, \bar{E}=E_{0} \cos (k z-\omega t) \hat{i}$
It is wave propagation along + z-axis after getting reflected from the plane (z = a) i.e x-y plane it will travel along -z-axis
$\therefore \bar{E}_{r}=E_{0} \cos (k z+\omega t) \hat{i}$

Hence, the answer is the option (4).

Q. 22 A circular disc of mass M and radius R is rotating about its axis with angular speed ω_1. Another stationary disc having radius $\frac{R}{2}$ and same mass M is dropped co-axially on the rotating disc. Gradually both the discs attain constant angular speed ω_2. The energy lost in the process is p% of the initial energy. Value of p is________

Option 1:
10

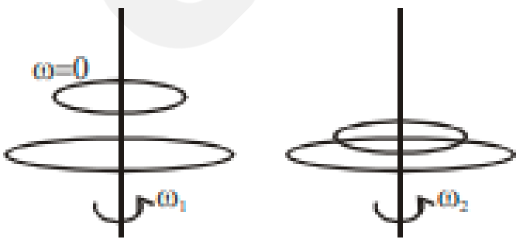
Option 2:
20

Option 3:
30

Option 4:
40

Correct Answer:
20

Solution:



Let the moment of inertia of bigger disc is $I=\frac{M R^{2}}{2}$

$$\Rightarrow \text{ MOI of small disc } = I_2 = \frac{M\left(\frac{R}{2}\right)^2}{2} = \frac{I}{4}$$

by angular momentum conservation

$$\mathrm{I}\omega_1 + \frac{\mathrm{I}}{4}(0) = \mathrm{I}\omega_2 + \frac{\mathrm{I}}{4}\omega_2 \Rightarrow \omega_2 = \frac{4\omega_1}{5}$$

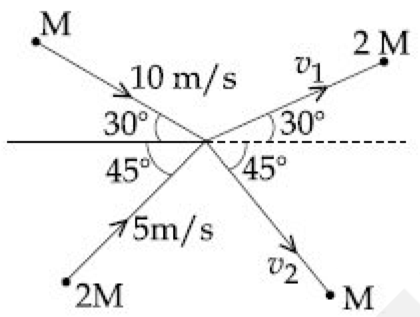
initial kinetic energy $= \mathrm{K}_1 = \frac{1}{2}\mathrm{I}\omega_1^2$

final kinetic energy

$$\mathrm{K}_2 = \frac{1}{2}\left(I + \frac{I}{4}\right)\left(\frac{4\omega_1}{5}\right)^2 = \frac{1}{2}I\omega_1^2\left(\frac{4}{5}\right)$$

$$\mathrm{P}\% = \frac{\mathrm{K}_1 - \mathrm{K}_2}{\mathrm{K}_1} \times 100\% = \frac{1 - 4/5}{1} \times 100 = 20\%$$

Q. 23 Two particles, of masses M and 2M, moving as shown, with speeds of 10 m/s and 5 m/s, collide elastically at the origin. After the collision, they move along the indicated directions with speeds v_1 and v_2. The values of v_1 and v_2 are approximately :



Option 1:
6.5 m/s and 6.3 m/s

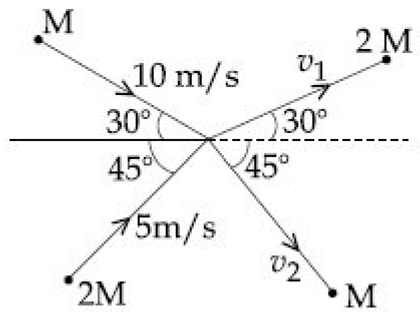
Option 2:
3.2 m/s and 6.3 m/s

Option 3:
6.5 m/s and 3.2 m/s

Option 4:
3.2 m/s and 12.6 m/s

Correct Answer:
6.5 m/s and 6.3 m/s

Solution:



u_1 $\quad u_{1x} = 10 \times \frac{\sqrt{3}}{2}\hat{i}$

$u_{1y} = 10 \times \frac{1}{2}\hat{j} = -5\hat{j}$ $\qquad M_1 = M$

u_2 $\quad u_{2x} = \frac{5}{\sqrt{2}}\hat{i}$

$u_{2y} = \frac{5}{\sqrt{2}}\hat{j}$ $\qquad M_2 = 2M$

v_1 $\quad v_{1x} = v_1 \times \frac{\sqrt{3}}{2}\hat{i}$

$v_{1y} = v_1 \times \frac{1}{2}\hat{j}$

v_2 $\quad v_{2x} = v_2 \times \frac{1}{\sqrt{2}}\hat{i}$

$v_{2y} = \frac{v_2}{\sqrt{2}}(-\hat{j})$

$$\Delta P_x = 0 => m \times (\frac{10\sqrt{3}}{2}) + (2m \times \frac{5}{\sqrt{2}}) = 2m(\frac{\sqrt{3}}{2})v_1 + m(\frac{v_2}{\sqrt{2}})$$

$$=> 5\sqrt{3} + 5\sqrt{2} = \sqrt{3}v_1 + \frac{v_2}{\sqrt{2}}$$ ....(1)

$$\Delta P_y = 0 => -m \times (5) + (2m \times \frac{5}{\sqrt{2}}) = 2m(\frac{v_1}{2}) - m(\frac{v_2}{\sqrt{2}})$$

$$=> 5\sqrt{2} - 5 = v_1 - \frac{v_2}{\sqrt{2}}$$ ....(2)

On adding (1) and (2)

$$5(\sqrt{3} - 1) + 10\sqrt{2} = (\sqrt{3} + 1)v_1$$

$$=> v_1 = \frac{5(\sqrt{3} - 1) + 10\sqrt{2}}{\sqrt{3} + 1} \approx 6.5\ m/s$$

$$=> v_2 \approx 6.3\ m/s$$

Q. 24 For a body projected at an angle with the horizontal from the ground, choose the correct statement.

Option 1:
The vertical component of momentum is maximum at the highest point.

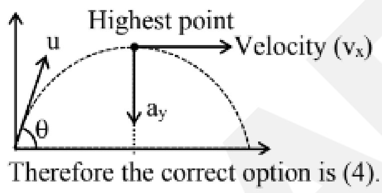
Option 2:
The Kinetic Energy (K.E.) is zero at the highest point of projectile motion.

Option 3:
The horizontal component of velocity is zero at the highest point.

Option 4:
Gravitational potential energy is maximum at the highest point.

Correct Answer:
Gravitational potential energy is maximum at the highest point.

Solution:
At highest point height is maximum and vertical component of velocity is zero.
So momentum is zero. At highest point horizontal component of velocity will not be zero but vertical component of velocity is equal to zero and because of this K.E. will not be equal to zero. Gravitational potential energy is maximum at highest point and equal to $\mathrm{mgH} = \mathrm{mg}\left(\frac{\mathrm{u}^2 \sin^2 \theta}{2\ \mathrm{g}}\right)$



Therefore the correct option is (4).

Q. 25 An object is taken to a height above the surface of earth at a distance $\frac{5}{4}\mathrm{R}$ from the centre of the earth. Where radius of earth, $\mathrm{R} = 6400\ \mathrm{km}$. The percentage decrease in the weight of the object will be :

Option 1:
36%

Option 2:
50%

Option 3:
64%

Option 4:
25%

Correct Answer:
36%

Solution:
The value of g from the centre and surface of the earth is given as :

$$R+h=\frac{5}{4}R \quad \text{(from the centre of the earth)}$$

$$h=\frac{R}{4} \quad \text{(from the surface of the earth)}$$

Now,

$$\frac{g_n}{g}=\frac{R^2}{(R+h)^2}=\frac{16R^2}{25R^2}$$

$$g_n=\frac{16}{25}g$$

Weight on the Earth's surface $= mg$

Weight of the object at a height $h = mg_n$

$$\frac{w}{w_n}=\frac{mg}{mg_n}=\frac{25}{16}$$

$$\frac{w-w_n}{W}=\frac{9}{25}$$

Percentage change in the value of g with height is :

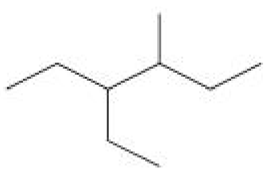
$$\frac{\Delta W}{W}=36\%$$

The percentage decrease in the weight of the object will be 36%

Hence, the answer is the option (1).

Chemistry

Q. 1 The correct IUPAC name of the following compound



is

Option 1:
4 - methyl - 3 - ethylhexane

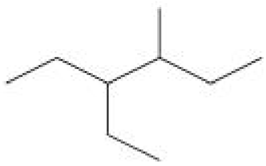
Option 2:
3 - ethyl - 4 - methylhexane

Option 3:
3, 4 - ethylmethylhexane

Option 4:
4 - ethyl - 3 - methylhexane

Correct Answer:
3 - ethyl - 4 - methylhexane

Solution:



Ethyl has more wight than methyl. So, ethyl will have more priority.

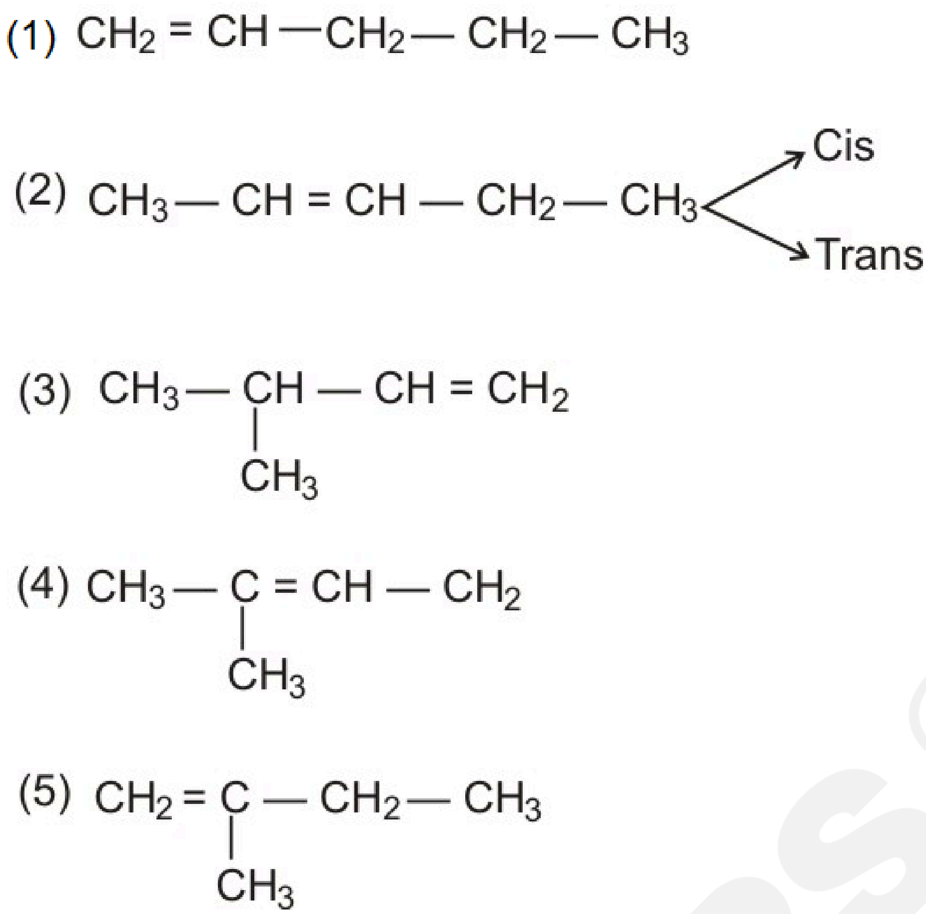
IUPAC name of the compound is 3-Ethyl-4-methyl hexane.

Therefore, **option (2) is correct.**

Q. 2 The number of acyclic structural isomers (including geometrical isomers) for pentene are_____

Correct Answer:
6

Solution:
The possible acyclic isomers of pentene are



Thus, there are 6 stereoisomers possible for acyclic forms of pentene.

Hence, the correct answer is 6

Q. 3 What is the standard reduction potential

(E°) for $Fe^{3+} \rightarrow Fe$?

Given that:

$Fe^{2+} + 2e^{-} \rightarrow Fe$; $E^{\circ}_{Fe^{2+}/Fe} = -0.47V$

$Fe^{3+} + e^{-} \rightarrow Fe^{2+}$; $E^{\circ}_{Fe^{3+}/Fe^{2+}} = +0.77V$

Option 1:
-0.057 V

Option 2:
+0.057 V

Option 3:
+0.30 V

Option 4:
−0.30 V

Correct Answer:
−0.057 V

Solution:

$$\begin{array}{lll} & Fe^{2+} + 2e^- \rightarrow Fe & \Delta G_1^\circ = -2F(-0.47\ V) \\ & Fe^{3+} + e^- \rightarrow Fe^{2+} & \Delta G_2^\circ = -F(0.77\ V) \\ \text{Add} & \overline{Fe^{3+} + 3e^- \rightarrow Fe} & \overline{\Delta G^\circ = -2F(-0.47\ V) - F(0.77\ V) = (0.17\ V)F} \end{array}$$

$$E^\circ_{Fe^{3+}|Fe} = -\frac{\Delta G^\circ}{3F} = -0.057\ V$$

Q. 4 The reaction of sulphur in alkaline medium is given below:

$$S_8(g) + aOH^-(aq) \rightarrow bS^{2-}(aq) + cS_2O_3^{2-} + dH_2O(l)$$

The value of 'a' is _________ (Integer answer)

Correct Answer:
12

Solution:
Need to first find out the oxidation state of each S.

Then balance all S.

Then balance all O and H.

$$S_8 + aOH^- \longrightarrow bS^{-2} + cS_2O_3^{-2} + dH_2O$$
$$S_8 + aOH^- \longrightarrow 4S^{-2} + 2S_2O_3^{-2} + dH_2O$$
$$S_8 + 12OH^- \longrightarrow 4S^{-2} + 2S_2O_3^{-2} + 6H_2O$$
$$a = 12$$

Q. 5 In $\overset{1}{C}H_2=\overset{2}{C}=\overset{3}{C}H-\overset{4}{C}H_3$ molecule, the hybridization of carbon 1,2,3 and 4 respectively are:

Option 1:
sp^2, sp, sp^2, sp^3

Option 2:
sp^2, sp^2, sp^2, sp^3

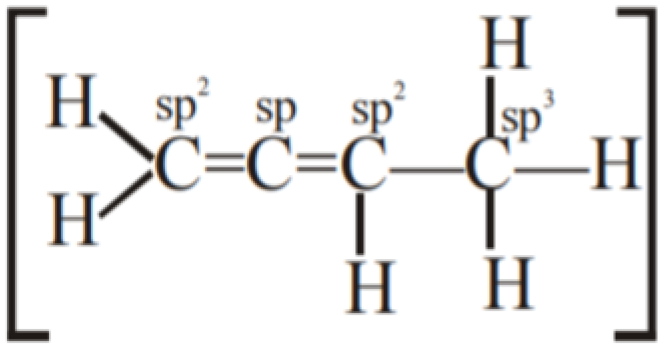
Option 3:
sp^3, sp, sp^3, sp^3

Option 4:
sp^2, sp^3, sp^2, sp^3

Correct Answer:
sp^2, sp, sp^2, sp^3

Solution:

$$[\overset{1}{C}H_2=\overset{2}{C}=\overset{3}{C}H—\overset{4}{C}H_3]$$

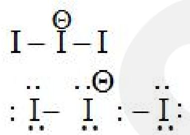


Q. 6 Total number of lone pair of electrons in I_3^- ion is :

Correct Answer:
9

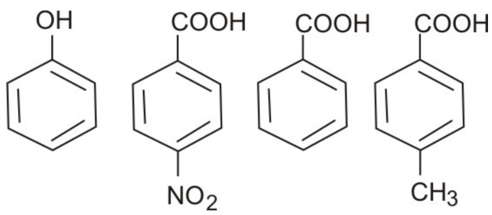
Solution:
Lewis structure -

I_3



No. of lone pairs =9

Q. 7 The correct order of acid character of the following compounds is :



I II III IV

Option 1:
III > II > I > IV

Option 2:
IV > III > II > I

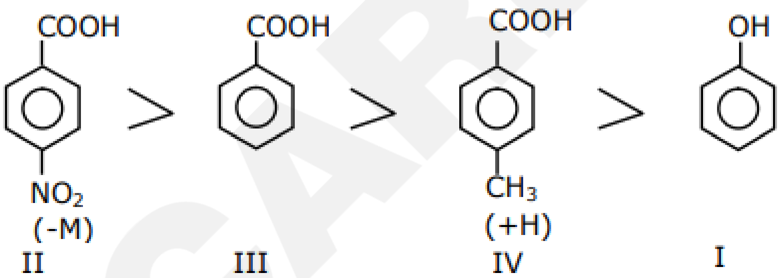
Option 3:
I > II > III > IV

Option 4:
II > III > IV > I

Correct Answer:
II > III > IV > I

Solution:

$$\text{Acidity of carboxylic acid } \propto -\mathrm{R} > -\mathrm{H} > -\mathrm{I} \propto \frac{1}{+\mathrm{R} > +\mathrm{H} > +\mathrm{I}}$$



Therefore, option(4) is correct.

Q. 8 The number of compound/s given below which contain/s $-COOH$ group is ______.

(A) Sulphanilic acid

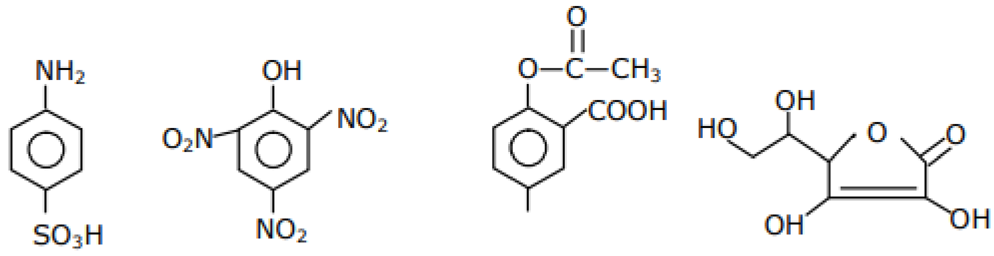
(B) Picric acid

(C) Aspirin

(D) Ascorbic acid

Correct Answer:
1

Solution:

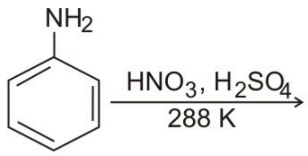


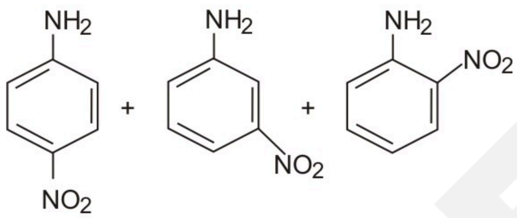
(A) (B) (C) (D)

Only 1 compound Aspirin contains -COOH group.

ANswer : 1

Q. 9





(A) (B) (C)

Correct statement about the given chemical reaction is :

Option 1:
$-\mathrm{NH_2}$ group is ortho and para directive, so product (B) is not possible.

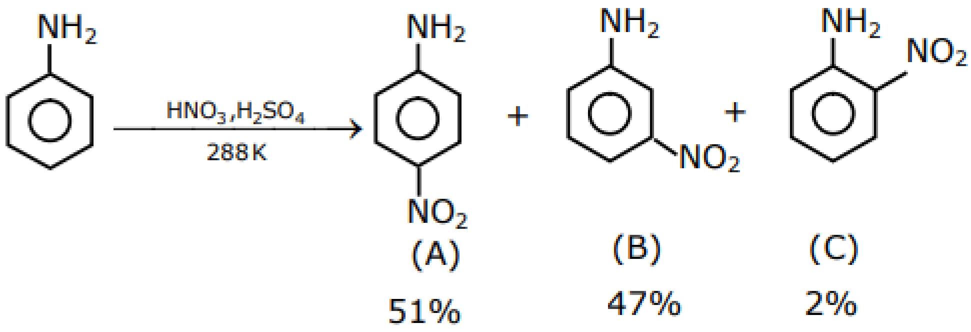
Option 2:
Reaction is possible and compound (B) will be the major product.

Option 3:
The reaction will form sulphonated product instead of nitration.

Option 4:
Reaction is possible and compound (A) will be major product.

Correct Answer:
Reaction is possible and compound (A) will be major product.

Solution:



The reaction is possible and compound (A) will be a major product. This statement is correct.

All other statements are wrong.

Therefore, option(4) is correct.

Q. 10 The total number of monohalogenated organic products in the following (including stereoisomers) reaction is ____

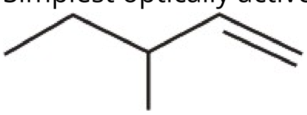
$$\underset{\text{(Simplest optically active alkene)}}{A} \xrightarrow[\text{(ii) } X_2/\Delta]{\text{(i) } H_2/Ni/\Delta}$$

Correct Answer:

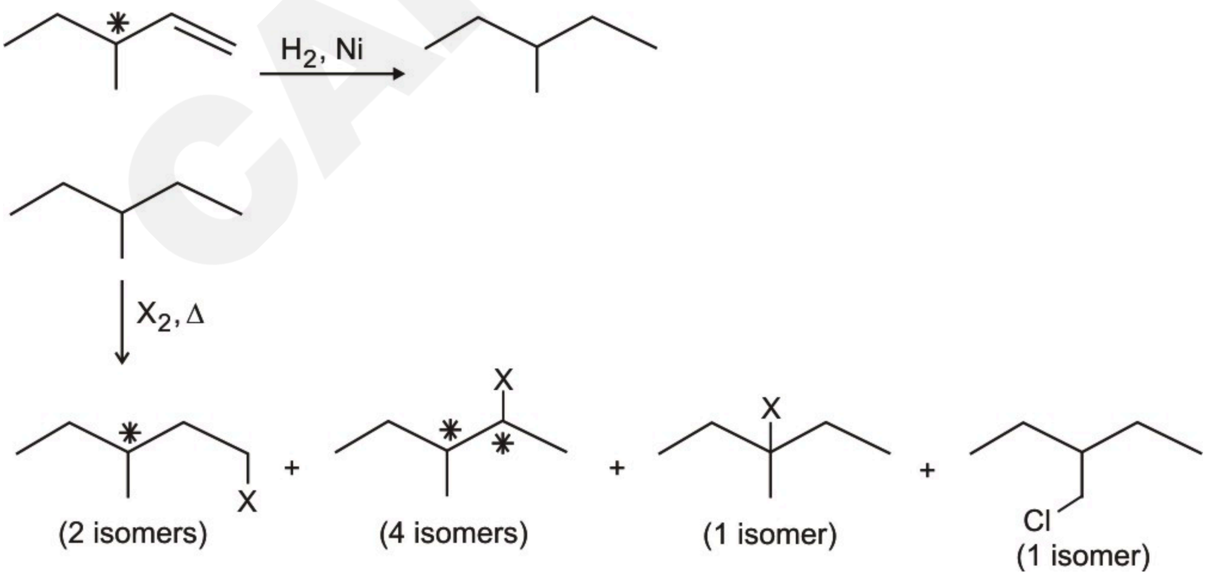
8

Solution:

Simplest optically active alkene is:



It undergoes the reactions as given below:



Hence, total number of isomers produced is 8.

Q. 11 Arrange the following metal complex/compounds in the increasing order of spin only magnetic moment. Presume all the three, high spins system.

(Atomic numbers Ce= 58, Gd= 64 and Eu = 63)

$(a)(NH_4)_2[Ce(NO_3)]_6$ $(b)Gd(NO_3)_3$ $(c)Eu(NO_3)_3$

Answer is:

Option 1:
c < a< b

Option 2:
a < b < c

Option 3:
a < c < b

Option 4:
b < a < c

Correct Answer:
a < c < b

Solution:

(a) ${}_{58}Ce \rightarrow [Xe]4f^2\ 5d^0\ 6s^2$

In complex $Ce^{4+} \rightarrow [Xe]\ 4f^0\ 5d^0\ 6s^0$

there is no unpaired electron so $\mu_m = 0$

(b) ${}_{64}Gd^{3+} \rightarrow [Xe]4f^7\ 5d^0\ 6s^0$

contain seven unpaired electrons so,

$\mu_m = \sqrt{7(7+2)} = \sqrt{63}$ B.M.

(c) ${}_{63}Eu^{3+} \rightarrow [{}_{54}Xe]4f^6\ 5d^0\ 6s^0$

contain six unpaired electron

so, $\mu_m = \sqrt{6(6+2)} = \sqrt{48}$ B.M.

Hence, order of spin only magnetic movement

$\boxed{b > c > a}$

Therefore, Option 3 is correct.

Q. 12 In the sixth period, the orbitals that are filled are :

Option 1:
6s, 4f, 5d, 6p

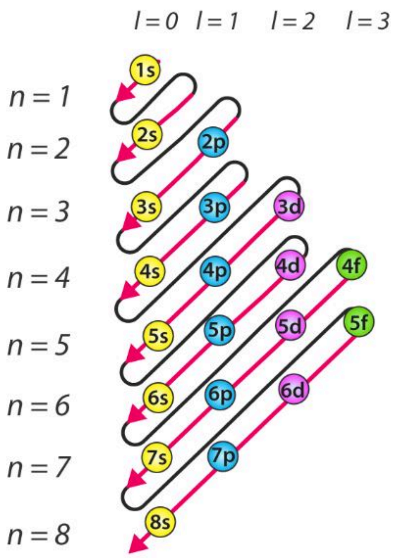
Option 2:
6s, 5d, 5f, 6p

Option 3:
6s, 5f, 6d, 6p

Option 4:
6s, 6p, 6d, 6f

Correct Answer:
6s, 4f, 5d, 6p

Solution:
Energy order of orbital's according to Aufbau principle-



The order of orbitals filling is 6s, 4f, 5d, 6p.

Therefore, the correct option is (1).

Q. 13 The number of non-ionisable hydrogen atoms present in the final product obtained from the hydrolysis of PCl_5 is :

Option 1:
0

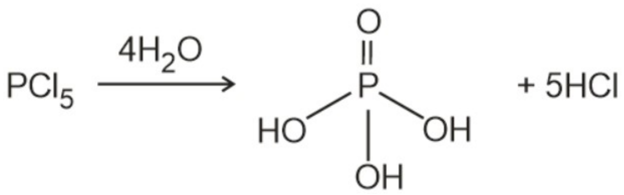
Option 2:
1

Option 3:
2

Option 4:
3

Correct Answer:
0

Solution:
Hydrolysis of PCl_5 produces Phosphoric Acid (H_3PO_4)



The Oxoacid of Phosphorus formed contains no unionisable Hydrogen.

Hence, the correct answer is option (1)

Q. 14 Consider the following reactions:

$$NaCl + K_2Cr_2O_7 + H_2SO_4(conc.) \rightarrow A + side\ products$$

$$A + NaOH \rightarrow B + side\ products$$

$$B + H_2SO_4(dil.) + H_2O_2 \rightarrow C + side\ products$$

The sum of the total number of atoms in one molecule each of A, B, and C is:

Option 1:
18

Option 2:
16

Option 3:
14

Option 4:
20

Correct Answer:
18

Solution:
(A) = CrO_2Cl_2

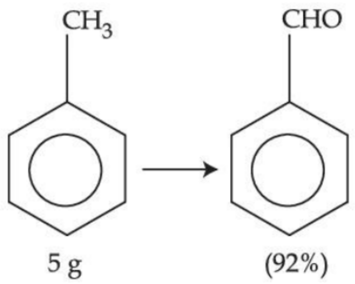
(B) = Na_2CrO_4

(C) = CrO_5

Total atoms = 5 (A) + 7 (B) + 6 (C) = 18 atoms

Hence, the answer is the option(1).

Q. 15



In the above reaction, $5\ \mathrm{g}$ of toluene is converted into benzaldehyde with 92% yield. The amount of benzaldehyde produced is __________$\times 10^{-2}\ \mathrm{g}$. (Nearest integer)

Correct Answer:
530

Solution:
From the reaction $1\ \mathrm{mol}$ of toluene is converted into $1\ \mathrm{mol}$ of benzaldehyde but yield is 92% so $0.92\ \mathrm{mol}$ of benzaldehyde form.

$$1\ \text{mole Ph} - \mathrm{CH_3} \longrightarrow 0.92\ \text{mole Ph} - \mathrm{CHO}$$
$$92\ \text{gram Ph}-\mathrm{CH_3} \longrightarrow 0.92\ \text{mol Ph} - \mathrm{CHO}$$
$$5\ \text{gram Ph}-\mathrm{CH_3} \longrightarrow 0.05\ \text{mol Ph} - \mathrm{CHO}$$
$$5\ \text{gram Ph}-\mathrm{CH_3} \longrightarrow 5.30\ \text{gram Ph} - \mathrm{CHO}$$

$$\text{Amount of benzaldehyde} = 530 \times 10^{-2}\ \mathrm{g}.$$
Ans = 530.

Q. 16 Excess of NaOH(aq) was added to 100 ml of $FeCl_3$(aq) resulting in 2.14 g of $Fe(OH)_3$. The molarity of $FeCl_3$(aq) is :
(Given molar mass of Fe=56 g mol^{-1} and molar mass of Cl=35.5 g mol^{-1})

Option 1:
0.2 M

Option 2:
0.3 M

Option 3:
0.6 M

Option 4:
1.8 M

Correct Answer:
0.2 M

Solution:

Given,

molar mass of Fe=56 g mol^{-1} and molar mass of Cl=35.5 g mol^{-1}

The chemical equation for the reaction is as follows:

$3NaOH(aq) + FeCl_3(aq) \longrightarrow 3NaCl(aq) + F(OH)_3(aq)$

Therefore, the mole ratio of $FeCl_3$ and $Fe(OH)_3$ is 1 : 1

Now moles of $Fe(OH)_3$ is given by:
Moles = Given mass / Molar mass

Molar mass of $Fe(OH)_3$ = 56 + 48 +3 = 107 grams / moles

2.14 g / (107g / mole) = 0.02 moles.

Therefore, the moles for $FeCl_3$ are also 0.02 moles

Now, Molarity = moles per volume(L)

The molarity of $FeCl_3$ is thus : (0.02 / 100) x 1000

Therefore, the molarity of $FeCl_3$ = 0.2M

Hence, the answer is an option (1).

Q. 17 The incorrect statement for the use of indicators in acid-base titration is :

Option 1:
Methyl orange may be used for a weak acid vs weak base titration.

Option 2:
Phenolphthalein is a suitable indicator for a weak acid vs strong base titration.

Option 3:
Methyl orange is a suitable indicator for a strong acid vs weak base titration.

Option 4:
Phenolphthalein may be used for a strong acid vs strong base titration.

Correct Answer:
Methyl orange may be used for a weak acid vs weak base titration.

Solution:
Weak acid – weak base:-
Neither phenolphthalein nor methyl orange is suitable.

Hence, the answer is the option (1).

Q. 18 Given

$(A) 2CO(g) + O_2(g) \rightarrow 2CO_2(g) \quad \Delta H_1^0 = -xKJmol^{-1}$

$(B) C \text{ (graphite) } + O_2(g) \rightarrow CO_2(g) \quad \Delta H_2^0 = -yKJmol^{-1}$

The ΔH° for the reaction

$C \text{ (graphite) } + \frac{1}{2} O_2(g) \rightarrow CO(g)$ is

Option 1:
$$\frac{x - 2y}{2}$$

Option 2:
$$\frac{x + 2y}{2}$$

Option 3:
$$\frac{2x - y}{2}$$

Option 4:
$2y - x$

Correct Answer:
$$\frac{x - 2y}{2}$$

Solution:

$2CO(g) + O_2(g) \rightarrow 2CO_2(g) \quad \Delta H_1^o = ykJ/mol \; \ldots (1)$

$C(\text{ graphite}) + O_2(g) \rightarrow CO_2(g) \quad \Delta H_2^o = -y \quad kJ/mol \; \ldots (2)$

$C(\text{ graphite}) + 1/2 O_2(g) \rightarrow CO(g) \quad \Delta H_3^o = ?$

$$\Delta H_3^o = H_2^o - \frac{H_1^o}{2} = -y - \frac{-x}{2}$$

$$\Delta H_3^o = \frac{x}{2} - y = \frac{x - 2y}{2}$$

Hence, the answer is the option (1).

Q. 19 Which of the following represents the correct order of metallic character of the given elements ?

Option 1:
$Si < Be < Mg < K$

Option 2:
$Be < Si < K < Mg$

Option 3:
$Be < Si < Mg < K$

Option 4:
$K < Mg < Be < Si$

Correct Answer:
$Si < Be < Mg < K$

Solution:
$Si < Be < Mg < K$
Si is having Non-metallic character.

Hence, the answer is the option (1).

Q. 20 Match List I with List II:

List I (Mixture)		List II (Separation Technique)	
A.	$CHCl_3 + C_6H_5NH_2$	I.	Steam distillation
B.	$C_6H_{14} + C_5H_{12}$	II.	Differential extraction
C.	$C_6H_5NH_2 + H_2O$	III.	Distillation
D.	Organic compound in H_2O	IV.	Fractional distillation

Option 1:
A-IV, B-I, C-III, D-II

Option 2:
A-III, B-IV, C-I, D-II

Option 3:
A-III, B-I, C-IV, D-II

Option 4:
A-II, B-I, C-III, D-IV

Correct Answer:
A-III, B-IV, C-I, D-II

Solution:
A. $CHCl_3 + C_6H_5NH_2 \rightarrow$ Distillation (III)
B. $C_6H_{14} + C_5H_{12} \rightarrow$ fractional distillation (IV)
c. $C_6H_5NH_2 \rightarrow H_2O \rightarrow$ steam distillation (I)
D. Organic compound in $H_2O \rightarrow$ Differential extraction (II)

Q. 21 The reaction $2NO + Br_2 \rightarrow 2NOBr$ takes places through the mechanism given below:

$NO + Br_2 \Leftrightarrow NOBr_2$ (fast)

$NOBr_2 + NO \rightarrow 2NOBr$ (slow)

The overall order of the reaction is ___________.

Correct Answer:

3

Solution:

$$\mathrm{RDS: NOBr_2 + NO \rightarrow 2NOBr}$$

$$\mathrm{r = K\,[NOBr_2]\,[NO]} \ldots\ldots\ (i)$$

$$\mathrm{Keq} = \frac{[\mathrm{NOBr_2}]}{[\mathrm{NO_2}]\,\mathrm{Br_2}]} \ldots\ldots\ldots\ (ii)$$

From (i) & (ii)

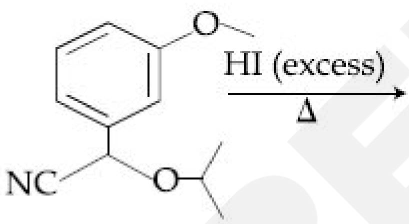
$$\mathrm{R = K.Keq.[NO]\,[Br_2]\,[NO]}$$

$$\mathrm{R = K,\,[NO]^2\,[Br_2]}$$

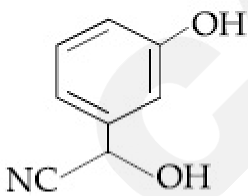
Overall order $= 3$

Hence, the answer is (3).

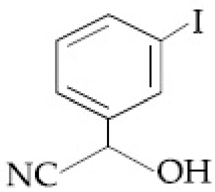
Q. 22 The major product of the following reaction is :



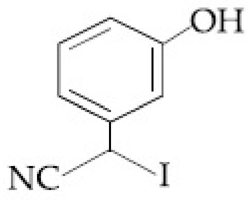
Option 1:



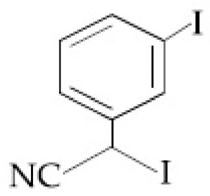
Option 2:



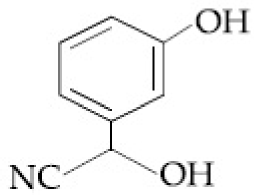
Option 3:



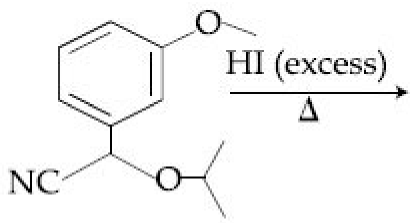
Option 4:



Correct Answer:



Solution:

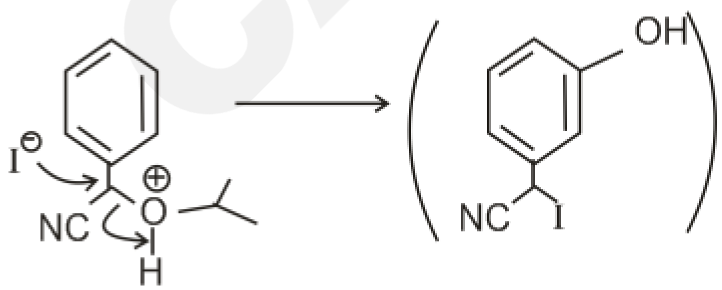


CN is strong electron withdrawing group, therefore it will create

δ^{+} charge on adjacent carbon and thus (I^{-}) will attack on this

carbon by S_N2 mechanism.

Thus,



NOTE: answer provided by NTA is wrong. The answer provided by us is correct.

Q. 23 Match **List - I** with **List - II.**

List-I	**List-II**
(A)Glucose + HI	(I) Gluconic acid
(B)Glucose + Br_2 water	(II) Glucose pentaacetate
(C) Glucose + acetic anhydride	(III) Saccharic acid
(D)Glucose + HNO_3	(IV) Hexane

Choose the correct answer from the options given below :

Option 1:
(A) - (IV), (B) - (I), (C) - (II), (D) - (III)

Option 2:
(A) - (IV), (B) - (III), (C) - (II), (D) - (I)

Option 3:
(A) - (III), (B) - (I), (C) - (IV), (D) - (II)

Option 4:
(A)- (I), (B) - (III), (C) - (IV), (D) - (II)

Correct Answer:
(A) - (IV), (B) - (I), (C) - (II), (D) - (III)

Solution:
(A) Grueose + HI → (IV) Hexane

(B) Glucose + Br_2 water → (I) Gluconic acid

(C) Glucose + acetic anhydride → (II) Glucose pentacetate

(D) Gslucose + HNO_3 → (III) Saccharic acid

So, (A) – (IV), (B) – (I), (C) – (II), (D) – (III)

Hence, the answer is the option (1).

Q. 24 Match List - I with List - II :

List-I **(Metal Ion)**	**List-II** **(Group in Qualitative analysis)**
$(a)Mn^{2+}$	(i) Group - III
$(b)As^{3+}$	(ii) Group - IIA
$(c)Cu^{2+}$	(iii) Group - IV
$(d)Al^{3+}$	(iv) Group - IIB

Choose the most appropriate answer from the options given below :

Option 1:
$(a)-(iii),(b)-(iv),(c)-(ii),(d)-(i)$

Option 2:
$(a)-(iv),(b)-(ii),(c)-(iii),(d)-(i)$

Option 3:
$(a)-(i),(b)-(iv),(c)-(ii),(d)-(iii)$

Option 4:
$(a)-(i),(b)-(ii),(c)-(iii),(d)-(iv)$

Correct Answer:
$(a)-(iii),(b)-(iv),(c)-(ii),(d)-(i)$

Solution:
The correct match of basic radicals and their groups in Qualitative analysis is given below

$(a)Mn^{2+}$: Group IV(iii)

$(b)As^{3+}$: Group IIB(iv)

$(c)Cu^{2+}$: Group IIA(ii)

$(d)Al^{3+}$: Group(III)(i)

Hence, the answer is the option (1).

Q. 25 A. Phenyl methanamine

B. N,N - Dimethylaniline

C. N- Methyl aniline

D. Benzenamine

Choose the correct order of basic nature of the above amines.

Option 1:
A>B>C>D

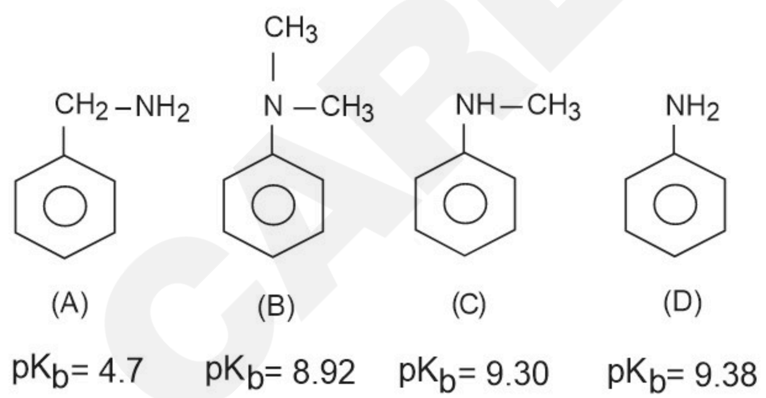
Option 2:
D> B> C> A

Option 3:
A> C> B > D

Option 4:
D > C> B> A

Correct Answer:
A>B>C>D

Solution:
The compounds along with their pK_b values are given below



Basic Strength order (A) > (B) > (C) > (D)

Hence, the correct answer is Option (1)

Maths

Q. 1 If the perpendicular bisector of the line segment joining the points P(1,4) and Q (k,3) has y-intercept equal to -4 then a value of k is:

Option 1:
-2

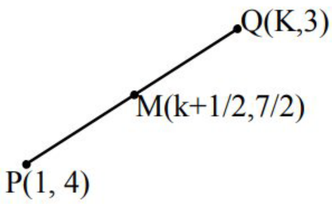
Option 2:
-4

Option 3:
$\sqrt{14}$

Option 4:
$\sqrt{15}$

Correct Answer:
-4

Solution:



$$\text{Slope } = \text{m} = \frac{1}{1-\text{k}}$$
equation of perpendicular bisector is
$$y+4=(k-1)(x-0)$$
$$\Rightarrow y+4=x(k-1)$$
$$\Rightarrow \frac{7}{2}+4=\frac{k+1}{2}(k-1)$$
$$\Rightarrow \frac{15}{2}=\frac{\text{k}^2-1}{2} \Rightarrow \text{k}^2=16 \Rightarrow \text{k}=4,-4$$

Q. 2 If the extremities of the base of an isosceles triangle are points $(2a, 0)$ and $(0, a)$ and the equation of one of the sides is x=2a, then the area of the triangle, in square units, is :

Option 1:
$\frac{5}{4}a^2$

Option 2:
$\frac{5}{2}a^2$

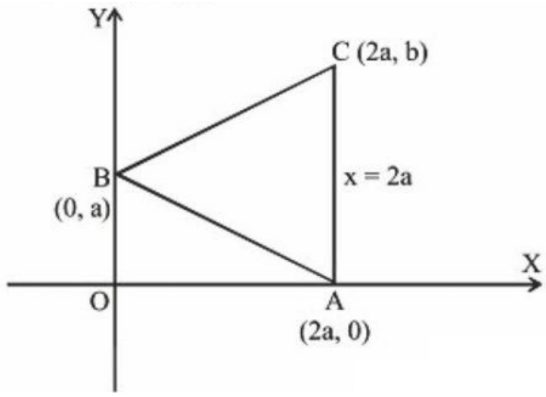
Option 3:
$\frac{25a^2}{4}$

Option 4:
$5a^2$

Correct Answer:
$\frac{5}{2}a^2$

Solution:
Let y-coordinate of C is b.

therefore, C = (2a,b)



$$AB = \sqrt{4a^2 + a^2} = \sqrt{5}a$$
$$\text{Now, } AC = BC \Rightarrow b = \sqrt{4a^2 + (b-a)^2}$$
$$\Rightarrow b^2 = 4a^2 + b^2 + a^2 - 2ab$$
$$\Rightarrow 2ab = 5a^2 \Rightarrow b = \frac{5a}{2}$$
$$\therefore C = \left(2a, \frac{5a}{2}\right)$$

Hence, the area of the triangle

$$= \frac{1}{2}\begin{vmatrix} 2a & 0 & 1 \\ 0 & a & 1 \\ 2a & \frac{5a}{2} & 1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 2a & 0 & 1 \\ 0 & a & 1 \\ 0 & \frac{5a}{2} & 0 \end{vmatrix}$$
$$= \frac{1}{2} \times 2a\left(-\frac{5a}{2}\right) = -\frac{5a^2}{2}$$

Since area can not be -ve, so

$$\text{the area of the triangle } = \frac{5a^2}{2}$$

Q. 3

Let $A(1,0),\ B(6,2)$ and $C\left(\frac{3}{2},6\right)$ be the vertices of a triangle ABC. If P is a point inside the triangle ABC such that the triangles APC, APB and BPC have equal areas, then the length of the line segment PQ, where Q is the point $\left(-\frac{7}{6},-\frac{1}{3}\right)$,is ____.

Correct Answer:
5

Solution:

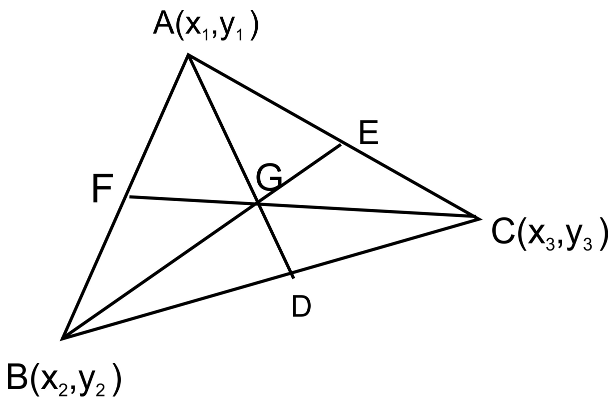
Centroid -

Centroid

Centroid of a triangle is the point of intersection of the medians of the triangle. A centroid divides the median in the ratio 2:1.

Whereas, the median is the line joining the mid-points of the sides and the opposite vertices.

The coordinates of the centroid of a triangle (G) whose vertices are A (x_1, y_1), B (x_2, y_2) and C(x_3, y_3), is given by



$$\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

If D (a_1, b_1), E (a_2, b_2) and F (a_3, b_3) are the mid point of ΔABC, then its centroid is given by

$$\left(\frac{a_1+a_2+a_3}{3}, \frac{b_1+b_2+b_3}{3}\right)$$

-

Distance between two points -

Distance between two points

Point A (x_1, y_1) and B (x_2, y_2) is two point on the plane then distance between them is given by

$$|AB| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

The distance of a point A (x, y) from the origin O (0, 0) is given by

$$|OA| = \sqrt{x^2+y^2}$$

-

A(1,0) B(6,2)   C(3/2,6)

Point P is the centroid of triangle ABC

P(17/6,8/3)

Distance between PQ is 5

Q. 4 Let $f:[0,3] \to \mathbf{R}$ be defined by

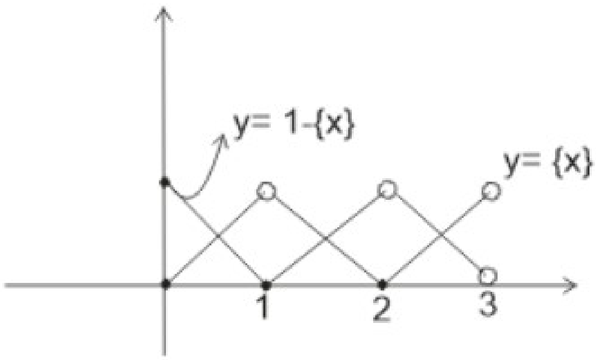
$f(x)=\min\{x-[x], 1+[x]-x\}$

where $[x]$ is the greatest integer less than or equal to x. Let P denote the set containing all $x \in [0,3]$ where f is discontinuous, and Q denote the set containing all $x \in (0,3)$ where f is not differentiable. Then the sum of number of elements in P and Q is equal to_______

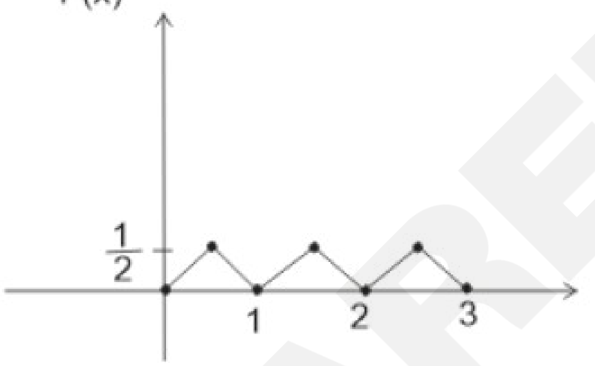
Correct Answer:
5

Solution:

$f(x)=\min(\{x\}, 1-\{x\})$



F(x)



No point of discontinuity $\Rightarrow n(P)=0$

5 points of non-differentiability $\Rightarrow n(Q)=5$

$\Rightarrow n(P)+n(Q)=5$

Q. 5 Let $f:[0,\infty) \to [0,3]$ be a function defined by

$$f(\mathrm{x})=\begin{cases}\max\{\sin t: 0 \leq \mathrm{t} \leq \mathrm{x}\}, & 0 \leq \mathrm{x} \leq \pi \\ 2+\cos \mathrm{x}, & \mathrm{x}>\pi\end{cases}$$

Then which of the following is true?

Option 1:
f is continuous everywhere but not differentiable exactly at one point in $(0,\infty)$

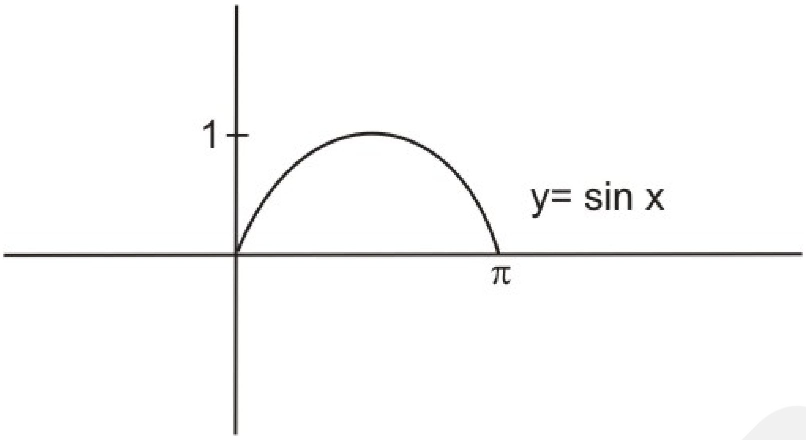
Option 2:
f is differentiable everywhere in $(0, \infty)$

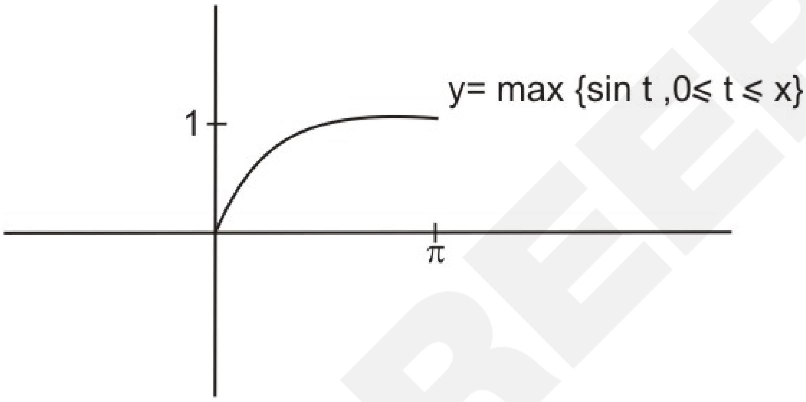
Option 3:
f is not continuous exactly at two points in $(0, \infty)$

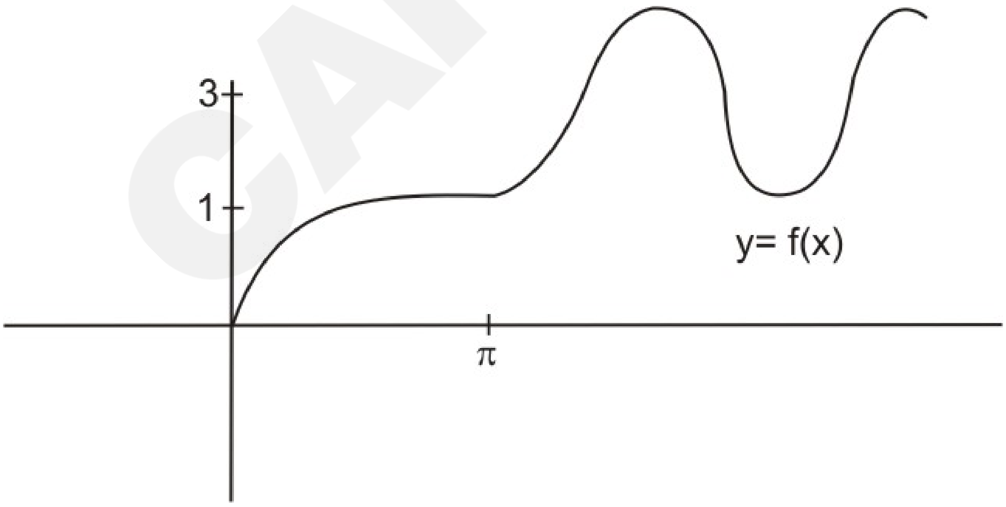
Option 4:
f is continuous everywhere but not differentiable exactly at two points in $(0, \infty)$

Correct Answer:
f is differentiable everywhere in $(0, \infty)$

Solution:







Clearly $f(x)$ is differentiable everywhere in $(0, \infty)$

The option (2) is correct.

Q. 6 Let $[t]$ denote the greatest integer less than or equal to t.

Let $f(x) = x - [x], g(x) = 1 - x + [x], and\, h(x) = min\{f(x), g(x)\}, x \in [-2, 2]$.

Then h is :

Option 1:
continuous in [-2,2] but not differentiable at more than four points in (-2,2)

Option 2:
continuous in [-2,2] but not differentiable at exactly three points in (-2,2)

Option 3:
not continuous at exactly four points in [-2,2]

Option 4:
not continuous at exactly three points in [-2,2]

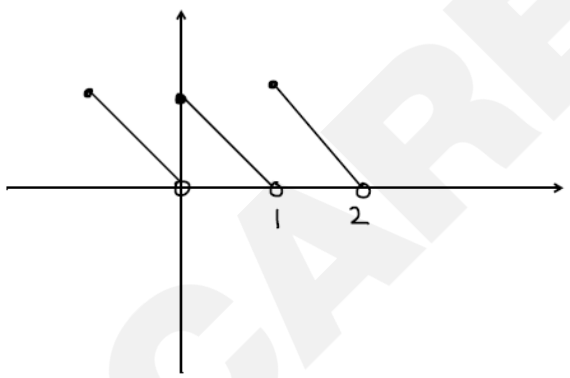
Correct Answer:
continuous in [-2,2] but not differentiable at more than four points in (-2,2)

Solution:

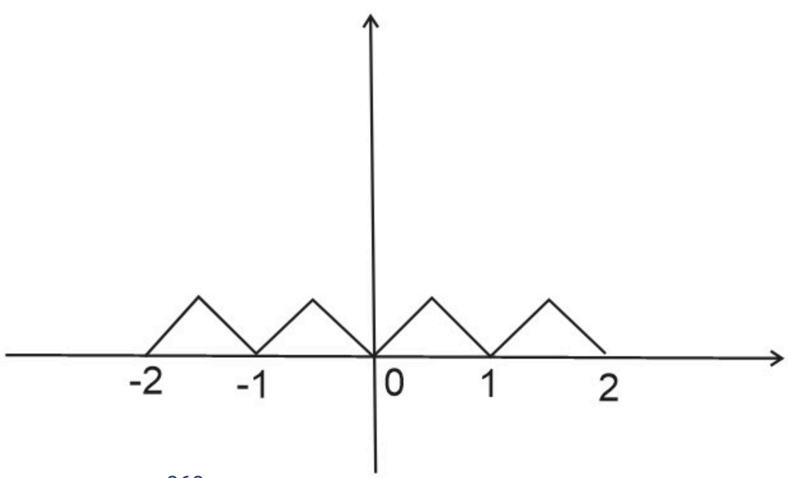
$f(x) = x - [x] = \{x\}$

$g(x) = 1 - (x - [x]) = 1 - \{x\}$

Using graphical tranformation, graph of $y = g(x)\ is$



$\therefore graph\, of\, h(x)\, is$

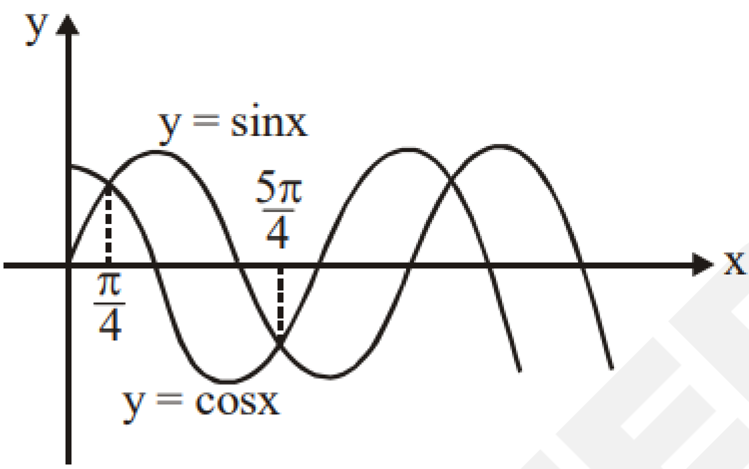


∴ Continuous at each point but not differentiable at 7 points in (-2,2)

Q. 7 The graph of sine and cosine functions, intersect each other at a number of points and between two consecutive points of intersection, the two graphs enclose the same area A. Then A^4 is equal to ____

Correct Answer:
64

Solution:



$$A = \int_{\pi/4}^{5\pi/4} (\sin x - \cos x) dx$$

$$= (-\cos x - \sin x)|_{z/4}^{5\pi/4}$$

$$= \left(-\left(\frac{-1}{\sqrt{2}}\right) - \left(\frac{-1}{\sqrt{2}}\right)\right) - \left(-\left(\frac{1}{\sqrt{2}}\right) - \left(\frac{1}{\sqrt{2}}\right)\right)$$

$$\Rightarrow A = \frac{2}{\sqrt{2}} + \frac{2}{\sqrt{2}} = 2\sqrt{2}$$

$$\Rightarrow A^4 = (2\sqrt{2})^4 = 16 \times 4 = 64$$

Q. 8 Consider a region $R = \{(x, y) \in \mathbb{R}^2 : x^2 \le 4 \le 2x\}$. if a line $y = \alpha$ divided the region R into two equal parts, then which of the following is true?

Option 1:
$3\alpha^2 - 8\alpha + 8 = 0$

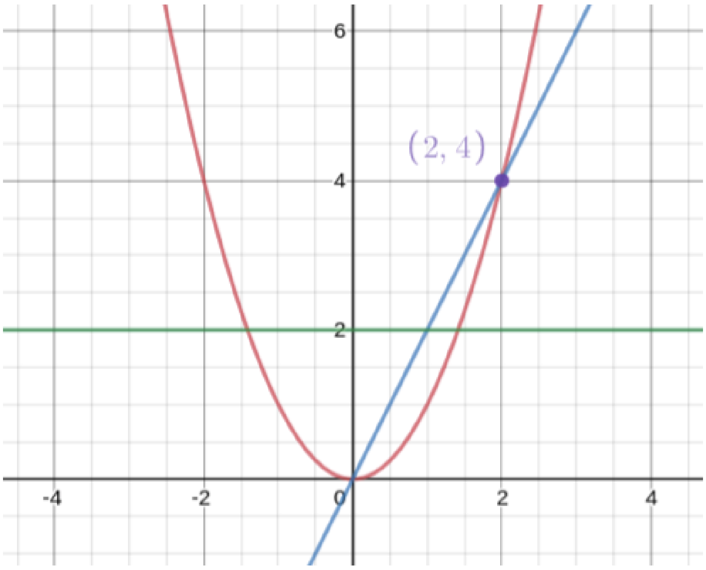
Option 2:
$\alpha^2 - 6\alpha + 16 = 0$

Option 3:
$\alpha^3 - 6\alpha^{\frac{3}{2}} - 16 = 0$

Option 4:
$3\alpha^2 - 8\alpha^{\frac{3}{2}} + 8 = 0$

Correct Answer:
$3\alpha^2 - 8\alpha^{\frac{3}{2}} + 8 = 0$

Solution:



$x^2 \leq y \leq 2x$

$$\int_0^\alpha \left(\sqrt{y} - \frac{y}{2}\right) dy = \frac{1}{2}\int_0^2 (2x - x^2)\, dx$$

$$\left.\frac{2}{3}y^{3/2} - \frac{y^2}{4}\right|_0^\alpha = \frac{1}{2}\left[x^2 - \frac{x^3}{3}\right]_0^2$$

$$\Rightarrow \frac{2}{3}\alpha^{3/2} - \frac{\alpha^2}{4} = \frac{1}{2}\left[4 - \frac{8}{3}\right]$$

$\Rightarrow \frac{8\alpha^{3/2} - 3\alpha^2}{12} = \frac{1}{2} \times \frac{4}{3}$
$\Rightarrow 8\alpha^{3/2} - 3\alpha^2 = 8$

Q. 9 Let z be those complex numbers which satisfy

$|z + 5| \leq 4 \text{ and } z(1 + i) + \bar{z}(1 - i) \geqslant -10, i = \sqrt{-1}$

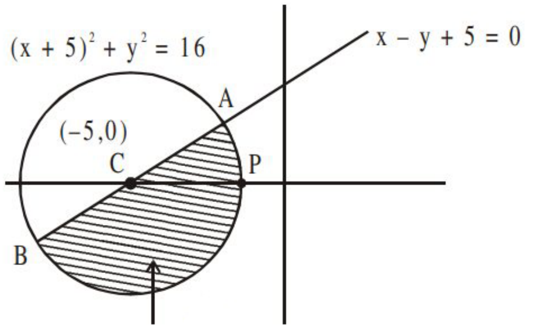
If the maximum value of $|z + 1|^2$ is $\alpha + \beta\sqrt{2}$, then the value of $(\alpha + \beta)$ is ______

Correct Answer:
48

Solution:
Let z = x + iy

$$|z+5| \leq 4$$
$$(x+5)^2+y^2 \leq 16$$
$$z(1+i)+\bar{z}(1-i) \geq -10$$

$$(z+\bar{z})+i(z-\bar{z}) \geq -10$$
$$(x+iy+x-iy)+i(x+iy-(x-iy)) \geq -10$$
$$2x+i(x+iy-x+iy) \geq -10$$
$$2x+2yi^2 \geq -10$$
$$2x-2y \geq -10$$
$$x-y \geq -5$$
$$x-y+5 \geq 0$$



Region bounded by inequalities (1) & (2)

$$|z+1|^2=|z-(-1)|^2$$

Let P = (-1,0)

$$|z+1|_{\max}^2=PB^2$$

For the point of intersection, solve

$$(x+5)^2+y^2=16$$
$$x-y+5=0$$

we get

$$\mathrm{A}(2\sqrt{2}-5, 2\sqrt{2}) \quad \mathrm{B}(-2\sqrt{2}-5,-2\sqrt{2})$$
$$\mathrm{PB}^2=(+2\sqrt{2}+4)^2+(2\sqrt{2})^2$$
$$|\mathrm{z}+1|^2=8+16+16\sqrt{2}+8$$
$$\alpha+\beta\sqrt{2}=32+16\sqrt{2}$$
$$\alpha=32, \beta=16 \Rightarrow \alpha+\beta=48$$

Q. 10 The number of roots of the equation, $(81)^{\sin^2 x}+(81)^{\cos^2 x}=30$ in the interval $[0, \pi]$ is equal to :

Option 1:
4

Option 2:
2

Option 3:
8

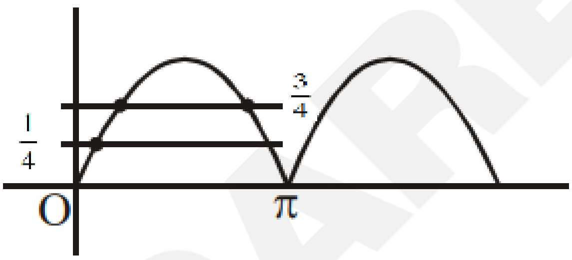
Option 4:
3

Correct Answer:
4

Solution:

$$(81)^{\sin^2 x} + (81)^{\cos^2 x} = 30$$
$$(81)^{\sin^2 x} + \frac{(81)^1}{(18)^{\sin^2 x}} = 30$$
$$(81)^{\sin^2 x} = t$$
$$t + \frac{81}{t} = 30$$
$$t^2 - 30t + 81 = 0$$
$$(t-3)(t-27) = 0$$

$$(81)^{\sin^2 x} = 3^1 \quad \text{or} \quad (81)^{\sin^2 x} = 3^3$$
$$3^{4\sin^2 x} = 3^1 \quad \text{or} \quad 3^{4\sin^2 x} = 3^3$$
$$\sin^2 x = \frac{1}{4} \quad \text{or} \quad \sin^2 x = \frac{3}{4}$$



Total sol. = 4

Q. 11 The area bounded by the lines $y = || x - 1 | - 2|$ and y = 2 is ______.

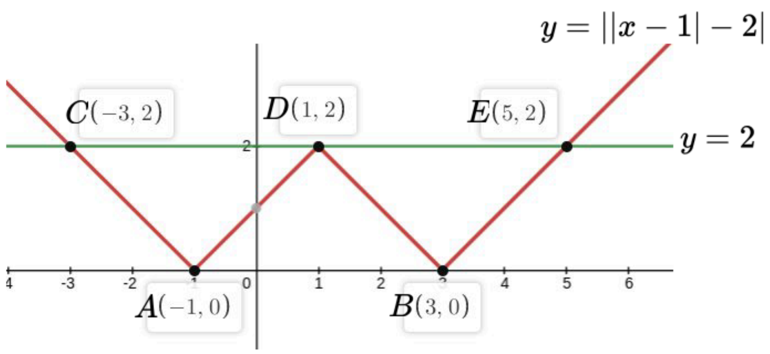
Correct Answer:
8

Solution:
Given the equation of curve are

y = | | x-1 | -2 |

and, y = 2

Plot the curve on the graph



We have to find area of triangle ACD and triangle BDE

$$\text{Area} = \frac{1}{2} \times 2 \times CD + \frac{1}{2} \times 2 \times DE$$
$$\text{Area} = CD + DE = 8$$

Q. 12 Let f be an odd function defined on the set of real numbers such that for $x \geqslant 0$,

$f(x) = 3\sin x + 4\cos x.$

Then $f(x)\ at x = -\frac{11\pi}{6}$ is equal to:

Option 1:
$\frac{3}{2} + 2\sqrt{3}$

Option 2:
$-\frac{3}{2} + 2\sqrt{3}$

Option 3:
$\frac{3}{2} - 2\sqrt{3}$

Option 4:
$-\frac{3}{2} - 2\sqrt{3}$

Correct Answer:
$\frac{3}{2} + 2\sqrt{3}$

Solution:
$f(x) = 3\sin x + 4\cos x.$

$$f\left(\frac{-11\pi}{6}\right) = 3\sin\left(\frac{-11\pi}{6}\right) + 4\cos\left(\frac{-11\pi}{6}\right)$$

$$f\left(\frac{-11\pi}{6}\right)=-3\sin\left(\frac{11\pi}{6}\right)+4\cos\left(\frac{11\pi}{6}\right)$$

$$f\left(\frac{-11\pi}{6}\right)=-3\sin\left(2\pi-\frac{\pi}{6}\right)+4\cos\left(2\pi-\frac{\pi}{6}\right)$$

$$f\left(\frac{-11\pi}{6}\right)=+3\sin\left(\frac{\pi}{6}\right)+4\cos\left(\frac{\pi}{6}\right)$$

$$f\left(\frac{-11\pi}{6}\right)=\frac{3}{2}+4\frac{\sqrt{3}}{2}$$

Q. 13

If $B=\begin{bmatrix}5 & 2\alpha & 1\\ 0 & 2 & 1\\ \alpha & 3 & -1\end{bmatrix}$ is the inverse of a 3×3 matrix A, then the sum of all values of α for which det $(A)+1=0$, is :

Correct Answer:

1

Solution:

Value of determinants of order 3 -

DETERMINANT OF ORDER 3

The determinant of a 3 X 3 matrix

$$\Delta=\begin{vmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{vmatrix}$$
$$=a_{11}(a_{22}a_{33}-a_{23}a_{32})$$
$$-a_{12}(a_{21}a_{33}-a_{31}a_{23})$$
$$+a_{13}(a_{21}a_{32}-a_{31}a_{22})$$

-

$$B=\begin{bmatrix}5 & 2\alpha & 1\\ 0 & 2 & 1\\ \alpha & 3 & -1\end{bmatrix}$$

$$|B|=5(-5)-2\alpha(-\alpha)-2\alpha$$

$$=2\alpha^2-2\alpha-25$$

$$Given,$$
$$|A|+1=0=>|A|=-1$$
$$Now, B=A^{-1}=>|B|=\frac{1}{|A|}=-1.$$
$$Hence, 1+|B|=0$$

$$1+|B|=2\alpha^2-2\alpha-25+1=0$$

$$=>2\alpha^2-2\alpha-24=0$$

$$=>\alpha^2-\alpha-12=0$$

$$=>\alpha^2-4\alpha+3\alpha-12=0$$

$$=>(\alpha-4)(\alpha+3)=0$$

$$root=4,-3$$

$$sum\ \ of\ \ root=4-3$$

$$=1$$

Q. 14 Let α and β be the roots of the equation $x^2+x+1=0$. Then for $y\neq 0$ in R, $\begin{vmatrix}y+1 & \alpha & \beta\\ \alpha & y+\beta & 1\\ \beta & 1 & y+\alpha\end{vmatrix}$ is equal to :

Option 1:
$y\left(y^2-1\right)$

Option 2:
$y\left(y^2-3\right)$

Option 3:
y^3

Option 4:
y^3-1

Correct Answer:
y^3

Solution:
Property of determinant -

If to each element of a line (row or column) of a determinant be added the equimultiples of the corresponding elements of one or more parallel lines , the determinant remains unaltered

- wherein

$$\text{i.e.} \begin{vmatrix} a_1 + Ia_2 + ma_3 & a_2 & a_3 \\ b_1 + Ib_2 + mb_3 & b_2 & b_3 \\ c_1 + Ic_2 + mc_3 & c_2 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}.$$

α, β are roots of $x^2 + x + 1 = 0$

$\alpha + \beta = -1, \alpha\beta = 1$

$$\begin{vmatrix} y+1 & \alpha & \beta \\ \alpha & y+\beta & 1 \\ \beta & 1 & y+\alpha \end{vmatrix}$$

$C_1 \rightarrow C_1 + C_2 + C_3$

$$\begin{vmatrix} y+1+\alpha+\beta & \alpha & \beta \\ \alpha+y+\beta+1 & y+\beta & 1 \\ \beta+1+y+\alpha & 1 & y+\alpha \end{vmatrix}$$

$$(y+1+\alpha+\beta) \begin{vmatrix} 1 & \alpha & \beta \\ 1 & y+\beta & 1 \\ 1 & 1 & y+\alpha \end{vmatrix}$$

$$= (y+1+\alpha+\beta)\left[(y+\beta)(y+\alpha) - \alpha(y+\alpha-1) + \beta(1-y-\beta)\right]$$

$$\Rightarrow (y+1+\alpha+\beta)\left[y^2 + y(\alpha+\beta) + \alpha\beta - 2y - 2^2 + 2 + \beta - y\beta - \beta^2\right]$$

$$\Rightarrow (y+1-1)\left[y^2 + y(-1) + 1 - y(-1) - \left(\alpha^2+\beta^2\right) - 1\right]$$

$$\Rightarrow y^3$$

Q. 15 Let a_1, a_2 , ...a_n be given A.P, whose common difference is an integer and $S_n = a_1 + a_2 + \ldots\ldots\ldots a_n$. If $a_1 = 1, a_n = 300$ and $15 \leq n \leq 50$, then the ordered pair (S_{n-4}, a_{n-4}) is equal to:

Option 1:
(2490, 249)

Option 2:
(2480, 249)

Option 3:
(2480, 248)

Option 4:
(2490, 248)

Correct Answer:
(2490, 248)

Solution:
$a_n = a_1 + (n-1)d$
$\Rightarrow 300 = 1 + (n-1)d$
$\Rightarrow (n-1)d = 299 = 13 \times 23$
$\text{since, } n \in [15, 50]$
$\therefore n = 24 \text{ and } d = 13$
$a_{n-4} = a_{20} = 1 + 19 \times 13 = 248$
$\Rightarrow a_{n-4} = 248$
$S_{n-4} = \frac{20}{2}\{1 + 248\} = 2490$

Hence, the answer is option (4).

Q. 16 If $2^{10} + 2^9 \cdot 3^1 + 2^8 \cdot 3^2 + \ldots\ldots + 2.3^9 + 3^{10} = S - 2^{11}$, then S is equal to.

Option 1:
$3^{11} - 2^{11}$

Option 2:
3^{11}

Option 3:
$\frac{3^{11}}{2} + 2^{10}$

Option 4:
2.3^{11}

Correct Answer:
3^{11}

Solution:

$$a = 2^{10}; r = \frac{3}{2}; n = 11(\text{G.P})$$
$$S = (2^{10}) \frac{\left(\left(\frac{3}{2}\right)^{11} - 1\right)}{\frac{3}{2} - 1} = 2^{11}\left(\frac{3^{11}}{2^{11}} - 1\right)$$
$$S = 3^{11} - 2^{11} = S - 2^{11} (\text{ Given })$$
$$\therefore S = 3^{11}$$

Hence, the answer is option (2).

Q. 17 If the mean deviation of the numbers 1, 1+d, ..., 1+100 *d* from their mean is 255, then a value of *d* is :

Option 1:
10.1

Option 2:
20.2

Option 3:
10

Option 4:
5.05

Correct Answer:
10.1

Solution:
As we learned in

Mean Deviation -

If $x_1, x_2, \ldots x_n$ are n observations then the mean deviation from point A is given by :

$$\frac{1}{n}\sum |x_i - A|$$

Mean$= \dfrac{1 + 1 + d + 1 + 2d + - - - - - - - - - - - - 1 + 100d}{101} = 1 + 50d$

Mean deviation

$$\Rightarrow \frac{1}{101}\sum_{r=0}^{100} |(I + rd) - (I + 50d)|$$

$$\Rightarrow \frac{1}{101} \times 2d \times \frac{50 \times 51}{2} = 255$$

d=10.1

Hence, the answer is the option 1.

Q. 18 If A and B are any two events such that P(A)=2/5 and $P(A \cap B) = 3/20$, then the conditional probability, $P(A/(A' \cup B'))$, where A' denotes the complement of A, is equal to :

Option 1:
1/4

Option 2:
5/17

Option 3:
8/17

Option 4:
11/20

Correct Answer:
5/17

Solution:
Given:

$$P(A) = \frac{2}{5}, \; P(A \cap B) = \frac{3}{20}$$

Now,

$$P\left(\frac{A}{A' \cup B'}\right) = \frac{P(A \cap (A' \cup B'))}{P(A' \cup B')}$$

Here,

$$P(A' \cup B') = P(A \cap B)' = 1 - P(A \cap B) = 1 - \frac{3}{20} = \frac{17}{20}$$

(Using De-Morgan's Law)

And $P(A \cap (A' \cup B')) = P((A \cap A') \cup (A \cap B')) = P(A \cap B') = P(A) - P(A \cap B)$

$$= \frac{2}{5} - \frac{3}{20} = \frac{5}{20}$$

$$P\left(\frac{A}{A' \cup B'}\right) = \frac{\frac{5}{20}}{\frac{17}{20}} = \frac{5}{17}$$

Hence, the answer is the option 2.

Q. 19 Let $\overrightarrow{a}=2\hat{i}-3\hat{j}+4\hat{k}$ and $\overrightarrow{b}=7\hat{i}+\hat{j}-6\hat{k}$.

If $\overrightarrow{r}\times\overrightarrow{a}=\overrightarrow{r}\times\overrightarrow{b}$, $\overrightarrow{r}.\left(\hat{i}+2\hat{j}+\hat{k}\right)=-3$, then $\overrightarrow{r}.\left(2\hat{i}-3\hat{j}+\hat{k}\right)$ is equal to :

Option 1:
10

Option 2:
12

Option 3:
8

Option 4:
13

Correct Answer:
12

Solution:

$\overrightarrow{r}\times\overrightarrow{a}-\overrightarrow{r}\times\overrightarrow{b}=0$

$\Rightarrow\overrightarrow{r}\times(\overrightarrow{a}-\overrightarrow{b})=0$

$\Rightarrow\overrightarrow{r}=\lambda(\overrightarrow{a}-\overrightarrow{b})$

$\Rightarrow\overrightarrow{r}=\lambda(-5\hat{i}-4\hat{j}+10\hat{k})$

$\text{Also }\overrightarrow{r}\cdot(\hat{i}+2\hat{j}+\hat{k})=-3$

$\Rightarrow\lambda(-5-8+10)=-3$

$\lambda=1$

$\text{Now }\overrightarrow{r}=-5\hat{i}-4\hat{j}+10\hat{k}$

$=\overrightarrow{r}\cdot(2\hat{i}-3\hat{j}+\hat{k})$

$=-10+12+10=12$

Hence, the answer is the option 2.

Q. 20 If $|\vec{a}|=2,|\vec{b}|=5\text{ and }|\vec{a}\times\vec{b}|=8,\text{ then }|\vec{a}\cdot\vec{b}|$ is equal to

Option 1:
6

Option 2:
4

Option 3:
3

Option 4:
5

Correct Answer:
6

Solution:

$$\left|\vec{a} \times \vec{b}\right| = |\vec{a}| \left|\vec{b}\right| \sin\theta$$
$$\Rightarrow 8 = 2 \cdot 5 \sin\theta$$
$$\Rightarrow \sin\theta = \frac{4}{5}$$
$$\Rightarrow \cos\theta = \frac{3}{5} \; or \; \frac{-3}{5}$$
$$\left|\vec{a} \cdot \vec{b}\right| = \left||\vec{a}| \left|\vec{b}\right| \cos\theta\right|$$
$$= \left|10 \cdot \left(\pm\frac{3}{5}\right)\right|$$
$$= |\pm 6|$$
$$= 6$$

Hence, the answer is the option 1.

Q. 21 Let $y = y(x)$ be the solution of the differential equation.

$xy' - y = x^2(x\cos x + \sin x), x > 0$. If $y(\pi) = \pi$, then $y"\left(\frac{\pi}{2}\right) + y\left(\frac{\pi}{2}\right)$ is equal to:

Option 1:
$2 + \frac{\pi}{2}$

Option 2:
$1 + \frac{\pi}{2} + \frac{\pi^2}{4}$

Option 3:
$2 + \frac{\pi}{2} + \frac{\pi^2}{4}$

Option 4:
$1 + \frac{\pi}{2}$

Correct Answer:
$2 + \frac{\pi}{2}$

Solution:

$$x\frac{dy}{dx}-y=x^2(x\cos x+\sin x), x>0$$
$$\frac{dy}{dx}-\frac{y}{x}=x(x\cos x+\sin x)\Rightarrow\frac{dy}{dx}+Py=Q$$
$$\text{so, I.F.}=e^{\int-\frac{1}{x}dx}=\frac{1}{|x|}=\frac{1}{x}(x>0)$$
$$\text{Thus, }\frac{y}{x}=\int\frac{1}{x}(x(x\cos x+\sin x))dx$$
$$\Rightarrow\frac{y}{x}=x\sin x+C$$
$$\because y(\pi)=\pi\Rightarrow C=1$$
$$\text{so, }y=x^2\sin x+x\Rightarrow(y)_{\pi/2}=\frac{\pi^2}{4}+\frac{\pi}{2}$$
$$\text{Also, }\frac{dy}{dx}=x^2\cos x+2x\sin x+1$$
$$\Rightarrow\frac{d^2y}{dx^2}=-x^2\sin x+4x\cos x+2\sin x$$
$$\Rightarrow\left.\frac{d^2y}{dx^2}\right|_{\frac{\pi}{2}}=-\frac{\pi^2}{4}+2$$
$$\text{Thus, }y_{(\frac{\pi}{2})}+\frac{d^2y}{dx^2}\Big|_{\frac{\pi}{2}}=\frac{\pi}{2}+2$$

Hence, the answer is option (1).

Q. 22 If the length of the perpendicular from the point $(\beta, 0, \beta)$

$(\beta \neq 0)$ to the line, $\frac{x}{1}=\frac{y-1}{0}=\frac{z+1}{-1}$ is $\sqrt{\frac{3}{2}}$,

then β is equal to :

Option 1:
1

Option 2:
2

Option 3:
-1

Option 4:
-2

Correct Answer:
-1

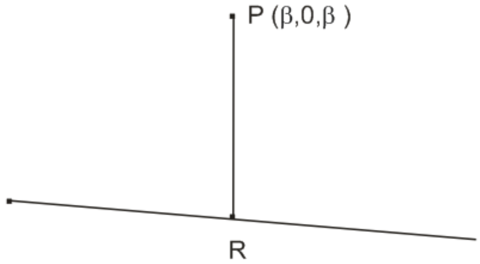
Solution:

Perpendicular Distance between a Point and a Line -

-

Let point $P(\beta, 0, \beta)$ given that length of perpendicular distance

from P to line is $\sqrt{\frac{3}{2}}$.



$$\frac{x}{1} = \frac{y-1}{0} = \frac{z+1}{-1}$$

$R = (\lambda, 1, -\lambda - 1)$

Direction ratio of $PR = (\lambda - \beta, 1, -\lambda - \beta - 1)$

PR is perpendicular to the line

$=> (\lambda - \beta)(1) + (1)0 + (-1)(-\lambda - \beta - 1) = 0$

$=> \lambda - \beta + \lambda + \beta + 1 = 0$

$=> \lambda = \frac{1}{2}$

$PR = \sqrt{(\lambda - \beta)^2 + 1^2 + (-\lambda - 1 - \beta)^2} = \sqrt{\frac{3}{2}}$

$=> 2\beta^2 + 2\beta = 0$

$=> \beta = 0, \beta = -1$

Correct option is (3)

Q. 23 If $\cos(\alpha + \beta) = \frac{3}{5}, \sin(\alpha - \beta) = \frac{5}{13}$ and $0 < \alpha, \beta < \frac{\pi}{4}$ then $\tan(2\alpha)$ is equal to :

Option 1:
$\frac{21}{16}$

Option 2:
$\frac{63}{16}$

Option 3:
$\frac{33}{52}$

Option 4:
$\frac{63}{52}$

Correct Answer:
$\frac{63}{16}$

Solution:

Properties of Inverse Trigonometric function -

$$\tan^{-1}x+\tan^{-1}y = \begin{cases} \tan^{-1}\left(\frac{x+y}{1-xy}\right), & If\ xy<1 \\ \pi+\tan^{-1}\left(\frac{x+y}{1-xy}\right) & If\ x>0, y>0\ and\ xy>1 \\ -\pi+\tan^{-1}\left(\frac{x+y}{1-xy}\right), & If\ x<0, y<0\ and\ xy>1 \end{cases}$$

$$\tan^{-1}x-\tan^{-1}y = \begin{cases} \tan^{-1}\left(\frac{x-y}{1+xy}\right), & If\ xy>-1 \\ \pi+\tan^{-1}\left(\frac{x-y}{1+xy}\right) & If\ x>0, y<0\ and\ xy<-1 \\ -\pi+\tan^{-1}\left(\frac{x-y}{1+xy}\right), & If\ x<0, y>0\ and\ xy<-1 \end{cases}$$

-

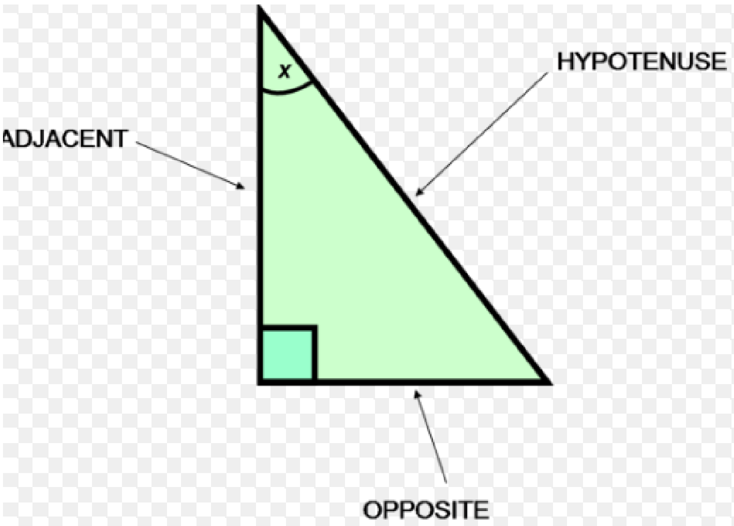
Trigonometric Ratios of Functions -

$$\sin\Theta=\frac{Opp}{Hyp}$$

$$\cos\Theta=\frac{Base}{Hyp}$$

$$\tan\Theta=\frac{Opp}{Base}$$

- wherein

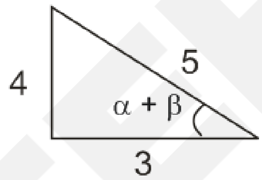


$$\cos(\alpha+\beta)=\frac{3}{5}$$

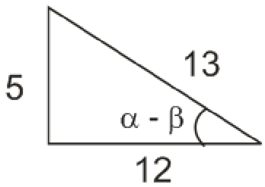
$$\sin(\alpha-\beta)=\frac{5}{13}$$

$$0<\alpha,\beta<\frac{4}{4} \quad then \quad \tan(2\alpha)=?$$

$$\cos(\alpha+\beta)=\frac{3}{5}$$



$$\tan(\alpha+\beta)=\frac{4}{3}$$



$$\sin(\alpha-\beta)=\frac{5}{13}$$

$$\tan(\alpha-\beta)=\frac{5}{12}$$

$$\alpha+\beta=\tan^{-1}\left(\frac{4}{3}\right)$$

$$\alpha-\beta=\tan^{-1}\left(\frac{5}{12}\right)$$

$$=2\alpha=\tan^{-1}\left(\frac{4}{3}\right)+\tan^{-1}\left(\frac{5}{12}\right)$$

$$=\tan^{-1}\left(\frac{\frac{4}{3}+\frac{5}{12}}{1-\frac{4}{3}\times\frac{5}{12}}\right)$$

$$= \tan^{-1}\left(\frac{\frac{21}{12}}{\frac{36-20}{36}}\right) = \frac{\frac{21}{12}}{\frac{16}{36}}$$

$$= \tan^{-1}\left(\frac{3 \times 21}{16}\right)$$

$$= \tan^{-1}\left(\frac{63}{16}\right)$$

Q. 24 The sum of all the four-digit numbers that can be formed using all the digits 2, 1, 2, 3 is equal to ______:

Correct Answer:
26664

Solution:

$- \quad - \quad \underline{1} \quad \frac{3!}{2!} = 3$

$- \quad - \quad \underline{2}\ 3! = 6$

$- \quad - \quad \underline{3} \quad \frac{3!}{2!} = 3$

The sum of digits of unit place $= 3 \times 1 + 6 \times 2 + 3 \times 3 = 24$

Required sum

$$= 24 \times 1000 + 24 \times 100 + 24 \times 10 + 24 \times 1$$
$$= 24 \times 1111$$
$$= 26664$$

Hence, the answer is 26664.

Q. 25 The value of $\sum_{r=0}^{20} {}^{50-r}C_6$ is equal to.

Option 1:
${}^{51}C_7 - {}^{30}C_7$

Option 2:
${}^{50}C_7 - {}^{30}C_7$

Option 3:
${}^{50}C_6 - {}^{30}C_6$

Option 4:
$^{51}C_7 +^{30} C_7$

Correct Answer:
$^{51}C_7 -^{30} C_7$

Solution:
As we have learnt

$^{\mathrm{n}}\mathrm{C}_{\mathrm{r}} + {}^{\mathrm{n}}\mathrm{C}_{\mathrm{r}-1} = {}^{\mathrm{n}+\mathrm{I}}\mathrm{C}_{\mathrm{r}}$

Now,

$$\sum_{r=0}^{20} {}^{50-r}C_6 = {}^{50}C_6 + {}^{49}C_6 + {}^{48}C_6 + \ldots. + {}^{30}C_6$$

$$= {}^{50}C_6 + {}^{49}C_6 + \ldots.. + {}^{31}C_6 + \left({}^{30}C_6 + {}^{30}C_7\right) - {}^{30}C_7$$

$$= {}^{50}C_6 + {}^{49}C_6 + \ldots.. + \left({}^{31}C_6 + {}^{31}C_7\right) - {}^{30}C_7$$

$$= {}^{50}C_6 + {}^{50}C_7 - {}^{30}C_7$$

$$= {}^{51}C_7 - {}^{30}C_7$$

Hence, the answer is option (1).